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The Aggregate Implications of Size Dependent Distortions

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The Aggregate Implications of Size Dependent Distortions

Nicolas Roys

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Abstract

This paper examines the aggregate implications of size-dependent distortions. These regulations misallocate labor across firms and hence reduce aggregate productivity. It then considers a case-study of labor laws in France where firms that have 50 employees or more face substantially more regulation than firms that have less than 50. The size distribution of firms is visibly distorted by these regulations: there are many firms with exactly 49 employees. A quantitative model is developed with a payroll tax of 0.15% that only applies to firm above 50 employees. Removing the regulation improves labor allocation across firms, leading in steady state to an increase in output per worker slightly less than 0.3%.

1 Introduction

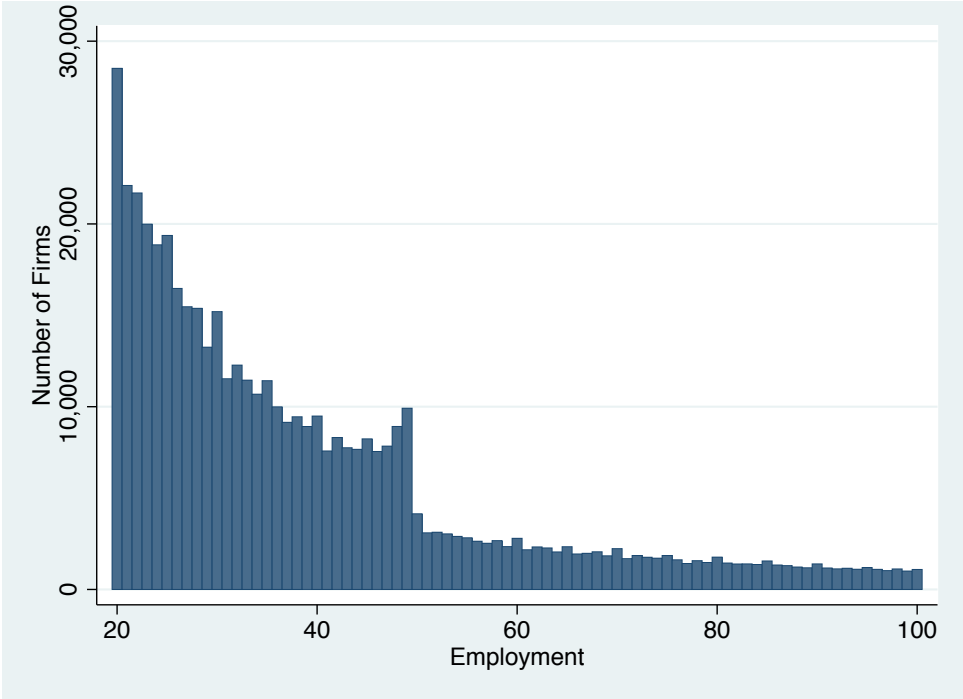
New firms created 2.9 million jobs per year on average over the period 1980-2010.[1] While new firms clearly play an important role in job creation, many fail after a short period of time or do not grow. Are regulations preventing young businesses to expand? Many regulators seem to share this view as small firms face lighter regulation than large firms in many countries. The rationale for exempting small firms from some regulations is that the compliance cost is too high relative to their sales. A necessary consequence, however, is that regulations are phased in as the firm grows, generating an implicit marginal tax. Because regulations are typically phased in at a few finite points, they are sometimes referred to as "threshold effects".

[1] See Decker et al. (2014).

Regulation, broadly defined, takes many forms, from hygiene and safety rules, to mandatory elections of employee representatives, to larger taxes. Under the Affordable Care Act, firms with 50 or more full-time equivalent employees are required to offer health insurance to their full-time employees. It raises concerns that firms cut employment to stay below the threshold or substitute some of their full-time workers with part-time workers. Similarly, regulations that altered the incentives to expand, explain the large number of small community banks in the United States.

These distortions have attracted attention in public policy circles. The common wisdom, as reflected in numerous reports by blue-ribbon panels, is that these regulations are a significant impediment to the growth of small firms, and should be suppressed or smoothed out. However, there is little work formally modeling these policies to understand and evaluate their effects. This paper proposes a simple model and give a quantitative evaluation of this common wisdom. What are the potential benefits of removing, or smoothing, the regulation thresholds? To answer this question, this paper considers a case study of regulations which only apply to firms with more than 50 employees in France. The firm size distribution is distorted: there are few firms with exactly 50 employees, and a large number of firms with 49 employees. Figure 1 plots the firm size distribution in our French data, illustrating this well-known pattern. The visibly distorted firm distribution suggests that productivity could

Figure 1: Distribution of firm employment between 20 and 100 employees in France.



be increased if firms close to the threshold grow, as labor would be reallocated towards more productive firms. Because these regulations depend on a precise threshold, the behavior of firms around the threshold is particularly informative on the effects of distortions.

The rest of the paper proceeds as follows. Section 2 presents a model to study regulations that limit firm scale. Section 3 presents a case-study of labor laws in France that differ depending on the side of the thresholds firms stand-by. Section 4 applies the model of Section 2 to study these distortions. Section 5 proposes a quantitative analysis. Section 6 concludes.

2 Model

This Section introduces a simple model of production and employment, based on Lucas (1978), to evaluate the impact of size-dependent distortions.

2.1 Environment

There is a continuum of firms with production function,

$$y = e^z n^\alpha,$$

where n is employment and e^z is a firm's productivity level (e denotes the exponential function). The distribution of productivity in the population is characterized by the density f. Production displays decreasing returns $\alpha \in (0, 1)$.[2] Aggregate output Y is defined as the integral of the production of each firm $y(z)$,

$$Y = \int_{-\infty}^{\infty} e^z n(z)^\alpha f(z) dz, \tag{1}$$

where $n(z)$ is the employment of firms with productivity z.

Firms hire labor in a competitive labor market where workers supply labor inelastically. Let total employment be denoted by N. The wage rate w, taken as given by each firm, is such that the labor market is in equilibrium,

$$\int_{-\infty}^{\infty} n(z) f(z) dz \leq N. \tag{2}$$

[2]This formulation is equivalent to a linear production technology where firms have some market power. In this case $\alpha < 1$ is equal to the inverse of the demand elasticity.

Labor costs for the firm are equal to the wage bill multiplied by a size-dependent tax $T(n)$.

2.2 Labor Demand

A firm with productivity z solves the optimization problem,

$$\max_n e^z n^\alpha - wn\left(1 + T(n)\right).$$

If T is differentiable, labor demand satisfies the first-order condition,

$$\alpha e^z n^{\alpha-1} = w\left(1 + T(n) + nT'(n)\right).$$

If $T(n) = 1 + \tau$, distortions are size independent and the first-order condition simplifies to $\alpha e^z n^{\alpha-1} = w\left(1+\tau\right)$. The following functional form will be used for the remaining of this section,

$$T(n) = cn^{\tau-1} - 1.$$

The left panel of Figure 2 displays this tax function for different values of τ. If $\tau = 0$, there are no distortions. If $\tau > 0$, distortions are size-dependent, and larger establishments face higher distortions than smaller ones. For instance with $\tau = 0.02$, a firm with less than 20 employees tax rate is at most 5% while firms with more than 100 employees tax rate is close to 10%.

Labor demand can be solved in closed-form,

$$n = \left(\frac{\alpha\lambda e^z}{wc\left(1+\tau\right)}\right)^{\frac{1}{1-\alpha-\tau}}. \tag{3}$$

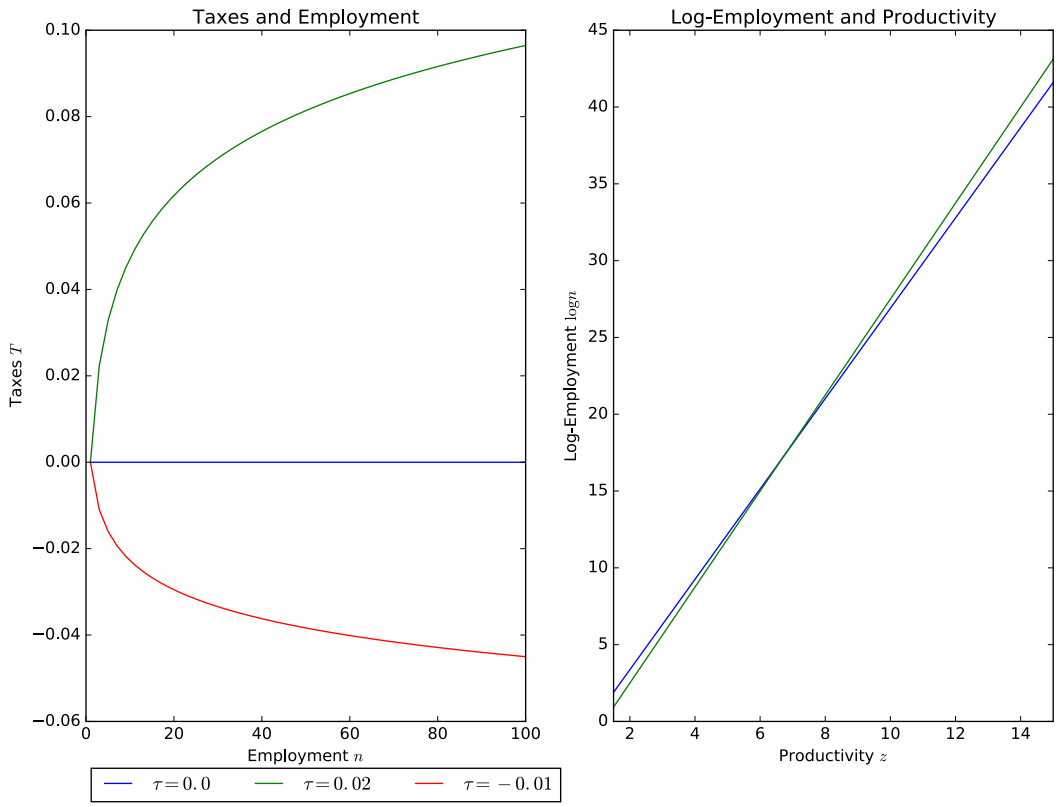
With distortions, the link between employment and productivity become weaker. More productive firm are relatively smaller and less productive firm are relatively larger as the right panel of Figure 2 shows.

2.3 Aggregates

Using labor demand (Equation 3) and the resource constraint (Equation 2), the equilibrium wage rate w can be expressed as

$$\left(1+\tau\right)cw = \alpha\left(\int e^{\frac{z}{1-\alpha-\tau}} f(z)dz\right) N^{-1}.$$

Figure 2: Taxes, Employment and Productivity



Using the equilibrium wage rate and inserting labor demand in Equation 1, aggregate output Y can be characterized in closed-form,

$$Y = \left(\int e^{\frac{z}{1-\alpha-\tau}} f(z) dz \right)^{1-\alpha} \left(\int e^{-\frac{z\tau}{1-\alpha-\tau}} f(z) dz \right) N^{\alpha}.$$

It is a Cobb-Douglas function in aggregate employment and a productivity index. The productivity index is a weighted average of the productivity level of each firm in the economy. The last term is equal to one without distortions and it is below one whenever $\tau > 0$.

How should the planner allocate labor across firms to maximize aggregate output Y? It corresponds to the competitive equilibrium when $\tau = 0$.[3] Then, aggregate output simplifies to,

$$Y = \left(\int_{-\infty}^{\infty} e^{\frac{z}{1-\alpha}} f(z) dz \right)^{1-\alpha} N^{\alpha}.$$

Further, when $\tau = 0$, the first-order condition of each firm is,

$$\alpha e^{z} n^{\alpha-1} = w.$$

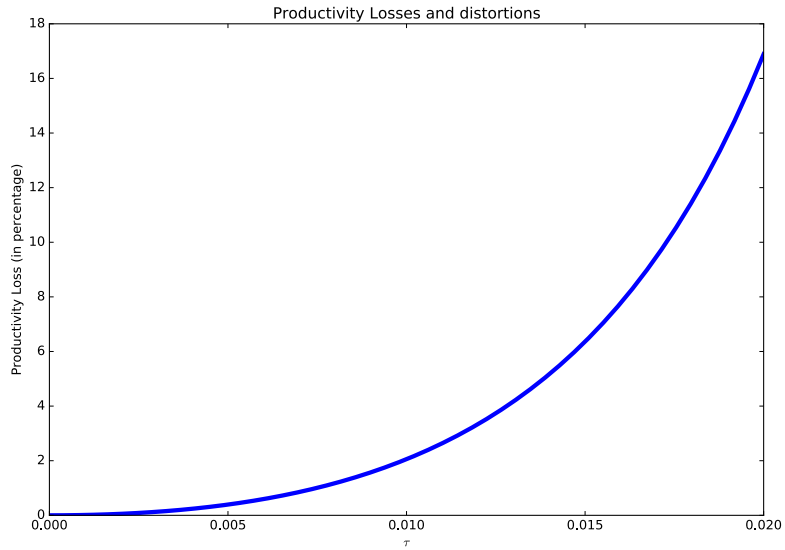
The efficient allocation equates marginal products $\alpha e^{z} n^{\alpha-1}$ across all firms. In other words, without distortions, high productivity firm and low productivity firm have the same marginal productivity of labor. In the distorted economy, there is dispersion across firm in average labor productivity. Formally, average labor productivity is,

$$\frac{y}{n} = \left(\frac{w\,(1+\tau)}{\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\tau}} e^{-\frac{\tau z}{1-\alpha-\tau}}.$$

How large are the output losses due to this distortions? Assume the distribution of productivity is exponential $f(z) = 3.1e^{-3.1z}, \forall z \in [0, \infty)$. The curvature parameter α is set to 0.66. And labor supply is normalized to one. Figure 3 reports the output loss (in percentage) for different values of τ. The upper-bound of $\tau = 0.02$ corresponds to tax rate of about 10% for a firm with 100 employees. One can see that distortions can lead to GDP losses above 15%.

[3]This holds independently of the value of c because c distorts all firms' decisions equally. With a fixed labor supply, aggregate output is unchanged as a lower wage rate restores equilibrium in the labor market.The parameter τ has an additional distortionary effect as it leads to an inefficient allocation of employment.

Figure 3: Productivity losses and distortions



3 Size-Dependent Regulations in France

The previous Section shows that the output losses due to distortions can potentially be large. It also begs the question of how large are these distortions in the real-world? The rest of the paper is devoted to quantify the effect of a particular distortion. It is a case-study of the impact of distortions on productivity by looking at size-dependent regulations in France. Because these regulations depend on a precise threshold, the behavior of firms around the threshold is particularly informative on the effects of distortions.

3.1 Institutional Background

Labor laws in France as well as various accounting and legal rules make special provisions for firms with more than 10, 11, 20, or 50 employees.

These regulations are not all based on the same definition of "employee". Labor laws, which are likely the most important, are based on the full-time equivalent workforce, computed as an average over the last twelve months. The full-time equivalent workforce takes into account part-time workers, as well a temporary workers, but not trainees or *contrats aidés* (a class of government-subsidized, limited duration contracts, which may be used to hire people that face "special difficulties" in finding employment, such the very long term unemployed or unskilled youth). Hence, it seems fairly difficult for firms to work around the

regulation. The main additional regulations as the firm reaches 50 employees are:

- possibly mandatory designation of an employee representative;

- a committee for hygiene, safety and work conditions must be formed and trained;

- a *comité d'entreprise* (works council) must be formed, that must meet at least every other month; this committee, that must have some office space and receives a subsidy equal to 0.2% of the total payroll, has both social objectives (e.g., organizing cultural or sports activities for employees) and an economic role (mostly on an advisory basis);

- higher payroll tax rate subsidizing training which goes from 0.9% to 1.5% (*formation professionelle*);

- in case of firing of more than 9 workers for "economic reasons", a special legal process must be followed (*plan social*). This process increases dismissal costs and creates legal uncertainty for the firm.

This list is not exhaustive, but clearly one would expect these costs to be significant. Some of these costs are also difficult to model in a tractable manner. In some cases - in particular, the *comité d'entreprise* - the firm is required to fund additional worker benefits. To the extent that the process is reasonably efficient, these rules might simply amount to a substitute form of compensation and have limited effects - the higher benefits may allow firms to attract better workers or to pay them less.

3.2 Data

The data is a panel of firms assembled by the French National Statistical Institute (INSEE), that covers the 1994-2000 period. This panel, known as BRN (Bénéfices Réels Normaux), contains employment as well as standard accounting information on total compensation costs, value added, current operating surplus, gross productive assets, etc. The BRN data are exhaustive of all private companies with a sales turnover of more than 3.5 million Francs (around 530,000 Euros) and liable to corporate taxes under the standard regime, and include some other smaller firms. The 3.5 million threshold implies that I have all firms with more than 30 employees or so. Hence I focus on the threshold at 50 employees, for which the data is essentially exhaustive. I removed from the sample firms with strictly less than 20 employees when I estimate the model. This generated a sample of 44,1890 firms that we follow for 7 years, or 309,323 firm-year observations.

Figure 1 plots the distribution of employment, pooling data for the entire period (1994-2000), and truncating at 100 employees. There is clearly a large discontinuity around the thresholds of 50 employees. Many surveys reveal "rounding" of employment, but this figure

Table 1: Distribution of firm employment between 40 and 59 employees.

	Fraction	S.E.	# Firms		Fraction	S.E.	# Firms
40	8.42	0.28	9,486	50	3.67	0.0029	4140
41	6.72	0.29	7,575	51	2.75	0.0029	3097
42	7.38	0.29	8,311	52	2.78	0.0029	3130
43	6.88	0.29	7,752	53	2.70	0.0029	3040
44	6.81	0.29	7,666	54	2.57	0.0029	2901
45	7.31	0.29	8,239	55	2.51	0.0029	2826
46	6.70	0.29	7,548	56	2.34	0.0029	2638
47	6.96	0.29	7,841	57	2.24	0.0029	2526
48	7.92	0.29	8,916	58	2.37	0.0029	2670
49	8.80	0.28	9,916	59	2.08	0.0029	2344

Notes: Fraction is the number of firms for each employment size over the range $40 - 59$, divided by the total number of firms between 40 and 59; S.E. is the associated standard error; and #Firms is the raw number of firms in each bin.

shows the opposite pattern.

Table 1 reports the size distribution of firms by employment over the range $40 - 59$. There is a clear drop in the number of firms after 49 employees. For example, there are more than three times as many firms with 49 employees as firms with 51 employees.

4 Model

I apply the model of Section 2 to the case of size distortions in France. I replace the smooth function T with a step function to mimic the regulations described in the previous section. Firms face a regulation which requires them to pay a higher proportional taxes on wages τ if they currently have more than $\underline{n}$ employees. Formally, if n is greater than $\underline{n}$, a proportional payroll tax τ applies. The proportional tax applies to all employment, including that below $\underline{n}$. For simplicity, there is only one threshold and $\underline{n} = 50$.

4.1 Labor Demand

To find the optimal labor demand and profit of the firm, I first solve the firm's problem conditional on operating below the threshold, then I find the solution conditional on operating above the threshold, and finally I find the overall solution by combining these results.

The current-period profit function for a firm which operates below the threshold is:

$$\pi^b(z) = \max_{0 \leq n < \underline{n}} \left\{ e^z n^\alpha - wn \right\}. \tag{4}$$

The superscript b stands for "below the threshold". Optimal employment is:

$$n^b(z) \quad = \begin{cases} \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} e^{\frac{z}{1-\alpha}} & , \text{ if } z < \underline{z} \\ \underline{n}^- & , \text{ if } z \geq \underline{z}. \end{cases}$$

where $\underline{z} = \log\left(\underline{n}^{1-\alpha}\frac{w}{\alpha}\right)$ and $\underline{n}^-$ indicates a value just below $\underline{n}$. Profits are given by the formula

$$\pi^b(z) \quad = \begin{cases} e^{\frac{z}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha), & \text{if } z < \underline{z} \\ e^z \underline{n}^\alpha - w\underline{n}, & \text{if } z \geq \underline{z}. \end{cases}$$

The current-period profit function for a firm that decides to operate above the threshold, and hence to face the regulation, is:

$$\pi^a(z) = \max_{n \geq \underline{n}} \left\{ e^z n^\alpha - w(1+\tau)n \right\}. \tag{5}$$

where the superscript a stands for "above the threshold". The firm operates strictly above the threshold if z is greater than a cutoff value $\overline{z}$, defined as the solution to

$$e^{\frac{z}{1-\alpha}} \left(\frac{\alpha}{w(1+\tau)}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) = e^z \underline{n}^\alpha - w\underline{n}.$$

It is easy to see that $\overline{z} > \underline{z}$, provided that there is a cost of operating above the threshold: $\tau w \overline{n} > 0$.

Summarizing, optimal employment if the firm decides to operate above the threshold is

$$n^a(z) \quad = \begin{cases} \underline{n} & \text{if } z < \overline{z}, \\ \left(\frac{\alpha}{w(1+\tau)}\right)^{\frac{1}{1-\alpha}} e^{\frac{z}{1-\alpha}} & , \text{ if } z \geq \overline{z}, \end{cases} \tag{6}$$

This leads to profits

$$\pi^a(z) \quad = \quad \begin{cases} e^{\frac{z}{1-\alpha}} \left(\frac{\alpha}{w(1+\tau)}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha), & \text{if } z \geq \overline{z}, \\ e^z \underline{n}^\alpha - w(1+\tau)\underline{n}, & \text{if } z < \overline{z}. \end{cases}$$

Combining the results of this section, the firm profit can be rewritten as a function of the current productivity. Mathematically,

$$\pi(z) = \max\left\{\pi^a(z), \pi^b(z)\right\}.$$

To obtain a formula for $\pi(z)$ note the following: (i) if $z < \underline{z}$, $\pi^b(z) > \pi^a(z)$, since the firm pays lower wages; (ii) for $z > \underline{z}$, the firm will decide to operate above the threshold; (iii) if $z \in (\underline{z}, \overline{z})$, it is optimal to remain just below the threshold. Hence,

$$\pi(z) \quad = \quad \begin{cases} e^{\frac{z}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) \text{ for } z < \underline{z}, \\ e^z \underline{n}^\alpha - w\underline{n} \text{ for } \underline{z} \leq z \leq \overline{z}, \\ e^{\frac{z}{1-\alpha}} \left(\frac{\alpha}{w(1+\tau)}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) - c_f \text{ for } z > \overline{z}. \end{cases}$$

For completeness, I also state the employment demand:

$$n(z) \quad = \quad \begin{cases} \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} e^{\frac{z}{1-\alpha}} \text{ for } z < \underline{z}, \\ \underline{n}^- \text{ for } \underline{z} \leq z \leq \overline{z}, \\ \left(\frac{\alpha}{w(1+\tau)}\right)^{\frac{1}{1-\alpha}} e^{\frac{z}{1-\alpha}} \text{ for } z \geq \overline{z}, \end{cases}$$

Overall, firms are distributed below the threshold or bunched exactly at (more precisely, just below) the threshold, or above the threshold.

4.2 Firm distribution

Firm productivity z has an exponential distribution with parameter λ. Since log employment is proportional to z, employment follow a Pareto distributions with parameter $\beta = \lambda(1-\alpha) + 1$.[4] Figure 4 displays the firm size distribution implied by the model around the threshold. There is a substantial "hole" in the distribution with no firms whatsoever

[4] Many studies have found that this is an good approximation of the firm-size distribution. See Gabaix (2016) for a review of Power Law in economics.

Figure 4: Distribution of employment between 40 and 59 in the model without measurement error.

Table 2: Economic Parameters

Parameters	Values	Definition	
r	0.05	interest rate	fixed
α	0.66	curvature profit function	fixed
σ_{mrn}	0.0324	measurement error	calibrated
τ	0.0015	payroll tax above $\underline{n}$	calibrated
λ	3.6829	exponential distribution	calibrated

between 50 and 55 employees. This is an empirical challenge, because in the data there are many firms with an employment level slightly greater than 49. I will attribute the presence of all these firms to measurement error.

5 Quantitative Analysis

This Section proposes a simple calibration of the model, and evaluates the aggregate effect of these distortions.

5.1 Calibration

I incorporate measurement error in (log) employment. Formally, measured employment is equal to product of the true value and a lognormal error term with standard deviation σ_{mrn} and mean equal to unity. Measurement error also helps capture model misspecification, which can take several forms. First, the measure of employment is the arithmetic average of the number of employees at the end of each quarter. This is the relevant measure of employment for some but not all of the regulations. Some regulations are based on employment measured in full-time equivalent and some other regulations apply if there is more than 50 employees in the firm for more than 12 months. Second, measurement error also captures adjustment cost or search frictions which lead to an imperfect control of the size of the workforce.

Table 3: Moments

	Data	Model
Std. Dev. $\Delta \log n$	0.1561	0.1561
Power Law coefficient	2.2522	2.2522
Density of firms in each bin		
40-46	0.0718	0.0666
47-49	0.0790	0.0783
50-52	0.0307	0.0341
53-59	0.0240	0.0281

The wage rate is normalized to 1. The real interest rate r is set to 5 percent. The curvature parameter α is set to 0.66. This last parameter is a reduced-form for the labor share, decreasing returns to scale and the elasticity of demand.

Table 3 lists the target moments: (i) the volatility of growth in employment, and the slope of the power law; (ii) the distribution of employment around the threshold, as approximated by the density of firms between 40 and 46 employees, between 47 and 49 employees, between 50 and 52 employees, and between 53 and 59 employees;[5] The rational for the first group of moments (i) is that I want the model to be consistent with key features of firm dynamics. The rationale for the distribution of employment around the threshold (ii) is that I want to reproduce well the discontinuity in the firm size distribution, which is the *prima facie* evidence that the regulation matters.

Table 3 evaluates the fit of the model. Overall, the data are consistent with a small, but significant proportional payroll tax of 0.15%. This value is lower than the taxes that are actually set in the law, which presents an apparent puzzle. One possible interpretation is that some of these regulations are indeed not as costly as they appear, and represent benefits that are valued by the workers. The model requires a measurement error of around 3%, or on average 2 workers around the threshold. In spite of its parsimony, the model is able to reproduce reasonably well all the targeted moments, and in particular the discontinuities in the distributions. A graphical illustration is provided in Figure 5.

[5]The distribution is the number of firms in each bin, divided by the length of the bin (7 or 3), and further divided by the total number of firms between 40 and 59.

Figure 5: Distribution of firm employment (between 40 and 59 employees), in the data and in the model. Full line is the data, dashed lines represent the model. The distributions are normalized by the total number of firms between 40 and 59 employees.

Table 4: Policy experiments

Experiment	Gains
Benchmark	0.30%
Apply the regulation to all firms	2.50%
Apply the regulation above 75 employees	0.06%

5.2 Policy Experiments

I use the calibrated model to infer the aggregate effect of the regulation on productivity. From the point of view of a social planner, the regulation misallocates labor across firms and hence reduces aggregate productivity. I now perform the same calculation as in Section 2. Precisely, I ask how much of an increase in output can be obtained, holding total employment constant, by reallocating labor across firms.

The gain in total output, holding total labor constant, is 0.3%, which is significant. Second, one might ask, how much of the efficiency gain can be achieved by extending the threshold to 75 employees rather than 50? The answer is, not much: the gains are reduced to 0.06%. Third, the motivation for the phase-in of the regulation at 50 employees is that it is too costly to impose the compliance cost on small firms. I evaluate this argument by considering the counterfactual: What would happen if all firms were subject to the regulation? It would reduce output by 2.5%. It is safe to say, then, that applying the regulation to all firms would be quite costly, which suggests that the phase-in is perhaps not

such a bad policy.

6 Concluding Remarks

This paper studies a particular regulation that clearly distorts the firm size distribution, leading to an obvious misallocation of labor—a channel that has been emphasized in the recent literature. The model fits the size distribution discontinuity around the threshold well. Removing the regulation leads to an increase of output close to 0.3%, holding employment fixed.

These results suggest that size distortions have a fairly moderate aggregate impact. What can explain the small benefits of Section 5 with the potentially large benefits in Section 2? Further research is needed to be conclusive on this issue. There are at least three reasons to believe the effects could be bigger. First, this is just one example of distortions among many others. France is characterized by, for instance, a stringent employment protection legislation, and more than 15% of workers are affected by minimum wage' increases. Second, the model abstracts from notion of match quality and assortative matching. It might be missing some of the negative effects of the regulation. For instance, some talented workers might be stuck in small unproductive firms because of these regulations while they would contribute more to aggregate output by working in larger firms. Last, the proposed framework is static and there may be dynamic effects of these policies that are missed by the current analysis. These distortions reduce the value of investment and the value of entry.

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