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# Trade and Labor Market Dynamics: 

# General Equilibrium Analysis of the China Trade Shock* 

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#### Abstract

We develop a dynamic trade model with spatially distinct labor markets facing varying exposure to international trade. The model captures the role of labor mobility frictions, goods mobility frictions, geographic factors, and input-output linkages in determining equilibrium allocations. We show how to solve the equilibrium of the model and take the model to the data without assuming that the economy is at a steady state and without estimating productivities, migration frictions, or trade costs, which can be difficult to identify. We calibrate the model to 22 sectors, 38 countries, and 50 U.S. states. We study how the rise in China's trade for the period 2000 to 2007 impacted U.S. households across more than a thousand U.S. labor markets distinguished by sector and state. We find that the China trade shock resulted in a loss of 0.8 million U.S. manufacturing jobs, about $25 \%$ of the observed decline in manufacturing employment from 2000 to 2007 . The U.S. gains in the aggregate but, due to trade and migration frictions, the welfare and employment effects vary across U.S. labor markets. Estimated transition costs to the new long-run equilibrium are also heterogeneous and reflect the importance of accounting for labor dynamics.


[^0]
## 1. INTRODUCTION

Understanding and quantifying the employment effects of trade shocks has been a central issue in recent research. A standard approach, relying on reduced-form analysis, has provided robust empirical evidence on the differential effects of trade shocks across local labor markets. These studies, however, say little about the effects on overall employment, welfare, or other aggregate outcomes, and cannot be used to study counterfactual policies. In this paper we study the general equilibrium effects on U.S. labor markets of a surge in China's productivity, a shock that accounts for the increase in Chinese import penetration into the U.S. market.

We develop a dynamic spatial trade and migration model to understand and quantify the disaggregate labor market effects resulting from changes in the economic environment. The model explicitly recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, input-output linkages, and international trade in shaping the effects of shocks across different labor markets. Hence, our model has intersectoral trade, interregional trade, international trade, and labor market dynamics.

In our economy, production takes place in spatially distinct markets. A market is a sector located in a particular region in a given country. ${ }^{1}$ In each market there is a continuum of heterogeneous firms producing intermediate goods a la Eaton and Kortum (2002, hereafter EK). Firms are competitive, have constant returns to scale technology, and demand labor, local factors, and materials from all other markets in the economy. ${ }^{2}$ The supply side of the economy features forward-looking households choosing whether to be employed or non-employed in the next period and in which labor market to supply labor, conditional on their location, the state of the economy, sectoral and spatial mobility costs, and an idiosyncratic shock a la Artuç, Chaudhuri and McLaren (2010, hereafter ACM). Employed households supply a unit of labor and receive the local competitive market wage; non-employed households obtain consumption in terms of home production. Incorporating these elements delivers a general equilibrium, dynamic discrete choice model, with realistic geographic features.

Taking a dynamic trade model with all these features to the data, and performing a counterfactual analysis, may seem unfeasible since it requires pinning down a large set of exogenous state variables, (hereafter referred as fundamentals), like productivity levels across sectors and regions, bilateral mobility (migration) costs across markets, bilateral international and domestic trade costs, and

[^1]endowments of immobile local factors. ${ }^{3}$ Our methodological contribution is to show that, under perfect foresight, by expressing the equilibrium conditions in relative time differences we are able to solve the model and perform large-scale counterfactual analyses without needing to estimate the fundamentals of the economy. Aside from data that directly map into the model's equilibrium conditions, the only parameters we need, in order to solve the full transition of the dynamic model, are the trade elasticities, the migration elasticity, and the intertemporal discount factor.

Our method relies on conditioning on the observed allocation. The intuition is that the observed allocation (namely data on production, employment, trade, and migration flows across markets) provides all the information we need on the levels of the fundamentals of the economy. Our result builds on Dekle, Eaton, and Kortum (2008, hereafter DEK), who have shown a similar result in the context of a static trade model. We show how to apply our method, which we label "dynamic hat algebra", to a dynamic discrete choice spatial trade model. ${ }^{4}$

We apply our model and solution method to study the effects of the rise in China's import competition on U.S. labor markets over the period 2000-2007, which we refer to as the China trade shock. U.S. imports from China more than doubled from 2000 to 2007. During the same period, manufacturing employment fell considerably while employment in other sectors, such as construction and services, grew. Several reduced-form studies (e.g. Autor, Dorn, and Hanson, 2013, hereafter ADH; Acemoglu et al., 2014; Pierce and Schott, 2016) document that an important part of the employment loss in manufacturing was a consequence of China's trade expansion, either as a consequence of technological improvements in the Chinese economy or reductions in trade costs. ${ }^{5}$ In most of these studies, the main reason that U.S. labor markets are differentially exposed to Chinese goods is their different degree of import competition.

We use our model to quantify how additional channels can also explain the employment loss in the manufacturing sector, and how other sectors of the economy, such as construction and services, were also exposed to the China shock. More importantly, we use our model to compute welfare effects across labor markets over time. In summary, we account for the distribution of winners and

[^2]losers across sectors and regions of the U.S. economy caused by the increase in Chinese competition.
We do this by calibrating a 38 -country, 50 -U.S.-state, and 22 -sector version of our model. ${ }^{6}$ We take the initial distribution of labor across markets in the U.S. economy and match the initial conditions of our model to those in the year 2000. We rely on the identification restriction suggested by ADH to measure China's shock; namely, we use the predicted changes in U.S. imports from China using as an instrument the change in imports from China by other high-income countries for the period 2000 to 2007 . Using our model, we compute the change in sectoral productivities in China between 2000 and 2007 that exactly matches the predicted changes in imports in the model. We label these changes in productivity the China trade shock and refer to them as such in the rest of the paper.

We find that increased Chinese competition reduces the aggregate manufacturing employment share by 0.5 percentage points in the long run, which is equivalent to a loss of about 0.8 million manufacturing jobs, or about $25 \%$ of the observed decline in manufacturing employment from 2000 to $2007 .{ }^{7,8}$ We also find that workers reallocate to construction and the services sectors, as these sectors benefit from the access to cheaper intermediate inputs from China. For instance, we find that about 75,000 jobs were created in construction as a result of the China shock.

With our model we can also quantify the relative contribution of different sectors, regions, and labor markets to the decline in manufacturing employment. We find that sectors with a higher exposure to import competition from China lose more manufacturing jobs. The computer and electronics industry, and the furniture industry accounted for about half of the decline in manufacturing employment, followed by the metal and textiles industries, which contributed another one-fourth. Some sectors, such as food, beverage, and tobacco, gained employment, as they were less exposed to China and benefited from cheaper intermediate goods. The fact that U.S. economic activity is not equally distributed across space, plus the differential sectoral exposure to China, imply that the impact of China's import competition varies across regions. We find that U.S. states with a larger concentration of sectors more exposed to China lose more manufacturing jobs. California, which by far accounts for the largest share of employment in computer and electronics (the sector most

[^3]exposed to China's import competition), accounted for about $12 \%$ of the decline. We also find that the change in employment shares across space is heterogeneous across industries. In particular, the reduction in local employment shares in manufacturing industries is more concentrated in a handful of states while the increase in local employment shares in non-manufacturing industries spread more evenly across U.S. states.

Our framework also allows us to quantify the welfare effects of the increased competition from China on the U.S. economy. Our results indicate that the China shock increased U.S. welfare by $0.35 \%$. Therefore, even when U.S. exposure to China decreases employment in the manufacturing sector, the U.S. economy is better off, as it benefits from access to cheaper goods from China. We also find a large dispersion in welfare effects across individual labor markets, ranging from $-1 \%$ to $4.8 \%$. Larger welfare gains are generally in labor markets that produce non-manufacturing goods as these industries do not suffer the direct adverse effects of the increased competition from China and at the same time benefit from access to cheaper intermediate manufacturing inputs from China used in production. Similarly, labor markets in states that trade more with the rest of the U.S. economy and purchase materials from sectors where Chinese productivity increases, tend to have larger welfare gains as they benefit from the access to cheaper inputs from China purchased from the rest of the U.S. economy. We also compute the welfare effects in the rest of the world and find that all countries gain from the China shock, with some countries having larger welfare gains and others having smaller welfare gains than the U.S. economy. Since reaching the new steady state after the China shock takes time due to mobility frictions, we compute the transition or adjustment costs to the new steady state and find substantial variation across labor markets.

We also extend the model to study the effects of increases in disability benefits, a type of nonemployment benefit aimed at mitigating some of the negative effects from import competition from China. We find that a gradual increase in the generosity of disability benefits to the levels observed in Europe, contribute to an additional decline in the manufacturing employment share of 0.24 percentage points, that is, to about 360.5 thousand additional manufacturing jobs lost. Importantly, we find that the employment effects are especially larger in those sectors and regions that have high exposure to the China shock, and we also find an increase in the non-employment rate in the long run.

We further extend our model in other dimensions by incorporating additional sources of persistence, time-varying fundamentals, CES preferences, and elastic labor supply. We show that the dynamic hat algebra works in these alternative models, and discuss their quantitative implications,
which are similar to our baseline results.
Finally, one extension that we do not consider in this paper is modelling the stochastic process of fundamentals. Such extension would require departing from the perfect foresight assumption. Our approach will not necessarily fail if one were to relax the assumption of perfect foresight, but adding rational expectations would imply solving the model for every possible realization of fundamentals in the future, which in our application, with more than 1000 endogenous state variables, is a computational constraint.

Our study is complementary to a large body of reduced-form empirical research aimed at identifying the disaggregate effects of changes in the economic environment. Our contribution is to introduce a framework to perform large-scale quantitative analysis which retains transparency about the main economic insights that deliver the results. Equally important, our model can speak about effects that are usually difficult to quantify or identify in reduced-form empirical research. For instance, we can study how the levels of aggregate employment for different countries and for specific labor markets respond to a change in economic fundamentals. ${ }^{9}$ Furthermore, we can explain how additional channels account for the change in welfare and many other economic outcomes at the aggregate and disaggregate levels and over time.

Our approach relates to a fast-growing strand of the literature that studies the impact of trade shocks on labor market dynamics. ${ }^{10}$ The work most closely related to ours is Artuç and McLaren (2010), ACM, and Dix-Carneiro (2014). We follow Artuç and McLaren (2010) and ACM in modeling the migration decisions of agents as a dynamic discrete choice. We depart from their assumption of a small open economy in partial equilibrium and introduce a multicountry, multiregion, multisector general equilibrium trade model with trade and migration costs. Our study is also complementary to Dix-Carneiro (2014), who focuses on measuring the frictions that workers face to move across sectors, and interpret their magnitude through the simulation of hypothetical trade liberalization episodes. Following Dix-Carneiro (2014), we use our general equilibrium model to quantify the dynamic effects of a trade shock across markets, but unlike him, we rely on our solution method to compute these effects at a more granular level.

Overall, we highlight three main departures of our paper from the previous literature. First,

[^4]relative to other recent dynamic discrete choice models of labor reallocation, we include all important general equilibrium mechanisms present in static quantitative trade and spatial models such as multiple countries, input-output linkages, multiple sectors, and multiple factors production. The resulting framework allows us to study a wider range of policy experiments compared to previous work. Second, we provide a method to compute the model and study counterfactuals without the need to estimate exogenous constant and time-varying fundamentals, which is key in order to take the model to a highly disaggregated level as we do. Finally, our paper complements reduced form studies on the effects of the China shock. We can not only measure the differential impact across labor markets but we can also compute employment effects and measure the welfare effects taking into account general equilibrium channels.

The paper is organized as follows. In Section 2 we present our dynamic spatial trade and migration model. In Section 3 we show how to solve the model and perform counterfactual analysis using the dynamic hat algebra. In Section 4 we explain how to take the model to the data, and how we estimate the China shock. In Section 5 we use our model to quantify the effects of increased Chinese competition on different U.S. labor markets. We also present different extension of the model and discuss additional results. Finally, we conclude in Section 6. All proofs are relegated to the appendix.

## 2. A DYNAMIC SPATIAL TRADE AND MIGRATION MODEL

We consider a world with $N$ locations, and $J$ sectors. We use the indexes $n$ or $i$ to identify a particular location and index sectors by $j$ or $k$. In each region-sector combination there is a competitive labor market. In each market there is a continuum of perfectly competitive firms producing intermediate goods.

Firms have a Cobb-Douglas constant returns to scale technology, demanding labor, a composite local factor that we refer to as structures, and materials from all sectors. We follow EK and assume that productivities are distributed Fréchet with a sector-specific productivity dispersion parameter $\theta^{j}$.

Time is discrete, and we denote it by $t=0,1,2, \ldots$ Households are forward looking, have perfect foresight, and optimally decide where to move given some initial distribution of labor across locations and sectors. Households face costs to move across markets and experience an idiosyncratic shock that affects their moving decision. The household's problem is closely related to the sectoral reallocation problem in ACM and to the competitive labor search model of Lucas and Prescott
(1974) and Dvorkin (2014). ${ }^{11}$

We first characterize the dynamic problem of a household deciding where to move conditional on a path of real wages across time and across labor markets. We then characterize the static subproblem to solve for prices and wages conditional on the supply of labor in a given market.

### 2.1 Households

At $t=0$ there is a mass $L_{0}^{n j}$ of households in each location $n$ and sector $j$. Households can be either employed or non-employed. An employed household in location $n$ and sector $j$ supplies a unit of labor inelastically and receives a competitive market wage $w_{t}^{n j}$. Given her income she decides how to allocate consumption over local final goods from all sectors with a Cobb-Douglas aggregator. Preferences, $U\left(C_{t}^{n j}\right)$, are over a basket of final local goods

$$
\begin{equation*}
C_{t}^{n j}=\prod_{k=1}^{J}\left(c_{t}^{n j, k}\right)^{\alpha^{k}} \tag{1}
\end{equation*}
$$

where $c_{t}^{n j, k}$ is the consumption of sector $k$ goods in market $n j$ at time $t$, and $\alpha^{k}$ is the final consumption share, with $\sum_{k=1}^{J} \alpha^{k}=1$. We denote the ideal price index by $P_{t}^{n}=\prod_{k=1}^{J}\left(P_{t}^{n k} / \alpha^{k}\right)^{\alpha^{k}}$. As in Dvorkin (2014), non-employed households obtain consumption in terms of home production $b^{n}>$ $0 .{ }^{12}$ To simplify the notation, we represent sector zero in each region as non-employment; hence, $C_{t}^{n 0}=b^{n} .{ }^{13}$

Assumption 1 Agents have logarithmic preferences, $U\left(C_{t}^{n j}\right) \equiv \log \left(C_{t}^{n j}\right)$.
The household's problem is dynamic. Households are forward looking and discount the future at rate $\beta \geq 0$. Migration decisions are subject to sectoral and spatial mobility costs.

Assumption 2 Labor reallocation costs $\tau^{n j, i k} \geq 0$ depend on the origin ( $n j$ ) and destination (ik), and are time invariant, additive, and measured in terms of utility.

[^5]In addition, households have additive idiosyncratic shocks for each choice, denoted by $\epsilon_{t}^{i k}$.
The timing for the households problem and decisions is as follows. Households observe the economic conditions in all labor markets and the realizations of their own idiosyncratic shocks. If they begin the period in a labor market, they work and earn the market wage. If they are non-employed in a region, they get home production. Then, both employed and non-employed households have the option to relocate. Formally,

$$
\begin{gathered}
\mathrm{v}_{t}^{n j}=U\left(C_{t}^{n j}\right)+\max _{\{i, k\}_{i=1, k=0}^{N, J}}\left\{\beta E\left[\mathrm{v}_{t+1}^{i k}\right]-\tau^{n j, i k}+\nu \epsilon_{t}^{i k}\right\}, \\
\text { s.t. } C_{t}^{n j} \equiv \begin{cases}b^{n} & \text { if } j=0, \\
w_{t}^{n j} / P_{t}^{n} & \text { otherwise }\end{cases}
\end{gathered}
$$

where $\mathrm{v}_{t}^{n j}$ is the lifetime utility of a household currently in region $n$ and sector $j$ at time $t$ and the expectation is taken over future realizations of the idiosyncratic shock. The parameter $\nu$ scales the variance of the idiosyncratic shocks. Note that households choose to relocate to the labor market that delivers the highest utility net of costs.

Assumption 3 The idiosyncratic shock $\epsilon$ is i.i.d. over time and distributed Type-I Extreme
Value with zero mean.

Assumption 3 is standard in dynamic discrete choice models. ${ }^{14}$ It allows for simple aggregation of idiosyncratic decisions made by households, as we now show. ${ }^{15}$

Let $V_{t}^{n j} \equiv E\left[\mathrm{v}_{t}^{n j}\right]$ be the expected lifetime utility of a representative agent in labor market $n j$, where the expectation is taken over the preference shocks. Then, given Assumption 3, we obtain (see Appendix 1)

$$
\begin{equation*}
V_{t}^{n j}=U\left(C_{t}^{n j}\right)+\nu \log \left(\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}\right) \tag{2}
\end{equation*}
$$

Equation (2) reflects the fact that the value of being in a particular labor market depends on the current-period utility and on the option value to move into any other market in the next period. ${ }^{16}$ $V_{t}^{n j}$ can be interpreted as the expected lifetime utility of a household before the realization of her

[^6]preference shocks or, alternatively, as the average utility of households in that market. ${ }^{17}$
Using Assumption 3 we can also show that the share of labor that transitions across markets has a closed-form analytical expression. In particular, denote by $\mu_{t}^{n j, i k}$ the fraction of households that relocate from market $n j$ to $i k$ (with $\mu_{t}^{n j, n j}$ the fraction who choose to remain in their original location); then (see Appendix 1)
\[

$$
\begin{equation*}
\mu_{t}^{n j, i k}=\frac{\exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}} . \tag{3}
\end{equation*}
$$

\]

Equation (3), which we refer to as the migration shares, has an intuitive interpretation. All other things being equal, markets with a higher lifetime utility (net of mobility costs) are the ones that attract more migrants. From this expression we can also see that $1 / \nu$ has the interpretation of a migration cost elasticity.

Equation (3) is a key equilibrium condition in this model because it conveys all the information needed to determine how the distribution of labor evolves over time. In particular, the dynamics of the distribution of households over markets are described by

$$
\begin{equation*}
L_{t+1}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{i k, n j} L_{t}^{i k} . \tag{4}
\end{equation*}
$$

The equilibrium condition (4) characterizes the evolution of the economy's state, the distribution of employment and non-employment across markets $L_{t}=\left\{L_{t}^{n j}\right\}_{n=1, j=0}^{N, J}$. Note that given our timing assumption, the supply of labor at each $t$ is fully determined by forward-looking decisions at period $t-1$. Now, conditional on labor supplied at each market, we can specify a static production structure of the economy that allows us to solve for equilibrium wages at each time $t$ such that labor markets clear. We now proceed to describe the production side of the economy.

### 2.2 Production

Production follows the multisector version in Caliendo and Parro (2015) and the spatial model of Caliendo et al. (2017). Firms in each sector and region are able to produce many varieties of intermediate goods, denoted by $q$. The technology to produce these intermediate goods requires labor and structures, which are the primary factors of production, and materials, which consist

[^7]of goods from all sectors. ${ }^{18}$ Total factor productivity (TFP) of an intermediate good is composed of two terms, a time-varying sectoral-regional component $\left(A_{t}^{n j}\right)$, which is common to all varieties in a region and sector, and a variety-specific component $\left(z^{n j}\right)$. Since an intermediate variety is identified by $z^{n j}$, we use it to index a variety.

## Intermediate Goods Producers

The technology for intermediate goods is described by

$$
q_{t}^{n j}=z^{n j}\left(A_{t}^{n j}\left(h_{t}^{n j}\right)^{\xi^{n}}\left(l_{t}^{n j}\right)^{1-\xi^{n}}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(M_{t}^{n j, n k}\right)^{\gamma^{n j, n k}},
$$

where $l_{t}^{n j}, h_{t}^{n j}$ are labor and structures inputs of firms in sector $j$ and region $n$, and $M_{t}^{n j, n k}$ is the material inputs from sector $k$ by firms in sector $j$ and region $n$. Material inputs are goods from sector $k$ produced in the same region $n$. The parameter $\gamma^{n j} \geq 0$ is the share of value added in the production of sector $j$ and region $n$, and $\gamma^{n j, n k} \geq 0$ is the share of materials from sector $k$ in the production of sector $j$ and region $n$. We assume that the production function exhibits constant returns to scale such that $\sum_{k=1}^{J} \gamma^{n j, n k}=1-\gamma^{n j}$. The parameter $\xi^{n}$ is the share of structures in value added. Structures are in fixed supply in each labor market.

We denote by $P_{t}^{n j}$ the price of materials, and by $r_{t}^{n j}$ the rental price of structures in region $n$ and sector $j$. The unit price of an input bundle is

$$
\begin{equation*}
x_{t}^{n j}=B^{n j}\left(\left(r_{t}^{n j}\right)^{\xi^{n}}\left(w_{t}^{n j}\right)^{1-\xi^{n}}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(P_{t}^{n k}\right)^{\gamma^{n j, n k}} \tag{5}
\end{equation*}
$$

where $B^{n j}$ is a constant. Then, the unit cost of an intermediate good $z^{n j}$ at time $t$ is $\frac{x_{t}^{n j}}{z^{n j}\left(A_{t}^{n j}\right)^{n j}}$.
Trade costs are represented by $\kappa_{t}^{n j, i j}$ and are of the "iceberg" type. One unit of any variety of intermediate good $j$ shipped from region $i$ to $n$ requires producing $\kappa_{t}^{n j, i j} \geq 1$ units in region $i$. If a good is nontradable, then $\kappa=\infty$. Competition implies that the price paid for a particular variety of good $j$ in region $n$ is given by the minimum unit cost across regions taking into account trade costs and where the vector of productivity draws received by the different regions is $z^{j}=$ $\left(z^{1 j}, z^{2 j}, \ldots, z^{N j}\right)$. That is,

$$
p_{t}^{n j}\left(z^{j}\right)=\min _{i}\left\{\kappa_{t}^{n j, i j} x_{t}^{i j} z^{i j}\left(A_{t}^{i j}\right)^{\gamma^{i j}}\right\} .
$$

[^8]
## Local Sectoral Aggregate Goods

Intermediate goods from sector $j$ from all regions are aggregated into a local sectoral good. Let $Q_{t}^{n j}$ be the quantity produced of aggregate sectoral goods $j$ in region $n$ and $\tilde{q}_{t}^{n j}\left(z^{j}\right)$ be the quantity demanded of an intermediate good of a given variety from the lowest cost supplier. The production of local sectoral goods is given by

$$
Q_{t}^{n j}=\left(\int\left(\tilde{q}_{t}^{n j}\left(z^{j}\right)\right)^{1-1 / \eta^{n j}} d \phi^{j}\left(z^{j}\right)\right)^{\eta^{n j} /\left(\eta^{n j}-1\right)}
$$

where $\phi^{j}\left(z^{j}\right)=\exp \left\{-\sum_{n=1}^{N}\left(z^{n j}\right)^{-\theta^{j}}\right\}$ is the joint distribution over the vector $z^{j}$, with marginal distribution given by $\phi^{n j}\left(z^{n j}\right)=\exp \left\{-\left(z^{n j}\right)^{-\theta^{j}}\right\}$ and the integral is over $\mathrm{R}_{+}^{N}$. For nontradable sectors the only relevant distribution is $\phi^{n j}\left(z^{n j}\right)$ since sectoral good producers use only local intermediate goods. There are no fixed costs or barriers to entry and exit in the production of intermediate and sectoral goods. Competitive behavior implies zero profits at all times.

Local sectoral aggregate goods are used as materials for the production of intermediate varieties as well as for final consumption. Note that the fact that local sectoral aggregate goods are not traded does not imply that consumers are not purchasing traded goods. On the contrary, both intermediate goods producers and households, via the direct purchase of the local sectoral aggregate goods, are purchasing tradable varieties.

Given the properties of the Fréchet distribution, the price of the sectoral aggregate good $j$ in region $n$ at time $t$ is

$$
\begin{equation*}
P_{t}^{n j}=\Gamma\left(\sum_{i=1}^{N}\left(x_{t}^{i j} \kappa_{t}^{n j, i j}\right)^{-\theta^{j}}\left(A_{t}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}}, \tag{6}
\end{equation*}
$$

where $\Gamma$ is a constant. ${ }^{19}$ To obtain (6), we assumed that $1+\theta^{j}>\eta^{n j}$. Following similar steps as earlier, we can solve for the share of total expenditure in market $(n, j)$ on goods $j$ from market $i .^{20}$ In particular,

$$
\begin{equation*}
\pi_{t}^{n j, i j}=\frac{\left(x_{t}^{i j} \kappa_{t}^{n j, i j}\right)^{-\theta^{j}}\left(A_{t}^{i j}\right)^{\theta^{j} \gamma^{i j}}}{\sum_{m=1}^{N}\left(x_{t}^{m j} \kappa_{t}^{n j, m j}\right)^{-\theta^{j}}\left(A_{t}^{m j}\right)^{\theta^{j} \gamma^{m j}}} . \tag{7}
\end{equation*}
$$

This equilibrium condition reflects that the more productive market $i j$ is, given factor costs, the cheaper is the cost of production in market $i j$, and therefore, the more region $n$ purchases sector $j$ goods from region $i$. In addition, the easier it is to ship sector $j$ goods from region $i$ to $n$ (lower $\kappa^{n j, i j}$ ), the more region $n$ purchases sector $j$ goods from region $i$. This equilibrium condition resembles a gravity equation.

[^9]
## Market Clearing and Unbalanced Trade

With an eye towards our application and to accommodate for observed trade imbalances, we assume there is a mass 1 of rentiers in each region. Rentiers cannot relocate to other regions. They own the local structures, rent them to local firms, and send all their local rents to a global portfolio. In return, rentiers receive a constant share $\iota^{n}$ from the global portfolio, with $\sum_{n=1}^{N} \iota^{n}=1$. The difference between the remittances and the income rentiers receive will generate imbalances, which change in magnitude as the rental prices change, and are given by $\sum_{k=1}^{J} r_{t}^{i k} H^{i k}-\iota^{n} \chi_{t}$, where $\chi_{t}=\sum_{i=1}^{N} \sum_{k=1}^{J} r_{t}^{i k} H^{i k}$ are the total revenues in the global portfolio. The local rentier owns this fraction of the global portfolio of structures and uses her income share from the global portfolio to buy goods produced in her own region using equation (1).

Let $X_{t}^{n j}$ be the total expenditure on sector $j$ good in region $n$. Then, goods market clearing implies

$$
\begin{equation*}
X_{t}^{n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t}^{i k, n k} X_{t}^{i k}+\alpha^{j}\left(\sum_{k=1}^{J} w_{t}^{n k} L_{t}^{n k}+\iota^{n} \chi_{t}\right) \tag{8}
\end{equation*}
$$

where the first term on the right-hand-side is the value of the total demand for sector $j$ goods produced in $n$ used as materials in all sectors and regions in the economy, and $\alpha^{j} \sum_{k=1}^{J}\left(w_{t}^{n k} L_{t}^{n k}+\right.$ $\left.\iota^{n} \chi_{t}\right)$ is the value of the final demand in region $n$.

Labor market clearing in region $n$ and sector $j$ is

$$
\begin{equation*}
L_{t}^{n j}=\frac{\gamma^{n j}\left(1-\xi^{n}\right)}{w_{t}^{n j}} \sum_{i=1}^{N} \pi_{t}^{i j, n j} X_{t}^{i j} \tag{9}
\end{equation*}
$$

while the market clearing for structures in region $n$ and sector $j$ must satisfy

$$
\begin{equation*}
H^{n j}=\frac{\gamma^{n j} \xi^{n}}{r_{t}^{n j}} \sum_{i=1}^{N} \pi_{t}^{i j, n j} X_{t}^{i j} \tag{10}
\end{equation*}
$$

### 2.3 Equilibrium

The endogenous state of the economy at any given moment in time is given by the distribution of labor across all markets $L_{t}$. The exogenous fundamentals are $\Theta_{t}$. The fundamentals of the economy are deterministic, some time-varying and some constant. The time-varying fundamentals of the economy are sectoral-regional productivities $A_{t}=\left\{A_{t}^{n j}\right\}_{n=1, j=1}^{N, J}$ and bilateral trade costs $\kappa_{t}=\left\{\kappa_{t}^{n j, i j}\right\}_{n=1, i=1, j=1}^{N, N, J}$. Constant fundamentals are the labor reallocation costs $\Upsilon=\left\{\tau^{n j, i k}\right\}_{n=1, j=0, i=1, k=0}^{N, J, J, N}$, the stock of land and structures across markets $H=\left\{H^{n j}\right\}_{n=1, j=1}^{N, J}$,
and home production across regions $b=\left\{b^{n}\right\}_{n=1}^{N}$. We can denote the fundamentals at date $t$ by $\Theta_{t} \equiv\left(\Theta_{1 t}, \Theta_{2}\right)$, where $\Theta_{1 t} \equiv\left(A_{t}, \kappa_{t}\right)$ and $\Theta_{2} \equiv(\Upsilon, H, b)$. We now proceed to formally define an equilibrium of the economy.

We seek to find equilibrium wages $w_{t}=\left\{w_{t}^{n j}\right\}_{n=1, j=1}^{N, J}$, and the equilibrium allocations $\pi_{t}=$ $\left\{\pi_{t}^{i j, n j}\right\}_{i=1, j=1, n=1}^{N, J, N}, X_{t}=\left\{X_{t}^{n j}\right\}_{n=1, j=1}^{N, J}$, given $\left(L_{t}, \Theta_{t}\right)$. We refer to this equilibrium as a temporary equilibrium. Formally,

Definition 1 Given $\left(L_{t}, \Theta_{t}\right)$, a temporary equilibrium is a vector of wages $w\left(L_{t}, \Theta_{t}\right)$ that satisfies the equilibrium conditions of the static subproblem, (5) to (10).

The temporary equilibrium of our model is the solution to a static multicountry interregional trade model. ${ }^{21}$ Suppose that for any $\left(L_{t}, \Theta_{t}\right)$ we can solve the temporary equilibrium. ${ }^{22}$ Then the wage rate can be expressed as $w_{t}=w\left(L_{t}, \Theta_{t}\right)$, and given that prices are all functions of wages, we can express real wages as $\omega^{n j}\left(L_{t}, \Theta_{t}\right)=w_{t}^{n j} / P_{t}^{n}$. After defining the temporary equilibrium, we can now define the sequential competitive equilibrium of the model given a path of exogenous fundamentals $\Theta=\left\{\Theta_{t}\right\}_{t=0}^{\infty}$. Let $\mu_{t}=\left\{\mu_{t}^{n j, i k}\right\}_{n=1, j=0, i=1, k=0}^{N, J, N, J} V_{t}=\left\{V_{t}^{n j}\right\}_{n=1, j=0}^{N, J}$ be the migration shares and lifetime utilities, respectively. The definition of a sequential competitive equilibrium is given as follows: ${ }^{23}$

Definition 2 Given $\left(L_{0}, \Theta\right)$, a sequential competitive equilibrium of the model is a sequence of $\left\{L_{t}, \mu_{t}, V_{t}, w\left(L_{t}, \Theta_{t}\right)\right\}_{t=0}^{\infty}$ that solves equilibrium conditions (2) to (4) and the temporary equilibrium at each $t$.

Finally, we define a stationary equilibrium of the model.

[^10]Definition 3 A stationary equilibrium of the model is a sequential competitive equilibrium such that $\left\{L_{t}, \mu_{t}, V_{t}, w\left(L_{t}, \Theta_{t}\right)\right\}_{t=0}^{\infty}$ are constant for all $t$.

A stationary equilibrium in this economy is a situation in which no aggregate variables change over time. It follows, that in a stationary equilibrium fundamentals need to be constant for all $t$. In such a stationary equilibrium, households may flow from one market to another, but inflows and outflows balance.

## 3. DYNAMIC HAT ALGEBRA

Solving for all the transitional dynamics in a dynamic discrete choice model with this rich spatial structure is difficult, and it also requires pinning-down the values of a large number of unknown fundamentals. Note from Definitions 1 to 3 that to solve for an equilibrium of the model it is necessary to condition on $\Theta_{t}$; namely, the level of the fundamentals of the economy (productivities, endowments of local structures, labor mobility costs, non-employment income, and trade costs) at each point in time. As we increase the dimension of the problem, for example by adding countries, regions, or sectors, the number of fundamentals grows geometrically. We now show how to compute the counterfactual changes in all endogenous variables across markets and time as the solution to a system of non-linear equations without needing to estimate the level of fundamentals, i.e. by employing dynamic hat algebra.

### 3.1 Solving the Model

We seek to use our model to perform various counterfactual experiments; i.e., to study the general equilibrium implications of a change in fundamentals relative to the fundamentals of a baseline economy. We now define formally the baseline economy.

Definition 4 The baseline economy is the allocation $\left\{L_{t}, \mu_{t-1}, \pi_{t}, X_{t}\right\}_{t=0}^{\infty}$ corresponding to the sequence of fundamentals $\left\{\Theta_{t}\right\}_{t=0}^{\infty}$.

We now show how to solve for the baseline economy in time differences. To ease the exposition we denote by $\dot{y}_{t+1} \equiv\left(y_{t+1}^{1} / y_{t}^{1}, y_{t+1}^{2} / y_{t}^{2}, \ldots\right)$ to the proportional change in any scalar or vector between periods $t$ and $t+1$. We start by showing how to solve for a temporary equilibrium of the baseline
economy at $t+1$, after a change in employment, $\dot{L}_{t+1}$, and fundamentals $\dot{\Theta}_{t+1}$, without needing estimates of $\Theta_{t}$.

Proposition 1 Given the allocation of the temporary equilibrium at $t$, $\left\{L_{t}, \pi_{t}, X_{t}\right\}$, the solution to the temporary equilibrium at $t+1$ for a given change in $\dot{L}_{t+1}$ and $\dot{\Theta}_{t+1}$ does not require information on the level of fundamentals at $t, \Theta_{t}$. In particular, it is obtained as the solution to the following system of non-linear equations:

$$
\begin{gather*}
\dot{x}_{t+1}^{n j}=\left(\dot{L}_{t+1}^{n j}\right)^{\gamma^{n j} \xi^{n}}\left(\dot{w}_{t+1}^{n j}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(\dot{P}_{t+1}^{n k}\right)^{\gamma^{n j, n k}},  \tag{11}\\
\dot{P}_{t+1}^{n j}=\left(\sum_{i=1}^{N} \pi_{t}^{n j, i j}\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j}} \gamma^{i j}\right)^{-1 / \theta^{j}},  \tag{12}\\
\pi_{t+1}^{n j, i j}=\pi_{t}^{n j, i j}\left(\frac{\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}}{\dot{P}_{t+1}^{n j}}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}},  \tag{13}\\
X_{t+1}^{n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{i k, n k} X_{t+1}^{i k}+\alpha^{j}\left(\sum_{k=1}^{J} \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k} w_{t}^{n k} L_{t}^{n k}+\iota^{n} \chi_{t+1}\right),  \tag{14}\\
\dot{w}_{t+1}^{n j} \dot{L}_{t+1}^{n j} w_{t}^{n j} L_{t}^{n j}=\gamma^{n j}\left(1-\xi^{n}\right) \sum_{i=1}^{N} \pi_{t+1}^{i j, n j} X_{t+1}^{i j}, \tag{15}
\end{gather*}
$$

where $\chi_{t+1}=\sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{i}} \dot{w}_{t+1}^{i k} \dot{L}_{t+1}^{i k} w_{t}^{i k} L_{t}^{i k}$.

Proposition 1 shows that given an allocation at time $t$ one can solve for the change in the temporary equilibrium as a consequence of a change in labor supply $\dot{L}_{t+1}$ and fundamentals $\dot{\Theta}_{t+1}$, without requiring information on the levels of fundamentals at time $t$. Note that Proposition 1 does not impose any restrictions on $\dot{\Theta}_{t+1}$. In particular, Proposition 1 says that for any changes in fundamentals (one by one or jointly) across time and space, one can solve for the change in real wages resulting from $\dot{\Theta}_{t+1}$.

Building on this last result, we can now characterize the solution of the dynamic model. The next proposition shows that, given an allocation at $t=0,\left\{L_{0}, \pi_{0}, X_{0}\right\}$, the matrix of gross migration flows at $t=-1, \mu_{-1}$, and a sequence of change in fundamentals, one can solve for the sequential equilibrium in time differences without needing to estimate the level of fundamentals. This result requires that the sequence of changes in fundamentals converges to one over time as the economy approaches the stationary equilibrium. Formally,

Definition 5 A converging sequence of changes in fundamentals is such that $\lim _{t \rightarrow \infty} \dot{\Theta}_{t}=1$.

To ease exposition, we denote by $u_{t}^{n j} \equiv \exp \left(V_{t}^{n j}\right)$. Moreover, we denote by $\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)$ (for all $n$ and $j$ ) the equilibrium real wages in time differences as functions of the change in labor $\dot{L}_{t+1}$ and time varying fundamentals $\dot{\Theta}_{t+1}$. Namely, $\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)$ is the solution to the system in Proposition 1.

Proposition 2 Conditional on an initial allocation of the economy, $\left(L_{0}, \pi_{0}, X_{0}, \mu_{-1}\right)$, given an anticipated convergent sequence of changes in fundamentals, $\left\{\dot{\Theta}_{t}\right\}_{t=1}^{\infty}$, the solution to the sequential equilibrium in time differences does not require information on the level of the fundamentals, $\left\{\Theta_{t}\right\}_{t=0}^{\infty}$ and solves the following system of non-linear equations:

$$
\begin{gather*}
\mu_{t+1}^{n j, i k}=\frac{\mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{n j, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}},  \tag{16}\\
\dot{u}_{t+1}^{n j}=\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}\right)^{\nu},  \tag{17}\\
L_{t+1}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{i k, n j} L_{t}^{i k} \tag{18}
\end{gather*}
$$

for all $j, n, i$ and $k$ at each $t$, where $\left\{\dot{\omega}^{n j}\left(\dot{L}_{t}, \dot{\Theta}_{t}\right)\right\}_{n=1, j=0, t=1}^{N, J, \infty}$ is the solution to the temporary equilibrium given $\left\{\dot{L}_{t}, \dot{\Theta}_{t}\right\}_{t=1}^{\infty}$.

Proposition 2 is one of our key results. It shows that by taking time differences we can solve the model for a given sequence of changes in fundamentals using data for the initial period (i.e., the initial value of the migration shares and the initial distribution of households across labor markets) without knowing the levels of fundamentals. For instance, suppose we want to solve the model with constant fundamentals. In this case, the set of fundamentals is given by $\Theta_{t} \equiv$ $\left(A_{t}, \kappa_{t}, \Upsilon, H, b\right)$ and in time differences is given by $\dot{\Theta}_{t} \equiv(1,1,1,1,1)$, and therefore, by computing the model in time differences we do not need to identify any fundamental of the economy. Of course, Proposition 2 can also be applied to compute the model with any sequence of fundamentals.

To gain intuition about how Proposition 2 works, consider the following example. Take migration shares (3) at time $t-1$. As we can see from (3) given $\beta$ and $\nu$, there are infinite combinations of values $V_{t}^{i k}$ and migration costs $\tau^{n j, i k}$ that can reconcile a given migration flow. So, in principle, there is no way we can uniquely solve for $V_{t}^{i k}$ without information on $\tau^{n j, i k}$. However, consider migration flows for the same market at time $t$ and take the relative time difference (3) between
time $t$ and $t-1$; namely,

$$
\frac{\mu_{t}^{n j, i k}}{\mu_{t-1}^{n j, i k}}=\frac{\exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu} / \exp \left(\beta V_{t}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \frac{\exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}{\sum_{m^{\prime}=1}^{N} \sum_{h^{\prime}=0}^{J} \exp \left(\beta V_{t}^{m^{\prime} h^{\prime}}-\tau^{n j, m^{\prime} h^{\prime}}\right)^{1 / \nu}}}
$$

Given the properties of the exponential function, the numerator of this last expression simplifies to $\exp \left(V_{t+1}^{i k}-V_{t}^{i k}\right)^{\beta / \nu}=\left(\dot{u}_{t+1}^{n j}\right)^{\beta / \nu}$. Now multiply and divide each element of the sum in the denominator by $\exp \left(\beta V_{t}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}$ and use migration flows at time $t-1$ to obtain (16). ${ }^{24}$ The procedure to derive equation (17) is similar and results from taking time differences between equation (2) expressed at time $t+1$ and at time $t$ (see Appendix 2). ${ }^{25}$

A couple of observations are noteworthy about the system of equilibrium conditions $(16),(17)$, and (18) in time differences. First, at the steady state $\left\{\dot{u}_{t}^{i k}\right\}_{i=1, j=0}^{N, J}=1$ for all $t$ regardless of the level of the fundamentals. This is an advantage since it simplifies considerably the computations of the model given that there is no need to solve for the steady state value functions. Second, we can use this system of equations conditioning on observables $\left(L_{0}, \pi_{0}, X_{0}, \mu_{-1}\right)$ and solve for the equilibrium even if the economy is not initially in a steady state. To see this in a simple way consider an economy with constant fundamentals $\left\{\dot{\Theta}_{t}\right\}_{t=1}^{\infty}=1$, let $\mu^{*}$ be the steady-state migration flow, and $L^{*}$ the steady-state employment distribution. Now suppose that $\mu_{-1}=\mu^{*}, L_{0}=L^{*}$, and $\left\{\dot{u}_{1}^{i k}\right\}_{i=1, j=0}^{N, J}=1$. From (16) note that since $\dot{u}_{1}^{i k}=1$, then $\mu_{0}=\mu_{-1}=\mu^{*}$. Then from (18) this implies that $L_{1}=L_{0}=L^{*}$ since $\mu^{*}$ is the steady-state migration flow; hence, $\dot{\omega}^{n j}(1,1)=1$. Finally, given that $\left\{\dot{u}_{1}^{i k}\right\}_{i=1, j=0}^{N, J}=1$, then only $\left\{\dot{u}_{2}^{i k}\right\}_{i=1, j=0}^{N, J}=1$ solves (17). Now condition on observed data $L_{0}$ and $\mu_{-1}$. If $L_{0}$, and $\mu_{-1}$ were at the steady state, then initiating the system at $\left\{\dot{u}_{1}^{i k}\right\}_{i=1, j=0}^{N, J}=1$ should solve the system of equations. However, if $L_{0}$ is not the steady-state distribution of labor of the economy, then after applying $\mu_{-1}$ to $L_{0}$ we will obtain $\dot{L}_{1} \neq 1$ and as a result $\dot{\omega}^{n j}\left(\dot{L}_{1}, 1\right) \neq 1$ and then $\left\{\dot{u}_{2}^{i k}\right\}_{i=1, j=0}^{N, J} \neq 1$ from (17). We use these observations to construct an algorithm that solves for the competitive equilibrium of the economy. In Appendix 4, Part I, we present the algorithm. ${ }^{26}$

[^11]
### 3.2 Solving for Counterfactuals

So far we have shown that we can take our model to the data and solve for the sequential competitive equilibrium of the economy. This might be interesting by itself; however, we also want to use the model to conduct counterfactuals. By counterfactuals we refer to the study of how allocations change across space and time, relative to a baseline economy, given a new sequence of fundamentals; which we denote by $\Theta^{\prime}=\left\{\Theta_{t}^{\prime}\right\}_{t=1}^{\infty}$.

From Proposition 2 we can solve for a baseline economy without knowing the level of fundamentals. Given this, we can then study the effects of a change in fundamentals from $\left\{\Theta_{t}\right\}_{t=1}^{\infty}$ to $\left\{\Theta_{t}^{\prime}\right\}_{t=1}^{\infty}$ (where $\left\{\Theta_{t}\right\}_{t=1}^{\infty}$ is the sequence of fundamentals of a baseline economy, and $\left\{\Theta_{t}^{\prime}\right\}_{t=1}^{\infty}$ is the sequence of counterfactual fundamentals), without explicitly knowing the level of $\Theta_{t}$. Of course, as in any dynamic model, when solving for the baseline economy, as well as for counterfactuals, we need to make an assumption of how agents anticipate the evolution of the fundamentals of the economy. For example, we can assume that the change in fundamentals is anticipated (or not) by agents at time 0 . Consistent with our perfect foresight assumption, we follow the convention that at the beginning of the period in the baseline economy agents anticipate the entire evolution of fundamentals. ${ }^{27}$ Then, to compute counterfactuals, we assume that agents at $t=0$ are not anticipating the change in the path of fundamentals and that at $t=1$ agents learn about the entire future counterfactual sequence of $\left\{\Theta_{t}^{\prime}\right\}_{t=1}^{\infty}$. This timing assumption allows us to use information about agents' actions before $t=1$ to solve for the sequential equilibrium, under the new fundamentals, in relative time differences.

The next proposition, defines how to solve for counterfactuals from unexpected changes in fundamentals. It shows that conditioning on the allocation of the baseline economy $\left\{L_{t}, \mu_{t-1}, \pi_{t}, X_{t}\right\}_{t=0}^{\infty}$, we can solve for counterfactuals without information on $\left\{\Theta_{t}\right\}_{t=0}^{\infty}$.

First, we introduce new notation. Let $\hat{y}_{t+1} \equiv \dot{y}_{t+1}^{\prime} / \dot{y}_{t+1}$ be the relative change in time between the counterfactual equilibrium, $\dot{y}_{t+1}^{\prime} \equiv y_{t+1}^{\prime} / y_{t}^{\prime}$, and the initial equilibrium, $\dot{y}_{t+1} \equiv y_{t+1} / y_{t}$. For instance, using this notation, $\hat{\Theta}_{t+1}$ refers to the counterfactual changes in fundamentals over time relative to the baseline economy, namely $\hat{\Theta}_{t+1}=\dot{\Theta}_{t+1}^{\prime} / \dot{\Theta}_{t+1}$. Note that $\hat{\Theta}_{t+1}=1$ does not mean that fundamentals are not changing, it means that fundamentals are changing in the same way as

[^12]in the baseline economy, namely $\Theta_{t+1}^{\prime} / \Theta_{t}^{\prime}=\Theta_{t+1} / \Theta_{t}$.

Proposition 3 Given a baseline economy, $\left\{L_{t}, \mu_{t-1}, \pi_{t}, X_{t}\right\}_{t=0}^{\infty}$, and a counterfactual convergent sequence of changes in fundamentals (relative to the baseline change), $\left\{\hat{\Theta}_{t}\right\}_{t=1}^{\infty}$, solving for the counterfactual sequential equilibrium $\left\{L_{t}^{\prime}, \mu_{t-1}^{\prime}, \pi_{t}^{\prime}, X_{t}^{\prime}\right\}_{t=1}^{\infty}$ does not require information on the baseline fundamentals $\left(\left\{\Theta_{1 t}\right\}_{t=0}^{\infty}, \Theta_{2}\right)$, and solves the following system of non-linear equations:

$$
\begin{gather*}
\mu_{t}^{\prime n j, i k}=\frac{\mu_{t-1}^{\prime n j, i k} \dot{\mu}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{\prime n j, m h} \dot{\mu}_{t}^{n j, m h}\left(\hat{u}_{t+1}^{m h}\right)^{\beta / \nu}},  \tag{19}\\
\hat{u}_{t}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t-1}^{\prime n j, i k} \dot{\mu}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}\right)^{\nu},  \tag{20}\\
L_{t+1}^{\prime n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{i k, n j} L_{t}^{\prime i k}, \tag{21}
\end{gather*}
$$

for all $j, n, i$ and $k$ at each $t$, where $\left\{\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)\right\}_{n=1, j=0, t=1}^{N, J, \infty}$ is the solution to the temporary equilibrium given $\left\{\hat{L}_{t}, \hat{\Theta}_{t}\right\}_{t=1}^{\infty}$, namely at each $t$, given $\left(\hat{L}_{t}, \hat{\Theta}_{t}\right), \hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)=\hat{w}_{t}^{n j} / \hat{P}_{t}^{n}$ solves,

$$
\begin{gather*}
\hat{x}_{t+1}^{n j}=\left(\hat{L}_{t+1}^{n j}\right)^{\gamma^{n j} \xi^{n}}\left(\hat{w}_{t+1}^{n j}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(\hat{P}_{t+1}^{n k}\right)^{\gamma^{n j, n k}},  \tag{22}\\
\hat{P}_{t+1}^{n j}=\left(\sum_{i=1}^{N} \pi_{t}^{\prime n j, i j} \dot{t}_{t+1}^{n j, i j}\left(\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}},  \tag{23}\\
\pi_{t+1}^{\prime n j, i j}=\pi_{t}^{\prime n j, i j} \dot{\pi}_{t+1}^{n j, i j}\left(\frac{\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}}{\hat{P}_{t+1}^{n j}}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j}} \gamma^{i j}  \tag{24}\\
X_{t+1}^{\prime n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{\prime i k, n k} X_{t+1}^{\prime i k}+\alpha^{j}\left(\sum_{k=1}^{J} \hat{w}_{t+1}^{n k} \hat{L}_{t+1}^{n k} w_{t}^{\prime n k} L_{t}^{\prime n k} \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k}+\iota^{n} \chi_{t+1}^{\prime}\right), \\
\hat{w}_{t+1}^{n k} \hat{L}_{t+1}^{n k}=\frac{\gamma^{n j}\left(1-\xi^{n}\right)}{w_{t}^{\prime n k} L_{t}^{\prime n k} w_{t+1}^{n k} \dot{L}_{t+1}^{n k}} \sum_{i=1}^{N} \pi_{t+1}^{i j, n j} X_{t+1}^{\prime i j}, \tag{25}
\end{gather*}
$$

where $\chi_{t+1}^{\prime}=\sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{i}} \hat{w}_{t+1}^{i k} \hat{L}_{t+1}^{i k} w_{t}^{i k} L_{t}^{i k} \dot{w}_{t}^{i k} \dot{L}_{t}^{i k}$.
Proposition 3 is another of our key results. It shows that we can compute counterfactuals from unanticipated changes to the baseline economy's fundamentals without knowing the levels or changes in fundamentals of the baseline economy. The baseline economy can contain either timevarying or constant fundamentals. For instance, if the baseline economy contains the factual changes in fundamentals the sequence of $\left\{\dot{L}_{t+1}, \dot{\mu}_{t}, \dot{\pi}_{t+1}, \dot{X}_{t+1}\right\}_{t=0}^{\infty}$ is the data; while if the baseline economy contains constant fundamentals the sequence of $\left\{\dot{L}_{t+1}, \dot{\mu}_{t}, \dot{\pi}_{t+1}, \dot{X}_{t+1}\right\}_{t=0}^{\infty}$ is computed using the
results from Proposition 2. In any case, by computing the model in relative time differences we do not need to identify any fundamentals of the baseline economy. As before, the proof of Proposition 3 is presented in Appendix 2. In Appendix 4, Algorithm Part II is the one we use to solve for counterfactuals - namely, for changes in fundamentals relative to the baseline.

It is worth emphasizing again that our solution method allows us to study the effects of changes in any element contained in the set $\Theta$, without having to estimate the entire set. This method has two main advantages. First, by conditioning on observed allocations at a given moment in time, one disciplines the model by making it match all cross-sectional moments in the data. Second, after conditioning on data, one can use the model to solve for counterfactuals without backing out the fundamentals of the economy. If the goal is to study the effects of a change in fundamentals relative to an economy with constant fundamentals, Proposition 2 shows that solving for the baseline economy with constant fundamentals requires cross-sectional data at the initial period of analysis. If instead the goal is to study the effects of a change in fundamentals relative to an economy with actual changes in fundamentals, Proposition 3 shows that we require cross-sectional data for the entire period of analysis. Ultimately, the choice between conducting counterfactuals with constant or time-varying fundamentals will depend on the question being asked and the data availability.

We now move to the empirical section of our paper where we use our model and apply the solution method. We first describe how to take the model to the data. After this, we evaluate the effects of the China shock with constant fundamentals. Later, in Section 5.3.2 we evaluate the effects of the shock with time-varying fundamentals, where the baseline economy is constructed using time-series data over the period 2000-2007 and then applying Proposition 2, assuming constant fundamentals from 2007 on.

## 4. TAKING THE MODEL TO THE DATA

Applying the solution method requires initial values of bilateral trade flows $\pi_{0}^{n j, i j}$, value added $w_{0}^{n j} L_{0}^{n j}+r_{0}^{n j} H_{0}^{n j}$, the distribution of employment $L_{0}$, and the initial period migration flows across regions and sectors, $\mu_{-1}$. We take the year 2000 as our initial period and match the model variables to the values observed in the data for that year. We also need to compute the share of value added in gross output $\gamma^{n j}$, the material shares $\gamma^{n j, n k}$, the share of structures in value added $\xi^{n}$, the final consumption shares $\alpha^{j}$, and the global portfolio shares $\iota^{n}$. Finally, we need estimates of the sectoral trade elasticities $\theta^{j}$, the migration elasticity $1 / \nu$, and the discount factor $\beta$. This section provides a summary of the data sources and measurements to calibrate the model, with further details
provided in Appendix 5.

Regions, sectors, and labor markets. We calibrate the model to the 50 U.S. states; 37 other countries, including China, and a constructed rest of the world. We consider 22 sectors, classified according to the North American Industry Classification System (NAICS). Of these 22, 12 are manufacturing sectors, 8 are service sectors, and we also include construction and a combined wholesale and retail trade. ${ }^{28}$ Our definition of a labor market in the U.S. economy is thus a statesector pair, including non-employment, leading to 1150 markets. For other countries, we assume a single labor market, but with the same set of productive sectors.

Trade and production data. We construct the bilateral trade shares $\pi_{0}^{n j, i j}$ for the year 2000 for the 38 countries in our sample, including the aggregate United States, from the World InputOutput Database (WIOD). We discipline the different uses in the data as follows. The WIOD has information on trade flows across countries as well as data on input-output linkages (purchases of materials across sectors). The bilateral trade flows in the model include both traded goods for use as intermediates, and traded goods for final consumption, and therefore, they match all bilateral trade flows in the WIOD. The sectoral bilateral trade flows between the 50 U.S. states were constructed by combining information from the WIOD database and the 2002 Commodity Flow Survey (CFS), which is the closest available year to 2000 . From the WIOD database we compute the total U.S. domestic sales for the year 2000 for our 22 sectors. From the 2002 CFS we compute the bilateral expenditure shares across regions and sectors. These two pieces of information allow us to construct the bilateral trade flows matrix for the 50 U.S. states across sectors, where the total U.S. domestic sales match the WIOD data for the year 2000 .

Bilateral trade flows between the 50 U.S. states and the rest of the countries in the world were constructed by combining information from the WIOD database and regional employment data from the Bureau of Economic Analysis (BEA). In our model, local labor markets have different exposures to international trade shocks because there is substantial geographic variation in industry specialization. Regions with a high concentration of production in a given industry should react more to international trade shocks hitting that industry. Therefore, following ADH, our measure for the exposure of local labor markets to international trade combines trade data with local industry employment. Specifically, we split the bilateral trade flows at the country level computed from WIOD into bilateral trade flows between the U.S. states and other countries by assuming that the

[^13]share of each state in total U.S. trade with any country in the world in each sector is determined by the regional share of total employment in that industry.

To construct the share of value added in gross output $\gamma^{n j}$, the material input shares $\gamma^{n j, n k}$, and the share of structure in value added $\xi^{n}$, we use data on gross output, value added, intermediate consumption, and labor compensation across sectors from the BEA for the U.S. states and from the WIOD for all other countries in our sample.

Finally, using the constructed trade and production data, we compute the final consumption shares $\alpha^{j}$, as described in Appendix 5; and we discipline the portfolio shares $\iota^{n}$ to match exactly the year 2000 observed trade imbalances.

The initial migration flow matrix and the initial distribution of labor. The initial distribution of workers in the year 2000 by U.S. states and sectors (and non-employment) is obtained from the 5 percent Public Use Microdata Sample (PUMS) of the decennial U.S. Census for the year 2000. Information on industry is classified according to the NAICS, which we aggregate to our 22 sectors and non-employment. ${ }^{29}$ We restrict the sample to people between 25 and 65 years of age who are either non-employed or employed in one of the sectors included in the analysis. Our sample contains almost 7 million observations.

Table 1: U.S. interstate and intersectoral labor mobility

| Probability | p25 | p50 | p75 |
| :--- | :---: | :---: | :---: |
| Changing sector but not state | $3.58 \%$ | $5.44 \%$ | $7.93 \%$ |
| Changing state but not sector | $0.04 \%$ | $0.42 \%$ | $0.73 \%$ |
| Changing state and sector | $0.02 \%$ | $0.03 \%$ | $0.05 \%$ |
| Staying in the same state and sector | $91.4 \%$ | $93.9 \%$ | $95.8 \%$ |

Note: Quarterly transitions. Data sources: ACS and CPS.

In our application we abstract from international migration. ${ }^{30}$ That is, we impose that $\tau^{n j, i k}=\infty$ for all $j, k$ such that regions $n$ and $i$ belong to different countries. Given this assumption, we need to measure the initial matrix of gross flows only for the U.S. economy. To construct the initial matrix of quarterly mobility across our regions and sectors $\left(\mu_{-1}\right)$, we combine information from the Current Population Survey (CPS) to compute intersectoral mobility and from the PUMS of the American Community Survey (ACS) to compute interstate mobility. Table A5.1 in Appendix

[^14]5 shows the information provided by these two datasets in terms of transition probabilities. ${ }^{31}$
Table 1 shows some moments of worker mobility across labor markets computed from our estimated transition matrix for the year 2000. Our numbers are consistent with the estimates by Molloy et al. (2011) and Kaplan and Schulhofer-Wohl (2012) for interstate moves and Kambourov and Manovskii (2008) for intersectoral mobility. ${ }^{32}$

One important observation from Table 1 is the large amount of heterogeneity in transition probabilities across labor markets, which indicates that workers in some industries and states are more likely to switch to a different labor market than other workers. In particular, the 25th and 75 th percentiles of the distribution of sectoral mobility probabilities by labor market are $40 \%$ lower and higher than the median, respectively. This dispersion is even larger for interstate moves. We interpret the observed low transition probabilities and their heterogeneity as evidence of substantial and heterogeneous costs of moving across labor markets, both spatially and sectorally.

Elasticities. We take a period in our model to correspond to one quarter, and therefore we calibrate the quarterly discount factor $\beta$ to 0.99 , implying a yearly interest rate of roughly $4 \%$. The sectoral trade elasticities $\theta^{j}$ are obtained from Caliendo and Parro (2015). We calibrate the migration elasticity, $1 / \nu$, by adapting the method and data used in ACM. From their model, they derive an estimating equation that relates current migration flows to future wages and future migration flows. Then, they estimate the equation by GMM and instrument using past values of flows and wages. ${ }^{33}$

In order to adapt ACM's procedure to our model and frequency, we have to deal with two issues. First, in our model agents have log utility while in ACM preferences are linear; and second, ACM estimate an annual elasticity while we are interested in a quarterly elasticity. Dealing with the first issue is not that difficult since from our model we obtain the analogous estimating equation

[^15]to ACM's preferred specification but with log utility, namely,
\[

$$
\begin{equation*}
\log \left(\mu_{t}^{n j, n k} / \mu_{t}^{n j, n j}\right)=\tilde{C}+\frac{\beta}{\nu} \log \left(w_{t+1}^{n k} / w_{t+1}^{n j}\right)+\beta \log \left(\mu_{t+1}^{n j, n k} / \mu_{t+1}^{n k, n k}\right)+\varpi_{t+1}, \tag{27}
\end{equation*}
$$

\]

where $\varpi_{t+1}$ is a random term, and $\tilde{C}$ is a constant. The relevant coefficient $\beta / \nu$ represents the elasticity of migration flows to changes in income, while in ACM it has the interpretation of a semi-elasticity. As pointed out by ACM, the disturbance term, $\varpi_{t+1}$, will in general be correlated with the regressors; thus, we require instrumental variables. As in ACM, our theory implies that past values of sectoral migration flows and wages are valid instruments; therefore, we use lagged flows and wages as instruments for the wage variable in (27). ${ }^{34}$

Dealing with the second issue is more involved. As ACM discuss, Kambourov and Manovskii (2013) point out a difficulty in interpreting flow rates that come out of the March CPS retrospective questions. They conclude that although superficially it appears to be annual, the mobility measured by the March CPS is less than annual. ACM correct for this bias, and conclude that the March CPS measures mobility at a five-month horizon. Then, they annualize the migration flow matrix by assuming that within a year the monthly flow rate matrix is constant. We transform the five-month migration flow matrices in ACM to quarterly matrices using the same procedure ACM but adapted to convert to quarterly flows.

After dealing with these two issues, we obtain a migration-elasticity of 0.2 , implying a value of $\nu=5.34$. This is our preferred estimate and we use this number in our empirical section below. To the best of our knowledge, there is no benchmark value for this quarterly elasticity in the literature. Yet, to put it in perspective, our estimate is consistent with the intuition that this elasticity should be smaller, thus $\nu$ larger, at higher frequencies. In fact, the implied annual inverse elasticity in our model is 2.02 at an annual frequency, and a larger value of 3.95 at a five-month frequency. ${ }^{35}$

[^16]
### 4.1 Identifying the China Trade Shock

In previous work, ADH and Acemoglu et al. (2014) argue that the increase in U.S. imports from China had asymmetric impacts across regions and sectors. In particular, labor markets with greater exposure to the increase in import competition from China saw a larger decrease in manufacturing employment. Given that the observed changes in U.S. imports from China are not necessarily the result of an exogenous shock to China (TFP or trade costs), we replicate the procedure of ADH to identify the supply-driven components of Chinese imports. To do so, we compute the predicted changes in U.S. imports from China using the change in imports from China by other advanced economies as an instrument. This procedure is related to the first-stage regression of the two-stage least squares estimation in ADH conducted under our definition of labor markets, that is, at our regional and sectoral disaggregation. ${ }^{36}$

We estimate the following regression

$$
\Delta M_{U S A, j}=a_{1}+a_{2} \Delta M_{\text {other }, j}+u_{j},
$$

where here $j$ is one of our 12 manufacturing sectors and $\Delta M_{U S A, j}$ is the change in U.S. imports from China, and $\Delta M_{o t h e r, j}$ is the change in imports from China by other advanced economies between 2000 and $2007 .{ }^{37}$

We then use the predicted changes in U.S. imports according to this regression to calibrate the size of the TFP changes for each of the manufacturing sectors in China that will deliver the same change in imports in the model as the predicted change in the data. ${ }^{38}$

In Appendix 6.1, Figure A6.2 shows the predicted change in U.S. manufacturing imports from China computed as in ADH and the implied sectoral productivity changes in China. Computers

[^17]and electronics is the sector most exposed to import competition from China, accounting for about $40 \%$ of the predicted total change in U.S. imports from China, followed by the textiles and furniture industries with about $12 \%$ each, and metal and machinery with $10 \%$ of the total import penetration growth each. On the other hand, the food, beverage, and tobacco industry, and the petroleum industry are the ones least exposed, accounting for less than $1.5 \%$ of the predicted total change in U.S. imports from China. ${ }^{39}$

## 5. THE EFFECTS OF THE CHINA TRADE SHOCK

In this section, we quantify the dynamic effects of China's import competition on the U.S. economy. We first compute the dynamic model, holding productivities in China constant, which is our baseline economy. We do this using the results from Proposition 2, assuming that agents foresee constant fundamentals over time. We then use the results from Proposition 3, solving for the changes in equilibrium allocations due to the China shock. We first discuss the effects on aggregate, sectoral, and regional employment in Section 5.1 and then analyze the effects on welfare across markets in Section 5.2. Section 5.3 then discusses the employment and welfare effects from the China shock when we allow for actual changes in fundamentals.

### 5.1 Employment Effects

Starting with sectoral employment, the upper-left panel in Figure 1 presents the dynamic response of the manufacturing share of employment both with and without the China shock. As the figure shows, there are transitional dynamics toward a steady-state equilibrium even in the absence of any change in Chinese productivity. These dynamics occur because the economy is not in a steady state in the year 2000. In other words, the observed employment in manufacturing in 2000 is the equilibrium result of a series of shocks and structural changes that hit the economy before that year; and, as a result, the economy is transitioning to a new steady state. For instance, U.S. manufacturing employment has experienced a secular decline over the past several decades, and in 2000 the economy was still adjusting to this structural change. Thus, we observe a decline in manufacturing employment even in the absence of productivity changes in China. ${ }^{40}$ The implication

[^18]of this observation is that calibrating the model under the assumption that the economy is in steady state would overestimate the impact of the increased import competition from China since part of the observed decline in manufacturing employment is not related to Chinese competition.

Fig. 1: The Evolution of Employment Shares


Note: The figure presents the evolution of employment in each sector (manufacturing, services, wholesale and retail and construction) over total employment. Total employment excludes farming, utilities, and the public sector. The dashed lines represent the shares from the baseline economy with no changes in fundamentals, what we denote by "No China-Shock", while the lines represent the shares from the economy with the China shock.

The upper-left panel in Figure 1 shows the transitional dynamics of manufacturing employment both with and without the China shock. The difference between the two is our account of the effect of China's import penetration growth on U.S. manufacturing employment. The figure shows that import competition from China contributed to a substantial decline in the share of manufacturing employment, a result that is in line with ADH . Our results indicate that increased competition from China reduced the share of manufacturing employment by 0.5 percentage point after 10 years, which is equivalent to about 0.8 million jobs or about $50 \%$ of the change in manufacturing employment that is not explained by a secular trend. ${ }^{41}$

[^19]As shown in the other three panels of Figure 1, increased import competition from China leads workers to relocate to other sectors; thus, the share of employment in services, wholesale and retail, and construction increases. We also find that Chinese competition reduced the U.S. nonemployment rate by 0.25 percentage point in the long run. The role of intermediate inputs and sectoral linkages is crucial to understanding these relocation effects. Import competition from China leads to decreased production among U.S. manufacturing sectors that compete with China, but it also affords the U.S. economy access to cheaper intermediate goods from China that are used as inputs in non-manufacturing sectors. Production and employment increase in the nonmanufacturing sectors as a result. Moreover, the increase in employment in these sectors more than offsets the decline in manufacturing employment so that the non-employment rate declines. In more isolated states such as Alaska, however, the non-employment rate increases, due to mobility frictions and because other sectors are not large enough to absorb all workers displaced from the manufacturing sector across different locations. Finally, employment in construction declines a bit in the short run after the China shock, which is explained, as mentioned earlier, by the fact that the economy was transitioning to a steady state when the change in Chinese productivity hit the U.S. economy. In the long run, we find that about 75 thousand jobs were created in construction as a result of the China shock. ${ }^{42}$

Our quantitative framework also allows us to further explore the decline in manufacturing employment caused by the China shock. In particular, we quantify the relative contribution of different sectors, regions, and local labor markets to the decline in the manufacturing share of employment.

Figure 2 shows the contribution of each manufacturing industry to the total decline in the manufacturing sector employment. Industries with higher exposure to import competition from China lost more employment. The computer and electronics and furniture industries contributed to about half of the decline in manufacturing employment, followed by the metal and textiles industries, which together contributed to about one-fourth of the total decline. Industries less exposed to import competition from China explain a smaller portion of the decline in manufacturing employment. In fact, these industries also benefit from access to cheaper intermediate goods from industries that

[^20]Fig. 2: Manufacturing employment declines (\% of total) due to the China trade shock


Note: The figure presents the contribution of each manufacturing industry to the total reduction in the manufacturing employment due to the China Shock.
experienced a substantial productivity increase in China. In some industries, such as food, beverage and tobacco, increased production from access to cheaper intermediate goods more than offset the negative effects of increased import competition, and employment increased as a result.

FIg. 3: Regional contribution to U.S. aggregate manufacturing employment decline (\%)


Note: The figure presents the contribution of each state to the total reduction of employment in the manufacturing sector due to the China shock.

The fact that the U.S. economic activity is not equally distributed across space, combined with its differential sectoral exposure to China, implies that the impact of import competition from China on manufacturing employment varies across regions.

Figure 3 presents the regional contribution to the total decline in manufacturing employment. States with a comparative advantage in industries more exposed to import competition from China lose more employment in manufacturing. For instance, California alone accounted for $20 \%$ of all employment in the computer and electronics industry in the year 2000. For comparison, the state with the next-largest share of employment in this industry is Texas with $8 \%$, while all other states had shares of employment in computer and electronics of less than $2 \%$. As a result, California is the state that contributed the most to the overall decline in manufacturing employment (about $12 \%$ ) followed by Texas. States with a comparative advantage in goods were less affected by import competition from China and states that benefited from the access to cheaper intermediate goods showed a smaller impact on employment.

Fig. 4: Regional contribution to U.S. agg. mfg. emp. decline normalized by regional emp. share


Note: The figure presents the contribution of each state to the U.S. aggregate reduction in the manufacturing sector employment, due to the China shock, normalized by the employment of each state relative to the U.S. aggregate employment.

While Figure 3 shows the spatial distribution of the aggregate decline in manufacturing employment, it is also informative to study the local impact in each region of the China shock. For instance, even when larger regions such as California are more exposed to the China shock because they concentrate a large fraction of U.S. employment in industries that have high exposure to foreign trade, larger regions also tend to be more diversified. That is, employment and production are also important in other sectors, such as services, with little direct exposure to trade. Therefore, although their contribution to the aggregate decline in manufacturing is large, the local impact
of the China shock could be mitigated compared with smaller and less diversified regions where manufacturing represents a higher share of local employment.

This local impact is shown in Figure 4, which displays the regional contribution to the total decline in manufacturing employment normalized by the employment share of the state in the U.S. economy. In the figure, a number greater than one means that the local change in manufacturing employment share is larger than the national change ( -0.5 percentage points). As we can see from this figure, the local impact in manufacturing employment in states like South Carolina and North Carolina was bigger than the impact for the whole U.S. economy. The figure also shows that in other bigger and more diversified states, such as California and Texas, the decline in manufacturing employment as a share of the state employment is similar to the aggregate U.S. decline in manufacturing employment share.

Fig. 5: Non-manufacturing employment increases (\% of total) due to the China trade shock


Note: The figure presents the contribution of each non-manufacturing sector to the total increase in the nonmanufacturing employment due to the China shock.

We now turn to the sectoral and spatial distribution of the employment gains in the non- manufacturing industries due to the China shock. The sectoral contribution to the change in nonmanufacturing employment is displayed in Figure 5. As we can see, all non-manufacturing industries absorbed workers displaced from manufacturing industries. In particular, besides the category other services, the health and education industries are the largest contributors among service industries, accounting for about 35 percent of the change in non-manufacturing employment share, followed by construction with a 10 percent contribution. Figure 6 shows that U.S. states with a larger service sector contribute more to the increase in non-manufacturing employment as they were able to absorb more workers displaced from the manufacturing industries. Specifically, New York is the largest contributor, accounting for about 9 percent of the total increase in non-manufacturing
employment, followed by California, which accounts for about 8 percent.
FIG. 6: Regional contribution to U.S. aggregate non-manufacturing employment increase (\%)


Note: The figure presents the contribution of each state to the total rise in the non-manufacturing employment due to the China shock.

Economic activity is unevenly distributed across space in the United States, and therefore, the sectoral employment effects in Figures 2 and 5 can mask different distributional effects across space in different industries. To study the regional employment effects from the China shock in different industries, Figures 7 and 8 present U.S. maps that show the changes in regional employment by industry. The first column of each figure presents the contribution of each region to the U.S. aggregate change in industry employment a consequence of the China shock (analogous to Figure 3). The second column presents, for each state, the contribution of each region to the U.S. aggregate change in industry employment normalized by the employment share of the state (analogous to Figure 4). Figure 7 presents the results for three selected manufacturing industries; furniture, machinery, and textiles, and Figure 8 presents the results for three selected non-manufacturing industries; construction, services, and wholesale and retail. In Appendix 7 we present the figures with the effects for all the other sectors.

From the figure we can see the unequal regional effects from the China shock in different industries. For instance, the decline in employment in furniture (Figure 7, panel a.1), an industry highly exposed to Chinese import competition, is concentrated in California while the decline in employment in machinery (Figure 7, panel b.1) is highly concentrated in the Midwestern states. Part of this concentration reflects that economic activity in these industries is mostly concentrated

Fig. 7: Regional employment declines in manufacturing industries

1. Contribution to industry employment decline in the U.S. (\%)
2. Normalized by regional employment share


Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the furniture mfg. industry. Panels b present the results for the machinery industry. Panels c present the results for the textiles industry.

Fig. 8: Regional employment increase in non-manufacturing industries
2. Normalized by regional employment share


Note: This figure presents the rise in local employment in non-manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate increase in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate increase in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the construction industry. Panels b present the results for all services industry. Panels c present the results for the whole. \& retail industry.
in these regions. After normalizing the contribution of each state by the employment share of the state in the U.S. economy, Figure 7, panels a.2, b. 2 and c.2, reveals the regions that had a larger local impact relative to the aggregate impact in the United States. For example, panel c. 2 shows that, as a consequence of the China shock, Alabama, Georgia, South Carolina and North Carolina experienced a reduction in the employment share in the textile industry that is more than twice as large as the reduction in the U.S. textile employment share. Panel b. 2 presents the case of the machinery industry, and we can see that even after controlling for size, the Midwestern states experienced the largest reduction in local employment share in the machinery industry relative to the national reduction.

Figure 8 presents the results for selected non-manufacturing industries. Recall from Figures 1 and 5 that non-manufacturing industries increased their employment share as a consequence of the China trade shock. We can see in Figure 8 panels a.1, b.1, and c.1, that, similar to the case of manufacturing industries, larger states such as California, New York, Texas and Florida are more important contributors to the overall change in employment. However, different from the manufacturing industries, after controlling for the relative size of the state, the local impact are much more evenly distributed across space. As a result, the reduction in local employment in manufacturing industries is more concentrated in a handful of states while the increase in local employment in non-manufacturing industries spread more evenly across U.S. states.

Finally notice that Figures 1, 2, 7, and 8 shed light on the contribution of each state/industry pair to the aggregate decline in manufacturing employment. For instance, in Figure 7 we have that California contributes 12.7 percent to the decline in employment in the furniture industry, while Figure 2 shows that the furniture industry contributes to about 27 percent to the decline in manufacturing employment. Given this, we have that the furniture industry in California accounts for about 3.5 percent of the total decline in manufacturing employment.

Overall, the contribution of each labor market to the total decline in manufacturing employment varies considerably across regions and industries. We find that most manufacturing labor markets lost jobs, although employment increased in some of them. The computer and electronics industry in California was the labor market that contributed the most to the decline in manufacturing employment, accounting for 4.1 percent of the total decline. Employment increased in labor markets such as food, beverage, and tobacco in Wisconsin, California, and Arkansas; and transportation equipment in New Hampshire, among others. Notice that even when California experienced a decline in manufacturing employment due to import competition from China, some labor markets
in California such as food, beverage and tobacco gained in employment, highlighting the importance of taking into account the spatial and sectoral distribution of economic activity. ${ }^{43}$

### 5.2 Welfare Effects

We now turn to the aggregate and disaggregate welfare effects of increased import competition from China on the U.S. economy. The change in welfare from a change in fundamentals $\hat{W}_{t}^{n j}$, measured in terms of consumption equivalent variation, can be expressed as

$$
\begin{equation*}
\hat{W}^{n j}=\sum_{s=1}^{\infty} \beta^{s} \log \left(\frac{\hat{C}_{s}^{n j}}{\left(\hat{\mu}_{s}^{n j, n j}\right)^{\nu}}\right) \tag{28}
\end{equation*}
$$

We compute the welfare effect of the China shock using equation (28), where $\hat{\Theta}$ incorporates the changes in TFP in the Chinese manufacturing sectors. ${ }^{44}$ In Appendix 1 we present the derivation of equation (28) and discuss the different mechanisms that shape the welfare effects of changes in fundamentals in our model in more detail.

We find that U.S. aggregate welfare increases by $0.35 \%$ due to China's import penetration growth. ${ }^{45}$ The aggregate change in welfare masks, however, an important heterogeneity in the welfare effects across different labor markets. Figure 9 presents a histogram with the changes in welfare across 1150 U.S. labor markets. An important takeaway from the figure is that there is a very heterogeneous response to the same aggregate shock across labor markets. Changes in welfare range from an increase of a 4.8 percent in plastics in New Mexico to a decrease of 1 percent in chemicals in Wyoming.

Welfare effects are more dispersed across labor markets that produce manufacturing goods than those that produce non-manufacturing goods, as manufacturing industries have different exposure to import competition from China. Also, all labor markets that produce service goods gain from the China shock, and welfare tends to be higher than for labor markets in the manufacturing sectors. Labor markets that produce non-manufacturing goods do not suffer the direct adverse effects of

[^21]Fig. 9: Welfare effects of the China Shock across labor markets


Note: The figure presents the change in welfare across all labor markets (central figure), for workers in manufacturing sectors (top right panel), and for workers in non-manufacturing sectors (bottom right panel) as a consequence of the China Shock. The largest and smallest 1 percentile are excluded in each figure. The percentage change in welfare is measured in terms of consumption equivalent variation.
increased competition from China and at the same time benefit from access to cheaper intermediate manufacturing inputs from China used in production in these industries. Similarly, labor markets located in states that trade more with the rest of the U.S. economy and purchase materials from sectors in which Chinese productivity increased more tend to have larger welfare gains because they benefit from access to cheaper inputs from China purchased from the rest of the U.S. economy. For instance, all labor markets located in California gain, even though California is highly exposed to China. The reason is that California benefits more than other states from the access to cheaper goods purchased from the rest of the U.S. economy and China. ${ }^{46}$

Migration costs are also important to understanding the differences between welfare effects of the China shock in the short run and in the long run. In the short run, migration costs prevent workers, in the labor markets most negatively affected by the China shock, from relocating to other industries. Therefore, real wages fall where labor market conditions worsen. In the long run, workers are able to relocate to industries or states with higher labor demand and real wages. As a result, we find that while in the long run only 1.5 percent of the labor markets experience welfare losses, real wages drop in about 45 percent of all labor markets when the China shock hits the U.S.

[^22]economy.
We also compute the welfare effects across countries. Figure 10 shows that all countries gain from the China shock, with some countries gaining more and others gaining less than the United States. Countries that are more open to trade, not only to China but to the world, such as Cyprus and Australia, experience bigger welfare gains, as they benefit from the access to cheaper intermediate goods from China as well as from purchasing cheaper goods from other countries that also benefit from purchasing cheaper intermediate goods from China.

Fig. 10: Welfare effects across countries


Note: The figure presents the change in welfare across countries in our sample from the effect of the China shock. The percentage change in welfare is measured as the percentage change in real consumption.

### 5.2.1 Adjustment Costs.-

Recent papers have highlighted the importance of the transitional dynamics for welfare evaluation; specifically, the fact that comparisons across steady-state equilibria can significantly overstate or understate welfare measures (i.e., Dix-Carneiro, 2014; Alessandria and Choi, 2014; Burstein and Melitz, 2011).

In order to provide a measure that accounts for the transition costs to the new steady state, we follow Dix-Carneiro (2014)'s measure of adjustment cost. Formally, we use

$$
A C^{n j}=\log \left(\frac{\hat{V}_{S S}^{n j}}{(1-\beta) \sum_{t=0}^{\infty} \beta^{t} \hat{V}_{t+1}^{n j}}\right),
$$

to measure the adjustment cost for market $n j$.

We find that transition costs burn $2.5 \%$ of the long-term aggregate welfare gains. ${ }^{47}$ However, the variation across individual labor markets is substantial. Figure 11 presents a histogram of the adjustment costs across individual labor markets.

Fig. 11: Adjustment costs


Note: The figure presents the transition costs across all labor markets (central figure), for workers in Manufacturing sectors (top right panel), and for workers in non-manufacturing sectors (bottom right panel) from the effects of the China shock. The largest and smallest 1 percentile are excluded in each figure.

The distribution has a long right tail, and several labor markets have adjustment costs substantially larger than the aggregate transition cost. We also find that some labor markets have negative adjustment costs as the welfare gains with transition dynamics overshoot the steady state. Similar to the welfare effects, adjustment costs in labor markets in the manufacturing sectors are more dispersed than in the non-manufacturing sectors, reflecting their varying exposure to import competition from China. Part of this heterogeneity in the adjustment costs across labor markets might capture human capital specificities that might vary across sectors ${ }^{48}$.

### 5.3 Additional Results

In this section, we discuss additional results of the China shock. In Section 5.3.1, we extend the model to study the effects of increases in non-employment benefits to help mitigate some of the negative effects from import competition from China. In Section 5.3.2, we quantify the welfare

[^23]and employment effects of the China shock allowing for actual changes in other fundamentals. Finally, in Section 5.3 .3 we extend the model to incorporate additional sources of persistence in the reallocation decisions of workers, and discuss the effects of the China shock in that alternative model.

### 5.3.1 Adding Disability Insurance to the Model.-

In this section, we extend the model to study the effects of increases in the generosity of the non-employment benefits that are aimed at alleviating the potential negative effects on workers in industries impacted by increased import competition. Specifically, we address the question: What would the impact of the China shock across U.S. markets have been if the government had increased the generosity of Social Security Disability Insurance (SSDI) at the same time?

To do this, we need to take a stand on how we introduce SSDI into the model. In particular, we need to model, a) the likelihood (or share) that a non-employed worker obtains benefits, b) how nonemployment benefits are redistributed to non-employed households across different labor markets, and c) how benefits are financed. In our model, we assume that all non-employed households are equally likely to obtain SSDI, that benefits vary across locations, and that benefits are financed locally. Specifically, we denote by $\delta$ the share of households that have access to SSDI and by $b_{t}^{1 n}$ the SSDI benefit that a household in $n$ obtains at period $t$. As any other household, the income coming from SSDI is spent on local goods. We assume that the fraction of non-employed that do not have access to SSDI, $1-\delta$, obtain consumption from home production $b^{2 n}$, as we assumed before. As a result, the instant utility of a representative non-employed household in region $n$ is given by

$$
\log b_{t}^{n}=\delta \log \left(b_{t}^{1 n} / P_{t}^{n}\right)+(1-\delta) \log b^{2 n}
$$

To close the model, we assume that there is a regional government in each region $n$ that finances the SSDI payments by levying taxes $\tau_{t}^{n}$ on the income of the owners of structures in that region such that it finances the full amount of the benefit. Revenues from taxes are then used to pay SSDI to the fraction of people receiving the insurance in that region. Therefore,

$$
\tau_{t}^{n}=\frac{b_{t}^{1 n} \delta L_{t}^{n 0}}{\iota^{n} \chi_{t}}
$$

where $L_{t}^{n 0}$ are the non-employed households in region $n$ at time $t$. The goods market clearing
condition then becomes

$$
X_{t}^{j n}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t}^{i k, n k} X_{t}^{i k}+\alpha^{j}\left(\sum_{k=1}^{J} w_{t}^{n k} L_{t}^{n k}+\delta b_{t}^{1 n} L_{t}^{n 0}+\iota^{n} \chi_{t}\left(1-\tau_{t}^{n}\right)\right) .
$$

As before, the equilibrium of this economy is defined by equations (5) to (10), and (2) to (4).
To perform counterfactual analysis that involves changes in the generosity of SSDI, $b_{t}^{1 n}$, we need to obtain a value for $\delta$. We define $\delta$ in the data as the fraction of non-employed workers between 18 and 64 years old receiving Social Security Income (SSI) and Disability Insurance (DI) in the year 2000. Using data on SSI and DI from the Social Security Administration for the year 2000 and data on non-employment by state for the year 2000 obtained from the American Community Survey we find that the fraction of non-employed workers receiving SSDI in the United States in the year 2000 is 17 percent, thus $\delta=0.17$.

Using this value, we perform counterfactual analysis to study the impact of changes in SSDI, $b_{t}^{1 n}$. To do so, we first compute the effect of the China shock in a model with constant SSDI, namely $b_{t}^{1 n}=b^{1 n}$ for all $t .^{49}$ We then solve for a counterfactual where we feed both the China shock and counterfactual changes in SSDI into the model, and the difference between both counterfactuals is what we interpret as the effect of changes in SSDI in the presence of the China shock. ${ }^{50}$

With constant SSDI, we find that the manufacturing employment share declines by 0.504 percent. This is similar to our result in Section 5, but we find that aggregate welfare increases more than in the model without SSDI as non-employed households experience an increase in real income coming from the decline in the price index. Specifically, we find aggregate welfare increases by 0.53 percent with constant SSDI. In other words, the presence of SSDI has minor implications on the changes in the allocations due to the exposure of Chinese import competition and it has a more important role in mitigating negative welfare effects in specific labor markets.

We then study the effect of an increase in SSDI in the United States to the level of other developed countries with more generous SSDI. In particular, we consider a gradual increase from 2000-2007 of the SSDI from 1.7 percent of GDP to the level in Europe of 2.7 percent of GDP. ${ }^{51}$ We find that a gradual increase in the generosity of SSDI contributes to an additional decline in

[^24]manufacturing employment share of 0.24 percent, that is, to about 360.5 thousand manufacturing jobs lost. Importantly, we find that the employment effects are larger in those sectors and regions that have high exposure to the China shock, and we also find an increase in the non-employment rate in the long run.

### 5.3.2 Effects of the China Shock with Time-Varying Fundamentals.-

In this section, we compute the employment and welfare effects of the China shock allowing for actual changes in fundamentals. Specifically, the counterfactual we conduct computes the effect of the China shock as the difference between a baseline economy where all fundamentals of the economy are changing as they do in the data (over the period from 2000-2007), and a counterfactual economy with the actual changes in fundamentals except for the productivities in China.

Fig. 12: The Evolution of Employment Shares


Note: The figure presents the evolution of employment in each sector (manufacturing, services, wholesale and retail and construction) over total employment. Total employment excludes farming, utilities, and the public sector. The dashed lines represent the shares from the baseline economy with time varying fundamentals, what we denote by "Actual", while the lines represent the shares from the economy without the China shock.

As described in Section 3, the dynamic hat algebra with time-varying fundamentals solves for the counterfactual equilibrium relative to the baseline economy that contains the actual changes in fundamentals as in the data, and therefore, requires to collect time series data on migration
flows and trade flows. We use the best available data to construct these time series over the period 2000-2007, and in Appendix 5 we describe in detail how we constructed these series. We decided to stop the data in 2007, the year before the global financial crisis started, and we assume constant fundamentals from our last data point on. We then compute a counterfactual economy where we keep China's productivity constant at its year 2000 level relative to the baseline economy with time-varying fundamentals.

Employment effects are displayed in Figure ??. The blue line in the panels displays the evolution of the employment shares in the baseline economy with time-varying fundamentals, and the green line displays the counterfactual economy in which all fundamentals are changing the same as in the baseline economy except for the productivities in China. ${ }^{52}$ We find that the China shock lead to a 0.31 percent decline in manufacturing employment share in the long run. Similarly to the results with constant fundamentals, we find that workers reallocate to other sectors. We find that the aggregate welfare increases by 0.14 percent as a consequence of the China shock, and we also find large heterogeneity in the welfare effects across labor markets. Overall, with timevarying fundamentals, welfare effects and employment effects are of a similar order of magnitude, and we find similar relocation effects across sectors when compared to our results with constant fundamentals.

### 5.3.3 Effect of the China Shock with Persistent Migration Decisions.-

In our model the i.i.d. nature of the idiosyncratic shocks, together with the migration costs, generates a gradual adjustment towards the steady state. In this section, we extend the model to incorporate an additional source of persistence in worker's decisions and we quantify the effects of the China shock using this alternative model. ${ }^{53}$ In Appendix 3.3 we show how we derive all the equilibrium conditions and how to apply the dynamic hat algebra to this model.

Suppose that at each moment in time households are subject to a Poisson process that determines the arrival of a new draw of the idiosyncratic shock. In particular, with probability $\rho$ the household does not receive a preference draw and stays in the same labor market, while with probability $1-\rho$ the household receives a new draw. We assume that the likelihood of this events are not location

[^25]specific. ${ }^{54}$ As before, let $V_{t}^{n j}=E\left[v_{t}^{n j}\right]$. The value function can be then written as
$$
V_{t}^{n j}=U\left(C_{t}^{n j}\right)+\rho \beta V_{t+1}^{n j}+(1-\rho) \nu \log \left(\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}\right),
$$
and then the fraction of households that stay in market $n j$ at time $t$ is now given by
$$
\mu_{t}^{n j, n j}=\rho+\frac{(1-\rho) \exp \left(\beta V_{t+1}^{n j}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}},
$$
while the fraction of workers that move to market $i k$ is given by
$$
\mu_{t}^{n j, i k}=\frac{(1-\rho) \exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}
$$

As we can see from these new equilibrium conditions, the fraction of households that decide to stay in a particular market is larger than $\rho$ given that some of the agents with a new draw still decide to stay. Also note that in the limit when $\rho=1$ the economy becomes static, there is no migration and we are back to a spatial trade model with no labor reallocation. On the other hand, when $\rho=0$ the model collapses to the one we had before.

Crucially, in this new set-up, both the migration cost elasticity $1 / \nu$ together with $\rho$ determine the flow of workers across markets. Recall from Section 4 that the cross sectional variation in migration flows and wages are used to identify the migration cost elasticity $1 / \nu$, but now this cross sectional variation is also going to depend on $\rho$. Given this, $\rho$ and $\nu$ cannot be separately identified from variation in wages and migration flows. Therefore, if we adjust the migration flow matrix by $\rho$, we can run regression (27) to identify $1 / \nu$. In doing so, however, we need to condition on $\rho$. So in order to evaluate how our results change as we add persistent households, we proceed to estimate three different values of $\nu$ conditioning on three different values of $\rho$. Specifically, we impose $\rho=0.1$, $\rho=0.2$, and $\rho=0.3$. Given these values for $\rho$, we obtain $\nu_{\rho=0.1}=5.0369, \nu_{\rho=0.2}=4.6973$, and $\nu_{\rho=0.3}=4.3189$ using our specification (equation 27) where we used $\tilde{\mu}_{t}^{n j, i k}=\left(\mu_{t}^{n j, n j}-\rho\right) /(1-\rho)$ instead of $\mu_{t}^{n j, i k}$ in order to be consistent with this new model.

Figure 13 shows the evolution of the employment shares in manufacturing, services, wholesale and retail, and construction for the case of $\rho=0.1$ and $\nu=5.0369$. The evolution of employment shares is remarkably similar to those where $\rho=0$ as seen in Figure 1. As discussed above, this

[^26]finding is consistent with the fact that, conditional to receiving an idiosyncratic preference draw, the migration cost elasticity is higher in the model with persistent idiosyncratic shocks than in the model in Section 2. Therefore, the higher mobility persistence coming from the parameter $\rho$ in the model is offset by a higher migration elasticity $1 / \nu$, and the resulting employment dynamics is similar to the one in the model with $\rho=0$.

Fig. 13: The Evolution of Employment Shares


Note: The figure presents the evolution of employment in each sector (manufacturing, services, wholesale and retail, and construction) over total employment. Total employment excludes farming, utilities, and the public sector. The dashed lines represent the shares from the baseline economy with no changes in fundamentals, what we denote by "No China-Shock", while the lines represent shares from the economy with the China shock. The results are computed with the model of persistent households with rho $=0.1$ and $n u=5.0369$.

This conclusion is robust to the choice of different values for $\rho$. Table 2 summarizes the effects on aggregate manufacturing employment shares and aggregate welfare under different values of $\rho$ and $\nu$. Although employment and welfare effects are similar, the manufacturing employment effect tends to be slightly smaller as the persistence parameter $\rho$ increases. That is, we observe somewhat less labor mobility and smaller welfare effects as a result.

Table 2: Aggregate effects across models with different degree of persistence

| Model |  | Change in |  |
| :--- | :---: | :---: | :---: |
| $\rho$ | $\nu$ | Mfg. emp. share | Welfare |
| 0 | 5.3436 | 0.5044 | 0.3449 |
| 0.1 | 5.0369 | 0.4819 | 0.3431 |
| 0.2 | 4.6973 | 0.4789 | 0.3401 |
| 0.3 | 4.3189 | 0.4834 | 0.3351 |

Note: This table presents long-run employment and welfare effects due to the China shock, under different values of $\rho$ and $\nu$.

## 6. CONCLUSION

Aggregate trade shocks can have varying effects across labor markets. One source of variation is the exposure to foreign trade, measured by the degree of import competition across labor markets. Another source of variation is the extent to which trade shocks impact the exchange of goods and the reallocation of labor across and within sectors and locations. Moreover, since labor movement across markets takes time, and mobility frictions depend on local characteristics, labor market outcomes adjust differently across industries, space, and over time to the same aggregate shock. Therefore, the study of the effects of shocks on the economy requires the understanding of the impact of trade on labor market dynamics.

In this paper, we build on ACM and EK to develop a dynamic and spatial trade model. The model explicitly recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, input-output linkages, and international trade in determining allocations. We calibrate the model to 38 countries, 50 U.S. states, and 22 sectors to quantify the impact of increased import competition from China over the period from 2000-2007 on employment and welfare across spatially different labor markets. Our results indicate that although exposure to import competition from China reduces manufacturing employment, aggregate U.S. welfare increases. Disaggregate effects on employment and welfare across regions, sectors, labor markets, and over time are shaped by all the mechanisms and ingredients mentioned previously.

We emphasize that our quantitative framework and solution method can be applied to an arbitrary number of sectors, regions, and countries. The framework can furthermore be used to address a broader set of questions, generating a promising research agenda. For instance, with our framework we can study the impact of changes in trade costs, or productivity, in any region of any country in the world. The framework can also be used to explore the effects of capital mobility across regions; to study the economic effects of different changes in government policies, such as
changes in taxes, subsidies or non-employment benefits; or to study policies that reduce mobility frictions. ${ }^{55}$

Other interesting topics to apply this framework are the quantification of the effects of trade agreements and other changes in trade policy on internal labor markets and the impact of migration across countries. In addition, it can be used to study the transmission of regional and sectoral shocks across a production network when trade and factor reallocation is subject to frictions. ${ }^{56}$ The model can also be computed at a more disaggregated level to study migration across metropolitan areas, or commuting zones, although the challenge in this case would be collecting the relevant trade and production data at these levels of disaggregation. Quantitative answers to some of these questions using dynamic models of the type developed here present an exciting avenue for future research.

Another important extension would be to depart from our perfect foresight assumption by modelling stochastic processes of fundamentals. This extension would widen the type of shocks that can be studied with our framework.

[^27]
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## APPENDIX 1: DERIVATIONS

In this appendix, we first derive the lifetime expected utility (2) and the gross migration flows described by equation (3). After doing so, we derive the welfare equation.

### 1.1 Derivations

The lifetime utility of a worker in market $n j$ is given by

$$
\mathrm{v}_{t}^{n j}=U\left(C_{t}^{n j}\right)+\max _{\{i, k\}_{i=1, k=0}^{N, J}}\left\{\beta E\left[\mathrm{v}_{t+1}^{i k}\right]-\tau^{n j, i k}+\nu \epsilon_{t}^{i k}\right\}
$$

Denote by $V_{t}^{n j} \equiv E\left[\mathrm{v}_{t}^{n j}\right]$ the expected lifetime utility of a worker, where the expectation is taken over the preference shocks. We assume that the idiosyncratic preference shock $\epsilon$ is i.i.d. over time and is a realization of a Type-I Extreme Value distribution with zero mean. In particular, $F(\epsilon)=\exp (-\exp (-\epsilon-\bar{\gamma}))$, where $\bar{\gamma} \equiv \int_{-\infty}^{\infty} x \exp (-x-\exp (-x)) d x$ is Euler's constant, and $f(\epsilon)=\partial F / \partial \epsilon$. We seek to solve for

$$
\Phi_{t}^{n j}=E\left[\max _{\{i, k\}_{i=1, k=0}^{N, J}}\left\{\beta E\left[\mathrm{v}_{t+1}^{i k}\right]-\tau^{n j, i k}+\nu \epsilon_{t}^{i k}\right\}\right] .
$$

Let $\bar{\epsilon}_{t}^{i k, m h}=\frac{\beta\left(V_{t+1}^{i k}-V_{t+1}^{m h}\right)-\left(\tau^{n j, i k}-\tau^{n j, m h}\right)}{\nu}$, note that

$$
\Phi_{t}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty}\left(\beta V_{t+1}^{i k}-\tau^{n j, i k}+\nu \epsilon_{t}^{i k}\right) f\left(\epsilon_{t}^{i k}\right) \prod_{m h \neq i k} F\left(\epsilon_{t}^{i k, m h}+\epsilon_{t}^{i k}\right) d \epsilon_{t}^{i k},
$$

Then substituting for $F(\epsilon)$, and $f(\epsilon)$ we obtain

$$
\Phi_{t}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty}\left(\beta V_{t+1}^{i k}-\tau^{n j, i k}+\nu \epsilon_{t}^{i k}\right) e^{\left(-\epsilon_{t}^{i k}-\bar{\gamma}\right)} e^{\left(-e^{\left(-\epsilon_{t}^{i k}-\bar{\gamma}\right)} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{\left(-\bar{\epsilon}_{t}^{i k, m h}\right)}\right)} d \epsilon_{t}^{i k}
$$

Defining $\lambda_{t}^{i k} \equiv \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(-\bar{\epsilon}_{t}^{i k, m h}\right)$ and considering the following change of variables, $\zeta_{t}^{i k}=\epsilon_{t}^{i k}+\bar{\gamma}$ we get

$$
\Phi_{t}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty}\left(\beta V_{t+1}^{i k}-\tau^{n j, i k}+\nu\left(\zeta_{t}^{i k}-\bar{\gamma}\right)\right) \exp \left(-\zeta_{t}^{i k}-\exp \left(-\left(\zeta_{t}^{i k}-\lambda_{t}^{i k}\right)\right)\right) d \zeta_{t}^{i k}
$$

Consider an additional change of variables; let $\tilde{y}_{t}^{i k}=\zeta_{t}^{i k}-\lambda_{t}^{i k}$. Hence, we obtain

$$
\Phi_{t}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(-\lambda_{t}^{i k}\right)\binom{\left(\beta V_{t+1}^{i k}-\tau^{n j, i k}+\nu\left(\lambda_{t}^{i k}-\bar{\gamma}\right)\right)}{+\nu \int_{-\infty}^{\infty} \tilde{y}_{t}^{k i k} \exp \left(-\tilde{y}_{t}^{i k}-\exp \left(-\tilde{y}_{t}^{i k}\right)\right) d \tilde{y}_{t}^{i k}},
$$

and using the definition of $\bar{\gamma}$, we get

$$
\Phi_{t}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(-\lambda_{t}^{i k}\right)\left(\beta V_{t+1}^{i k}-\tau^{n j, i k}+\nu \lambda_{t}^{i k}\right),
$$

and replacing the definition of $\lambda_{t}^{i k}$, we get

$$
\Phi_{t}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(-\log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(-\bar{\epsilon}_{t}^{i k, m h}\right)\right)\binom{\beta V_{t+1}^{i k}-\tau^{n j, i k}}{+\nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(-\bar{\epsilon}_{t}^{i k, m h}\right)}
$$

Substituting the definition of $\bar{\epsilon}_{t}^{i k, m h}$, we get,

$$
\Phi_{t}^{n j}=\nu\left(\log \sum_{m=1}^{N} \sum_{h=0}^{J} e^{\left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}\right) \sum_{i=1}^{N} \sum_{k=0}^{J} e^{\left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{\left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}
$$

which implies

$$
\Phi_{t}^{n j}=\nu\left(\log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}\right)
$$

and therefore

$$
V_{t}^{n j}=U\left(C_{t}^{n j}\right)+\nu\left(\log \sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}\right)
$$

We now derive equation (3). Define $\mu_{t}^{n j, i k}$ as the fraction of workers that reallocate from labor market $n j$ to labor market $i k$. This fraction is equal to the probability that a given worker moves from labor market $n j$ to labor market $i k$ at time $t$; that is, the probability that the expected utility of moving to $i k$ is higher than the expected utility in any other location. Formally,

$$
\mu_{t}^{n j, i k}=\operatorname{Pr}\left(\frac{\beta V_{t+1}^{i k}-\tau^{n j, i k}}{\nu}+\epsilon_{t}^{i k} \geq \max _{m h \neq i k}\left\{\frac{\beta V_{t+1}^{m h}-\tau^{n j, m h}}{\nu}+\epsilon_{t}^{m h}\right\}\right)
$$

Given our assumptions on the idiosyncratic preference shock,

$$
\mu_{t}^{n j, i k}=\int_{-\infty}^{\infty} f\left(\epsilon_{t}^{i k}\right) \prod_{m h \neq i k} F\left(\beta\left(V_{t+1}^{i k}-V_{t+1}^{m h}\right)-\left(\tau^{n j, i k}-\tau^{n j, m h}\right)+\epsilon_{t}^{i k}\right) d \epsilon_{t}^{i k}
$$

From the above derivations, we know that

$$
\mu_{t}^{n j, i k}=\int_{-\infty}^{\infty} \exp \left(-\epsilon_{t}^{i k}-\bar{\gamma}\right) e^{\left(-e^{\left(-\epsilon_{t}^{i k}-\bar{\gamma}\right)} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{\left(-\bar{\epsilon}_{t}^{i k, m h}\right)}\right)} d \epsilon_{t}^{i k}
$$

Using the definitions from above, we get

$$
\mu_{t}^{n j, i k}=\exp \left(-\lambda_{t}^{i k}\right) \int_{-\infty}^{\infty} \exp \left(-\tilde{y}_{t}-\exp \left(-\tilde{y}_{t}\right)\right) d \tilde{y}_{t}
$$

and solving for this integral we obtain

$$
\mu_{t}^{n j, i k}=\frac{\exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}
$$

### 1.2 The Option Value and Welfare Equations

In this section, we discuss the welfare effects resulting from changes in fundamentals in our economy.

To begin, let $V_{t}^{\prime n j}$ be the present discounted value of utility at time $t$ in market $n j$ under the counterfactual change in fundamentals $\left\{\Theta_{t}^{\prime}\right\}_{t=0}^{\infty}$, and let $V_{t}^{n j}$ denote the same object for the case of the baseline economy given a sequence of fundamentals $\left\{\Theta_{t}\right\}_{t=0}^{\infty}$. Now, write the expected lifetime utility of being at market $n j$ at time $t$ as

$$
\begin{equation*}
V_{t}^{n j}=\log C_{t}^{n j}+\beta V_{t+1}^{n j}+\nu \log \left(\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta\left(V_{t+1}^{i k}-V_{t+1}^{n j}\right)-\tau^{n j, i k}\right)^{1 / \nu}\right), \tag{A1-1}
\end{equation*}
$$

where the second term on the right hand side of equation $(A 1-1)$ is the option value. From equation (3) we know that

$$
\mu_{t}^{n j, n j}=\frac{\exp \left(\beta V_{t+1}^{n j}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}},
$$

and therefore the option value is given by

$$
\nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta\left(V_{t+1}^{m h}-V_{t+1}^{n h}\right)-\tau^{n j, m h}\right)^{1 / \nu}=-\nu \log \mu_{t}^{n j, n j} .
$$

Plugging this equation into the value function, we get

$$
V_{t}^{n j}=\log C_{t}^{n j}+\beta V_{t+1}^{n j}-\nu \log \mu_{t}^{n j, n j} .
$$

Finally, iterating this equation forward we obtain

$$
V_{t}^{n j}=\sum_{s=t}^{\infty} \beta^{s-t} \log C_{s}^{n j}-\nu \sum_{s=t}^{\infty} \beta^{s-t} \log \mu_{s}^{n j, n j} .
$$

Given this we obtain that the expected lifetime utilities in the counterfactual and in the baseline economy are given by,

$$
\begin{aligned}
V_{t}^{\prime n j} & =\sum_{s=t}^{\infty} \beta^{s-t} \log \left(\frac{C_{s}^{\prime n j}}{\left(\mu_{s}^{\prime n j, n j}\right)^{\nu}}\right), \\
V_{t}^{n j} & =\sum_{s=t}^{\infty} \beta^{s-t} \log \left(\frac{C_{s}^{n j}}{\left(\mu_{s}^{n j, n j}\right)^{\nu}}\right) .
\end{aligned}
$$

We define the compensating variation in consumption for market $n j$ at time $t$ to be the scalar $\delta^{n j}$ such that

$$
\begin{aligned}
V_{t}^{\prime n j} & =V_{t}^{n j}+\sum_{s=t}^{\infty} \beta^{s-t} \log \left(\delta^{n, j}\right), \\
& =\sum_{s=t}^{\infty} \beta^{s-t} \log \left(\frac{C_{s}^{n j}}{\left(\mu_{s}^{n, j ; n, j}\right)^{\nu}} \delta^{n, j}\right) .
\end{aligned}
$$

Re-arranging this we have that $\log \left(\delta^{n, j}\right)=(1-\beta)\left(V_{t}^{\prime n j}-V_{t}^{n j}\right)$, or

$$
\begin{equation*}
\log \left(\delta^{n, j}\right)=(1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} \log \left(\frac{C_{s}^{\prime n j} / C_{s}^{n j}}{\left(\mu_{s}^{\prime n j, n j} / \mu_{s}^{n j, n j}\right)^{\nu}}\right), \tag{A1-2}
\end{equation*}
$$

which can also be written as

$$
\begin{aligned}
\log \left(\delta^{n, j}\right) & =\sum_{s=0}^{\infty} \beta^{s} \log \left(\frac{C_{s}^{\prime n j} / C_{s}^{n j}}{\left(\mu_{s}^{\prime n j, n j} / \mu_{s}^{n j, n j}\right)^{\nu}}\right)-\sum_{s=0}^{\infty} \beta^{s+1} \log \left(\frac{C_{s}^{\prime n j} / C_{s}^{n j}}{\left(\mu_{s}^{n j, n j} / \mu_{s}^{n j, n j}\right)^{\nu}}\right) \\
& =\log \left(\frac{C_{0}^{\prime n j} / C_{0}^{n j}}{\left(\mu_{0}^{n j, n j} / \mu_{0}^{n j, n j}\right)^{\nu}}\right)+\sum_{s=1}^{\infty} \beta^{s} \log \left(\frac{\left(C_{s}^{\prime n j} / C_{s}^{n j}\right) /\left(C_{s-1}^{\prime n j} / C_{s-1}^{n j}\right)}{\left(\left(\mu_{s}^{\prime n j, n j} / \mu_{s}^{n j, n j}\right) /\left(\mu_{s-1}^{\prime n j, n j} / \mu_{s-1}^{n j, n j}\right)\right)^{\nu}}\right) \\
& =\log \left(\frac{C_{0}^{\prime n j} / C_{0}^{n j}}{\left(\mu_{0}^{n j, n j} / \mu_{0}^{n j, n j}\right)^{\nu}}\right)+\sum_{s=1}^{\infty} \beta^{s} \log \left(\frac{\hat{C}_{s}^{n j}}{\left(\hat{\mu}_{s}^{n j, n j}\right)^{\nu}}\right)
\end{aligned}
$$

Given that $C_{0}^{\prime n j}=C_{0}^{n j}$, and $\mu_{0}^{\prime n j, n j}=\mu_{0}^{n j, n j}$. we obtain

$$
\log \left(\delta^{n, j}\right)=\sum_{s=1}^{\infty} \beta^{s} \log \left(\frac{\hat{C}_{s}^{n j}}{\left(\hat{\mu}_{s}^{n j, n j}\right)^{\nu}}\right)
$$

which is our measure of consumption equivalent change in welfare in equation (28).
Note that the change in welfare in market $n j$ from a change in fundamentals relative to the baseline economy is given by the present discounted value of the expected change in real consumption, and the change in the option value. Equation $(A 1-2)$ shows that the change in the option value is summarized by the change in the fraction of workers that do not reallocate, $\hat{\mu}_{t}^{n j, n j}$, and the variance of the taste shocks $\nu$. The intuition is that higher $\hat{\mu}_{s}^{n j, n j}$ means that fewer workers in market $n j$ move to a market with higher expected value. Notice that if the cost of moving to a different labor market is infinite, then $\hat{\mu}_{t}^{n j, n j}=1$, and the option value is zero.

In our model, the change in real consumption in market $n j, \hat{C}_{s}^{n j}$ is given by the change in the real wage earned in that market, $\hat{w}_{t}^{n j} / \hat{P}_{t}^{n}$, and can be expressed as ${ }^{57}$

$$
\begin{equation*}
\hat{C}_{t}^{n j}=\frac{\hat{w}_{t}^{n j}}{\prod_{k=1}^{J}\left(\hat{w}_{t}^{n k}\right)^{\alpha^{k}}} \prod_{k=1}^{J}\left(\frac{\hat{w}_{t}^{n k}}{\hat{P}_{t}^{n k}}\right)^{\alpha^{k}} . \tag{A1-3}
\end{equation*}
$$

The first component denotes the unequal welfare effects for households working in different sectors within the same region $n$; and reflects the fact that workers in sectors that pay higher wages have more purchasing power in that region. The second component is common to all households residing in region $n$ and captures the change in the cost of living in that region. This second component is a measure of the change in the average real wage across labor markets in region $n$, weighted by the importance of each sector in the consumption bundle, and it is shaped by several mechanisms in our model. Specifically,

$$
\begin{equation*}
\prod_{k=1}^{J}\left(\frac{\hat{w}_{t}^{n k}}{\hat{P}_{t}^{n k}}\right)^{\alpha^{k}}=\sum_{k=1}^{J} \alpha^{k}\left(\log \left(\hat{\pi}_{t}^{n k, n k}\right)^{-1 / \theta^{k}}+\log \frac{\hat{w}_{t}^{n k}}{\hat{x}_{t}^{n k}}\right) \tag{A1-4}
\end{equation*}
$$

[^28]The first term in equation $(A 1-4)$ is the change in trade openness, $\log \left(\hat{\pi}_{t}^{n k, n k}\right)$, that gives households in region $n$ access to cheaper imported goods. The second term in equation ( $A 1-4$ ) is the change in factor prices, $\log \frac{\hat{w}_{t}^{n k}}{\hat{x}_{t}^{n k}}$, and captures the effects of migration, local factors, and intersectoral trade.

To fix ideas, consider the case where we abstract from materials in the model, $\log \frac{\hat{w}_{t}^{n k}}{\hat{x}_{t}^{n k}}=$ $-\xi^{n} \log \left(\hat{L}_{t}^{n k} / \hat{H}^{n k}\right)$. Migration into region $n$ may have a positive or negative effect on factor prices depending on how $L_{t}^{n k}$ changes relative to the stock of structures $H^{n k}$. In our model structures are in fixed supply, thus, migration has a negative effect on real wages because the inflow of workers strains local fixed factors and raises the relative price of structures and the cost of living in region $n$. This is a congestion effect as in Caliendo et al. (2017). ${ }^{58}$ Finally, material inputs and input-output linkages impact welfare through changes in the cost of the input bundle as in Caliendo and Parro (2015).

Now consider the case of a one-sector-model (more details are presented in Appendix 3.1) with $N$ labor markets indexed by $\ell$, and households in location $\ell$ consume local goods. In this setup, the welfare equation $(A 1-2)$ takes the form

$$
\hat{W}^{\ell}=\sum_{s=1}^{\infty} \beta^{s} \log \frac{\hat{w}_{s}^{\ell} / \hat{P}_{s}^{\ell}}{\left(\hat{\mu}_{s}^{\ell, \ell}\right)^{\prime}},
$$

and the change in real wages is given by $\log \left(\hat{w}_{t}^{\ell} / \hat{P}_{t}^{\ell}\right)=-\left(1 / \theta^{j} \gamma\right) \log \hat{\pi}_{t}^{\ell, \ell}-\beta \log \left(\hat{L}_{t}^{\ell} / \hat{H}_{t}^{\ell}\right)$. It follows then, that in a one-sector model with no materials and structures, the welfare equation reduces to

$$
\hat{W}^{\ell}=\sum_{s=1}^{\infty} \beta^{s} \log \frac{\left(\hat{\pi}_{s}^{\ell, \ell}\right)^{-1 / \theta}}{\left(\hat{\mu}_{s}^{\ell, \ell}\right)^{\nu}}
$$

which combines the welfare formulas in ACM (2010), and ACR (2012).

[^29]
## APPENDIX 2: PROOF OF PROPOSITIONS

This appendix presents the proofs of the propositions presented in the main text.
Proposition 1 Given the allocation of the temporary equilibrium at $t$, $\left\{L_{t}, \pi_{t}, X_{t}\right\}$, the solution to the temporary equilibrium at $t+1$ for a given change in $\dot{L}_{t+1}$ and $\dot{\Theta}_{t+1}$ does not require information on the level of fundamentals at $t, \Theta_{t}$, and solve the following system of non-linear equations

$$
\begin{gather*}
\dot{x}_{t+1}^{n j}=\left(\dot{L}_{t+1}^{n j}\right)^{\gamma^{n j} \xi^{n}}\left(\dot{w}_{t+1}^{n j}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(\dot{P}_{t+1}^{n k}\right)^{\gamma^{n j, n k}},  \tag{A2-1}\\
\dot{P}_{t+1}^{n j}=\left(\sum_{i=1}^{N} \pi_{t}^{n j, i j}\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}},  \tag{A2-2}\\
\pi_{t+1}^{n j, i j}=\pi_{t}^{n j, i j}\left(\frac{\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}}{\dot{P}_{t+1}^{n j}}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}},  \tag{A2-3}\\
X_{t+1}^{n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{i k, n k} X_{t+1}^{i k}+\alpha^{j}\left(\sum_{k=1}^{J} \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k} w_{t}^{n k} L_{t}^{n k}+\iota^{n} \chi_{t+1}\right),  \tag{A2-4}\\
\dot{w}_{t+1}^{n j} \dot{L}_{t+1}^{n j} w_{t}^{n j} L_{t}^{n j}=\gamma^{n j}\left(1-\xi^{n}\right) \sum_{i=1}^{N} \pi_{t+1}^{i j, n j} X_{t+1}^{i j}, \tag{A2-5}
\end{gather*}
$$

where $\chi_{t+1}=\sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{i}} \dot{w}_{t+1}^{i k} \dot{L}_{t+1}^{i k} w_{t}^{i k} L_{t}^{i k}$.
Proof: Let $\left\{L_{t}, \pi_{t}, X_{t}\right\}$ be the allocation of the temporary equilibrium associated to $\Theta_{t}$. Consider a given change in $L_{t}$ to $L_{t+1}$ and $\Theta_{t}=\left\{\left(A_{t}, \kappa_{t}\right), \Theta_{2}\right\}$ to $\Theta_{t+1}=\left\{\left(A_{t+1}, \kappa_{t+1}\right), \Theta_{2}\right\}$. Denote these changes in time differences as $\dot{L}_{t+1}$ and $\dot{\Theta}_{t+1}$. First we show how to express the equilibrium conditions that define a temporary equilibrium under $L_{t}$ and under $L_{t+1}$ in time differences, namely we derive equations $(A 2-1)$ to $(A 2-5)$. Recall that we have defined the operator "." over a variable $y_{t+1}$ as $\dot{y}_{t+1}=\frac{y_{t+1}}{y_{t}}$.

From the first order conditions of the intermediate goods producers problem we obtain that $\frac{r_{t}^{n j} H^{n j}}{\xi^{n}}=\frac{w_{t}^{n j} L_{t}^{n j}}{1-\xi^{n}}$, and expressing this condition in time difference we obtain

$$
\begin{equation*}
\frac{\dot{r}_{t+1}^{n j}}{\xi^{n}}=\frac{\dot{w}_{t+1}^{n j} \dot{L}_{t+1}^{n j}}{1-\xi^{n}} \tag{A2-6}
\end{equation*}
$$

now use the definition of the input bundle (5) at time $t\left(x_{t}^{n j}\right)$ and $t+1\left(x_{t+1}^{n j}\right)$. Taking the ratio of these expressions and substituting $\dot{r}_{t+1}^{n j}$ using $(A 2-6)$ we obtain $(A 2-1)$.

Use equilibrium conditions (6) and (7) at time $t\left(P_{t}^{n j}\right.$ and $\left.\pi_{t}^{n j, i j}\right)$ and at $t+1\left(P_{t+1}^{n j}\right.$ and $\left.\pi_{t+1}^{n j, i j}\right)$ and express this conditions relative to each other, namely

$$
\begin{equation*}
\frac{P_{t+1}^{n j}}{P_{t}^{n j}}=\left(\sum_{i=1}^{N} \frac{\left(x_{t+1}^{i j} \kappa_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(A_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}}{\sum_{m=1}^{N}\left(x_{t}^{m j} \kappa_{t}^{n j, m j}\right)^{-\theta^{j}}\left(A_{t}^{m j}\right)^{\theta^{j} \gamma^{m j}}}\right)^{-1 / \theta^{j}} \tag{A2-7}
\end{equation*}
$$

Now multiply and divide each element in the summation by $\left(x_{t}^{i j} \kappa_{t}^{n j, i j}\right)^{-\theta^{j}}\left(A_{t}^{i j}\right)^{\theta^{j}} \gamma^{i j}$, and then
using $\pi_{t}^{n j, i j}$, we obtain

$$
\begin{equation*}
\frac{P_{t+1}^{n j}}{P_{t}^{n j}}=\left(\sum_{i=1}^{N} \pi_{t}^{n j, i j}\left(\frac{x_{t+1}^{i j} \kappa_{t+1}^{n j, i j}}{x_{t}^{i j} \kappa_{t}^{n j, i j}}\right)^{-\theta^{j}}\left(\frac{A_{t+1}^{i j}}{A_{t}^{i j}}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}} . \tag{A2-8}
\end{equation*}
$$

Finally use the "." notation and we arrive at $(A 2-2)$.
Similarly, multiplying and dividing the numerator of $\pi_{t+1}^{n j, i j}$ by $\left(x_{t}^{i j} \kappa_{t}^{n j, i j}\right)^{-\theta^{j}}\left(A_{t}^{i j}\right)^{\theta j} \gamma^{i j}$ and then multiplying and dividing each element in the summation of the denominator of $\pi_{t+1}^{n j, i j}$ by $\left(x_{t}^{i j} \kappa_{t}^{n j, i j}\right)^{-\theta^{j}}$ $\left(A_{t}^{i j}\right)^{\theta^{j} \gamma^{i j}}$ and then using $\pi_{t}^{n j, i j}$, we obtain

$$
\begin{equation*}
\pi_{t+1}^{n j, i j}=\frac{\pi_{t}^{n j, i j}\left(\frac{x_{t+1}^{i j} \kappa_{t+1, i j}^{n j, i j}}{x_{t}^{i} \kappa_{t}^{n, \lambda j}}\right)^{-\theta^{j}}\left(\frac{A_{t+1}^{i j}}{A_{t}^{i j}}\right)^{\theta^{j} \gamma^{i j}}}{\sum_{m=1}^{N} \pi_{t}^{n j, m j}\left(\frac{x_{t+1}^{m j} \kappa_{t+1}^{n j, m j}}{x_{t}^{m j} \kappa_{t}^{j n, m j}}\right)^{-\theta^{j}}\left(\frac{A_{t+1}^{m j}}{A_{t}^{m j}}\right)^{\theta^{j} \gamma^{m j}}} . \tag{A2-9}
\end{equation*}
$$

Now substitute the denominator with $(A 2-2)$ and we arrive at $(A 2-3)$.
To derive $(A 2-4)$, start with the market clearing at $t+1$,

$$
\begin{equation*}
X_{t+1}^{n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{i k, n k} X_{t+1}^{i k}+\alpha^{j}\left(\sum_{k=1}^{J} w_{t+1}^{n k} L_{t+1}^{n k}+\iota^{n} \chi_{t+1}\right) \tag{A2-10}
\end{equation*}
$$

and now multiply and divide $\sum_{k=1}^{J} w_{t+1}^{n k} L_{t+1}^{n k}$ by $w_{t}^{n k} L_{t}^{n k}$ to obtain, $\sum_{k=1}^{J} \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k} w_{t}^{n k} L_{t}^{n k}$. Substitute this expression to obtain $(A 2-4)$, where $\chi_{t+1}=\sum_{i=1}^{N} \sum_{k=1}^{J} \dot{r}_{t}^{i k} r_{t}^{i k} H^{i k}$, and using $(A 2-6)$ we can express this as $\chi_{t+1}=\sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{2}} \dot{w}_{t+1}^{i k} \dot{L}_{t+1}^{i k} w_{t}^{i k} L_{t}^{i k}$.

Finally, to obtain $(A 2-5)$, start with the labor market clearing condition at $t+1$,

$$
\begin{equation*}
w_{t+1}^{n j} L_{t+1}^{n j}=\gamma^{n j}\left(1-\xi^{n}\right) \sum_{i=1}^{N} \pi_{t+1}^{i j, n j} X_{t+1}^{i j}, \tag{A2-11}
\end{equation*}
$$

and multiply and divide the left hand side by $w_{t+1}^{n j} L_{t+1}^{n j}$ to obtain $(A 2-5)$.
Now, inspecting equations $(A 2-1)$ to $(A 2-5)$, we see that with information on the allocation at $t,\left\{L_{t}, \pi_{t}, X_{t}\right\}$, we can solve for $\left\{\dot{w}_{t+1}^{n j}, \dot{x}_{t+1}, \dot{P}_{t+1}^{n j}, \pi_{t+1}^{n j, i j}, X_{t+1}^{n j}\right\}_{n=1, i=1, j=1}^{N, N, J}$, given $\dot{\Theta}_{t+1}=\left\{\dot{\kappa}_{t+1}^{n j, i j}\right.$, $\left.\dot{A}_{t+1}^{n j}\right\}_{n=1, i=1, j=1}^{N, N, J}$, without estimates of $\Theta_{t}$.

Proposition 2 Conditional on an initial allocation of the economy, $\left(L_{0}, \pi_{0}, X_{0}, \mu_{-1}\right)$, given an anticipated sequence of changes in fundamentals, $\left\{\dot{\Theta}_{t}\right\}_{t=1}^{\infty}$, with $\lim _{t \rightarrow \infty} \Theta_{t}=1$, the solution to the sequential equilibrium in time differences does not require information on the level of the fundamentals, $\left\{\Theta_{t}\right\}_{t=0}^{\infty}$, and solves the following system of non-linear equations:

$$
\begin{gather*}
\mu_{t+1}^{n j, i k}=\frac{\mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{n, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}},  \tag{A2-12}\\
\dot{u}_{t+1}^{n j}=\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}\right)^{\nu},  \tag{A2-13}\\
L_{t+1}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{i k, n j} L_{t}^{i k}, \tag{A2-14}
\end{gather*}
$$

for all $j, n, i$ and $k$ at each $t$, where $\left\{\dot{\omega}^{n j}\left(\dot{L}_{t}, \dot{\Theta}_{t}\right)\right\}_{n=1, j=0, t=1}^{N, J, \infty}$ is the solution to the temporary equilibrium given $\left\{\dot{L}_{t}, \dot{\Theta}_{t}\right\}_{t=1}^{\infty}$.

Proof: Consider the fraction of workers who reallocate from market $n, j$ to $i, k$, at $t=0$; that is, equilibrium condition (3) at $t=0$ :

$$
\mu_{0}^{n j, i k}=\frac{\exp \left(\beta V_{1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}
$$

Taking the relative time differences (between $t=-1$ and $t=0$ ) of this equation, we get

$$
\frac{\mu_{0}^{n j, i k}}{\mu_{-1}^{n j, i k}}=\frac{\frac{\exp \left(\beta V_{1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}}{\frac{\exp \left(\beta V_{0}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{0}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}}
$$

Given that mobility costs do not change over time, this expression can be expressed as

$$
\frac{\mu_{0}^{n j, i k}}{\mu_{-1}^{n j, i k}}=\frac{\exp \left(\beta V_{1}^{i k}-\beta V_{0}^{i k}\right)^{1 / \nu}}{\frac{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu} \frac{\exp \left(\beta V_{0}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}{\exp \left(\beta V_{0}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{0}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}},}
$$

which is equivalent to

$$
\frac{\mu_{0}^{n j, i k}}{\mu_{-1}^{n j, i k}}=\frac{\exp \left(V_{1}^{i k}-V_{0}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{-1}^{n j, m h} \exp \left(V_{1}^{m h}-V_{0}^{m h}\right)^{\beta / \nu}}
$$

Using the definition of $u_{t}^{i k}$ we get

$$
\mu_{0}^{n j, i k}=\frac{\mu_{-1}^{n j, i k}\left(\dot{u}_{1}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{-1}^{n j, m h}\left(\dot{u}_{1}^{m h}\right)^{\beta / \nu}}
$$

where we express the migration flows at $t=0$ as a function of data at $t=-1$. Following similar steps, we can express the migration flows at any $t$, as

$$
\begin{equation*}
\mu_{t}^{n j, i k}=\frac{\mu_{t-1}^{n j, i k}\left(\dot{u}_{t+1}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{n j, m h}\left(\dot{u}_{t+1}^{m h}\right)^{\beta / \nu}} \tag{A2-15}
\end{equation*}
$$

which is equilibrium condition (16) in the main text.
Now take the equilibrium condition (2) in time differences at region $n$ and sector $j$ between periods 0 and 1 ,
$V_{1}^{n j}-V_{0}^{n j}=U\left(C_{1}^{n j}\right)-U\left(C_{0}^{n j}\right)+\nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{2}^{m h}-\tau^{n j, m h}\right)^{1 / \nu} \sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}$.
Multiplying and dividing each term in the numerator by $\exp \left(\beta V_{1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}$ and using (3),
we obtain

$$
V_{1}^{n j}-V_{0}^{n j}=U\left(C_{1}^{n j}\right)-U\left(C_{0}^{n j}\right)+\nu \log \left(\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{0}^{n j, m h} \exp \left(\beta V_{2}^{m h}-\beta V_{1}^{m h}\right)^{1 / \nu}\right)
$$

Taking exponential from both sides and using the definition of $u_{t+1}^{i, k}$ and Assumption 1, we obtain

$$
\dot{u}_{1}^{n j}=\dot{\omega}^{n j}\left(\dot{L}_{1}, \dot{\Theta}_{1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{0}^{n j, i k}\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}\right)^{\nu}
$$

where $\dot{\omega}^{n j}\left(\dot{L}_{1}, \dot{\Theta}_{1}\right)=\dot{w}^{n j}\left(\dot{L}_{1}, \dot{\Theta}_{1}\right) / \dot{P}^{n}\left(\dot{L}_{1}, \dot{\Theta}_{1}\right)$ solves the temporary equilibrium at $t=1$. Finally, for all $t$, we get,

$$
\begin{equation*}
\dot{u}_{t+1}^{n j}=\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}\right)^{\nu} \tag{A2-16}
\end{equation*}
$$

where $\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)=\dot{w}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right) / \dot{P}^{n}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)$ solves the temporary equilibrium at $t+1$.
Note that by Proposition 1, the sequence of temporary equilibria given $\dot{\Theta}_{t+1}$ does not depend on the level of $\Theta_{t}$. The equilibrium conditions $(A 2-15)$ and $(A 2-16)$ do not depend on the level of $\Theta_{t}$ either. Therefore, given a sequence $\left\{\dot{\Theta}_{t}\right\}_{t=1}^{\infty}$, with $\dot{\Theta}_{\infty}=1$, the solution to the change in the sequential equilibrium of the model given $\dot{\Theta}_{t}$ does not require knowing the level of $\Theta_{t}$.

Proposition 3 Given a baseline economy, $\left\{L_{t}, \mu_{t-1}, \pi_{t}, X_{t}\right\}_{t=0}^{\infty}$, and a counterfactual convergent sequence of changes in fundamentals, $\left\{\hat{\Theta}_{t}\right\}_{t=1}^{\infty}$, solving for the counterfactual sequential equilibrium $\left\{L_{t}^{\prime}, \mu_{t-1}^{\prime}, \pi_{t}^{\prime}, X_{t}^{\prime}\right\}_{t=1}^{\infty}$ does not require information on the fundamentals $\left(\left\{\Theta_{1 t}\right\}_{t=0}^{\infty}, \Theta_{2}\right)$, and solves the following system of non-linear equations:

$$
\begin{gather*}
\mu_{t}^{\prime n j, i k}=\frac{\mu_{t-1}^{m j, i k} \dot{\mu}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{\prime n j, m h} \dot{\mu}_{t}^{n j, m h}\left(\hat{u}_{t+1}^{m h}\right)^{\beta / \nu}},  \tag{A2-17}\\
\hat{u}_{t}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t-1}^{n j, i k} \dot{\mu}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}\right)^{\nu},  \tag{A2-18}\\
L_{t+1}^{\prime n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{i k, n j} L_{t}^{i k}, \tag{A2-19}
\end{gather*}
$$

for all $j, n, i$ and $k$ at each $t$, where $\left\{\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)\right\}_{n=1, j=0, t=1}^{N, J, \infty}$ is the solution to the temporary equilibrium given $\left\{\hat{L}_{t}, \hat{\Theta}_{t}\right\}_{t=1}^{\infty}$.

Proof: Given a baseline economy, $\left\{L_{t}, \mu_{t-1}, \pi_{t}, X_{t}\right\}_{t=0}^{\infty}$, we first show how to obtain real wages across labor markets, $\left\{\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)\right\}_{n=1, j=0, t=1}^{N, J, \infty}$, given $\left\{\hat{L}_{t}, \hat{\Theta}_{t}\right\}_{t=1}^{\infty}$. After this, we show how to obtain the equilibrium conditions $(A 2-17),(A 2-18)$, and $(A 2-19)$.

Take as given $\left\{\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right\}$ for any given $t$. We want to obtain the solution to $\left\{\hat{\omega}^{n j}\left(\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right)\right\}_{n=1}^{N, J}$, recalling that $\hat{\omega}^{n j}\left(\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right) \equiv \hat{w}_{t+1}^{n j} / \hat{P}_{t+1}^{n}$. We now derive that the equilibrium conditions to solve for $\hat{w}_{t+1}^{n j}$ are given by

$$
\begin{equation*}
\hat{x}_{t+1}^{n j}=\left(\hat{L}_{t+1}^{n j}\right)^{\gamma^{n j} \xi^{n}}\left(\hat{w}_{t+1}^{n j}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(\hat{P}_{t+1}^{n k}\right)^{\gamma^{n j, n k}}, \tag{A2-20}
\end{equation*}
$$

$$
\begin{gather*}
\hat{P}_{t+1}^{n j}=\left(\sum_{i=1}^{N} \pi_{t}^{\prime n j, i j} \dot{\pi}_{t+1}^{n j, i j}\left(\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}},  \tag{A2-21}\\
\pi_{t+1}^{\prime n j, i j}=\pi_{t}^{n j, i j} \dot{\pi}_{t+1}^{n j, i j}\left(\frac{\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}}{\hat{P}_{t+1}^{n j}}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}},  \tag{A2-22}\\
X_{t+1}^{\prime n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{i k k, n k} X_{t+1}^{\prime i k}+\alpha^{j}\left(\sum_{k=1}^{J} \hat{w}_{t+1}^{n k} \hat{L}_{t+1}^{n k} w_{t}^{\prime n k} L_{t}^{\prime n k} w_{t+1}^{n k} \dot{L}_{t+1}^{n k}+\iota^{n} \chi_{t+1}^{\prime}\right), \tag{A2-23}
\end{gather*}
$$

where $\chi_{t+1}^{\prime}=\sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{\imath}} \hat{w}_{t+1}^{i k} \hat{L}_{t+1}^{i k} w_{t}^{\prime i k} L_{t}^{\prime i k} \dot{w}_{t}^{i k} \dot{L}_{t}^{i k}$, and labor market equilibrium is

$$
\begin{equation*}
\hat{w}_{t+1}^{n k} \hat{L}_{t+1}^{n k}=\frac{\gamma^{n j}\left(1-\xi^{n}\right)}{w_{t}^{n k} L_{t}^{n k} \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k}} \sum_{i=1}^{N} \pi_{t+1}^{\prime i j, n j} X_{t+1}^{\prime i j} \tag{A2-24}
\end{equation*}
$$

Equilibrium condition $(A 2-20)$ is derived by taking the ratio between equilibrium condition ( $A 2-1$ ) in the counterfactual economy $\dot{x}_{t+1}^{\prime n j}$ and $\dot{x}_{t+1}^{n j}$ from the baseline economy, using the notation $\hat{x}_{t+1}^{n j}=\dot{x}_{t+1}^{\prime n j} / \dot{x}_{t+1}^{n j}$.

The equilibrium condition $(A 2-21)$ requires more work. Start from the counterfactual evolution of prices

$$
\begin{equation*}
\dot{P}_{t+1}^{\prime n j}=\left(\sum_{i=1}^{N} \pi_{t}^{\prime n j, i j}\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{\prime n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{\prime i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}} . \tag{A2-25}
\end{equation*}
$$

Now multiply and divide each expression in the parenthesis by $\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}$ and then use equilibrium condition $(A 2-3)$ to rewrite $\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}=\dot{\pi}_{t+1}^{n j, i j}\left(\dot{P}_{t+1}^{n j}\right)^{-\theta^{j}}$. It immediately follows that

$$
\begin{aligned}
\dot{P}_{t+1}^{\prime n j} & =\left(\sum_{i=1}^{N} \pi_{t}^{\prime n j, i j} \dot{\pi}_{t+1}^{n j, i j}\left(\dot{P}_{t+1}^{n j}\right)^{-\theta^{j}}\left(\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}} \\
\dot{P}_{t+1}^{\prime n j} & =\dot{P}_{t+1}^{n j}\left(\sum_{i=1}^{N} \pi_{t}^{\prime n j, i j} \dot{\pi}_{t+1}^{n j, i j}\left(\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}}
\end{aligned}
$$

and then we obtain $(A 2-21)$.
To solve for $(A 2-22)$, start from $(A 2-3)$ for the case of the counterfactual economy, namely

$$
\pi_{t+1}^{\prime n j, i j}=\pi_{t}^{\prime n j, i j}\left(\frac{\dot{x}_{t+1}^{\prime i j} \dot{\kappa}_{t+1}^{\prime n j, i j}}{\dot{P}_{t+1}^{\prime n j}}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{\prime i j}\right)^{\theta^{j}} \gamma^{i j}
$$

and now multiply and divide the right-hand-side by $\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j}} \gamma^{i j}$ and again use equilibrium condition $(A 2-3)$ to rewrite $\left(\dot{x}_{t+1}^{i j} \dot{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\dot{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}=\dot{\pi}_{t+1}^{n j, i j}\left(\dot{P}_{t+1}^{n j}\right)^{-\theta^{j}}$ and we immediately obtain $(A 2-22)$.

To obtain $(A 2-23)$ start from $(A 2-4)$ for the case of the counterfactual economy,

$$
X_{t+1}^{\prime n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{\prime i k, n k} X_{t+1}^{\prime i k}+\alpha^{j}\left(\sum_{k=1}^{J} \dot{w}_{t+1}^{\prime n k} \dot{L}_{t+1}^{\prime n k} w_{t}^{\prime n k} L_{t}^{\prime n k}+\iota^{n} \chi_{t+1}^{\prime}\right)
$$

and now multiply and divide $\dot{w}_{t+1}^{\prime n k} \dot{L}_{t+1}^{\prime n k} w_{t}^{\text {nk }} L_{t}^{\text {nk }}$ by $\dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k}$ to obtain $(A 2-23)$. Following this last step one also obtains $\chi_{t+1}^{\prime}$ and $(A 2-24)$.

Note that $(A 2-20)-(A 2-24)$ form a system of non-linear equations that given the baseline economy, $\left(\dot{\pi}_{t+1}^{n j, i j}, \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k}\right)$, the solution for the counterfactual economy at time $t,\left(w_{t}^{\prime n k} L_{t}^{\prime n k}\right)$ and the counterfactual change in fundamentals $\left(\hat{\kappa}_{t+1}^{n j, i j}, \hat{A}_{t+1}^{i j}\right)$ can be used to solve for $\hat{w}_{t+1}^{n j}$ and hence, $\hat{\omega}^{n j}\left(\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right) \equiv \hat{w}_{t+1}^{n j} / \hat{P}_{t+1}^{n}$. Note that for the case of $t=0$, we have that $w_{t}^{\prime n k} L_{t}^{\prime n k}=w_{t}^{n k} L_{t}^{n k}$.

Now we show how to obtain $(A 2-17),(A 2-18)$, and $(A 2-19)$.
Start from $(A 2-12)$ for the case of the counterfactual economy,

$$
\mu_{t+1}^{\prime n j, i k}=\frac{\mu_{t}^{\prime n j, i k}\left(\dot{u}_{t+2}^{\prime i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{\prime n j, m h}\left(\dot{u}_{t+2}^{\prime m h}\right)^{\beta / \nu}} .
$$

Now take the ratio between this equilibrium condition and $(A 2-12)$ to obtain

$$
\frac{\mu_{t+1}^{\prime n j, i k}}{\mu_{t+1}^{n j, i k}}=\frac{\frac{\mu_{t}^{\prime n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}}{\frac{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{\prime n j, m h}\left(\dot{u}_{t+2}^{\prime \prime h}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J=} \mu_{t}^{n j, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}},}
$$

which can be written as

$$
\mu_{t+1}^{\prime n j, i k}=\frac{\mu_{t}^{\prime n j, i k} \dot{\mu}_{t+1}^{n j, i k}\left(\hat{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \frac{\mu_{t}^{\prime n j, m h}\left(\dot{u}_{t+2}^{\prime m h}\right)^{\beta / \nu}}{\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{j j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}},
$$

and now take each expression in the summation term of the denominator and multiply and divide by $\mu_{t}^{n j, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}$

$$
\mu_{t+1}^{\prime n j, i k}=\frac{\mu_{t}^{\prime n j, i k} \dot{\mu}_{t+1}^{n j, i k}\left(\hat{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J}\left(\frac{\mu_{t}^{\prime n j, m h}}{\mu_{t}^{n j, m h}}\right) \frac{\mu_{t}^{n j, m h}\left(\hat{u}_{t+2}^{m h}\right)^{\beta / \nu}}{\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{j j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}} .
$$

Use $(A 2-12)$ in the denominator to obtain

$$
\mu_{t+1}^{\prime n j, i k}=\frac{\mu_{t}^{\prime n j, i k} \dot{\mu}_{t+1}^{n j, i k}\left(\hat{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J}\left(\frac{\mu_{t}^{\prime n j, m h}}{\mu_{t}^{n j, m h}}\right) \mu_{t+1}^{n j, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}},
$$

which gives us $(A 2-17)$.
To obtain $(A 2-18)$, start from $(A 2-13)$ for the counterfactual economy,

$$
\dot{u}_{t+1}^{\prime n j}=\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)^{\prime}\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{\prime n j, i k}\left(\dot{u}_{t+2}^{\prime i k}\right)^{\beta / \nu}\right)^{\nu},
$$

and take ratio of this expression relative to $(A 2-13)$ to obtain

$$
\frac{\dot{u}_{t+1}^{\prime n j}}{\dot{u}_{t+1}^{n j}}=\frac{\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)^{\prime}}{\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)}\left(\frac{\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{\prime n j}, i k}{\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}\right)^{\nu}
$$

using the "hat" notation

$$
\hat{u}_{t+1}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\mu_{t}^{\prime n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{n j, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}}\right)^{\nu}
$$

Now multiply and divide each term in the summation of the right-hand-side by $\mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}$ to obtain

$$
\hat{u}_{t+1}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J}\left(\frac{\mu_{t}^{m n j, i k}}{\mu_{t}^{n j, i k}}\right) \frac{\mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t}^{n j, m h}\left(\dot{u}_{t+2}^{m h}\right)^{\beta / \nu}}\left(\hat{u}_{t+2}^{i k}\right)^{\beta / \nu}\right)^{\nu},
$$

and now use $(A 2-12)$ to obtain

$$
\hat{u}_{t+1}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{t+1}, \hat{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J}\left(\frac{\mu_{t}^{n j, i k}}{\mu_{t}^{n j, i k}}\right) \mu_{t+1}^{n j, i k}\left(\hat{u}_{t+2}^{i k}\right)^{\beta / \nu}\right)^{\nu}
$$

which is equivalent to $(A 2-18)$.
The equilibrium condition $(A 2-19)$ is simply the evolution of labor for the counterfactual economy, namely $(A 2-14)$ with the "prime" notation.

At $t=1$ the equilibrium conditions are slightly different. This is the result of the timing assumption that the counterfactual fundamentals are unknown before $t=1$. This means that at $t=0, \hat{u}_{0}^{n j}=1, \mu_{0}^{i n j, i k}=\mu_{0}^{n j, i k}$, and $L_{1}^{\prime n j}=L_{1}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{0}^{i k, n j} L_{0}^{i k}$. To account for the unexpected change in fundamentals at $t=1$, we need to solve for,

$$
\begin{equation*}
\mu_{1}^{\prime n j, i k}=\frac{\vartheta_{0}^{n j, i k}\left(\hat{u}_{2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \vartheta_{0}^{n j, m h}\left(\hat{u}_{2}^{m h}\right)^{\beta / \nu}}, \tag{A2-26}
\end{equation*}
$$

and

$$
\begin{equation*}
\hat{u}_{1}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{1}, \hat{\Theta}_{1}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \vartheta_{0}^{n j, i k}\left(\hat{u}_{2}^{i k}\right)^{\beta / \nu}\right)^{\nu}, \tag{A2-27}
\end{equation*}
$$

where

$$
\vartheta_{0}^{n j, i k} \equiv \mu_{1}^{n j, i k}\left(\hat{u}_{1}^{i k}\right)^{\beta / \nu}
$$

To obtain this expression, take the lifetime utility at period $t=0$ for the economy with no shock,

$$
u_{0}^{n j}=\left(w_{0}^{n j} / P_{0}^{n}\right)\left(\sum_{m=1}^{N} \sum_{h=0}^{J}\left(u_{1}^{m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}\right)^{\nu}
$$

multiply and divide by $u_{1}^{\prime m h}$, to obtain

$$
u_{0}^{n j}=\left(w_{0}^{n j} / P_{0}^{n}\right)\left(\sum_{m=1}^{N} \sum_{h=0}^{J}\left(\frac{u_{1}^{m h}}{u_{1}^{\prime m h}}\right)^{\beta / \nu}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}\right)^{\nu}
$$

define

$$
\phi_{1}^{m h} \equiv\left(u_{1}^{m h} / u_{1}^{\prime m h}\right)^{\beta / \nu},
$$

then

$$
u_{0}^{n j}=\left(w_{0}^{n j} / P_{0}^{n}\right)\left(\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{m h}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}\right)^{\nu}
$$

Take the lifetime utility at period $t=1$ in the counterfactual economy,

$$
u_{1}^{\prime m h}=\left(w_{1}^{\prime n j} / P_{1}^{\prime n}\right)\left(\sum_{m=1}^{N} \sum_{h=0}^{J}\left(u_{2}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}\right)^{\nu}
$$

and take the difference between $u_{1}^{\prime n j}$ and $u_{0}^{n j}$, to get

$$
\begin{gather*}
u_{0}^{n j}=\left(w_{0}^{n j} / P_{0}^{n}\right)\left(\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{m h}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}\right)^{\nu}, \\
\frac{u_{1}^{\prime m h}}{u_{0}^{n j}}=\frac{\left(w_{1}^{\prime n j} / P_{1}^{\prime n}\right)}{\left(w_{0}^{n j} / P_{0}^{n}\right)}\left(\frac{\sum_{m=1}^{N} \sum_{h=0}^{J}\left(u_{2}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{m h}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}}\right)^{\nu}, \tag{A2-28}
\end{gather*}
$$

Note that we can re-write $\mu_{0}^{n j, i k}$ as

$$
\begin{align*}
\mu_{0}^{n j, i k} & =\frac{\left(u_{1}^{i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J}\left(u_{1}^{m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}} \\
& =\frac{\left(u_{1}^{i k} / u_{1}^{i k}\right)^{\beta / \nu}\left(u_{1}^{i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J}\left(u_{1}^{m h} / u_{1}^{\prime m h}\right)^{\beta / \nu}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}} \\
& =\frac{\phi_{1}^{i k}\left(u_{1}^{i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{m h}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}} . \tag{A2-29}
\end{align*}
$$

Given this, we can take equation $(A 2-28)$ and multiply and divide each term in the summation by $\phi_{1}^{i k}\left(u_{1}^{i k}\right)^{\beta / \nu}$ to obtain
$\frac{u_{1}^{\prime m h}}{u_{0}^{n j}}=\frac{\left(w_{1}^{\prime n j} / P_{1}^{\prime n}\right)}{\left(w_{0}^{n j} / P_{0}^{n}\right)}\left[\sum_{i=1}^{N} \sum_{k=0}^{J}\left(\frac{\phi_{1}^{i k}\left(u_{1}^{\prime i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \phi_{1}^{m h}\left(u_{1}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}}\right) \frac{\left(u_{2}^{i k}\right)^{\beta / \nu}}{\phi_{1}^{i k}\left(u_{1}^{\prime i k}\right)^{\beta / \nu}}\right]^{\nu}$.
We then substitute $\mu_{0}^{n j, i k}$ to obtain

$$
\frac{u_{1}^{\prime m h}}{u_{0}^{n j}}=\frac{\left(w_{1}^{\prime n j} / P_{1}^{\prime n}\right)}{\left(w_{0}^{n j} / P_{0}^{n}\right)}\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\mu_{0}^{n j, i k}}{\phi_{1}^{i k}}\left(\frac{u_{2}^{i k}}{u_{1}^{i k}}\right)^{\beta / \nu}\right)^{\nu}
$$

and using the "dot" notation we obtain

$$
\dot{u}_{1}^{\prime m h}=\left(\dot{w}_{1}^{\prime n j} / \dot{P}_{1}^{\prime n}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\mu_{0}^{n j, i k}}{\phi_{1}^{i k}}\left(\dot{u}_{2}^{\prime i k}\right)^{\beta / \nu}\right)^{\nu}
$$

This last step uses the fact that $\left(w_{0}^{\prime n j} / P_{0}^{\prime n}\right)=\left(w_{0}^{n j} / P_{0}^{n}\right)$, and $u_{0}^{\prime m h}=u_{0}^{m h}$. Now take this expression for $\dot{u}_{1}^{\prime m h}$ relative to the equilibrium condition for $\dot{u}_{1}^{m h}$, namely

$$
\dot{u}_{1}^{m h}=\left(\dot{w}_{1}^{n j} / \dot{P}_{1}^{n}\right)\left[\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{0}^{n j, i k}\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}\right]^{\nu}
$$

to obtain

$$
\frac{\dot{u}_{1}^{\prime m h}}{\dot{u}_{1}^{m h}}=\frac{\left(\dot{w}_{1}^{\prime n j} / \dot{P}_{1}^{\prime n}\right)}{\left(\dot{w}_{1}^{n j} / \dot{P}_{1}^{n}\right)}\left(\frac{\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\mu_{0}^{n j, i k}}{\phi_{1}^{i k}}\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}}{\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{0}^{n j, i k}\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}}\right)^{\nu},
$$

or

$$
\hat{u}_{1}^{m h}=\left(\hat{w}_{1}^{n j} / \hat{P}_{1}^{n}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\mu_{0}^{n j, i k}}{\phi_{1}^{i k}} \frac{\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{0}^{n j, m h}\left(\dot{u}_{2}^{m h}\right)^{\beta / \nu}}\right)^{\nu} .
$$

Now multiply and divide each term in the summation by $\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}$ to obtain

$$
\hat{u}_{1}^{m h}=\left(\hat{w}_{1}^{n j} / \hat{P}_{1}^{n}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\left(\dot{u}_{2}^{i k} / \dot{u}_{2}^{i k}\right)^{\beta / \nu}}{\phi_{1}^{i k}} \frac{\mu_{0}^{n j, i k}\left(\dot{u}_{2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{0}^{n j, m h}\left(\dot{u}_{2}^{m h}\right)^{\beta / \nu}}\right)^{\nu},
$$

and use the equilibrium condition for $\mu_{1}^{n j, i k}$ to get

$$
\hat{u}_{1}^{m h}=\left(\hat{w}_{1}^{n j} / \hat{P}_{1}^{n}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \frac{\mu_{1}^{n j, i k}}{\phi_{1}^{i k}}\left(\hat{u}_{2}^{i k}\right)^{\beta / \nu}\right)^{\nu} .
$$

Finally note that $\left(\mu_{1}^{n j, i k} / \phi_{1}^{i k}\right)=\vartheta_{0}^{n j, i k}$, and that substituting this we obtain $(A 2-27)$.
To obtain ( $A 2-26$ ) take

$$
\mu_{1}^{\prime n j, i k}=\frac{\left(u_{2}^{\prime i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J}\left(u_{2}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}},
$$

then use the equilibrium condition for $\mu_{1}^{n j, i k}$

$$
\begin{aligned}
\frac{\mu_{1}^{\prime n j, i k}}{\mu_{1}^{n j, i k}} & =\frac{\frac{\left(u_{2}^{\prime i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}{\left(u_{2}^{i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \frac{\left(u_{2}^{\prime m h}\right)^{\beta / \nu} \exp \left(\tau^{n j, m h}\right)^{-1 / \nu}}{\sum_{i=1}^{N} \sum_{k=0}^{J}\left(u_{2}^{i k}\right)^{\beta / \nu} \exp \left(\tau^{n j, i k}\right)^{-1 / \nu}}} \\
& =\frac{\left(u_{2}^{\prime i k} / u_{2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{1}^{n j, m h}\left(u_{2}^{\prime m h} / u_{2}^{m h}\right)^{\beta / \nu}}
\end{aligned}
$$

and then multiply and divide the numerator and each expression in the summation of the denomi-
nator by $\left(u_{1}^{i k} / u_{1}^{i k}\right)^{\beta / \nu}$ to obtain,

$$
\frac{\mu_{1}^{\prime n j, i k}}{\mu_{1}^{n j, i k}}=\frac{\left(u_{1}^{\prime i k} / u_{1}^{i k}\right)^{\beta / \nu}\left(\hat{u}_{2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{1}^{n j, m h}\left(u_{1}^{\prime m h} / u_{1}^{m h}\right)^{\beta / \nu}\left(\hat{u}_{2}^{m h}\right)^{\beta / \nu}}
$$

using the definition of $\vartheta_{0}^{n j, i k}$ we obtain $(A 2-26)$.

## APPENDIX 3: EXTENSIONS

### 3.1 The One-Sector Trade and Migration Model

In this appendix, we present the one-sector model. To simplify our notation, we index the $N$ labor markets by $\ell, m$ and $n$. As in the main text, we let $\ell=0$ denote non-employment status.

## Households (Dynamic Problem)

The problem of the agent is as follows:

$$
\mathrm{v}_{t}^{\ell}=\log \left(w_{t}^{\ell} / P_{t}^{\ell}\right)+\max _{\{m\}_{m=1}^{N}}\left\{\beta E\left[\mathrm{v}_{t+1}^{m}\right]-\tau^{\ell, m}+\nu \epsilon_{t}^{m}\right\}
$$

After using the properties of the Extreme Value distribution, we find that the expected lifetime utility of a worker is given by

$$
V_{t}^{\ell}=\log \left(w_{t}^{\ell} / P_{t}^{\ell}\right)+\nu \log \left(\sum_{m=1}^{N} \exp \left(\beta V_{t+1}^{m}-\tau^{\ell, m}\right)^{1 / \nu}\right)
$$

Similarly, the transition matrix, or choice probability, is given by

$$
\mu_{t}^{\ell, m}=\frac{\exp \left(\beta V_{t+1}^{m}-\tau^{\ell, m}\right)^{1 / \nu}}{\sum_{n=1}^{N} \exp \left(\beta V_{t+1}^{n}-\tau^{\ell, n}\right)^{1 / \nu}}
$$

and the evolution of the distribution of labor across markets is given by

$$
L_{t+1}^{\ell}=\sum_{m=1}^{N} \mu_{t}^{m, \ell} L_{t}^{m}
$$

## Production (Temporary Equilibrium)

As in the main text, at each $\ell$ there is a continuum of perfectly competitive intermediate good producers with constant returns to scale technology and idiosyncratic productivity $z^{\ell} \sim$ Fréchet $(1, \theta)$. In particular, the problem of an intermediate good producer is as follows,

$$
\min _{\left\{l_{t}^{\ell}, M_{t}^{\ell}\right\}} w_{t}^{\ell} l_{t}^{\ell}+P_{t}^{\ell} M_{t}^{\ell}, \text { subject to } q_{t}^{\ell}\left(z^{\ell}\right)=z^{\ell} A^{\ell}\left(l_{t}^{\ell}\right)^{\gamma}\left(M_{t}^{\ell}\right)^{1-\gamma}
$$

where $M_{t}^{\ell}$ is the demand for material inputs, and $A^{\ell}$ is fundamental TFP in $\ell$. As it is shown shortly, material inputs are produced with intermediates from every other market in the world. Denote by $P_{t}^{\ell}$ the price of materials produce in $\ell$. Therefore, the unit price of an input bundle is given by

$$
x_{t}^{\ell}=B^{\ell}\left(w_{t}^{\ell}\right)^{\gamma}\left(P_{t}^{\ell}\right)^{1-\gamma}
$$

where $B^{\ell}$ is a constant.
The unit cost of an intermediate good $z^{\ell}$ at time $t$ is

$$
\frac{x_{t}^{\ell}}{z^{\ell} A^{\ell}}
$$

Competition implies that the price paid for a particular variety is in market $\ell$ is given by

$$
p_{t}^{\ell}(z)=\min _{m \in N} \kappa^{\ell, m} x_{t}^{m} z^{\ell} A^{\ell}
$$

Final goods in $\ell$ are produced by aggregating intermediate inputs from all $\ell$. Let $Q_{t}^{\ell}$ be the quantity of final goods in $\ell$ and $\tilde{q}_{t}^{\ell}(z)$ the quantity demanded of an intermediate variety such that the vector of productivity draws received by the different $\ell$ is $z=\left(z^{1}, z^{2}, \ldots, z^{N}\right)$. The production of final goods is given by

$$
Q_{t}^{\ell}=\left(\int_{R_{++}^{N}}\left(\tilde{q}_{t}^{\ell}(z)\right)^{1-1 / \eta} d \phi(z)\right)^{\eta /(\eta-1)},
$$

where $\phi(z)=\exp \left\{-\sum_{\ell=1}^{N}\left(z^{\ell}\right)^{-\theta}\right\}$ is the joint distribution function over the vector $z$. Given the properties of the Fréchet distribution, the price of the final good $\ell$ at time $t$ is

$$
P_{t}^{\ell}=\Gamma\left(\sum_{m=1}^{N}\left(\frac{x_{t}^{m} \kappa^{\ell, m}}{A^{m}}\right)^{-\theta}\right)^{-1 / \theta}
$$

where $\Gamma$ is a constant given by the value of a Gamma function evaluated at $1+(1-\eta / \theta)$ and we assume that $1+\theta>\eta$. The share of total expenditure in market $\ell$ on goods from $m$, is given by

$$
\pi_{t}^{\ell, m}=\frac{\left(x_{t}^{m} \kappa^{\ell, m} / A^{m}\right)^{-\theta}}{\sum_{n=1}^{N}\left(x_{t}^{n} \kappa^{\ell, n} / A^{n}\right)^{-\theta}} .
$$

## Market Clearing

Let $X_{t}^{\ell}$ denote the total expenditure on final goods in $\ell$. Then, the goods market clearing condition is given by

$$
X_{t}^{\ell}=(1-\gamma) \sum_{m=1}^{N} \pi_{t}^{m, \ell} X_{t}^{m}+w_{t}^{\ell} L_{t}^{\ell}
$$

Labor market clearing in $\ell$ is

$$
w_{t}^{\ell} L_{t}^{\ell}=\gamma \sum_{m=1}^{N} \pi_{t}^{m, \ell} X_{t}^{m}
$$

We now provide a formal definition of the equilibrium together with the equilibrium conditions.
Definition Given $\left(L_{0}, \Theta\right)$, a sequential competitive equilibrium of the one sector model is a sequence of $\left\{L_{t}, \mu_{t}, V_{t}, w\left(L_{t}, \Theta\right)\right\}_{t=0}^{\infty}$ that solves

$$
\begin{gathered}
V_{t}^{\ell}=\log \left(w_{t}^{\ell} / P_{t}^{\ell}\right)+\nu \log \left(\sum_{m=1}^{N} \exp \left(\beta V_{t+1}^{m}-\tau^{\ell, m}\right)^{1 / \nu}\right) \\
\mu_{t}^{\ell, m}=\frac{\exp \left(\beta V_{t+1}^{m}-\tau^{\ell, m}\right)^{1 / \nu}}{\sum_{n=1}^{N} \exp \left(\beta V_{t+1}^{n}-\tau^{\ell, n}\right)^{1 / \nu}} \\
L_{t+1}^{\ell}=\sum_{m=1}^{N} \mu_{t}^{m, \ell} L_{t}^{m}
\end{gathered}
$$

where $w_{t}^{\ell} / P_{t}^{\ell}$ is the solution to the temporary equilibrium at each $t$ and solves

$$
x_{t}^{\ell}=B^{\ell}\left(w_{t}^{\ell}\right)^{\gamma}\left(P_{t}^{\ell}\right)^{1-\gamma}
$$

$$
\begin{gathered}
P_{t}^{\ell}=\Gamma\left(\sum_{m=1}^{N}\left(B^{m}\left(w_{t}^{m}\right)^{\gamma}\left(P_{t}^{m}\right)^{1-\gamma}\right)^{-\theta}\left(\kappa^{\ell, m} / A^{m}\right)^{-\theta}\right)^{-1 / \theta} \\
\pi_{t}^{\ell, m}=\frac{\left(x_{t}^{m} \kappa^{\ell, m} / A^{m}\right)^{-\theta}}{\sum_{n=1}^{N}\left(x_{t}^{n} \kappa^{\ell, n} / A^{n}\right)^{-\theta}} \\
w_{t}^{m} L_{t}^{m}=\sum_{\ell=1}^{N} \frac{\left(x_{t}^{m} \kappa^{\ell, m} / A^{m}\right)^{-\theta}}{\sum_{n=1}^{N}\left(x_{t}^{n} \kappa^{\ell, n} / A^{n}\right)^{-\theta}} w_{t}^{\ell} L_{t}^{\ell}
\end{gathered}
$$

### 3.2 The CES Version of the Model

In this appendix, we extend to model to the case of a constant elasticity of substitution (CES) utility function. In particular, we allow for different degree of substitutability across manufacturing and non-manufacturing industries. Preferences over the basket of final local goods is given by $U\left(C_{t}^{n j}\right)$ where

$$
\begin{equation*}
C_{t}^{n j}=\left(\left(\varkappa^{n j}\right)^{1 / \eta}\left(c_{t}^{n j, M}\right)^{\frac{\eta-1}{\eta}}+\left(1-\varkappa^{n j}\right)^{1 / \eta}\left(c_{t}^{n j, S}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{A7-1}
\end{equation*}
$$

where $c_{t}^{n j, M}$ and $c_{t}^{n j, S}$ are Cobb-Douglas aggregates of consumption of manufacturing goods and non-manufacturing goods, respectively, in market $n j$ at time $t$, given by

$$
c_{t}^{n j, M}=\prod_{k \in M}\left(c_{t}^{n j, k}\right)^{\alpha^{k}} ; c_{t}^{n j, S}=\prod_{k \in S}\left(c_{t}^{n j, k}\right)^{\alpha^{k}},
$$

with $\sum_{k \in M} \alpha^{k}=1 ; \sum_{k \in S} \alpha^{k}=1$. The price index of final goods in market $n j$ is the given by

$$
\begin{aligned}
& P_{t}^{n j}=\left(\varkappa^{n j}\left(p_{t}^{n j, M}\right)^{1-\eta}+\left(1-\varkappa^{n j}\right)\left(p_{t}^{n j, S}\right)^{1-\eta}\right)^{\frac{1}{\eta-1}} \\
& p_{t}^{n j, M}=\prod_{k \in M}\left(p_{t}^{n j, k} / \alpha^{k}\right)^{\alpha^{k}} ; p_{t}^{n j, S}=\prod_{k \in S}\left(p_{t}^{n j, k} / \alpha^{k}\right)^{\alpha^{k}},
\end{aligned}
$$

As in Section 2, the equilibrium of the economy is given by equations (5) to (10), and (2) to (4) subject to the utility function given by $U\left(C_{t}^{n j}\right)$ with $C_{t}^{n j}$ given by equation $(A 7-1)$.

## Equilibrium Conditions in Relative Time Differences.-

As before, we denote by $\dot{y}_{t+1} \equiv y_{t+1} / y_{t}$ the change in any variable between to periods of time in the baseline economy, and by $\dot{y}_{t+1}^{\prime} \equiv y_{t+1}^{\prime} / y_{t}^{\prime}$ the change in time in the counterfactual economy. The relative change in variable $y$ between the counterfactual economy and the baseline economy is given by $\hat{y}_{t+1} \equiv \dot{y}_{t+1}^{\prime} / \dot{y}_{t}$. Therefore, the relative change in the local price index between the counterfactual economy and the baseline economy is given by

$$
\hat{P}_{t+1}^{n j}=\left(\alpha_{t}^{\prime n j, M} \dot{\alpha}_{t+1}^{n j, M}\left(\hat{p}_{t+1}^{n j, M}\right)^{1-\eta}+\alpha_{t}^{\prime n j, S} \dot{\alpha}_{t+1}^{n j, S}\left(\hat{p}_{t+1}^{n j, S}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}},
$$

where $\alpha_{t}^{n j, M}$ and $\alpha_{t}^{n j, S}$ are the final expenditure share of manufacturing and non-manufacturing
goods, respectively, given by

$$
\begin{gathered}
\alpha_{t}^{n j, M}=\frac{p_{t}^{n j, M} c_{t}^{n j, M}}{P_{t}^{n j} C_{t}^{n j}}=\varkappa^{n j}\left(\frac{p_{t}^{n j, M}}{P_{t}^{n j}}\right)^{1-\eta}, \\
\alpha_{t}^{n j, S}=\frac{p_{t}^{n j, S} c_{t}^{n j, S}}{P_{t}^{n j} C_{t}^{n j}}=\left(1-\varkappa^{n j}\right)\left(\frac{p_{t}^{n j, S}}{P_{t}^{n j}}\right)^{1-\eta},
\end{gathered}
$$

with $\alpha_{t}^{n j, M}+\alpha_{t}^{n j, S}=1$. It follows that $\alpha_{t}^{\prime n j, M}=\alpha_{t-1}^{\prime n j, M} \dot{\alpha}_{t}^{n j, M}\left(\frac{\hat{p}_{t}^{n j, M}}{\hat{P}_{t}^{n j}}\right)^{1-\eta}$, and that $\alpha_{t}^{\prime n j, S}=$ $\alpha_{t-1}^{\prime n j, S} \dot{\alpha}_{t}^{n j, S}\left(\frac{\hat{p}_{t}^{n j, S}}{\hat{P}_{t}^{n j}}\right)^{1-\eta}$. Finally, we have

$$
\begin{aligned}
& \hat{p}_{t+1}^{n j, M}=\prod_{k \in M}\left(\hat{p}_{t+1}^{n j, k}\right)^{\alpha^{k}} \\
& \hat{p}_{t+1}^{n j, S}=\prod_{k \in S}\left(\hat{p}_{t+1}^{n j, k}\right)^{\alpha^{k}} .
\end{aligned}
$$

The rest of the equilibrium condition in relative time differences are the same as those derived in Section 3.

### 3.3 Additional Sources of Persistence to the Model

In the model developed in Section 2, the i.i.d taste shocks as well as the asymmetric migration costs are a source of persistence in the migration choice. There is, therefore, a gradual adjustment of shocks to the new steady state in the model. In this section, we extend the model to incorporate additional sources of persistence, and as a robustness exercise, we quantify the effects of the China shock in these alternative models. Importantly, we show how dynamic hat algebra can be applied to these alternative models.

### 3.3.1 Persistence Due to Local Preferences (Amenities).-

In the first extension of our model, we add additional persistence by introducing a fixed individual heterogeneity to preferences. Concretely, we assume that the utility of residing in a particular location includes preferences for amenities, which are location specific and time invariant. Therefore, we now have that

$$
U\left(C_{t}^{n j}, B^{n}\right)=\log \left(C_{t}^{n j}\right)+\log B^{n}
$$

where $B^{n}$ is a local, time invariant amenity in location $n$. As we can see, this additional preference for a location adds more persistence to the migration decision, as agents are going to command a larger wage differential, and a larger idiosyncratic draw in order to find it optimal to migrate. Notice also, that a model with fixed preferences over locations is isomorphic to the model in Section 2 if we apply a suitable renormalization of migration $\operatorname{costs} \tau^{n j, i k}$. In particular, the value of a household in location $n j$ at time $t$ is now given by

$$
v_{t}^{n j}=\log C_{t}^{n j}+\log B^{n}+\max _{\{i, k\}_{i=1, k=0}^{N J,}}\left\{\beta E\left[v_{t+1}^{i k}\right]-\tau^{n j, i k}+\nu \epsilon_{t}^{i k}\right\} .
$$

We can now define $\bar{\tau}^{n j, i k}=\tau^{n j, i k}-\log B^{n}$, so that the value function becomes isomorphic that in Section 2. The only distinction is that the implied level of migration costs in the model with fixed
preferences for locations will be lower than in the model of Section 2. This distinction is important when estimating the model in levels. However, dynamic hat algebra will differentiate out the levels of $\tau^{n j, i k}$ and $B^{n}$, so that all propositions in Section 3 still hold.

### 3.3.2 Additional Source of Persistence in Household Choices.-

An alternative extension of our model is to consider the case in which agents have a more persistent idiosyncratic shock, that is, their idiosyncratic preferences for locations do not change every period. We now proceed to characterize the problem allowing for a particular type of serial correlation of shocks. Consider the value of an agent located at $n j$, and assume that we start the economy with a given allocation of workers across markets. This initial allocation is assumed to be determined by an initial draw of idiosyncratic shocks $\epsilon_{0}^{i k}$. Now suppose that at each moment in time agents are subject to a Poisson process that determines the arrival of a new draw of the idiosyncratic shock. In particular, we assume with probability $\rho$ that the household does not receive a preference draw, and therefore stays in the same labor market. On the other hand, we assume a probability of $1-\rho$ that the household receives a new draw, although not all agents with a new draw will migrate. We assume that the likelihood of these events are not location specific.

As before, let $V_{t}^{n j}=E\left[v_{t}^{n j}\right]$. The value function can be then written as

$$
V_{t}^{n j}=U\left(C_{t}^{n j}\right)+\rho \beta V_{t+1}^{n j}+(1-\rho) \nu \log \left(\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}\right) .
$$

The fraction of households that stay in market $n j$ at time $t$ is now given by

$$
\mu_{t}^{n j, n j}=\rho+\frac{(1-\rho) \exp \left(\beta V_{t+1}^{n j}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}},
$$

while the fraction of workers that move to market $i k$ is given by

$$
\mu_{t}^{n j, i k}=\frac{(1-\rho) \exp \left(\beta V_{t+1}^{i k}-\tau^{n j, i k}\right)^{1 / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{m h}-\tau^{n j, m h}\right)^{1 / \nu}}
$$

We then define the choice probabilities conditional on receiving a new idiosyncratic preference draw as

$$
\begin{gathered}
\tilde{\mu}_{t}^{n j, n j}=\frac{\mu_{t}^{n j, n j}-\rho}{1-\rho} \\
\tilde{\mu}_{t}^{n j, i k}=\frac{\mu_{t}^{n j, i k}}{1-\rho}
\end{gathered}
$$

The evolution of employment at market $n j$ is given by

$$
L_{t+1}^{n j}=\rho L_{t}^{n j}+(1-\rho) \sum_{i=1}^{N} \sum_{k=0}^{J} \tilde{\mu}_{t}^{i k, n j} L_{t}^{i k} .
$$

This is the system of equations that defines the equilibrium of the household's dynamic system in a model with persistent idiosyncratic shocks. This equilibrium condition shows how adding persistence affects the evolution of the state variable of the economy. It is precisely from the fact that only a share $(1-\rho)$ of households have a new idiosyncratic draw that this is the share of agents that decide to reallocate across markets over time. Of course, not all of the agents with a new draw migrate. In fact, a fraction $(1-\rho) \tilde{\mu}_{t}^{n j, n j}$ decides to stay.

Note also that the value function can be re-expressed as

$$
V_{t}^{n j}=U\left(C_{t}^{n j}\right)+\beta V_{t+1}^{n j}-(1-\rho) \nu \log \tilde{\mu}_{t}^{n j, n j} .
$$

This equation shows how the persistent parameter $\rho$ re-scales the option value of migration. Importantly, notice that in the model with this additional shock, $1 / \nu$ is the migration-cost elasticity conditional on receiving an idiosyncratic preference draw, while in the model where $\rho=0,1 / \nu$ is the unconditional migration-cost elasticity. We now show these equilibrium conditions in relative time differences and that all propositions in Section 3 still hold.

### 3.3.3 Equilibrium Conditions in Relative Time Differences.-

As before, let $\hat{y}_{t+1} \equiv \dot{y}_{t+1}^{\prime} / \dot{y}_{t+1}$ be the proportional change between the counterfactual equilibrium $\dot{y}_{t+1}^{\prime} \equiv y_{t+1}^{\prime} / y_{t}^{\prime}$, and the baseline equilibrium $\dot{y}_{t+1} \equiv y_{t+1} / y_{t}$ across time. The expected value of a household in market $n j$ at time $t$ in a model with the additional shock, expressed in relative time differences is then given by

$$
\hat{u}_{t}^{n j}=\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)\left(\hat{u}_{t+1}^{n j}\right)^{\beta \rho / \nu}\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \tilde{\mu}_{t-1}^{n j, i k} \dot{\tilde{\mu}}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}\right)^{(1-\rho) \nu}
$$

The probability choice $\mu_{t}^{n j, i k}$ in relative time differences is given by

$$
\tilde{\mu}_{t}^{n j, i k}=\frac{\tilde{\mu}_{t-1}^{\prime n j, i k} \dot{\tilde{\mu}}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}}{\sum_{h=1}^{N} \sum_{m=0}^{J} \tilde{\mu}_{t-1}^{\prime n j, m h} \dot{\tilde{\mu}}_{t}^{n j, m h}\left(\hat{u}_{t+1}^{m h}\right)^{\beta / \nu}} .
$$

The evolution of the state variable $L_{t+1}^{n j}$ is given by

$$
L_{t+1}^{n j}=\rho L_{t}^{n j}+(1-\rho) \sum_{i=1}^{N} \sum_{k=0}^{J} \tilde{\mu}_{t}^{i k, n j} L_{t}^{i k}
$$

where $\hat{\omega}^{n j}\left(\hat{L}_{t}, \hat{\Theta}_{t}\right)$ solves the temporary equilibrium expressed in relative time differences as before. Given that we do not need to estimate levels of migration costs in this dynamic system, and that the equilibrium conditions of the static subproblem have not changed, all propositions of Section 3 still hold.

### 3.4 Intensive Margin: Elastic Labor Supply

In this appendix, we extend the model to allow for an elastic labor supply by each household. Specifically, we introduce labor-leisure decisions into each household's utility function. As before, we denote $\dot{y}_{t+1} \equiv y_{t+1} / y_{t}$ to be the change in any variable between to periods of time in the baseline economy, and $\dot{y}_{t+1}^{\prime} \equiv y_{t+1}^{\prime} / y_{t}^{\prime}$ to be the change in time in the counterfactual economy. The relative change in variable $y$ between the counterfactual economy and the baseline economy is given by $\hat{y}_{t+1} \equiv \dot{y}_{t+1}^{\prime} / \dot{y}_{t}$. We also define $\hat{U}_{t+1}=\left(U_{t+1:}^{\ell^{\prime}}-U_{t:}^{\ell^{\prime}}\right)-\left(U_{t+1:}^{\ell}-U_{t:}^{\ell}\right)$. Therefore, the relative change in utility between the counterfactual economy and the baseline economy is given by

$$
\begin{equation*}
\hat{U}_{t+1}=\log \frac{\hat{w}_{t+1}^{\ell}}{\hat{P}_{t+1}^{\ell}} \tag{A7-2}
\end{equation*}
$$

and the rest of the equilibrium condition in relative time differences are the same as those derived in Section 3.

In what follows, we present alternative specifications for the utility function that have been considered in the macro literature.

### 3.4.1 Case 1.-

Consider the following alternative utility function

$$
U\left(C_{t}^{\ell}, l_{t}^{\ell}\right)=\log C_{t}^{\ell}+\frac{\left(l_{t}^{\ell}\right)^{1+1 / \phi}}{1+1 / \phi}
$$

The household's problem is given by

$$
\max _{\left\{C_{t}^{\ell}, l_{t}^{\ell}\right\}} \log C_{t}^{\ell}+\frac{\left(l_{t}^{\ell}\right)^{1+1 / \phi}}{1+1 / \phi} \text { s.t. } P_{t}^{\ell} C_{t}^{\ell}=w_{t}^{\ell} l_{t}^{\ell} \text {, with } 0 \leq l_{t}^{\ell} \leq 1 \text {, }
$$

and the optimality conditions are given by

$$
C_{t}^{\ell}=\frac{w_{t}^{\ell}}{P_{t}^{\ell}}, \text { and } l_{t}^{\ell}=1
$$

Using the optimality conditions, we can express the indirect utility as

$$
U_{t:}^{\ell}=\log \frac{w_{t}^{\ell}}{P_{t}^{\ell}} .
$$

The indirect utility in relative time differences is given by

$$
\hat{U}_{t+1}=\log \frac{\hat{w}_{t+1}^{\ell}}{\hat{P}_{t+1}^{\ell}} .
$$

### 3.4.2 Case 2.-

Consider the following utility function

$$
U\left(C_{t}^{\ell}, l_{t}^{\ell}\right)=\log C_{t}^{\ell}+\phi \log \left(1-l_{t}^{\ell}\right),
$$

where $C_{t}^{\ell}$ is the amount of consumption by households located at $\ell$ at time $t$. Households are endowed with one unit of labor; thus, $1-l_{t}^{\ell}$ is the amount of leisure consumed in location $\ell$ at time $t$. The elasticity of utility with respect to leisure is given by $\phi$. At each time $t$ households decide consumption and the amount of time devoted to leisure, and the household's problem is then given by:

$$
\max _{\left\{C_{t}^{\ell}, l_{t}^{\ell}\right\}} \log C_{t}^{\ell}+\phi \log \left(1-l_{t}^{\ell}\right) \text { s.t. } P_{t}^{\ell} C_{t}^{\ell}=w_{t}^{\ell} t_{t}^{\ell} \text {, with } 0 \leq l_{t}^{\ell} \leq 1
$$

The optimality conditions are given by

$$
\begin{aligned}
C_{t}^{\ell} & =\frac{1}{1+\phi} \frac{w_{t}^{\ell}}{P_{t}^{\ell}} \\
l_{t}^{\ell} & =\frac{1}{1+\phi} .
\end{aligned}
$$

Using the optimality conditions, we can express the indirect utility as

$$
U_{t:}^{\ell}=\log \frac{1}{1+\phi} \frac{w_{t}^{\ell}}{P_{t}^{\ell}}+\phi \log \frac{1}{1+\phi} .
$$

The indirect utility in relative time differences is given by

$$
\hat{U}_{t+1}=\log \frac{\hat{w}_{t+1}^{\ell}}{\hat{P}_{t+1}^{\ell}}
$$

### 3.4.3 Case 3.-

Consider the following alternative utility function

$$
U\left(C_{t}^{\ell}, l_{t}^{\ell}\right)=\log C_{t}^{\ell}+B l_{t}^{\ell}
$$

In this case, the household's problem is given by

$$
\max _{\left\{C_{t}^{\ell}, \nu_{t}^{\ell}\right\}} \log C_{t}^{\ell}+B l_{t}^{\ell} \text { s.t. } P_{t}^{\ell} C_{t}^{\ell}=w_{t}^{\ell} l_{t}^{\ell} \text {, with } 0 \leq l_{t}^{\ell} \leq 1,
$$

and the optimality conditions are given by

$$
C_{t}^{\ell}=\frac{1}{B} \frac{w_{t}^{\ell}}{P_{t}^{\ell}}, l_{t}^{\ell}=\frac{1}{B} .
$$

In this case, the indirect utility is given by

$$
U_{t:}^{\ell}=\log \frac{1}{B} \frac{w_{t}^{\ell}}{P_{t}^{\ell}}+\log \frac{1}{B}
$$

The indirect utility in relative time differences is given by

$$
\hat{U}_{t+1}=\log \frac{\hat{w}_{t+1}^{\ell}}{\hat{P}_{t+1}^{\ell}}
$$

## APPENDIX 4: SOLUTION ALGORITHM

## Part I: Solving for the sequential competitive equilibrium

The strategy to solve the model given an initial allocation of the economy, $\left(L_{0}, \pi_{0}, X_{0}, \mu_{-1}\right)$, and given an anticipated convergent sequence of changes in fundamentals, $\left\{\dot{\Theta}_{t}\right\}_{t=1}^{\infty}$, is as follows:

1. Initiate the algorithm at $t=0$ with a guess for the path of $\left\{\dot{u}_{t+1}^{n j}(0)\right\}_{t=0}^{T}$, where the superscript (0) indicates that it is a guess. The path should converge to $\dot{u}_{T+1}^{n j(0)}=1$ for a sufficiently large $T$. Take as given the set of initial conditions $L_{0}^{n j}, \mu_{-1}^{n j, i k}, \pi_{0}^{n i, n j}, w_{0}^{n j} L_{0}^{n j}, r_{0}^{n j} H_{0}^{n j}$.
2. For all $t \geq 0$, use $\left\{\dot{u}_{t+1}^{n j(0)}\right\}_{t=0}^{T}$ and $\mu_{-1}^{n j, i k}$ to solve for the path of $\left\{\mu_{t}^{n j, i k}\right\}_{t=0}^{T}$ using equation (16).
3. Use the path for $\left\{\mu_{t}^{n j, i k}\right\}_{t=0}^{T}$ and $L_{0}^{n j}$ to get the path for $\left\{L_{t+1}^{n j}\right\}_{t=0}^{T}$ using equation (18).
4. Solving for the temporary equilibrium:
(a) For each $t \geq 0$, given $\dot{L}_{t+1}^{n j}$, guess a value for $\dot{w}_{t+1}^{n j}$.
(b) Obtain $\dot{x}_{t+1}^{n j}, \dot{P}_{t+1}^{n j}$, and $\pi_{t+1}^{n j, i j}$ using equations (11), (12), and (13). ${ }^{59}$
(c) Use $\pi_{t+1}^{n j, i j}, \dot{w}_{t+1}^{n j}$, and $\dot{L}_{t+1}^{n j}$ to get $X_{t+1}^{n j}$ using equation (14).
(d) Check if the labor market is in equilibrium using equation (15), and if not, go back to step (a) and adjust the initial guess for $\dot{w}_{t+1}^{n j}$ until labor markets clear.
(e) Repeat steps (a) through (d) for each period $t$ and obtain paths for $\left\{\dot{w}_{t+1}^{n j}, \dot{P}_{t+1}^{n j}\right\}_{t=0}^{T}$.
5. For each $t$, use $\mu_{t}^{n j, i k}, \dot{w}_{t+1}^{n j}, \dot{P}_{t+1}^{n j}$, and $\dot{u}_{t+2}^{n j(0)}$ to solve backwards for $\dot{u}_{t+1}^{n j(1)}$ using equation (17). This delivers a new path for $\left\{\dot{u}_{t+1}^{n j(1)}\right\}_{t=0}^{T}$, where the superscript 1 indicates an updated value for $u$.
6. Take the path for $\left\{\dot{u}_{t+1}^{n j(1)}\right\}_{t=0}^{T}$ as the new set of initial conditions.
7. Check if $\left\{\dot{u}_{t+1}^{n j(1)}\right\}_{t=0}^{T} \simeq\left\{\dot{u}_{t+1}^{n j(0)}\right\}_{t=0}^{T}$. If not, go back to step 1 and update the initial guess.

## Part II: Solving for counterfactuals

Denote by $\hat{y}_{t+1} \equiv \dot{y}_{t+1}^{\prime} / \dot{y}_{t+1}$ to the proportional change between the counterfactual equilibrium, $\dot{y}_{t+1}^{\prime} \equiv y_{t+1}^{\prime} / y_{t}^{\prime}$, to the baseline economy, $\dot{y}_{t+1} \equiv y_{t+1} / y_{t}$ across time. With this notation, $\hat{\Theta}_{t+1}$ is the proportional counterfactual changes in fundamentals across time relative to the baseline economy, namely $\hat{\Theta}_{t+1}=\dot{\Theta}_{t+1}^{\prime} / \dot{\Theta}_{t+1}$.

To compute counterfactuals we assume that agents at $t=0$ are not anticipating the change in the path of fundamentals and that at $t=1$ agents learn about the entire future counterfactual sequence of $\left\{\Theta_{t}^{\prime}\right\}_{t=1}^{\infty}$.

[^30]Take as given a baseline economy, $\left\{L_{t}, \mu_{t-1}, \pi_{t}, X_{t}\right\}_{t=0}^{\infty}$ and a counterfactual convergent sequence of changes in fundamentals, $\left\{\hat{\Theta}_{t}\right\}_{t=1}^{\infty}$.

To solve for the counterfactual equilibrium, proceed as follows:

1. Initiate the algorithm at $t=0$ with a guess for the path of $\left\{\hat{u}_{t+1}^{n, j(0)}\right\}_{t=0}^{T}$, where the superscript (0) indicates it is a guess. The path should converge to $\hat{u}_{T+1}^{n j(0)}=1$ for a sufficiently large $T$. Take as given the initial conditions $L_{0}^{n j}, \mu_{-1}^{n j, i k}, \pi_{0}^{n j, i j}, w_{0}^{n j} L_{0}^{n j}, r_{0}^{n j} H_{0}^{n j}$, the baseline economy, $\left\{\dot{L}_{t}, \dot{\mu}_{t-1}, \dot{\pi}_{t}, \dot{X}_{t}\right\}_{t=0}^{\infty}$ and the solution to the sequential competitive equilibrium of the baseline economy.
2. For all $t \geq 0$, use $\left\{\hat{u}_{t+1}^{n j(0)}\right\}_{t=0}^{T}$ and $\left\{\dot{\mu}_{t-1}\right\}_{t=0}^{\infty}$ to solve for the path of $\left\{\mu_{t}^{\prime n j}\right\}_{t=0}^{T}$ using equations: For $t=0$

$$
\begin{gathered}
\hat{u}_{0}^{n j(0)}=1 \\
\mu_{0}^{n j, i k}=\mu_{0}^{n j, i k} \\
L_{1}^{\prime n j}=L_{1}^{n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{0}^{i k, n j} L_{0}^{i k}
\end{gathered}
$$

For period $t=1$

$$
\mu_{1}^{\prime n j, i k}=\frac{\vartheta_{0}^{n j, i k}\left(\hat{u}_{2}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \vartheta_{0}^{n j, m h}\left(\hat{u}_{2}^{m h}\right)^{\beta / \nu}}
$$

where

$$
\vartheta_{0}^{n j, i k(0)}=\mu_{1}^{n j, i k}\left(\hat{u}_{1}^{i k(0)}\right)^{\beta / \nu}
$$

For period $t \geq 1$ :

$$
\mu_{t}^{\prime n j, i k}=\frac{\mu_{t-1}^{\prime n j, i k} \dot{\mu}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k}\right)^{\beta / \nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{\prime n j, m h} \dot{\mu}_{t}^{n j, m h}\left(\hat{u}_{t+1}^{m h}\right)^{\beta / \nu}} .
$$

3. Use the path for $\left\{\mu_{t}^{\prime n j, i k}\right\}_{t=0}^{T}$ and $L_{0}^{\prime n j}$ to get the path for $\left\{L_{t+1}^{\prime n j}\right\}_{t=0}^{T}$ using the equation (21) in the paper. That is,

$$
L_{t+1}^{\prime n j}=\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{\prime n j, i k} L_{t}^{\prime i k}
$$

4. Solving for the temporary equilibrium
(a) For each $t \geq 0$, given $\hat{L}_{t+1}^{n j}$, guess a value for $\left\{\hat{w}_{t+1}^{n j}\right\}_{n=1, j=0}^{N, J}$
(b) Obtain $\hat{x}_{t+1}^{n j}, \hat{P}_{t+1}^{n j}$, and $\hat{\pi}_{t+1}^{n j, i j}$ using

$$
\begin{gathered}
\hat{x}_{t+1}^{n j}=\left(\hat{L}_{t+1}^{n j}\right)^{\gamma^{n j} \xi^{n}}\left(\hat{w}_{t+1}^{n j}\right)^{\gamma^{n j}} \prod_{k=1}^{J}\left(\hat{P}_{t+1}^{n k}\right)^{\gamma^{n j, n k}}, \\
\hat{P}_{t+1}^{n j}=\left(\sum_{i=1}^{N} \pi_{t}^{n j, i j} \dot{\pi}_{t+1}^{n j, i j}\left(\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}\right)^{-1 / \theta^{j}},
\end{gathered}
$$

and

$$
\pi_{t+1}^{\prime n j, i j}=\pi_{t}^{\prime n j, i j} \pi_{t+1}^{n j, i j}\left(\frac{\hat{x}_{t+1}^{i j} \hat{\kappa}_{t+1}^{n j, i j}}{\hat{P}_{t+1}^{n j}}\right)^{-\theta^{j}}\left(\hat{A}_{t+1}^{i j}\right)^{\theta^{j} \gamma^{i j}}
$$

(c) Use $\pi_{t+1}^{\prime n j, i j}, w_{t}^{\prime n k} L_{t}^{\prime n k}, \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k}, \hat{w}_{t+1}^{n j}$, and $\hat{L}_{t+1}^{n j}$ to get $X_{t+1}^{\prime n j}$ using equation

$$
X_{t+1}^{\prime n j}=\sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi_{t+1}^{\prime i k, n k} X_{t+1}^{\prime i k}+\alpha^{j}\left(\sum_{k=1}^{J} \hat{w}_{t+1}^{n k} \hat{L}_{t+1}^{n k} w_{t}^{\prime n k} L_{t}^{\prime n k} \dot{w}_{t+1}^{n k} \dot{L}_{t+1}^{n k}+\iota^{n} \chi_{t+1}^{\prime}\right)
$$

where $\chi_{t+1}^{\prime}=\sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{i}} \hat{w}_{t+1}^{i k} \hat{L}_{t+1}^{i k} w_{t}^{\prime i k} L_{t}^{\prime i k} \dot{w}_{t}^{i k} \dot{L}_{t}^{i k}$
(d) Check if the labor market is in equilibrium using a slightly modified version of equation (15), namely

$$
\hat{w}_{t+1}^{n k} \hat{L}_{t+1}^{n k}=\frac{\gamma^{n j}\left(1-\xi^{n}\right)}{w_{t}^{\prime n k} L_{t}^{\prime n k} w_{t+1}^{n k} \dot{L}_{t+1}^{n k}} \sum_{i=1}^{N} \pi_{t+1}^{\prime i j, n j} X_{t+1}^{\prime i j},
$$

and if not go back to step (a) and adjust the initial guess for $\left\{\hat{w}_{t+1}^{n j}\right\}_{n=1, j=0}^{N, J}$ until labor markets clear.
(e) Repeat steps (a) though (d) for each period $t$ and obtain paths for $\left\{\hat{w}_{t+1}^{n j}, \hat{P}_{t+1}^{n j}\right\}_{n=1, j=0, t=0}^{N, J, T}$.
5. For each $t$, use $\mu_{t}^{n j, i k}, \hat{w}_{t+1}^{n j}, \hat{P}_{t+1}^{n j}$, and $\hat{u}_{t+2}^{n j(0)}$ to solve for backwards $\hat{u}_{t+1}^{n j(1)}$ using equations: For periods $t$ where $t \geq 2$

$$
\hat{u}_{t}^{n j(1)}=\left(\frac{\hat{w}_{t}^{n j}}{\hat{P}_{t}^{n}}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t-1}^{\prime n j, i k} \dot{\mu}_{t}^{n j, i k}\left(\hat{u}_{t+1}^{i k(0)}\right)^{\beta / \nu}\right)^{\nu}
$$

For period 1:

$$
\hat{u}_{1}^{n j(1)}=\left(\frac{\hat{w}_{1}^{n j}}{\hat{P}_{1}^{n}}\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \vartheta_{0}^{n j, i k(0)}\left(\hat{u}_{2}^{i k}\right)^{\beta / \nu}\right)^{\nu}
$$

This delivers a new path for $\left\{\hat{u}_{t+1}^{n j(1)}\right\}$, where the superscript 1 indicates an updated value for $\hat{u}$.
6. Take the path for $\left\{\hat{u}_{t+1}^{n j(1)}\right\}$ as the new set of initial conditions.
7. Check if $\left\{\hat{u}_{t+1}^{n j(1)}\right\} \simeq\left\{\hat{u}_{t+1}^{n j(0)}\right\}$. If not, go back to step 1 and update the initial guess.

## APPENDIX 5: DATA

### 5.1 Data Description

5.1.1 List of sectors and countries We calibrate the model to the 50 U.S. states, 37 other countries including a constructed rest of the world, and a total of 22 sectors classified according to the North American Industry Classification System (NAICS) for the year 2000. The list includes 12 manufacturing sectors, 8 service sectors, wholesale and retail trade, and the construction sector. Our selection of the number of sectors and countries was guided by the maximum level of disaggregation at which we were able to collect the production and trade data needed to compute our model. The 12 manufacturing sectors are Food, Beverage, and Tobacco Products (NAICS 311-312); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (NAICS 313-316); Wood Products, Paper, Printing, and Related Support Activities (NAICS 321-323); Petroleum and Coal Products (NAICS 324); Chemical (NAICS 325); Plastics and Rubber Products (NAICS 326); Nonmetallic Mineral Products (NAICS 327); Primary Metal and Fabricated Metal Products (NAICS 331-332); Machinery (NAICS 333); Computer and Electronic Products, and Electrical Equipment and Appliance (NAICS 334-335); Transportation Equipment (NAICS 336); Furniture and Related Products, and Miscellaneous Manufacturing (NAICS 337-339). The 8 service sectors are Transport Services (NAICS 481-488); Information Services (NAICS 511-518); Finance and Insurance (NAICS 521-525); Real Estate (NAICS 531-533); Education (NAICS 61); Health Care (NAICS 621-624); Accommodation and Food Services (NAICS 721-722); Other Services (NAICS $493,541,55,561,562,711-713,811-814)$. We also include the Wholesale and Retail Trade sector (NAICS 42-45), and the Construction sector, as mentioned earlier.

The countries in addition to the United States are Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, Slovak Republic, Slovenia, South Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world.
5.1.2 International trade, production, and input shares across countries International trade flows across sectors and the 38 countries including the United States for the year 2000, $X_{0}^{n j, i j}$ where $n, i$ are the 38 countries in our sample, are obtained from the World Input-Output Database (WIOD). The WIOD provides world input-output tables from 1995 onward. National input-output tables of 40 major countries and a constructed rest of the world are linked through international trade statistics for 35 sectors. For three countries in the database, Luxembourg, Malta, and Latvia, value added and/or gross output data were missing for some sectors; thus, we decided to aggregate these three countries with the constructed rest of the world, which gives us the 38 countries (37 countries and the United States) we used in the paper. From the world input-output table, we know total purchases made by a given country from any other country, including domestic sales, which gives us the bilateral trade flows. ${ }^{60}$

We construct the share of value added in gross output $\gamma^{n j}$, and the material input shares $\gamma^{n j, n k}$ across countries and sectors using data on value added, gross output data, and intermediate consumption from the WIOD.

The sectors, indexed by $c i$ for sector $i$ in the WIOD database, were mapped into our 22 sectors

[^31]as follows: Food Products, Beverage, and Tobacco Products (c3); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4-c5); Wood Products, Paper, Printing, and Related Support Activities (c6-c7); Petroleum and Coal Products (c8); Chemical (c9); Plastics and Rubber Products (c10); Nonmetallic Mineral Products (c11); Primary Metal and Fabricated Metal Products (c12); Machinery (c13); Computer and Electronic Products, and Electrical Equipment and Appliances (c14); Transportation Equipment (c15); Furniture and Related Products, and Miscellaneous Manufacturing (c16); Construction (c18); Wholesale and Retail Trade (c19-c21); Transport Services (c23-c26); Information Services (c27); Finance and Insurance (c28); Real Estate (c29-c30); Education (c32); Health Care (c33); Accommodation and Food Services (c22); and Other Services (c34).

### 5.1.3 Regional trade, production data, and input shares

Interregional Trade Flows The sectoral bilateral trade flows across the 50 U.S. states, $X_{0}^{n j, i j}$ for all $n, i=U . S$. states, were constructed by combining information from the WIOD database and the 2002 Commodity Flow Survey (CFS). From the WIOD database we compute the total U.S. domestic sales for the year 2000 for our 22 sectors. We use information from the CFS for the year 2002, which is the closest available year to 2000 , to compute the bilateral expenditure shares across U.S. states, as well as the share of each state in sectoral total expenditure. The CFS survey for the year 2002 tracks pairwise trade flows across all 50 U.S. states for 43 commodities classified according to the Standard Classification of Transported Goods (SCTG). These commodities were mapped into our 22 NAICS sectors by using the CFS tables for the year 2007, which present such mapping. The 2007 CFS includes data tables that cross-tabulate establishments by their assigned NAICS codes against commodities (SCTG) shipped by establishments within each of the NAICS codes. These tables allow for mapping of NAICS to SCTG and vice versa. Having constructed the bilateral trade flows for the NAICS sectors, we first compute how much of the total U.S. domestic sales in each sector is spent by each state. To do so, we multiply the total U.S. domestic sales in each sector by the expenditure share of each state in each sector. Then we compute how much of this sectoral expenditure by each state is spent on goods from each of the 50 U.S. states. We do so by applying the bilateral trade shares computed with the 2002 CFS to the regional total spending in each sector. The final product is a bilateral trade flows matrix for the 50 U.S. states across sectors, where the bilateral trade shares across U.S. states are the same as those in the 2002 CFS, and the total U.S. domestic sales match those from the WIOD for the year 2000.

Regional production data and input shares We compute the share of value added in gross output $\gamma^{n j}$, and the material input shares $\gamma^{n j, n k}$ for all $n, i=U . S$. states, for each state and sector in the United States for the year 2000, using data on value added, gross output, and intermediate consumption. We obtain data on sectoral and regional value added from the Bureau of Economic Analysis (BEA). Value added for each of the 50 U.S. states and 22 sectors is obtained from the Bureau of Economic Analysis (BEA) by subtracting taxes and subsidies from GDP data. Gross outputs for the U.S. states in the 12 manufacturing sectors are computed from our constructed bilateral trade flows matrix as the sum of domestic sales and total exports. ${ }^{61}$ With the valueadded data and gross output data for all U.S. states and sectors, we compute the share of value added in gross output $\gamma^{n j}$. For the eight service sectors, the wholesale and retail trade sector, and the construction sector, we have only the aggregate U.S. gross output computed from the

[^32]WIOD database; thus, we assume that the share of value added in gross output is constant across states and equal to the national share of value added in gross output; that is, $\gamma^{n j}=\gamma^{U S j}$ for each non-manufacturing sector $j$, and $n=U . S$. states.

While material input shares are available by sector at the country level, they are not disaggregated by state in the WIOD database. We assume therefore that the share of materials in total intermediate consumption varies across sectors but not across regions. Note, however, that the material-input shares in gross output are still sector and region specific as the share of total material expenditure in gross output varies by sector and region.
5.1.4 Trade between U.S. states and the rest of the world. The bilateral trade flows between each U.S. state and the rest of the countries in our sample were computed as follows. In our paper, local labor markets have different exposure to international trade shocks because there is substantial geographic variation in industry specialization. Labor markets that are more important in the production in a given industry should react more to international trade shocks in that industry. Therefore, our measure for the exposure of local labor markets to international trade combines trade data with local industry employment. Specifically, following ADH, we assume that the share of each state in total U.S. trade with any country in the world in each sector is determined by the regional share of total employment in that industry. The employment shares used to compute the bilateral trade shares between the U.S. states and the rest of the countries are constructed using employment data across sectors and states from the BEA. ${ }^{62}$ Using this procedure, we obtain $X_{0}^{n j, i j}$ for all $n=U . S$. states, $i \neq U . S$. states, and $n \neq U . S$. states, $i=U . S$. states .
5.1.5 Bilateral trade shares Having obtained the bilateral trade flows $X_{0}^{n j, i j}$ for all $n, i$, we construct the bilateral trade shares $\pi_{0}^{n j, i j}$ as $\pi_{0}^{n j, i j}=X_{0}^{n j, i j} / \sum_{m=1}^{N} X_{0}^{n j, m j}$.
5.1.6 Share of final goods expenditure The share of income spent on goods from different sectors is calculated as follows,

$$
\alpha^{j}=\frac{\sum_{n=1}^{N} \sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi^{i k, n k} X^{i k}}{\sum_{n=1}^{N} \sum_{k=1}^{J} w^{n k} L^{n k}+\sum_{n=1}^{N} \iota^{n} \chi},
$$

where $\sum_{n=1}^{N} \sum_{k=1}^{J} \gamma^{n k, n j} \sum_{i=1}^{N} \pi^{i k, n k} X^{i k}$ denotes total spending in intermediate goods across all countries and regions, and $\sum_{n=1}^{N} \sum_{k=1}^{J} w^{n k} L^{n k}+\sum_{n=1}^{N} \iota^{n} \chi$ is the total world income.
5.1.7 Share of labor compensation in value added Disaggregated data on labor compensation are generally very incomplete. Therefore, we compute the share of labor compensation in value added, $1-\xi^{n}$, at the national level and assume that it is constant across sectors. For the United States, data on labor compensation and value added for each state for the year 2000 are obtained from the BEA. For the rest of the countries, data are obtained from the OECD inputoutput table for 2000 or the closest year. For India, Cyprus, and the constructed rest of the world, labor compensation data were not available. In these cases, we input the median share across all countries from the other 34 countries that are part of the rest of the world.

[^33]5.1.8 Local shares from global portfolio We need to calibrate $\iota^{n}$. The way we do so is as follows. Denote by $D^{n}$ to the imbalance of location (region/country) $n$. Data on $D_{n}$ comes directly from bilateral trade data for the year 2000. Using data on value added by sector and location, $V A^{n k}$, and labor compensation shares $1-\xi^{n}$, we solve for the local shares from the global portfolio as follows
$$
\iota^{n}=\frac{\sum_{k=1}^{J} \xi^{n} V A^{n k}-D^{n}}{\sum_{i=1}^{N} \sum_{k=1}^{J} \sum_{k=1}^{J} \xi^{n} V A^{n k}} .
$$

Note that trivially, $\sum_{i=1}^{N} \iota^{n}=1$, since $\sum_{i=1}^{N} D^{n}=0$.
5.1.9 The initial labor mobility matrix and the initial distribution of labor To determine the initial distribution of workers in the year 2000 by U.S. states and sectors (and non-employment), we use the $5 \%$ Public Use Microdata Sample (PUMS) of the decennial U.S. Census for the year 2000. As we mentioned before, information on industry is classified according to the NAICS, which we aggregate to our 22 sectors and non-employment. We restrict the sample to people between 25 and 65 years of age who are either non-employed or employed in one of the sectors included in the analysis. Our sample contains almost 7 million observations.

We combine information from the PUMS of the American Community Survey (ACS) and the Current Population Survey (CPS) to construct the initial matrix of quarterly mobility across our states and sectors $\left(\mu_{-1}\right){ }^{63}$ Our goal is to construct a transition matrix describing how individuals move between state-sector pairs from one quarter to the next (from $t$ to $t+1$ ). The ACS has partial information on this; in particular, the ACS asks people about their current state and industry (or non-employment) and the state in which they lived during the previous year. We use the year 2001 since this is the first year for which data on interstate mobility at a yearly frequency are available. ${ }^{64}$ After selecting the sample as we did before in terms of age range and the industries in our analysis, we have around 600,000 observations. We find that around $2 \%$ of the U.S. population moves across states in a year in this time period. Unfortunately, the ACS does not have information on workers' past employment status or the industries in which people worked during the previous period, so we resort to other data for this information.

We use the PUMS from the monthly CPS to obtain information on past industry of employment (or non-employment) at the quarterly frequency. The main advantage of the CPS is that it is the source of official labor market statistics and has a relatively large sample size at a monthly frequency. In the CPS, individuals living in the same address can be followed month to month for a small number of periods. ${ }^{65}$ We match individuals surveyed three months apart and compute their employment or non-employment status and work industry, accounting for any change between interviews as a transition. ${ }^{66}$ The main limitation with the CPS is that individuals who move to a different residence, which of course includes interstate moves, cannot be matched. Our threemonth match rate is close to $90 \% .{ }^{67}$ As the monthly CPS does not have information on interstate moves, we use this information to compute the industry and non-employment transitions within each state-that is, a set of 50 transition matrices, each with $23 \times 23$ cells. After restricting the

[^34]sample as discussed earlier, in any given month we have around 12,800 observations for the entire United States. To more precisely estimate the transitions, we use all months from October 1998 to September 2001, leading to a sample of over 475,000 matched records. Since for this time period the CPS uses the Standard Industry Classification, we translate this classification into NAICS, using the crosswalk in Table A6.3.

Table A5.1 summarizes the information used to construct a quarterly transition matrix across state, industry, and non-employment. The letter $x$ in the table denotes information available in the matched CPS, and the letter $y$ denotes information available in the ACS. The information missing from the above discussion is the past industry history of interstate movers. To have a full transition matrix, we assume that workers who move across states and are in the second period in state $i$ and sector $j$ have a past industry history similar to workers who did not switch states and are in the second period in state $i$ and sector $j .{ }^{68}$

Table A5.1: Information Available on ACS and CPS

| $$ | State A |  |  |  |  | State B |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Ind 1 <br> Ind 2 | Ind 1 | nd 2 | ... | Ind J | Ind 1 | Ind 2 | $\ldots$ | Ind J |
|  |  | x | x | $\ldots$ | x |  |  |  |  |
|  |  | x | x | ... | x |  |  |  |  |
|  | Ind J <br> Total | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |  |  |  |
|  |  | x | x |  | x |  |  |  |  |
|  |  | y | y | $\ldots$ | y | y | y | ... | y |
| $$ | Ind 1 |  |  |  |  | x | x | $\ldots$ | x |
|  | Ind 2 |  |  |  |  | x | x | $\ldots$ | x |
|  | Ind J |  |  |  |  | x | . x |  | . x |
|  | Total | y | y | ... | y | y | y | ... | y |

As mentioned earlier, information on interstate mobility in the ACS is for moves over the year. To calculate quarterly mobility we assume that interstate moves are evenly distributed over the year and we rule out more than one interstate move per year. In this case, our adjustment consists of keeping only one-fourth of these interstate moves and imputing three-fourths as non-moves. After this correction, we impute the past industry history for people with interstate moves from state $i$ to state $n$ and industry $j$ according to the intrastate sectoral transition matrix for state $n$ conditional on industry $j$.

Our computed value for the initial labor transition matrix is consistent with aggregate magnitudes of interstate and industry mobility for the yearly frequency estimated in Molloy et al. (2011) and Kamborouv and Manovskii (2008). We obtain a mobility transition matrix with over 1.3 million elements. ${ }^{69}$

### 5.2 Constructing the Actual Baseline Economy.-

In this section of the appendix we describe the data sources and assumptions used to construct the

[^35]time series data needed to compute the dynamic counterfactuals with time-varying fundamentals described in Section 5.
5.2.1 Trade, production, and input shares across countries International trade flows across sectors and the 38 countries in our sample over the period 2000-2007 are obtained from the WIOD database. ${ }^{70}$ To construct the sectoral bilateral trade flows across the 50 U.S. states over 2000-2007 we proceed as follows. The CFS releases sectoral bilateral trade data for the U.S. states every five years, and therefore we use the 2002 and 2007 releases to construct the bilateral trade flows for those years. We then interpolate the years 2003 through 2006 using a linear growth. As we explained above, and because of the lack of bilateral trade data in the CFS before 2002, we assume that the sectoral bilateral trade shares across U.S. states in 2000 are the same as in 2002; and therefore, we also assume that bilateral trade shares in 2001 are the same as in 2002. Finally, and as we did for the year 2000, to match the bilateral expenditures across states from the CFS with the aggregate U.S. domestic sales from WIOD, we multiply the total U.S. sectoral domestic sales from WIOD for every year over 2000-2007 by the expenditure share of each state in each sector. Then we compute how much of this sectoral expenditure by each state is spent on goods from each of the 50 U.S. states using the bilateral trade shares constructed for each year as explained above. The time series of the bilateral trade flows between each U.S. state and the rest of the countries in our sample were computed in the same way as we proceed for the year 2000. The employment shares used to compute U.S. states exposure to international trade in each industry are constructed using employment data across sectors and states from the BEA for each year over the period 2000-2007.
5.2.2 Migration flows and employment Migration flows for each quarter over the period 2000-2007 were constructed using the same procedure described in Appendix 5.1.9. With the time series of migration flows and the initial distribution of employment for the year 2000, we are able to recover the distribution of employment across U.S. labor markets for 2000-2007.

### 5.3 LEHD migration flow data.-

As described in this Appendix, we use multiple periods to construct some of our labor market flows data. We combine three years of monthly matched CPS records to obtain information on sectoral mobility patterns and flows in-and-out of non-employment. Our records are matched three months apart (one quarter). In any given month of the years 1998-2000, we have around 12,800 matched records and when we pool three years of data we have 475,440 individuals in that sample. Despite the relatively large sample size, measurement error and empty cells could still be a source of concern. To gain information on how our constructed transitions and labor market flows compare to the data, we construct a matrix of interstate and intersectoral transitions using data from the Census Bureau's Longitudinal Employer-Household Dynamics (LEHD), in particular, the Job-to-Job Flows data (J2J).The data we use can be obtained at http://lehd.ces.census.gov/data/j2j beta.html. As described by the Census Bureau, the Job-to-Job Flows data is a beta release of new national statistics on quarterly job mobility in the United States. The data include statistics on: (1) the job-to-job transition rate, (2) hires and separations to and from employment, and (3) characteristics of origin and destination jobs for job-to-job transitions. These statistics are available nationally and at the state level and contain origin and destination state, as well as origin and destination industry. This J2J data is readily available to the public with no restrictions. The main advantage of the LEHD data is that it combines administrative data from the state's Unemployment Insurance program, the Quarterly Census of Employment and Wages, and additional administrative data and

[^36]data from censuses and surveys. As such, sample size is probably not an issue. However, these data present some limitations. (1) In the early 2000s, a large number of states are not included in the data. States have joined gradually over time into the LEHD program but even today data for Massachusetts are unavailable. (2) Manufactures are aggregated as a single sector and without access to the micro-data, which is restricted, individual industries cannot be identified. (3) There is very limited information on origin-destination for flows involving non-employment.

Due to these limitations, we prefer to use our own constructed flows. However, we use the J2J data to gauge how our transitions compare to those in the J2J. For this, we aggregate our manufactures as a single sector and do not compare transitions involving non-employment. Moreover, we only compare the flows for the groups of states that are available in the J2J data in the year 2000, since this is the year for which we construct our flows. ${ }^{71}$

We find that the migration flows constructed using data from the ACS and CPS are highly correlated with the transition probabilities from the LEHD J2J data. The overall correlation is 0.99 , and the correlations across location and across industries are also 0.99 . If we take out the stayers, the correlations are still quite high; the overall correlation is 0.7 , the correlation across locations is 0.81 and the correlation across industries is 0.96 . Therefore, our computed mobility rates are very close to those in the LEHD J2J dataset. Finally, we want to highlight that we conducted robustness checks in which we add a very small number to any of our zero probability transitions. We find that our results remain largely unaltered. The reason is that these type of transitions typically involve a small labor market either as origin or destination (or both). Thus, quantitatively, as we aggregate results at the level of sectors or states, whether transitions are exactly zero or approximately zero do not seem to affect the results much.

### 5.4 Comparing Migration Flows: Data Versus Model.-

We evaluate if the iid assumption on preference shocks delivers too much mobility compared to the data. To do so, we simulated data from our model and compared the outcomes to the data. In particular, we simulated from our model a panel of one million individuals over 120 quarters and kept track of their labor market history. The initial distribution of workers matches that of the year 2000 and the simulation is performed under our baseline economy (without the China shock).

Table A5.2: Actual and simulated mobility rates percent

|  | Data | Model |
| :--- | :---: | :---: |
| Quarterly sector switching rate | 6.1 | 5.4 |
| Yearly state mobility rate | 2.3 | 2.4 |

Note: Model values are computed with simulated individual histories over 120 periods. Data on yearly state mobility rate computed using the ACS, 2001-2007. Data on quarterly sector mobility rate computed using matched CPS, 2000-2007. Sector mobility excludes non-employment

Table A5.2 shows the probability a worker switches one of the 22 sectors from one quarter to the next and the probability the worker moves to a different state from one year to the next. The simulations are largely consistent with the data. Thus, while workers receive a shock every period, only a small fraction decide to move. The numbers reported in Table A5.2 align well with mobility rates computed in other studies in the literature, like Molloy et al. (2011) and Kaplan and Schulhofer-Wohl (2012) for interstate mobility, and Kambourov and Manovskii (2008) for intersectoral mobility.

[^37]
## APPENDIX 6: ESTIMATION

### 6.1 Predicting Import Changes from China

To identify the China shock, we use the international trade data from ADH. ${ }^{72}$ Specifically, we use data measuring the value of trade between several countries from 1991-2007. ADH retrieve these data from the UN Comrade Database and concord them from six-digit Harmonized System (HS) product codes to a 1987 Standard Industrial Classification (SIC) manufacturing industry code scheme. ${ }^{73}$ Their scheme is essentially the same as the SIC 1987 classification scheme, except for a few four-digit industries that did not map directly from the HS-codes. These industries are aggregated into other four-digit industry codes so that each of the ADH's resulting 397 industries maps directly from a HS trade code. ${ }^{74}$ Once the data are in this SIC 1987 structure, the authors deflate the import values into real 2007 US dollars using the personal consumption expenditure deflator and aggregate the country-level data into importing and exporting regions. The final data are reported over two importing regions (the United States and an aggregate of eight other developed countries - namely, Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland- and four exporting regions (China and other low-income countries). For our purposes, we use the two data series that measure imports from China by the United States, and imports from China by the other advanced economies.

To make these data comparable with the rest of our analysis, we developed a crosswalk to map the data from ADH's SIC coding into our NAICS sectors. Because their SIC codes include only manufacturing industries, they only intersect with 13 of our 22 NAICS sectors -our 12 manufacturing sectors and also the information and communications sector. ${ }^{75}$ Table A6.3 shows the exact mapping between the two industry schemes. The SIC 1987 codes are a hierarchical system, in which the first two numbers represent the broader groups, and as extra digits are added the industry, the system becomes more narrowly defined. Many of the SIC codes matched our sectors on the two-digit level, in other words, the broad groups were the same.

After this redefinition of sectors, we compute the changes in the level of imports from China between 2000 and 2007 by the United States and the other advanced economies. The change in U.S. imports from China during this period can, in part, be the result of domestic U.S. shocks, but we are looking for a measure of changes in imports that are mostly the result of shocks that originate in China. Inspired by ADH's instrumental variable strategy, we run the following regression

$$
\Delta M_{U S A, j}=a_{1}+a_{2} \Delta M_{o t h e r, j}+u_{j}
$$

where $j$ is one of our 12 manufacturing sectors, and $\Delta M_{U S A, j}$ and $\Delta M_{o t h e r, j}$ are the changes in real U.S. imports from China and imports by the other advanced economies from China between 2000 and 2007.

The coefficient of the regression is estimated $a_{2}=1.27$ with a robust standard error of 0.011 . We want to emphasize that our motivation for the choice of our sample of countries is to closely follow Autor et al. (2013), where the authors include eight high-income countries (other than the

[^38]Table A6.3: Concordance SIC87dd - NAICS

| NAICS | NAICS Sector Description | SIC87dd Codes |
| :--- | :--- | :--- |
| 1 | Food, Beverage, and Tobacco Products | $20^{* *}, 21^{* *}$ |
| 2 | Textiles and Apparel Products | $22^{* *}, 23^{* *}, 31^{* *}$ |
| 3 | Wood, Paper, Printing and Related Products | $24^{* *}$ exc. $241^{*}, 26^{* *}, 274^{*}-279^{*}$ |
| 4 | Petroleum and Coal Products | $29^{* *}$ |
| 5 | Chemical | $28^{* *}$ |
| 6 | Plastics and Rubber Products | $30^{* *}$ |
| 7 | Nonmetallic Mineral Products | $32^{* *}$ |
| 8 | Primary and Fabricated Metal Products | $33^{* *}, 34^{* *}$ |
| 9 | Machinery | $351^{*-356^{*}, 3578-3599}$ |
| 10 | Computer, Electrical, and Appliance | $3571-3577,365^{*}-366^{*}$, |
|  |  | $3812-3826,3829,386^{*}-387^{*}$, |
|  |  | $361^{*}-364^{*}, 367^{*}-369^{*}$ |
| 11 | Transportation Equipment | $37^{* *}$ |
| 12 | Furniture and Miscellaneous Products | $25^{* *}, 3827,384^{*}-385^{*}, 39^{* *}$ |
| 16 | Information and Communication | $271^{*}-273^{*}$ |

Note: an entire broad group was mapped into the NAICS sector by substituting the last one or two digits with an asterisk. All intervals listed in the table are inclusive.

United States) to construct their instrument: Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland, in the estimation of the above regression. Figure A6.1 shows the actual and predicted change in U.S. imports from China constructed with this set of countries. As can be seen from the figure, the predicted power of the of the regressor is very strong. The R -squared of the regression is 0.98 , with an extremely large $F$ statistic.

This regression is related to the first-stage regression in AHD's two-stage least square estimation. Using this result we construct the changes in U.S. imports from China for each industry that are predicted by the change in imports in other advanced economies from China.

To measure the China Trade Shock we find the changes in fundamental productivity in the 12 manufacturing sectors in China that match the sectoral predicted changes in U.S. import from China from the years 2000 to 2007 . We first did this with a static version of our model so that we obtained the changes in productivity from 2000 to 2007, which we then interpolated across all quarters. We then feed into our dynamic model the TFP measures obtained from the static version of our model and solved for the TFP changes that minimize the sum of squares of the difference between the relative change of the predicted U.S. imports from China over 2000-2007 in the data and the ones from the dynamic model. After minimizing the sum of squares of the difference, the correlation between the model and the data is 0.98 . Figure A6.2 shows the predicted change in U.S. manufacturing imports from China computed as in ADH and the implied sectoral productivity changes in China.

In Figure A6.2, measured TFP is defined as $\left(A_{t}^{n j}\right)^{\gamma^{n j}} /\left(\pi_{t}^{n j, n j}\right)^{1 / \theta^{j}}$, see Caliendo et al. (2017) for details. Our model estimates that TFP increased in all manufacturing industries in China. While our estimated changes in Chinese TFP are correlated with the changes in U.S. imports from China by sector, this correlation is not perfect.

Fig. A6.1: Actual and predicted import changes 2000-2007 (billions of dollars of 2009)


Note: The figure presents the contribution of each state to the total increase of employment share in the nonmanufacturing sector due to the China shock.

FIG. A6.2: Predicted change in imports vs. China's TFP changes (2000-2007)


Note: The figure presents the predicted change in imports from using the ADH specification and the change in China's measured TFP by sector for the period 2000-2007.

### 6.2 Reduced-Form Analysis

In the previous paragraphs we described how we followed ADH to compute the change in U.S. imports from China. We now take one step forward and reproduce some of the results in ADH but under our definition of labor market and under our sample selection criteria. ${ }^{76}$

We follow the same methodology as ADH to impute the U.S. total imports to state-industry units, except where ADH used commuting zones and SIC codes we use states and our 12 manufacturing sectors. Total U.S. manufacturing imports are allocated to states by weighting total imports according to the number of employees in a certain local industry relative to the total national employment. Following the example of ADH, we use County Business Patterns (CBP) data for the year 2000 from the Census Bureau to measure local industry employment. The CBP is a county-level, annual data set that provides details on local firm-level employment by industry. The data are compiled from the Census Bureau's Business Register, and include almost all employment at known companies.

To avoid giving away identifiable information about specific firms, the census bureau will sometimes report county-industry level data in an interval instead of one point. ADH establish a methodology of imputing employment within these intervals, which we follow to get the most accurate estimate of local industry employment. ADH start by using the employment distribution of known firms within a particular size interval and the aggregated employment in a firm's industry to narrow the employment interval. Once the possibility of values is narrowed, they set employment to the midpoint of the bracket and run a regression using a sample of similar firms. Finally, they add up and proportionally adjust the imputed numbers based on the aggregate employment in that industry. ${ }^{77}$ To actually perform the imputations we use ADH's publicly available code, and only adapt a few lines at the end that aggregate employment to state-sector levels instead of commuting zone-industry levels.

Once we have the 12 -sector state-level industry employment data, we allocate the national import data to the worker level using the following formula proposed by ADH (see their equation 3):

$$
\Delta I P W_{u i t}=\sum_{j} \frac{L_{i j t}}{L_{u j t}} \frac{\Delta M_{u c j t}}{L_{i t}} .
$$

The expression above states that the change in U.S. imports per worker from China is defined based on each state's industry employment structure in the starting year. Following ADH's notation, $L_{i t}$ is the total employment at state $i$ at time $t, j$ represents one of our 12 manufacturing sectors, and the $u$ stands for a U.S. related variable (as opposed to a variable constructed using other countries imports, for which they use an o). For example, $\Delta M_{u c j t}$ means the change in U.S. imports from China for industry $j$ time $t .^{78}$

We also followed ADH in constructing our dependent variable: the change in local manufacturing employment as a share of the working age population . Data for local manufacturing employment comes from the 2000 census $5 \%$ PUMS and from the 2006, 2007, and 2008 ACS $1 \%$ PUMS. To make the data samples more comparable, we followed ADH in pooling 2006-2008 ACS samples together and treating them all as 2007. Both the census and ACS data come from the Minnesota IPUMS service. Industry data from these sources are originally coded according to census industry codes under a NAICS classification that we aggregate to our 22 NAICS sectors. As in our study,

[^39]we restrict the sample to those individuals between ages 25 to 65 that are either employed or non-employed. ${ }^{79}$ As a last step, we augment the microdata weights by multiplying the PUMS sampling weights with the ADH labor weight (see data ADH Data appendix for details). We finish by collapsing the data to the state-level and taking the difference in the share of manufacturing labor as a percent of the labor force (ages 25 to 65) between 2000 and 2007. We use the constructed variables to run a regression relating the change in local manufacturing employment from 2000 to $2007\left(\Delta L_{i t}^{m}\right)$ to the change in imports per worker $\left(\Delta I P W_{u i t}\right)$ :
$$
\Delta L_{i t}^{m}=b_{1}+b_{2} \Delta I P W_{u i t}+e_{i t}
$$

In this regression the unit of observation is a U.S. state. We include D.C. as a state but exclude Hawaii and Alaska since they are not part of ADH analysis. As in ADH, we perform a Two Stage Least Squares regression instrumenting $\triangle I P W_{\text {uit }}$ with $\triangle I P W_{\text {oit }}$, which is other advanced economies' change in imports from China per worker. ${ }^{80}$

In addition, we also run the following regression,

$$
\Delta \bar{u}_{i t}=c_{1}+c_{2} \Delta I P W_{u i t}+e_{i t}
$$

where $\bar{u}_{i t}$ is the change in the non-employment rate of state $i$ for the age groups in our sample. ADH perform a similar regression in their Table 5. Once again, we perform the same type of regression but using our definitions and time period and do not have additional controls in the regression.

Table A6.4 presents the results. As in ADH, we find that the change in $I P W_{u i t}$, negatively affects the share of employment in manufactures and positively affects unemployment. Our estimates of $b_{2}$ are -1.72 with a robust standard error of $0.19 .{ }^{81}$ The regression results in columns (1) and (3) are somewhat different from those reported by ADH. Our reduced-form results using our data are largely aligned with theirs, both in terms of the sign and significance. The differences stem from the different time periods we use (we use only changes between 2000 to 2007 while in several of ADH's specifications they use 1990 to 2007), the use of additional controls in the regressions, the definition of geographic areas and industries (we use U.S. states and NAICS sectors), and sample selection criteria (population ages and labor force).

In columns (2) and (4), we run the same type of regressions but with model generated data. The coefficients we estimate with the model generated data are close to those estimated with actual data, displaying the same sign and significance. Our estimate of the effects of Chinese import penetration on unemployment is positive, as in ADH. However, this is a relative effect. States with a relatively higher import penetration will tend to have a relatively higher non-employment rate.

[^40]Table A6.4: Reduced-form regression results

|  | $\Delta L_{i t}^{m}$ |  |  | $\Delta \bar{u}_{i t}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | data | model |  | data | model |
|  | $(1)$ | $(2)$ |  | $(3)$ | $(4)$ |
| $\Delta I P W_{\text {uit }}$ | -1.718 | -0.977 |  | 1.146 | 1.469 |
|  | $(0.194)$ | $(0.219)$ |  | $(0.334)$ | $(0.564)$ |
| Obs |  |  |  |  |  |
| $R^{2}$ | 0.51 | 50 |  | 49 | 50 |

Note: Results from Two Stage Least Squares using $I P W_{\text {oit }}$ (imports of other advanced economies per worker) as instrument.. Regressions in columns 1 and 2 have the change in the share of manufacturing employment as the dependent variable and regressions in columns 3 and 4 have the change in the share of the population nonemployed as the dependent variable. Data stands for the regression using observed data and model stands for the same regression using model generated data given our counterfactual experiment. Changes are between 2000 and 2007. Estimated standard error is in parentheses. Model includes the 50 U.S. states, where D.C. has been merged with Virginia. Data include the 48 U.S. continental states and D.C. as a separate state. All regressions include a constant but no other controls. Results differ slightly from ADH due to different time periods, the use of additional controls in the regression, the definition of geographic area and industries used, and sample selection criteria.

However, we know from our model that non-employment tends to fall on average on almost all states.

## APPENDIX 7: ADDITIONAL RESULTS

### 7.1 Regional Employment Effects

In this appendix, we present the U.S. states' contributions to the change in the employment share in different industries The key finding in these figures is the large spatial heterogeneity in the employment effects from the China shock across different industries.

Fig. A7.1: Regional employment declines in manufacturing industries
2. Normalized by regional employment share

1. Contribution to industry employment decline in the U.S. (\%)
a.1: Petroleum, Coal

b.1: Wood paper
a.2: Petroleum, Coal

b.2: Wood paper


Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the petroleum, coal industry, Panels b present the results for the wood paper industry.

Fig. A7.2: Regional employment declines in manufacturing industries

1. Contribution to industry employment decline in the U.S. (\%)

## a.1: Chemicals


b.1: Non Metallic

c.1: Transport Mfg.
2. Normalized by regional employment share
a.2: Chemicals

b.2: Non Metallic

c.2: Transport Mfg.


Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the chemicals industry. Panels b present the results for the non metallic industry. Panels c present the results for the transport mfg. industry.

Fig. A7.3: Regional employment declines in manufacturing industries

1. Contribution to industry employment decline in the U.S. (\%)

## a.1: Plastics, Rubber


b.1: Metal

c.1: Computers electronics

2. Normalized by regional employment share

a.2: Plastics, Rubber


b.2: Metal

c.2: Computers electronics

Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the plastics, rubber industry. Panels b present the results for the metal industry. Panels c present the results for the computers electronics industry.

Fig. A7.4: Regional employment increases in mfg. and non-mfg. industries

1. Contribution to industry employment increase in the U.S. (\%)
2. Normalized by regional employment share
b.1: Information Serv.

c.1: Real Estate


## a.1: Food Beverage Tobacco




## a.2: Food Beverage Tobacco


b.2: Information Serv.

c.2: Real Estate


Note: This figure presents the rise in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the food beverage and tobacco industry. Panels b present the results for the information serv. industry. Panels c present the results for the real estate industry.

Fig. A7.5: Regional employment increases in non-manufacturing industries

1. Contribution to industry employment increase in the U.S. (\%)
a.1: Transport services

b.1: Finance

c.1: Education


Note: This figure presents the rise in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the transport services sector, Panels b present the results for the finance sector. Panels c present the results for the education sector.

Fig. A7.6: Regional employment increases in non-manufacturing industries

1. Contribution to industry employment increase in the U.S. (\%)
a.1: Health Care

b.1: Accom. \& Food

c.1: Other Serv.


b.2: Accom. \& Food

c.2: Other Serv.

Note: This figure presents the rise in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in the industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in the industry employment normalized by the employment size of each state relative to the U.S. aggregate employment. Panels a present the results for the health care industry. Panels b present the results for the accom. \& food industry. Panels c present the results for the other serv. industry.


[^0]:    * First draft: March 2015. We thank Alex Bick, Ariel Burstein, Carlos Carrillo-Tudela, Arnaud Costinot, Jonathan Eaton, Rafael Dix-Carneiro, Pablo Fajgelbaum, Penny Goldberg, Sam Kortum, Juan Pablo Nicolini, Eduardo Morales, Giuseppe Moscarini, Alexander Monge-Naranjo, Juan Sanchez, Joe Shapiro, Derek Stacey, Peter Schott, Guillaume Vandenbroucke, Jonathan Vogel, and seminar participants for useful conversations and comments. Hannah Shell provided excellent research assistance. All views and opinions expressed here are the authors' and do not necessarily reflect those of the Federal Reserve Bank of St. Louis. Previously circulated under "Trade and Labor Market Dynamics." Correspondence: Caliendo: lorenzo.caliendo@yale.edu; Dvorkin: maximiliano.a.dvorkin@stls.frb.org; Parro: fparro1@jhu.edu.
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[^1]:    ${ }^{1}$ Our setup can accommodate an arbitrary number of sectors, regions, and countries.
    ${ }^{2}$ The production structure of the model builds on multicountry international trade models a la EK. We introduce dynamics, international trade, and labor mobility frictions to the rich spatial model of Caliendo, et al. (2017).

[^2]:    ${ }^{3}$ Our model belongs to a class of dynamic discrete choice models in which estimation and identification of these large sets of fundamentals is, in general, challenging. For more details, see Rust $(1987,1994)$. For recent studies that estimate fundamentals in a similar context to ours, see Artuç, Chaudhuri, and McLaren (2010), and Dix-Carneiro (2014).
    ${ }^{4}$ Costinot and Rodriguez-Clare (2014) coin the term "exact hat algebra," and show that this technique also holds in a large variety of trade models even under the presence of fixed costs. Other recent applications of the exact hat algebra method are Caliendo and Parro (2015), and Burstein, Morales, and Vogel (2016). Eaton, et al. (2015) show how to apply DEK in the context of multi-country trade model with capital accumulation.
    ${ }^{5} \mathrm{ADH}$ argue that structural reforms in the Chinese economy resulted in large technological improvements in export-led sectors. As a result, China's import penetration to the Unites States increased. Handley and Limao (2014) and Pierce and Schott (2016) argue that the U.S.' elimination of uncertainty about tariff increases on Chinese goods was another important reason why U.S. imports from China grew.

[^3]:    ${ }^{6}$ It is worth noting that for an application of this dimension not using our solution method will require estimating: $N \times R \times J$ productivity levels, $N^{2} \times R^{2} \times J$ asymmetric bilateral trade costs, $N^{2} \times R^{2} \times J^{2}$ labor mobility costs, and $N \times R \times J$ stocks of local factors. Where $N, R$, and $J$ are countries, regions and sectors, respectively.
    ${ }^{7}$ The observed change in manufacturing employment in the U.S. from 2000 to 2007 was 3.4 millions according to the Department of Labor, Bureau of Labor Statistics.
    ${ }^{8}$ The figure of 0.8 million is about $50 \%$ of the change in the aggregate manufacturing employment share unexplained by a secular trend. We compute the secular trend for the U.S. manufacturing employment share of total private employment as a linear trend from the year 1967 to 1999 , the year before the China shock. The trend predicts a share of $12.83 \%$ for the year 2007 , while the observed share was $11.85 \%$. More details are provided in Section 5 .

[^4]:    ${ }^{9}$ More broadly, through the lens of our model, we can study the effects of changes in many economic conditions, for instance, how changes in trade costs, labor migration costs, local structures, productivity, non-employment benefits (or home production), and local policies affect the rest of the economy. In addition, we can analyze how aggregate changes in economic circumstances can have heterogeneous disaggregate effects.
    ${ }^{10}$ For instance, see Artuç and McLaren (2010); Artuç Chaudhuri and McLaren (2010); Dix-Carneiro (2014); DixCarneiro and Novak (2015); Cosar (2013); Cosar, Guner, and Tybout (2014); Kondo (2013); Menezes-Filho and Muendler (2011); and the references therein.

[^5]:    ${ }^{11}$ Another related model of labor reallocation is Coen-Pirani (2010). Idiosyncratic preference shocks are widely used in the literature on worker reallocation. See, for example, Dix-Carneiro (2014), Kennan and Walker (2011), Monte (2015), Pilossoph (2014), and Redding (2012).
    ${ }^{12}$ Alternatively, one could assume that non-employed households use income to buy market goods. In this case, consumption of non-employed households in region $n$ is given by $b^{n} / P_{t}^{n}$. We consider this alternative specification later on in our quantitative analysis. We will also extend it to include a particular form of non-employment insurance financed with local taxes.
    ${ }^{13}$ To simplify the notation, we ignore local amenities, which can vary both by sectors and regions. As it will become clear later, our exercise and results are invariant to including these amenities under the assumption that they enter the period utility additively and are constant over time. More general types of amenities, including congestion or agglomeration effects, can also be handled by the solution method we propose, but we abstract from them here.

[^6]:    ${ }^{14}$ For a survey on this literature, see Aguirregabiria and Mira (2010).
    ${ }^{15}$ In Appendix 3.4, we extend our model for the case of elastic labor supply. In particular, we incorporate laborleisure decisions in each household's utility function, using alternative specifications.
    ${ }^{16}$ For an example of a model that delivers a similar expression, refer to Artuç and McLaren (2010), ACM, and Dix-Carneiro (2014). ACM also provide an economic interpretation of the different components of the option value to move across sectors.

[^7]:    ${ }^{17}$ In our case, the measure of this representative agent evolves endogenously with the change in economic conditions. See Dvorkin (2014) for further details.

[^8]:    ${ }^{18}$ For example, a sector/industry is computer and electronic product manufacturing (NAICS 334 in the data), which is an aggregate of many varieties like electronic computers (334111), audio and video equipment (33431), and circuit boards (NAICS 334412). Computer and electronic products are purchased by households for final consumption and by firms as materials for production. When we calibrate the model we show how the share of expenditure by households and firms is guided by the data.

[^9]:    ${ }^{19}$ In particular, the constant $\Gamma$ is the Gamma function evaluated at $1+\left(1-\eta^{n j} / \theta^{j}\right)$.
    ${ }^{20}$ For detailed derivations, please refer to Caliendo et al. (2017).

[^10]:    ${ }^{21}$ It is important to emphasize that the temporary equilibrium described in Definition 1 is not specific to a multisector EK model, but it can also be the equilibrium of other trade models such as Melitz (2003). In other words, an economy has a temporary equilibrium if one can solve for equilibrium prices given the distribution of employment.
    ${ }^{22}$ In Appendix 3.1 we present a one sector version of our model that maps into Alvarez and Lucas' (2007) model. Alvarez and Lucas (2007) show existence and uniqueness of the equilibrium. For a proof and characterization of the conditions for existence and uniqueness of a more general static model than that of Alvarez and Lucas (2007), refer to Allen and Arkolakis (2014), and for a proof of existence and uniqueness of a static model more similar to our static sub-problem, see Redding (2012).
    ${ }^{23}$ Proposition 8 from Cameron, Chaudhuri, and McLaren (2007) shows the existence and uniqueness of the sequential competitive equilibrium of a simplified version of our model. Using the results from Alvarez and Lucas (2007) together with proposition 8 from Cameron, Chaudhuri, and McLaren (2007), there exists a unique sequential equilibrium of the one sector model in Appendix 3.1.

[^11]:    ${ }^{24}$ Another way to understand our method is by relating it to Hotz and Miller (1993) and Berry (1994). They show that choice probabilities provide information on payoffs and parameters, and by inverting choice probabilities it is possible to estimate the parameters. We show that by taking time differences of choice probabilities and inverting them we can solve for the model, and perform counterfactuals, without estimating the parameters.
    ${ }^{25}$ It is worth noting that given Assumption 2, we do not require information on the level of wages and local prices across markets in the initial period to solve the model. If instead we had linear utility, then equation (17) would be given by

    $$
    \dot{u}_{t+1}^{n j}=\omega_{t}^{n j}\left(\dot{\omega}^{n j}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)-1\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t}^{n j, i k}\left(\dot{u}_{t+2}^{i k}\right)^{\beta / \nu}\right)^{\nu}
    $$

    which, as we can see, would require conditioning on the level of real wages $\omega_{t}^{n j}$ in the first period.
    ${ }^{26}$ It should be clear at this point that our solution method requires actual data on migration flows, trade, employment, and production to compute the model. In our quantitative application, we initialize the economy with data for

[^12]:    production, trade, migration and employment for the U.S. economy and the world in the year 2000 . Therefore, we are not assuming the economy is in a steady state, and our initial data reflects exactly the state of the U.S. economy in the year 2000, which is not necessarily a steady state.
    ${ }^{27}$ Note that the sequence of fundamentals that defines the baseline economy does not need to be constant. There can be any converging evolution of fundamentals in the baseline economy. The only requirement for the baseline economy is that the initial allocation will reflect this informational assumption.

[^13]:    ${ }^{28}$ Agriculture, mining, utilities, and the public sector are excluded from the analysis.

[^14]:    ${ }^{29}$ When we construct the matrix of mobility flows across our labor markets, all of the workers that, in the initial period, are not employed in an industry, are part of the pool of non-employed workers.
    ${ }^{30}$ This simplification is a consequence of data availability. As we discussed previously, our model can accommodate international migration.

[^15]:    ${ }^{31}$ In Appendix 5, we compare our constructed migration flows with an alternative dataset from the Census Bureau; Longitudinal Employer-Household Dynamics (LEHS), in particular, the Job-to-Job Flows data (J2J). We find that the migration flows constructed using data from the ACS and CPS are highly correlated with the transition probabilities from the LEHD J2J data.
    ${ }^{32}$ Since our period is a quarter, our rates are not directly comparable with the yearly mobility rates for state and industry from these studies. Moreover, our sample selects workers from ages 25 to 65 , who tend to have lower mobility rates than younger workers.
    ${ }^{33}$ ACM construct migration flow measures and real wages for 26 years between 1975-2000, using U.S. Census Bureau's March Current Population Surveys (CPS). We use ACM data in our estimation and do not proceed to disaggregate their data forward. Due to its small sample size, using the March CPS to construct interregional and intersectoral migration flows could bias down the amount of mobility. For further details, see ACM and Appendix 5 .

[^16]:    ${ }^{34}$ The exclusion restriction is that the error term, $\varpi_{t+1}$, is not correlated over time. Naturally, depending on the context, this is a strong assumption which in some cases could be violated. For example, if there are unobservable serially correlated characteristics of some labor markets, they are going to be subsumed in the residual. We rely on ACM's strategy but note that future research should focus on finding a different instrument, or a different estimation strategy, that is not subject to this criticism. See ACM for a discussion on other strengths and weaknesses of this approach.
    ${ }^{35}$ As mentioned above, ACM's model has linear utility, and therefore $1 / \nu$ is a semi-elasticity in ACM. They estimate $\nu=1.88$ at a annual frequency.

[^17]:    ${ }^{36}$ See Appendix 6 for more details on the data construction and estimation. One might be concerned that with our data and at our level of disaggregation the specification from ADH might not deliver employment effects comparable to ADH. Therefore, in Appendix 6 we also run the second-stage regression in ADH with our data and the results we obtain are largely aligned with those in ADH.
    ${ }^{37}$ In particular, the set of countries used by ADH in the construction of $\Delta M_{\text {other }, j}$ are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. The coefficient $a_{2}$ in the regression is estimated to be 1.27 with a robust standard error of 0.01 . The predictive power of the regressor is large with an R-squared of 0.98 . Including additional countries in the construction of $\Delta M_{o t h e r, j}$ has very small effects on the predicted values for $\Delta M_{U S A, j}$. See Appendix 6 for further details.
    ${ }^{38}$ To do so, we proceed in two steps. We first employ a static multicountry, multi-sector version of our model and calibrate the TFP changes to our 12 manufacturing sectors of the Chinese economy $\left\{\hat{A}^{C h i n a, j}\right\}_{j=1}^{12}$ that match exactly the change in U.S. manufacturing imports from China from 2000 to 2007. Second, we feed into our dynamic model the TFP measures obtained from the static version of our model, and solve for the TFP changes that minimize the sum of squares of the difference between the relative change of the predicted U.S. imports from China over 2000-2007 in the data and the ones from the dynamic model. Since the change in U.S. imports from China is evenly distributed over this period, we interpolated the estimated TFP changes over 2000-2007 across all quarters and feed in this sequence of TFP shocks into our dynamic model.

[^18]:    ${ }^{39}$ We compared our measured TFP with estimates from the literature. Brandt, Van Biesebroeck, and Zhang (2012) estimate and annual growth in Chinese manufacturing TFP of about 8 percent over the period 1998-2007, while we obtain an average TFP growth in manufacturing of 7.9 percent over 2000-2007.
    ${ }^{40}$ Recall that in this study we refer to the China shock as the change in productivity in China from the years 2000 to 2007. Of course, part of the contraction in manufacturing employment share that the model predicts may actually

[^19]:    be caused by increases in productivity in China occurring in the 1980s and 1990s.
    ${ }^{41}$ The difference between the observed share of manufacturing employment in the U.S. economy in 2007 and its predicted value using a simple linear trend on this share between 1965 and 2000 is $1 \%$. In other words, the change in the U.S. manufacturing share that is unexplained by a linear trend is $1 \%$. To compute the implied levels of manufacturing employment loss in 2007, we take data on total employment from the BEA for the year 2007 (Table

[^20]:    SA25N: Total Full-Time and Part-Time Employment by NAICS Industries). To match the sectors in our model, we subtract employment in farming, mining, utilities, and the public sector, which yields a level of employment of 151.4 million. We multiply by our model's implied change in manufacturing employment share and get 0.76 million jobs.
    ${ }^{42}$ In Appendix 3.2 we extend our model for the case of a CES utility function with an elasticity of substitution between manufacturing and non-manufacturing different from one. Our main results are robust to changes in the value of this elasticity. For instance, we find that in the range of an elasticity of substitution between 0.1 and 2, the manufacturing employment share declines about 0.5 percentage point as a consequence of the China shock, and aggregate welfare increases by about 0.3 percent. The stability of these effects is due to the fact manufacturing expenditure shares move little in the counterfactual economy relative to the baseline economy.

[^21]:    ${ }^{43}$ ADH show evidence that higher exposure to Chinese imports in a labor market cause a larger increase in unemployment in that market. In our model, non-employment falls due to the China shock, but we constructed a measure of import changes per worker in each U.S. state over the period from 2000-2007 and find that states with a lower import penetration experience a larger fall in non-employment. Similarly, in states with higher import penetration non-employment does not fall as much. Therefore, our model also accounts for the positive relation between import penetration and non-employment in a labor market.
    ${ }^{44}$ In a one-sector model with no materials and structures, equation (28) reduces to $W_{t}^{n j}=\sum_{s=1}^{\infty} \beta^{s} \log \frac{\left(\hat{\pi}_{s}^{n n}\right)^{-1 / \theta}}{\left(\hat{\mu}_{s}^{n n}\right)^{\nu}}$, which combines the welfare formulas in ACM, and Arkolakis, Costinot, and Rodriguez-Clare (2012).
    ${ }^{45}$ We aggregate welfare across labor markets using the employment shares at the initial year. In other words, we use an Utilitarian approach to aggregate welfare of heterogeneous workers.

[^22]:    ${ }^{46}$ We performed a series of robustness exercises where we recomputed the allocation and welfare results using different values of $v$, ranging from $\nu=3$ to $\nu=5.34$. We find that the effect of the China shock on manufacturing employment shares and aggregate welfare are very robust to the value of $\nu$, although the value of this parameter has a moderate effect on the dispersion of the welfare effects across labor markets.

[^23]:    ${ }^{47}$ As we did before with welfare measures, we use the $t=0$ labor shares as weights to aggregate across labor markets.
    ${ }^{48}$ For instance, some workers could experience a reduction in the market value of their skills because the same skills are embodied in cheaper labor in China. One way to think about this in our model is that the sectoral migration costs capture, in part, the skill composition in each industry, and therefore, how costly it is for certain skill groups to switch across industries that require a different human capital specificity.

[^24]:    ${ }^{49}$ Notice that this counterfactual is not the same as that in Section 5 since with constant SSDI we have that $\log \dot{b}_{t}^{n}=\delta \log \left(1 / \dot{P}_{t}^{n}\right)$ while in Section 5 we had that $\log \dot{b}_{t}^{n}=0$.
    ${ }^{50}$ Alternatively, note that a model with constant $\mathrm{SSDI}, \dot{b}_{t}^{1 n}=0$ and $\delta=1$ is equivalent to a model where nonemployed households spend all the non-market income $b^{n}$ on market goods. In such model, we find that the China shock results in a decline in manufacturing employment share of 0.504 percent, and that aggregate welfare increases by 1.42 percent.
    ${ }^{51}$ These values are obtained from the report of "Trends in the Social Security and Supplemental Security Income Disability Programs" elaborated by the Social Security Administration. This report can be found at https://www.ssa.gov/policy/docs/chartbooks/disability_trends/.

[^25]:    ${ }^{52}$ We want to clarify that our data excludes agriculture, mining, utilities, and the public sector. As a result, the manufacturing employment share in Panel (a) is higher than if we were to include the above four industries. For instance, in the year 2000 , the manufacturing share is about 16.5 percent while it is 14.5 percent when including all industries.
    ${ }^{53}$ One way to add persistence is by including preferences to local amenities that are time-invariant. In Appendix 3.3 we extend the model by incorporating into the households moving decisions the preference for local amenities. We also show that all our quantitative results are robust to the presence of additive and time-invariant amenities.

[^26]:    ${ }^{54}$ There is an alternative interpretation that can be given to this specification. Consider the model where households only take an idiosyncratic draw when they are born. In this model, at each moment in time a fraction $\rho$ of agents survives to the next period, while a fraction $1-\rho$ is replaced with new agents (possibly the offspring of the agents that die) and these are the agents that when born take a new draw.

[^27]:    ${ }^{55}$ There is a rapid and growing interest to answer these type of questions; see for instance, Fajgelbaum, Morales, Suárez-Serrato, Zidar (2015), Ossa (2015), and Tombe and Zhu (2015).
    ${ }^{56}$ We can therefore extend the analysis of Acemoglu et al. (2012) to a frictional economy. Moreover, we could incorporate local natural disaster shocks and quantify their effect, as recently analyzed in Carvalho et al. (2014).

[^28]:    ${ }^{57} \hat{C}_{t}^{n, 0}=1$ if the household in region $n$ at time $t$ is non-employed.

[^29]:    ${ }^{58}$ Dix-Carneiro (2014) studies the impact of capital mobility on the reallocation of labor.

[^30]:    ${ }^{59}$ Notice that $\dot{w}_{t}^{n j}=\dot{w}_{t}^{n}=\dot{r}_{t}^{n j}=\dot{r}_{t}^{n}$ for all $n$ such that $\tau^{n j, n k}=0$, and $\dot{r}_{t}^{n j}=\dot{w}_{t}^{n j} \dot{L}_{t}^{n j}$ for all $n$ such that $\tau^{n j, n k} \neq 0$.

[^31]:    ${ }^{60}$ In a few cases (12 of 30,118 observations), the bilateral trade flows have small negative values due to negative change in inventories. Most of these observations involve bilateral trade flows between the constructed rest of the world and some other countries, and in two cases, bilateral trade flows of Indonesia. We input zero trade flows when we observe these small negative bilateral trade flows that in any way represent a negligible portion of total trade.

[^32]:    ${ }^{61}$ In a few cases ( 34 observations), gross output was determined to be a bit smaller than value added (probably due to some small discrepancies between trade and production data-for instance, a few missing trade shipments in the CFS database); in these cases we constrain value added to be equal to gross output.

[^33]:    ${ }^{62}$ In 22 cases, data are missing, and in these cases we search for employment data in the closest available year. Still, in three cases (Alaska in the plastics and rubber industry, and North Dakota and Vermont in the petroleum and coal industries, we could not find employment data) thus, we input zero employment. The 19 cases in which we find employment data in years different from 2000 represent in total less than $0.01 \%$ of U.S. employment in 2000 .

[^34]:    ${ }^{63}$ The ACS interviews provide a representative sample of the U.S. population for every year since 2000. For the year 2001, the sample consists of $0.5 \%$ of the U.S. population. The survey is mandatory and is a complement to the decennial Census.
    ${ }^{64}$ The 2000 Census asked people about the state in which they lived five years before but not the previous year; thus, we do not use the Census data despite the much larger sample.
    ${ }^{65}$ In particular, the CPS collects information on all individuals at the same address for four consecutive months, stops for eight months, and then surveys them again for another four months.
    ${ }^{66}$ We observe individuals three months apart using, on the one hand, their first and fourth interviews, and on the other, their fifth and eighth interviews.
    ${ }^{67}$ Mortality, residence change, and nonresponse rates are the main drivers of the $10 \%$ mismatch rate.

[^35]:    ${ }^{68}$ Mechanically, we distribute the interstate movers according to the intersectoral mobility matrix for the state in which they currently live.
    ${ }^{69}$ With the exception of one element, all zero transitions occur out of the diagonal. In fact, the diagonal of the matrix typically accumulates the largest probability transition values, which just reflects the fact that staying in one's current labor market is a high probability event. However, we do find that one of the estimated transition probabilities in the diagonal is zero. Only in this case we replace this value with the minimum value of the other elements in the diagonal and re-normalize such that the conditional transition probabilities add to one.

[^36]:    ${ }^{70}$ Gross output data for Cyprus was not available for 2007 in the petroleum industry; thus we input its value for the year 2004, which is the closest year with available data.

[^37]:    ${ }^{71}$ We use four quarters of data in the J2J dataset, from 2000Q2 to 2001Q1.

[^38]:    ${ }^{72}$ The data for their analysis is publicly available on David Dorn's website http://www.ddorn.net/data.htm.
    ${ }^{73}$ For more details about this crosswalk, see ADH's Online Data and Theory Appendix.
    ${ }^{74}$ Details about the industry coding scheme (referred to as sic 87 dd by the authors) can be found on David Dorn's website.
    ${ }^{75}$ Because of the different definitions between SIC and NAICS, some industries classified as manufacturing in SIC are now part of the information and communications sector in NAICS. The value of imports for these industries is very small and we drop them from our calculations.

[^39]:    ${ }^{76}$ That is, we use U.S. states instead of commuting zones, and we use 12 manufacturing sectors classified by NAICS instead of the 397 SIC manufacturing industries that ADH use. Moreover, we restrict the sample to people within ages 25 to 65 that are in the labor force, while ADH use people 16 to 64 that worked the previous year.
    ${ }^{77}$ For more details on the imputation process, see the ADH online data dictionary.
    ${ }^{78}$ In ADH this equation varied over commuting zones $(i)$ and SIC industries $(j)$.

[^40]:    ${ }^{79} \mathrm{ADH}$ restrict the sample to those individuals aged 16 to 64 who had worked in the past year and were not institutionalized.
    ${ }^{80}$ Note that, as in ADH , the formula for $\triangle I P W_{o i t}$ contains the imports from other advanced economies, but the employment of the different U.S. states and sectors. We calibrated our model with data on other countries from the WIOD. Unfortunately, the WIOD does not contain data from New Zealand and Switzerland. Therefore, our definition of other advanced economies uses data from Australia, Denmark, Finland, Germany, Japan, and Spain. Thus, we only use these 6 countries instead of the 8 used by ADH.
    ${ }^{81}$ Using ADH's codes and data we are able to replicate their results exactly. We are particularly interested in their estimates of column 2 of their Table 2, which under their definitions of commuting zones and SIC industries delivers $b_{2}=-0.72$ with their codes and data. Unfortunately, we cannot directly use their data to aggregate to our definitions of sectors and U.S. states. We obtained the data from the original sources and followed ADH's steps. With this data and under their definitions of commuting zones, SIC industries and sample selection, we estimate $b_{2}=-0.8$ and significant. Keeping their definitions of SIC industries and sample selection but using U.S. states instead of commuting zones, we estimate $b_{2}=-0.97$ and significant. On the other hand, keeping their commuting zones and sample selection but aggregating industries to our 12 NAICS sectors we estimate $b_{2}=-1.07$ and significant. Finally, changing both the geographic and industry definitions to ours, but keeping their sample selection criteria we find $b_{2}=-1.51$ and significant. Thus, the differences in the definitions that we use tend to amplify the estimated coefficient relative to theirs.

