Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

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Working Paper 2015-005B
http://research.stlouisfed.org/wp/2016/2016-005.pdf

March 2015
Revised
November 2015

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Self-Fulfilling Credit Cycles*

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Abstract

In U.S. data 1981–2012, unsecured firm credit moves procyclically and tends to lead GDP, while secured firm credit is acyclical; similarly, shocks to unsecured firm credit explain a far larger fraction of output fluctuations than shocks to secured credit. In this paper we develop a tractable dynamic general equilibrium model in which unsecured firm credit arises from self-enforcing borrowing constraints, preventing an efficient capital allocation among heterogeneous firms. Unsecured credit rests on the value that borrowers attach to a good credit reputation which is a forward-looking variable. We argue that self-fulfilling beliefs over future credit conditions naturally generate endogenously persistent business cycle dynamics. A dynamic complementarity between current and future borrowing limits permits uncorrelated sunspot shocks to unsecured debt to trigger persistent aggregate fluctuations in both secured and unsecured debt, factor productivity and output. We show that these sunspot shocks are quantitatively important, accounting for around half of output volatility.

JEL classification: D92, E32
Keywords: Unsecured firm credit; Credit cycles; Sunspots

*The paper is a substantially revised and extended version of our earlier working paper (FRB St. Louis Working Paper No. 2012-047A). We would like to thank Andrew Atkeson, Martin Barbie, Aleksander Berentsen, Tom Krebs, Patrick Pintus, Vincenzo Quadrini and Ctirad Slavik for helpful comments and discussions, and seminar and conference audiences at Aix-Marseille, Berlin, Bielefeld, Cologne Macroeconomics Workshop, CREST (Paris), CRETE Conference, Durham, EEA-ESEM (Toulouse), Einaudi Institute Rome, Frankfurt, Mainz, SED (Toronto), Vienna Macroeconomics Workshop, UCL, UCLA and Zurich for their comments. We especially thank Maria Arias for research assistance. Leo Kaas thanks the German Research Foundation (grant No. KA 1519/3) for financial support. The usual disclaimer applies.

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1 Introduction

Over the past two decades, important advances in macroeconomic research illustrated how financial market conditions can play a key role in business cycle fluctuations. Starting with seminal contributions of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), much of this research shows how frictions in financial markets amplify and propagate disruptions to macroeconomic fundamentals, such as shocks to total factor productivity or to monetary policy.[1] More recently, and to some extent motivated by the events of the last financial crisis, several theoretical and quantitative contributions argue that shocks to the financial sector itself may not only lead to severe macroeconomic consequences but can also contribute significantly to business cycle movements. For example, Jermann and Quadrini (2012) develop a model with stochastic collateral constraints which they identify as residuals from aggregate time series of firm debt and collateral capital. Estimating a joint stochastic process for total factor productivity and borrowing constraints, they find that both variables are highly autocorrelated and that financial shocks play an important role in business cycle fluctuations.[2] But what drives these shocks to financial conditions and to aggregate productivity? And what makes their responses so highly persistent?

This paper argues that unsecured firm credit is of key importance for answering these questions. We first document new facts about secured versus unsecured firm credit. Most strikingly, for the U.S. economy over the period 1981–2012, we find that unsecured debt is strongly procyclical, with some tendency to lead GDP, while secured debt is at best acyclical, thus not contributing to the well-documented procyclicality of total debt. This finding provides some challenge for business-cycle theories based on the conventional view of Kiyotaki and Moore (1997) that collateralized debt amplifies and even generates the business cycle. When credit is secured by collateral, a credit boom is associated with not only a higher leverage ratio but also a higher value of the collateralized assets. Conversely, an economic slump is associated with deleveraging and a decrease in the value of collateral. This suggests that secured debt, such as the mortgage debt, should be strongly correlated with GDP. But this is not what we find; to the contrary, based on firm-level data from Compustat and on aggregate data from the Flow of Funds Accounts of the Federal Reserve Board, we show that it is the unsecured part of firm credit which strongly comoves with output.

[1] For recent surveys, see Quadrini (2011) and Brunnermeier et al. (2012).

[2] Other examples of financial shocks are Kiyotaki and Moore (2012) who introduce shocks to asset resaleability, Gertler and Karadi (2011) who consider shocks to the asset quality of financial intermediaries, and Christiano et al. (2014) who use risk shocks originating in the financial sector. These papers also impose or estimate highly persistent shock processes.

To examine the macroeconomic role of unsecured firm debt, we develop and analyze a parsimonious dynamic general equilibrium model with heterogeneous firms and limited credit enforcement. In the model, credit constraints and aggregate productivity are endogenous variables. Constraints on unsecured credit depend on the value that borrowers attach to future credit market conditions which is a forward-looking variable. Aggregate productivity depends on the reallocation of existing capital among heterogeneous firms which, among others, depends on current credit constraints. When these constraints bind, they slow down capital reallocation between firms and push aggregate factor productivity below its frontier. We show that this model exhibits a very natural equilibrium indeterminacy which gives rise to endogenous cycles driven by self-fulfilling beliefs in credit market conditions (sunspot shocks). In particular, a one-time sunspot shock triggers an endogenous and persistent response of credit, productivity and output.

Intuitively, the explanation for sunspot cycles and persistence is a dynamic complementarity in endogenous constraints on unsecured credit. Borrowers' incentives to default depend on their expectations of future credit market conditions, which in turn influence current credit constraints. If borrowers expect a credit tightening over the next few periods, their current default incentives become larger which triggers a tightening of current credit. This insight also explains why a one-time sunspot shock *must be followed* by a long-lasting response of credit market conditions (and thus of macroeconomic outcomes): if market participants expect that a credit boom (or a credit slump) will die out quickly, these expectations could not be powerful enough to generate a sizable current credit boom (or slump).

The model is a standard stochastic growth model which comprises a large number of firms facing idiosyncratic productivity shocks. In each period, productive firms wish to borrow from their less productive counterparts. Besides possibly borrowing against collateral, the firms exchange unsecured credit which rests on reputation. Building upon Bulow and Rogoff (1989) and Kehoe and Levine (1993), we assume that a defaulting borrower is excluded from future credit for a stochastic number of periods. As in Alvarez and Jermann (2000), endogenous forward-looking credit limits prevent default. These credit limits depend on the value that a borrower attaches to a good reputation which itself depends on future credit market conditions.

An important contribution of this paper is the tractability of our framework which permits us to derive a number of insightful analytical results in Section 3. With standard and convenient specifications of preferences and technology, we characterize any equilibrium by one backward-looking and one forward-looking equation (Proposition 1).[3] With this characterization, we

[3]Much of the literature on limited enforceability of unsecured credit does not allow for such simple representations and therefore resorts to rather sophisticated computational techniques (see e.g. Kehoe and Perri (2002), Krueger and Perri (2006) and Marcet and Marimon (2011)).

prove that unsecured credit cannot support first-best allocations, thereby extending related findings of Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) to a growth model with idiosyncratic productivity (Proposition 2). We then show the existence of multiple stationary equilibria for a range of parameter configurations (Proposition 3). While there is always an equilibrium without unsecured credit, there can also exist one or two stationary equilibria with a positive volume of unsecured credit. One of these equilibria supports an efficient allocation of capital between firms, and another one features a misallocation of capital. The latter equilibrium is the one that provides the most interesting insights, since unsecured credit is traded and yet factor productivity falls short of the technology frontier.[4] We show that this equilibrium is always locally indeterminate, and hence permits the existence of sunspot cycles fluctuating around the stationary equilibrium (Proposition 4). Moreover, output and credit respond persistently to a one-time sunspot shock.

In Section 4 we calibrate an extended model to the U.S. economy. While sunspot shocks are the main driving force for fluctuations in unsecured credit, we also introduce fundamental shocks to collateral and to aggregate technology. This allows us to analyze to which extent different financial shocks, separately affecting secured and unsecured credit, as well as independent aggregate productivity shocks, contribute to the observed output movements in the recent business-cycle episodes. We find that sunspot shocks generate around half of the total output volatility. We further demonstrate that sunspot shocks generate highly persistent responses of several macroeconomic variables. Similarly persistent responses are neither generated by shocks to collateral nor by aggregate technology shocks. Thus, the propagation of sunspot shocks is an inherent feature of the endogenous model dynamics of unsecured credit.

One way to understand the role of expectations is that unsecured credit is like a bubble sustained by self-fulfilling beliefs, as has been argued by Hellwig and Lorenzoni (2009). Transitions from a "good" macroeconomic outcome with plenty of unsecured credit to a "bad" outcome with low volumes of unsecured credit can be triggered by widespread skepticism about the ability of financial markets to continue the provision of unsecured credit at the volume needed to support socially desirable outcomes, which is similar to the collapse of a speculative bubble.[5] The emergence and the bursting of rational bubbles in financially constrained economies

[4]The other, determinate steady states of this model either do not sustain unsecured credit (and hence resemble similar dynamics as in a Kiyotaki-Moore-type model with binding collateral constraints) or they have an efficient allocation of capital (and hence exhibit the same business cycle properties as a frictionless model).

[5]Although we use a similar enforcement mechanism as Hellwig and Lorenzoni (2009), the existence of multiple equilibria does not hinge on this specification. In fact, multiple equilibria with different levels of unsecured credit would also emerge if we used the stronger enforcement of Kehoe and Levine (1993) (i.e. two-sided market exclusion of defaulters in perpetuity).

has received attention in a number of recent contributions, e.g. Caballero and Krishnamurthy (2006), Kocherlakota (2009), Farhi and Tirole (2012) and Miao and Wang (2012). One difficulty with many of the existing macroeconomic models with bubbles is that the no-bubble equilibrium is an attracting steady state, so that they can only account for the bursting of bubbles but not for their buildup. Although there are no asset-price bubbles in our model, its equilibrium dynamics account for recurrent episodes of credit booms and busts which are solely driven by self-fulfilling beliefs. In a recent contribution, Martin and Ventura (2012) construct a model with permanent stochastic bubbles and they discuss the economy's response to belief shocks (investor sentiments), like we do. But in their model bubbles arise in an overlapping generations model with two-period lived investors for similar reasons as in Tirole (1985), whereas we consider a standard business cycle model with infinitely-lived households that permits a quantitative application.

Our work is also related to a literature on sunspot cycles arising from financial frictions. In an early contribution, Woodford (1986) shows that a simple borrowing constraint makes infinitely-lived agents behave like two-period-lived overlapping generations, so that endogenous cycles can occur with sufficiently strong income effects or with increasing returns in production (see e.g. Benhabib and Farmer (1999) for a survey).[6] Harrison and Weder (2013) introduce a production externality in a Kiyotaki-Moore (1997) model and show that sunspots emerge for reasonable values of returns to scale. Benhabib and Wang (2013) show how the interaction between collateral constraints and endogenous markups can lead to indeterminacy for plausibly calibrated parameters. Liu and Wang (2014) find that the financial multiplier arising from credit constraints gives rise to increasing returns at the aggregate level which facilitates indeterminacy. Unlike our contribution, this literature does not make a distinction between secured and unsecured borrowing, hence does not address the empirical fact we present in this paper: unsecured credit is far more important than secured credit in driving the business cycle.

Other recent contributions find equilibrium multiplicity and indeterminacy in endowment economies with limited credit enforcement under specific assumptions about trading arrangements (Gu et al. (2013)) and on the enforcement technology (Azariadis and Kaas (2013)). Azariadis and Kaas (2014) study a related model with limited enforcement, also documenting equilibrium multiplicity. That paper builds on a stylized model with linear production technologies which is not suited for a quantitative analysis, it does not consider sunspot shocks and focuses on a multi-sector economy without firm-specific risk.

[6] Although earlier work on indeterminacy has shown that sunspot shocks can induce persistent macroeconomic responses (e.g. Farmer and Guo (1994) and Wen (1998)), the adjustment dynamics are typically sensitive to the particular specifications of technologies and preferences. In our model, persistent responses arise necessarily due to the dynamic complementarity in unsecured credit conditions.

The rest of this paper is organized as follows. The next section documents empirical evidence about secured and unsecured firm credit in the U.S. economy. In Section 3, we lay out the model framework, we characterize all equilibria by a forward-looking equation in the reputation values of borrowers, and we derive our main results on equilibrium multiplicity, indeterminacy and sunspot cycles. In Section 4 we extend the model in a few dimensions and conduct a quantitative analysis to explore the impacts of sunspot shocks and fundamental shocks on business cycle dynamics. Section 5 concludes.

2 Unsecured versus Secured Firm Debt

This section summarizes evidence about firms' debt structure and its cyclical properties. We explore different firm-level data sets, covering distinct firm types, and we relate our findings to evidence obtained from the Flow of Funds Accounts. In line with previous literature,[7] we show that unsecured debt constitutes a substantial part of firms' total debt and is typically lower for samples including smaller firms. Time-series variation, whenever available, further indicates that unsecured debt plays a much stronger role for aggregate output dynamics than debt secured by collateral. We first describe the data and the variables measuring unsecured and secured debt, and then report business cycle features.

2.1 The Share of Unsecured Debt

We start with the publicly traded U.S. firms covered by Compustat for the period 1981–2012 for which Compustat provides the item "dm: debt mortgages and other secured debt". In line with Giambona and Golec (2012), we use this item to measure secured debt and we attribute the residual to unsecured debt.[8] The *unsecured debt share* is then defined as the ratio between unsecured debt and total debt. To clean the data, we remove financial firms and utilities, and we also remove those firm-year observations where total debt is negative, where item "dm" is missing or where "dm" exceeds total debt. Since Compustat aggregates can easily be biased by the effect of the largest firms in the sample (cf. Covas and den Haan (2011), we also consider subsamples where we remove the largest 1% or 5% of the firms by their asset size.[9] To see the

[7] See, in particular, recent corporate finance contributions examining heterogeneity in the debt structure across firms (e.g. Rauh and Sufi (2010), Giambona and Golec (2012) and Colla et al. (2013)), which do not address business cycles, however.

[8] This classification means that unsecured debt is not explicitly backed by collateral; it does not mean that it has zero (or little) recovery value in the case of default; see also footnote 32 below.

[9] In Appendix A, we also consider series for which all firm-level variables are winsorized at the 1% and 99% levels in order to remove the effects of outliers. We find that all results are robust to this adjustment.

impact of the largest firms for unsecured borrowing, Figure 1 shows the series of the unsecured debt share for the three samples obtained from Compustat. The role of the largest firms is quite important for the level of the unsecured debt share, although much less for the time variation.[10] The very biggest firms are likely to have better access to bond markets and hence borrow substantially more unsecured. Removing the largest 1% (5%) of firms, however, cuts out 45% (75%) of the aggregate firm debt in the sample. Interestingly, in the years prior to the financial crisis of 2007-08, the unsecured debt share fell substantially, as firms expanded their mortgage borrowing relatively faster than other types of debt, with some reversal after 2008.

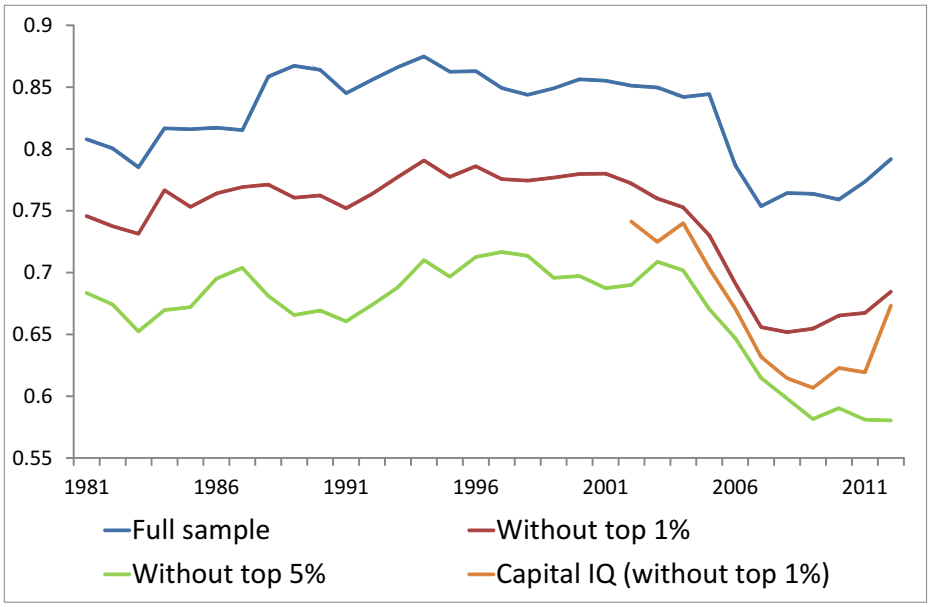

Figure 1: The share of unsecured debt in total debt for firms in Compustat and in Capital IQ.

While Compustat covers public firms, the vast majority of U.S. firms is privately owned. To complement the above evidence, we also explore two data sets to obtain debt information for private firms. We first look at firms included in the database of Capital IQ which is an affiliate of Standard and Poor's that produces the Compustat database but covers a broader set of firms. Since coverage by Capital IQ is comprehensive only from 2002 onwards, we report these statistics for the period 2002–2012. We clean the data in the same way as above and consider aggregates for the full sample (without financials and utilities) and for the sample without the 1% (5%) of the largest firms. Similar to the Compustat definition, we use Capital IQ

[10] While the effect of the largest firms is also important for total debt *growth*, it is not important for its cyclicality, as we show in the Appendix.

item "SEC: Secured Debt" and the residual "DLC+DLTT-SEC" to measure unsecured debt. The resulting unsecured debt shares show a similar cyclical pattern as those from Compustat during the same period. For visual clarity, Figure 1 only includes the series with the largest 1% of firms removed. We note that including larger firms or removing the top 5% of firms has similar effects as in Compustat, though it does not affect the U-shaped cyclical pattern in the graph. Relative to the corresponding series in Compustat, firms in Capital IQ borrow more secured in all years, which is possibly explained by the fact that these firms have a lower market transparency and hence less access to bond markets.[11]

It is worth to emphasize that even the private firms included in the Capital IQ database are relatively large firms with some access to capital markets, so they are also not fully representative for the U.S. business sector. To obtain evidence on the debt structure of small firms, we utilize the data collected in the Survey of Small Business Finances (SSBF) conducted by the Federal Reserve Board in 2003. Earlier surveys, conducted in the years 1987, 1993 and 1998, do not contain comparably comprehensive information on collateral requirements, so that we cannot obtain evidence across time. Firms in this survey report their balances in different debt categories (and within each category for up to three financial institutions). For each loan, they report whether collateral is required and which type of collateral is used (real estate, equipment and others). We aggregate across firms for each debt category and measure as secured debt all the loans for which collateral is required, while unsecured debt comprises credit card balances and all loans without reported collateral requirements. We minimally clean the data by only removing observations with zero or negative assets or equity. Table 1 shows the results of this analysis. While mortgages and credit lines constitute the largest debt categories of small firms, accounting for almost three quarters of the total, significant fractions of the other three loan categories are unsecured. This results in an unsecured debt share of 19.3 percent for firms in the SSBF.[12]

The evidence presented in Figure 1 and in Table 1 suggests that the unsecured debt share varies between 20% (for the smallest firms) and 75% (for Compustat firms excluding the largest 1%).[13] To obtain a rough estimate for the average share of unsecured debt, we can further

[11]Firms in our Capital IQ sample are actually bigger than Compustat firms. In the period 2002-2012, the average asset size of Compustat firms in the full (bottom 99%; bottom 95%) samples are 2,602 (1,230; 550) Mio. Dollars, whereas Capital IQ firms in the full (bottom 99%; bottom 95%) samples have average asset size 3,391 (2,028; 1,142). In total, there are about twice as many observations in Compustat than in Capital IQ in each year.

[12]Because collateral requirement is a dummy variable, only a fraction of these loans might actually be secured by collateral. This measure of unsecured credit should therefore be regarded as a lower bound.

[13]Note that the latter number is consistent with those found in two other studies about the debt structure of Compustat firms. Rauh and Sufi (2010) examine the financial footnotes of 305 randomly sampled non-financial

Table 1: Secured and unsecured debt in the Survey of Small Business Finances (2003)

Debt category	Share of debt (%)	Secured by real estate/equipment (%)	Secured by other collateral (%)	Unsecured (%)
Credit cards	0.6	0.0	0.0	100.0
Lines of credit	36.5	39.4	38.5	22.1
Mortgages	38.0	98.0	0.4	1.7
Motor vehicle loans	4.8	52.1	2.1	45.8
Equipment loans	6.5	62.0	1.7	36.4
Other loans	13.6	53.6	6.3	40.1
Total	100.0	65.4	15.2	19.3

utilize the information in the Flow of Funds Accounts in which firm debt is categorized into several broad categories. About 95% of all credit market liabilities of non-financial firms are either attributed to mortgages (31%), loans (31%) or corporate bonds (33%). While mortgages are clearly secured and bonds are unsecured types of debt, the security status classification is ambiguous for loans. Among the non-mortgage loans in Table 1, around 30% are unsecured; this is a similar fraction as found in other studies.[14] Taken together, this suggests that around 45% ($\approx (0.33 + 0.31 \cdot 0.3)/(0.95)$) of the credit liabilities of non-financial firms is unsecured. In Section 4, we use an unsecured debt share of 0.5 as a calibration target.

2.2 Business Cycle Features

2.2.1 Compustat

We consider the time series from Compustat, deflate them by the price index for business value added, and linearly detrend the real series.[15] Table 2 reports the volatility of secured and unsecured debt (relative to output) as well as the contemporaneous correlations with

firms in Compustat. Based on different measures, their unsecured debt share (defined as senior unsecured plus subordinated debt relative to total debt) is 70.3%. Giambona and Golec (2012) look at the distribution of unsecured debt shares for Compustat firms, reporting mean (median) values of 0.63 (0.75).

[14] Using bank survey data, Berger and Udell (1990) find that around 70% of all commercial and industrial loans in the U.S. are secured. Booth and Booth (2006) find that 75% of their sample of syndicated loans are secured.

[15] We use a linear trend to capture the low-frequency movements in credit and output that are quite significant over the period 1981–2012.

output. Secured debt is weakly negatively correlated with GDP in the full sample, it becomes zero and weakly positive once we exclude the top 1% or 5% firms. In sharp contrast, unsecured debt is always strongly positively correlated with GDP. Thus, the well-known procyclicality of total firm credit is driven by the independent role of unsecured debt. Both secured and unsecured debt are about three to four times as volatile as output.

Table 2: Relative Volatility and Comovement with Output (Compustat)

	Volatility relative to GDP			Correlation with GDP		
	full	w/o top 1%	w/o top 5%	full	w/o top 1%	w/o top 5%
Secured debt	3.61	3.39	2.76	-0.15	-0.05	0.15
Unsecured debt	4.19	3.73	4.43	0.70	0.70	0.75

Figure 2 shows the detrended time series of unsecured and secured debt for the full sample over the observation period, together with GDP.[16] While unsecured debt comoves strongly with output, secured debt is only weakly related. Between the mid 1990s and the mid 2000s, both debt series move together, but they exhibit quite different patterns before and after this period. Unsecured debt falls much more sharply than secured debt during all recessions except the one in 2008/09.

Figure 3 graphs the correlations between current GDP and lagged (future) real debt levels. The top panel pertains to the full sample, the middle panel to the sample without the largest 1% of firms, and the bottom panel to the sample without the largest 5% of firms. Regardless of the sample, unsecured debt (i) is strongly positively correlated with GDP, and (ii) tends to lead GDP by one year (the peak correlation is about 0.75 at one year lead). In sharp contrast, secured debt (i) is uncorrelated or negatively correlated with GDP, and (ii) tends to lag GDP when the contemporaneous correlation is weakly positive (bottom panel).

To obtain some indication about causality, we conduct a Granger causality test to explore if secured or unsecured debt contain superior information to help predict output. To do so, we estimate the equation

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \gamma d^u_{t-1} + \tau d^s_{t-1} + \varepsilon_t$$

by OLS, where d^s_t and d^u_t are secured and unsecured debt and y_t is real GDP. We note that two lags of GDP provide the best fit for the benchmark model ($R^2 = 0.835$) before including any lagged debt as additional independent variables. We find that the coefficient on unsecured

[16] See Figure 11 in Appendix A for the subsample series which are very similar.

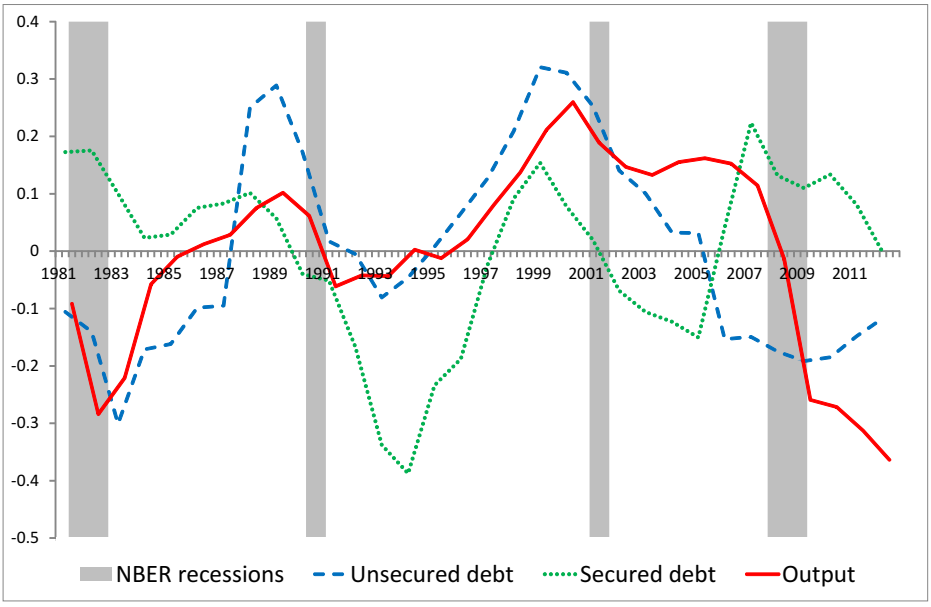

Figure 2: Unsecured and secured debt for Compustat firms, and GDP multiplied by factor four (annual linearly detrended series, 1981–2012). Quarters with NBER recessions are in grey bars.

debt is significantly positive in all sample series, whereas that on secured debt is negative but insignificantly different from zero. We thus conclude that unsecured debt helps predict future GDP movements, while this is not the case for secured debt. This result suggests that in the Great Moderation period (including the recent financial crisis period), the so-called "credit cycle" and its intimate relation to the business cycle is not driven by movements in secured debt or the value of collateral, which much of the existing macro-finance literature often attribute to as the culprit of aggregate booms and busts. In Appendix A we complement these findings by a SVAR analysis showing how shocks to unsecured credit affect output significantly whereas shocks to secured credit do not.

2.2.2 Flow of Funds Accounts

One limitation of applying evidence from Compustat in a macroeconomic context is that it only contains information about publicly traded firms. Our analysis shows that the results are not driven by the largest firms in the sample or by outliers. Furthermore, for the private firms covered in Capital IQ, the cyclical patterns look very similar during the shorter period for which these data are available. On the other hand, aggregate data from the Flow of Funds Accounts, though covering the full non-financial business sector, are not completely informative

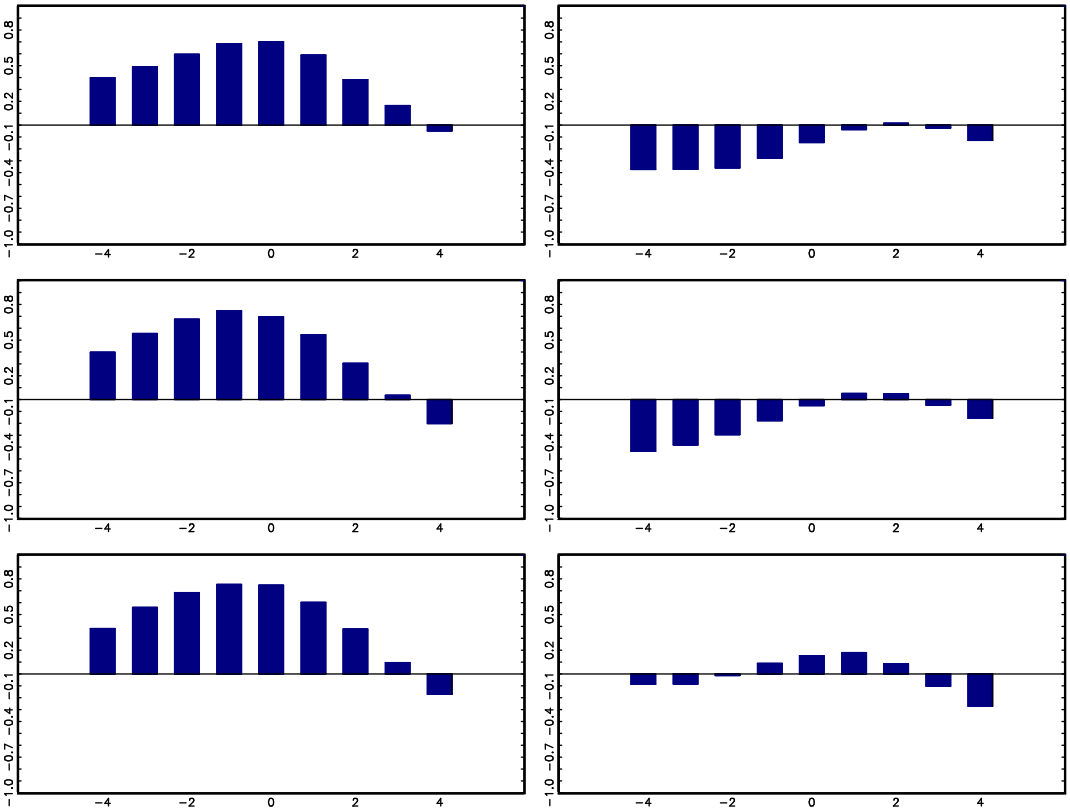

Figure 3: Correlations between GDP y_t and debt category d_{t+j} for $j \in [-4, 4]$ (left unsecured, right secured). The top (middle, bottom) graphs are for the full (bottom 99%, bottom 95%) Compustat samples. All variables are deflated and linearly detrended.

Table 3: Granger Causality Test

	R^2	Unsecured debt (γ)	Secured debt (τ)
Benchmark	0.835	N/A	N/A
Full sample	0.845	0.025**	-0.017
w/o top 1%	0.872	0.075***	-0.046
w/o top 5%	0.889	0.093***	-0.071***

Notes: *** (**) indicates significance at the 5% (10%) level.

regarding the distinction between secured and unsecured debt, as they only break the firms' credit market liabilities in several broad categories. Nonetheless, when we use those categories as proxies for secured and unsecured debt components, we confirm the main insights obtained

above.

Since mortgages can be classified as secured debt, while corporate bonds add to unsecured debt, we use those series as proxies for these two debt categories.[17] Table 4 confirms our previous findings: While mortgages are acyclical, corporate bonds as a proxy for unsecured debt are strongly procyclical.[18]

Table 4: Relative Volatility and Comovement with Output (Flow of Funds, 1981–2012)

	Volatility relative to GDP	Correlation with GDP
Mortgages	3.52	0.00
Corporate bonds	1.58	0.53

We also obtain similar findings about lead-lag relations as above. Figure 4 shows the lead-lag correlations for the annualized series: Corporate bonds are strongly correlated with output, with a peak correlation of 0.6 at a one-year lead, while mortgages show much weaker cyclicality, lagging GDP by about two years.

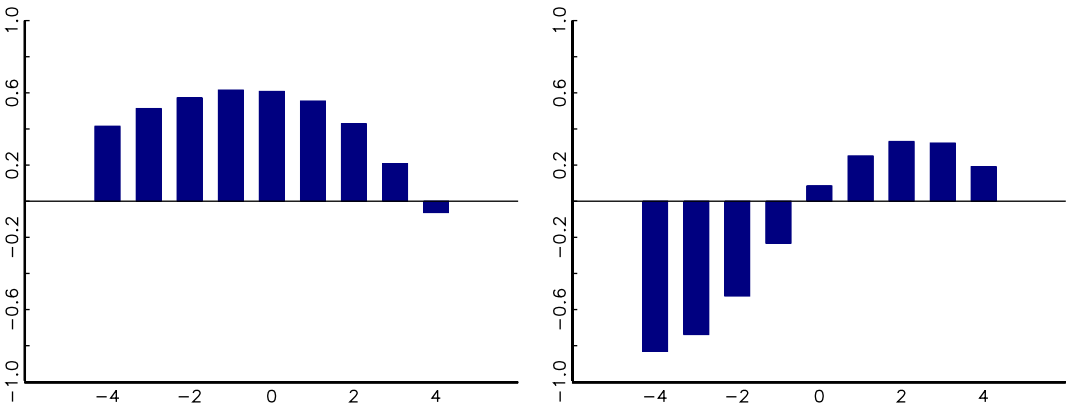

Figure 4: Correlations between GDP at year t and corporate bonds (left) and mortgages (right) at year $t + j$ for $j \in [-4, 4]$.

We briefly remark that those findings do not apply to the period before 1980 where the role of debt structure over the business cycle seems to be quite different. In fact, in the period 1952-1980, mortgages appear to be strongly correlated with output, which is more consistent with conventional macro-finance theories where the value of collateral determines

[17] As argued before, loans cannot be attributed to either proxy series.

[18] The table is based on quarterly data, deflated and detrended in the same way as for the Compustat series.

firms' borrowing capacity over the cycle. At the same time, corporate bonds show a weaker (positive) correlation with output. Although we do not have more precise measures for secured and unsecured credit prior to 1981, this observation suggests that there is a structural break around this time, possibly induced by regulatory changes and financial innovations that had a major impact on firms' debt policies. The sharp increases in unsecured debt during the expansionary phases in the 1980s and 1990s likely reflect the strong growth in corporate bond markets which are followed by sharp contractions at the onset of subsequent recessions.

3 A Model of Unsecured Firm Credit

To capture the prominent role of unsecured firm credit, we develop in this section a macroeconomic model in which heterogeneous firms face idiosyncratic productivity shocks and borrow up to endogenous credit limits which preclude default in equilibrium. For expositional reasons, we present first a benchmark model featuring only unsecured credit, along with fixed labor supply and i.i.d. firm-specific productivity shocks. We also do not consider aggregate shocks to economic fundamentals. All these assumptions will be relaxed in the next section. Tractability and the main theoretical findings are preserved in these extensions, as we show in the Appendix.

3.1 The Setup

The model has a continuum $i \in [0, 1]$ of firms, each owned by a representative owner, and a unit mass of workers. At any time t, all individuals maximize expected discounted utility

$$\mathbb{E}_t(1-\beta)\sum_{\tau \geq t}\beta^{\tau-t}\ln(c_\tau)$$

over future consumption streams. Workers are perfectly mobile across firms; they supply one unit of labor per period, have no capital endowment, and do not participate in credit markets. Firm owners hold capital and have no labor endowment.[19] They produce a consumption and investment good y_t using capital k'_t and labor ℓ_t with a common constant-returns technology $y_t = (k'_t)^\alpha (A\ell_t)^{1-\alpha}$. Aggregate labor efficiency A is constant for now, which will be relaxed in Section 4.

[19]The assumption of a representative owner by no means restricts this model to single-owner businesses. All it requires is that the firm's owners desire a smooth dividend stream for which there is ample evidence (e.g. Leary and Michaely (2011)).

Firms differ in their ability to operate capital investment k_t. Some firms are able to enhance their invested capital according to $k'_t = a^p k_t$; they are labeled "productive". The remaining, "unproductive" firms deplete some of their capital investment such that $k'_t = a^u k_t$. We assume that $a^p > 1 > a^u$ and write $\gamma \equiv a^u/a^p (< 1)$ for the relative productivity gap.[20] Productivity realizations are independent across agents and uncorrelated across time; firms are productive with probability π and unproductive with probability $1-\pi$. Thus, a fraction π of the aggregate capital stock K_t is owned by productive firms in any period. Uncorrelated productivity simplifies the model; it also implies that the dynamics of borrowers' net worth does not propagate shocks as in, e.g., Kiyotaki and Moore (1997) and Bernanke and Gertler (1989).[21] At the end of a period, all capital depreciates at common rate δ.

Timing within each period is as follows. *First*, firm owners observe the productivity of their business, they borrow and lend in a centralized credit market at gross interest rate R_t, and they hire labor in a centralized labor market at wage w_t. *Second*, production takes place. *Third*, firm owners redeem their debt; they consume and save for the next period. All prices and credit constraints (as defined below) possibly depend on the realization of sunspot shocks.

In the credit market, productive firms borrow from unproductive firms. All credit is unsecured and is only available to borrowing firms with a clean credit history. If a firm decides to default in some period, the credit reputation deteriorates and the firm is banned from unsecured credit.[22] Defaulting firms can continue to operate their business; hence they are able to produce or to lend their assets to other firms.[23] Each period after default, the firm recovers its credit reputation with probability ψ (≥ 0) in which case it regains full access to credit markets.[24]

Since no shocks arrive during a credit contract (that is, debt is redeemed at the end of the period before the next productivity shock is realized), there exist default-deterring credit limits, defined similarly as in the pure-exchange model of Alvarez and Jermann (2000). These

[20] This specification corresponds to the capital quality shocks considered by Gertler and Kiyotaki (2010) and by Christiano et al. (2014) and is used for tractability reasons (see footnote 28 below).

[21] See subsection 4.5 and Appendix D for an extension to a framework with correlated productivity shocks.

[22] That is, lenders receive no payment in a default event. In the next section and in Appendix C, we relax this assumption by introducing collateral assets and secured credit. In this extension, a fraction of unsecured borrowing can also be recovered.

[23] We can think of such default events as either a liquidation, in which case the firm owners can start a new firm which needs to build up reputation, or as a reorganization in which case the firm continues operation (see also footnote 36 below).

[24] With permanent exclusion of defaulters ($\psi = 0$), this enforcement technology corresponds to the one discussed by Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) who assume that defaulters are excluded from future credit but are still allowed to save.

limits are the highest values of credit that prevent default. Unsecured borrowing is founded on a borrower's desire to maintain a good credit reputation and continued access to future credit. Below we prove that credit constraints are necessarily binding in equilibrium (see Proposition 2).

Workers do not participate in the credit market and hence consume their labor income w_t in every period. This assumption is not as strong as it may seem; in the steady-state equilibrium it only requires that workers are not permitted to borrow. This is because the steady-state gross interest rate R satisfies $R < 1/\beta$ (see Corollary 1 below), which means that workers are borrowing-constrained and do not desire to save.[25]

At the beginning of the initial period $t = 0$, firm owner i is endowed with capital (equity) e_0^i, hence the initial equity distribution $(e_0^i)_{i\in[0,1]}$ is given. In any period $t \geq 0$, let θ_t denote the constraint on a borrower's debt-equity ratio in period t. This value is common for all borrowing firms, as we show below. It is endogenously determined to prevent default; cf. property (iii) of the following equilibrium definition. A productive firm i entering the period with equity (capital) e_t^i can borrow up to $b_t^i = \theta_t e_t^i$ and invest $k_t^i = e_t^i + b_t^i$. An unproductive firm lends out capital, so $b_t^i \leq 0$, and investment is $k_t^i = e_t^i + b_t^i \leq e_t^i$. Although the constraints $b_t^i \leq \theta_t e_t^i$ seem to resemble the collateral limits in the literature emanating from Kiyotaki and Moore (1997), we emphasize that θ_t here has very different features: it is a forward-looking variable that reacts to changes in credit market expectations.

The budget constraint for firm i with capital productivity $a^i \in \{a^p, a^u\}$ reads as

$$c_t^i + e_{t+1}^i = (a^i k_t^i)^\alpha (A\ell_t^i)^{1-\alpha} + (1-\delta)a^i k_t^i - w_t \ell_t^i - R_t b_t^i \ . \tag{1}$$

We are now ready to define equilibrium.

Definition: *A competitive equilibrium is a list of consumption, savings, and production plans for all firm owners, $(c_t^i, e_t^i, b_t^i, k_t^i, \ell_t^i)_{i\in[0,1],t\geq 0}$, conditional on realizations of idiosyncratic productivities and sunspot shocks, consumption of workers, $c_t^w = w_t$, factor prices for labor and capital (w_t, R_t), and debt-equity constraints θ_t, such that:*

(i) *$(c_t^i, e_t^i, b_t^i, k_t^i, \ell_t^i)$ maximizes firm owner i's expected discounted utility $\mathbb{E}\sum_{t\geq 0}\beta^t \ln(c_t^i)$ subject to budget constraints (1) and credit constraints $b_t^i \leq \theta_t e_t^i$.*

(ii) *The labor market and the credit market clear in all periods $t \geq 0$;*

$$\int_0^1 \ell_t^i \, di = 1 \ , \qquad \int_0^1 b_t^i \, di = 0 \ .$$

[25] Outside the steady state, the workers' first-order condition $\mathbb{E}_t[\beta R_t w_t / w_{t+1}] < 1$ is satisfied in the log-linear approximation of our model for the calibrated parameters and for shocks of reasonable magnitude.

(iii) *If $b_t^i \leq \theta_t e_t^i$ is binding in problem (i), firm owner i is exactly indifferent between debt redemption and default in period t, where default entails exclusion from credit for a stochastic number of periods with readmission probability ψ in each period following default.*

3.2 Equilibrium Characterization

Our model permits a tractable characterization. This is because individual firms' policies (i.e., borrowing/lending, saving, employment) are all linear in the firms' equity and independent of the firms' history, which in turn implies that these decisions can be easily aggregated. Furthermore, default incentives are also independent of the current size of the firm which implies that all borrowing firms face the same constraint on their debt-equity ratio. Uncorrelated idiosyncratic productivities simplify the model further because all firms have the same chance to become productive in each period, so that the distribution of wealth is irrelevant.[26]

Since firms hire labor so as to equate the marginal product to the real wage, all productive (unproductive) firms have identical capital-labor ratios; these are linked by a no-arbitrage condition implied by perfect labor mobility:

$$\frac{k_t^p}{\ell_t^p} = \gamma \frac{k_t^u}{\ell_t^u} \ . \tag{2}$$

With binding credit constraints, a fraction $z_t \equiv \min[1, \pi(1+\theta_t)]$ of the aggregate capital stock K_t is operated by productive firms. It follows from (2) and labor market clearing that

$$\frac{k_t^p}{\ell_t^p} = \frac{a_t K_t}{a^p} \leq K_t < \frac{a_t K_t}{a^u} = \frac{k_t^u}{\ell_t^u} \ ,$$

where $a_t \equiv a^p z_t + a^u(1-z_t)$ is the average capital productivity. The gross return on capital for a firm with capital productivity $a^s \in \{a^u, a^p\}$ is then $a^s R_t^*$ with $R_t^* \equiv [1-\delta+\alpha A^{1-\alpha}(a_t K_t)^{\alpha-1}]$ (see Appendix B for a detailed derivation).

In any equilibrium, the gross interest rate cannot exceed the capital return of productive firms $a^p R_t^*$ and it cannot fall below the capital return of unproductive firms $a^u R_t^*$. Thus it is convenient to write $R_t = \rho_t a^p R_t^*$ with $\rho_t \in [\gamma, 1]$. When $\rho_t < 1$, borrowers are credit constrained. In this case the leveraged equity return $[1+\theta_t(1-\rho_t)]a^p R_t^*$ exceeds the capital return $a^p R_t^*$. Unproductive firms, on the other hand, lend out all their capital when $\rho_t > \gamma$;

[26] If productivity shocks are autocorrelated, the wealth distribution becomes a state variable, but the model remains tractable since only a single variable, the wealth share of borrowing firms, matters for aggregate dynamics. This follows again because linear policy functions permit aggregation; see subsection 4.5 and Appendix D.

they only invest in their own inferior technology if $\rho_t = \gamma$. Therefore, credit market equilibrium is equivalent to the complementary-slackness conditions

$$\rho_t \geq \gamma \quad , \quad \pi(1+\theta_t) \leq 1 \ . \tag{3}$$

With this notation, the firm owner's budget constraints (1) simplify to $e_{t+1} + c_t = R_t e_t$ when the firm is unproductive in t, and to $e_{t+1}+c_t = [1+\theta_t(1-\rho_t)]a^p R_t^* e_t$ when the firm is productive. It follows from logarithmic utility that every firm owner consumes a fraction $(1-\beta)$ of wealth and saves the rest.

To derive the endogenous credit limits, let $V_t(W)$ denote the continuation value of a firm owner with a clean credit reputation who has wealth W at the end of period t, prior to deciding consumption and saving. These values satisfy the recursive equation[27]

$$V_t(W) = (1-\beta)\ln[(1-\beta)W] + \beta\pi\mathbb{E}_t\Big[V_{t+1}\Big([1+\theta_{t+1}(1-\rho_{t+1})]a^p R_{t+1}^*\beta W\Big) + \beta(1-\pi)V_{t+1}(R_{t+1}\beta W)\Big] \ .$$

The first term in this equation represents utility from consuming $(1-\beta)W$ in the current period. For the next period $t+1$, the firm owner saves equity βW which earns leveraged return $[1+\theta_{t+1}(1-\rho_{t+1})]a^p R_{t+1}^*$ with probability π and return R_{t+1} with probability $1-\pi$. It follows that continuation values take the form $V_t(W) = \ln(W) + V_t$ where V_t is independent of wealth, satisfying the recursive relation

$$V_t = (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta\mathbb{E}_t\Big[\pi\ln\Big([1+\theta_{t+1}(1-\rho_{t+1})]a^p R_{t+1}^*\Big) + (1-\pi)\ln(R_{t+1}) + V_{t+1}\Big] \ . \tag{4}$$

For a firm owner with a default flag and no access to credit, the continuation value is $V_t^d(W) = \ln(W) + V_t^d$, where V_t^d satisfies, analogously to equation (4), the recursion

$$V_t^d = (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta\mathbb{E}_t\Big[\pi\ln(a^p R_{t+1}^*) + (1-\pi)\ln(R_{t+1}) + V_{t+1}^d + \psi(V_{t+1} - V_{t+1}^d)\Big] \ . \tag{5}$$

This firm owner cannot borrow in period $t+1$ so that the equity return is $a^p R_{t+1}^*$ with probability π and R_{t+1} with probability $1-\pi$. At the end of period $t+1$, the credit reputation recovers with probability ψ in which case the continuation utility increases from V_{t+1}^d to V_{t+1}.

If a borrower has a clean credit reputation and enters period t with equity e_t, the debt-equity constraint θ_t makes him exactly indifferent between default and debt redemption if

$$\ln\Big([1+\theta_t(1-\rho_t)]a^p R_t^* e_t\Big) + V_t = \ln\Big(a^p R_t^*(1+\theta_t)e_t\Big) + V_t^d \ .$$

Here the right-hand side is the continuation value after default: the firm owner invests $(1+\theta_t)e_t$, earns return $a^p R_t^*$ and does not redeem debt. The left-hand side is the continuation value under

[27] In the absence of sunspot shocks, the expectations operator could be dropped from this and from subsequent equations because we abstract from aggregate shocks to economic fundamentals in this section.

solvency, where the borrower earns the leveraged equity return $[1+\theta_t(1-\rho_t)]a^p R_t^*$. Defining $v_t \equiv V_t - V_t^d \geq 0$ as the "value of reputation", this equation can be solved for the default-deterring constraint on the debt-equity ratio

$$\theta_t = \frac{e^{v_t} - 1}{1 - e^{v_t}(1-\rho_t)} \ . \tag{6}$$

This constraint is increasing in the reputation value v_t: a greater expected payoff from access to unsecured credit makes debt redemption more valuable, which relaxes the self-enforcing debt limit. In the extreme case when the reputation value is zero, unsecured credit cannot be sustained so that $\theta_t = 0$.

Using (4) and (5), reputation values satisfy the recursive identity

$$v_t = \beta\mathbb{E}_t\Big[\pi \ln\Big(1+\theta_{t+1}(1-\rho_{t+1})\Big)+(1-\psi)v_{t+1}\Big] = \beta\mathbb{E}_t\Big[\pi \ln\Big(\frac{\rho_{t+1}}{1-e^{v_{t+1}}(1-\rho_{t+1})}\Big)+(1-\psi)v_{t+1}\Big] \ . \tag{7}$$

We summarize this equilibrium characterization as follows.

Proposition 1 *Any solution $(\rho_t, \theta_t, v_t)_{t\geq 0}$ to the system of equations (3), (6) and (7) gives rise to a competitive equilibrium with interest rates $R_t = \rho_t a^p R_t^*$, capital returns $R_t^* = 1-\delta+\alpha A^{1-\alpha}(a_t K_t)^{\alpha-1}$ and average capital productivities $a_t = a^u + (a^p - a^u)\cdot \min[1, \pi(1+\theta_t)]$. The capital stock evolves according to*

$$K_{t+1} = \beta\Big[(1-\delta)+\alpha A^{1-\alpha}(a_t K_t)^{\alpha-1}\Big]a_t K_t \ . \tag{8}$$

An implication of this proposition is that any equilibrium follows two dynamic equations, the backward-looking dynamics of aggregate capital, equation (8), and the forward-looking dynamics of reputation values, equation (7) or, equivalently, equation (9) below. The latter identity is independent of the aggregate state K_t, and hence permits a particularly simple equilibrium analysis.[28]

Using Proposition 1, we obtain two immediate results. First, an equilibrium with no unsecured credit always exists ($v_t = 0$, $\theta_t = 0$ and $\rho_t = \gamma$ in all periods). Intuitively, there is no value to reputation, any borrower prefers to default on unsecured credit so that debt limits must be zero. Second, we show that constraints on unsecured credit are necessarily binding. This is in line with earlier results by Bulow and Rogoff (1989) and Hellwig and Lorenzoni

[28] Reputation values are independent of aggregate capital since all returns are multiples of R_t^* which is due to our specification of capital productivity shocks, $k_t' = a^s k_t$ for $s = u, p$.

(2009) who show that the first best[29] cannot be implemented by limited enforcement mechanisms which ban defaulting agents from future borrowing but not from future lending. It differs decisively from environments with two-sided exclusion, as in Kehoe and Levine (1993) and Alvarez and Jermann (2000), where first-best allocations can be sustained with unsecured credit under certain circumstances.[30] The intuition for this result is as follows. If borrowers were unconstrained, the interest rate would coincide with the borrowers' capital return. Hence there is no leverage gain, so that access to credit has no value. In turn, every borrower would default on an unsecured loan, no matter how small. We summarize this finding in

Proposition 2 *Any equilibrium features binding borrowing constraints. Specifically, given any time and history, there exists some future time and continuation history in which the borrowing constraint is binding.*

It follows immediately that the equilibrium interest rate is smaller than the rate of time preference.

Corollary 1 *In any steady state equilibrium,* $R < 1/\beta$.

3.3 Multiplicity and Cycles

Although borrowers must be constrained, the credit market may nonetheless be able to allocate capital efficiently. In particular, when the reputation value v_t is sufficiently large, credit constraints relax and the interest rate exceeds the capital return of unproductive firms who then lend out all their capital. Formally, when v_t exceeds the threshold value

$$\overline{v} \equiv \ln\left[\frac{1}{1-\gamma(1-\pi)}\right] > 0 \ ,$$

the equilibrium conditions (3) and (6) are solved by $\theta_t = (1-\pi)/\pi$ and $\rho_t = [1-e^{-v_t}]/(1-\pi) > \gamma$. Conversely, when v_t falls short of $\overline{v}$, credit constraints tighten, the interest rate equals the capital return of unproductive firms ($\rho_t = \gamma$), who are then indifferent between lending out

[29] In the first best equilibrium of this economy, there are no credit constraints, the interest rate equals the capital return of productive firms, $R_t = a^p R_t^*$, so that all firms (productive and unproductive) earn the same return. All capital is employed at productive firms, and the model is thus isomorphic to a standard growth model with a representative firm.

[30] In endowment economies with permanent exclusion of defaulters, it is well known that perfect risk sharing can be implemented if the discount factor is sufficiently large, if risk aversion is sufficiently strong or if the endowment gap between agents is large enough (see e.g. Kehoe and Levine (2001)). Azariadis and Kaas (2013) show that the role of the discount factor changes decisively if market exclusion is temporary. We remark that the multiplicity results discussed in this paper do not change under permanent exclusion of defaulters.

capital or investing in their own technology, so that some capital is inefficiently allocated. We can use this insight to rewrite the forward-looking equation (7) as

$$v_t = \mathbb{E}_t f(v_{t+1}) \ , \tag{9}$$

with

$$f(v) \equiv \begin{cases} \beta(1-\psi)v + \beta\pi \ln\left[\dfrac{\gamma}{1-e^v(1-\gamma)}\right] & , \ \text{if } v \in [0, \overline{v}] \ , \\ \beta(1-\pi-\psi)v + \beta\pi \ln(1/\pi) & , \ \text{if } v \in [\overline{v}, v_{max}] \ . \end{cases}$$

Here $v = v_{max} = \ln(1/\pi)$ is the reputation value where the interest rate reaches $\rho = 1$ and borrowers are unconstrained. It is straightforward to verify that f is strictly increasing if $\pi + \psi < 1$, convex in $v < \overline{v}$, and it satisfies $f(0) = 0$, $f(\overline{v}) > \overline{v}$ if γ is small enough, and $f(v_{max}) < v_{max}$. This reconfirms that the absence of unsecured credit ($v = 0$) is a stationary equilibrium. Depending on economic fundamentals, there can also exist one or two steady states exhibiting positive trading of unsecured credit. Figure 5(a) shows a situation in which function f has three intersections with the 45-degree line: $v = 0$, $v^* \in (0, \overline{v})$ and $v^{**} \in (\overline{v}, v_{max})$. The steady states at $v = 0$ and at v^* have an inefficient capital allocation, whereas capital is efficiently allocated at $v^{**} > \overline{v}$. Figure 5(b) shows a possibility with only two steady states, at $v = 0$ and at $v^{**} > \overline{v}$. A third possibility (not shown in the figure) is that $v = 0$ is the unique steady state so that unsecured credit is not enforceable. The following proposition describes how the set of stationary equilibria changes as the productivity ratio $\gamma = a^u/a^p$ varies.

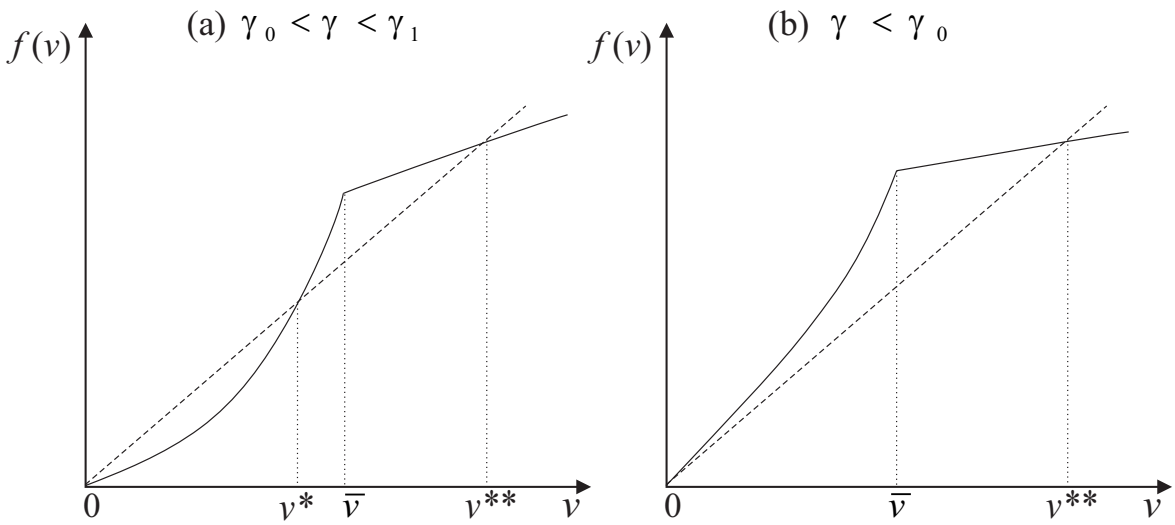

Figure 5: Steady states at $v = 0, v^*, v^{**}$.

Proposition 3 *For all parameter values $(\beta, \pi, \psi, \gamma)$ there exists a stationary equilibrium without unsecured credit and with inefficient capital allocation. In addition, there are threshold values $\gamma_0, \gamma_1 \in (0,1)$ with $\gamma_0 < \gamma_1$ for the productivity ratio γ such that:*

(a) *For $\gamma \in (\gamma_0, \gamma_1)$, there are two stationary equilibria with unsecured credit: one at $v^* \in (0, \overline{v})$ with inefficient capital allocation and one at $v^{**} \in (\overline{v}, v_{max})$ with efficient capital allocation.*

(b) *For $\gamma \leq \gamma_0$, there exists a unique stationary equilibrium with unsecured credit and efficient capital allocation at the reputation value $v^{**} \in (\overline{v}, v_{max})$.*

(c) *For $\gamma > \gamma_1$, there is no stationary equilibrium with unsecured credit.*

For small enough idiosyncratic productivity fluctuations ($\gamma > \gamma_1$), unsecured credit is not enforceable because firm owners value participation in credit markets too little. Conversely, for larger idiosyncratic shocks, exclusion from future credit is a sufficiently strong threat so that unsecured credit is enforceable without commitment. When idiosyncratic productivity shocks are sufficiently dispersed, the unique steady state with unsecured credit has an efficient factor allocation, while for intermediate values of γ, a third equilibrium emerges with unsecured credit and some misallocation of capital.

The explanation for equilibrium multiplicity is a *dynamic complementarity* between endogenous credit constraints which are directly linked to reputation values. Borrowers' expectations of future credit market conditions affect their incentives to default now which in turn determines current credit constraints. If future constraints are tight, the payoff of a clean credit reputation is modest so that access to unsecured credit has low value. In turn, current default-deterring credit limits must be small. Conversely, if borrowers expect future credit markets to work well, a good credit reputation has high value, and this relaxes current constraints.

As Figure 5 shows, multiplicity follows from a specific non-linearity between expected and current reputation values. To understand this non-linearity, it is important to highlight the different impact of market expectations on borrowing constraints and on interest rates. In the inefficient regime $v \leq \overline{v}$, improvements in credit market expectations relax credit constraints without changes in the interest rate which leads to particularly large gains from participation and hence to a strong impact on the current value of reputation. Conversely, if $v > \overline{v}$, beliefs in better credit conditions also raise the interest rate which dampens the positive effect and hence mitigates the increase in the current reputation value.

Even when unsecured credit is available and possibly supports efficient allocations of capital, that efficiency rests upon the confidence of market participants in future credit market

conditions. When market participants expect credit constraints to tighten rapidly, the value of reputation shrinks over time which triggers a self-fulfilling collapse of the market for unsecured credit. For instance, if $\gamma < \gamma_0$, the steady state at v^{**} is determinate and the one at $v = 0$ is indeterminate; see Figure 5(b). That is, there exists an infinity of non-stationary equilibria $v_t = f(v_{t+1}) \to 0$ where the value of reputation vanishes asymptotically. These equilibria are mathematically similar to the bubble-bursting equilibria in overlapping-generation models or in Kocherlakota (2009). If $\gamma \in (\gamma_0, \gamma_1)$, the two steady states at $v = 0$ and at v^{**} are determinate, whereas the one at v^* is indeterminate. In that situation, a self-fulfilling collapse of the credit market would be described by an equilibrium with $v_t \to v^*$ where a positive level of unsecured credit is still sustained in the limit.

In both of these events, a one-time belief shock can lead to a permanent collapse of the credit market. But in the latter case, indeterminacy also permits stochastic business cycle dynamics driven by self-fulfilling beliefs (sunspots).[31] Sunspot fluctuations vanish asymptotically if $\gamma < \gamma_0$, but they give rise to permanent volatility around the indeterminate steady state v^* if $\gamma \in (\gamma_0, \gamma_1)$.

Proposition 4 *Suppose that $\gamma \in (\gamma_0, \gamma_1)$ as defined in Proposition 3. Then there exist sunspot cycles featuring permanent fluctuations in credit, output and total factor productivity.*

The dynamic complementarity between current and future endogenous credit constraints not only creates expectations-driven business cycles, it also generates an endogenous propagation mechanism: because of $f'(v^*) > 1$, a one-time belief shock in period t triggers a persistent adjustment dynamics of reputation values v_{t+k} (and thus of credit, investment and output) in subsequent periods. Intuitively, a self-fulfilling boom (slump) in unsecured credit in period t can only emerge if the boom (slump) is expected to last for several periods.

Corollary 2 *A one-time sunspot shock $\varepsilon_t > 0$ ($\varepsilon_t < 0$) in period t induces a persistent positive (negative) response of firm credit and output.*

Although an endogenous propagation mechanism is not a necessary feature of any sunspot model, it tends to be associated with a large class of neoclassical models with local indeterminacy, such as Benhabib and Farmer (1994). Local indeterminacy introduces additional state

[31] Proposition 4 is a direct consequence of Theorem 3 in Chiappori et al. (1992) establishing the existence of local sunspot equilibria in the neighborhood of an indeterminate steady state. For a formal proof in the context of this model, consider an arbitrary sequence of random variables $\varepsilon_{t+1} \in (-v_t, v^{**} - v_t)$, $t \geq 1$, satisfying $\mathbb{E}_t(\varepsilon_{t+1}) = 0$, and define the stochastic process $v_{t+1} = f^{-1}(v_t + \varepsilon_{t+1}) \in (0, v^{**})$. This sunspot process solves equation (9).

variables that tend to generate endogenous propagation mechanisms. Our model differs from other sunspot models in that it uses the borrowers' reputation as an additional state variable. The difference this makes is that sunspots are tied specifically to confidence in credit markets. We show in the next section that self-fulfilling beliefs in future credit conditions can indeed generate output fluctuations broadly similar to the data.

4 Quantitative Analysis

The previous section demonstrates how self-fulfilling belief shocks can generate procyclical responses of unsecured credit, with potentially sluggish adjustment dynamics. In this section we introduce some additional features to this model and calibrate it to the U.S. economy in order to examine the business-cycle features of sunspot shocks as well as of fundamental shocks.

4.1 Model Extension

We extend the model in three directions. First, we include variable labor supply. Second, we allow firms to issue debt secured by collateral. Third, we introduce aggregate fundamental shocks to technology and to the the firms' collateral capacity. We still assume that firms' idiosyncratic productivity process is i.i.d.; this will be relaxed in subsection 4.5.

Specifically, we modify workers' period utility to $\ln(C_t - \frac{\varphi}{1+\varphi} L_t^{(1+\varphi)/\varphi})$ where L_t is labor supply and φ is the Frisch elasticity. Regarding secured borrowing, we assume that a fraction $\lambda_t < 1$ of a firm's end-of-period assets can be recovered by creditors in a default event. Since all firms can pledge collateral to their creditors, the relevant outside option of a defaulter is the exclusion from unsecured credit while retaining access to collateralized credit. As before, all credit is within the period and no default occurs in equilibrium, which implies that secured and unsecured credit carry the same interest rate R_t. Besides sunspot shocks, we allow for shocks to λ_t and to aggregate labor efficiency A_t. The first type of shock directly affects the tightness of borrowing constraints, much like the financial shocks considered by Jermann and Quadrini (2012). Shocks to labor efficiency account for those movements in aggregate output which are not generated by the endogenous response of aggregate productivity to changes in the allocation of capital.

All productive firms in period t can borrow secured up to the debt-equity limit θ_t^s which is determined from $R_t\theta_t^s = \lambda_t a^p R_t^*(1+\theta_t^s)$. For each unit of equity the firm borrows θ_t^s so that a fraction λ_t of the end-of-period assets $a^p R_t^*(1+\theta_t^s)$ fully protect the lenders who provide secured credit. On top of that, firms can borrow unsecured up to the endogenous debt-equity

limit θ_t^u. In Appendix C, we solve this extended model and show that the constraint on the total debt-equity ratio $\theta_t = \theta_t^s + \theta_t^u$ which precludes default is

$$\theta_t = \frac{e^{v_t} - 1 + \lambda_t}{1 - \lambda_t - e^{v_t}(1 - \rho_t)} \ , \tag{10}$$

where v_t is again the value of reputation, i.e. the utility benefit of a clean credit reputation, and $\rho_t = R_t/(a^p R_t^*)$. This relationship extends equation (6) to the case where some assets can be collateralized. Observe that all borrowing must be secured, i.e. $\theta_t = \theta_t^s$, if reputation has no value ($v_t = 0$). If $v_t > 0$, borrowing in excess of θ_t^s is unsecured. Note, however, that the share λ_t of the unsecured debt obligation $R_t\theta_t^u$ could be recovered if a firm opted for default. This is certainly a realistic feature since bond holders, for example, can recover a substantial fraction of their assets after a default.[32] We also generalize (9) to a forward-looking equation

$$v_t = \mathbb{E}_t f(v_{t+1}, \lambda_{t+1}) \ . \tag{11}$$

Therefore, we obtain a similar dichotomy as before: the dynamics of reputation values is independent of the capital stock, of labor market variables or technology shocks.[33] We also confirm that, for specific parameter constellations, a steady state with unsecured credit and inefficient capital allocations exists; we choose this equilibrium for the calibration of model parameters.[34] This steady state is again indeterminate so that self-fulfilling belief shocks impact on the dynamics of unsecured credit.

[32] No unsecured debt can be recovered in Section 3 where we assume $\lambda_t = 0$. Here, recovery of unsecured debt hinges on our specification that a positive share of assets is pledgeable, and this share determines both the secured debt limit θ_t^s (of which 100% can be recovered) defined above and the recovery rate of unsecured debt. Although we do not use the latter recovery rate as a calibration target, we note that our calibrated value $\lambda = 0.43$ is consistent with empirical debt recovery rates on corporate bonds. Jankowitsch et al. (2014) examine recovery rates for formal and informal default events, reporting mean recovery rates of 37.1% for Chapter 11 restructuring, 40.7% for Chapter 11 liquidation, and 51.3% for "distressed exchanges".

[33] While this property is useful to characterize equilibrium and to provide intuition for the main relationships, it is by no means essential for our theory. Alternative formulations of the collateral constraint can give rise to an equation of the form $v_t = \mathbb{E}_t f(v_{t+1}, \lambda_{t+1}, K_{t+1}, A_{t+1})$, so that technology shocks feed (positively) into reputation values. For example, if collateral assets are share λ_t of capital $(1-\delta)k_t$ (instead of wealth $a^p R_t^* k_t$), then the level of the interest rate R_t (which depends on K_t and A_t) would enter into the equations for the debt-equity ratio as well as into the recursive equation for v_t, equations (20) and (21).

[34] As in the simpler model of the previous section, the other (determinate) steady states either feature efficient factor allocations or do not sustain unsecured credit. Hence their business-cycle properties either resemble those of a standard frictionless model or those of an economy with collateral-based credit constraints. We prove these assertions formally in Appendix D for the extended model with possibly autocorrelated idiosyncratic shocks. We also discuss how a higher value of collateral assets impacts on the availability of unsecured credit. Interestingly, a reduction of λ may have detrimental consequences on the enforceability of unsecured credit.

4.2 Calibration

We calibrate this model to the U.S. economy, choosing parameters so that the indeterminate steady state equilibrium matches suitable long-run properties. The calibration targets correspond to statistics obtained for the U.S. business sector in the period 1981–2012. As our best available data source on unsecured versus secured credit is available at annual frequency, we calibrate the model annually and set δ, α and β in a standard fashion to match plausible values of capital depreciation, factor income shares and the capital-output ratio.[35] The Frisch elasticity is set to $\varphi = 1$. We normalize average capital productivity in steady state to $a = 1$, as well as steady-state labor efficiency to $A = 1$. We set the exclusion parameter $\psi = 0.1$ so that a defaulting firm owner has difficulty obtaining unsecured credit for a period of 10 years after default.[36] We choose the remaining parameters π, λ and a^u to match the following three targets:[37] (1) Credit to non-financial firms is 0.82 of annual GDP; (2) the debt-equity ratio of constrained firms is set to $\theta = 3$; (3) unsecured credit is 50 percent of total firm credit.[38] Given that this model has a two-point distribution of firm productivity (and hence of debt-equity ratios), the choice of target (2) is somewhat arbitrary. We also calibrated the model with $\theta = 2$ and obtain very similar results. All parameters are listed in Table 5.

Despite the simplicity of this model, it is worth to note that this calibration has a reasonably low share of credit-constrained firms ($\pi = 18\%$) and that the mean debt-to-capital ratio ($\theta\pi = 54\%$) is in line with empirical findings (cf. Rajan and Zingales (1995)). We further remark that our parameterization produces a plausible cross-firm dispersion of total factor productivity (TFP). With firm-level output equal to $y^i = (a^i - 1)k^i + (A\ell^i)^{1-\alpha}(a^i k^i)^\alpha$, we calculate a standard deviation of log TFP equal to 0.33 which is close to the within-industry average 0.39 reported in Bartelsman et al. (2013).

[35] Output is real value added in the business sector, and the capital stock is obtained from the perpetual inventory method based on total capital expenditures in the business sector. This yields 1.49 as our target for the capital-output ratio.

[36] This 10-year default flag corresponds to the bankruptcy regulation for individual firm owners who file for bankruptcy under Chapter 7 of the U.S. Bankruptcy Code. Generally, business firms in the U.S. can file for bankruptcy under either Chapter 7 (which leads to liquidation) or Chapter 11 (which allows to continue operation after reorganization). In either case, it is plausible to assume that the reputation loss from default inhibits full access to credit for an extended period.

[37] The normalization $a = a^u + \pi(1 + \theta)(a^p - a^u) = 1$ then yields parameters a^p and $\gamma = a^u/a^p$.

[38] (1) Credit market liabilities of non-financial business are 0.82 of annual output (average over 1981-2012, Flow of Funds Accounts of the Federal Reserve Board, Z.1 Table L.101). (2) Debt-equity ratios below 3 are usually required to qualify for commercial loans (see Herranz et al. (2012)). Further, in our SSBF (Capital IQ, Compustat) samples, the mean debt-equity ratios are 3.04 (3.15, 2.43). Regarding (3), see the discussion at the end of subsection 2.1.

Table 5: Parameter choices.

Parameter	Value	Explanation/Target
δ	0.078	Depreciation rate
α	0.3	Capital income share
β	0.89	Capital-output ratio
φ	1	Frisch elasticity
ψ	0.1	10-year default flag
π	0.18	Share of productive firms (Credit volume)
λ	0.43	Recovery parameter (Unsecured debt share)
a^u	0.779	Lowest productivity (Debt-equity ratio $\theta = 3$)
a^p	1.080	Highest productivity (Normalization $a = 1$)

4.3 Persistence of sunspot shocks

For illustrative purposes, we first suppose that fundamental shocks are absent, i.e. λ_t and A_t are at their steady-state values, while sunspot shocks are the only source of business cycle dynamics. In this case, the log-linearized dynamics of the credit-to-capital ratio[39] follows

$$\hat{\theta}_{t+1} = \frac{1}{\varphi_2}\hat{\theta}_t + d_1\varepsilon^s_{t+1} \ ,$$

where coefficients d_1, φ_2 are specified in Appendix C and ε^s_{t+1} is a sunspot shock. In particular, we find that the autocorrelation coefficient is

$$\frac{1}{\varphi_2} = \frac{1}{\beta(1-\psi) + \beta\pi(1+\theta)\frac{a^p - a^u}{a^u}} \ ,$$

which equals 0.949 for the calibrated model parameters. That is, when we feed the model with uncorrelated sunspot shocks, the endogenous dynamics of credit is highly persistent, actually more so than in the data.[40] Table 6 confirms this finding and reports business-cycle statistics under sunspot shocks. Most importantly, uncorrelated sunspot shocks generate persistent business-cycle dynamics with autocorrelation coefficients which are somewhat above

[39]The hat symbol over a variable indicates the log deviation of that variable from steady state. The credit-to-capital ratio in the model is $\theta_t\pi$ whose log deviation equals the one of the borrowers' credit-equity ratio θ_t because π is constant.

[40]To obtain data analogues for the (linearly detrended) credit-to-capital ratio, we can either use firm credit from the Flow of Funds Accounts or from Compustat. This yields annual auto-correlation coefficients of 0.883 (Flow of Funds) and 0.817 (Compustat).

their data counterparts. Volatilities and co-movement of consumption and investment are plausible, whereas credit is too strongly correlated with output, which comes as no surprise since all output dynamics is induced by the sunspot-driven dynamics of credit.[41]

Table 6: Model statistics with uncorrelated sunspot shocks.

	Output	Credit	Investment	Consumption	Employment
U.S. data (1981–2012)					
Rel. volatility	1	2.73	2.43	0.80	0.69
Autocorrelation	0.848	0.832	0.618	0.899	0.893
Corr. with output	1	0.620	0.715	0.969	0.910
Model					
Rel. volatility	1	2.59	3.28	0.84	0.35
Autocorrelation	0.925	0.903	0.791	0.978	0.978
Corr. with output	1	0.993	0.771	0.923	0.923

Notes: Output and investment are for the U.S. business sector. Credit is for the Compustat firm sample considered in Section 2 without the largest 1% of firms. All variables are deflated, logged and linearly detrended. Model statistics are based on 100,000 simulations of 32 periods. The volatility of sunspot shocks is set so that the model-generated output volatility matches the one in the data.

4.4 Multiple shocks

To evaluate the relative importance of sunspot shocks for the overall business cycle dynamics, we include fundamental shocks to the financial sector (collateral parameter λ_t) as well as to the real sector (labor efficiency parameter A_t). We identify sunspot shocks as well as fundamental shocks as follows (see Appendix C for details). We use the Compustat series for secured credit to compute the secured-credit-to-capital ratio whose cyclical component measures $\hat{\theta}_t^s$. Similarly, all Compustat credit (secured and unsecured) identifies the series $\hat{\theta}_t$. We then use those two series to back out the (log deviations of) reputation values $\hat{v}_t$ and collateral parameters $\hat{\lambda}_t$. Labor efficiency $\hat{A}_t$ is identified so as to match the cyclical component of output. Hence it

[41] When we decompose total credit into secured and unsecured components, we find that both are strongly procyclical when sunspots are the only source of shocks; correlations with output are 0.83 (0.95) for secured (unsecured) credit. Secured credit is however much less volatile; relative standard deviations are 1.36 (4.13) for secured (unsecured) credit. In a decomposition analysis that follows in the next section and in Appendix C we elaborate on this issue, showing in particular that sunspot shocks are most important for fluctuations in unsecured debt while collateral shocks are the main driving force of secured debt.

picks up all output dynamics left unexplained by financial shocks (shocks to collateral $\hat{\lambda}$ and to unsecured credit $\hat{v}$). Therefore, all three shocks together generate by construction the output dynamics of the data. We can therefore measure how each of them contributes to the total volatility and how it accounts for output movements in specific episodes.

We consider the following structural vector autoregression (SVAR):

$$\begin{pmatrix} \hat{A}_t \\ \hat{\lambda}_t \\ \hat{v}_t \end{pmatrix} = B \begin{pmatrix} \hat{A}_{t-1} \\ \hat{\lambda}_{t-1} \\ \hat{v}_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{pmatrix} \tag{12}$$

with coefficient matrix B, and apply the Choleski decomposition such that $e_t = (e_{1t}, e_{2t}, e_{3t})' = C(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})'$ with lower triangular matrix C. We call ε_{1t} the technology shock, ε_{2t} the collateral shock and ε_{3t} the sunspot shock. By ordering the sunspot shock as the last variable in the SVAR, we assume that those shocks can impact only credit market expectations contemporaneously, while all correlations in the innovations to $(\hat{A}_t, \hat{\lambda}_t, \hat{v}_t)$ are attributed to technology shocks and to collateral shocks. In other words, we may be attributing too much influence to technology and collateral shocks, thus providing a lower bound on the contribution of sunspot shocks. We take into account that the forward-looking equation for reputation values (11) imposes a restriction on the last row in (12); see equation (31) for a log-linearized version in which all coefficients are given from the calibrated parameters. We therefore only estimate the first two equations and impose the model restriction on the last row in (12).[42]

Figure 6 shows the implied time series decomposition of output into the three components associated with the three identified structural shocks $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})$, where the red solid line in each window represents the data output and the blue dashed line represents the predicted output when only one of the structural shocks is active. The lower-right graph puts all three shocks together which, by construction, explains all output variation. Sunspot shocks ε_{3t} account for the broad business cycle features of output quite well (lower-left window); this is despite the fact that we have attributed all the contemporaneous correlations of the three innovations to technology and collateral shocks. Collateral shocks seem to matter for the credit-expansion periods in the late 1990s and in the mid 2000s, while they only account for a moderate portion of the decline in 2008-2009. Technology shocks do not appear to matter much for output movements since the 1990s, although they are responsible for a substantial fraction of the output drop after the Great Recession.

[42] We also perform a similar analysis in which we estimate (12) without any restrictions on matrix B. Our main findings are similar and attribute an even larger role to sunspot shocks. Particularly, we find that sunspot shocks account for around 70% of the variance of output, employment, consumption and investment. They also induce similarly persistent impulse responses.

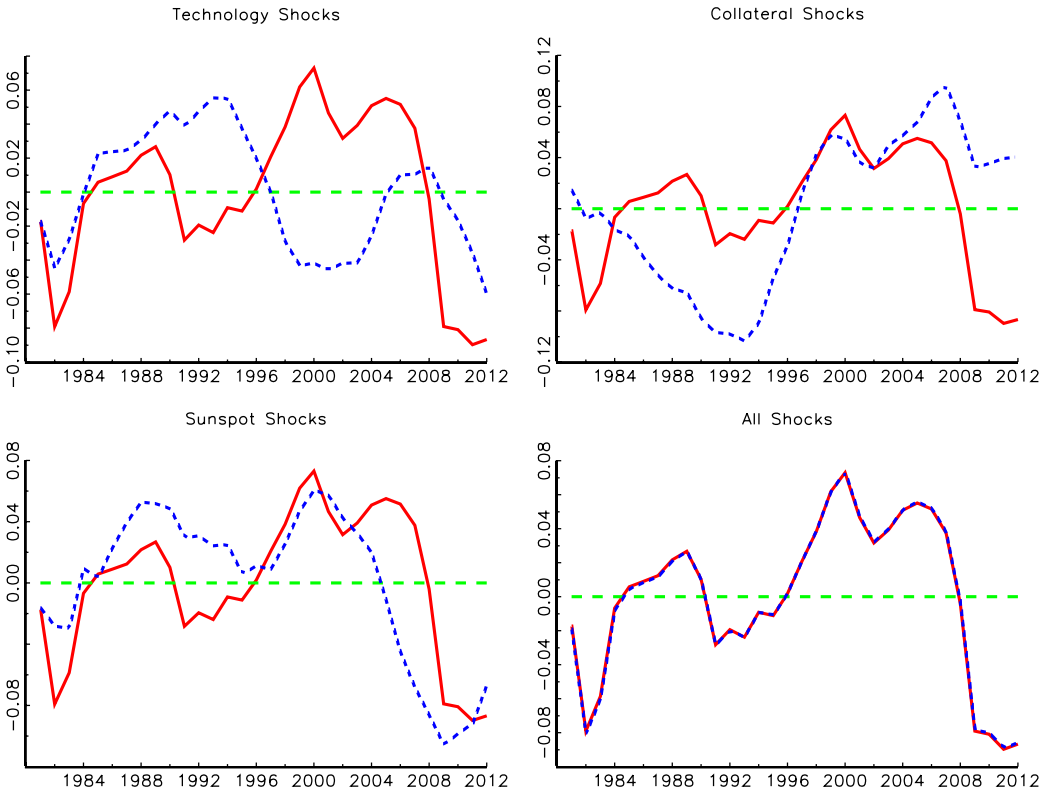

Figure 6: Decomposition of output in the three shocks. The red (solid) curve is data output, the blue (dashed) curves are model-generated output dynamics if only one shock is active (first three graphs). In the bottom right graph all three shocks are active.

We can also decompose the total variance of output (more specifically, the power spectrum) into the three structural components, with each contributed separately from the three identified shocks. We find that sunspot shocks account for 51% of the total output variance, collateral shocks explain 44%, and technology shocks only explain the remaining 5%. This result is quite striking: Even though the $\hat{A}_t$ series is constructed to match all output dynamics that is not explained by financial shocks, shocks to $\hat{A}_t$ play a rather minor role for the total output variance. The result that the two financial shocks account for the vast majority of output dynamics differs markedly from Jermann and Quadrini (2012) who find that productivity shocks and financial shocks both explain around half of output fluctuations. But our model generates a similar result when we shut down sunspot shocks. Precisely, when we set $\hat{v}_t = 0$ and identify $\hat{\lambda}_t$ and $\hat{A}_t$ to account for the dynamics of total firm credit and output, we find that structural shocks to collateral and to technology each account for around half of output

volatility. Put differently, technology shocks pick up a large fraction of the output dynamics that is coming from self-fulfilling belief shocks driving unsecured credit in our model.[43] Sunspot shocks not only matter for output, but also for the dynamics of other macroeconomic variables: in a variance decomposition we find that sunspot shocks account for around 50% of the variance of employment, consumption, investment, and firm credit, whereas technology shocks account for less than 10%;[44] see Figure 13 and Table 9 in Appendix C for the time series decompositions and business cycle statistics.

We can write the output series as $\hat{Y}_t = \hat{Y}_{1,t} + \hat{Y}_{2,t} + \hat{Y}_{3,t}$, where each component is contributed from each of the three structural shocks respectively shown in Figure 6, and then compute the lead-lag correlations $\text{corr}(\hat{Y}_{i,t+j}, \hat{Y}_t)$ for $i = 1, 2, 3$, and $j = -4, \ldots, 4$. Figure 7 shows that the part of output driven by sunspot shocks is the most highly correlated with data output and it also leads output by one year, whereas technology-driven output is either insignificant or it correlates negatively with data output. Collateral-driven output correlates positively and it lags data output. These findings are broadly consistent with the lead-lag observations that we present in Section 2.

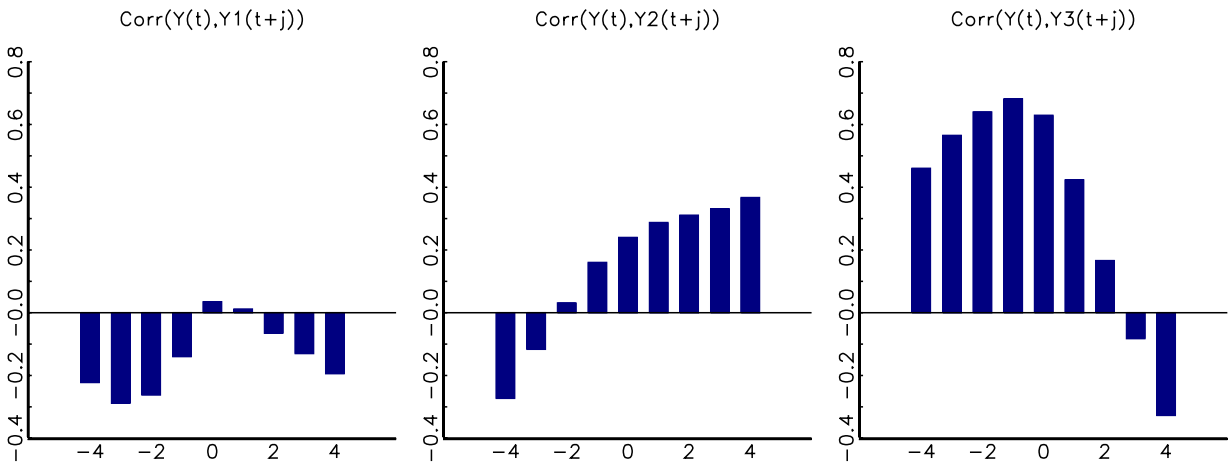

Figure 7: Lead-lag correlation between output $\hat{Y}_t$ and the three shock components $\hat{Y}_{i,t+j}$, where $i = 1$ is technology shocks, $i = 2$ is collateral shocks and $i = 3$ is sunspot shocks.

By construction, the three structural shocks not only describe output fluctuations but also the secured and unsecured credit dynamics. It is interesting to examine to what extent the

[43] Our model further differs from Jermann and Quadrini (2012) in that aggregate productivity is partly endogenous and hence correlates positively with financial conditions.

[44] The standard errors for these point estimates are small. For example, the one-standard error bands for the sunspot contributions to any of these variables are between 1 and 11 percentage points wide.

three shocks contribute to these separate credit cycles and how these components relate to output dynamics. Unsurprisingly, we find that shocks to the collateral parameter λ are most important in explaining movement in secured credit, whereas technology shocks or sunspot shocks do not track the secured-credit cycle well; see Figure 14 in Appendix C. When we correlate the component of secured debt driven by collateral shocks (shown in the top right graph of Figure 14) with output, we find that it is acyclical and lags output (see the middle graph in Figure 16). So the reason that secured credit is not procyclical becomes clear: Its dynamics is dominated by collateral shocks, and this structural shock-induced movement in secured credit is not procyclical. In contrast, the sunspot-induced movement in secured credit is strongly procyclical (Figure 16, right graph) but this shock does not dominate fluctuations in secured credit (Figure 14, bottom left graph). A similar decomposition analysis can be undertaken for the unsecured credit cycle; see Figure 15. Here we find that sunspot shocks account for the major movements in unsecured credit, while technology shocks and collateral shocks do not capture the pattern of unsecured credit well. Given this finding and that unsecured credit is strongly procyclical, we can infer that sunspot shocks are the major driving force of the credit cycle.

Lastly, in Figure 8 we show the impulse responses of output, investment, consumption and employment to the three orthogonal shocks (one standard deviation). Sunspot shocks generate a stronger and more persistent response than the other two shocks. In particular, a collateral shock implies that the positive output response turns negative only two years after the shock which is at odds with the VAR evidence on the real effects of credit market shocks (e.g., Lown and Morgan (2006) and Gilchrist et al. (2009)). This suggests that sunspot shocks (on top of or independent of collateral shocks) are an important contributing factor.

4.5 Autocorrelated productivity

A strong simplifying assumption in our benchmark model is that firm productivity is drawn each period independently from a two-point distribution. The main benefit is that this makes the model very tractable, permitting a complete analytical characterization of the global dynamics in Section 3. We now show that this framework can be readily extended to account for an autocorrelated idiosyncratic productivity process (still on a two-point distribution). We establish in Appendix D that our main results survive. In particular, there are multiple steady-state equilibria; further, a steady-state equilibrium with unsecured credit and some misallocation of capital is typically indeterminate and hence gives rise to sunspot-driven dynamics.

We write π_p (π_u) for the probability that a currently productive (unproductive) firm becomes

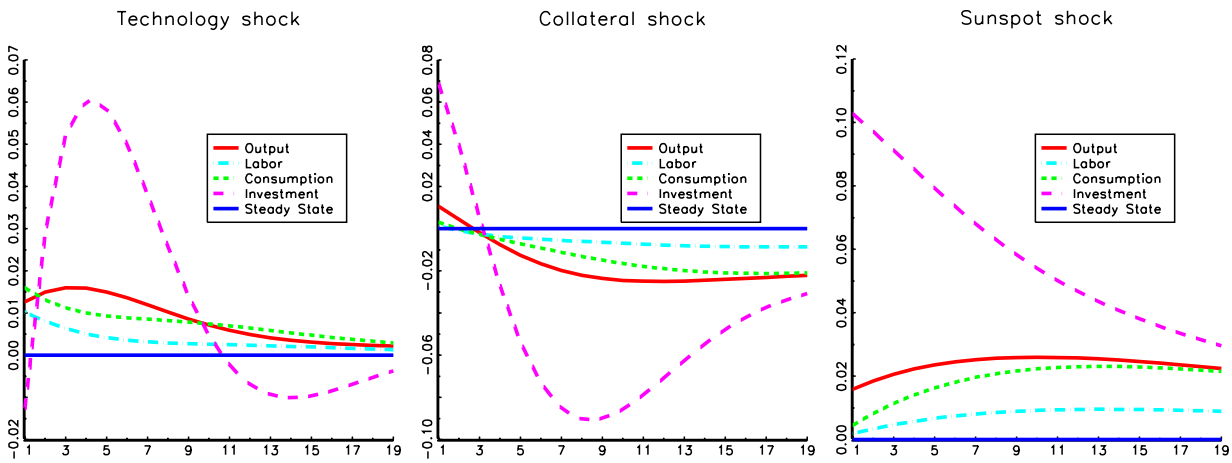

Figure 8: Impulse responses to the three shocks.

productive next period. With probability $1-\pi_p$ ($1-\pi_u$, resp.), this firm becomes unproductive next period. The i.i.d. case considered thus far is the special case where $\pi_p = \pi_u = \pi$. To calibrate π_p and π_u, together with parameters λ, a_u and a_p, we use the same calibration targets as before, but now also require that the annual autocorrelation of firm productivity is 0.39. We take this calibration target from Abraham and White (2006) who estimate plant-level productivity dynamics for the U.S. manufacturing sector. This yields the parameter values $\pi_p = 0.425$, $\pi_u = 0.035$, $a^p = 1.087$, $a^u = 0.759$, $\lambda = 0.419$. All other parameters are the same as in Table 5.

There are two substantial changes in this model extension. The first is that the capital distribution among firms becomes a state variable. Because of the two-point distribution and the fact that all policy functions are linear in the firm's wealth, the only relevant state variable is the share of capital owned by productive firms which we label x_t.[45] When θ_t denotes again the debt-equity ratio of borrowing firms, the dynamics of x_t takes the simple form $x_{t+1} = X(x_t, \theta_t)$. The second change is that the forward-looking dynamics of reputation values has an additional lag/lead and takes the form $v_t = \mathbb{E}_t f(v_{t+1}, \lambda_{t+1}, v_{t+2}, \lambda_{t+2})$. At the indeterminate steady-state equilibrium, this equation permits sunspot solutions. For details about these variations and the log-linearization, see Appendix D.

The main quantitative findings are robust to this change. When we feed this model with

[45]Because firm growth rates alternate stochastically between two values, the wealth distribution does not follow an ergodic process and therefore has no stationary distribution (as in the model with uncorrelated shocks). This does not matter for aggregate dynamics for which only the aggregate wealth of borrowing firms is relevant.

uncorrelated sunspot shocks as the only source of aggregate uncertainty, we obtain very similar volatilities and co-movement patterns as in Table 6. But because the wealth distribution responds itself persistently to shocks, the autocorrelation coefficients increase slightly. For example, output has annual autocorrelation of 0.947. When we also consider shocks to technology and to the collateral constraint (in addition to sunspot shocks), we confirm our previous finding: sunspot shocks account for a large portion of output dynamics. This is illustrated in Figure 9 which shows similar patterns as Figure 6 for the model with uncorrelated firm-specific shocks. In a variance decomposition, we find that sunspot shocks account for 40% of output dynamics, while collateral shocks explain 58% and technology shocks only 2%. The contribution of sunspot shocks to the dynamics of employment, consumption, investment and credit is also around 40% and hence a bit lower than in the model with uncorrelated shocks. We further find similar lead-lag patterns and impulse responses; see Appendix D for the corresponding figures.

5 Conclusions

Two enduring characteristics of the business cycle are the high autocorrelations of credit and output time series, and the strong cross-correlation between those two statistics. Understanding these correlations, without the help of large and persistent shocks to the productivity of financial intermediaries and to the technical efficiency of final goods producers, has been a long-standing goal of macroeconomic research and the motivation for the seminal contributions mentioned in the first paragraph of the introduction to this paper. Is it possible that cycles in credit, factor productivity and output are not the work of large and persistent productivity shocks that afflict all sectors of the economy simultaneously? Could these cycles instead come from shocks to people's confidence in the credit market?

This paper gives an affirmative answer to both questions within an economy in which part of the credit firms require to finance investment is secured by collateral, and the remainder is based on reputation. Unsecured firm credit in the U.S. economy from 1981 to 2012 is strongly correlated with GDP and leads it by about a year. In our model, unsecured credit improves debt limits, facilitates capital reallocation and helps aggregate productivity, provided that borrowers expect plentiful unsecured credit in the future. Favorable expectations of future debt limits increase the value of remaining solvent and on good terms with one's lenders. Widespread doubts, on the other hand, about future credit will lead to long-lasting credit tightening with severe macroeconomic consequences.

It is this dynamic complementarity of current with future lending that connects macroeco-

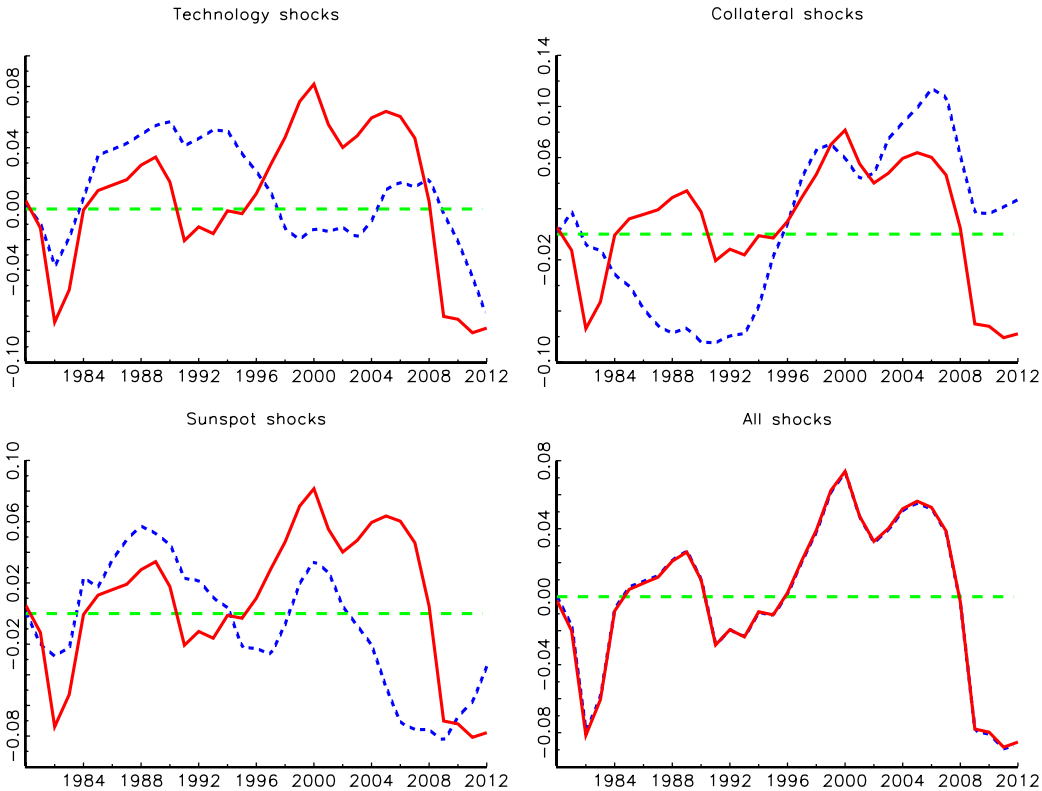

Figure 9: Decomposition of output in the three shocks for the model with autocorrelated firm-specific shocks. The red (solid) curve is data output, the blue (dashed) curves are model-generated output dynamics if only one shock is active (first three graphs). In the bottom right graph all three shocks are active.

nomic performance over time and endows one-time expectational impulses with long lasting responses. A calibrated version of our economy matches well with the observed autocorrelations and cross-correlations of output, firm credit and investment. Using our model to identify structural shocks to collateral credit, unsecured credit and aggregate technology, we find that sunspot shocks to unsecured credit account for around half the variance in all major time series, while technology shocks play a rather minor role. On the other hand, if the endogenous influence of sunspots on credit conditions is excluded a priori, our results show that too much output volatility would be incorrectly attributed to exogenous movements in aggregate technology – a standard result in the literature. We conclude that self-fulfilling and endogenously propagated credit shocks are quite important in U.S. business cycles.

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Appendix A: Further Empirical Findings

A.1 Winsorized Data

In Section 2 we consider aggregate series for different samples from Compustat and from Capital IQ. To account for the possible impact of outliers, we also consider aggregate series where all firm-level variables are winsorized at the 1% and 99% levels. Again we compare samples containing all firms with those where the largest 1% or the largest 5% of firms are removed. Figure 10 shows the series of the unsecured debt share for the different samples obtained from Compustat and also for the one from Capital IQ (without the largest 1% of firms). As in Figure 1 we see that the effect of the largest firms if important for the level of the unsecured debt share, but not much for the cyclicality. The cyclical pattern of the Capital IQ series during 2002-2012 is also similar as for the non-winsorized series.

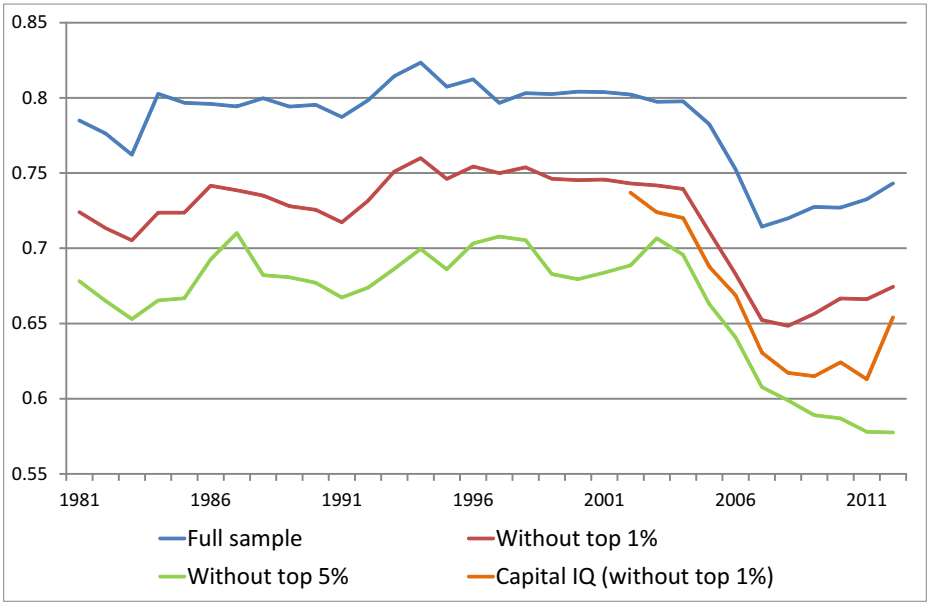

Figure 10: The share of unsecured debt in total debt for firms in Compustat and in Capital IQ (winsorized data).

Table 7 confirms the main insights about business-cycle features of secured and unsecured debt (again deflated and linearly detrended). As in Table 2, both secured and unsecured debt are three to four times as volatile as GDP, and unsecured debt shows a much greater procyclicality than secured debt which is now weakly positively correlated with GDP for all three sample series.

Table 7: Relative Volatility and Comovement with Output (Compustat, winsorized data)

	Volatility relative to GDP			Correlation with GDP		
	full	w/o top 1%	w/o top 5%	full	w/o top 1%	w/o top 5%
Secured debt	2.82	2.79	2.74	0.11	0.14	0.22
Unsecured debt	3.27	3.52	4.40	0.75	0.71	0.75

A.2 The Impact of the Largest Firms

As Figure 1 shows, the largest firms have a strong effect on the unsecured debt share, although much less on the cyclical features of this share. Regarding the series of secured and unsecured debt, Figure 11 shows that their cyclical components are also very similar in the three Compustat samples, and they are further in line with the respective Capital IQ series in the overlapping sample period.

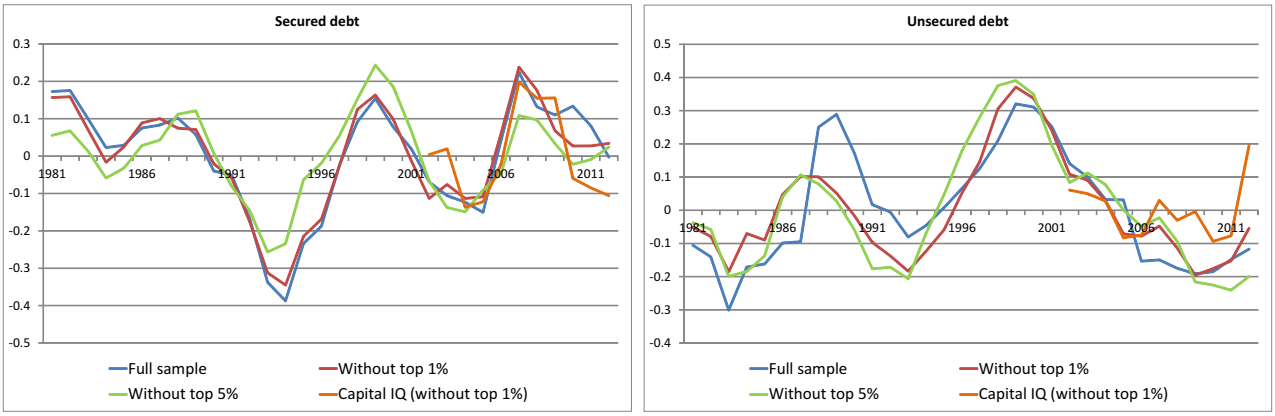

Figure 11: Secured and unsecured debt for Compustat (full sample, without 1% and 5% of largest firms) and Capital IQ (without 1% largest firms). All series are linearly detrended and are based on non-winsorized data.

While cycles are similar, debt growth varies decisively when the largest firms are removed from the sample, as is shown in Table 8. Apparently, the largest firms in the Compustat sample accumulated more debt than smaller firms, and this difference is particularly strong for unsecured debt which grew only by 1% for the bottom 95% of firms relative to 3.6% for the full sample.

Table 8: Average annual debt growth (Compustat, 1981-2012)

	Debt growth		
	full	w/o top 1%	w/o top 5%
Secured debt	3.9	3.8	2.4
Unsecured debt	3.6	2.8	1.0

A.3 SVAR analysis

As a parsimonious diagnostic analysis, we complement the business-cycle observations in Section 2 by a very simple two-variable SVAR model to study impulse responses of U.S. GDP to different debt shocks, assuming that shocks to debt have no contemporaneous impact on output. The findings shown in Figure 12 (right column) are consistent with those above. Shocks to unsecured debt account for significant impulse responses of output (explaining about 35%-45% of total output variance, lower-right panel), while shocks to secured debt generate no significant output response (explaining about 0%-10% of total output variance, upper-right panel).

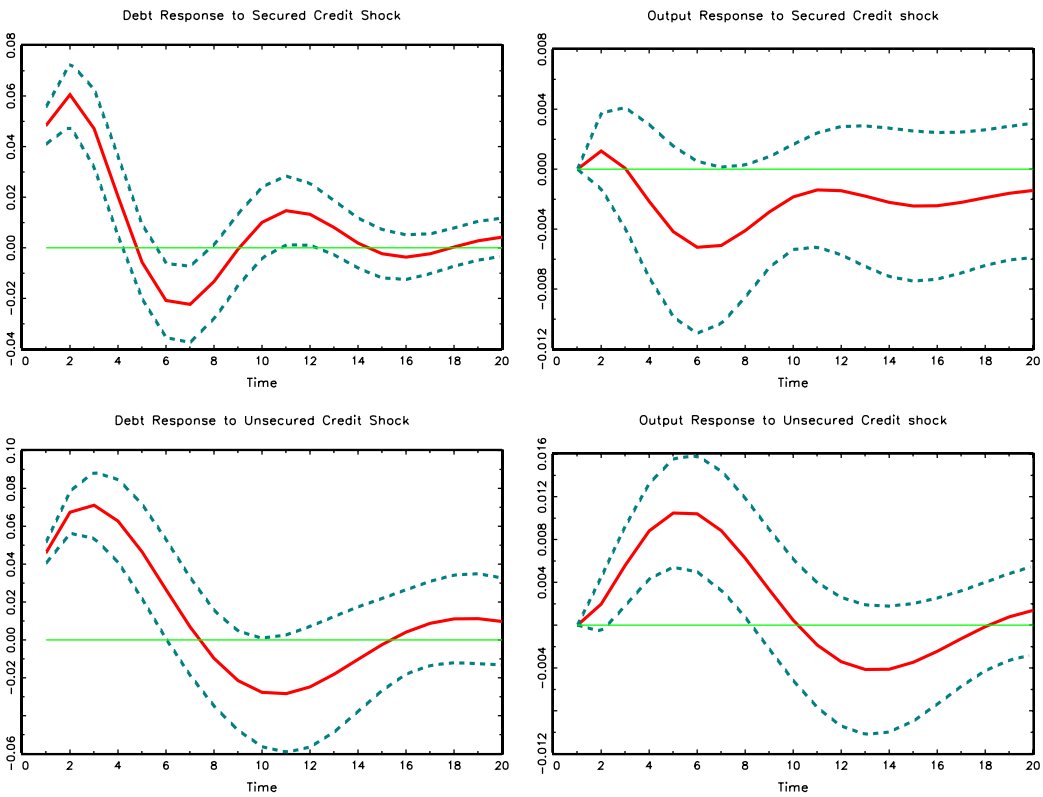

Figure 12: Impulse responses of output and debt to shocks to secured credit (top) and unsecured credit (bottom).

Appendix B: Proofs

Derivation of the capital return R_t^*:

Consider a firm of type $s \in \{p, u\}$ with capital k_t^s. It employs ℓ_t^s workers so that the marginal product of labor equals the real wage:

$$(1-\alpha)A\Big(\frac{a^s k_t^s}{A\ell_t^s}\Big)^{\alpha} = w_t \ .$$

It follows for all firms

$$a^p \frac{k_t^p}{\ell_t^p} = a^u \frac{k_t^u}{\ell_t^u} \equiv \kappa_t \ ,$$

where κ_t is independent of firm type. Let L_t^s denote aggregate employment of type-s firms. Thus,

$$L_t^p = \frac{a^p}{\kappa_t} z_t K_t \ , \ L_t^u = \frac{a^u}{\kappa_t}(1-z_t)K_t \ ,$$

where $z_t = \min(1, \pi(1+\theta_t))$ is the share of capital operated by productive firms. Then labor market clearing $L_t^p + L_t^u = 1$ implies that

$$\kappa_t = a_t K_t \text{ and } w_t = (1-\alpha)A^{1-\alpha}(a_t K_t)^{\alpha} \ ,$$

with $a_t = z_t a^p + (1-z_t)a^u$. Therefore, firm s employs $\ell_t^s = \frac{a^s}{a_t K_t} k_t^s$ workers, and its gross output net of labor costs is

$$(a^s k_t^s)^{\alpha}(A\ell_t^s)^{1-\alpha} + (1-\delta)a^s k_t^s - w_t \ell_t^s = a^s k_t^s \Big[\Big(\frac{A}{a_t K_t}\Big)^{1-\alpha} + 1 - \delta - \frac{w_t}{a_t K_t}\Big] = a^s k_t^s \Big[\alpha\Big(\frac{A}{a_t K_t}\Big)^{1-\alpha} + 1 - \delta\Big] \ .$$

This shows that $a^s R_t^*$ with $R_t^* = [1 - \delta + \alpha(A/(a_t K_t))^{1-\alpha}]$ is the capital return of a type-s firm.

Proof of Proposition 2: Suppose that, for some time T and a given history of sunspot shocks, borrowers were unconstrained in all future periods $t \geq T$ and continuation histories. That is, in any period $t \geq T$ and future event history, unproductive firms lend out all their capital to productive firms who borrow $(1-\pi)K_t$ in the aggregate, and the interest rate equals the capital return of productive firms, $R_t = a^p R_t^*$. It follows that $\rho_t = 1$ for all $t \geq T$ and all continuation histories, so that there are no gains from leverage. This implies that the only solution to equation (7) is $v_t = 0$ for all $t \geq T$ and all realizations of shocks. But then it follows from equation (6) that debt-equity constraints are $\theta_t = 0$, which together with $\rho_t = 1$ contradicts equation (3). □

Proof of Corollary 1: In steady state, $K_{t+1} = K_t$ implies that $1/\beta = aR^*$ where $a = a^u + (a^p - a^u)\min(1, \pi(1+\theta))$ is average capital productivity. From Proposition 2, any steady

state has binding credit constraints, so that $R = \rho a^p R^* < a^p R^*$. Then either $\rho = \gamma = a^u/a^p$ implies $R = a^u R^* < aR^*$, or $\rho > \gamma$ and (3) implies $\pi(1+\theta) = 1$, so that $a = a^p$ and again $R < aR^*$. In any case, $R < 1/\beta$ follows. □

Proof of Proposition 3: Because of $f(v_{max}) < v_{max}$ and continuity, a solution $f(v) = v \in (\overline{v}, v_{max})$ exists iff $f(\overline{v}) > \overline{v}$. This condition is

$$[1 - \gamma(1-\pi)]^{1+\Phi} > \pi^{\Phi} \ , \tag{13}$$

with $\Phi = \beta\pi/(1-\beta(1-\psi))$. The LHS in (13) is decreasing in γ, LHS<RHS at $\gamma = 1$, and LHS>RHS at $\gamma = 0$. Therefore there exists a solution $\gamma_1 \in (0,1)$ where LHS=RHS. It follows that the steady state $v^{**} \in (\overline{v}, v_{max})$ exists if $\gamma < \gamma_1$.

Since f is strictly convex in $v \in (0, \overline{v})$, a steady state $v^* \in (0, \overline{v})$ exists if $\gamma < \gamma_1$ (implying $f(\overline{v}) > \overline{v}$) and if $f'(0) < 1$. The latter condition is equivalent to $\gamma > \gamma_0 \equiv \frac{\Phi}{\Phi+1} \in (0,1)$. It is also straightforward to verify that γ_0 satisfies inequality (13) which proves $\gamma_0 < \gamma_1$. This completes the proof. □

Appendix C: Extended Model and Log-Linearization

We first derive the dynamic equilibrium equations, for arbitrary stochastic processes for labor efficiency A_t and for the collateral share λ_t. Then we log-linearize the model at the indeterminate steady state.

Labor market equilibrium

Labor demand of firm i is

$$\ell_t^i = a_t^i k_t^i \Big(\frac{(1-\alpha) A_t^{1-\alpha}}{w_t} \Big)^{1/\alpha} ,$$

so that aggregate labor demand is $L_t^d = a_t K_t \Big(\frac{(1-\alpha) A_t^{1-\alpha}}{w_t} \Big)^{1/\alpha}$ with average capital productivity

$$a_t = a^u + \min(1, \pi(1+\theta_t))(a^p - a^u) . \tag{14}$$

With labor supply $L_t^s = w_t^{\varphi}$, the market-clearing wage is

$$w_t = (1-\alpha)^{\frac{1}{1+\varphi\alpha}} (a_t K_t)^{\frac{\alpha}{1+\varphi\alpha}} A_t^{\frac{1-\alpha}{1+\varphi\alpha}} .$$

This yields the equilibrium labor-to-capital ratio in efficiency units

$$\frac{A_t L_t}{a_t K_t} = (1-\alpha)^{\frac{\varphi}{1+\varphi\alpha}} (a_t K_t)^{-\frac{1}{1+\varphi\alpha}} A_t^{\frac{1+\varphi}{1+\varphi\alpha}} .$$

Credit market equilibrium

The return on capital for firm $i = p, u$ is

$$(a_t^i k_t^i)^{\alpha} (A_t \ell_t^i)^{1-\alpha} + a_t^i (1-\delta) k_t^i - w_t \ell_t^i = \underbrace{[1 - \delta + r_t]}_{\equiv R_t^*} a_t^i k_t^i ,$$

where

$$r_t \equiv \alpha \Big(\frac{A_t L_t}{a_t K_t} \Big)^{1-\alpha} \tag{15}$$

is the average capital return. A productive firm that borrows θ_t per unit of equity has leveraged equity return

$$\tilde{R}_t \equiv a^p R_t^* + \theta_t (a^p R_t^* - R_t) .$$

A firm without access to unsecured credit can still borrow secured, such that the debt does not exceed the value of collateral assets which equal share λ_t of end-of-period wealth. Thus, the debt-equity constraint for secured borrowing θ_t^s is determined from $R_t \theta_t^s = \lambda_t a^p R_t^* (1 + \theta_t^s)$, so that

$$\theta_t^s = \frac{\lambda_t}{\frac{R_t}{a^p R_t^*} - \lambda_t} . \tag{16}$$

As in Section 3, the continuation utility of a borrower with a clean credit reputation can be written $\ln(W) + V_t$ with end-of-period wealth W. Similarly, $\ln(W) + V_t^d$ is the continuation utility of a borrower with a default flag. Then a borrower with equity e decides not to default at the end of the period if

$$\ln[\tilde{R}_t e] + V_t \geq \ln[(1+\theta_t)a^p R_t^*(1-\lambda_t)e] + V_t^d \ .$$

With $v_t = V_t - V_t^d$, the default-deterring debt-equity ratio follows from this equation as

$$\theta_t = \frac{\lambda_t + e^{v_t} - 1}{1 - \lambda_t - (1 - \frac{R_t}{a^p R_t^*})e^{v_t}} \ . \tag{17}$$

Clearly, θ_t increases in both the collateral share λ_t and in the reputation value v_t. Moreover, $\theta_t = \theta_t^s$ if $v_t = 0$.

If a borrower decides to default, he is punished by exclusion from unsecured credit, retaining full access to secured credit. With probability ψ, the credit reputation recovers, and the borrower can also borrow unsecured. Exactly as in the model without secured borrowing, we derive a forward-looking equation for the reputation value:

$$v_t = \beta \mathbb{E}_t \Big\{ \pi \ln \Big[\frac{a^p R_{t+1}^* + \theta_{t+1}(a^p R_{t+1}^* - R_{t+1})}{a^p R_{t+1}^* + \theta_{t+1}^s(a^p R_{t+1}^* - R_{t+1})} \Big] + (1-\psi)v_{t+1} \Big\} \ . \tag{18}$$

Here the expression in the term $\ln(.)$ is the excess leverage return that a borrower with a clean credit reputation enjoys relative to a defaulter who has access to secured borrowing only.

Consider a credit market equilibrium with $R_t = a^u R_t^*$, so that unproductive firm owners are indifferent between lending and investing in their own technology. In such situations, we have $R_t/(a^p R_t^*) = a^u/a^p = \gamma$, so that equations (16), (17) and (18) simplify to

$$\theta_t^s = \frac{\lambda_t}{\gamma - \lambda_t} \ , \tag{19}$$

$$\theta_t = \frac{\lambda_t + e^{v_t} - 1}{1 - \lambda_t - (1-\gamma)e^{v_t}} \ , \tag{20}$$

$$v_t = \mathbb{E}_t \Big\{ \beta\pi \ln \Big[\frac{\gamma - \lambda_{t+1}}{1 - \lambda_{t+1} - (1-\gamma)e^{v_{t+1}}} \Big] + \beta(1-\psi)v_{t+1} \Big\} \ . \tag{21}$$

Capital accumulation and output

The aggregate capital stock evolves according to

$$K_{t+1} = \beta \Big[(1-\delta)a_t K_t + \alpha(A_t L_t)^{1-\alpha}(a_t K_t)^\alpha \Big] = \beta a_t K_t [1 - \delta + r_t] \ . \tag{22}$$

Since $a^p > 1 > a^u$, productive firms enhance their capital stock by $(a^p - 1)k_t^i$ while unproductive firms deplete capital $(1 - a^u)k_t^i$. In the aggregate, therefore, the term $(a_t - 1)K_t$ (which may be positive or negative outside the steady state) adds to aggregate investment. Total output is

$$Y_t = (a_t - 1)K_t + (A_t L_t)^{1-\alpha}(a_t K_t)^\alpha = (a_t - 1)K_t + a_t K_t \frac{r_t}{\alpha} \ . \tag{23}$$

Consumption and investment are

$$\begin{aligned} C_t &= w_t L_t + (1-\beta)a_t K_t[1 - \delta + r_t] = a_t K_t \Big[\frac{1-\alpha}{\alpha} r_t + (1-\beta)(1-\delta+r_t)\Big] \ , \\ I_t &= Y_t - C_t \ . \end{aligned}$$

Steady state equilibrium and calibration of parameters

The main theoretical results of Section 3 can be extended to this more general setup. In particular, credit constraints are binding in equilibrium, provided that the collateral share parameter λ is sufficiently low. Further, an indeterminate steady state with unsecured credit and an inefficient capital allocation exists for specific parameter values. For a proof of these assertions in a more general framework that also incorporates autocorrelated productivity shocks, see Appendix D. Given the calibration target for K/Y, $A = a = 1$ and parameters α and φ, we can solve for steady-state values

$$K = (1-\alpha)^\varphi (K/Y)^{\frac{1+\varphi\alpha}{1-\alpha}} \ , \ r = \alpha Y/K \ ,$$

as well as for output, consumption and investment. Given calibration targets for the aggregate credit-to-capital ratio $\pi\theta$ and for the borrowers' debt-equity ratio θ, we can solve for π.

For any choice of a^u and $a^p = a^u + \frac{1-a^u}{\pi(1+\theta)}$, and given the calibration target for the secured-credit-to-capital ratio $\theta^s\pi$, we obtain λ from $\theta^s = \lambda/(\gamma - \lambda)$, $\gamma = a^u/a^p$, and the reputation value in steady state from (20):

$$v = \ln\Big[\frac{(1-\lambda)(1+\theta)}{1+\theta(1-\gamma)}\Big] \ .$$

We then choose a^u so that also equation (21) is satisfied in steady state, namely

$$v[1 - \beta(1-\psi)] = \beta\pi \ln\Big[\frac{\gamma - \lambda}{1 - \lambda - (1-\gamma)e^v}\Big] \ .$$

Log linearization

Log-linearize equations (14), (15), (19), (20), (21), (22), (23) to obtain

$$\begin{aligned}
\hat{a}_t &= \pi\theta(a^p - a^u)\hat{\theta}_t \ , && (24)\\
\hat{r}_t &= r_1\hat{A}_t + r_2[\hat{a}_t + \hat{K}_t] \ , && (25)\\
\hat{\theta^s}_t &= \frac{\gamma}{\gamma-\lambda}\hat{\lambda}_t \ , && (26)\\
\hat{\theta}_t &= d_1\hat{v}_t + d_2\hat{\lambda}_t \ , && (27)\\
\hat{v}_t &= \mathbb{E}_t[\varphi_1\hat{\lambda}_{t+1} + \varphi_2\hat{v}_{t+1}] \ , && (28)\\
\hat{K}_{t+1} &= \hat{K}_t + \hat{a}_t + \frac{r}{1-\delta+r}\hat{r}_t \ , && (29)\\
\hat{Y}_t &= \hat{K}_t + (1+K/Y)\hat{a}_t + \hat{r}_t \ , && (30)
\end{aligned}$$

where

$$\begin{aligned}
r_1 &= \frac{(1-\alpha)(1+\varphi)}{1+\varphi\alpha} \ ,\\
r_2 &= -\frac{1-\alpha}{1+\varphi\alpha} \ ,\\
d_1 &= \frac{e^v v}{\lambda+e^v-1} + \frac{(1-\gamma)e^v v}{1-\lambda-(1-\gamma)e^v} \ ,\\
d_2 &= \frac{\lambda}{\lambda+e^v-1} + \frac{\lambda}{1-\lambda-(1-\gamma)e^v} \ ,\\
\varphi_1 &= \frac{\beta\pi}{v}\frac{\lambda(1-\gamma)(e^v-1)}{(\gamma-\lambda)(1-\lambda-(1-\gamma)e^v)} \ ,\\
\varphi_2 &= \beta(1-\psi) + \frac{\beta\pi(1-\gamma)e^v}{1-\lambda-(1-\gamma)e^v} \ .
\end{aligned}$$

Because $\varphi_2 > 1$ at the indeterminate steady state, we obtain from (28) a stationary forward solution with sunspot shocks ε^s_{t+1} satisfying $\mathbb{E}_t(\varepsilon^s_{t+1}) = 0$:

$$\hat{v}_{t+1} = \frac{1}{\varphi_2}\hat{v}_t - \frac{\varphi_1}{\varphi_2}\hat{\lambda}_{t+1} + \varepsilon^s_{t+1} \ . \qquad (31)$$

Identification of shocks

Given the series for the credit-to-capital ratio and for the secured-credit-to-capital ratio, we obtain $\hat{\theta}_t$ and $\hat{\theta^s}_t$. Then, we can solve for $\hat{\lambda}_t$ and $\hat{v}_t$ from (26) and (27). Finally, we choose $\hat{A}_t$ to match the output series $\hat{Y}_t$.

We then consider the VAR $(\hat{A}_t, \hat{\lambda}_t, \hat{v}_t)' = B(\hat{A}_{t-1}, \hat{\lambda}_{t-1}, \hat{v}_{t-1})' + e_t$ where we estimate the first two equations and impose restriction (31) on the last one. For the structural shocks, we apply

the Choleski decomposition $e_t = C\varepsilon_t$ with lower-triangular matrix C. We obtain

$$B = \begin{pmatrix} 0.5602 & -0.2291 & -0.0034 \\ 0.4466 & 0.9731 & 0.0058 \\ -0.1447 & -0.3153 & 0.9478 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 0.0211 & 0.0000 & 0.0000 \\ -0.0162 & 0.0263 & 0.0000 \\ 0.0099 & -0.0407 & 0.0683 \end{pmatrix} .$$

Macroeconomic dynamics

We show in Section 4 how the three structural shocks account for the U.S. output dynamics since the 1980s (see Figure 6). Sunspot shocks also account for a substantial fraction of the dynamics of other macroeconomic variables, as shown in Figure 13. While the model matches by construction the data output series perfectly in all episodes of the business cycle, it does not match employment, consumption, credit and investment perfectly, as these variables are not included in our SVAR and we only consider three aggregate shocks contained in $(\hat{A}_t, \hat{\lambda}_t, \hat{v}_t)$. The fit is not too bad, however, since the broad business-cycle pattern and the second moments of these data series are matched reasonably well by the model (see Table 9). Also, sunspot shocks are key to achieving this reasonably good match.

Table 9: Model statistics with all three shocks.

	Output	Credit	Investment	Consumption	Employment
U.S. data (1981–2012)					
Rel. volatility	1	2.73	2.43	0.80	0.69
Autocorrelation	0.848	0.832	0.618	0.899	0.893
Corr. with output	1	0.620	0.715	0.969	0.910
Model					
Rel. volatility	1	2.52	2.34	0.89	0.37
Autocorrelation	0.971	0.955	0.850	0.981	0.968
Corr. with output	1	0.852	0.852	0.984	0.981

Notes: Output and investment are for the U.S. business sector. Credit is for the Compustat firm sample considered in Section 2 without the largest 1% of firms. All variables are deflated, logged and linearly detrended. Model statistics are analytical moments based on impulse response functions.

Complementing the decomposition of output dynamics in Figure 6, Figures 14 and 15 show the decompositions of secured and unsecured credit into the three shock contributions. By construction, all three shocks together fully account for the separate credit cycles. Technology shocks do not matter much for either series, collateral shocks capture the the major patterns of secured credit, whereas sunspot shocks account mostly for the movements of unsecured credit.

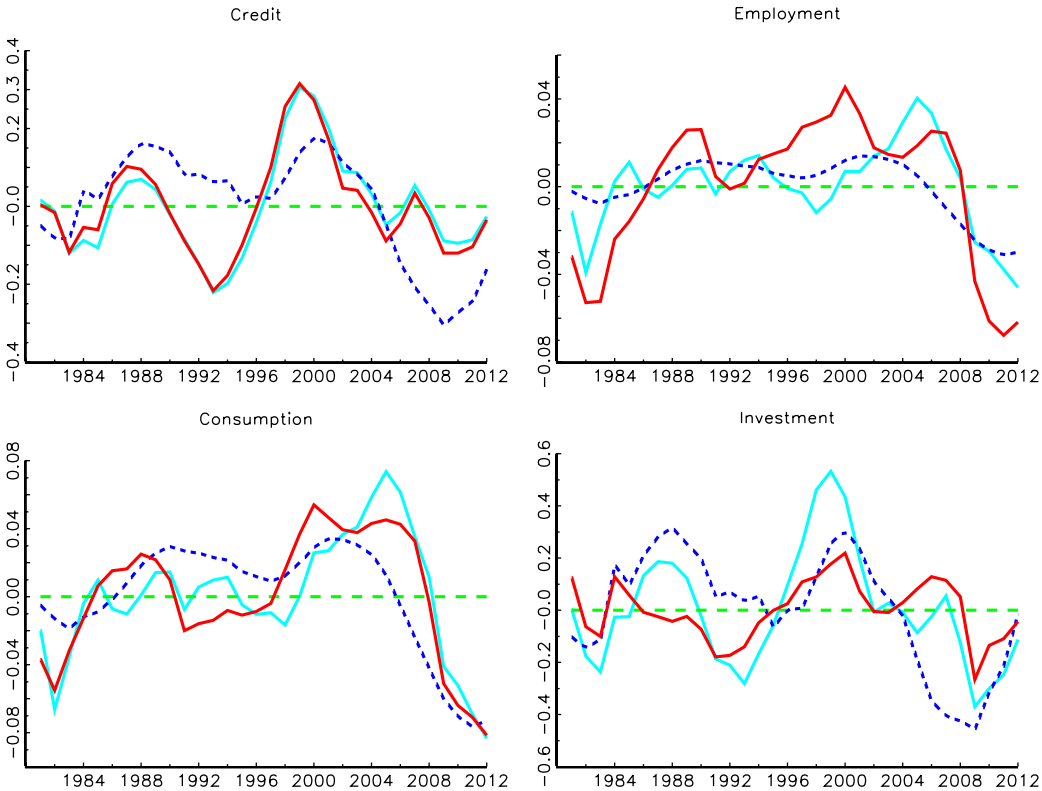

Figure 13: Dynamics of credit, employment, consumption and investment. The red (solid) curves are U.S. data, the light blue (solid) curves are model dynamics if all shocks are active, the blue (dashed) curves are model dynamics with only sunspot shocks.

Given that secured credit is acyclical in the data, it is interesting to explore whether this applies to the components of secured credit induced by each of the three structural shocks. To this end we compute the lead-lag correlations $\text{corr}(\hat{Y}_t, \hat{X}_{i,t+j})$ for $j = -4, \ldots, 4$, where $\hat{X}_{i,t}$ is the component of secured credit induced by shock $i = 1$ (technology), $i = 2$ (collateral) and $i = 3$ (sunspot). Figure 16 shows that the part of output driven by sunspot shocks is highly correlated with output. At the same time, however, sunspot shocks do not account for the main movements of secured credit (see Figure 14). In contrast, the collateral-induced component of secured credit matters most for the secured-credit cycle, but this component is not procyclical and lags output (see the middle graph in Figure 16). The technology-driven component of secured credit neither matters much for the secured-credit cycle (Figure 14), nor does it correlate positively with output (left graph in Figure 16).

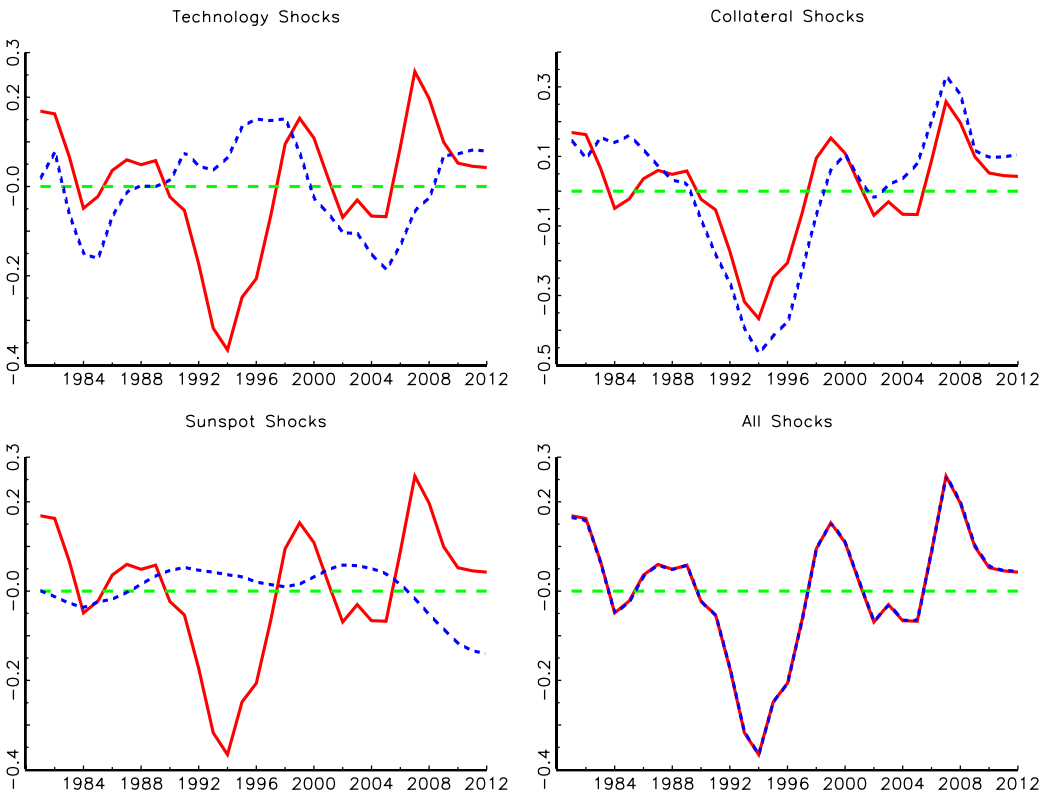

Figure 14: Decomposition of secured credit in the three shocks. The red (solid) curve is the secured credit to capital ratio, the blue (dashed) curves are model-generated dynamics if only one shock is active (first three graphs). In the bottom right graph all three shocks are active.

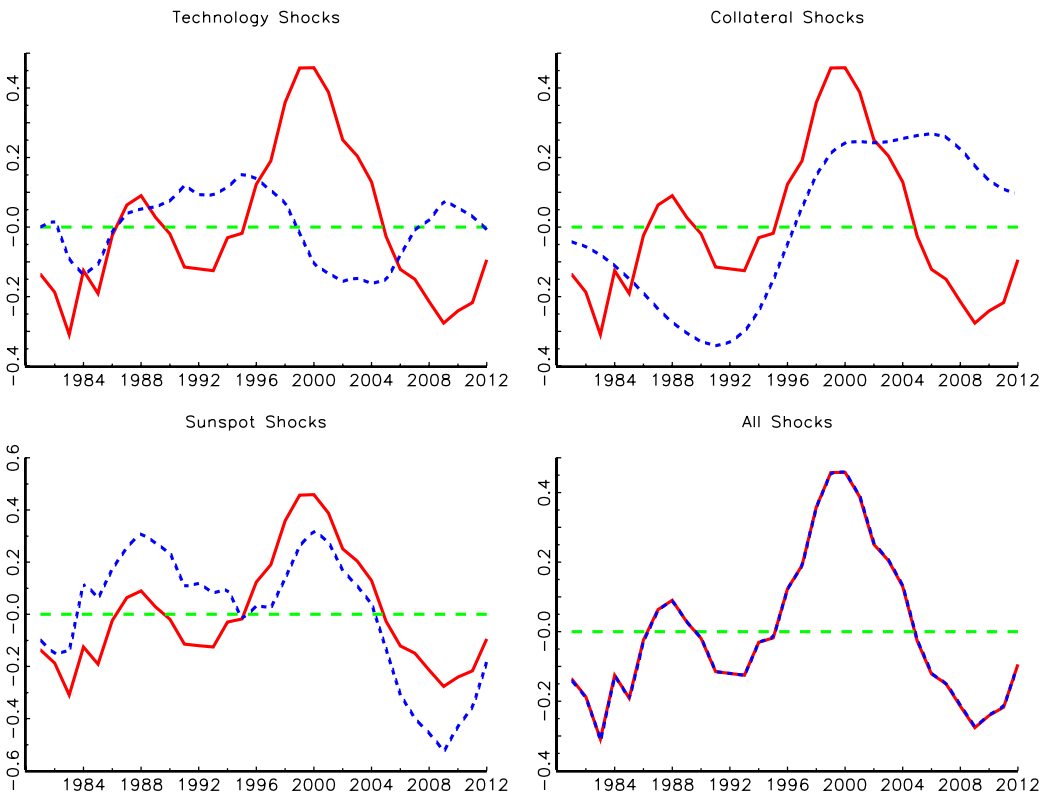

Figure 15: Decomposition of unsecured credit in the three shocks. The red (solid) curve is the unsecured credit to capital ratio, the blue (dashed) curves are model-generated dynamics if only one shock is active (first three graphs). In the bottom right graph all three shocks are active.

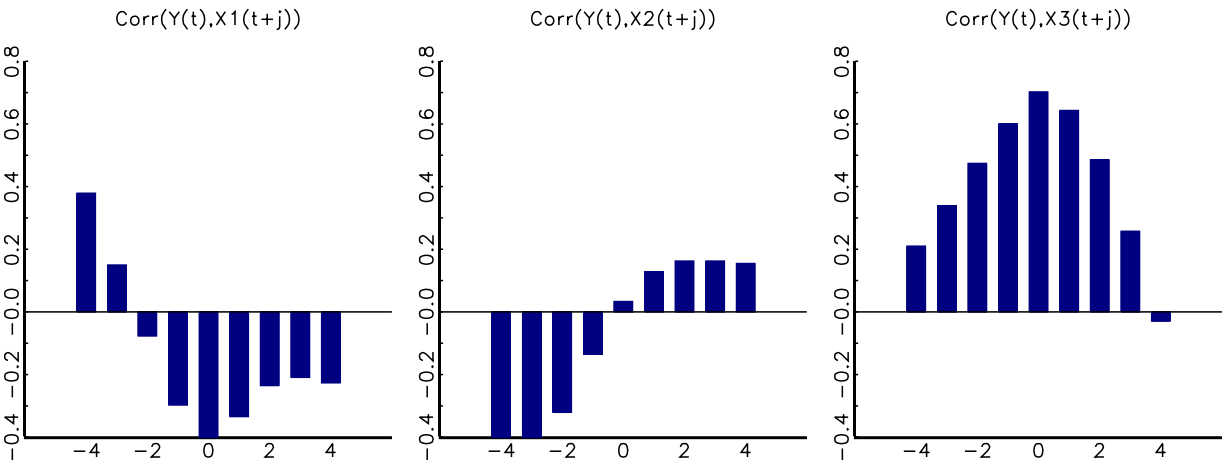

Figure 16: Lead-lag correlation between output $\hat{Y}_t$ and the components of secured credit $\hat{X}_{i,t+j}$, where $i = 1$ is technology shocks, $i = 2$ is collateral shocks and $i = 3$ is sunspot shocks.

Appendix D: Autocorrelated Firm-Specific Productivity

This appendix extends the model and the main theoretical results to an autocorrelated idiosyncratic productivity process. Specifically, suppose that productive firms stay productive with probability π_p and become unproductive otherwise, whereas unproductive firms become productive with probability π_u and stay unproductive otherwise. Productivities are positively autocorrelated when $\pi_p > \pi_u$. The i.i.d. benchmark considered in the main text corresponds to the case $\pi_p = \pi_u = \pi$. We assume that the collateral share is sufficiently low so as to ensure binding credit constraints and a capital misallocation in the absence of unsecured credit:

$$\lambda < \frac{\gamma(1-\pi_p)}{1-\gamma(\pi_p-\pi_u)} \ . \tag{32}$$

One major difference to the benchmark model is that the share of capital in the hands of productive firms at the beginning of a period, denoted x_t, is a state variable which adjusts sluggishly over time (see Kiyotaki (1998)) according to

$$x_{t+1} = \frac{\pi_p \tilde{R}_t x_t + \pi_u R_t(1-x_t)}{\tilde{R}_t x_t + R_t(1-x_t)} \ , \tag{33}$$

where $R_t = \rho_t a^p R_t^*$ is the gross interest rate (the equity return of unproductive firms) and $\tilde{R}_t = [1+\theta_t(1-\rho_t)]a^p R_t^*$ is the equity return of productive firms. Given x_t, fraction $z_t = \min(1, x_t(1+\theta_t))$ of capital is operated by productive firms, $a_t = z_t a^p + (1-z_t)a^u$ is average capital productivity. Capital market equilibrium reduces to the complementary-slackness condition

$$\rho_t \geq \gamma \quad , \quad x_t(1+\theta_t) \leq 1 \ . \tag{34}$$

To derive the endogenous debt-equity ratio θ_t, define $V_t(W)$ ($V_t^d(W)$) for the continuation values of a *productive* firm owner with a clean credit reputation (with a default flag) who has wealth W at the end of period t. Similarly, define continuation values for *unproductive* firm owners as $U_t(W)$ ($U_t^d(W)$). Borrowers with a default flag can still borrow secured, so their equity return is $\tilde{R}_t^d \equiv [1+\theta_t^s(1-\rho_t)]a^p R_t^*$, where θ_t^s is the debt-equity limit of secured borrowing, given by (16). Because of logarithmic utility, all firm owners save fraction β of wealth and continuation utilities can be written in the form $V_t(W) = \ln(W) + V_t$ etc. where V_t, V_t^d, U_t, U_t^d are independent of wealth and satisfy the recursive equations (with constant

$C \equiv (1-\beta)\ln(1-\beta) + \beta \ln \beta$):

$$\begin{aligned}
V_t &= C + \beta \mathbb{E}_t \Big[\pi_p (\ln \tilde{R}_{t+1} + V_{t+1}) + (1-\pi_p)(\ln R_{t+1} + U_{t+1}) \Big] \ , \\
V_t^d &= C + \beta \mathbb{E}_t \Big[\pi_p (\ln \tilde{R}_{t+1}^d + V_{t+1}^d + \psi(V_{t+1} - V_{t+1}^d)) \\
&\quad + (1-\pi_p)(\ln R_{t+1} + U_{t+1}^d + \psi(U_{t+1} - U_{t+1}^d)) \Big] \ , \\
U_t &= C + \beta \mathbb{E}_t \Big[\pi_u (\ln \tilde{R}_{t+1} + V_{t+1}) + (1-\pi_u)(\ln R_{t+1} + U_{t+1}) \Big] \ , \\
U_t^d &= C + \beta \mathbb{E}_t \Big[\pi_u (\ln \tilde{R}_{t+1}^d + V_{t+1}^d + \psi(V_{t+1} - V_{t+1}^d)) \\
&\quad + (1-\pi_u)(\ln R_{t+1} + U_{t+1}^d + \psi(U_{t+1} - U_{t+1}^d)) \Big] \ .
\end{aligned}$$

Define $v_t \equiv V_t - V_t^d$ and $u_t \equiv U_t - U_t^d$ as reputation values for productive and unproductive firm owners, satisfying

$$\begin{aligned}
v_t &= \beta \mathbb{E}_t \Big[\pi_p \Big(\ln \frac{\tilde{R}_{t+1}}{\tilde{R}_{t+1}^d} + (1-\psi) v_{t+1} \Big) + (1-\pi_p)(1-\psi) u_{t+1} \Big] \ , \\
u_t &= \beta \mathbb{E}_t \Big[\pi_u \Big(\ln \frac{\tilde{R}_{t+1}}{\tilde{R}_{t+1}^d} + (1-\psi) v_{t+1} \Big) + (1-\pi_u)(1-\psi) u_{t+1} \Big] \ .
\end{aligned}$$

These equations can be reduced to one in v_t with two forward lags, generalizing equation (18):

$$v_t = \beta \mathbb{E}_t \Big[\pi_p \ln \frac{\tilde{R}_{t+1}}{\tilde{R}_{t+1}^d} + (1-\psi)[\pi_p + 1 - \pi_u] v_{t+1} \Big] - \beta^2 (1-\psi)[\pi_p - \pi_u] \mathbb{E}_t \Big[\ln \frac{\tilde{R}_{t+2}}{\tilde{R}_{t+2}^d} + (1-\psi) v_{t+2} \Big] \ . \quad (35)$$

Default-deterring debt limits are linked to reputation values v_t and to the collateral share λ_t according to the same equation (17) as in the model with uncorrelated productivity. This again permits a simple equilibrium characterization as solutions $(\rho_t, \theta_t, v_t, x_t)$ to the system of equations (33), (34), (35) and (17).

It is straightforward to check that credit constraints are binding if (32) holds, which generalizes Proposition 2. If constraints were slack in all periods, $\rho_t = 1$ and $\tilde{R}_t = \tilde{R}_t^d = R_t$ would imply that $v_t = 0$ in all periods t, so that default-deterring debt-equity ratios are $\theta_t = \lambda/(1-\lambda)$. On the other hand, because of (33), the capital share of productive firm owners would converge to the stationary population share which is $x_t \to \overline{x}_{FB} \equiv \frac{\pi_u}{1+\pi_u-\pi_p}$. Capital market equilibrium with non-binding constraints requires however that the debt capacity of borrowers exceeds capital supply of lenders, $\theta_t x_t \geq 1 - x_t$ which boils down to $\lambda \geq (1-\pi_p)/(1-\pi_p+\pi_u)$, contradicting condition (32).

Condition (32) furthermore implies that there exists an equilibrium without unsecured credit ($v_t = 0$ for all t) where capital is inefficiently allocated. In this equilibrium, $\rho_t = \gamma$, $\theta_t = \overline{\theta} \equiv$

$\frac{\lambda}{\lambda-\gamma}$, and the stationary capital share $\overline{x}$ solves the quadratic

$$\overline{x}\Big[(1-\gamma)\overline{x}+\gamma-\lambda\Big] = \pi_p(1-\lambda)\overline{x}+\pi_u(\gamma-\lambda)(1-\overline{x}) \ ,$$

which has a unique solution $\overline{x} \in (0,1)$. A credit market equilibrium with inefficient capital allocation at $\rho = \gamma$ requires that $\overline{x}\overline{\theta} < 1-\overline{x}$. It is straightforward to verify that this is equivalent to condition (32).

We can generalize Proposition 3 as follows.

Proposition 5 *Suppose that $\pi_p \geq \pi_u$ (i.e., non-negative autocorrelation). For all parameter values there exists a stationary equilibrium without unsecured credit and with inefficient capital allocation. Provided that λ is sufficiently small, there are threshold values $\gamma_0 < \gamma_1 < 1$ such that:*

(a) *For $\gamma \in (\gamma_0, \gamma_1)$, there are two stationary equilibria with unsecured credit, one of them with inefficient capital allocation and the other one with efficient capital allocation.*

(b) *For $\gamma > \gamma_1$, there is no stationary equilibrium with unsecured credit.*

(c) *For $\gamma \leq \gamma_0$, there exists a unique stationary equilibrium with unsecured credit and efficient capital allocation.*

Proof: The existence of the equilibrium without unsecured credit has already been established above. Consider first a steady-state equilibrium v^* with an inefficient capital allocation ($\theta x < 1-x$ and $\rho = \gamma$) and unsecured credit ($v^* > 0$). Because of $\tilde{R}/\tilde{R}^d = \frac{\gamma-\lambda}{1-\lambda-e^v(1-\gamma)}$, equation (35) implies in steady state that

$$e^v = F(e^v) \equiv \left(\frac{\gamma-\lambda}{1-\lambda-e^v(1-\gamma)}\right)^{\Phi} \ , \tag{36}$$

with parameter $\Phi \equiv \frac{\beta\pi_p-\beta^2(1-\psi)(\pi_p-\pi_u)}{1-\beta(1-\psi)[\pi_p+1-\pi_u]+\beta^2(1-\psi)^2(\pi_p-\pi_u)} > 0$. Redefine $\varphi = e^v > 1$ and note that F is increasing and strictly convex with $F(\varphi) \to \infty$ for $\varphi \to (1-\lambda)/(1-\gamma) > 1$. We also have that $F(1) = 1$ (which corresponds to the steady state $v = 0$ without unsecured credit). This implies that equation (36) has a solution $\varphi = e^v > 1$ if and only if $F'(1) < 1$ which is equivalent to $\gamma > \gamma_0 \equiv \frac{\lambda+\Phi}{1+\Phi}$. The stationary capital share x solves

$$x = H(x) \equiv \frac{\pi_p[1+\theta(1-\gamma)]x+\pi_u\gamma(1-x)}{[1+\theta(1-\gamma)]x+\gamma(1-x)} \ ,$$

where function H is (weakly) increasing (because of $\pi_p \geq \pi_u$). This equation has a unique solution $x \in (0,1)$ which satisfies $\theta x < 1-x$ if and only if $1/(1+\theta) > H(1/(1+\theta))$ which is equivalent to

$$\theta < \frac{1-\pi_p}{\pi_p(1-\gamma)+\pi_u\gamma} \ .$$

Using $\theta = \frac{\varphi-1+\lambda}{1-\lambda-\varphi(1-\gamma)}$, this is equivalent to

$$\varphi < \overline{\varphi} \equiv \frac{(1-\lambda)(1-\gamma(\pi_p-\pi_u))}{1-\gamma+\pi_u\gamma} \ .$$

Since F is increasing and convex with $F'(\varphi) > 1$, this holds if and only if $F(\overline{\varphi}) > \overline{\varphi}$ which is equivalent to

$$[1-\gamma(1-\pi_u)]^{1+\Phi} > (1-\lambda)^{1+\Phi}\left(\frac{\gamma}{\gamma-\lambda}\right)^{\Phi}[\pi_p-\gamma(\pi_p-\pi_u)]^{\Phi}[1-\gamma(\pi_p-\pi_u)] \ . \qquad (37)$$

In this inequality, both the LHS and the RHS are decreasing functions of γ such that LHS(1) < RHS(1) (because of (32)) and LHS($\underline{\gamma}$) = RHS($\underline{\gamma}$) at $\underline{\gamma} \equiv \lambda/(1-\pi_p+\lambda(\pi_p-\pi_u)) < 1$. Moreover, we have $0 > \text{LHS}'(\underline{\gamma}) > \text{RHS}'(\underline{\gamma})$ if and only if

$$\lambda[\pi_p-\lambda(\pi_p-\pi_u)](1-\pi_u)(1+\Phi) < (1-\lambda)(1-\pi_p)\Phi[1-\pi_p+\lambda(\pi_p-\pi_u)] \ .$$

This inequality is true if λ is sufficiently small, so that we can conclude that there exists $\gamma_1 \in (\underline{\gamma}, 1)$ such that inequality (37) is satisfied for all $\gamma \in (\underline{\gamma}, \gamma_1)$ (see Figure 17. Since also $\gamma_0 \in (\underline{\gamma}, \gamma_1)$, we conclude that there exists a steady state with inefficient capital allocation and unsecured credit if and only if $\gamma \in (\gamma_0, \gamma_1)$.

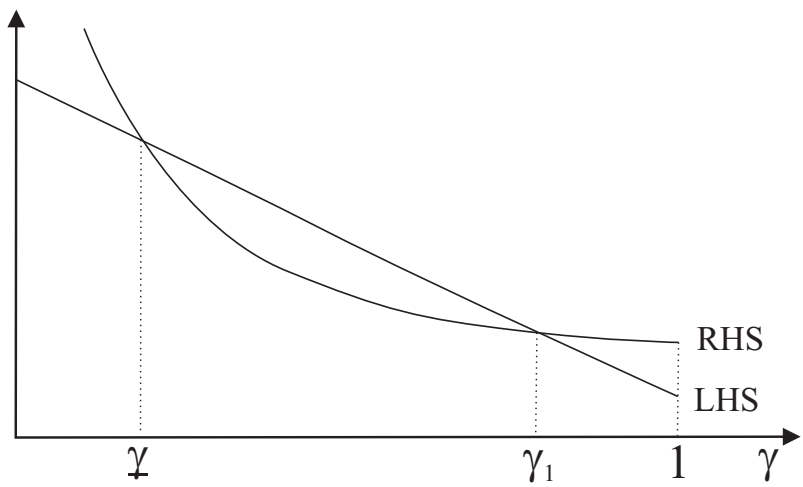

Figure 17: Existence of the threshold γ_1.

Second, consider an equilibrium at $v = v^{**}$ with unsecured credit and efficient capital allocation, so that $\rho > \gamma$ and $\theta = \frac{e^v-1+\lambda}{1-\lambda-e^v(1-\rho)}$. The stationary capital share in such an equilibrium is $x = \frac{\pi_p(1-\rho)+\pi_u\rho}{1-\rho(\pi_p-\pi_u)}$, and capital market equilibrium requires that $x\theta = 1-x$.

Combining these equations establishes the equilibrium interest rate at given reputation value v:

$$\rho = \frac{e^v - 1 + \lambda}{e^v(1-\pi_u) - (1-\lambda)(\pi_p - \pi_u)} \ . \tag{38}$$

On the other hand, equation (35) yields the stationary reputation value, analogously to (36),

$$e^v = \left(\frac{\rho - \lambda}{1 - \lambda - e^v(1-\rho)}\right)^{\Phi} \ . \tag{39}$$

Solving (38) for e^v and substitution into (39) yields the following equation for the equilibrium value of ρ:

$$[1-\rho(1-\pi_u)]^{1+\Phi} = (1-\lambda)^{1+\Phi}\left(\frac{\rho}{\rho-\lambda}\right)^{\Phi}[\pi_p - \rho(\pi_p - \pi_u)]^{\Phi}[1-\rho(\pi_p-\pi_u)] \ . \tag{40}$$

In this equation, both sides (functions of ρ) are the same as both sides in inequality (37) (functions of γ). We conclude, again for λ sufficiently small, that $\rho = \gamma_1 < 1$ solves equation (40). In turn, for every $\gamma < \gamma_1 = \rho$, a steady-state equilibrium with efficient capital allocation and unsecured credit exists. This completes the proof. □

There is also an interesting interaction between secured and unsecured credit in this model. Indeed, changes in the collateral parameter λ may have sizable consequences for unsecured credit. If the interest rate is low and capital is inefficiently allocated ($\rho = \gamma$), an increase of the collateral parameter λ raises both leveraged returns $\tilde{R}$ (for borrowers with a good credit reputation) and $\tilde{R}^d$ (for borrowers with a default flag), but the first return increases more. This makes default a less attractive option for borrowers which increases the value of reputation and improves the enforceability of unsecured credit. Mathematically, both threshold values γ_0 and γ_1 are increasing in λ.[46]

Corollary 3 *The availability of unsecured credit can be promoted by an increase of λ: both threshold values γ_0 and γ_1 are increasing in λ. Conversely, unsecured credit can become unenforceable if λ is too low.*

Proof: It is obvious from the proof of Proposition 5 that γ_0 is increasing in λ. γ_1 is increasing in λ if the RHS of (37) is decreasing in λ (see Figure 17). But the derivative of the RHS w.r.t. λ has the same sign as $-1 + \dfrac{\Phi(1-\gamma)}{\gamma - \lambda}$ which is negative since $\gamma > \gamma_0$. Since both thresholds are increasing in λ, the proof of the corollary follows directly from Proposition 5. □

[46] Note, however, that both steady-state values of v^* and of v^{**} shift down if λ increases, so that secured credit crowds out unsecured credit in any steady–state equilibrium. Graphically in Figure 5, an increase of λ shifts $f(v)$ up for $v < \overline{v}$ and down for $v > \overline{v}$ (while $\overline{v}$ falls). If λ rises sufficiently, capital is efficiently allocated and all credit is secured (i.e., both v^* and v^{**} eventually collapse to $v = 0$).

Steady state and log linearization

We can calibrate a steady-state equilibrium in the same way as in Appendix C. The log-linearized equations (25), (26), (27), (29) and (30) are the same as before. Because of $a_t = a^u + x_t(1+\theta_t)(a^p - a^u)$, (24) generalizes to

$$\hat{a}_t = x(a^p - a^u)\Big[\theta\hat{\theta}_t + (1+\theta)\hat{x}_t\Big] \ . \tag{41}$$

To generalize (28), write $\dfrac{\tilde{R}_t}{\tilde{R}_t^d} = \dfrac{\gamma - \lambda_t}{1 - \lambda_t - (1-\gamma)e^{v_t}}$ and log-linearize (35):

$$\hat{v}_t = \mathbb{E}_t[\varphi_1\hat{\lambda}_{t+1} + \varphi_2\hat{v}_{t+1} + \varphi_3\hat{\lambda}_{t+2} + \varphi_4\hat{v}_{t+2}] \ , \tag{42}$$

where

$$\begin{aligned}
\varphi_1 &= \frac{\beta\pi_p}{v}\cdot\frac{\lambda(1-\gamma)(e^v-1)}{(\gamma-\lambda)(1-\lambda-(1-\gamma)e^v)} \ , \\
\varphi_2 &= \beta(1-\psi)(\pi_p + 1 - \pi_u) + \beta\pi_p\frac{(1-\gamma)e^v}{1-\lambda-(1-\gamma)e^v} \ , \\
\varphi_3 &= -\frac{\beta^2(1-\psi)(\pi_p-\pi_u)}{v}\cdot\frac{\lambda(1-\gamma)(e^v-1)}{(\gamma-\lambda)(1-\lambda-(1-\gamma)e^v)} \ , \\
\varphi_4 &= -\beta^2(1-\psi)(\pi^p-\pi^u)\Big[1-\psi+\frac{(1-\gamma)e^v}{1-\lambda-(1-\gamma)e^v}\Big] \ .
\end{aligned}$$

Finally, log linearization of (33) yields

$$\hat{x}_{t+1} = x_1\hat{x}_t + x_2\hat{\theta}_t \ , \tag{43}$$

with

$$\begin{aligned}
x_1 &= \frac{\pi_p(1+\theta(1-\gamma))x - \pi_u\gamma}{\pi_p(1+\theta(1-\gamma))x + \pi_u\gamma(1-x)} - \frac{(1-\gamma)(1+\theta)x}{(1+\theta(1-\gamma))x + \gamma(1-x)} \ , \\
x_2 &= \frac{\pi_p\theta(1-\gamma)x}{\pi_p(1+\theta(1-\gamma))x + \pi_u\gamma(1-x)} - \frac{\theta(1-\gamma)x}{(1+\theta(1-\gamma))x + \gamma(1-x)} \ .
\end{aligned}$$

Sunspot dynamics

The steady state is indeterminate if the forward-looking equation (42) has multiple stationary solutions. In the absence of fundamental shocks ($\hat{A}_t = \hat{\lambda}_t = 0$), consider the sunspot dynamics $\hat{v}_{t+1} = \rho\hat{v}_t + \varepsilon^s_{t+1}$ with $\mathbb{E}(\varepsilon^s_{t+1}) = 0$. Then (42) implies that ρ must satisfy $1 = \varphi_2\rho + \varphi_4\rho^2$ which has two solutions. For the calibrated parameter values we verify that one of those solutions is stationary, with autocorrelation coefficient $\rho = 0.903$ (the other solution is non-stationary with $\rho > 1$).

In the presence of fundamental shocks, we again consider the SVAR (12), rewritten as $z_t = Bz_{t-1} + e_t$ with $z_t = (\hat{A}_t, \hat{\lambda}_t, \hat{v}_t)'$ and $e_t \in \mathbb{R}^3$. Rewrite (42) in the form

$$\hat{v}_t = \mathbb{E}_t\Big[\Phi_1 z_{t+1} + \Phi_2 z_{t+2}\Big] \ ,$$

with $\Phi_1 = (0, \varphi_1, \varphi_2)$ and $\Phi_2 = (0, \varphi_3, \varphi_4)$. This condition imposes the following restriction on matrix B:

$$\Phi_1 B + \Phi_2 B^2 = (0, 0, 1) \ .$$

For given coefficients in the first two rows of (12), this restriction defines a quadratic for the coefficients in the last row. Again, for the calibrated parameters and for the estimated coefficients in the first two rows of B, we verify that this restriction gives rise to one solution with stationary matrix B.

Additional results

Complementing the results presented in subsection 4.5, Figure 18 and Figure 19 show that lead-lag relations and impulse responses are broadly in line with our findings for the model with i.i.d. firm-specific productivity shocks. The part of output driven by sunspot shocks is again positively correlated with data output and it leads output significantly by one year, now with a lower peak correlation at 0.41. Collateral-driven output correlates more strongly with data output than before but again it lags output. Regarding impulse responses, we again find that sunspot shocks induce more persistent and volatile responses than the other two shocks.

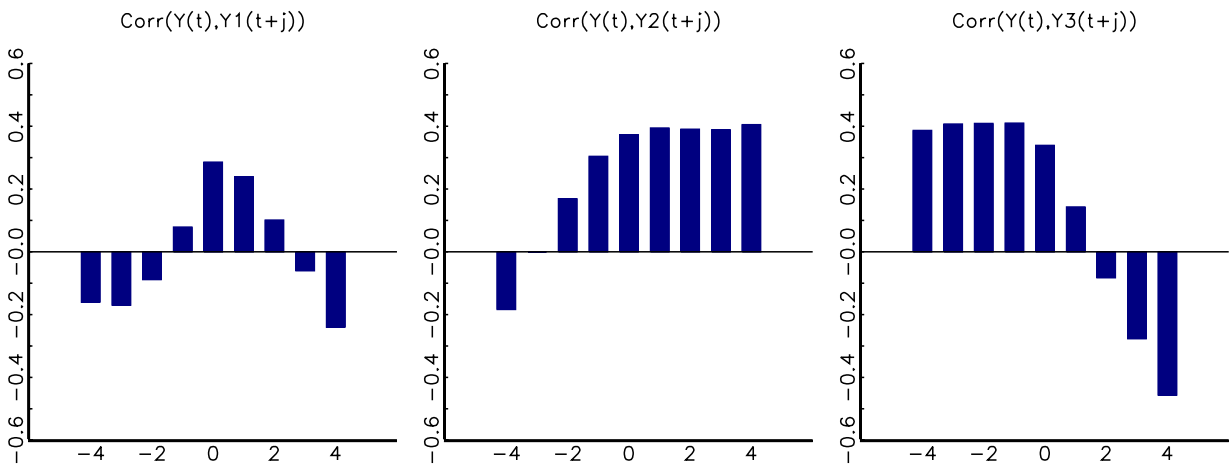

Figure 18: Lead-lag correlation between output $\hat{Y}_t$ and the three shock components $\hat{Y}_{i,t+j}$ for the model with autocorrelated firm-specific shocks, where $i = 1$ is technology shocks, $i = 2$ is collateral shocks and $i = 3$ is sunspot shocks.

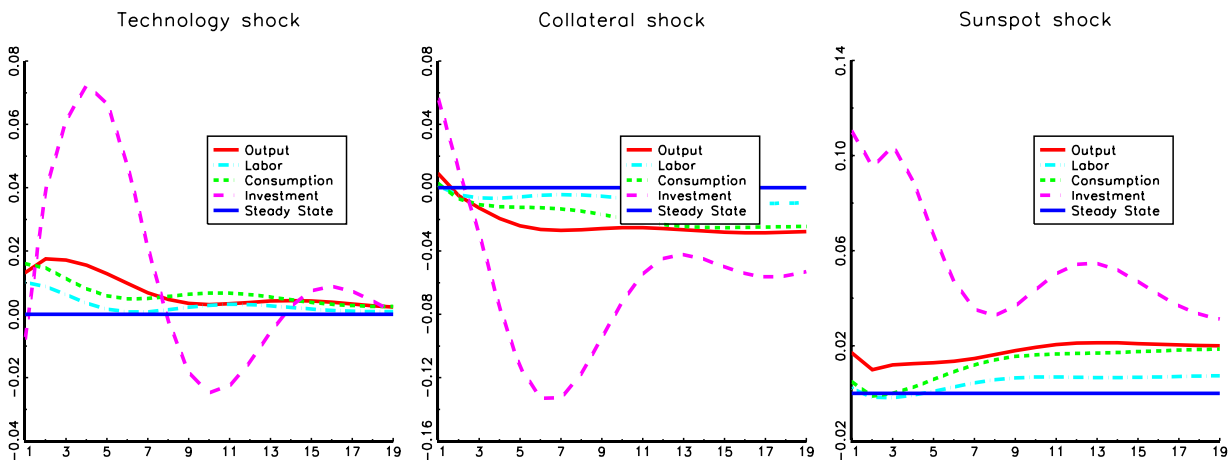

Figure 19: Impulse responses to the three shocks in the model with autocorrelated firm-specific shocks.