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Rehypothecation and Liquidity*

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Abstract

Rehypothecation refers to the practice of spending a borrowed security that is ostensibly assigned as collateral in a lending arrangement. We develop a dynamic general equilibrium monetary model where an "asset shortage" and incomplete markets motivates the formation of credit relationships and the rehypothecation of assets. Rehypothecation generally improves welfare by permitting liquidity to flow where it is needed. We find that rehypothecation confers greater welfare benefits in high inflation rate economies, but that there is generally too much rehypothecation. Away from the Friedman rule, a regulatory restriction specifically in the form of SEC rule 15c3-3 can improve welfare by enhancing the demand for real cash balances.

Keywords: rehypothecation, money, collateral, credit relationship.

JEL codes: E4, E5

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1 Introduction

For someone who wants to borrow money, unsecured credit is limited and expensive. But credit limits are relaxed, and credit costs lowered, when the borrower offers the lender a security or an asset as collateral. The act of using collateral to support a loan is called *hypothecation*. Collateralized lending arrangements involving all sorts of assets are commonplace.

In any collateralized lending arrangement, various property rights must be assigned over the money and collateral. Typically, the borrower is granted full use-rights over the cash that has been lent.[1] The assignment of property rights over collateral, however, are somewhat more varied. Use-rights frequently remain with the borrower. Consider, for example, collateral in the form of a vehicle, presently operated by the borrower, but subject to seizure in the event of default. Alternatively, a lending arrangement may assign to the lender property rights (but not use-rights) over the collateral. In the context of the previous example, imagine the borrower permitting the lender use of the vehicle's "pink slip" as collateral for her own borrowing activity. The terms of a loan will naturally depend on the assignment of property rights over money and collateral. The assignment of property rights over collateral to the lender is called a rehypothecation right. *Rehypothecation* occurs when the rehypothecation right is exercised. The rehypothecation right is a standard clause in most client-broker margin account agreements and rehypothecation is common practice in the repo market.[2]

Rehypothecation has a certain mystique about it. The idea of spending somebody else's property sounds bizarre.[3] Using it to support a "daisy chain" of loans conjures up images vaguely reminiscent of fractional reserve banking. The practice is widely thought to have played a major role in the 2008-09 financial crisis. According to a prominent group of regulators commenting on the events of 2008, "custody of assets and rehypothecation practices were

[1] In some cases, the use-rights may be conditional. For example, a consumer loan may be extended for home improvement but not a vacation. Of course, this is true even for unsecured loans.

[2] Singh and Aitken (2010) provide measures of the practice and Monnet (2011) provides a useful primer.

[3] Monnet (2011) emphasizes the bizarre nature of rehypothecation when he begins his review of the practice with the question "How would you feel if even though you were making regular monthly payments, your mortgage bank sold your house?"

dominant drivers of contagion, transmitting liquidity risks to other firms."[4] In light of such views, it is not surprising that rehypothecation has been subject to intense regulatory scrutiny since 2008. The Dodd-Frank Act, for example, restricts the rehypothecation of assets in credit derivatives markets–swap contracts must now be cleared by central counterparties who are required to hold collateral in segregated accounts. Of course, many restrictions on the practice were already in place prior to the financial crisis. For example, in the United States, SEC rule 15c3-3 limits how much client collateral can be rehypothecated by broker-dealers in a margin account.

Our own view is that rehypothecation is not as mysterious or dangerous as it is often made out to be. This view is informed by the following observation. When a person borrows money from a creditor, they are expected to rehypothecate (i.e., spend) the borrowed money. What else is one *supposed* to do with borrowed money? Moreover, the idea that money–whether borrowed or not–should circulate as an exchange medium is something that is generally viewed in a favorable light. Is this not the purpose of a circulating exchange medium to facilitate economic transactions that might not otherwise have taken place? From the perspective of the lender, all that really matters is whether the money is returned on schedule and with the interest promised. The fact that a money loan is "re-used" in a sequence of transactions seems conceptually distinct from the prospect of default.

If the rehypothecation of a money loan is viewed with favor (or innocuity), should the same not be true of a borrowed security? While the rehypothecation right is sometimes buried in the fine print of a contract, it is unreasonable to suppose that sophisticated players like investment banks and brokerage houses are unaware of the practice. Agencies presumably borrow securities for the same reason people borrow money–because it suits their purposes. And what is the point of borrowing a security if one is not permitted to spend it? What is the point of borrowing a vehicle if you are not permitted to drive it?

The reasoning above forms the basis of our model of rehypothecation. It appears that agencies sometimes want to borrow money and at other times, securities. There is the question of why agencies possessed with both do not liquidate securities (if they want money) or sell their money (if they want

[4] "Risk Management Lessons from the Global Banking Crisis of 2008," Senior Supervisors Group, October 21, 2009, p. 6.

securities). We do not delve deeply into this question. Duffie, Gârleanu, Pederson (2005) and Geanakoplos (2010) each describe reasons for why it may not be desirable to dispose of an asset in an outright sale. Monnet and Narajabad (2012) invoke a form of market incompleteness–pairwise trading, where traders potentially value the security differently–to explain the decision to "rent" (borrow) securities rather than buy them outright. We also assume pairwise trading, so that financial trades must occur within the context of a credit relationship characteristic of (but not exclusive to) repo arrangements. The device of pairwise trading permits us to model an unsecured credit facility within a credit relationship. It also permits us to focus on what motivates rehypothecation rather than what motivates repo arrangements.[5]

Most theoretical investigations of rehypothecation are located in the finance literature. The recent papers there seem quite naturally focused on explaining the events that occurred during the financial crisis of 2008-09. Eren (2014) and Infante (2015), for example, each examine two-period models where dealers act as intermediaries for cash lenders (e.g., money market funds) and borrowers (e.g., hedge funds). Dealers profit (obtain funding) through the spread that they earn through haircut differentials. Eren (2014) shows how rehypothecation and over-collateralization can expose hedge funds to the bankruptcy risk of dealer banks. In a similar vein, Infante (2015) shows how dealers with higher default risk are more exposed to runs by collateral providers than to runs by cash lenders.

The purpose of our paper is to study the phenomenon of rehypothecation from the perspective of a dynamic general equilibrium monetary model. We think a monetary model is appropriate because rehypothecation is closely related to liquidity provision. Moreover, real world collateralized lending arrangements typically involve a swap of *money* and (rights over) securities. Among other things, we are able to address the question of how monetary policy might affect the practice and desirability of rehypothecation. But because we abstract from aggregate risk and default, we are not in a position to examine the role rehypothecation may play in a financial crisis–we leave this interesting avenue of investigation for another day. However, the model is well-suited to exploring the economic impact and welfare consequences of

[5] While rehypothecation frequently arises in repo markets, it is not exclusive to such markets. Consider a retail investor with a margin account at a discount broker. The broker is permitted to rehypothecate the client's securities (up to some limit), but there is no repo agreement between broker and client.

regulatory interventions, like SEC rule 15c3-3 in the United States.

Our framework of analysis is based on the quasilinear model of Lagos and Wright (2005) and Geromichalos, Licari and Suárez-Lledó (2007). The economy consists of investors and workers who trade on a sequence of competitive spot markets. Because investors and workers do not trust each other (Gale, 1978), goods and services are exchanged on a *quid pro quo* basis for money and/or securities (claims to a Lucas tree). Money is generally dominated in rate of return by securities, but is nevertheless held because of its liquidity properties. In particular, only money is acceptable as a payment instrument in a subset of trading opportunities. Investors are subject to idiosyncratic liquidity risk: at the beginning of each period they may find themselves in a cash-only market with workers.

Consider an investor who enters a period with a portfolio of wealth consisting of money and securities. The investor wants to hire workers, but it turns out that these workers only accept cash. If the investor is short of cash, he might try to borrow it or sell securities. In reality, these avenues are not always conveniently available when an opportunity presents itself. Or, if a securities market is available, it is thinly traded and the security must be sold at a steep discount. For these and other reasons, investors can be expected to form credit relationships. In the present context, a credit relationship among investors can help mitigate liquidity risk. Investors in need of cash borrow it from investors in need of securities (alternatively, investors in need of securities borrow them from investors in need of cash). The resulting swap of assets resembles a repo arrangement. Reputational considerations permit a degree of unsecured credit among investors, so that what appear to be over-collateralized loans are possible. Moreover, even if investors could be perfectly trusted to repay debts among themselves, it would make sense to attach a rehypothecation right to the "collateral" flowing to the investors wanting to borrow securities. The reason is simple: these investors may come across a profitable trading opportunity that can be financed with the security in question.

Our paper is the first (that we know of) to study rehypothecation in the context of a dynamic general equilibrium monetary model. Away from the Friedman rule, the level of activity in the cash market is depressed. Activity in the credit market depends on the supply of liquid securities. For some parameters, there is an "asset shortage" (Caballero, 2006) so that credit market

activity is constrained and securities trade at a liquidity premium. The role of rehypothecation in this context is to relieve the asset shortage by facilitating the movement of liquid securities toward the agents that can make use of them (see also Maurin, 2014). To the extent that this added liquidity helps facilitate exchanges that would not otherwise take place, rehypothecation serves to improve market efficiency. We find that rehypothecation does generally serve to improve economic welfare, but that there is generally too much of it (relative to an ideal benchmark). We also find that the economic benefits associated with rehypothecation are larger in high inflation, high interest rate economies.

We use our model to examine the implications of SEC rule 15c3-3, which restricts how much client collateral can be rehypothecated by a broker-dealer. Because there is generally too much rehypothecation, it is not surprising to learn that the restriction can improve welfare.[6] But discerning the exact mechanism by which this happens is not so obvious. As it turns out, the specific form of SEC rule 15c3-3 is such that it enhances the demand for real cash balances, which is inefficiently low in a high inflation environment (there is an incentive to accumulate additional cash to overcome the regulatory restriction). In other words, modeling the details of the actual regulation matters. Other hypothetical (but reasonable) regulatory restrictions do not generally influence the demand for real cash balances.[7] While it is unlikely that the designers of this regulation had this particular effect in mind, it is nevertheless interesting to discover its theoretical existence.

The outline of our paper is as follows. In Section 2, we describe the physical environment and characterize the set of Pareto optimal (first-best) allocations. In Section 3, we describe the market structure, the frictions that make exchange media necessary, and monetary policy. We formalize the economic problems that agents solve in Section 4 and characterize a stationary monetary equilibrium in Section 5 for different regions of the parameter space. Section 5 also contains our main results, both analytical and numeri-

[6] Kocherlakota (2003) constitutes an example of how a trading restriction can improve welfare. More recently, the models of Berentsen, Huber, and Marchesiani (2014) and Geromichalos and Herrenbrueck (2014) also have the property that liquidity restrictions can improve economic welfare. The idea that a trading restriction might improve welfare when the economy is away from a Pareto optimum is due to Lipsey and Lancaster (1956).

[7] One such alternative was suggested to us by a referee. We discuss this alternative specification in the appendix.

cal, concerning the effects of SEC rule 15c3-3 and how the rule interacts with inflation. We find that, to be effective, a regulation limiting rehypothecation needs to be stricter in economies with lower inflation or lower rate of return on securities. Furthermore, the welfare gains from both rehypothecation and the regulation restricting it are increasing in inflation. In Section 6 we verify under which circumstances the equilibrium characterized in the preceding sections can still be supported when we allow for default in credit transactions, i.e., limited commitment between investors. We find that restricting rehypothecation directly through regulation or indirectly with inflation has opposite effects on default incentives. Section 7 concludes.

2 Environment

In this section we describe the physical environment and characterize a Pareto optimal allocation, which will be used as a benchmark in what follows. Time is discrete and the horizon is infinite, $t = 0, 1, 2, .., \infty$. Each date t is divided into three subperiods. The first subperiod is immaterial as far as a planner allocation is concerned. In the second subperiod, there are two segmented locations, where production (and exchange) can potentially take place. In the third subperiod, production (and exchange) may again occur–this time in a centralized location.

The economy is populated by two types of infinitely-lived agents labeled *investors* and *workers*. There is a continuum of each type of agent, with the population mass of each normalized to unity. In the first subperiod of each date, investors and workers are exogenously assigned to one of the two segmented goods markets that operate in the second subperiod. For simplicity, we assume that a half of all investors and workers are allocated to each of the two second subperiod goods markets. At the individual level, this assignment is viewed as an *i.i.d.* idiosyncratic shock in the first subperiod (there is no aggregate risk).

Let $(c_{k,t}, y_{k,t})$ denote the per capita output consumed and produced in goods market $k = 1, 2$ at date t. Output is nonstorable. Let $u(c)$ denote the flow utility payoff from consuming c and let $-h(y)$ denote the utility cost of producing y. Assume $u'' < 0 < u'$, $u(0) = 0$ and $h' > 0$, $h'' \geq 0$ with $h'(0) = h(0) = 0$. In the third subperiod, all agents have linear preferences defined over a nonstorable good, $x \in \mathbb{R}$ (negative values of x are interpreted

as production). Thus, our model adopts the quasilinear preference and timing structure of Lagos and Wright (2005).

Preferences for investor $i \in [0,1]$ are given by

$$E_0^i \sum_{t=0}^{\infty} \beta^t \left[0.5u(c_{1,t}(i)) + 0.5u(c_{2,t}(i)) + x_t^b(i)\right] \tag{1}$$

and preferences for worker $i \in [0,1]$ are given by

$$E_0^i \sum_{t=0}^{\infty} \beta^t \left[-0.5h(y_{1,t}(i)) - 0.5h(y_{2,t}(i)) + x_t^w(i)\right] \tag{2}$$

where $0 < \beta < 1$. Finally, there is a single fixed productive asset–a Lucas tree that generates a constant nonstorable income flow $\omega \geq 0$ at the beginning of each evening.

A Pareto optimal allocation is a feasible allocation that maximizes a weighted sum of *ex ante* utilities (1) and (2). If u is strictly concave, then efficiency dictates $c_{k,t}(i) = c_t$ for all investors. If h is strictly convex, then efficiency dictates $y_{k,t}(i) = y_t$ for all workers.[8] Since there are equal numbers of investors and workers in each segmented goods markets, a resource constraint implies $c_t = y_t$ for all t. Clearly, the efficient goods market allocation is stationary and satisfies $u'(y^*) = h'(y^*)$.

The resource constraint in the evening is given by

$$\int_0^1 x_t^b(i)di + \int_0^1 x_t^w(i)di = \omega$$

One may, without loss, treat investors and workers symmetrically, so let $x = x_t^w(i)$ and $z = x_t^b(i)$ for all i and t. The resource constraint may therefore be written as $x + z = \omega$. The choice of x (and z) serves to only distribute utility. Since the total surplus is proportional to $u(y^*) - h(y^*)$, the *ex ante* participation constraints are satisfied for any x such that $h(y^*) \leq x \leq u(y^*)$.

3 Market structure and frictions

Investors and workers are anonymous to each other, so that unsecured credit arrangements involving parties from these two groups are not possible. This

[8] If h is linear, then each worker is assigned output level y_t in expectation.

friction motivates the need for a medium of exchange, the natural candidate here being the Lucas tree (or securities representing claims to the dividend flow of this asset). In the afternoon, investors will dispose of their securities in exchange for output supplied by workers. In the evening, investors will be motivated to produce output, to be sold to workers in exchange for securities, which investors then carry into the next afternoon.

Since rehypothecation typically occurs within the context of a credit arrangement involving money and securities, we need to introduce a demand for money. We do so by assuming that workers will only accept money as payment in location 1–the *cash market*. Workers in location 2–the *credit market*–accept both money and securities as payment.[9]

When investors are rebalancing their wealth portfolios in evening trade, they do not know beforehand which of the two locations they will visit the next afternoon. This would not be an issue if a well-functioning financial market was available in the morning. In that case, investors could just dispose of securities in outright sales if they needed cash and vice-versa. Moreover, if they need to borrow, they could do so. We assume that these markets are unavailable to investors in the morning–a restriction meant to capture the fact that investors are not always in contact with centralized financial markets. Investors are therefore subject to a form of liquidity risk. Accumulating low-return cash is useful because it hedges against a liquidity shock–the risk of visiting the cash market.

To bypass the lack of a centralized financial market, we assume that investors form bilateral credit relationships, which we call *partnerships*. For simplicity, we assume that these partnerships are perfectly durable and that they can commit to honor obligations that they make to each other. What we have to say below continues to hold if we replace full commitment with limited commitment subject to a strong enough punishment for noncompliance.[10] In

[9] The label "credit market" is chosen because the sale of assets here is equivalent to a fully collateralized lending arrangement. That is, investors could borrow output from workers in the afternoon, using the security as collateral that is legally seizable in the event of default. Note that technically, such a collateralized loan arrangement need not violate the assumption of anonymity (the knowledge of private credit histories). One could imagine, in particular, a mechanical protocol that executes collateralized loan arrangements among anonymous agents. In fact, the Bitcoin-related platform *Ethereum* is a protocol that permits exactly this type of exchange to take place.

[10] In Section 6 we study the case when punishment for noncompliance is perpetual exclusion from the partnership.

either case, the important thing to note is that the partnership permits some degree of unsecured credit between investors.

To make things even simpler than they need to be, assume that the idiosyncratic "location shock" is perfectly negatively correlated within a partnership. Thus, in the morning, partners learn of their itinerary: each investor will be travelling to a different segmented (cash or credit) market in the afternoon subperiod. Both investors then meet in the centralized evening market where they work to rebalance the partnership's wealth portfolio (the net debtor in the partnership works harder to pay off his debt).

Apart from exchanges that may occur in the morning (between investment partners), all markets are competitive. We assume throughout that the supply of fiat money grows at a constant gross rate $\mu \geq \beta$ and that new money is injected (or withdrawn) via lump-sum transfers (or taxes) to the investors in the third subperiod. Let τ denote the real transfer per investor.

Given this market structure and trading frictions, a meaningful form of rehypothecation can occur in equilibrium. In particular, investor partners with *ex ante* identical wealth portfolios in the morning are motivated to rearrange money and securities subsequent to the realization of the liquidity shock. The investor travelling to the cash market "borrows" cash from his partner. The investor travelling to the credit market "borrows" the security from his partner. If the value of cash sent to the cash investor is less than the value of the security sent to the credit investor, the operation looks (to an econometrician) like an over-collateralized cash loan. However, since the two partners trust each other (in the benchmark case), this exchange of assets is in no way motivated by the need for collateral to support the intra-partner credit arrangement. It just turns out to be efficient to send the assets to the partner who can make best use of them as exchange media. For any such exchange to make economic sense, it is absolutely critical that a rehypothecation right be attached to the security that is sent to the investor destined for the credit market.

We should like to emphasize that the security identified as "collateral" in the intra-partner credit relationship is not meant to securitize the cash loan–it is meant to be rehypothecated. The way to see this clearly is to imagine a world without cash. In this case, no trade ever occurs in the cash market. However, the investor travelling to the credit market may still want to borrow his partner's security and spend it. In such a transaction, there

can be no sense in which the security is used as collateral for a loan within the credit relationship. There is, however, a sense in which the security may serve as collateral when it is rehypothecated since there is an equivalence in the model between an outright sale of the security in the afternoon market and a fully collateralized loan between investor and worker (see footnote 9).

4 Decision-making

Let (p_1, p_2) denote the price of output, measured in units of money, in the afternoon cash and credit markets, respectively. Let q denote the real price of securities in the afternoon credit market and let ϕ denote the ex-dividend real price of securities in the evening. Finally, let p denote the nominal price of output (transferable utility) in the evening.

4.1 Workers

Let $W^k(m, a)$ denote the value of a worker who enters the afternoon cash market ($k = 1$) and credit market ($k = 2$) with money balances m and securities a. Let $N(m', a')$ denote the value of a worker who enters the evening with money balances m' and securities a'. Assume that W^k, N are increasing and strictly concave in each of their arguments. Let W^k_j, N_j denote the first derivatives of these functions with respect to arguments $j = \{m, a\}$.

4.1.1 Afternoon

A worker in the cash market sells product for cash. Since securities are illiquid in the cash market ($a' = a$), his choice problem is given by:

$$W^1(m, a) \equiv \max_{y_1} \{-h(y_1) + N(m + p_1 y_1, a)\}$$

where $m'_1 = m + p_1 y_1 \geq 0$. Product supply satisfies the first-order condition:

$$(1/p_1)h'(y_1) = N_m(m'_1, a) \tag{3}$$

By the envelope theorem:

$$\begin{aligned} W^1_m(m, a) &= (1/p_1)h'(y_1) \\ W^1_a(m, a) &= N_a(m'_1, a) \end{aligned}$$

In the credit market, workers supply product for money and/or assets:

$$W^2(m,a) \equiv \max_{y_2, a_2'} \{-h(y_2) + N(m_2', a_2')\}$$

where $m_2' = m + p_2 y_2 + p_2 q(a - a_2')$. While there are non-negativity constraints to consider here ($m_2', a_2' \geq 0$), we anticipate that these will not bind since workers are at this stage wishing to accumulate assets, not spend them. Consequently, optimal behavior is characterized by:

$$\begin{aligned} (1/p_2)h'(y_2) &= N_m(m_2', a_2') && (4) \\ p_2 q N_1(m_2', a_2') &= N_a(m_2', a_2') && (5) \end{aligned}$$

By the envelope theorem:

$$\begin{aligned} W_m^2(m,a) &= (1/p_2)h'(y_2) \\ W_a^2(m,a) &= p_2 q N_m(m_2', a_2') \end{aligned}$$

4.1.2 Evening

Let $W(m,a)$ denote the value of entering the morning as a worker with portfolio (m,a); i.e.,

$$W(m,a) \equiv 0.5W^1(m,a) + 0.5W^2(m,a)$$

For a worker that enters the evening with portfolio (m', a'), the choice problem is given by:

$$N(m',a') \equiv \max_{m^+, a^+} \left\{(\phi + \omega)a' + (1/p)(m' - m^+) - \phi a^+ + \beta W(m^+, a^+)\right\}$$

We anticipate that workers will want to dispose of their money and asset holdings in the evening, so here we make explicit the non-negativity constraints $m^+, a^+ \geq 0$. With associated Lagrange multipliers ξ_m and ξ_a on the respective non-negativity constraints, optimality requires:

$$\begin{aligned} (1/p) &= \beta W_m(m^+, a^+) + \xi_m \\ \phi &= \beta W_a(m^+, a^+) + \xi_a \end{aligned}$$

By the envelope theorem:

$$\begin{aligned} N_m(m',a') &= (1/p) && (6) \\ N_a(m',a') &= (\phi + \omega) && (7) \end{aligned}$$

4.1.3 A simplifying assumption: linear cost

For the remainder of the paper, we assume linear cost of production in the afternoon.

Assumption 1 $h(y) = y$.

In this case, the first-order conditions for the worker's problem derived above should be re-interpreted as equilibrium no-arbitrage conditions. These conditions are imposed when we characterize the decisions of investors. The linear cost structure permits us to proceed in this manner and aids greatly in exposition.

Since $h'(y) = 1$, conditions (3) and (6) imply $p_1 = p$. Likewise, conditions (4) and (6) imply $p = p_2$. Finally, conditions (5), (6) and (7) imply $q = (\phi + \omega)$.

4.2 Investors

4.2.1 Morning

Because investors acquire money and securities in the evening, they will be carrying assets into the morning of the next period.[11] Consider an investor partnership that enters the morning with a *combined* asset portfolio (m, a). *Ex ante*, individuals in an investor partnership do not know whether they will be travelling to the cash market or the credit market. After the location shock is realized in the morning, investor partners have an opportunity to reallocate their combined assets between them.

As investors are *ex ante* identical in their attributes, each will have an equal share of the coalition portfolio $(m/2, a/2)$. The value of the partnership lies in the opportunity for "asset trades" in the morning, conditional on type realizations, as well as the value associated with a bilateral credit facility. Let (m_2, a_2) denote the portfolio allocated to the investor travelling to the

[11] Workers on the other hand will have depleted their assets in the evening and so enter the next period with zero assets. As a consequence, there is no need for workers to rebalance their portfolios in the morning. They will simply work to acquire the assets they want in the afternoon with the intent of spending them in the evening.

credit market, where:

$$m \geq m_2 \geq 0 \tag{8}$$

$$a \geq a_2 \geq 0 \tag{9}$$

Given our setup, a natural exchange would have cash flowing to the cash-investor and securities flowing to the credit-investor, i.e., $m_2 < m/2$ and $a_2 > a/2$. If $m_2 < m/2$, then the credit-investor is in effect sending $[m/2 - m_2]$ dollars to the cash-investor. If $a_2 > a/2$, then the cash-investor is in effect sending $p(\phi + \omega)[a_2 - a/2]$ dollars worth of securities to the credit-investor, where $p(\phi + \omega)$ is the nominal price of the security in the afternoon. If the value of what is exchanged is equated, then the transaction can be considered either as an outright purchase of securities by the credit-investor, or as a collateralized cash loan to the cash-investor. (Alternatively, one might view the transaction as a collateralized *securities* loan to the credit-investor.)

Of course, there is no reason to believe *a priori* that a "balanced" trade $[m/2 - m_2] = p(\phi + \omega)[a_2 - a/2]$ is necessarily optimal in this partnership. In fact, we anticipate that an optimal intra-investor allocation will sometimes have the property $0 < [m/2 - m_2] < p(\phi + \omega)[a_2 - a/2]$. In this case, the transaction looks like an overcollateralized cash loan (from credit-investor to cash-investor) or, equivalently, an undercollateralized securities loan (from cash-investor to credit-investor). Thus, in principle, the "haircut" applied to the security may be either positive or negative.

One way to map our model into reality is to interpret the investor partnership as the type of relationship that is formed between different broker-dealers, or broker-dealers and other securities lenders (e.g., central banks, pension funds, insurers).[12] Interpret the investor traveling to the cash market as a client and the investor traveling to the credit market as a broker-dealer. The client wants to "borrow" $[m/2 - m_2]$ dollars from his margin account held with the broker-dealer, and is willing to "pledge" securities worth up to $p(\phi + \omega)[a_2 - a/2]$ dollars as "collateral." As is the case in reality, the broker-dealer agreement permits the rehypothecation of collateral for use in proprietary trades. Clearly, if the value of cash and securities passing hands is not the same, then some amount of unsecured credit is involved. For the

[12] Singh (2013, figure 1) reports that the volume of repo trades between broker-dealers (inter-dealer trade) and between dealers and securities lenders was about seven trillion dollars in 2011.

moment, we assume that broker-dealers and their clients can be trusted to repay unsecured debt. In reality, reputational concerns (the threat of punishments for default) can support some positive level of unsecured credit. In any case, the credit arrangements described here are unwound each evening. If the broker-dealer rehypothecated the client's collateral in the afternoon, either in a short-sale or as collateral for a proprietary lending arrangement (with workers), then the collateral–or its value equivalent–is returned in the evening.

The undercollateralized securities loan (overcollateralized cash loan) seems to trouble regulators concerned with the rehypothecation of securities. As such, some jurisdictions place restrictions on the rehypothecation of borrowed securities. In the United States, SEC rule 15c3-3 imposes the following restriction

$$\theta \left[m/2 - m_2 \right] \geq p \left(\phi + \omega \right) \left[a_2 - a/2 \right] \tag{10}$$

for some policy parameter $\theta \geq 0$. The *regulatory constraint* (10) is a requirement on cash margin for borrowed securities with rehypothecation right. It restricts the value of borrowed securities that can be rehypothecated by the credit-investor to be a multiple of the value of money lent to the cash-investor. In general, think of the cash-investor depositing all his securities with the credit-investor and placing a fraction of these in a segregated account. Securities in this segregated account may still serve as collateral for the cash loan but cannot be reused by the credit-investor, i.e., they do not carry rehypothecation rights. The regulatory constraint places a limit on the amount of securities that can be rehypothecated.[13]

In what follows, we study the economic consequences of imposing the regulatory constraint (10) on financial transactions between investors, i.e., of limiting the rehypothecation of borrowed securities according to SEC rule 15c3-3. In the appendix, we consider an alternative restriction to rehypothecation, following a more standard approach of exogenously limiting the value of assets that can be reused.

[13] In Canada, rehypothecation is apparently prohibited; in which case $\theta = 0$. In the U.K., there are apparently no legal limits to rehypothecation; in which case $\theta = \infty$. In the U.S., $\theta = 1.4$.

4.3 Afternoon

Recall that (m, a) represents the partnership's combined assets in the morning and when entering the afternoon. Recall as well that (m_2, a_2) denotes the portfolio allocated to the investor travelling to the credit market in the afternoon. Let (m', a') denote the partnership's combined asset portfolio entering the evening.

The afternoon flow budget constraint for the partnership is given by:

$$m - m' + p(\phi + \omega)(a - a') - py_1 - py_2 \geq 0 \tag{11}$$

Individual investors are subject to liquidity constraints depending on their itinerary. The cash-investor is subject to the following liquidity constraint:

$$m - m_2 - py_1 \geq 0 \tag{12}$$

while the credit-investor is subject to:

$$m_2 + p(\phi + \omega) a_2 - py_2 \geq 0 \tag{13}$$

There are a number of non-negativity constraints that need to be made explicit. Individual investor asset holdings must be non-negative. Since these constraints hold individually, they will automatically hold at the partnership level; i.e., $m', a' \geq 0$.

Condition (12) restricts the cash-investor's expenditures in the afternoon, py_1 to not exceed his allocated cash, $m - m_2$. Thus, it ensures that the cash-investor's money holdings are non-negative when entering the evening. His securities holdings are non-negative since he doesn't get to spend non-lent securities in the afternoon.

The liquidity constraint of the credit-investor, (13) imposes a non-negativity restriction on his combined asset holdings. That is, afternoon expenditures, py_2 cannot exceed total allocated assets, $m_2 + p(\phi + \omega)a_2$. Hence, we need to impose two additional conditions that restrict his cash and securities holdings to be non-negative, respectively.

Since m' is the total cash brought into the evening by the partnership and $m - m_2 - py_1$ is the cash brought into the evening by the cash-investor, the difference between these two objects, representing the cash brought into the evening by the credit-investor, must be non-negative:

$$m' - [m - m_2 - py_1] \geq 0 \tag{14}$$

A similar argument applied to the credit-investor's securities holdings implies:

$$a' - [a - a_2] \geq 0 \tag{15}$$

Recall that a' represents the partnership's total security holdings entering the evening. The difference $a - a_2$ represents the (unspent) securities held by the cash-investor or, equivalently, deposited in a segregated account with the credit-investor, with no rehypothecation rights. Thus, (15) restricts the credit-investor's security holdings to be non-negative.

We will now show that if the liquidity constraint of the cash-investor holds with equality (which is always the case when $\mu > \beta$ and without loss of generality when $\mu = \beta$), then restrictions (11), (12), (14) and (15) imply (13). Given that (15) implies $a_2 \geq a - a'$, the flow budget constraint (11) implies $m - m' + p(\phi + \omega)a_2 - py_1 - py_2 \geq m - m' + p(\phi + \omega)(a - a') - py_1 - py_2 \geq 0$. Note that (12) and (14) imply $m' \geq 0$. Thus, $m + p(\phi + \omega)a_2 - py_1 - py_2 \geq 0$. Assuming (12) holds with equality, $m - py_1 = m_2$, which yields (13).

4.3.1 Evening

Let (m^+, a^+) denote the partnership's combined assets carried into the next period. The partnership is subject to the following evening budget constraint:

$$x = (\phi + \omega)\, a' + (1/p)(m' - m^+) - \phi a^+ - 2\tau \tag{16}$$

Recall that τ is the real value of new cash injections per investor.

4.4 Investor partnership problem

Let $B(m, a)$ denote the value of the investor partnership entering the morning with combined assets (m, a). Let $V(m', a')$ denote the value of the partnership entering the evening with combined assets (m', a'). These value functions must satisfy the recursion:

$$B(m, a) \equiv \max_{y_1, y_2, m_2, a_2, m', a'} \{u(y_1) + u(y_2) + V(m', a')\}$$

subject to (10), (11), (12) and the non-negativity constraints (8), (9), (14), (15). Note that since we anticipate morning cash flowing to the cash-investor

and morning securities flowing to the credit-investor, the non-negativity constraints (8), (9) simplify to $m_2 \geq 0$ and $a - a_2 \geq 0$. Recall that (13) is implied by the other constraints.

Lagrange multipliers are assigned as follows. Let ψ be the multiplier associated with the budget constraint, (11); λ with the liquidity constraint, (12); ξ with the regulatory constraint, (10); ζ_1 and ζ_2 with the credit-investor's non-negativity constraints, (14) and (15), respectively; ζ_3 with $m_2 \geq 0$; and ζ_4 with $a - a_2 \geq 0$.

The necessary first-order conditions for an optimum are:

$$\begin{aligned} u'(y_1) - p\left[\psi + \lambda - \zeta_1\right] &= 0 && (17) \\ u'(y_2) - p\psi &= 0 && (18) \\ -\chi\theta - \lambda + \zeta_1 + \zeta_3 &= 0 && (19) \\ -p\left(\phi + \omega\right)\chi + \zeta_2 - \zeta_4 &= 0 && (20) \\ V_m(m', a') - \psi + \zeta_1 &= 0 && (21) \\ V_a(m', a') - \psi p\left(\phi + \omega\right) + \zeta_2 &= 0 && (22) \end{aligned}$$

By the envelope theorem:

$$\begin{aligned} B_m(m, a) &= \chi\theta/2 + \psi + \lambda - \zeta_1 && (23) \\ B_a(m, a) &= p\left(\phi + \omega\right)\left[\chi/2 + \psi\right] - \zeta_2 + \zeta_4 && (24) \end{aligned}$$

In the evening, the choice problem of the investor partnership solves:

$$V(m', a') \equiv \max_{m^+, a^+} \left\{\left(\phi + \omega\right) a' + (1/p)(m' - m^+) - \phi a^+ + 2\tau + \beta B(m^+, a^+)\right\}$$

There are also the non-negativity constraints $m^+, a^+ \geq 0$, but we anticipate that these will not bind for investors in the evening.[14] The demands for money and securities in the evening must satisfy:

$$\begin{aligned} (1/p) &= \beta B_m(m^+, a^+) && (25) \\ \phi &= \beta B_a(m^+, a^+) && (26) \end{aligned}$$

By the envelope theorem:

$$\begin{aligned} V_m(m', a') &= (1/p) && (27) \\ V_a(m', a') &= (\phi + \omega) && (28) \end{aligned}$$

[14] Investors will want to rebuild their asset positions in order to finance their consumption expenditures in the following afternoon.

4.5 Gathering restrictions

We restrict attention to stationary allocations where all real variables are constant over time and nominal variables grow at rate μ. Combine (17), (23) and (25) to form:

$$\mu = \beta[u'(y_1) + \theta p\chi/2] \tag{29}$$

When the regulatory constraint is slack ($\chi = 0$), we get the standard result that $\mu > \beta$ implies $y_1 < y^*$.

Now combine (18), (20), (22), (24) and (26) to form:

$$\phi = \beta(\phi + \omega)\left[u'(y_2) - p\chi/2\right] \tag{30}$$

From conditions (17), (21) and (27), we have

$$p\lambda = u'(y_1) - 1 \tag{31}$$

which implies $\lambda > 0$ iff $y_1 < y^*$. From (18) we have

$$p\psi = u'(y_2) \tag{32}$$

Clearly, $p\psi \geq 1$, i.e., $\psi > 0$, so that the budget constraint (11) binds. Using (18), (21), (22), (27) and (28) we get

$$p\zeta_1 = u'(y_2) - 1 \tag{33}$$

$$(\phi + \omega)^{-1}\zeta_2 = u'(y_2) - 1 \tag{34}$$

There are two cases to consider: $y_2 = y^*$ and $y_2 < y^*$. If $y_2 = y^*$, then $u'(y_2) = 1$ and so (33)–(34) imply $\zeta_1 = \zeta_2 = 0$. If $y_2 < y^*$, then $u'(y_2) > 1$ which again, from (33)–(34) implies $\zeta_1 > 0$, and $\zeta_2 > 0$.

Finally, using (19), (20), (31)–(34) we get

$$p\zeta_3 = u'(y_1) - u'(y_2) + \theta p\chi \tag{35}$$

$$(\phi + \omega)^{-1}\zeta_4 = u'(y_2) - 1 - p\chi \tag{36}$$

We now invoke the market-clearing condition $m = 2M$. Cash-investors spend all of their cash (when $\mu = \beta$, they weakly prefer to do so). Thus, (12) holds with equality. Together with the market-clearing condition, we have:

$$2M - m_2 = py_1 \tag{37}$$

Finally, the regulatory constraint needs to be satisfied in equilibrium. Thus,

$$\theta[M - m_2] \geq p(\phi + \omega)(a_2 - 1) \tag{38}$$

where here, we have invoked the equilibrium conditions $m = 2M$ and $a = 2$. If this constraint remains slack, then $\chi = 0$; otherwise $\chi > 0$ and the condition above holds with equality.

5 Equilibrium characterization

We begin our characterization of the equilibrium by deriving some general results that will guide our analysis below. First, the credit-investor consumes the first-best level of output only if the regulatory constraint is slack.

Lemma 1 *$y_2 = y^*$ only if $\chi = 0$.*

Proof. Follows from (36) and $\zeta_4 \geq 0$. ■

Depending on parameter values, $y_2 < y^*$ is possible when the regulatory constraint does not bind. As we will show below, the consumption allocation to the credit-investor depends critically on the dividend of the security (which influences the market value of collateral).

Second, as is commonly the case in monetary economies, the Friedman rule, $\mu = \beta$, implements the first-best.

Lemma 2 *If $\mu = \beta$ then $y_1 = y_2 = y^*$ and $\chi = 0$.*

Proof. Suppose $\mu = \beta$. We need to show that $y_1 = y_2 = y^*$ is implementable in equilibrium. From Lemma 1 we know that $y_2 = y^*$ only if $\chi = 0$. Thus, we impose and verify that $\chi = 0$ in equilibrium. From (29), we get $y_1 = y^*$. Since $y_2 = y^*$ and $\chi = 0$ we get $\zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$. Without loss of generality, set $m_2 = M$ and $a_2 = 1$. Then (38) is automatically satisfied since there is no asset exchange between investors and so we verify that $\chi = 0$ Prices ϕ and p are given by (30) and (37), respectively. ■

At the Friedman rule, cash is sufficient to finance first-best consumption in the afternoon and no asset exchange or credit among investors is necessary.

Furthermore, securities are priced at their fundamental value, $\phi^* \equiv \beta\omega/(1-\beta)$.

Our third result is that when $\theta \geq 1$ and the regulatory constraint binds, the credit-investor carries no cash into the afternoon market. That is, the partnership allocates all the cash to the cash-investor.

Lemma 3 *If $\theta \geq 1$ and $\chi > 0$ then $\zeta_3 > 0$.*

Proof. Fix $\theta \geq 1$ and assume $\chi > 0$. Suppose $\zeta_3 = 0$. From Lemma 2 we know that $\mu = \beta$ implies $\chi = 0$, so for this case to be an equilibrium it must be that $\mu > \beta$. From (29) we have $y_1 < y^*$ and from Lemma 1 we have $y_2 < y^*$. Given $y_1 < y^*$, (31) implies $\lambda > 0$ and thus, (12) holds with equality. Given $y_2 < y^*$, (33) and (34) imply $\zeta_1 > 0$ and $\zeta_2 > 0$, respectively. Thus, $m' = 0$ and $a' = a - a_2$. In addition, since (11) always holds with equality, these results imply that (13) holds with equality.

Given $\zeta_3 = 0$ and $\chi > 0$, (35) implies $\theta p \chi = u'(y_2) - u'(y_1) > 0$. Thus, $y_1 > y_2$. Using (12) and (13) with both holding with equality implies $m - m_2 > m_2 + p(\phi+\omega)a_2$, which in equilibrium can be rearranged as $2(M - m_2) > p(\phi+\omega)a_2$. Since $\chi > 0$, the regulatory constraint (38) holds with equality: $\theta(M - m_2) = p(\phi+\omega)(a_2 - 1)$. These two expression put together imply $a_2(1 - \theta/2) > 1$. Given $a_2 \in [0, 2]$, we need $\theta < 1$ to satisfy this inequality, a contradiction with $\theta \geq 1$. ■

From this point on we study economies where the rate of return on cash is too low (the money growth rate is too high) to implement the first-best.

Assumption 2 *$\mu > \beta$.*

5.1 High-dividend economies

We first focus our analysis on an economy where the dividend is high enough so that the credit-investor consumes the first-best allocation when the regulatory constraint is slack or, equivalently, when there is no regulation.

5.1.1 Regulatory constraint is slack

Assume that the regulatory constraint is slack ($\chi = 0$). Then, from (29) we have a condition that determines y_1 solely as a function of inflation μ; i.e.,

$\mu = \beta u'(y_1)$. Given $\mu > \beta$, we have $u'(y_1) > 1$. Let us assume (and then verify) that $y_2 = y^*$, so that $u'(y_2) = 1$. Since $\chi = 0$, from (35) we have $\zeta_3 > 0$. This, in turn, implies $m_2 = 0$, so that (37) determines the evening price-level $p = 2M/y_1$. Note, as well that $\zeta_4 = 0$ from (36) but that without loss, we can assign all securities to the credit-investor, i.e., $a_2 = 2$.

We now identify the circumstances under which the assumption $y_2 = y^*$ is legitimate. Essentially, we need condition (13) to hold at $y_2 = y^*$. That is, since $m_2 = 0$, we need $(\phi + \omega)\, a_2 \geq y^*$. Given $y_2 = y^*$, from (30) we have the standard asset-pricing formula for the Lucas tree; i.e, $\phi^* \equiv \beta\omega/(1-\beta)$. With the asset priced at fundamental value ϕ^* and with all assets allocated to the credit-investor ($a_2 = a = 2$), this latter condition implies

$$\omega \geq 0.5\,(1-\beta)\, y^* \equiv \hat{\omega} \tag{39}$$

Definition 1 *A high-dividend economy is defined as $\omega \geq \hat{\omega}$.*

Let us now fix $\omega \geq \hat{\omega}$ and identify what is needed to ensure that the regulatory constraint does not bind. Recall that $p = 2M/y_1$, $m_2 = 0$, $a_2 = 2$ and $\phi = \phi^*$. Combining these conditions with (38), we derive the minimum value for θ that would ensure the regulatory constraint remains slack:

$$\theta \geq \left[\frac{2\omega}{(1-\beta)y_1}\right] \equiv \Theta_H(\omega, \mu) \tag{40}$$

where y_1 solves $\mu = \beta u'(y_1)$.

The results derived above are summarized in the following proposition.

Proposition 1 *In a high-dividend economy, if $\theta \geq \Theta_H(\omega, \mu)$ then $y_1 < y_2 = y^*$ and $\chi = 0$. The critical regulatory parameter $\Theta_H(\omega, \mu)$ as defined in* (40) *is increasing in ω and μ.*

Recall that higher values of θ imply that it is easier to satisfy the regulatory restriction on rehypothecation. Larger values of ω imply a higher value for securities and hence, a higher value for the securities that may potentially be rehypothecated. For the regulatory constraint to remain slack as ω is increased, the parameter θ must be increased to relax the regulatory restriction. Note that $\Theta_H(\hat{\omega}, \mu) = y^*/y_1 > 1$. That is, at the lowest dividend that allows for $y_2 = y^*$, θ must be above unity to keep the regulatory

constraint slack. As dividends increase, so does the required θ. Hence, a regulator that sets $\theta \leq 1$ will always succeed in restricting rehypothecation, regardless of the rates of return on cash and securities.

The critical value $\Theta_H(\omega, \mu)$ is increasing in μ because higher rates of inflation reduce the demand for real money balances y_1. Since $p = 2M/y_1$, the price-level increases which again, from (38), makes the regulatory constraint more difficult to satisfy. Higher μ reduces the real value of cash balances or, equivalently, raises the nominal value of securities that may be rehypothecated.

5.1.2 Regulatory constraint binds

Now assume that we have $\theta < \Theta_H(\omega, \mu)$ so that the regulatory constraint can no longer be satisfied when $y_2 = y^*$.

Lemma 4 *In a high-dividend economy, if $\theta < \Theta_H(\omega, \mu)$ then $\chi > 0$ and $y_2 < y^*$.*

Proof. If $y_2 = y^*$ and $\chi = 0$, then condition (40) cannot be satisfied for $\theta < \Theta_H(\mu, \omega)$. From Lemma 1, we have $y_2 = y^*$ only if $\chi = 0$. Thus, when $\theta < \Theta_H(\omega, \mu)$ we either have: (i) $y_2 < y^*$ and $\chi = 0$; or (ii) $y_2 < y^*$ and $\chi > 0$. We will show that (i) leads to a contradiction so that (ii) must be true.

Suppose $y_2 < y^*$ and $\chi = 0$. Condition (39) implies that when $\omega \geq \hat{\omega}$, $(\phi^* + \omega)2 \geq y^*$, i.e., the credit investor can purchase the first-best level of output when securities are priced at their fundamental value and $a_2 = a = 2$. When $\chi = 0$, (30) implies that $\phi > \phi^*$ (the real value of the Lucas tree is above fundamental) and (36) implies that $\zeta_4 > 0$ so that $a_2 = a = 2$ (all the partnership's securities are allocated to the credit-investor). Thus, $(\phi + \omega)2 \geq y^*$, i.e, the credit-investor can more than afford to purchase y^*, a contradiction with $y_2 < y^*$. ■

From (33) and (34) we see that $y_2 < y^*$ implies $\zeta_1 > 0$ and $\zeta_2 > 0$, which in turn implies $m' = 0$ (since (12) holds with equality) and $a' = a - a_2$. In other words, the partnership carries no cash into the evening and the credit-investor spends all of his assigned securities.[15]

[15] Note that when the regulatory constraint binds, the credit-investor may not be allowed to spend all of the partnership's assets. Since $y_2 < y^*$, the credit-investor will spend all

Using (29), we can express $p\chi$ in terms of y_1,

$$p\chi = (2/\theta)[\mu/\beta - u'(y_1)] \tag{41}$$

which allows us to solve the asset price ϕ from (30) as a function of (y_1, y_2):

$$\phi = \left[\frac{\beta L(y_1, y_2)}{1 - \beta L(y_1, y_2)}\right]\omega \tag{42}$$

where

$$L(y_1, y_2) \equiv (1/\theta)[u'(y_1) - \mu/\beta] + u'(y_2)$$

Note that $L(y_1, y_2) < 1/\beta$ can be interpreted as a liquidity premium on the security.

Recall that ζ_3 and ζ_4 are the Lagrange multipliers associated with the constraints $m_2 \geq 0$ and $a - a_2 \geq 0$, respectively. Recall that when $\chi = 0$ and $y_2 = y^*$, we showed that $m_2 = 0$ and $a_2 = a$. From (35) and (36), we see that $\chi > 0$ ($\Rightarrow y_2 < y^*$) does not necessarily imply that one or both of these constraints bind. Combine (41) and (42) with conditions (35) and (36) to form:

$$p\zeta_3 = \mu/\beta - [\theta L(y_1, y_2) + (1-\theta)u'(y_2)] \geq 0 \tag{43}$$

$$\frac{\omega\theta\zeta_4}{1 - \beta L(y_1, y_2)} = [u'(y_1) - \mu/\beta] + \theta[L(y_1, y_2) - 1] \geq 0 \tag{44}$$

Next, we combine the market-clearing conditions $m = 2M$ and $a = 2$ with the liquidity constraints (12) and (13) which, together with the regulatory constraint (38) and (42), implies

$$y_1 = 2M/p - m_2/p \tag{45}$$

$$y_2 = m_2/p + \frac{a_2\omega}{1 - \beta L(y_1, y_2)} \tag{46}$$

$$\theta[M/p - m_2/p] = \frac{(a_2 - 1)\omega}{1 - \beta L(y_1, y_2)} \tag{47}$$

With ϕ determined by (42), conditions (43)–(47), together with the non-negativity constraints $m_2 \geq 0$, $a - a_2 \geq 0$, characterize the equilibrium values $y_1, y_2, m_2, a_2, p, \zeta_3, \zeta_4$.

securities at his disposal and the partnership carries the unspent securities $a - a_2$ into the evening. It does not matter which investor physically holds the unspent securities, only that the amount $a - a_2$ cannot be spent by the credit-investor since it carries no rehypothecation rights.

Proposition 2 *In a high-dividend economy, if $\theta < \Theta_H(\omega, \mu)$ then the following is true: (i) $u'(y_1) < \mu/\beta$; (ii) $y_2 < y^*$; (iii) $a_2 < 2$ so that $\zeta_4 = 0$. Furthermore if $\theta \geq 1$ then: (iv) $m_2 = 0$ with $\zeta_3 > 0$; and (v) $a_2 > 1$.*

Proof. Lemma 4 implies $\chi > 0$.

(i) $u'(y_1) < \mu/\beta$ follows from (41) and $\chi > 0$.

(ii) $y_2 < y^*$, follows from Lemma 4.

(iii) First, we need to show that $L(y_1, y_2) \in (1, 1/\beta)$. $L(y_1, y_2) < 1/\beta$ follows from (42) and $\phi > 0$ in equilibrium. To show $L(y_1, y_2) > 1$ we use (44). A simple rearrangement implies $L(y_1, y_2) \geq 1 + (1/\theta)[\mu/\beta - u'(y_1)] > 1$, where the last inequality follows from (i).

To show $a_2 < 2$ (and hence, $\zeta_4 = 0$), note that (46) can be rearranged as

$$\frac{[1 - \beta L(y_1, y_2)]y_2}{1 - \beta} = \frac{[1 - \beta L(y_1, y_2)]m_2/p}{1 - \beta} + \frac{a_2\omega}{1 - \beta}.$$

Given $m_2 \geq 0$ and $\omega \geq \hat{\omega}$ we obtain, using (39)

$$\frac{[1 - \beta L(y_1, y_2)]y_2}{1 - \beta} \geq \frac{a_2\omega}{1 - \beta} \geq 0.5a_2y^*.$$

Suppose $a_2 = 2$. Since $L(y_1, y_2) \in (1, 1/\beta)$, we get $y_2 > y^*$, a contradiction with (ii). Thus, $a_2 < 2$ and $\zeta_4 = 0$.

(iv) Follows from Lemma 3.

(v) $a_2 > 1$ follows from (47), $m_2 = 0$ and $L(y_1, y_2) < 1/\beta$. ■

We can use the results in Proposition 2 to provide a sharper characterization of the equilibrium. Given $\zeta_4 = 0$, condition (44) implies $L(y_1, y_2) = \ell(y_1)$ where:

$$\ell(y_1) \equiv 1 + (1/\theta)[\mu/\beta - u'(y_1)].$$

From Lemma 3 we know that $\theta \geq 1$ is sufficient to guarantee $\zeta_3 > 0$ when $\chi > 0$. However, it is easy to construct examples where $\theta < 1$ and $\zeta_3 > 0$. So, in general, when parameters are such that $\zeta_3 > 0$ and thus, $m_2 = 0$, the equilibrium allocation (y_1, y_2) is characterized by:

$$\begin{aligned} \mu/\beta - u'(y_1) &= 0.5\theta[u'(y_2) - 1] \\ y_2 &= \frac{\omega}{1 - \beta\ell(y_1)} + 0.5\theta y_1 \end{aligned}$$

Given the equilibrium allocation (y_1, y_2), we can solve for (p, a_2):

$$\begin{aligned} p &= \frac{2M}{y_1} \\ a_2 &= \frac{y_2[1-\beta\ell(y_1)]}{\omega}. \end{aligned}$$

The effect of a binding regulatory constraint is to *increase y_1 toward its first-best level* at the cost of lowering y_2. Thus, the regulatory constraint may improve welfare due to consumption smoothing across types. When the regulatory constraint binds, the investor partnership attaches additional value to cash accumulation because more cash helps to overcome the regulatory constraint on future rehypothecation. The resulting additional demand for real cash balances reduces the price-level and permits more consumption in the cash market, at the expense of the credit market.

5.2 Equilibrium in low-dividend economies

We now assume that the dividend is low enough so that the credit-investor always consumes below the first-best allocation. That is, assume $\omega < \hat{\omega}$ as defined in (39).

Definition 2 *A low-dividend economy is defined as $\omega < \hat{\omega}$.*

In this case, (39) is not satisfied and thus, $y_2 = y^*$ cannot satisfy the credit-investor liquidity constraint, (13). Therefore, in what follows, it is always the case that the credit-investor consumes below the first-best allocation, i.e., $y_2 < y^*$.

5.2.1 Regulatory constraint is slack

Suppose that parameters are such that the regulatory constraint is slack in a low-dividend economy. Given $\chi = 0$, y_1 solves (29), i.e., $\mu = \beta u'(y_1)$. Hence, assuming the regulatory constraint does not bind, the consumption of a cash-investor is the same in low-dividend and high-dividend economies, and depends solely on the rate of return on cash, β/μ.

Several results follow from $u'(y_2) > u'(y^*) = 1$. Condition (30) implies $\phi = \beta(\phi + \omega)u'(y_2) > \phi^*$. From (31), (32) and (34) we have $\zeta_1 > 0$, $\zeta_2 > 0$ and $\zeta_4 > 0$, respectively. Thus, $m' = 0$, $a' = 0$ and $a_2 = a$. From (33), $p\zeta_3 = \mu/\beta - u'(y_2)$, so we have two cases: (i) $y_1 = y_2$ and so $\zeta_3 = 0$, $m_2 \geq 0$; and (ii) $y_1 < y_2$ and so $\zeta_3 > 0$, $m_2 = 0$. The former case implies $\phi = (\phi+\omega)\mu$ and therefore, can only exist if $\mu \in (\beta, 1)$ (otherwise, $\phi < 0$). Let us then focus on the latter case.

Assume $\zeta_3 > 0$ and thus, $m_2 = 0$. Then, (11) implies the consumption of the credit-investor solves

$$y_2[1 - \beta u'(y_2)] = 2\omega \tag{48}$$

Given $m_2 = 0$, (37) implies $M/p = y_1/2$. Thus, we can verify the conditions for which the regulatory constraint is satisfied. Using the results derived so far, (38) implies $\theta y_1 \geq y_2$. This condition can be restated as:

$$\theta \geq \frac{y_2}{y_1} \equiv \Theta_L(\omega, \mu) \tag{49}$$

where y_1 solves $\mu = \beta u'(y_1)$ and y_2 solves (48). $\Theta_L(\omega, \mu)$ is the minimum value of θ that ensures the regulatory constraint does not bind in a low-dividend economy. Note that, from (48), y_2 is increasing in ω, while from $\mu = \beta u'(y_1)$, y_1 is decreasing in μ. Thus, $\Theta_L(\omega, \mu)$ is increasing in both arguments.

If we couple (49) with $y_1 < y_2$, which ensures $\zeta_3 > 0$ as assumed, we can summarize these results in the following proposition.

Proposition 3 *In a low-dividend economy, if $\theta \geq \Theta_L(\omega, \mu) > 1$ then: $y_1 < y_2 < y^*$; $m_2 = 0$; and $\chi = 0$. The critical regulatory parameter $\Theta_L(\omega, \mu)$ as defined in* (49) *is increasing in ω and μ.*

As in the case of high-dividend economies, a regulator that sets $\theta \leq 1$ will always succeed in restricting rehypothecation, regardless of the rates of return on assets. The intuition for why the critical value $\Theta_L(\omega, \mu)$ is increasing in both arguments is the same as the one provided above for high-dividend economies.

For the case of log utility, $u(y) = \ln y$, the characterization provided here can be written explicitly in terms of parameters. Specifically, $\Theta_L \equiv \mu(1 + 2\omega/\beta)$. Thus, if $\theta \geq \mu(1+2\omega/\beta) > 1$ then $y_1 = \beta/\mu < y_2 = \beta + 2\omega < y^* = 1$.

5.2.2 Regulatory constraint binds

Now assume that $\theta < \Theta_L(\omega, \mu)$ so that the regulatory constraint binds in a low-dividend economy. The equilibrium characterization leading up to Proposition 2 also applies to low-dividend economies with a binding regulatory constraint. That is, conditions (43)–(47), together with the non-negativity constraints $m_2 \geq 0$, $a - a_2 \geq 0$, characterize the equilibrium values $y_1, y_2, m_2, a_2, p, \zeta_3, \zeta_4$, regardless of the dividend ω. An important difference between low and high-dividend economies is that when $\omega < \hat{\omega}$ it is possible that $\zeta_4 > 0$, i.e., $a_2 = a$. This is not the case in high-dividend economies, where $a_2 < a$ when the regulatory constraint binds. To see this possibility, note that the proof of Proposition 2(iii) relies on $\omega \geq \hat{\omega}$.

When parameters and policy variables are such that $\zeta_4 = 0$ when $\omega < \hat{\omega}$ and $\theta < \Theta_L(\omega, \mu)$, then the characterization provided in Proposition 2 also applies to low-dividend economies.

As mentioned above, in low-dividend economies with a binding regulatory constraint, it is possible to have both $\zeta_3 > 0$ and $\zeta_4 > 0$, i.e., $m_2 = 0$ and $a_2 = 2$. We leave the explanation for why this is possible to the numerical section below. Here, we proceed with a characterization of this case. Suppose then that $\zeta_4 > 0$ so that $a_2 = a = 2$ in equilibrium. Furthermore, to simplify the analysis, suppose parameters are such that $\zeta_3 > 0$ and thus, $m_2 = 0$. From Lemma 3 we know that $\theta \geq 1$ is sufficient to guarantee $\zeta_3 > 0$ when $\chi > 0$, but the result can hold for other parameterizations. The equilibrium allocation (y_1, y_2) is then characterized by:

$$\begin{aligned} y_1 &= \frac{2\omega}{\theta[1 - \beta L(y_1, y_2)]} \\ y_2 &= \frac{2\omega}{1 - \beta L(y_1, y_2)} \end{aligned}$$

and the price level is given by $p = 2M/y_1$. Note that $y_2 = \theta y_1$, which is related to the critical value defined in (49). The gap between allocations is increasing in θ. Thus, as the regulatory constraint tightens (lower θ), the allocations of cash and credit investors get closer to each other, which due to concave utility, may improve welfare. We explore this result further in the following sections.

5.3 Regulation and welfare

In a stationary equilibrium, the *ex ante* welfare of an investor is given by $W \equiv (1-\beta)^{-1}0.5[u(y_1)+x_1+u(y_2)+x_2]$, where x_1 and x_2 are the evening allocations of the cash-investor and the credit-investor, respectively. Note that the evening allocation for the partnership, as defined in (16), is equal to the expected allocation of the individual investors, i.e., $x = 0.5[x_1+x_2]$. Taking the afternoon budget constraint (11) we can write $m'/p+a' = m/p+(\phi+\omega)a-y_1-y_2$. Replacing the left-hand side of this expression in the evening budget constraint (16), we get $x = m/p+(\phi+\omega)a-y_1-y_2-m^+/p-\phi a^+ + 2\tau$. Using equilibrium conditions $m = 2M$, $m^+ = 2\mu M$, $a = a^+ = 2$ and $\tau = (\mu-1)M/p$ we get $x = 2\omega - y_1 - y_2$ and thus,

$$(1-\beta)W = 0.5[u(y_1)-y_1+u(y_2)-y_2]+\omega \tag{50}$$

Suppose $u(c) = \ln c$. Then, we can solve the model analytically. The expressions, however, are rather complex and cannot be easily interpreted. Consider instead expressing y_1 in terms of y_2. We get

$$y_1 = \frac{2\beta}{2\mu - \beta\theta(1/y_2 - 1)}.$$

Since $y_2 < y^* = 1$ when $\chi > 0$, we have that y_1 is decreasing in y_2. Therefore, if we start in an equilibrium where $y_2 = y^*$ and the regulatory constraint is just satisfied in a high-dividend economy, i.e., where $\omega = \hat{\omega}$ and $\theta = \Theta_H(\hat{\omega}, \mu)$, then any change in policy that makes the regulatory constraint bind implies y_1 increases and y_2 decreases. Since investors' preferences are strictly concave, any small such perturbation improves their *ex ante* welfare. Using (50) and fixing $\omega = \hat{\omega}$ we get

$$\left.\frac{dW}{dy_2}\right|_{y_2=y^*} = -\frac{\beta\theta(\mu-\beta)}{4\mu^2(1-\beta)} < 0.$$

That is, in high-dividend economies where the regulatory constraint is just satisfied, (small) changes in policy that make this constraint bind are welfare improving.

The intuition for the welfare result derived above is that the regulatory constraint corrects the inefficiency generated by being away from the Friedman rule. Starting from an equilibrium with $\mu > \beta$ and $y_1 < y^*, y_2 = y^*$,

imposing a binding regulatory constraint increases the rate of return on cash, since it is now valued higher than before due its ability to relax the regulatory constraint.

5.4 Numerical analysis

We now illustrate the effects of tightening regulation and increasing inflation. Throughout this section, we assume $U(c) = \ln c$, $\beta = 0.96$ and $\omega = 0.015$. This parameterization implies $\hat{\omega} = 0.02$, so we will be analyzing a low-dividend economy. When we vary regulation tightness θ, we assume $\mu = 1$. When we vary inflation, we fix $\theta = 1.05$. In all cases considered, we obtain $\zeta_3 > 0$ and thus, $m_2 = 0$. In some charts, we include a case labeled "unregulated" which corresponds to an equilibrium in which there is no regulatory constraint. Welfare is measured as the equivalent afternoon-consumption compensation relative to the first-best allocation. That is, how much an investor would have to be compensated every period, in terms of afternoon consumption, in order to be indifferent between living in the corresponding economy and the first-best. Given log-utility the welfare cost simplifies to the proportion Δ solving $\ln(1+\Delta) = (1-\beta)[W(y^*, y^*) - W(y_1, y_2)]$.

Figures 1, 2 and 3 show the effects of tightening the regulatory constraint, i.e., reducing θ. The x-axis of each figure is in terms of $1/\theta$, so that we tighten the constraint as we move to the right.

Figure 1 consists of four panels showing the effects of θ on (χ, ζ_4), p, a_2 and ϕ, respectively. For θ sufficiently high, the regulatory constraint does not bind ($\chi = 0$); since this is a low-dividend economy, $y_2 < y^*$ and thus, $\zeta_4 > 0$–all securities are allocated to the credit-investor. For θ low enough, the regulatory constraint binds, $\chi > 0$. Initially, although the regulatory constraint binds, all securities are still allocated to the credit-investor. This is possible since the value of securities ϕ decreases, which allows the partnership to still satisfy the constraint at $a_2 = 2$. In this region, ϕ decreases as we decrease θ since the premium on securities dictated by the constraint $a - a_2 \geq 0$ is becoming less important as the regulatory constraint becomes tighter (i.e., ζ_4 decreases as θ decreases). Eventually, however, for θ sufficiently low, we obtain $\zeta_4 = 0$ and $a_2 < 0$. That is, the regulatory constraint is tight enough that the value of the cash loan can no longer support allocating all the securities to the credit-investor. As we lower θ further the regulatory

constraint binds more tightly and securities start carrying a higher premium.

Figure 1: The Effects of Tighter Regulation

Figure 2 shows how afternoon output (y_1, y_2) varies with θ. As we discussed above, imposing a binding regulation brings the allocations of the cash-investor and the credit-investor closer together. In other words, restricting rehypothecation increases risk diversification. Note, however, that there are limits to regulation: if θ is too low, it is possible to have $y_1 > y_2$ and, eventually, lower welfare than without regulation.

Figure 2: The Effects of Tighter Regulation on Output

Figure 3 shows welfare as a function of θ and includes the unregulated case as a reference point. There is always a welfare loss relative to the first-best due to monetary policy being away from the Friedman rule, $\mu > \beta$. As the regulation binds, welfare gets closer to the first-best, indicating that restricting rehypothecation improves welfare. However, if the regulation is too tight (θ too low), then welfare can be lower than in the unregulated case.

Figure 3: The Effects of Tighter Regulation on Welfare

Figures 4, 5 and 6 show the effects of increasing inflation. In an unregulated economy, monetary policy directly affects the consumption of the cash-investor: y_1 is decreasing in μ. However, given that all the cash is allocated to the cash-investor and all the securities to the credit-investor, consumption of the credit-investor is unaffected by inflation. In contrast, in an economy subject to restrictions on rehypothecation, the effects of increasing inflation are similar to those of tightening the regulatory constraint: increasing μ raises the prices level, which in turn, tightens the regulatory constraint and thus, works similarly to lowering θ. Given that higher inflation has two effects, lowering cash-investor's consumption and tightening the regulatory constraint, Figures 4 and 5 show both the regulated and the unregulated cases, to highlight the impact of each.

Figure 4: The Effects of Higher Inflation

Figure 5: The Effects of Higher Inflation on Output

Figure 6 shows welfare for a given inflation rate relative to the first-best (which is implemented at the Friedman rule, $\mu = \beta$). It compares the cases of no partnership, unregulated partnership and regulated partnership.[16] The cost of 10% inflation in the no partnership case is in the range of costs derived in previous studies (e.g., see Lucas, 2000 and Lagos and Wright, 2005). The value of allowing the practice of rehypothecation is roughly the distance between the no partnership and unregulated cases. As we can see, rehypothecation, which allows for risk-sharing between investors in a partnership, is especially useful in high inflation economies. In our example, the welfare gain is about 1.1% of consumption when inflation is 10%.

Why does the value of rehypothecation increase with inflation? In the no partnership case, both types of investors use cash to finance afternoon consumption. In some cases, the credit-investor has excess assets–for our parameterization, in fact, he consumes the first-best. When we allow for partnerships, cash flows to the cash-investor, who is in higher need of liquidity, at the cost of lowering the consumption of the credit-investor somewhat. This is why rehypothecation improves welfare, as previously explained. Now, when we allow for rehypothecation, as we increase inflation, the marginal value of the extra unit of cash flowing to the cash-investor increases, while consumption for the credit-investor remains unaffected. Thus, the value of the partnership increases.

The gains from a binding regulation are also increasing in inflation, as the regulation mitigates the inefficiency of a low rate of return on cash. At 10% inflation, setting $\theta = 1.05$ yields a further 0.1% welfare gain. Although the welfare gain from restricting rehypothecation is an order of magnitude lower than altogether allowing the practice, the gain is still seizable.

Figure 6: Welfare Cost of Inflation

[16] In the no partnership case, individual investors go at it alone. Their choice problems in this case are straightforward, so we omit writing them down formally.

6 Limited commitment

We now verify under which circumstances the equilibrium characterized in Section 5 can be supported under limited commitment and how default incentives vary with regulation tightness, inflation and securities valuation.

Assume that investors in a partnership cannot commit to repaying in the evening any asset (cash or securities) loans acquired in the morning.[17] When a default occurs, loans made in both directions within the partnership are not repaid, regardless of which investor is the defaulting party. We assume that the punishment for default in the evening is autarky, i.e., an investor is forced to operate forever on its own after defaulting. In this modified environment, any securities not lent by the cash-investor (i.e., $a - a_2$) are safely in his possession. Equivalently, these securities could be deposited with the credit-investor in a segregated account with no rehypothecation rights. As such, they cannot be defaulted on and are always returned to the cash-investor.

For now, we assume (and later verify) that investors have no incentives to default. Note that a stationary equilibrium where default is not optimal is equivalent to the equilibrium with commitment characterized in the previous section. To simplify exposition, for the remaining of this section, we make the following assumption.

Assumption 3 *$\theta \in [1, \Theta_i(\omega, \mu))$, where $i = H$ if $\omega \geq \hat{w}$ and $i = L$ if $\omega < \hat{w}$.*

By Lemma 3, $\theta \geq 1$ ensures that $\zeta_3 > 0$ and thus, $m_2 = 0$, i.e., all the cash in a partnership is allocated to the cash-investor.[18] The policy parameter θ is assumed to be low enough so that the regulatory constraint binds. Hence, $y_2 < y^*$ and the credit-investor spends all his allocated assets.

In equilibrium, an investor who arrives to the evening market with any unspent assets, repays assets borrowed in the morning, receives payment for asset lent in the morning and acquires cash and securities to be used in the following period. The only unspent assets are the securities that the credit investor was not allowed to rehypothecate, $a - a_2$. Given $m_2 = 0$, the evening value of the cash loan is equal to m/p. The evening value of the securities

[17] Because these loans are intraday, we assume they bear zero interest.

[18] As we shall make clear below, the analysis in this section relies on $m_2 = 0$ and can be generalized to values of θ below 1 as long as parameters ensure $m_2 = 0$.

loan is $\phi(a_2 - a/2)$. The net loan repayment is the difference between the two loans, the sign depending on investor's type. Each investor acquires $0.5(\phi a^+ + m^+/p) - \tau$ for use in the following period. Finally, recall that in equilibrium, $m = 2M$, $m^+ = 2\mu M$, $a = a^+ = 2$ and $\tau = (\mu - 1)M/p$.

The evening value for a cash-investor remaining in the partnership is

$$\underbrace{(\phi + \omega)(2 - a_2)}_{\substack{\text{non-rehypothecated} \\ \text{securities}}} + \underbrace{\phi(a_2 - 1) - M/p}_{\text{net loan repayment}} - \underbrace{(\phi + M/p)}_{\text{new assets}} + \beta W$$

whereas the evening value for a credit-investor remaining in the partnership is

$$\underbrace{-\phi(a_2 - 1) + M/p}_{\text{net loan repayment}} - \underbrace{(\phi + M/p)}_{\text{new assets}} + \beta W$$

where W is the continuation value for an investor remaining in the partnership, as expressed in (50), for a given equilibrium allocation (y_1, y_2).

We now verify when investors have no incentives to default in equilibrium. The value of autarky is given by the function $B^d(m, a) \equiv 0.5[B_1^d(m, a) + B_2^d(m, a)]$ satisfying the following recursion:

$$\begin{aligned}
B_1^d(m, a) &\equiv \max_{y_1} \ \{u(y_1) + V^d(m - py_1, a)\} \\
B_2^d(m, a) &\equiv \max_{y_2, a_2'} \ \{u(y_2) + V^d(m + p(\phi + \omega)(a - a_2') - py_2, a_2')\} \\
V^d(m', a') &\equiv \max_{m^+, a^+} \ \{(\phi + \omega)a' + (1/p)(m'^+) - \phi a^+ + \tau + \beta B^d(m^+, a^+)\}
\end{aligned}$$

subject to the usual non-negativity constraints, and where prices p and ϕ and transfers τ correspond to the equilibrium with no default. Note that in the problems stated above, we are anticipating that liquidity constraints will bind.

Since flow utility is linear in the evening, the choice assets of a defaulting investor is independent of the value of assets he gets to run away with. Thus, the portfolio choice of a defaulting investor is given by

$$\max_{m^+, a^+} \ -m^+/p - \phi a^+ + \tau + \beta B^d(m^+, a^+)$$

Note the evening asset choice when defaulting is the same as in autarky. Thus, the expression above is equal to the value in autarky of entering the evening market with zero assets, $V^d(0, 0)$.

A defaulting cash-investor gets to keep the non-rehypothecated (non-lent) securities. However, having defaulted on the cash-loan, the investor is not entitled to receive back his lent securities. Thus, the net loan repayment is zero. The evening value for a defaulting cash-investor is thus $V^d(0, 2 - a_2)$. Since $V_a^d = \phi + \omega$, this value can be expressed as $V^d(0,0) + (\phi + \omega)(2 - a_2)$. For the credit-investor, the evening value of defaulting is simply equal to $V^d(0,0)$. The cash-investor has a higher default payoff than the credit-investor when $a_2 < 2$.

Default in the evening occurs when the value of remaining in the partnership is lower than the value of defaulting. Hence, there in no default in equilibrium when

$$\phi(a_2 - 2) - 2M/p + \beta W \geq V^d(0,0) \tag{51}$$

$$-\phi a_2 + \beta W \geq V^d(0,0) \tag{52}$$

Both sides of these conditions depend on policy variables θ and μ. Notably, $V^d(0,0)$ depends on these variables through their effect on equilibrium prices.

The following result states conditions under which the cash-investor has more incentives to default than the credit-investor.

Proposition 4 *If $\theta < [\beta L(y_1, y_2)]^{-1}$, then $\phi(a_2 - 2) - 2M/p < -\phi a_2$, i.e., the incentives to default are stronger for the cash-investor than for the credit-investor.*

Proof. From (51) and (52), the default incentives for the cash-investor are stronger than for the credit-investor when $\phi(a_2 - 2) - 2M/p < -\phi a_2$, as stated in proposition statement. After a simple rearrangement, this expression can be written as $\phi(a_2 - 1) - M/p < 0$. Given $m_2 = 0$, a binding regulatory constraint implies $M/p = (1/\theta)(\phi + \omega)(a_2 - 1)$. Thus, $\phi(a_2 - 1) - (1/\theta)(\phi + \omega)(a_2 - 1) < 0$. From Proposition 2, which characterizes high-dividend economies and also low-dividend economies when $\zeta_4 = 0$, we have $a_2 > 1$ (when $\zeta_4 > 0$ in low-dividend economies, $a_2 = 2$). Thus, the inequality simplifies to $\phi - (1/\theta)(\phi + \omega) < 0$ or $(\theta - 1)\phi < \omega$. Using (42), which holds in all economies with a binding regulatory constraint, we get $(\theta - 1)\beta L(y_1, y_2)[1 - \beta L(y_1, y_2)]^{-1} < 1$, which implies the assumption made in the proposition statement. ■

Note that the proposition also holds for $\theta < 1$, as long as parameters are still such that $m_2 = 0$–follows from the inequality $(\theta - 1)\phi < \omega$ in the

proof. Given this result, in the numerical analysis that follows we focus on the default incentives of a cash-investor.

Figures 7 and 8 show how regulation, inflation and dividends affect the incentive to default for the cash-investor. Both figures show the expression $\phi(a_2 - 2) - 2M/p + \beta W - V^d(0,0)$, as a function of θ and assuming different values of μ and ω. As explained above, when this expression is positive, the cash-investor has no incentives to default; conversely, when it is negative, the no-default equilibrium cannot be supported as the cash-investor would find it optimal to default in the evening. In all cases considered, the incentives to default are decreasing in θ, i.e., a tighter regulation (lower θ) makes default more attractive. The main mechanism that explains this result is the fact that a tighter regulation implies a smaller amount of securities can be rehypothecated; thus, the cash-investor arrives to the evening market with a larger (and secure) asset position, which he gets to keep in the event of a default.

Figure 7 shows the effects of inflation on the default incentives of the cash-investor. Although increasing μ tightens the regulatory constraint in the same way as decreasing θ, the effects on default incentives are actually opposite. As mentioned above, a tighter regulation due to lower θ increases default incentives. A higher μ lowers the rate or return on cash, which makes autarky (the punishment for default) relatively less attractive. From a policy perspective, there is a point after which it may be more desirable to restrict rehypothecation indirectly through inflation rather than regulation, as the direct approach through θ would increase (rather than decrease) the incentives for default. This comes at a cost, of course, as higher inflation reduces reduces welfare.

Figure 7: Effect of Inflation on Default Incentives of Cash-Investor

Figure 8 shows that lower dividends also increase the default incentives for the cash-investor. The reason is simple: the lower fundamental value for securities makes the investor-partnership less valuable. Thus, the punishment of autarky is less severe.

Figure 8: Effect of Dividends on Default Incentives of Cash-Investor

7 Conclusions

Rehypothecation turns collateral into exchange media, which is desirable when liquidity is scarce. The practice generally improves welfare, especially in high inflation (interest rate) environments. However, there is some reason to believe that there is generally too much of it. We find that interventions, specifically in the form of SEC rule 15c3-3, can improve welfare by enhancing the demand for real cash balances, which is too low when monetary policy is away from the Friedman rule.

It would be of some interest to extend the model developed here to incorporate aggregate uncertainty over returns on securities when commitment between investors is limited. Imagine, for example, a "news shock" arriving in the afternoon that leads to a rational revision in the conditional forecast of the security's payoff in the evening. In the case of an overcollateralized loan, a good news shock may increase the value of collateral to a point where the lender (security borrower) finds it too costly to reacquire the collateral and make good on his obligation to return it (or its market value equivalent). It would then be possible to investigate how policy might be designed to accommodate spikes in "repo failures" in times of financial crisis (see Hördahl and King, 2008).

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Appendix

A Alternative restriction on rehypothecation

We modeled the regulatory constraint (10) explicitly after SEC rule 15c3-3. A critical element of this regulation is the link between cash lent and securities borrowed. To better understand the effects of this specific constraint, we now look at an alternative restriction, motivated by the type of liquidity constraints proposed by Kiyotaki and Moore (2002).[19] In particular, suppose that credit-investors can only use a fraction $\vartheta \in [0,1]$ of their borrowed assets. The value of assets the credit-investor is allowed to spend in the afternoon is thus, $a/2 + \vartheta(a_2 - a/2)$, which can be rearranged to $\vartheta a_2 + (1-\vartheta)a/2$. The credit-investor faces now the following afternoon liquidity constraint:

$$m_2 + p(\phi + \omega)[\vartheta a_2 + (1-\vartheta)a/2] - py_2 \geq 0 \tag{53}$$

Note that if $\vartheta = 1$ then (53) simplifies to (13).

The partnership problem in the afternoon is then

$$B(m,a) \equiv \max_{y_1, y_2, m_2, a_2, m', a'} \{u(y_1) + u(y_2) + V(m', a')\}$$

subject to (11), (12), (53) and the non-negativity constraints (8), (9), (14), (15). Again, since we anticipate morning cash flowing to the cash-investor and morning securities flowing to the credit-investor, the non-negativity constraints (8), (9) simplify to $m_2 \geq 0$ and $a - a_2 \geq 0$, respectively. We assign the same Lagrange multipliers as before. Let ξ be the multiplier associated with the credit-investor's liquidity constraint (53). If $\vartheta = 1$ then $\xi = 0$ as (53) simplifies to (13), which, as argued previously, is implied by the other inequality constraints.

We can follow the same steps as in the main text to characterize an equilibrium in this environment (note that the evening problem remains the

[19] For applications to monetary economies see, for example, Li and Li (2013) and He, Wright and Zhu (2014).

same). The first-order conditions of the partnership problem are:

$$\begin{aligned} u'(y_1) - p[\psi + \lambda - \zeta_1] &= 0 \\ u'(y_2) - p[\psi + \xi] &= 0 \\ -\lambda + \xi + \zeta_1 + \zeta_3 &= 0 \\ \xi p(\phi + \omega)\vartheta + \zeta_2 - \zeta_4 &= 0 \\ V_m(m', a') - \psi + \zeta_1 &= 0 \\ V_a(m', a') - \psi p(\phi + \omega) + \zeta_2 &= 0 \end{aligned}$$

and by the Envelope Theorem:

$$\begin{aligned} B_m(m, a) &= \psi + \lambda - \zeta_1 \\ B_a(m, a) &= p(\phi + \omega)[\psi + \xi(1 - \vartheta)/2] + \zeta_2 \end{aligned}$$

After some work we obtain:

$$\mu = \beta u'(y_1) \tag{54}$$

which solves for y_1 as a function of μ. Compare this equation to condition (29) in our model. An important result in the paper is that consumption for the cash-investor depends on whether the regulatory constraint binds or not, as cash helps relax this constraint. In contrast, (54) is independent of ϑ, since there is no link between the borrowed securities that the credit-investor can reuse and the cash that the cash-investor can spend. Hence, we obtain the standard result that consumption of the "cash-good" is solely a function of the rate of return on cash.

We characterize the remaining conditions of this alternative environment to highlight other similarities and differences with our previous analysis. The price of securities satisfies:

$$\phi = \beta(\phi + \omega)[u'(y_2) + p\xi(1 - \vartheta)/2] \tag{55}$$

where the term $p\xi(1-\vartheta)/2$ plays the same role as the term $-p\chi/2$ in condition (30). The multipliers for the budget and liquidity constraints satisfy:

$$p\lambda = \mu/\beta - 1 \tag{56}$$

$$p(\psi + \xi) = u'(y_2) \tag{57}$$

which implies $\lambda > 0$ (when $\mu > \beta$) and $\psi + \xi > 0$. The remaining Lagrange multipliers solve:

$$
\begin{aligned}
p\zeta_1 &= u'(y_2) - 1 - p\xi && (58)\\
(\phi+\omega)^{-1}\zeta_2 &= u'(y_2) - 1 - p\xi && (59)\\
p\zeta_3 &= \mu/\beta - u'(y_2) && (60)\\
(\phi+\omega)^{-1}\zeta_4 &= u'(y_2) - 1 - p\xi(1-\vartheta) && (61)
\end{aligned}
$$

Clearly, when $\xi = 0$ equations (54)–(61) are identical to (29)–(36) when $\chi = 0$.

Suppose now $\xi > 0$. Then, $y_2 < y^*$ and $\zeta_4 > 0$, so that $a_2 = a$. Given that the liquidity constraint of the cash-investor is always satisfied with equality, $\xi > 0$ implies $a' > 0$ and thus, $\zeta_1 = \zeta_2 = 0$. Hence, $p\xi = u'(y_2) - 1 > 0$ and $\psi = 1$. Market clearing implies $m = 2M$ and $a = 2$. Thus, $a_2 = 2$. We also have $m' = 0$ (without loss of generality when $\mu = \beta$). The liquidity and budget constraints become:

$$
\begin{aligned}
2M/p + (\phi+\omega)(2-a') - y_1 - y_2 &= 0 && (62)\\
2M/p - m_2/p - y_1 &= 0 && (63)\\
m_2/p + (\phi+\omega)(1+\vartheta) - y_2 &= 0 && (64)
\end{aligned}
$$

The unknowns are $M/p, m_2/p, a', y_1, y_2, \phi$ solved by (54), (55), (62)–(64). By condition (60), there are two cases: (i) $m_2 = 0$, $y_1 < y_2 < y^*$; and (ii) $m_2 > 0$, $y_1 = y_2 < y^*$. Focus on the case with $m_2 = 0$. The solution is characterized by:

$$
\begin{aligned}
u'(y_1) &= \mu/\beta\\
y_2[1 - \beta H(y_2, \vartheta)] &= \omega(1+\vartheta)\\
a' &= 1 - \vartheta\\
2M/p &= y_1\\
\phi &= \omega\beta H(y_2, \vartheta)/[1 - \beta H(y_2, \vartheta)]
\end{aligned}
$$

where $H(y_2, \vartheta) \equiv u'(y_2) + (u'(y_2) - 1)(1-\vartheta)/2 > 0$, which is strictly decreasing in both arguments.

What are the welfare effects of tightening the credit-investor's liquidity constraint, i.e., of lowering ϑ? As mentioned above, when $\vartheta = 1$, the

environment coincides with one in which there is no regulatory constraint (equivalently, where the constraint is slack, $\chi = 0$). As we vary ϑ, y_1 remains constant, so all the welfare effects come from how ϑ affects y_2. From (50), we know welfare is increasing in y_2. Totally differentiating the second condition above with respect to ϑ and rearranging we obtain

$$(dy_2/d\vartheta)\{\omega(1+\vartheta)/y_2 - \beta y_2 u''(y_2)[1+(1-\vartheta)/2]\} = \omega - \beta y_2[u'(y_2)-1)]/2$$

Since u is concave, the sign of $dy_2/d\vartheta$ is the same as the sign of the right-hand side of the equation above. As long as $\omega - \beta y_2[u'(y_2)-1)]/2 > 0$, y_2 is increasing in ϑ. Thus, tightening the liquidity constraint of the credit-investor (lowering ϑ) lowers welfare. This is in contrast to tightening the regulatory constraint (lowering θ), which leads to higher welfare, as long as the regulation is not too tight.

Figure 9 shows how welfare depends on ϑ, using the same parameterization as in the main body of the paper. The unregulated case is presented as a benchmark, same as in Figure 3. The case considered in this section is labeled "constrained". When $\vartheta = 1$ the two cases coincide. As we lower ϑ (move to the left), the liquidity constraint of the credit-investor binds more tightly, which lowers his consumption and thus, overall welfare. As shown in Figure 3, a binding regulatory constraint can lead to higher welfare, underscores our choice of modeling it after real-world SEC rule 15c3-3 rather than an exogenously imposed constrain on the liquidity of borrowed securities

Figure 9: The Effects of Borrowed Securities' Liquidity on Welfare

Figures

Figure 1: The Effects of Tighter Regulation

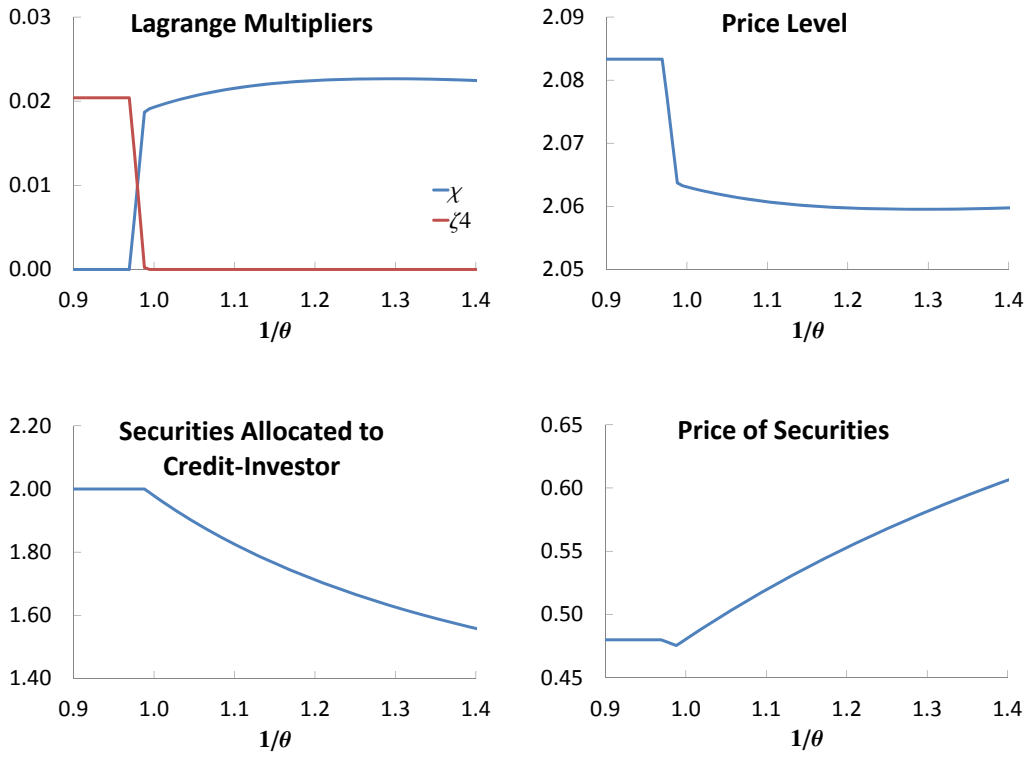


Figure 2: The Effects of Tighter Regulation on Output

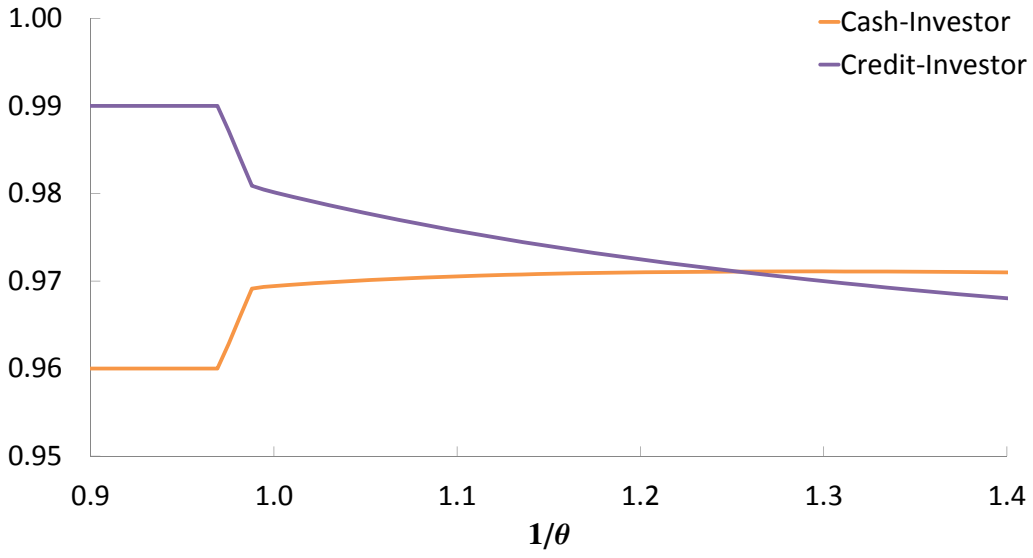
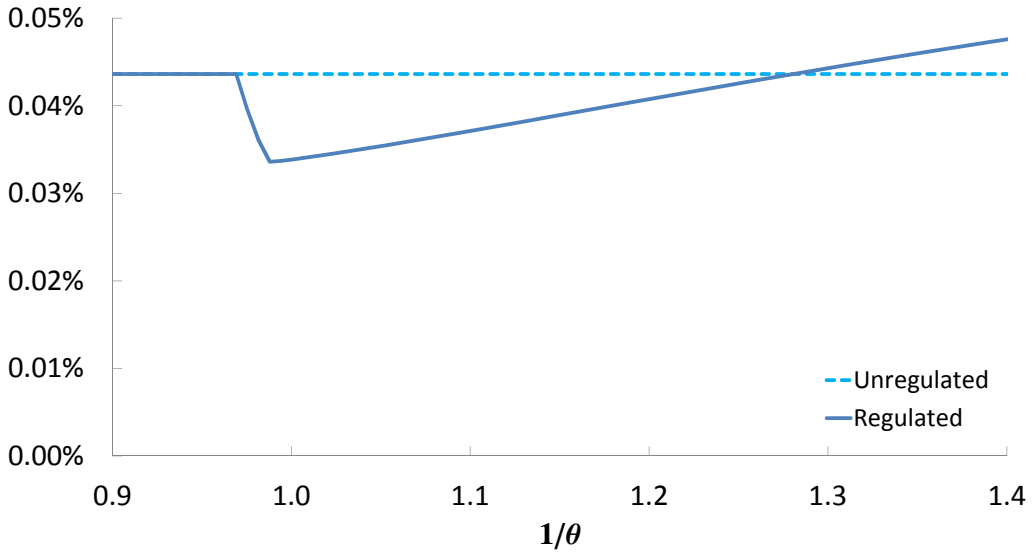


Figure 3: The Effects of Tighter Regulation on Welfare



Note: lines display equivalent consumption compensation relative to the first-best.

Figure 4: The Effects of Higher Inflation

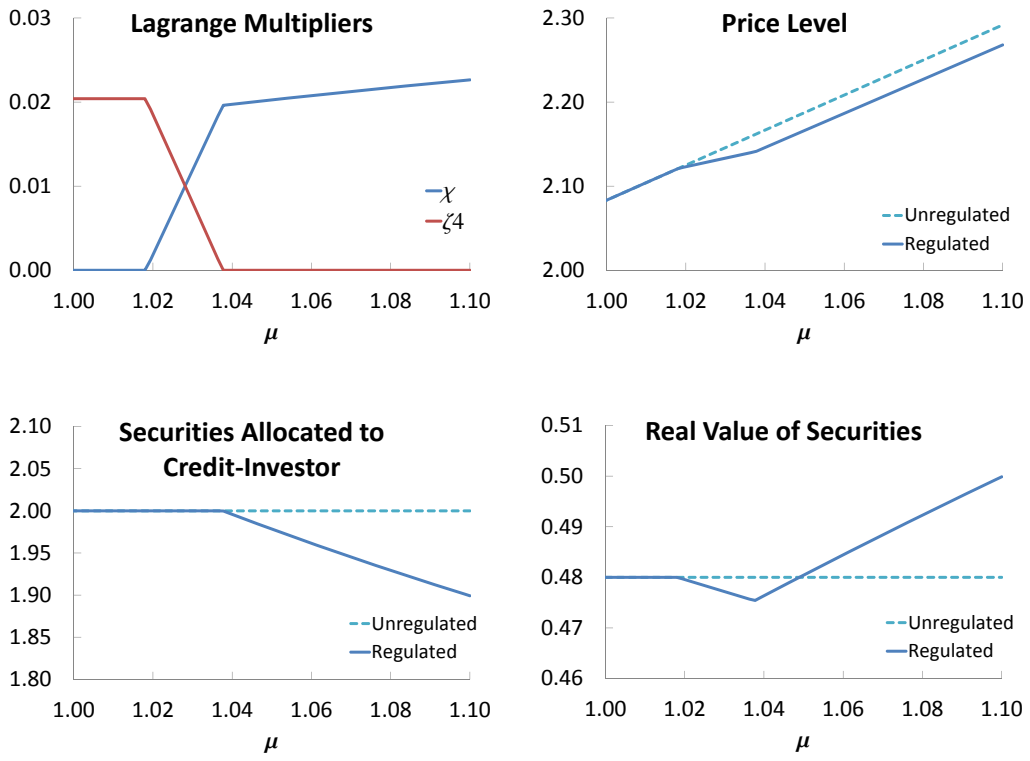


Figure 5: The Effects of Higher Inflation on Output

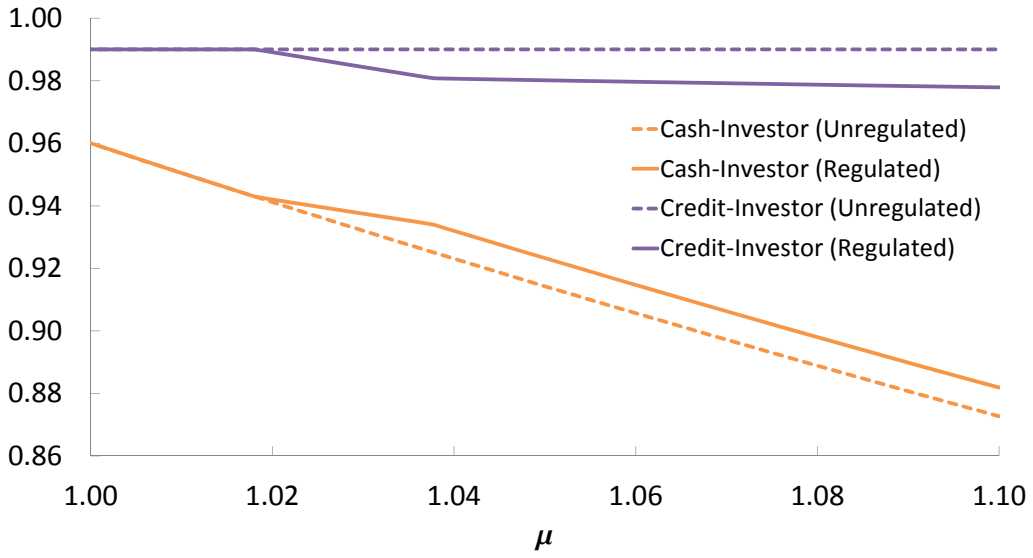
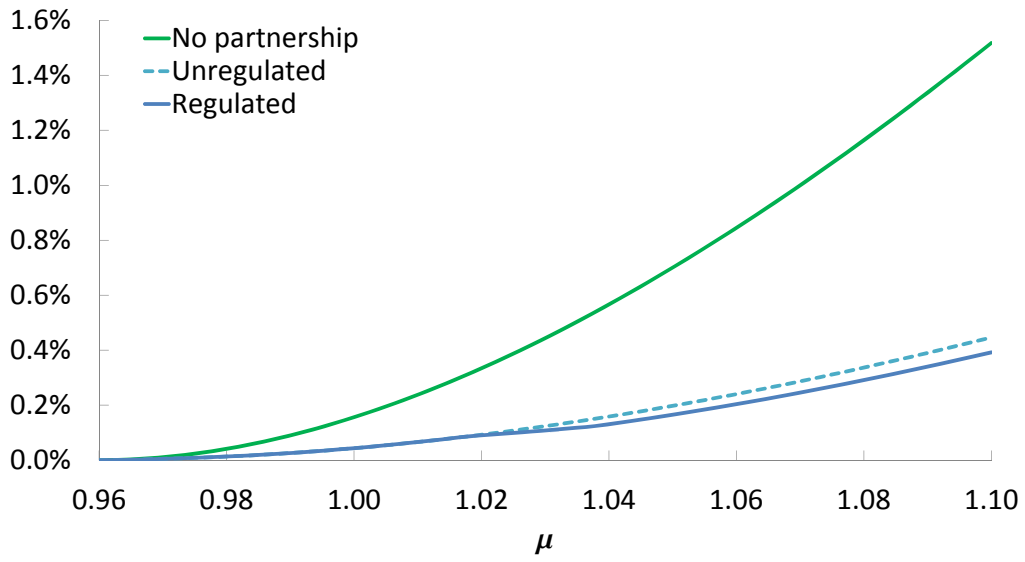
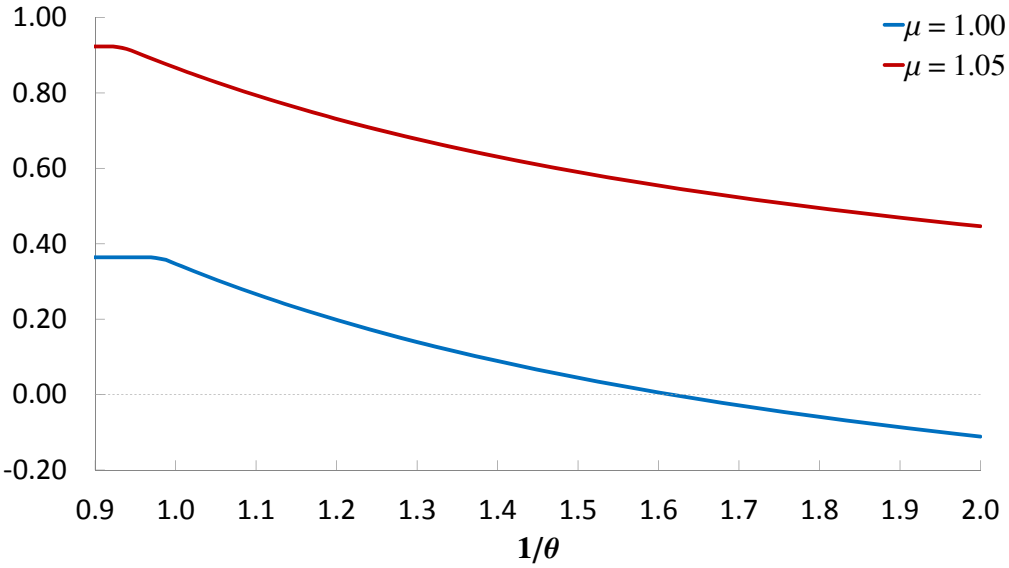


Figure 6: Welfare Cost of Inflation



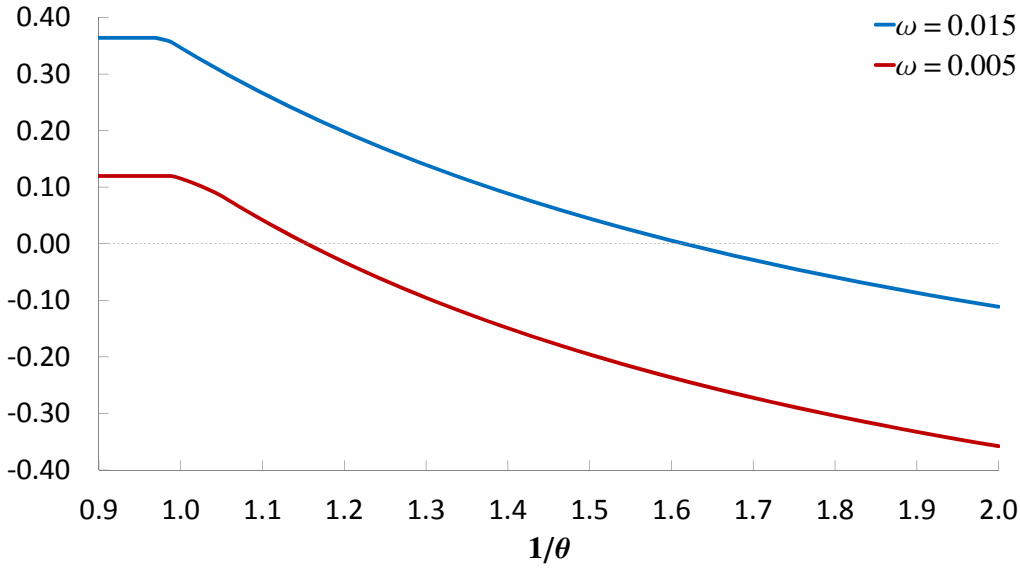
Note: lines display equivalent consumption compensation relative to the first-best.

Figure 7: Effect of Inflation on Default Incentives of Cash-Investor



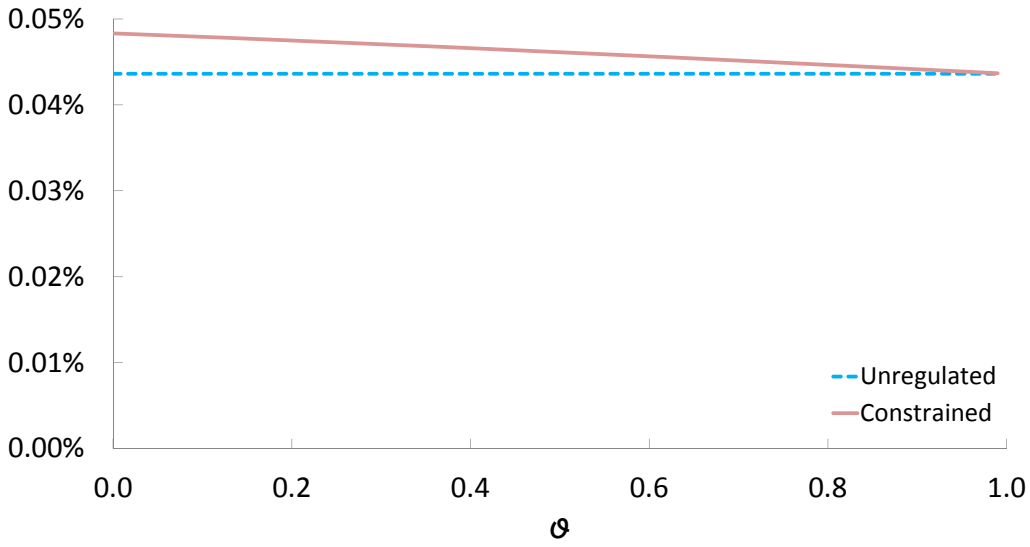
Note: lines display $\phi(a_2 - 2) - 2M/p + \beta W - V^d(0,0)$.

Figure 8: Effect of Dividends on Default Incentives of Cash-Investor



Note: lines display $\phi(a_2 - 2) - 2M/p + \beta W - V^d(0,0)$.

Figure 9: The Effects of Borrowed Securities' Liquidity on Welfare



Note: lines display equivalent consumption compensation relative to the first-best.