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Abstract

We develop a quantitative model of sovereign debt maturity choice and the term structure of bond yields in the presence of default risk. Our new approach allows the sovereign to choose an arbitrary maturity structure, and rationalizes various stylized facts about the debt maturity and the yield spread curves: first, the duration and the maturity of sovereign debt generally exceed one year, and they comove positively with the borrowing country's business cycle. Second, yield spread curves exhibit an upward slope in good times and are inverted during periods of credit-market stress. Third, the sovereign yield spread curve is non-linear and may become non-monotonic close to a default episode, an under-studied aspect of sovereign defaults. We also use our novel framework to explore how key country-specific and international financial market features, such as GDP volatility, the presence of sudden stops, the degree of impatience in the economy and the level of risk aversion affect debt duration, maturity, and the term structure of sovereign bond spreads, and we successfully confront the model with cross-country evidence.

JEL Classification: F34, F41, G15
Keywords: Crises, Default, Yield Curve, Spreads, Bond Duration.

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1 Introduction

Our paper proposes a new approach to studying the term structure of interest rate spreads and the maturity of sovereign bonds. Our new framework helps rationalize the maturity choice for sovereign debt and the pricing of this debt at each maturity. Consistent with debt pricing observed in the data, our model shows that when economic growth weakens (bad times), sovereign default risk and interest rate spreads over risk-free debt increase, while term spreads decline, often resulting in a negative-sloped yield spread curve for the borrowing country.

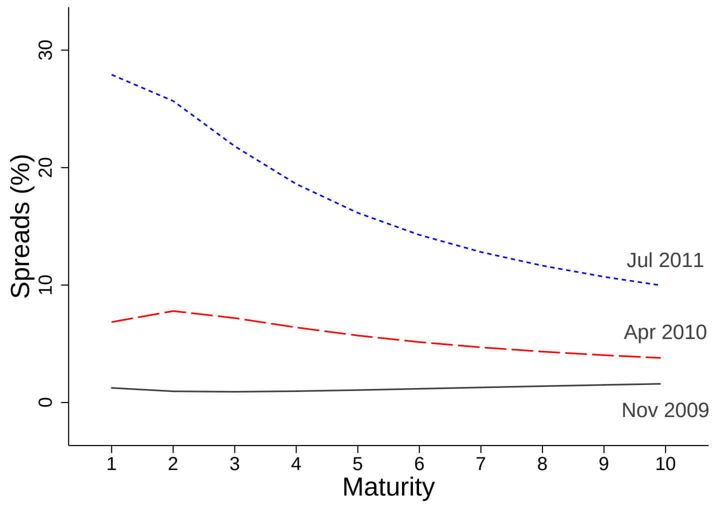
Our sovereign debt management setup also helps explain debt pricing behavior often observed during debt crisis that had not yet been addressed in the literature, in particular, the non-monotonic yield spread curves, as in the case of the Greek spread curve during 2009-2011, shown in Figure 1. The solid black line in the figure corresponds to November 2009, when Greece's troubles were mounting but their full severity had not yet been clearly perceived. Greek sovereign yield spreads over German bonds of comparable maturity were historically low, and the Greek curve was flat. The dashed red line corresponds to April 2010, after the credit rating agency Fitch downgraded Greece's credit rating to $BBB-$.[1] In April 2010 the curve was higher than in November 2009 and was hump-shaped, peaking at a 2-year maturity. The blue short-dashed line corresponds to July 2011, near the peak of the Greek debt crisis; the Fitch rating for Greece's credit was CCC, just one notch above default. The yield spread curve at the time was higher than ever and downward sloping.[2]

The literature on macroeconomics and the term structure of interest rates has long recognized both the relevance of non-linearities, i.e., curvature, in characterizing the yield curve (see, for instance, Ang and Piazzesi, 2003) and the importance of considering the whole term structure of interest rates for aggregate macroeconomic variables (Gurkaynak and Wright, 2012). The non-linearity and non-monotonicity of the spread curves shown in Figure 1 suggest that analyzing only the 1-year vs 10-year yield term spread, the focus of the current models of sovereign debt maturity and default, can leave out pricing information that is relevant to understand the dynamics of

[1] A $BBB-$ rating means that the capacity for payment of financial commitments is considered adequate, but adverse business or economic conditions are more likely to impair such payment capacity.

[2] A description of the changes in the Greek yield curve during this period can be found in Neely (2012).

Figure 1: Greek Yield Spread Curves



Source: See Appendix.

debt maturity, and it may even be misleading. The Greek yield spread curves for November 2009 and for April 2010 in particular, help illustrate this point. The slope exhibits a very different magnitude and even a different sign if one uses a term spread between 5 and 1 vs 5 and 2 years, potentially leading to opposite economic readings from the short end of the curve. The example highlights that to accurately reflect the pricing information provided by sovereign bonds, it is not sufficient to focus on the term spread for a single maturity pair, and it may be necessary to instead look at the spread curve. Interestingly, the sovereign default literature has long recognized the non-linear dynamics of sovereign debt pricing, but neither the non-linearity nor the non-monotonicity of the yield spread curve was addressed in this literature, where debt maturity models have discussed term yield spreads considering a one-period bond and a perpetuity with a counterfactual decay in coupon payments.

The behavior of yield spreads, debt duration and maturity over the business cycle using data for several economies is summarized in Table 1. The first two columns of the table show that 10 year and 1-year sovereign bond yield spreads are countercyclical. Additionally, bad times are linked to the shortening of the average debt maturity and, in particular, of duration. Such procyclical patterns for the maturity and duration of sovereign debt are shown in the last two columns of the table. Finally, the table also helps to highlight that even during bad times,

Table 1: Yield Spread, Duration, and Maturity over the Business Cycle

	1-year spread	10-year spread	Duration	Maturity
Overall	1.14	1.65	4.93	7.70
Good times	0.71	1.39	5.19	8.07
Bad times	1.40	2.04	4.81	7.13
Corr. with log(output)	-0.23	-0.35	0.47	0.12

Note: Averages across country-specific medians for several emerging markets. Good (bad) times are the years when the detrended log-output is above (below) 0. Duration is derived following the Macaulay definition. Details are given in the Appendix.

countries tend to sustain debt profiles with maturity and duration that significantly exceed one year.

Our new modeling approach provides a useful framework to assess quantitatively the role of four economic and financial market features that have been linked to sovereign debt maturity in the literature. The first factor is the volatility of GDP growth, where we find that a country with a more volatile process for income growth seemingly seeks to mitigate the higher yield spreads from higher default risk by both deleveraging and lowering its debt maturity. We document this type of debt management equilibrium outcome on the level and time profile of debt, in both our model and our empirical analysis.

The other factor is the possibility of a sudden stop. The presence of sudden stops as a long-standing feature of international credit markets has been well-documented in the literature (see for instance Edwards, 2007). Sudden stops may significantly shape the maturity profile of sovereign debt, as the possibility of a sudden withdrawal of funding that makes the sovereign unable to repay its immediate debt obligations creates a strong incentive for the sovereign to borrow at longer maturities. Other things constant, economies with less open capital accounts are less exposed to sudden stops. We show, using data for several economies, that countries with more open capital accounts have a longer maturity of their sovereign debt.

Another economic feature that can significantly affect the choice of sovereign debt maturity is the degree of risk aversion of the borrower. The higher the risk aversion, the higher the incentive of the sovereign to insure itself against movements in market interest rates or other shocks that may deteriorate the country's borrowing conditions, by fixing the terms of the debt contract. Interestingly, the higher risk aversion of the sovereign borrower may also be interpreted as an economy with a higher level of after-tax income inequality among heterogeneous households, as argued by Ferriere (2015), which also associated inequality to stronger incentives to default. Our model thus suggests that more unequal economies tend to borrow at longer maturities, a result that we also find in the data.

Finally, a fourth economic characteristic with implications for the choice of debt maturity by the sovereign is the degree of impatience in the economy, which may be proxied in the data by the fraction of youngsters in the overall population of the country. Our model predicts that a country with a higher level of patience should exhibit less outstanding debt and yield spreads, but longer maturity and duration, which we confirm in the country dataset. Intuitively, a more patient country can partially trade the lower average yield spreads from a lower outstanding debt balance, by extending its maturity profile to improve its rollover risk profile.

Our analysis borrows from different strands of the literature on sovereign debt, default, and debt maturity. Following the seminal work in international sovereign debt by Eaton and Gersovitz (1981), a large portion of the literature on quantitative models of sovereign debt default has used only one-period debt (Arellano, 2008; Aguiar and Gopinath, 2006; D'Erasmo, 2008; Yue, 2010; Mendoza and Yue, 2012, among others). Models of long debt duration, such as Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), feature exogenous maturity. In contrast, our quantitative model features endogenous sovereign debt maturity and repayment. The work of Arellano and Ramanarayanan (2012) also includes the choice of maturity, but there are several important differences.[3] First, debt in that model consists of a one-period bond and a long-term bond. The latter is a perpetuity bond with a duration defined by an exogenous parameter of coupon payment decay. The equilibrium duration in these models depends heavily on this parameter of the long-term bond, which should not be the case if duration were fully endogenous. Hatchondo et al. (2014) calibrate the decay parameter of the long-term bond to be 3.4% so that debt has an average duration of 4.2 years in their model simulations. Second, as pointed out in Bai et al. (2014), the perpetuity bond is restricted to have a front-loaded repayment schedule, which is at odds with the data but technically required to keep the discounted present value of repayments bounded. While the work by Bai et al. (2014) tackles the problem of a front-loaded

[3]See also Hatchondo and Martinez (2013) and Hatchondo et al. (2014).

repayment schedule, its framework does not consider debt dilution, i.e., that additional borrowing decreases the price of outstanding debt, a feature considered by the literature to be important to understand maturity choice and term yield spread (Hatchondo et al., 2014). Finally, the models of endogenous maturity with two bonds with fixed maturity, like Arellano and Ramanarayanan (2012), generate the 1-10 years term spread, but their spreads are not directly comparable with data because the 10-year spread is for a coupon bond, and in an empirical analysis of yield curves considers the price of zero-coupon bonds of different maturity. Additionally, these studies are silent about the pricing of intermediate tenors or the zero-coupon spread curve.

Cole and Kehoe (2000) study self-fulfilling debt crisis equilibria considering a different timing from Eaton and Gersovitz (1981), where the government issues new bonds before it decides to default within the period. Cole and Kehoe (2000) characterize crisis zones, i.e., the values and maturity structure of sovereign debt under which a financial crisis can arise due to a loss of confidence in the government. We model sudden stops as exogenous shocks and focus on the country's choice of maturity. In our framework, countries choose longer maturity to prevent the cost of sudden stops.

Aguiar and Amador (2013) also point to the optimality of strategies involving only short-term debt in the context of sovereign default risk models. Their study finds that during periods of deleveraging it is optimal for sovereigns to engage in strategies that use only one-period bonds. Aguiar and Amador (2013) argue that active shifts in the maturity structure by the sovereign may affect deleveraging incentives, and hence change the equilibrium price of long-term bonds. The price movements of long-term bonds will shrink the budget set of the sovereign, creating the incentive to use only one-period bonds during a period of deleveraging. Our model generates shorter maturity in the times of high risk of default, which resembles their mechanism.

The remainder of the paper proceeds as follows. In Section 2, we present the economic environment and the benchmark model, and we define the equilibrium. In Section 3, we perform a quantitative analysis of the benchmark model. In Section 4, we study quantitatively the role of key country-specific and international credit market features on the choice of maturity and the yield spread curve. In Section 5, we conclude.

2 Model

We consider a small open economy model with households, a benevolent government, and foreign lenders. The government trades in bonds of different maturities with risk-neutral foreign lenders. Debt contracts are not enforceable, as the government has the option to default on them. Default is costly for the country, and the foreign lenders charge a premium to account for the probability of not being paid back by the government.

2.1 Preferences and endowments

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. Each period the small open economy has a stochastic endowment y_t that follows a finite-state first-order Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition probability $\Pr\{y_{t+1} = y' \mid y_t = y\}$.

The benevolent government in the country maximizes the expected utility over consumption sequences of the representative household. The discount factor is $\beta \in (0, 1)$ and the risk aversion coefficient is $\gamma \geq 1$. The momentary utility function is

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}. \tag{1}$$

2.2 Debt markets

A country's bond portfolio is described by a fixed coupon payment, b, and its maturity, m. The portfolio may consist of any number of bonds.[4] In this section, we introduce the notation that we use to write the price of those bonds.

Given a country's portfolio, characterized by (b, m), the market value of a bond that pays $\tilde{b}$ for n periods is

$$\text{bond value} = \tilde{b}q(y, b, m; n).$$

As default occurs on the entire portfolio, this bond price, q, depends on the portfolio characteristics $\{b, m\}$. When the portfolio maturity, m, coincides with the bond maturity, n, the price q

[4]Whether the portfolio is composed of one bond or several bonds is irrelevant in this framework.

represents the unit price of the portfolio. The market value of the portfolio can be written as

$$\text{portfolio value} = bq(y, b, m; n = m).$$

To illustrate the notation further, consider the following example. Suppose that the country has the portfolio:

$$\text{portfolio} = \{b, b, b, b\}.$$

Then,

$$\text{portfolio value} = b \times q(y, b, 4; 4).$$

Given that the country is holding that portfolio, we can also compute the value of a promise to pay only a part of the coupon, $\underline{b}$, for the following four periods.[5] That is,

$$\text{holding part of the coupon} = \{\underline{b}, \underline{b}, \underline{b}, \underline{b}\} \rightarrow \text{value} = \underline{b} \times q(y, b, 4; 4).$$

This pricing is possible because q is the price per unit of the coupon. More importantly, this notation also allows us to price a subset of the portfolio that consists of payments of $\underline{b}$ for only the first three periods.[6] That is,

$$\text{subset of portfolio} = \{\underline{b}, \underline{b}, \underline{b}, 0\} \rightarrow \text{value} = \underline{b} \times q(y, b, 4; 3).$$

Similarly, we can obtain the market value of a (zero coupon) bond that pays $\underline{b}$ in the fourth period given that the country's portfolio is the same than above. That is,

$$\text{four-period zero coupon bond} = \{0, 0, 0, \underline{b}\} \rightarrow \text{value} = \underline{b} \times \left[q(y, b, 4; 4) - q(y, b, 4; 3)\right].$$

These prices provide very useful notation for the country's choice of maturity.

[5]For instance, this may be the case because the country has two outstanding bonds, one paying $\underline{b}$ and another one paying $b - \underline{b}$.

[6]For instance, this may be the case because the country has three bonds outstanding, one for $\{\underline{b}, \underline{b}, \underline{b}, 0\}$, other for $\{b - \underline{b}, b - \underline{b}, b - \underline{b}, 0\}$, and a zero-coupon bond $\{0, 0, 0, b\}$.

2.3 Decision problem

A country with an outstanding amount of assets, b (debt if $b < 0$), has two actions to choose from. The first option is to make the payment (G, for good credit status). The second option for the country is to default (D, for default). The country's choice to either pay or default is represented by

$$V(y, a, b, m) = \max \left[V^G(y, a, b, m), V^D(y) \right], \tag{2}$$

where the policy function $D(y, a, b, m)$ is 1 if default is preferred and 0 otherwise. If the country does not receive a sudden stop shock ($a = 1$), and decides not to default, it selects the maturity of the new portfolio, m', and the debt level, b'. The value in this case is:

$$V^G(y, 1, b, m) = \max_{b', m'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y', a'|y} V(y', a', b', m')$$

subject to

$$c = y + b - \underbrace{q(y, b', m'; m')b'}_{\text{issue new debt}} + \underbrace{q(y, b', m'; m-1)b}_{\text{retire old debt}}$$

$$b' \in \mathbb{R}_-, m' \in \mathbf{M}(m).$$

In contrast, a country that receives a sudden stop shock and therefore has no access to credit markets ($a = 0$), may pay but cannot issue new debt:

$$V^G(y, 0, b, m) = \frac{(y + b)^{1-\gamma}}{1-\gamma} + \beta E_{y', a'|y} V(y', a', b, m-1).$$

The policy functions for the amount of debt and the maturity are $B(y, a, b, m)$ and $M(y, a, b, m)$. Notice that when a country makes only its debt payment, the policies are $B(y, a, b, m) = b$ and $M(y, a, b, m) = m - 1$. It then follows that if $a = 0$, we must have $B(y, 0, b, m) = b$ and $M(y, 0, b, m) = m - 1$.

Default brings immediate financial autarky for a stochastic number of periods and a direct output loss to the defaulting country. Formally, the value of default is:

$$V^D(y) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y}\left[(1-\lambda)V^D(y') + \lambda V(y',a',0,1)\right].$$

2.4 Equilibrium

Given the world interest rate r, the price of the country's debt must be consistent with zero expected discounted profits. The price of a bond of maturity $n > 0$ of a country with income y, new debt $-b'$, and maturity $m' > 0$ can be represented by

$$q(y,b',m';n) =$$
$$\frac{E_{y',a'|y}}{1+r}\{(1 - D(y',a',b',m'))[1 + q(B(y',a',b',m'),y',M(y',a',b',m');n-1)]\}.$$

3 Quantitative analysis

We solve the model numerically, calibrating the parameters based on the literature and the available data. In this section, we describe the calibration strategy and we compare the results with the data. As we focus on sovereign default risk, we consider sovereign bond yield spreads over risk-free debt instruments, where the spread at each maturity is the difference between the yield on a zero-coupon bond with that maturity and default risk, and the yield on a bond with the same characteristics but with negligible default risk. We present the details of the model computations in the Appendix.

3.1 Calibration

We set the maximum possible debt maturity to 15 years, significantly larger than the maturity observed for most emerging markets.[7] Each year, the country can change the maturity by at

[7]Our results are robust to allowing for longer maximum maturities. In the simulations for our benchmark setup, the frequency of observations at maturity 15 years is 2.9×10^{-5} , and that for maturity 14 is 2.9×10^{-4}.

most one year: i.e., $\mathbf{M}(m) = \{m-1, m, m+1\}$.[8]

We use the standard function for the income loss in case of default:

$$\Phi(y) = \begin{cases} 0, & \text{if } y < \phi \\ \phi, & \text{otherwise.} \end{cases}$$

We follow Arellano and Ramanarayanan (2012) in setting most of the model parameters, which are summarized in Table 2. In particular, the yearly risk-free interest rate is set to 0.032, the probability of redemption if the country is excluded from the financial markets is set to 0.17, the standard deviation of the income shock is set to 0.017, and the persistence is set to 0.9. Consistent with the estimates of Edwards (2007) and Bianchi et al. (2013), we consider a sudden stop probability of 10 percent.

Table 2: Calibrated Parameters

Parameter	Value	Basis
Interest rate, r	0.032	Arellano and Ramanarayanan (2012)
Redemption prob., λ	0.17	"
Income shock std, σ_y	0.017	"
Income autocorrelation, ρ_y	0.90	"
Sudden stop prob., p_s	0.10	Edwards (2007) and Bianchi et al. (2013)
Discount factor, β	0.89	Debt-to-output ratio = 42%
Cost of default, ϕ	0.90	Default rate = 2%
Risk aversion, γ	5.00	$\rho(\log(c), \log(y)) = 0.83$

The remaining parameters are calibrated to match specific targets. In particular, we calibrate the discount factor, β, the threshold of income in the default cost function, ϕ, and the risk aversion parameter, γ, to match three important moments in the data: the debt-to-output ratio, the default rate, and the correlation of output with consumption. Table 3 shows that the model reproduces the targeted moments closely.

[8] It saves computing time and is consistent with observed annual variations in maturity. Our results are robust to considering $\mathbf{M}(n) = \{m-2, m-1, m, m+1, m+2\}$.

Table 3: Fit of Targeted Moments

	Target	Model
Default (%)	2.00	1.76
Face value of debt / Output, nb/y	0.42	0.48
$\rho(\log(c), \log(y))$	0.83	0.86

3.2 Results

Our model can closely match several key empirical stylized facts. As illustrated in Table 4, in addition to capturing the moments usually discussed in the literature, such as the higher volatility of consumption than output, our model statistics mimic extremely well the statistics for the average sovereign debt duration and maturity from the data, as well as their cyclical behavior.[9]

Table 4: Maturity and Duration

	Data	Model
Duration	4.93	4.07
Duration (good times)	5.19	4.36
Duration (bad times)	4.81	3.73
Maturity	7.70	7.88
Maturity (good times)	8.07	8.25
Maturity (bad times)	7.13	7.28
$\rho(n, log(y))$	0.12	0.29
$\rho(dur, log(y))$	0.47	0.39
$\sigma(log(c))/\sigma(log(y))$	1.22	1.10
$\sigma(log(TB/y))/\sigma(log(y))$	0.70	0.56
$\rho(TB/y, log(y))$	-0.02	0.10

Our framework is also able to capture the dynamics of yield spreads for different bond maturities over the business cycle. As summarized in Table 5, yield spreads are countercyclical, and spreads for short term bonds are lower than those for longer-term instruments. In addition, the term spread declines in bad times, reflecting the flattening of the curve observed during bad times, i.e., during periods of higher economic and financial stress.

[9]Consistent with the data, we use the Macaulay definition to derive the duration of sovereign debt. See the Appendix for definitions of debt duration and yield spreads.

Table 5: Sovereign Yield Spreads

	Data	Model
1-year spread	1.14	0.10
1-year spread (good times)	0.71	0.01
1-year spread (bad times)	1.40	1.38
10-year spread	1.65	2.00
10-year spread (good times)	1.39	1.51
10-year spread (bad times)	2.04	2.93
$\rho(1YS, log(y))$	-0.23	-0.32
$\rho(10YS, log(y))$	-0.35	-0.45

To understand why default risk may explain changes in the yield spread curve, notice that the yield of a zero-coupon bond that pays in j periods is,

$$i_t^0(j) = \frac{1+r}{(\prod_{i=1}^{j} P_{t+i})^{1/j}},$$

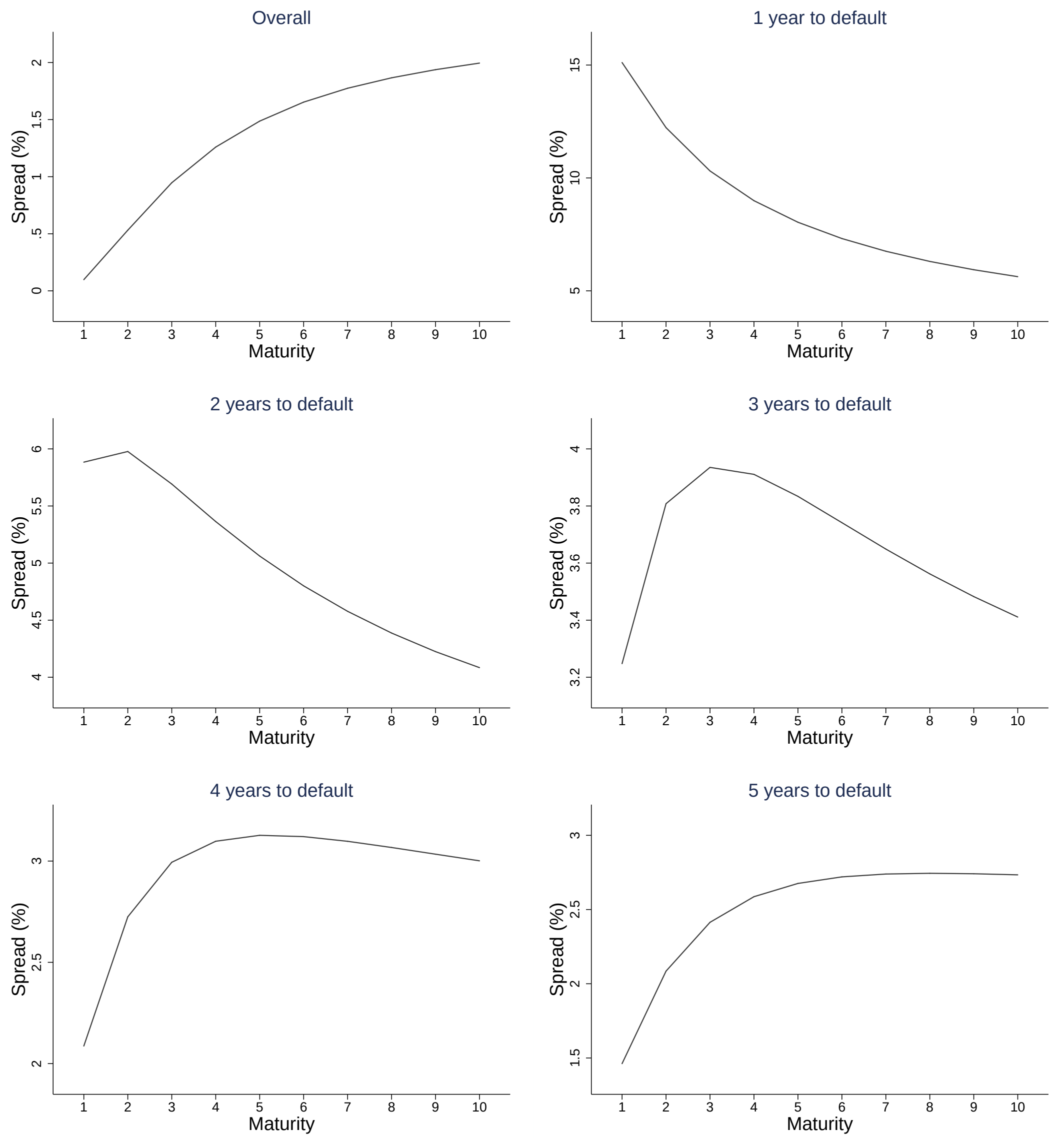
where $P_{t+1}, P_{t+2}, ..., P_{t+j}$ denote the repayment probabilities in $1, 2, ..., j$ periods ahead and r is the risk-free interest rate.[10] As a result, depending on the future repayment probabilities P_{t+i}, the yield curve may be increasing, decreasing, or non-monotone, following the patterns observed in the data, as illustrated earlier in Figure 1 for the case of Greece. In our model, these probabilities are endogenous, and depend on the country's income, as well as on the level, the maturity and the duration of its debt.

In line with the data, the yield spread curves generated by the model are typically upward sloped and concave, as illustrated in the top left panel of Figure 2. Additionally, the data show that the shape of the curve for a country changes significantly, which is consistent with the country experiencing time-varying debt repayment probabilities at different maturities. Interestingly, our model economy can capture very well the changes in the yield spread curve found in the data as countries approach default, as illustrated by the other panels of Figure 2. The top right panel shows that yields are high and decreasing with maturity in the year before default episodes. The middle panel shows that a few years before a default the yield spread curve becomes non-monotone, peaking at the period in which default is most likely, as already described for the case

[10] The derivation of this formula is presented in the Appendix.

of the Greek yield spread curve during 2011. Finally, the bottom panel shows that when default is further away, the curve is increasing and concave. The concavity and non-monotonicity of the yield spread curve in the presence of sovereign default risk are stylized facts not addressed before in the literature.

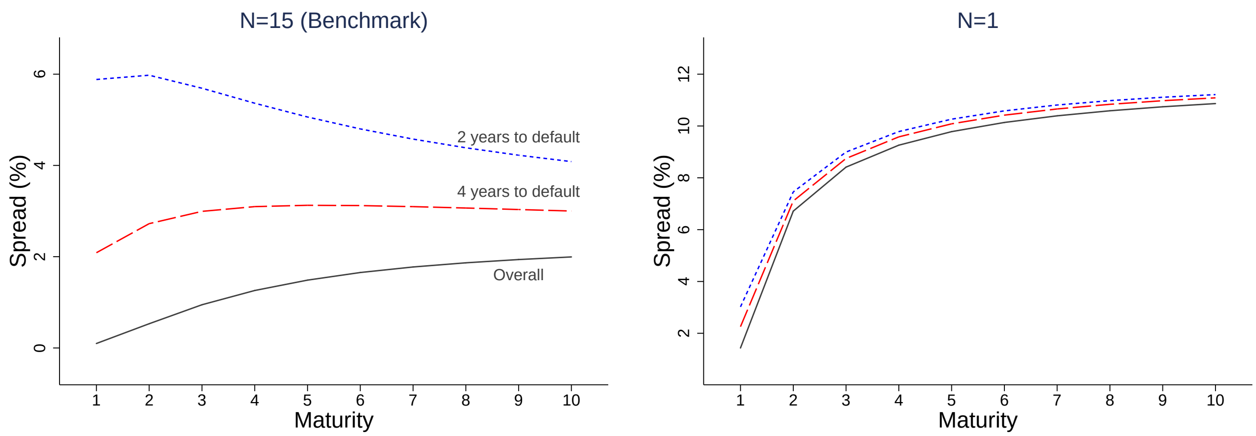
Figure 2: Sovereign Yield Spread Curves



Note: Averages across the median spreads of each sample.

One can generate the yield spread curves implicitly defined in previous sovereign default models in the same way we computed the curves in our model, even when some of those bond

Figure 3: Sovereign Yield Spread Curves, Alternative Models

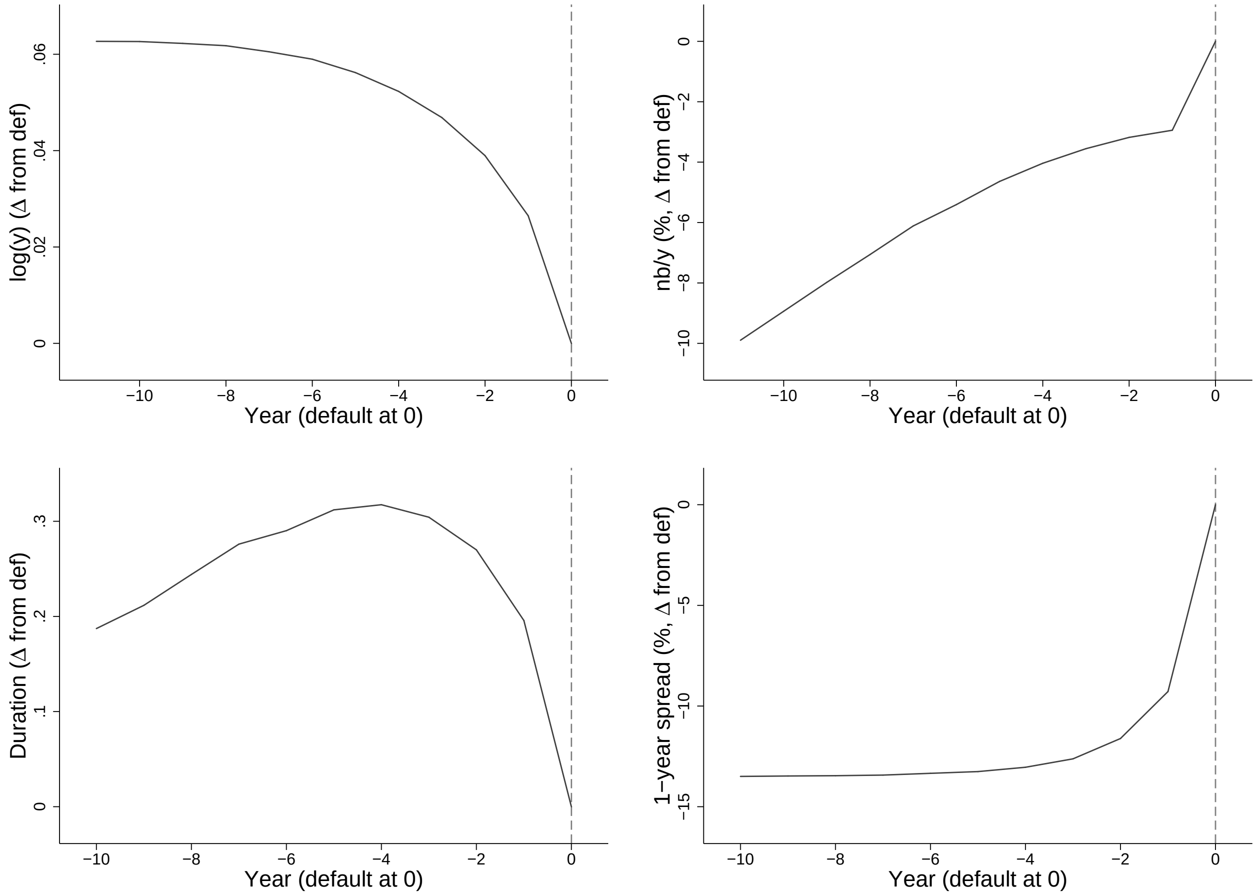


Note: Averages across the median spreads of each sample. For the $N = 1$ model, the benchmark calibration gives no default in equilibrium, hence we recalibrated β to get a default rate comparable with our benchmark setting. The debt-to-output in that alternative model is about 20 percent.

maturities are not allowed in those other models. In Figure 3, we compare the yield curves implied by our benchmark model with those that would arise in a model in which the debt maturity is forced to be equal to one in every period (Arellano, 2008; Aguiar and Gopinath, 2006). Notice that, in contrast to our benchmark model, the yield spread curve implied by the allocations in the model with $N = 1$ is very steep, and it does not change much across states of the world.

To better understand the dynamics of the yield spread curve, it is useful to look at the evolution of the main debt and economic activity indicators as the economy approaches a debt crisis episode. Figure 4 summarizes the behavior of several key variables according to the model in the ten periods leading to a default, with the values for the variables measuring deviations from the average values at default. As the top left panel on output growth shows, in the years before a crisis, economic activity deteriorates at an increasing pace, especially in the three years preceding the event. The decline in output contributes to an increase in the value of the debt-to-GDP ratio (in the upper right panel), which accelerates just prior to the crisis. As the debt burden increases, initially the economy extends the duration of its debt, but as the debt burden weighs further, the incentives for debt dilution becomes stronger and starting about three years before the crisis the country increasingly reduces its debt duration (bottom left panel), which

Figure 4: Behavior around Default



Note: For these plots, we first get the within-sample median across default episodes of a statistic $j = \{-10, ..., 0\}$ years before the default. Then, we take the average across samples for every j, and plot the deviations from the average at default, i.e. $j = 0$.

helps fuel the observed dramatic rise in short-term yield spreads, especially during those last three years preceding the default, as illustrated in the lower right panel.

4 The cross-section of maturity choices

In this section, we study the cross section of sovereign debt maturity. We highlight four important factors that may be affecting the debt maturity choices of countries, and we show that our model offers an explanation for these findings.

We collected data on several countries' macroeconomic characteristics that can be mapped to different parameters in our model:

(i) GDP volatility: we study variations in debt maturity and other variables in our model as we change income volatility. We use the standard deviation of GDP growth as the counterpart in the data.

(ii) Sudden stop probability: as the empirical counterpart of the model's sudden stop probability, we consider the number of sudden stop episodes in the country documented over the most recent 10 years. For this purpose, we follow the list of sudden stops episodes in Comelli (2015).

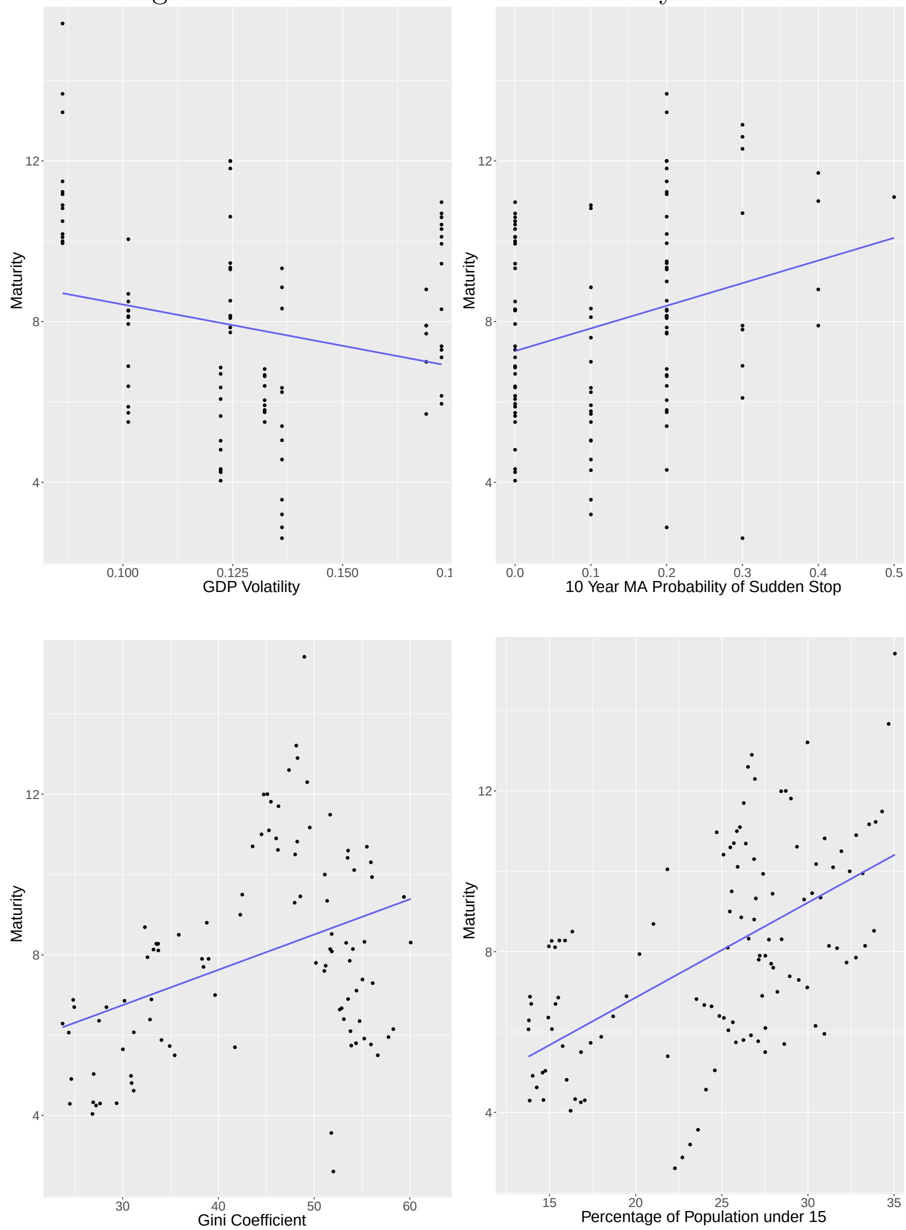
(iii) Inequality-Risk aversion: Ferriere (2015) argues that higher risk aversion of the sovereign borrower may be interpreted as a higher level of income inequality in an economy populated by heterogeneous hand-to-mouth consumers.[11] Therefore, we use data on income inequality in the borrowing country as a proxy for a country's risk aversion.

(iv) Discount factor: the share of the population with age under 15 years old is used as a proxy for the model's discount factor, β, where a younger population will place a larger weight in the future than an older population. That is, a larger proportion of young individuals in the economy is associated with a larger value of β.

Figure 5 shows the relationship observed in the data between the four variables introduced above and the sovereign debt maturity. The top left panel shows that countries with higher income volatility have lower maturity. The relationship between the probability of a sudden stop and debt maturity is positive, as displayed in the top right panel. The two bottom panels show a positive relationship between income inequality and debt maturity, and between the share of population under 15 years old and debt maturity.

[11] Ferriere (2015) associates the higher inequality to stronger incentives to default, but there is no analysis of maturity choice.

Figure 5: The Cross-Section of Maturity Choices



In these figures, we are not controlling for other factors that may also be affecting sovereign debt maturity. However, the Appendix Table 9 shows that the same relationships hold in regressions with other control variables, including year and regions fixed effects. Note that since the effect on maturity operates through other macroeconomic variables (for instance, higher risk aversion implies a higher risk of default, less debt, and higher maturity), regressions controlling for other macroeconomic variables would not capture as well the mechanism operating in the model.

Next, as illustrated in Table 6, we analyze the changes of the four macroeconomic variables in the context of our model. First, we show the results with lower and higher variance of the endowment shock. We find that other things equal, higher income volatility implies lower maturity, higher spreads and default, and a lower level of debt. An increase in the volatility of income for a given level of debt and its maturity would imply a sharp increase in the probability of default and thus in the cost of debt, i.e., the yield spreads. To limit the rise in borrowing costs, in equilibrium the country reduces the amount of debt and its maturity.

Second, Table 6 illustrates quantitatively how changing the probability of a sudden stop affects the key statistics. The maturities across good and bad times lengthen substantially as the probability of a sudden stop increases. The model also exhibits larger pro-cyclicality of both debt duration and maturity, as observed in the data. In addition, the model captures the pro-cyclicality of the term spread generally found in the data, as well as the cyclicality of most macroeconomic variables described in the table, especially with a 10% probability of a sudden stop. The recent episode of Ireland helps illustrate these relationships. The likelihood of a sudden stop in an environment of increased sovereign default risk is relevant to explain the recent debt dynamics of peripheral euro-area economies. Ireland exited its 2010 international bailout at the end of 2013 without any European Union or International Monetary Fund precautionary credit line as an insurance policy. The country's sole reliance on market funding without such a safety net, together with tight budget conditions, provided the Irish government with strong incentives to borrow short term. However, policymakers opted to issue long-term debt. Despite the issuance of long-term bonds, the 10-2 years term spread for Ireland declined about 600 basis points to 350 basis points from 2014Q1 to 2015Q1. During this period, the higher risk of a credit event in Greece increased the likelihood of a sharp fall in short-term funding to Ireland. Issuing more long maturity debt helped Ireland to reduce its rollover risk.

Third, Table 6 also shows the role of risk aversion. As mentioned earlier, an increase in risk aversion can also be interpreted as a rise in inequality if the economy were populated by heterogeneous hand-to-mouth consumers (Ferriere, 2015). If the country is more risk averse, it chooses a longer maturity to take advantage of the hedging properties of long debt maturity

Table 6: The Role of Macroeconomic Factors

	Benchmark	Income shock sd		Pr. sudden stop		Risk aversion		Discount factor	
σ_y, std. dev. y	0.017	0.016	0.018	0.017	0.017	0.017	0.017	0.017	0.017
p_s, sudden stop pr.	10%	10%	10%	5%	15%	10%	10%	10%	10%
γ, risk aversion	5	5	5	5	5	4	6	5	5
β, discount factor	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.85	0.93
Duration	4.07	4.27	3.96	3.57	4.40	3.59	4.54	3.37	5.44
Duration (good times)	4.36	4.47	4.33	3.85	4.74	3.84	4.92	3.65	5.68
Duration (bad times)	3.73	4.04	3.51	3.31	4.03	3.32	4.12	3.12	5.20
Maturity	7.88	8.07	7.72	6.64	8.76	6.76	9.00	6.26	10.81
Maturity (good times)	8.25	8.49	8.23	7.02	9.18	7.07	9.55	6.82	11.11
Maturity (bad times)	7.28	7.83	6.96	6.13	8.09	6.19	8.31	5.94	10.47
$\rho(n, log(y))$	0.29	0.25	0.33	0.30	0.29	0.30	0.30	0.30	0.18
$\rho(dur, log(y))$	0.39	0.32	0.45	0.39	0.39	0.38	0.41	0.38	0.28
1-year spread	0.10	0.09	0.13	0.13	0.08	0.16	0.06	0.40	0.00
1-year spr. (good times)	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.09	0.00
1-year spr. (bad times)	1.38	0.89	2.09	1.48	1.25	1.47	1.30	2.56	0.19
10-year spread	2.00	1.43	2.65	1.85	2.04	1.91	2.04	3.24	0.64
10-year spr. (good times)	1.51	1.14	1.93	1.38	1.54	1.49	1.49	2.76	0.30
10-year spr. (bad times)	2.93	1.95	4.02	2.69	2.99	2.63	3.16	4.21	1.26
$\rho(1YS, log(y))$	-0.32	-0.30	-0.33	-0.34	-0.30	-0.33	-0.30	-0.28	-0.35
$\rho(10YS, log(y))$	-0.45	-0.40	-0.49	-0.51	-0.42	-0.47	-0.44	-0.38	-0.61
$\sigma(log(c))/\sigma(log(y))$	1.10	1.21	1.04	1.10	1.10	1.15	1.06	1.15	1.01
$\sigma(log(TB/y))/\sigma(log(y))$	0.56	0.76	0.43	0.53	0.58	0.62	0.52	0.69	0.33
$\rho(TB/y, log(y))$	0.10	0.08	0.12	0.07	0.11	0.06	0.14	0.12	0.15
Default (%)	1.76	1.27	2.23	1.65	1.78	1.67	1.81	2.45	0.81
Face val. of debt / y	0.48	0.56	0.42	0.45	0.49	0.45	0.50	0.48	0.39
$\rho(log(c), log(y))$	0.86	0.77	0.91	0.87	0.85	0.84	0.87	0.80	0.94

Note: For all the columns, the same parameters are used as in the benchmark (see Table 2).

(Arellano and Ramanarayanan, 2012).

Finally, the last two columns of Table 6 show the effect of changing the discount factor. As expected, more patient countries have less outstanding debt. Without any changes in debt maturity, this would imply a very drastic decline in yield spreads. The country must trade-off some of the decline in the average yield spread by choosing a longer maturity that reduces the risk of default due to a sudden stop. In our quantitative analysis, this strategic reduction in debt rollover risk is reflected in the more than 10-fold decline in the 1-year spread in bad times from 256 basis points to 19 basis points, while the 10-year spread for the same state of nature also falls but proportionately less, from 421 basis points to 126 basis points.

Overall, the results show that sovereign debt maturity and duration are determined by the trade-off between the benefits of long-term debt via the hedging of changes in the rollover costs of the debt, and its costs via higher spreads due to debt dilution. Changes in economic conditions or country characteristics that increase the incentives to hedge, such as increasing the risk aversion and the probability of sudden stops, imply a longer equilibrium debt maturity. Changes that tend to reduce sovereign borrowing costs, such as increasing the discount factor, lead to an increase in equilibrium debt maturity because it becomes relatively cheaper for the economy to borrow longer term. In contrast, changes that tend to increase sovereign borrowing costs, such as higher income volatility, lead to a reduction in debt maturity to partially offset the increase in yield spreads.

5 Conclusions

We develop a novel approach to handling the choice of sovereign debt maturity in a tractable manner in the presence of default risk. In our framework, the sovereign chooses the maturity structure instead of constraining it to a combination of a one-period bond and a perpetuity bond, as done in the literature, thus providing a more flexible and realistic portrayal of debt maturity relative to other models. Our study also rationalizes and quantitatively mimics key properties of the sovereign debt maturity, duration and yield spread curve found in the data: first, the duration and the maturity of sovereign debt generally exceed one year and comove positively with the borrowing country's business cycle. Second, yield spread curves show an upward slope in good times and are inverted during periods of credit-market stress. Third, the curve is non-linear and may become non-monotonic close to a default episode.

Our novel approach to the quantitative modeling of debt maturity choice also helps assess the role of country-specific and international financial market features in driving the stylized facts described above. In particular, the presence of sudden stops, a country's risk aversion, income volatility and degree of impatience, have a large impact on countries' maturity choice.

Finally, our model also provides an excellent framework to explore other aspects of sovereign debt that are not addressed in this paper. For instance, one quantitatively assesses alternative debt rescheduling mechanisms by extending our baseline model to incorporate debt restructurings in default episodes. We leave this type of extensions for future research.

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A Appendix

A.1 Computation

Approximating the solution using additional shocks

It is well-known that numerical solutions of models of defaultable sovereign debt are prone to convergence issues. In this paper, we overcome such problems by introducing two idiosyncratic shocks to our economy, both with small variances. First, there is an i.i.d. income shock that adds to the persistent component, as highlighted in Chatterjee and Eyigungor (2012). This component of income, x_t, has a truncated normal distribution that is i.i.d. with mean 0 and variance σ_x and the bounds are given by $\underline{x}$ and $\overline{x}$.

In addition to the transitory income shocks, there is a stochastic value shifter, χ_t, that affects the value of increasing the maturity of the portfolio. This variable is also i.i.d., normally distributed with mean 0 and variance σ_χ.

Decision problem and timing

At the beginning of each period, the persistent component of the endowment, y, the sudden stop shock and the value shifter, χ, are realized. If the country did not receive a sudden stop shock, it selects maturity of the new portfolio, m', that will be in effect only if the country ends up not defaulting by the end of the period. The value at that point is:

$$V(y,1,b,m,\chi) = \max_{m' \in \{\max\{m-1,1\},m,\min\{m+1,N\}\}} \{\hat{V}(y,b,m,m') + \mathbb{I}(m'=m)\chi + \mathbb{I}(m'=m+1)2\chi\}.$$

where the value of committing (in case of not defaulting) to $m' \in \{\min\{m+1,N\},m,\max\{m-1,1\}\}$ is given by

$$\hat{V}(y,b,m,m') = E_x\left[\max\left(V^G(y,b,m,m',x), V^D(y)\right)\right].$$

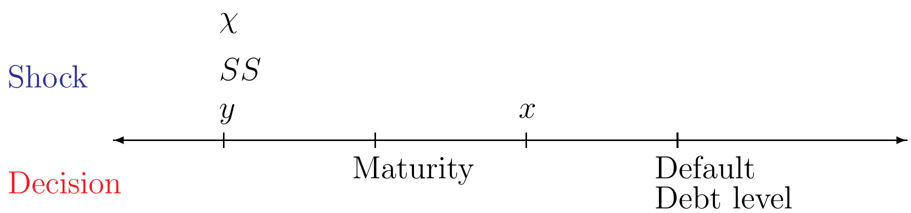
After the decision on the maturity, the idiosyncratic component of the endowment, x, is realized. Then the country decides on whether or not to default on the existing debt, and in case of not defaulting, on the debt level, b'. Given the decision on the maturity of the portfolio, and the transitory income shock, the value is:

$$V^G(y,b,m,m',x) = \max_{b'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y} V(y',a',b',m',\chi')$$

subject to

$$c = y + x + b - q(y,b',m';m')b' + q(y,b',m';m-1)b$$
$$b' \in \mathbb{R}_-.$$

Figure 6: Timing of Shocks and Decisions within a Period



A default brings financial autarky for a stochastic number of periods and a direct output loss to the defaulting country. Formally, the value of default is:

$$V^D(y) = \frac{(y - \Phi(y) + \underline{x})^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y}\left[(1-\lambda)V^D(y') + \lambda V(y', a', 0, 1, \chi')\right].$$

If the country has received the sudden stop shock in the beginning of the period, it has no decisions to make until x is realized. Then it has two options to choose from. First is to pay the debt promises and continue with the existing portfolio, and the second option is to default.

$$V(y, 0, b, m, \chi) = E_x\left[\max\left(V^P(y, b, m, x), V^D(y)\right)\right].$$

$$V^P(y, b, m, x) = \frac{(y + x + b)^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y} V(y', a', b, m-1, \chi')$$

if $m > 1$ and,

$$V^P(y, b, 1, x) = \frac{(y + b)^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y} V(y', a', 0, 1, \chi').$$

Figure 6 illustrates the timing of shocks and decisions in a given period. Notice that if the two additional shocks are degenerate, i.e. with zero variance, then the model we are actually solving coincides with the model described in Section 2.

Equilibrium

Given the world interest rate r, the price of the country's debt must be consistent with zero expected discounted profits. The price of a bond of maturity $n > 0$ in a country with income y, new debt $-b'$, and maturity $m' > 0$ can be represented by

$$q(y, b', m'; n) = \frac{E_{y',a',\chi',x'|y}}{1+r}\{(1-D)[1 + q(y', B, M; n-1)]\},$$

where we denote $M(y', b', m', \chi', a')$ by M, $B(y', b', m', M, a', x')$ by B, and $D(y', b', m', M, a', x')$ by D for simplicity.

Solution algorithm

We start an iteration solving the optimal debt levels given guesses for (i) the price function and (ii) the expected values for next period value function (for both good and bad financial standing). For every component j on the persistent income shock grid $\mathbf{y}$, k on the transitory income shock grid $\mathbf{x}$, i on the debt grid $\mathbf{b}$, and for every maturity level m, we solve the optimal debt level of next period, for each possible maturity choice, m'. In finding the optimal debt level, we use a discrete grid search method, and we apply the divide-and-conquer algorithm highlighted by Gordon and Qiu (2015) to limit the boundaries of the debt selection. Chatterjee and Eyigungor (2012) have shown that optimal debt level is increasing with respect to the transitory income shock in their framework, and it is also the case in ours. In turn, we use this monotonicity and the optimal debt decisions for each $(\mathbf{y}(j), \mathbf{b}(i), m, m', \mathbf{x}(k))$ to get the threshold levels of the transitory income shock that correspond to each different choice of the debt level. Formally, if the points $\{\tilde{b}_1, .., \tilde{b}_h\}$ on the debt grid is chosen for $(\mathbf{y}(j), \mathbf{b}(i), m, m')$, we find the $h-1$ threshold levels of the transitory shock, $\{\bar{x}_1, .., \bar{x}_{h-1}\}$ such that $\tilde{b}_1$ is optimal for all $x \leq \bar{x}_1$, $\tilde{b}_2$ is optimal for all $x \in (\bar{x}_1, x_2]$, ... , and $\tilde{b}_h$ is optimal for all $x > \bar{x}_{h-1}$. In addition to the optimal debt level, we also find the threshold level $\hat{x}_{def}$ that gives the indifference between defaulting or not, using the fact that $V^G(y, b, m, m', x)$ is increasing in x and $V^D(y)$ does not depend on it. Then, we get the value before the realization of the transitory income shock, $\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m')$ using these thresholds. Using these values, we then get the relevant thresholds for the maturity choice:[12]

$$\bar{\chi}_+ \equiv \max\{\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1), \\ (\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m-1) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1))/2\}$$

$$\bar{\chi}_{-,1} \equiv (\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m))/2$$

$$\bar{\chi}_{-,2} \equiv \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m-1) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m).$$

Then the probabilities of the sovereign increasing the maturity by one, keeping it constant, and decreasing it by one, are $P_+ \equiv 1 - \Phi(\bar{\chi}_+)$, $P_0 \equiv \max\{0, \Phi(\bar{\chi}_{-,1}) - \Phi(\bar{\chi}_{-,2})\}$, and $P_- \equiv 1 - P_+ - P_-$. Hence, the value of the country that has not received a sudden stop shock is given by:

$$\begin{aligned} E_\chi\left(V(y, 1, b, m, \chi)\right) = P_+ \times &\left[\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1) + E\left(\chi | \chi > \bar{\chi}_+\right)\right] \\ + P_0 \times &\left[\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m) + E\left(\chi | \bar{\chi}_{-,1} > \chi > \bar{\chi}_{-,2}\right)\right] \\ + P_- \times &\,\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m-1). \end{aligned}$$

We follow the same approach to find the default policy and the expected value under the event that the country receives a sudden stop, except that the only threshold level to find in that case is the one of default threshold for x.

[12] For simplicity, suppose here that $1 < m < N$.

Then, using the thresholds of χ, above which the country chooses to increase the maturity, the threshold levels of x that determines the optimal debt levels and the default policies of the country we derive the price matrix q.

The algorithm described so far is about solving for one iteration of the model. We start the iterations with a guess $W_0(y, b, m)$ for the expected value of being included in the financial markets; a guess $W_0^d(y)$ for the value of being excluded, i.e. $V(y)$; and a guess $Q_0(y, b, m; n)$ for the price function, i.e. $q(y, b, m; n)$. Using these guesses, we solve the policy, value and price functions in the first iterations, which gives us W_1, W_1^d, and Q_1. We follow the same step for 15000 iterations and take the iteration that gives the "closest" price function to that of the following iteration. Our convergence criteria considers the maximum relative distance:

$$\delta_h^Q \equiv \max_{j,k,m,n} \frac{|Q_h(j,k,m,n) - Q_{h+1}(j,k,m,n)|}{\frac{Q_h(j,k,m,n)+Q_{h+1}(j,k,m,n)+0.001}{2}}.$$

Basics

We solve the model numerically using value function iteration on a discrete state space for debt and income shocks. For the debt level, we use maturity-specific grids of 201 points. We use a grid of 101 points for the persistent income shock. We approximate the process of persistent income component using the Tauchen method. Table 7 shows the robustness of the results to reducing the number of points on each grid.[13]

Once we solved for the policy and value functions, we simulate 1500 countries (paths) for 500 years and take out the first 100 periods before calculating the moments. The model counterparts of the correlation and standard deviation statistics are averages across samples. For the first-order moments, we first take country-specific medians before averaging across countries. For the data, we follow the same approach when generating statistics.

Role of the two additional idiosyncratic shocks

The non convex nature of the value functions is often behind the difficulty to obtain convergence in defaultable sovereign debt models. In particular, the objective function of the sovereign, in the case of not defaulting, might have multiple local maxima, associated with potentially similar implied values. Hence, little differences in the guessed price functions can cause jumps in the optimal debt level, even if the maturity decision does not change. This is a problem common to most sovereign debt models—unless taken care of with a technique specifically designed to mitigate this issue. Since our model also involves another layer of discrete choice, which is that of the maturity, non-convexity issues are magnified in our baseline.

The additional shocks we introduce for numerical purposes help for convergence because they randomize the decision on the portfolio, and on default, hence convexifying the value and price functions. Intuitively, when default decisions, optimal debt level, and optimal maturity depend on a continuous random variable, then small changes in the price function do not lead

[13] For the computation with 51 points on the income grid, we proportionally reduce the number of points above and below the median by a half, i.e. to 45 points below and 5 points above the median.

to discrete changes in the default decision or choosing one debt level-maturity combination over another. Instead, these small changes in the price function lead to continuous and potentially small changes in the probability of a particular decision being made. Hence, over two iterations, smaller changes in the price function lead to smaller changes in the optimal policies, and in turn, smaller changes in the price function in the next iteration.

Chatterjee and Eyigungor (2012) document that the presence of the income shock x helps significantly to obtain convergence in solving sovereign debt models, by randomizing the decision on default and the optimal debt level. However, the presence of such a shock in our model would not be sufficient for two reasons related to the maturity choice structure: First, maturity choice makes the optimal debt level non-monotone with respect to the i.i.d. income shock. This is the reason why we assume that these shocks are realized after the maturity choice being made, as given the maturity choice, the debt level is monotone with respect to x. Second, the discrete choice of maturity calls for additional randomization by itself, as changes in the price function could affect the choice of m'. This is why we introduce the maturity shocks, and why they are actually useful.

For the results shown in sections 3 to 4, we use a standard deviation for the i.i.d. income shocks, σ_x, equal to 0.05 percent of the mean income. The standard deviation for the i.i.d. maturity shocks, σ_χ, are set to 0.3 percent of the absolute flow utility of consuming the mean income, $\bar{y}$. In Table 8 we show the moments implied by alternative specifications for the sizes of the additional shocks, around the values we use for the benchmark. Moreover, we include convergence outcomes for each of the models. In particular, in the bottom panel of the table, (i) the first row corresponds to the measure of our criteria, δ^Q, which is the maximum relative distance in the price function between two iterations, (ii) the second row considers the maximum absolute distance in the price function, and (iii) the third row shows the average absolute distance in the price function over all the points. In the final three rows, we present the distance measures using the solution for the value function, W, instead of that for the price function, Q. From the size of the shocks that we use in the benchmark calibration, neither reducing the variance of each shock by a half nor doubling it significantly change in the moments implied by the model.

A.2 Notes on variable definitions

Duration. For the duration of a bond, we use the Macaulay definition (as in Hatchondo and Martinez, 2009) which is a weighted sum of future coupon payments:

$$\frac{q(y,b',m';1) + 2\times(q(y,b',m';2)-q(y,b',m';1)) + ... + n\times(q(y,b',m';n)-q(y,b',m';n-1))}{q(y,b',m';n)}.$$

Yield to maturity. Yield for maturity n for an observation with borrowing level b', income y and maturity of the held bond m is calculated as follows:

$$YTM(y,b',m';n) \equiv \left(\frac{1}{q(y,b',m';n)-q(y,b',m';n-1)}\right)^{\frac{1}{n}} - 1.$$

Then the spread for maturity m is $YTM(y,b',m';n) - r$.

A.3 Yield curve and repayment probability

Consider bonds that make constant unitary coupon payments for m periods. In this section we explain how the unit bond price depends on the default probabilities. For simplicity, we consider the repayment probabilities in periods $1, 2, ..., j$ denoted by $P_{t+1}, P_{t+2}, ..., P_{t+j}$. Then, the price of a bond that pays one unit the next period is

$$q_t(1) = \frac{P_{t+1}}{1+r},$$

where r is the risk free rate. Similarly, the price for bonds that pay one unit for two periods is

$$q_t(2) = \frac{P_{t+1}}{1+r} + \frac{P_{t+1}P_{t+2}}{(1+r)^2},$$

and more generally, the price for a bonds that pays for j periods is

$$q_t(j) = \frac{P_{t+1}}{1+r} + \frac{P_{t+1}P_{t+2}}{(1+r)^2} + ... + \frac{\prod_{i=1}^{j} P_{t+i}}{(1+r)^j}.$$

With these prices at hand, we can write prices of zero-coupon bonds that pay in $1, 2, ..., j$ periods

$$\begin{aligned} q_t^0(1) &= q_t(1) = \frac{P_{t+1}}{1+r}, \\ q_t^0(2) &= q_t(2) - q_t(1) = \frac{P_{t+1}P_{t+2}}{(1+r)^2}, \\ &... \\ q_t^0(j) &= q_t(j) - q_t(j-1) = \frac{\prod_{i=1}^{j} P_{t+i}}{(1+r)^j}. \end{aligned} \tag{3}$$

Then, the yield of a zero-coupon bond that pays in j period is $i_t^0(j)$,

$$\begin{aligned} i_t^0(1) &= \frac{1+r}{P_{t+1}}, \\ i_t^0(2) &= \frac{1+r}{(P_{t+1}P_{t+2})^{1/2}}, \\ &... \\ i_t^0(j) &= \frac{1+r}{(\prod_{i=1}^{j} P_{t+i})^{1/j}}. \end{aligned} \tag{4}$$

B Empirical Analysis

The statistics for maturity, duration, output, consumption and debt are for Argentina, Brazil, Chile, Colombia, Hungary, Mexico, Peru, Poland, Slovenia and Turkey. For spreads, we only have availability for Brazil, Chile and Mexico among these countries.

Output is “GDP in constant 2005 USD”, consumption is “Household final consumption expenditure in constant 2005 USD”, and debt level is “External debt stocks”, all provided by the World Development Indicators (WDI).

The information on duration for Chile (1990-2010), Hungary (1999-2010), Mexico (2007-2010), Poland (2000-2010), Slovenia (2003-2010) and Turkey (2001-2010) is from the OECD database, and it is follows the Macaulay definition. For Argentina and Brazil, we use the numbers from Cruces et al. (2002). For Peru, we get the statistics from the Ministerio de Economia y Finanzas (MEF, 2001-2013). For Colombia, the corresponding data comes from the HAVER database (2001-2015). Data on maturity is also provided by the OECD for Chile (1990-2010), Hungary (1999-2010), Mexico (2007-2010), Poland (1997-2010), Slovenia (2003-2010) and Turkey (2005-2010). For Peru, we consult to MEF (2001-2013), for Colombia (2001-2015) and Brazil (2005-2015) we use data provided by the HAVER database. Maturity statistics for Argentina are provided by Ministerio de Economia y Finanzas Publicas (2000-2014). All the information for duration and debt is for external debt, except the maturity for Argentina which is for gross debt. When data is given in higher frequency than a year, we take the median within a year.

The statistics for spreads are for a subset of the countries above: Brazil, Chile, Colombia, Mexico, Slovenia and Turkey. This data is provided by the Federal Reserve Board.

We detrend log-output and log-consumption using an HP-filter with smoothing parameter of 1600 as in Hatchondo and Martinez (2009). For the statistics we report from the data, we use country-specific medians and then compute the average across countries.[14] We classify a year as a good (bad) time if the detrended log-output of the country is above (below) 0.

For the cross-country relationships between maturity and country-level characteristics, we use the following variables:

1. log-maturity: Log maturity of sovereign debt. Various sources.
2. Sudden stops in the last 10 years: Probability of a sudden stop event for a given country as calculated from a 10 year moving average of the number of years in which each country experienced a sudden stop event. Source: International Monetary Fund
3. Per Capita GDP: Ratio of nominal GDP to population calculated by ICRG from World Bank data.
4. Poverty: Level of poverty as calculated by ICRG from IMF, CIA, and World Bank data.
5. Population under 15 years old: Percentage of population under 15 years of age, from the World Bank's World Development Indicators: Population Dynamics database.
6. Gini Coeff.: Gini Coefficient from the World Bank's World Development Indicators: Population Dynamics database.
7. Std. Dev. Growth: Standard deviation of the growth rate.

[14]This is consistent with our approach to the simulation results.

Table 7: Robustness to Alternative Sizes for Debt and Income Grids

	BM	Alternative grids		
		Debt		Income
Grids for b	201	161	201	201
Grids for b (coarse)	41	41	26	41
Grids for y	101	101	101	51
Duration	4.07	4.07	4.19	4.06
Duration (good times)	4.36	4.36	4.47	4.36
Duration (bad times)	3.73	3.74	3.85	3.73
Maturity	7.88	7.87	8.07	7.83
Maturity (good times)	8.25	8.24	8.51	8.24
Maturity (bad times)	7.28	7.28	7.58	7.23
$\rho(n, log(y))$	0.29	0.29	0.27	0.29
$\rho(dur, log(y))$	0.39	0.39	0.38	0.39
1-year spread	0.10	0.10	0.12	0.13
1-year spread (good times)	0.01	0.01	0.01	0.01
1-year spread (bad times)	1.38	1.39	1.53	1.47
10-year spread	2.00	1.99	2.06	2.06
10-year spread (good times)	1.51	1.50	1.55	1.51
10-year spread (bad times)	2.93	2.93	3.05	2.98
$\rho(1YS, log(y))$	-0.32	-0.32	-0.32	-0.32
$\rho(10YS, log(y))$	-0.45	-0.46	-0.46	-0.48
$\sigma(log(c))/\sigma(log(y))$	1.10	1.10	1.09	1.09
$\sigma(log(TB/y))/\sigma(log(y))$	0.56	0.56	0.55	0.53
$\rho(TB/y, log(y))$	0.10	0.10	0.10	0.09
Default (%)	1.76	1.75	1.75	1.80
Value of debt / Output	0.35	0.35	0.35	0.34
Face value of debt / Output	0.48	0.48	0.48	0.46
$\rho(log(c), log(y))$	0.86	0.86	0.86	0.87

Note: For all the columns, the same parameters are used as in the benchmark (see Table 2).

Table 8: Analyzing the Role of the Additional Shocks

	Benchmark	Alternative variances for IID shocks on			
		Income		Maturity	
$\sigma_x/\bar{y}$	0.0005	0.00025	0.001	0.0005	0.0005
$\sigma_\chi/\lvert u(\bar{y})\rvert$	0.003	0.003	0.003	0.0015	0.006
Duration	4.07	4.06	4.06	4.11	3.98
Duration (good times)	4.36	4.36	4.35	4.42	4.26
Duration (bad times)	3.73	3.73	3.73	3.73	3.69
Maturity	7.88	7.86	7.84	7.92	7.63
Maturity (good times)	8.25	8.25	8.22	8.38	8.03
Maturity (bad times)	7.28	7.27	7.24	7.26	7.12
$\rho(n, log(y))$	0.29	0.29	0.30	0.35	0.25
$\rho(dur, log(y))$	0.39	0.39	0.39	0.44	0.35
1-year spread	0.10	0.10	0.10	0.10	0.10
1-year spread (good times)	0.01	0.01	0.01	0.01	0.01
1-year spread (bad times)	1.38	1.37	1.38	1.39	1.34
10-year spread	2.00	1.99	1.99	2.03	1.92
10-year spread (good times)	1.51	1.51	1.50	1.55	1.43
10-year spread (bad times)	2.93	2.92	2.91	2.94	2.82
$\rho(1YS, log(y))$	-0.32	-0.32	-0.32	-0.32	-0.32
$\rho(10YS, log(y))$	-0.45	-0.45	-0.46	-0.46	-0.45
$\sigma(log(c))/\sigma(log(y))$	1.10	1.10	1.10	1.10	1.11
$\sigma(log(TB/y))/\sigma(log(y))$	0.56	0.56	0.56	0.55	0.56
$\rho(TB/y, log(y))$	0.10	0.10	0.10	0.10	0.09
Default (%)	1.76	1.76	1.75	1.77	1.70
Value of debt / Output	0.35	0.35	0.35	0.35	0.35
Face value of debt / Output	0.48	0.47	0.47	0.48	0.47
$\rho(log(c), log(y))$	0.86	0.86	0.86	0.86	0.86
Distance in price, relative	4.7×10^{-10}	1.6×10^{-9}	7.1×10^{-10}	4.2×10^{-10}	3.2×10^{-10}
Distance in price, max	2.2×10^{-9}	2.7×10^{-9}	2.7×10^{-9}	2.7×10^{-9}	1.8×10^{-9}
Distance in price, per state	2.5×10^{-12}	3.5×10^{-12}	4.9×10^{-12}	2.7×10^{-12}	1.8×10^{-12}
Distance in value, relative	1.6×10^{-13}	3.2×10^{-13}	2.9×10^{-13}	4.7×10^{-13}	1.3×10^{-13}
Distance in value, max	3.7×10^{-13}	7.9×10^{-13}	7.3×10^{-13}	1.2×10^{-12}	3.4×10^{-13}
Distance in value, per state	2.5×10^{-14}	5.9×10^{-15}	5.6×10^{-15}	9.8×10^{-15}	6.9×10^{-15}

Note: For all the columns, the same parameters are used as in the benchmark (see Table 2).

Table 9: Various Regression Specifications, log(maturity)

	Dependent variable: log-maturity														
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Sudden stops in the last 10 years	0.932***				0.656***	0.656***	-0.920				0.939***	-0.866	-0.424		-0.421
	(0.203)				(0.241)	(0.241)	(0.701)				(0.279)	(0.653)	(0.585)		(0.600)
Population under 15 years old		0.0253***			0.0204***	0.0204***		0.0305***	0.0238***		0.0123	0.0235***		0.00242	0.00178
		(0.00445)			(0.00508)	(0.00508)		(0.00545)	(0.00477)		(0.00773)	(0.00522)		(0.0149)	(0.0157)
Gini Coeff.			0.0123**					-0.00353		0.0194***	0.00967		0.0191***	0.0180*	0.0181*
			(0.00541)					(0.00525)		(0.00315)	(0.00602)		(0.00317)	(0.00966)	(0.00984)
Std. Dev. Growth				-5.377***			-9.237***		-1.429	-4.637***		-5.109	-6.359**	-4.299*	-6.096
				(1.381)			(3.465)		(1.031)	(0.965)		(3.111)	(2.663)	(2.550)	(4.096)
Per Capita GDP	-0.0344**	-0.00184	-0.00143	-0.0151	-0.0162	-0.0162	0.00112	-0.000560	-0.0161	0.00101	-0.0120	-0.000804	0.00761	-0.000597	0.00637
	(0.0163)	(0.0155)	(0.0182)	(0.0192)	(0.0167)	(0.0167)	(0.0226)	(0.0176)	(0.0178)	(0.0181)	(0.0173)	(0.0216)	(0.0200)	(0.0204)	(0.0233)
Poverty	-0.244***	-0.237***	-0.307***	-0.299***	-0.222***	-0.222***	-0.333***	-0.231***	-0.200***	-0.336***	-0.267***	-0.234***	-0.345***	-0.324***	-0.337***
	(0.0521)	(0.0501)	(0.0632)	(0.0590)	(0.0504)	(0.0504)	(0.0632)	(0.0594)	(0.0512)	(0.0601)	(0.0669)	(0.0557)	(0.0629)	(0.0958)	(0.102)
constant	2.283***	1.569***	1.640***	3.120***	1.638***	1.638***	3.793***	1.479***	1.709***	1.842***	1.275***	2.359***	2.154***	1.783***	2.108***
	(0.160)	(0.218)	(0.304)	(0.250)	(0.225)	(0.225)	(0.626)	(0.260)	(0.298)	(0.228)	(0.219)	(0.614)	(0.473)	(0.448)	(0.711)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	97	97	82	83	97	97	83	82	83	69	82	83	69	69	69
R^2	0.624	0.646	0.587	0.669	0.676	0.676	0.681	0.650	0.725	0.786	0.705	0.736	0.789	0.786	0.789

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$