Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

How Has Empirical Monetary Policy Analysis Changed After the Financial Crisis?

Neville R. Francis
Laura E. Jackson
and
Michael T. Owyang

Working Paper 2014-019B
http://research.stlouisfed.org/wp/2014/2014-019.pdf

Revised April 27, 2016

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Federal Reserve Bank of St. Louis Working Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.

How Has Empirical Monetary Policy Analysis Changed After the Financial Crisis?*

Neville R. Francis
Department of Economics
University of North Carolina, Chapel Hill

Laura E. Jackson†
Department of Economics
Bentley University

Michael T. Owyang
Research Division
Federal Reserve Bank of St. Louis

Keywords: zero lower bound, affine term structure

April 27, 2016

Abstract

In the wake of the Great Recession, the Federal Reserve lowered the federal funds rate (FFR) target essentially to zero and resorted to unconventional monetary policy. With the nominal FFR constrained by the zero lower bound (ZLB) for an extended period, empirical monetary models cannot be estimated as usual. In this paper, we consider whether the standard empirical model of monetary policy can be preserved without breaks. We consider whether alternative policy instruments (e.g., a long-term interest rate) can be considered substitutes for the FFR over the ZLB period. Furthermore, we compare the shadow rates proposed in Krippner [2012] and Wu and Xia [2016] as alternative measures of the stance of monetary policy. We ask whether the shadow rate is a sufficient representation of the policy instrument or if the financial crisis requires other modifications. We find that, when using a dataset that spans both the pre-ZLB and ZLB periods, the shadow rate acts as a fairly good proxy for monetary policy by producing impulse responses of macro indicators similar to what we'd expect based on the post-WWII, non-ZLB benchmark and by displaying stable parameter estimates when compared to this benchmark. [JEL codes: E43, E44, E52]

*The authors benefitted from conversations with Anusha Chari, Leo Krippner, Cynthia Wu, and conference participants at the SNDE, CEF, and Midwest Macro. Diana Cooke, Hannah G. Shell, and Kate Vermann provided research assistance. The views expressed here are the authors' alone and do not reflect the opinions of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†ljackson@bentley.edu

1 Introduction

Since the onset of the Financial Crisis, monetary policy in the U.S. has taken unprecedented measures in an effort to stimulate the economy. The Federal Reserve lowered its primary policy instrument—the target for the federal funds rate—essentially to zero.[1] At that point, this instrument became ineffective due to the nominal bound at zero and the Fed was forced to resort to unconventional monetary policy.[2]

From the standpoint of academics, this period presents an important problem for assessing the effects of monetary policy. Many monetary models use the effective nominal federal funds rate (FFR) to represent the stance of policy. Nominal interest rates, however, cannot take on negative values in a world in which cash is a viable alternative to standard short-term assets. Because the nominal FFR has been constrained by the zero lower bound (ZLB) for an extended period, empirical monetary models cannot be estimated as usual. Even in models for which determining the effects of monetary policy is not the primary objective (e.g., measuring the effects of fiscal policy), a proper measure of the stance of monetary policy is often necessary.

The empirical literature offers a number of potential remedies. First, we could treat the ZLB period as special, using either breaks or dummies to represent changes in economic relationships.[3] Second, we could include alternative policy instruments, such as the size of the balance sheet or dummies representing the implementation of unconventional policies. These two alternatives have the disadvantage of increasing the number of estimated parameters for a period that, presumably, represents a short sample. Third, we could replace the FFR as the conventional stance of monetary policy with a proxy that is allowed to violate the ZLB and that captures the effects of both conventional and unconventional policy.

Since the financial crisis, academics have proposed such proxies of the accommodative stance of monetary policy when the short rate is at the ZLB. Recently, Krippner [2012] and Wu and Xia

[1] Friedman [2010] describes a detailed timeline of the steps taken by the Fed along with significant market events. Williams [2011] reviews the unconventional monetary policies employed during the financial crisis.

[2] Unconventional monetary policy used in the U.S. included quantitiative easing (QE), large scale asset purchases (LSAPs), and forward guidance. Walsh [2010] discusses the channels through which QE could stabilize the economy. Wright [2011] analyzes how long-term interest rates respond to LSAP's in a ZLB environment. Gagnon et al. [2011] present the mechanisms through which these purchases affect the overall macroeconomy. Campbell et al. [2012] discuss the effects of forward guidance.

[3] This option could be considered the most extreme as it suggests that the effect of monetary policy is potentially time-varying [see, for example, Aastveit et al. [2014]]. If the effectiveness of policy varies, the Fed must reconsider at each moment the conduct of policy and the appropriate instruments.

[2016] have used the shadow rate methodology to construct alternative measures of the stance of policy. Krippner [2012] builds on Black [1995] and Gorovoi and Linetsky [2004], modeling interest rates as options by calculating the value of a call option to hold cash.[4] The modifications in Krippner [2012] generate closed-form solutions for bond prices and yields. Rather than describing yields in a ZLB environment directly, Wu and Xia [2016] construct an analytical approximation of forward rates in discrete time, allowing for a more straightforward estimation approach than the other shadow rate methodologies and produce closed-form expressions for the shadow forward rates. Both models calculate a shadow short-term interest rate which would be seen in financial markets if the cash option did not exist. In principle, the Fed may have dropped the FFR further if not for the nominal bound at zero. The shadow rate has been considered a proxy for the stance of monetary policy in an environment in which the zero lower bound is binding. From this foundation one can develop a full model of the shadow term structure based upon the shadow short rate depicting the fundamental policy objectives.

In this paper, we compare these approaches to modeling monetary policy at the ZLB in commonly-used, benchmark linear or piecewise-linear models. Our results show that, using the various monetary policy measures, our VARs fail to find any discernible differences in the macroeconomic effects of monetary policy. That is, the impulse responses using the shadow rate measures (that control for the ZLB period) are all contained within the confidence bands of similar impulse responses when we naively apply the FFR as policy instrument without any such ZLB controls. However, we also find that the method proposed by Krippner [2012] delivers stable parameter estimates when the ZLB period is included in the sample. Under the assumption of no structural shift in the conduct of monetary policy, we view the stability of the parameters in favor of using the Krippner shadow rate to represent policy during the unique environment at the ZLB.

The balance of the paper is organized as follows: Section 2 establishes the benchmark VAR that describes monetary policy in the pre-ZLB environment and for the full sample through the end of 2015. In this section, we also test for parameter stability just prior to the onset of the ZLB period. Section 3 examines how we can model some of the actions taken by the Fed at the zero lower bound in these standard models. Finally, Section 4 concludes.

[4] Black [1995] modified the Gaussian affine term structure model (GATSM) to relax the nominal ZLB by introducing a fictitious shadow instantaneous (zero maturity) interest rate that can take on negative values. When the nominal rate is above the ZLB, the shadow rate takes on the nominal value.

2 The Benchmark VAR

The short rate—often, an overnight rate—is one of the primary instruments for conducting monetary policy. When adverse shocks are large, monetary accommodation can drive the short rate close to zero. In practice, no-arbitrage conditions prevent nominal rates from falling below zero since agents can substitute out of bonds into cash. This feature of nominal interest rates can prevent a proper evaluation of the stance of monetary policy when the short rate is at or near zero and other instruments must be relied upon to conduct policy. In this section, we estimate a standard monetary VAR for the pre-crisis period (1960:I-2007:IV) and then naïvely extend the analysis with data for the financial crisis period.

2.1 The Pre-ZLB VAR

Before we can determine whether empirical models of monetary policy have changed, we must first establish a benchmark. We estimate a quarterly four-variable VAR(4) in output, inflation, commodity prices, and a policy instrument.[5] For the benchmark model, we include the effective FFR as the policy instrument:

$$Y_t = A(L)\, Y_{t-1} + \varepsilon_t,$$

where $A(L)$ is a polynomial in the lag operator, $\varepsilon_t \sim N(0, \Sigma)$, and we suppress the constant and any trends for notational simplicity. The monetary shock is identified by assuming that FFR can react to macro variables but the macro variables cannot contemporaneously react to shocks to the FFR. Partitioning $A(L)$ into blocks will facilitate exposition: Let X_t represent the macro variables of interest and R_t represent the FFR. Then, we can rewrite the VAR as:

$$\begin{bmatrix} X_t \\ R_t \end{bmatrix} = \begin{bmatrix} A^X(L) & A^{RX}(L) \\ A^{XR}(L) & A^R(L) \end{bmatrix} \begin{bmatrix} X_{t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^r \end{bmatrix},$$

where $A^X(L)$ represents the effects of the lagged macro variables on each other, $A^{RX}(L)$ represents the effects of policy on the macro variables, $A^{XR}(L)$ represents the feedback from macro variables

[5] Our measure of output is the annualized quarterly difference in the log of GDP taken from the BEA. Inflation is taken as the difference in the log of the CPI taken from the BLS. Commodity prices are the log differences in the Producer Price Index: All Commodities. All data are seasonally adjusted.

to policy, and $A^R(L)$ represents the possible persistence in FFR.[6]

The benchmark data sample covers the period from 1960:I to 2007:IV, which starts after the Korean War price control period and ends prior to the financial crisis and generally resembles the standard VAR used for monetary analysis prior to the FFR hitting the ZLB.

The first column of Figure 1 shows the impulse responses for the VAR(4) outlined above using the effective nominal FFR as the policy instrument and data for the period ending in 2007:IV. The responses shown are to a 25-basis-point shock to the effective nominal FFR ordered last in the VAR and identified using the Cholesky decomposition. The responses are computed for each draw of the sampler, generating the posterior coverage. The plots show the median response (blue line) as well as the 68-percent posterior coverage intervals (blue shaded regions). The impulse responses are as expected: An increase in the policy rate causes output to fall and, after some time, inflation to eventually fall.

2.2 A Naïve Approach to the ZLB Period

A challenge posed by the ZLB period is how to model the use of new monetary instruments that may be omitted from the VAR. One approach would be to ignore the change in the monetary environment and naïvely extend the sample through the ZLB period without accounting for the use of alternative policy instruments. Thus, we reestimate the benchmark VAR using data from 1960:I-2015:II to determine how the impulse responses would change if one naïvely extended the sample.

The second column of Figure 1 shows the impulse responses for the VAR(4) outlined above using the effective nominal FFR as the policy instrument through the ZLB period. The plots show the median responses (dark green line) as well their associated 68-percent posterior coverage intervals (green shaded region) for the naive full-sample VAR extending the data through 2015:II. The thick dark blue lines and blue shaded regions give the impulse response median point estimates and their 68-percent posterior coverage for the benchmark specification in which the data end in 2007:IV. The responses resemble those of the benchmark in both quantitative and qualitative terms—a contractionary policy shock results in a decrease in output, a (somewhat delayed) eventual decline

[6] To preserve consistency with results in later sections, the VAR presented here is estimated with Bayesian methods. The prior is a zero mean Normal-inverse Wishart distribution. The posterior distributions are simulated using the Gibbs sampler.

in inflation, and falling commodity prices.[7]

The result is not surprising given that preponderance of data in the pre-crisis period. Even though the nominal funds rate does not change for a number of years, adding the ZLB period does not qualitatively change the resulting impulse responses to a monetary shock. There are slight quantitative differences from the benchmark, but the impulse responses do not change all that much. Consequently, very little bias results from computing the impulse responses naively by including data from both non-ZLB and ZLB periods. However, this exercise does not tell us much about the ZLB period itself since the conventional policy instrument effectively does nothing during this period. While the estimates are slightly different, we can still effectively describe what happens in normal times but we do not know what happens in the ZLB period.

2.3 Testing for Parameter Instability

If one believed that the policy environment changed dramatically at the ZLB, a first reaction might be to impose a break at, say, 2007:IV, just before the nominal funds rate hit the bound. Let M_0 and M_1 represent the no-break and break models, respectively. To determine whether the data prefer the break model to the no-break model, we compute twice the log of the Bayes Factor to obtain a statistic $\mathcal{B}$ that has scale comparable to that of the likelihood ratio test statistic:

$$\mathcal{B} = 2 \ln \left(\frac{\pi \left(Y | M_1 \right)}{\pi \left(Y | M_0 \right)} \right), \tag{1}$$

where $\pi \left(Y | M_i \right)$ be the marginal likelihood of the data, given model M_i.[8]

We construct a dummy variable to indicate data from the ZLB period during the crisis and post-crisis recovery, taking on a value of 0 for dates prior to and including 2007:IV and a value of 1 after 2007:IV. We test for parameter instability in five subsets of the model parameters: (1) a break in the variance/covariance matrix (Σ) only; (2) a break in the response of macro variables to the policy instrument, $A^{RX}(L)$; (3) a combination of (1) and (2) - breaks in Σ and $A^{RX}(L)$; (4) a break the full set of VAR coefficients, $A(L)$; and (5) a combination of (1) and (4) - breaks in Σ and

[7] In Section 3.4, we more formally measure the distance between the posterior distributions of the benchmark parameters and those estimated with the ZLB data included.

[8] We compute the marginal likelihood using the output of the Gibbs sampler with the method described in Chib [1995]. We use the scale suggested in Jeffreys [1961] to interpret the strength of evidence against model M_0 and in favor of model M_1, where values of $\mathcal{B} > 6$ are considered strong evidence in favor of the break model. Negative values of $\mathcal{B}$ indicate that the no-break model is preferred.

$A(L)$. To impose the breaks in (2) and (3), we include an interaction between the ZLB dummy variable and all lags of the policy rate in each of the first three VAR equations. Therefore, the break model allows for a change in the VAR coefficients on the policy rate in each of the equations describing the behavior of the fundamental macroeconomic variables. To impose the breaks in (4) and (5), we include an interaction between the ZLB dummy variable and all lags of the Y_t vector in all four equations of the VAR.

The first lines of the two panels of Table 1 show the values of $\mathcal{B}$ comparing models for a VAR(4) using the effective FFR as the monetary instrument for the entire post-war sample (1960:I-2015:II, top panel) and for the post-Great Moderation sample (1984:I-2015:II, bottom panel).[9] Evidence in favor of breaks is mixed: While we find multiple cases of strong evidence against the model with constant parameters, the model with parameter instability is favored over the baseline model when allowing for variation in the response of macro variables to the FFR, $A^{RX}(L)$, and variation in Σ. The break in $A^{RX}(L)$ results from the fact that the fed funds rate exhibits no variation in the post break period, perhaps biasing the coefficients in the no break model. Likewise, the variance of the FFR innovation (and the associated covariances) change in the ZLB period. However, a (simultaneous) break in the full set of model parameters (cases (4) and (5) described above) is not favored. Allowing for parameter instability in the VAR(4) models introduces an excessive number of new parameters to estimate with limited data over the ZLB period; thus, we do not find definitive evidence supporting the models with breaks.[10]

3 Monetary Policy at the ZLB

Ideally, we want to be able to account for the effects of the Fed's unconventional monetary policy during the times in which the FFR is at the ZLB. Using the FFR alone to represent policy would erroneously suggest that the Fed was inactive during the depths of the financial crisis and did little to stimulate the recovery. Therefore, we need a way to incorporate the policy accommodation associated with the balance sheet liquidity programs and the use of forward guidance that were

[9] Because the number of breaking parameters is potentially large and the sample for the post-break period is short, we estimated a VAR(1) to verify robustness. Results were similar and are available from the authors upon request.

[10] In the VAR(1), this result is overturned. This suggests that breaking a small set of parameters might produce more desirable results than imposing constant parameters. However, breaking the sizable number of parameters in a VAR(4) is not supported.

employed by the Fed during the crisis. We pose the following question: Is there a policy instrument that summarizes the stance of monetary during the ZLB period that retain's the linearity of the VAR? Ideally, the new instrument would map back into some observable indicators, allowing one to address policy questions that can be answered in the same linear framework without requiring additional variables or parameters. We first augment the VAR with a long-term interest rate to incorporate how the Fed's unconventional policies effectively flattened the yield curve by influencing rates at the longer horizon and compare the responses to those of the benchmark VAR model. Finally, we will use the two shadow rate measures that explicitly account for the ZLB period (one adopted from Krippner [2012] and the other from Wu and Xia [2016]) as proxies for the policy instrument and again compare the impulse responses associated with these measures to the benchmark responses.

3.1 Adding Alternative Monetary Instruments

As we mentioned above, during the ZLB period, the Fed began to utilize alternative policy measures intended to provide temporary injections of liquidity and often targeted yields for longer maturity assets. These policies also represented a substantial increase in the Fed's balance sheet. One way to model the effects of these unconventional policies is to include a long-term interest rate directly in the VAR.

Figure 2 compares the impulse responses to a shock to the FFR in the benchmark to the VAR that includes the long-term interest rate. As in the previous analysis, the first column shows the median point estimate and 68-percent posterior coverage intervals for the impulse responses to a 25-basis-point increase in the FFR, estimated using the benchmark VAR. The second column shows the full-sample responses. The dark blue lines and blue shaded regions replicate the benchmark results. The median point estimates of the full-sample are almost identical the benchmark, with wider posterior coverage around the responses of inflation and commodity prices. To analyze the effects of the inclusion of the long-term interest rate, the third column shows the benchmark impulse responses to a 25-basis-point increase in the 10-year Treasury Bond rate using data through 2007:IV. The full-sample responses in the fourth column illustrate how the macroeconomic variables respond in almost identical fashion to changes in long-term interest rates over the non-ZLB and ZLB periods

In the ZLB period, we can measure the response of the macro variables to a shock to the long-

term interest rate, accounting for the fact that the FFR is already at zero and does not fluctuate after these shocks. Koop et al. [1996] suggested the computation of generalized impulse responses (GIRFs) which use Monte Carlo methods to compute the difference in two indirect forecasts—one conditional on the shock and one conditional on no shock. Conditioning on the funds rate hitting the ZLB is a straightforward extension of this method. We construct the GIRFs of all variables in the VAR, conditioning on being in the ZLB period and restricting the FFR to remain at its initial value at the time of the shock. In this way, we capture how aggregate activity fluctutates as long-term interest rates vary, imposing that the short-term policy instrument is constrained.

Figure 3 compares (1) the GIRF to a 25-basis-point increase in the 10-year Treasury Bond rate in the ZLB with a traditional 25-basis-point increase in the FFR in normal times, and (2) the GIRF of the 10-year T-Bond shock in the ZLB period with a similar T-Bond shock in normal times, allowing the FFR to respond without restriction. The response of GDP growth to the 10-year T-Bond shock in the ZLB resembles the response to the benchmark FFR shock. This suggests that the channel through which policy affects real activity through long rates is similar to the effect via short rates under normal conditions. However, in the ZLB period, the responses of nominal variables, inflation and commodity prices, look more similar to those in response to benchmark long-rate shocks, with no restrictions, rather than conventional FFR shocks. As depicted in Figure 3, the ZLB responses of the FFR are restricted to be zero at all horizons, compared with the richer dynamics in the benchmark when the policy rate is left unconstrained. Finally, the persistence of a shock to the 10-year T-Bond rate under normal or ZLB conditions is almost identical, thus unaffected by the constraints imposed on the short end of the term structure.

If one wanted to augment the FFR with the 10-year T-bond as a combined monetary instrument, we might want to ensure that the addition resolves the need to incorporate breaks. The second lines of each panel in Table 1 show $\mathcal{B}$ comparing VARs listing the 10-year Treasury Bond rate after the FFR. We find comparable, and sometimes even stronger, evidence favoring a break in $A^{RX}(L)$ and Σ for the full-sample model than when using only the FFR. The explanatory value from substantial variation in longer-term interest rates comes into play primarily around the early stages of the financial crisis and over the period witnessing drastic cuts in the FFR towards the ZLB. Incorporating these additional dynamics emphasizes the variation underlying the structural form of the model and amplifies the importance of allowing for parameter instability. This very strong

evidence favoring changing coefficients over constant coefficients on lagged values of the policy instrument in the equations for the macro variables suggests that accounting for the unconventional policy via long-term interest rates is not sufficient to maintain VAR linearity.[11]

Accounting for shocks to the long-term interest rate produce a sufficient representation of policy for the full post-war period, including a majority of non-ZLB data. However, augmenting the model in this way may not diminsh the need for parameter breaks to model the ZLB period and also might prove to be an inefficient approach for future empirical work once we return to a normal, non-ZLB environment and the longer-term interest rate may no longer be important for measuring conventional policy effects. The Fed has a variety of alternative policy programs in its arsenal but does not need to use them when it can adjust the FFR effectively. Including the 10-year rate in addition to the FFR introduces more parameters to estimate and more structure in the model associated with the ordering of variables in the VAR. In response to this, we pose the question: Can we find a proxy measurement of R_t that captures the stance of policy across all periods? We attempt to answer this question with the use of shadow interest rates.

3.2 Shadow Short Rates

One of the Fed's stated objectives in conducting unconventional monetary policy was to affect interest rates for longer maturity assets, suggesting that examining the term structure of interest rates could uncover a potential alternative policy instrument. Because the nominal short rate is constrained during the ZLB period, Black [1995] proposed a model with a fictitious shadow bond with the same maturity as the policy instrument and an unconstrained shadow interest rate. The nominal short rate, R_t, can then be expressed as the maximum of the shadow short rate, r_t, and zero:

$$R_t = \max\{r_t, 0\}. \tag{2}$$

When the nominal rate binds at the ZLB, the shadow rate is unconstrained and can fall below zero.[12]

[11] We find some evidence supporting a break in the Σ matrix in both the full-sample and post-Great Moderation VAR(1) model. This further supports the need to allow for variation in the volatility of reduced-form shocks at the ZLB.

[12] Krippner [2012], Wu and Xia [2016], and Christensen and Rudebusch [2013] estimate different versions of this shadow rate using financial market data spanning the full term structure.

Krippner [2012] and Wu and Xia [2016] argue that the shadow rate can be used to measure the stance of monetary policy when nominal rates hit the ZLB. The shadow rate, however, is a purely financial construct that does not take into account its effects on macro variables. If we are to use the shadow rate in empirical models of monetary policy, we need to know whether standard VAR models can be extended through the ZLB period by replacing the effective FFR with the exogenously constructed shadow rate or if the ZLB period, in and of itself, requires an alternative model.

The shadow rate developed by Krippner [2012], and discussed subsequently in Krippner [2013a] and Krippner [2013b], is publicly available on the Reserve Bank of New Zealand website. Meanwhile the shadow rate developed by Wu and Xia [2016] is publicly available on Federal Reserve Bank of Atlanta website. Both shadow rates are available at a monthly frequency. We aggregate the monthly shadow rate data to quarterly by averaging the estimated values for the shadow rate over each quarter.

Figure 4 shows a sub-sample from 2006:I-2015:II of the quarterly policy instruments used for estimation in the VAR. Prior to 2009:I, all policy instruments use observed values of the nominal FFR. From 2009:I through the end of the sample, we substitute the two shadow rate measures for the effective fed funds rate in separate VARs. By construction, when the nominal FFR is sufficiently far above zero, it and the shadow rates coincide. The shadow rate should be equal to the nominal short rate when the nominal short rate is positive and the model preserves this relationship up to a small measurement error. However, once the FFR effectively reaches the ZLB in 2008:IV, the shadow rate becomes increasingly more negative as the Fed took action to jump-start the economy.

Therefore, as a third alternative treatment option for the ZLB period, we estimate the same VAR(4) as above with these two new hybrid monetary policy measures. The third and fourth panels of Figure 1 show the impulse responses of the VAR using the shadow rates substituted for the effective funds rate at the ZLB. The third column estimates the VAR with the Krippner [2012] shadow rate as the policy instrument from 2009:I-2015:II and the fourth column estimates the VAR with the Wu and Xia [2016] shadow rate as the policy instrument. Using either shadow rate to represent policy generates posterior coverages for the responses of all macro variables similar to the benchmark. The median benchmark responses fall within the posterior coverage for the full-sample analyses in all cases except for medium- and longer-horizon responses of commodity prices.

As before, we consider whether replacing the fed funds rate with the shadow short rates can diminish the need for parameter breaks during the ZLB period. Lines 3 and 4 of the panels in Table 1 show $\mathcal{B}$ comparing models using the Krippner [2012] and Wu and Xia [2016] shadow rates as the policy instrument, respectively. There is very slight evidence suggesting the need for a shift in the macro responses to the Wu and Xia [2016] shadow rate in the VAR(4) model over the entire post-war sample. However, the evidence supporting a break in $A^{RX}(L)$ no longer exists for the post-Great Moderation subsample. Conversely, when using the Krippner [2012] shadow rate, the VAR(4) results always favor the constant parameter model. This shadow rate exhibits much richer dynamics over the crisis and ZLB periods, taking on greater negative values. It appears that this variation, in contrast to the FFR or the smoother path of the Wu and Xia [2016] shadow rate, allows the Krippner [2012] shadow rate to provide a better proxy for the policy instrument, under the presumption of no structural shift in the effects of monetary policy, in the VAR setting.[13]

3.3 Policy Effects During the ZLB Period

These results suggest that replacing the fed funds rate with the shadow rate is one method to maintaining a consistent linear VAR when estimating a model with data that includes the ZLB period. But are the effects of a shadow rate shock during the ZLB period different if we only use data from the ZLB period? A substantial downward movement of the shadow rate occurs in the early stages of the Great Recession, with increasingly negative shadow rates while the FFR is near zero. Does this movement in the shadow rate represent a significant policy stimulus associated with unconventional policies when the FFR does not deviate from the ZLB?

Wu and Xia [2016] treat this period differently than the subsequent ZLB period once the economy is no longer in recession but the FFR is still near zero and unconventional policies are still in use. They argue for using the shadow rate to model policy action only after the recessionary conditions subside. We would like to construct a comprehensive measure of policy even during the recessionary period. The large negative shocks that pushed the economy into recession and drove the FFR towards zero are important for determining the validity of the shadow rate approach to

[13] Hännikäinen [2016] compares the predictive content of the Krippner [2012] and Wu and Xia [2016] shadow rates for U.S. real activity and inflation. Interestingly, the study finds that while both shadow rates have out-of-sample predictive power for inflation in ZLB and non-ZLB periods, the Wu and Xia [2016] shadow rate produces more accurate forecasts. Alternatively, neither shadow rate has predictive power for real activity.

modeling policy in these abnormal environments.

We re-estimate the VAR and include only the period after the onset of the ZLB to look specifically at economic conditions when the shadow rate should provide more information than the FFR alone. Due to the limited amount of data for this period, we shorten the number of lags in the VAR to include only one lag of the vector of dependent variables.[14] Figure 5 shows both the impulse responses of the benchmark VAR(1) and and a VAR(1) where the shadow rate replaces the FFR over the period from 2008:I through 2015:II. During this time, the nominal FFR remained near zero and, thus, provides little explanatory value regarding the effectiveness of countercyclical monetary policy. The shadow rates incorporate both the influence of unconventional policy measures on current economic conditions as well as market expectations of future policy. Thus, the responses of macro aggregates to shocks to this alternative policy measurement capture more comprehensive policy action during the severe economic contraction.

The impulse responses are estimated with much less precision and the posterior coverage is considerably wider for the VAR shadow rate estimation using only ZLB data than for the benchmark. As a result, the median benchmark responses tend to fall within the considerably wide posterior coverage. The median point estimate of the response of output when using the Wu-Xia shadow rate over the ZLB period suggests a much more substantial decline than that from the Krippner shadow rate, which still suggests a slightly deeper decline than the benchmark. Not surprisingly, the median responses of inflation and commodity prices fluctuate substantially from the benchmark. Caution must be taken when interpreting these results as the response of inflation implies a price puzzle in the benchmark VAR(1) but neither of the shadow rates replicate this type of response during the ZLB period. Contractionary policy shocks are associated with falling inflation and commodity prices using the Krippner shadow rate but falling inflation and rising commodity prices with the Wu-Xia shadow rate. As we previously discussed, the shadow rate may incorporate future expectations as it extracts data from interest rates and investment decisions.[15]

[14] Of course, given the data limitations for this period, error bands are expected to be large and results will be only suggestive.

[15] For robustness, we repeat these impulse response comparisons after adjusting the benchmark to begin after the end of the Great Moderation (1984:I-2007:IV) rather than using the entire post-war sample. We reach the same qualititative conclusions regarding the deviation from the benchmark for full-sample and ZLB sub-sample responses using the FFR and the shadow rates.

3.4 Quantifying Distance from the Benchmark Model

Finally, we test whether there are appreciable differences in the impulse responses to a shock to the shadow short rate estimated over the full sample versus the responses to a shock to the FFR in our benchmark pre-ZLB sample. In order to do this, we construct the Kullback-Leibler Divergence (KLD) between the posterior distribution of the benchmark VAR parameters and the posterior distribution of the VAR parameters estimated using the alternative policy instruments.[16] We can think of the KLD as a type of loss function that measures deviations between distributions. We take the benchmark, pre-ZLB posterior parameter distributions as the truth and measure the extent to which the posterior distributions differ from this when using the shadow rate proxies. Ideally, we want to look at the difference in the distributions of the impulse responses themselves. However, since there is a one-to-one mapping between the impulse responses functions and the VAR coefficient and covariance matrices, we can use the output of the Gibbs sampler to analyze the posterior distribution of the parameter estimates directly.

Table 2 gives the values of the KLD between the post-war, pre-ZLB benchmark (1960:I-2007:IV) and the full-sample (1960:I-2015:II) when using the FFR and each of the two shadow rates. When including both the pre-ZLB and ZLB periods within the sample, the model in which the FFR is used to represent policy the entire time produces the smallest KLD, thus showing the smallest deviation from the benchmark. This is in agreement with our previous results using the full-sample with a majority of non-ZLB data. However, when looking specifically at the ZLB period, the posterior distribution of VAR parameters estimated with the shadow rate of Krippner [2012] has the smallest KLD, thus exhibiting less variation than the distributions using the FFR or the shadow rate of Wu and Xia [2016]. In accordance, the results remain the same if we adjust the benchmark to only consider data after the end of the Great Moderation, therefore estimating the VAR using data from 1984:I-2007:IV to establish our basis for comparison. The KLD produced from the FFR is smaller than those of the two shadow-rate-models for the full, post-Great Moderation (1984:I-2015:II) sample. Furthermore, the KLD using the Krippner [2012] shadow rate is smallest for the ZLB sub-sample. Removing the influence of pre-ZLB data in the dataset by excluding the first 24 years of data allows for the shadow rate modifications at the ZLB to achieve greater success at

[16] The Kullback-Leibler Distance is a metric to assess the deviation of one distribution from another. See Kullback and Leibler [1951] for more details regarding how to construct this distance.

merging a comprehensive, continuous representation of policy between these two periods. However, there is little consistency in terms of which measurement of the policy rate produces the most cohesive measurement of policy when merging together non-ZLB and ZLB subsamples.

We cannot compute the impulse responses of macroeconomic variables to the FFR during the ZLB period alone as the policy instrument did not exhibit meaningful variation over this time. When at the ZLB, the shadow rates more closely recover some of the benchmark macro dynamics of the pre-ZLB period and these are further preserved if we employ a full dataset over the entire post-war period, including the years at the ZLB. Having these anomalous years of data between extended episodes of normal economic activity does not seem to prohibit the use of standard VAR analyses for the effects of monetary policy.

4 Conclusions

Researchers attempting to measure the effects of monetary policy during the financial crisis and subsequent recession beginning in 2008 have encountered difficulties when trying to use the FFR which essentially flatlines at zero for much of the period under consideration. We have proposed using the shadow rate as a measurement of policy which is able to fluctuate to negative values when the effective central bank policy rate faces a binding constraint at zero. Our results suggest that the shadow rate acts as a good proxy for monetary policy throughout the ZLB environment only if using a dataset that spans both the pre-ZLB and ZLB periods.

Examining the FFR alone may suggest that policy has become inactive or ineffective but the monetary authority has indeed been successful at implementing expansionary policy albeit through alternative mechanisms. An important point to note is that the economy has witnessed a break in the instrument used to enact policy but not a break in the effects of monetary policy on the macroeconomy. Economic researchers use the FFR as a measurement of the policy instrument for the post-WWII era even though the Fed targeted non-borrowed reserves from 1979-1982 and borrowed reserves from late 1982 through the mid-1980's. It did not stop targeting M1 until 1987 and M2 until 1993 and began announcing formal targets for the FFR only in 1994. Similarly to that change in the behavior of central bankers, the ZLB period beginning in December 2008 has rendered the traditional policy tool impotent for stimulating economic activity. The Fed has suc-

cessfully utilized balance sheet items as instruments and introduced a much more expansive period of alternative policy measures than the time spent targeting non-borrowed/borrowed reserves. In order to accurately represent monetary policy during this period, we need a surrogate measurement such as the shadow rate.

References

Knut Are Aastveit, Andrea Carriero, Todd E. Clark, and Massimiliano Marcellino. Have standard vars remained stable since the crisis? February 2014.

Fischer Black. Interest rates as options. *The Journal of Finance*, 50(5):1371–1376, Dec. 1995.

Jeffrey R. Campbell, Charles Evans, Jonas D.M. Fisher, and Alejandro Justiano. Macroeconomic effects of federal reserve forward guidance. *Brookings Papers on Economic Activity*, 44:1–80, 2012.

Siddhartha Chib. Marginal likelihood from the gibbs output. 90(432):1313–1321, Dec. 1995.

Jens H.E. Christensen and Glenn D. Rudebusch. Estimating shadow-rate term structure models with near-zero yields. *Federal Reserve Bank of San Francisco Working Paper Series*, June 2013.

Benjamin M. Friedman. Learning from the crisis: What can central banks do? Technical report, February 2010.

Joseph Gagnon, Matthew Raskin, Julie Remache, and Brian Sack. The financial market effects of the federal reserve's large-scale asset purchases. *International Journal of Central Banking*, March 2011.

Viatcheslav Gorovoi and Vadim Linetsky. Black's model of interest rates as options, eigenfunction expansions and japanese interest rates. *Mathematical Finance*, 14(1):49–78, Jan. 2004.

Jari Hännikäinen. The shadow rate as a predictor of real activity and inflation: Evidence from a data-rich environment. *Tampere Economic Working Papers*, 2016. No. 106.

H. Jeffreys. *Theory of Probability*. Oxford University Press, 3rd edition, 1961.

Gary Koop, Hashem M. Pesaran, and Simon M. Potter. Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics*, 74(1):119–147, 1996.

Leo Krippner. Modifying gaussian term structure models when interest rates are near the zero lower bound. *Reserve Bank of New Zealand Discussion Paper Series*, Mar. 2012.

Leo Krippner. A tractable framework for zero lower bound gaussian term structure models. *Reserve Bank of New Zealand Discussion Paper Series*, 2013a.

Leo Krippner. Measuring the stance of monetary policy in zero lower bound environments. *Economics Letters*, 118:135–138, 2013b.

S. Kullback and R.A. Leibler. On information and sufficiency. 22(1):79–86, Mar. 1951.

Carl E. Walsh. Using monetary policy to stabilize economic activity. In *Financial Stability and Macroeconomic Policy*, pages 245–296. 2010.

John C. Williams. Unconventional monetary policy: Lessons from the past three years. Presentation, Swiss National Bank Research Conference, September 2011.

Jonathan H. Wright. What does monetary policy do to long-term interest rates at the zero lower bound? *NBER Working Papers*, June 2011. Working Paper 17154.

Jing Cynthia Wu and Fan Dora Xia. Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit, and Banking*, 48:253–291, 2016.

A Tables and Figures

Table 1: Bayes Factor Test for Parameter Instability in U.S. Monetary Policy					
Jeffrey's scale for evidence in favor of parameter instability in VAR(4) coefficients					
	Full-Sample $(1960-2015)$				
	$p=4$				
Break in:	Σ	$A^{RX}(L)$	Σ and $A^{RX}(L)$	$A(L)$	Σ and $A(L)$
Federal Funds Rate	-55.8	11.1^{***}	19.4^{***}	-177.6	-178.8
FFR + 10-year T-Bond Rate	-59.2	15.5^{***}	16.5^{***}	-337.7	-343.2
Krippner Shadow Rate	-76.9	-59.7	-77.9	-230.4	-237.9
Wu-Xia Shadow Rate	-69.8	1.3^{*}	-5.7	-180.3	-190.8
	Post-Great Moderation $(1984-2015)$				
	$p=4$				
Break in:	Σ	$A^{RX}(L)$	Σ and $A^{RX}(L)$	$A(L)$	Σ and $A(L)$
Federal Funds Rate	-70.6	9.1^{**}	-18.7	-215.2	-236.4
FFR + 10-year T-Bond Rate	-71.3	10.6^{***}	-17.3	-379.5	-392.9
Krippner Shadow Rate	-34.2	-50.5	-50.2	-253.1	-225.0
Wu-Xia Shadow Rate	-65.6	-6.7	-28.2	-215.5	-233.9

Table 1: Bayes Factor Comparison for Parameter Instability. Comparison of log marginal likelihoods of models allowing for parameter instability in the VAR coefficients and/or covariance matrix, using various measures of the policy rate: 1. the effective nominal FFR, 2. the FFR and a long-term interest rate (10-year Treasury Bond rate) in the VAR, 3. the shadow rate of Krippner (2012), and 4. the shadow rate of Wu and Xia (2014). We compare one model assuming constant parameters in the pre-ZLB and ZLB periods with another allowing for a break in some subset of the model parameters. We compute an adjusted Bayes Factor, as described in the text, to convert it to a scale similar to that used in the well-known Likelihood Ratio tests. Interpretation of the Bayes Factors is as follows: * indicates positive evidence in favor of the model with parameter instability, ** indicates strong evidence, and *** indicates very strong evidence.

Table 2: Kullback-Leibler Divergence		
Benchmark:	**1960:I-2007:IV**	
Alternative:	**Full-Sample: VAR(4)**	**ZLB: VAR(1)**
	(1960 – 2015)	(2008 – 2015)
Federal Funds Rate	**22.92**	135.51
Krippner Shadow Rate	34.15	**85.95**
Wu-Xia Shadow Rate	26.32	90.62
Benchmark:	**1984:I-2007:IV**	
Alternative:	**Post-Great Moderation**	**ZLB**
	(1984 – 2015)	(2008 – 2015)
Federal Funds Rate	**44.12**	81.04
Krippner Shadow Rate	61.76	**38.44**
Wu-Xia Shadow Rate	49.53	65.38

Table 2: Kullback-Leibler Divergence between the posterior distributions of the VAR parameters estimated with the benchmark data, using the FFR, and the posterior distributions when estimating the model using full-sample instrument combining the FFR and shadow rate policy measurements. We compare the distributions to either the full pre-ZLB benchmark (1960:I-2007:IV) or the Post-Great Moderation benchmark (1984:I-2007:IV). When estimating the parameters using only ZLB data, we shorten the VAR to include only one lag of the vector of dependent variables and compare the results to VAR(1) results from the benchmark periods.

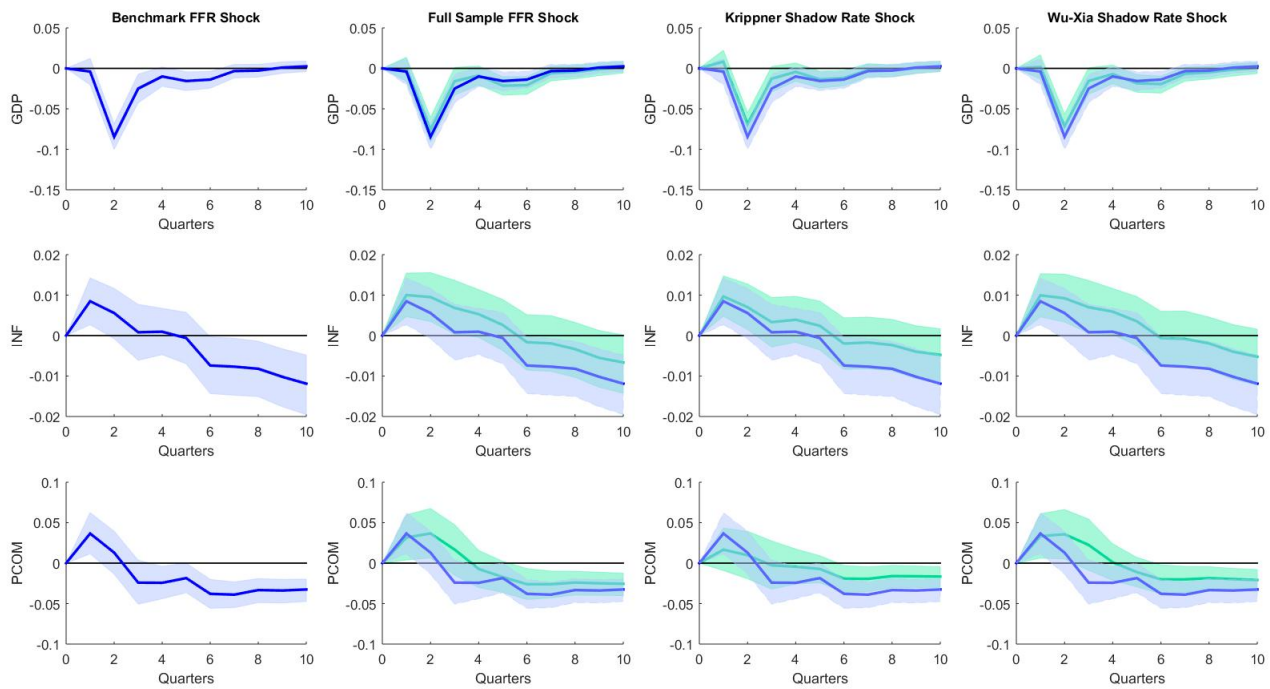

Figure 1: Impulse Responses Over the Full Sample with the FFR and Shadow Rates - First panel: Benchmark IRF (1960:I-2007:IV): VAR(4) with GDP growth, GDP Deflator Inflation, Commodity Price Inflation, and the FFR as the policy instrument. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis-point shock to the policy instrument. Second panel: IRF of full-sample VAR (1960:I-2015:II) using the FFR as the policy instrument through the ZLB period. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, incorporating data through the ZLB period without accounting for any potential changing macroeconomic dynamics. The thick blue line and the blue shaded region replicate the plots from the benchmark model. Third panel: IRF of full-sample VAR using the Krippner (2012) shadow rate as the policy instrument through the ZLB period. Fourth panel: IRF of full-sample VAR using the Wu and Xia (2014) shadow rate as the policy instrument through the ZLB period.

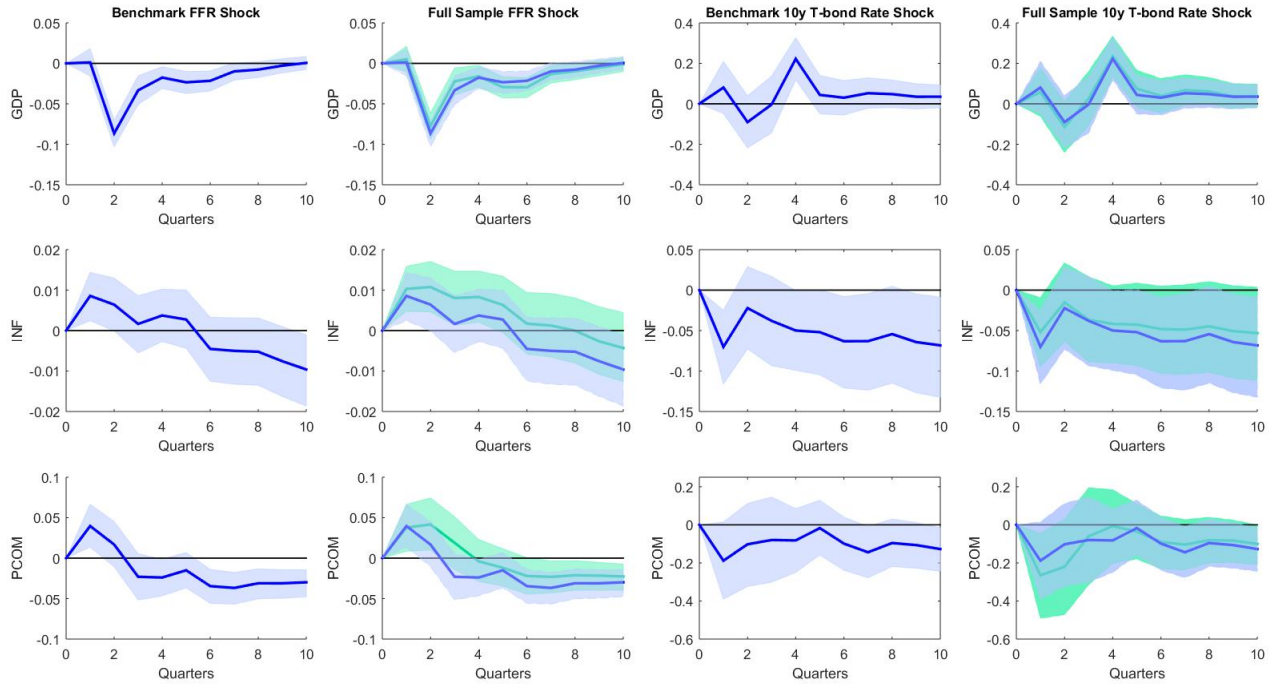

Figure 2: Impulse Responses Over the Full Sample with the FFR and 10-year Treasury Bond Rate - First panel: Benchmark IRF (1960:I-2007:IV): VAR(4) with GDP growth, GDP Deflator Inflation, Commodity Price Inflation, the FFR as the policy instrument, and the 10-year Treasury Bond Rate. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the FFR. Second panel: IRF of full-sample VAR (1960:I-2015:II) using the FFR to represent the policy instrument during the ZLB period. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, incorporating data through the ZLB period without accounting for any potential changing macroeconomic dynamics. The thick blue line and the blue shaded region replicate the plots from the benchmark model. Third panel: Benchmark IRF depicting the responses to a 25 basis point increase in the 10-year Treasury Bond Rate duing the non-ZLB environment. Fourth Panel: IRF of full-sample VAR depicting responses to a 25 basis point increase in the 10-year Treasury Bond Rate. This captures the emphasis on influencing longer-term interest rates with unconventional policy during the ZLB period.

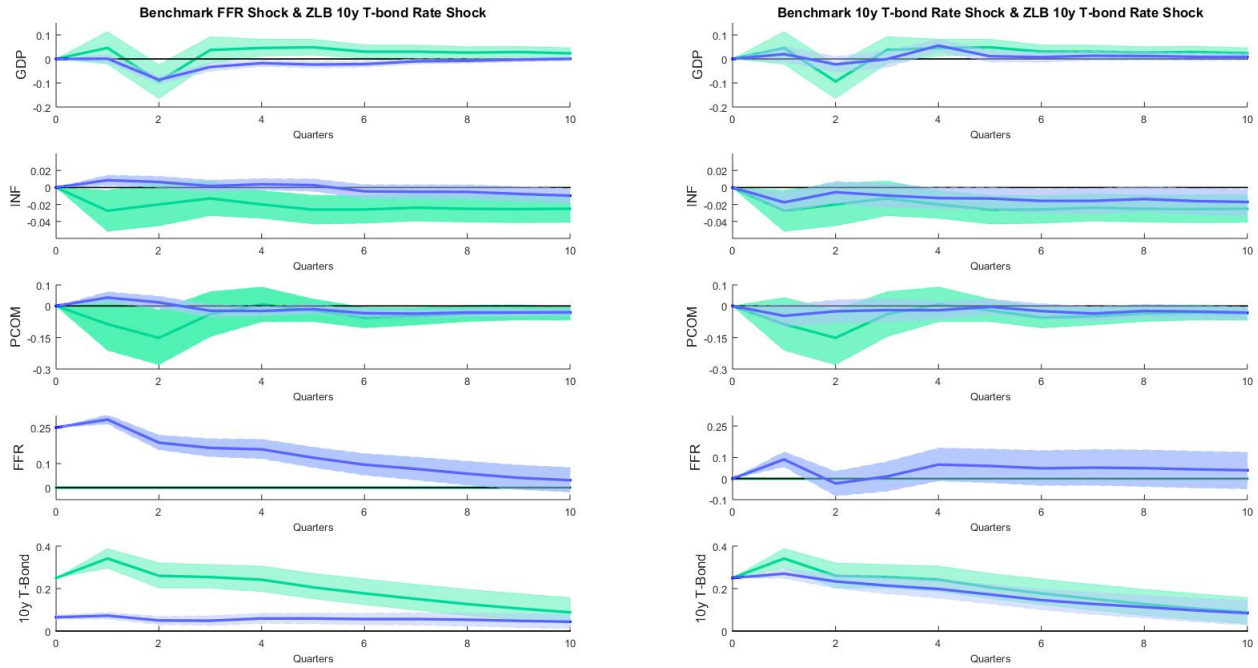

Figure 3: Generalized Impulse Responses in the ZLB Environment - Left panel: VAR(4) with GDP growth, GDP Deflator Inflation, Commodity Price Inflation, the FFR, and the 10-year Treasury Bond Rate. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the FFR in the non-ZLB environment (1960:I-2007:IV). The thick green line and the green shaded area give the median generalized IRF point estimates and 68% posterior coverage, respectively, of the responses to a 25 basis point increase in the 10-year Treasury Bond Rate duing the ZLB environment, restricting the response of the FFR to be zero. Right panel: The blue areas now show the unrestricted responses to a 25 basis point shock to the10-year T-Bond Rate in the non-ZLB period, compared with the green areas showing the restricted generalized IRF responses to a 25 basis point shock to the 10-year T-Bond Rate in the ZLB period.

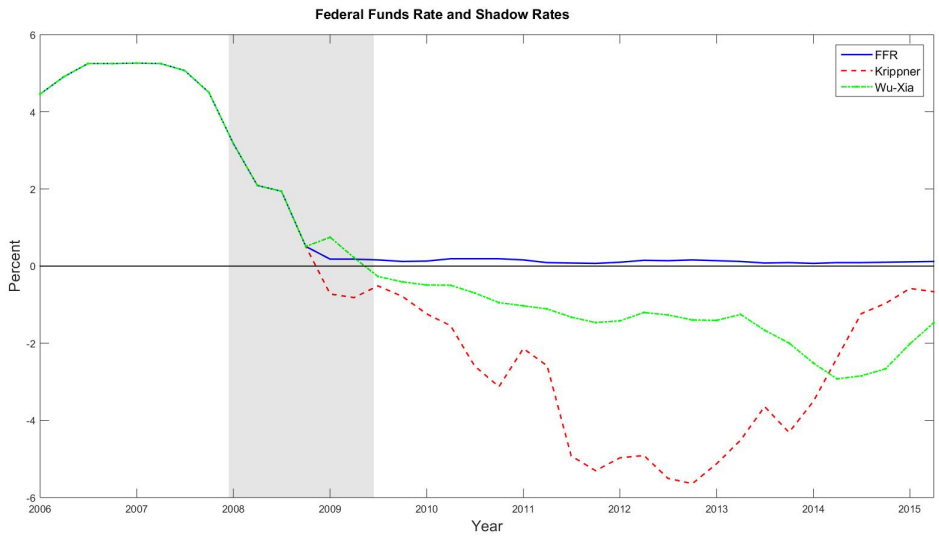

Figure 4: Plot of quarterly nominal FFR and estimated shadow rates over the period from 2006:I - 2015:II. The shaded columns indicated US recessions.

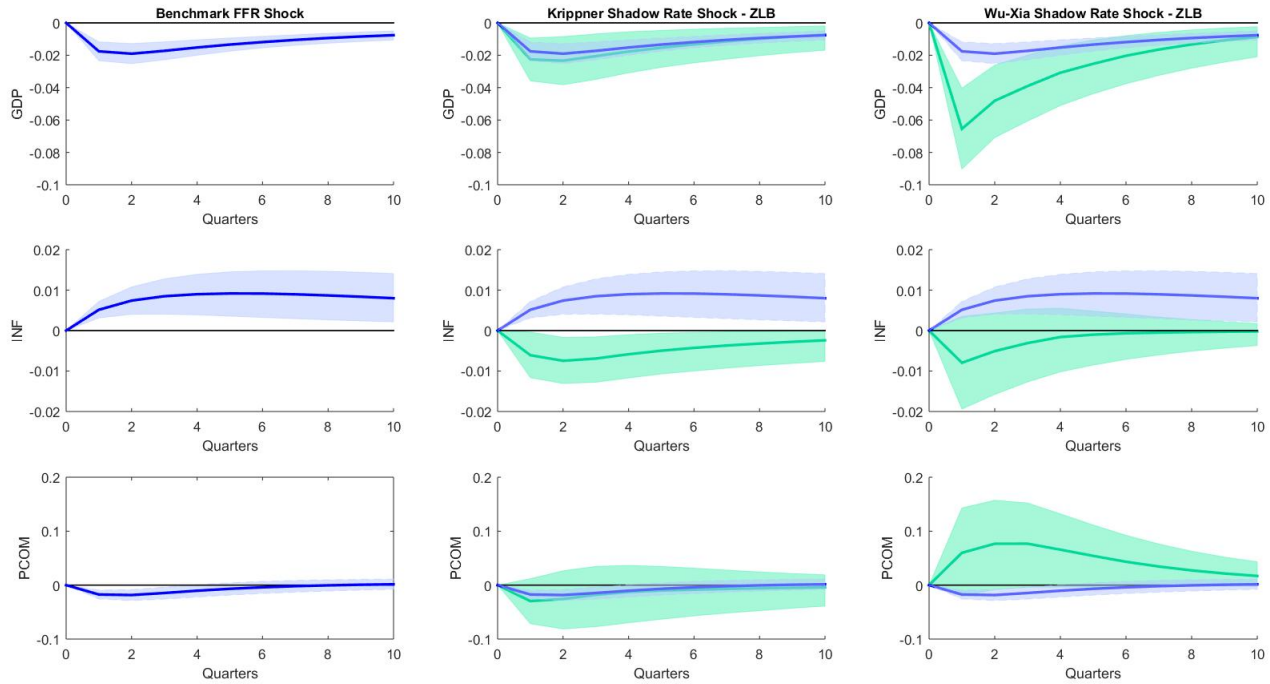

Figure 5: Impulse Responses in the ZLB Environment with Shadow Rates- First panel: Benchmark IRF (1960:I-2007:IV): VAR(1) with GDP growth, GDP Deflator Inflation, Commodity Price Inflation, and the FFR as the policy instrument. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the policy instrument. Second panel: IRF of VAR(1) using data from the ZLB period only, 2008:I-2015:II, and using the Krippner (2012) shadow rate as the policy instrument. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, using only data from the ZLB period. The thick blue line and the blue shaded region replicate the plots from the benchmark model. Third panel: IRF of VAR(1) using data from the ZLB period only and using the Wu and Xia (2014) shadow rate as the policy instrument.