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Abstract

In this paper we provide analytical and Monte Carlo evidence that Chow and Predictive tests can be consistent against alternatives that allow structural change to occur at either end of the sample. Attention is restricted to linear regression models that may have a break in the intercept. The results are based on a novel reparameterization of the actual and potential break point locations. Standard methods parameterize both of these locations as fixed fractions of the sample size. We parameterize these locations as more general integer valued functions. Power at the ends of the sample is evaluated by letting both locations, as a percentage of the sample size, converge to zero or one. We find that for a potential break point function, the tests are consistent against alternatives that converge to zero or one at sufficiently slow rates and are inconsistent against alternatives that converge sufficiently quickly. Monte Carlo evidence supports the theory though large samples are sometimes needed for reasonable power.

Keywords: structural change, Chow test, Predictive test, intercept correction.

J.E.L. categories: C53, C12, C52.

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1. Introduction

In this paper we establish the consistency of tests designed to detect structural breaks in the intercept of a linear regression at the beginning and end of a sample of observations. Our results constrast with comments made within the structural break literature wherein it seems to be common knowledge that these tests are inconsistent against alternatives that allow breaks at the ends of the sample (e.g. Dufour, Ghysels and Hall, 1994). Whether or not structural break tests are consistent is a serious issue since as documented by Stock and Watson (1996), a large percentage of economic variables exhibit structural breaks across time. Whether or not tests for structural breaks in an intercept are consistent is of particular interest for forecasting agents since as noted by Clements and Hendry (1996), structural breaks in the intercept is one of the most common reasons for real-time predictive failure. It is for this reason they recommend the use of intercept-corrections when constructing forecasts.

In this paper we apply novel asymptotics to standard Chow (1960) and Predictive (Ghysels and Hall, 1990) tests for structural change in the intercept of linear regression models. To derive the asymptotic behavior of these tests one has to specify both the location of the actual and the potential break. The standard approach is to define the location of the actual break as $T_B = [\lambda_B T]$ and the potential break as $R = [\lambda T]$ for fixed fractions $0 < \lambda_B, \lambda < 1$of the sample size T. Asymptotics are derived by letting T diverge holding these fractions fixed. Power at the ends of the sample is derived by allowing λ_B to approach either zero or one.

The approach we take to detecting structural breaks at the ends of the sample is methodologically distinct. We parameterize the actual and potential break points as more general integer valued functions. As an example, suppose that we parameterize the location of the actual break as $T_B = T - [(1-\lambda)T^b]$, $0 < b < 1$ and $0 < \lambda < 1$. By using this parameterization we are able to refine the notion of the "end" of the sample to the notion of "local to the end" of the sample. Note that using this parameterization, the ratio T_B/T is allowed to converge to one in the same fashion as considered in previous work. The difference is that we can control the rates of convergence more delicately by allowing b to vary. Similarly, we can parameterize the location of the potential break as $R = T - [(1-\lambda)T^a]$ $0 < a \leq 1$ and $0 < \lambda < 1$. By taking this approach we also allow the location of the potential break to be "local to the end" of the sample. Note that by letting a = 1 we retain the standard method of selecting the potential break point.

Allowing for these more general integer valued functions we first derive the limiting null distributions of Chow and Predictive tests of structural change. We show that these tests are both asymptotically chi-square. As a corollary we are able to derive the limiting distribution of a max-Chow test designed to detect structural breaks at either the beginning or end of the sample.

We then derive the limiting behavior of these tests under the "local to the end" alternatives discussed above. We are able to show that Chow, max-Chow and Predictive tests can be consistent against such alternatives if the choice of potential break is chosen appropriately. We obtain the intuitive result that power increases as the distance between the actual and potential break decreases. Our results make clear that whether or not a test for structural change is consistent depends crucially on the particular definition of the "end" of the sample.

We conclude by examining the finite sample size and power of the tests using Monte Carlo experiments. As the theory suggests power increases the closer the potential break point is to the actual break point. In accordance with that result, for a fixed choice of the potential break we see that power of the test decreases as the actual break gets closer to the ends of the sample.

2. Theory

In this section we provide analytical results for Chow, max-Chow and Predictive tests for structural change. Throughout we maintain that there are no breaks on the interior of the sample and hence if a break occurs it does so at a location T_B satisfying $\lim T_B/T \in \{0, 1\}$. In this way we consider our results as complementary to existing results on testing for structural breaks over the interior of the sample by Chow (1960), Kramer, Ploberger and Alt (1988), Ploberger, Kramer and Kontrus (1989), Ghysels and Hall (1990), Andrews (1993), Sowell (1996) and Bai (1997).

The following notation will be used. Forecasts and/or predictions of the scalar y_{t+1}, $t = 1,\ldots,T$, are generated using a $(k_1 + 1 = k\times 1)$ vector of covariates $x_{2,t} = (x_{1,t}', d_{t,R})' = (1, z_t', d_{t,R})'$. For the potential break location R the scalar $d_{t,R}$ denotes an indicator function that takes the value 1 if $t > R$ and zero otherwise. A dummy variable for the actual break location d_{t,T_B} is defined similarly. The $((k_1 - 1) \times 1)$ vector z_t denotes the subset of predictors (other than a constant) that do not depend upon the sample size T. Since, under the alternative, we treat the actual break location T_B as distinct from the potential break location R, it is useful to define the $(k_1 + 1 = k\times 1)$ vector $x_{3,t} = (x_{1,t}', d_{t,T_B})' = (1, z_t', d_{t,T_B})'$.

In all results we allow T_B, R, $P \equiv T - R$ and $P_B \equiv T - T_B$ to diverge as the sample size T diverges. By using this asymptotic approximation we distinguish our results from others that treat P and P_B as finite while still allowing R and T_B to diverge. For example, using this approximation Andrews (2002) and Andrews and Kim (2003) establish the asymptotic behavior of several Predictive tests for structural change. Asymptotically valid critical values are constructed using parametric subsampling methods. Since they treat P and P_B as finite their tests are inconsistent. Even so, Monte Carlo experiments show the tests can have reasonable power.

We address the Chow and Predictive tests in separate sub-sections. For each we first show that the test can be asymptotically chi-square under the null hypothesis even when the potential break point location is allowed to be at the end of the sample in the sense that either limR/T = 0 or 1. We then proceed to show that the test can be consistent against alternatives that are at the end of the sample in the sense that $\lim T_B/T = 0$ or 1. Before doing so we need first provide a set of assumptions sufficient for the results.

<u>Assumption 1:</u> (a) The DGP satisfies $y_{t+1} = x_{3,t}'\beta_3^* + u_{t+1}$ with $Ex_{3,t}u_{t+1} \equiv Eh_{3,t+1} = 0$ for all t with $\beta_3^* = (\beta_{3,1}^{*\prime}, \beta_{3,2}^*)'$, (b) Under the null hypothesis $\beta_{3,1}^* = \beta_1^*$ and $\beta_{3,2}^* = 0$, (c) Under the alternative hypothesis $\beta_{3,2}^* \neq 0$, (d) The parameters are estimated using OLS.

<u>Assumption 2:</u> Maintain the null hypothesis. (a) $U_t = [u_{t+1}, u_{t+1}z_t, z_t']'$ is covariance stationary with $Eu_{t+1}^2 = \sigma^2$, (b) $E(u_{t+1}| x_{1,t}, u_{t+1-j}\ j \geq 1) = 0$, (c) For some r > 8 U_t is uniformly L^r bounded, (d) For some r > d > 2, U_t is strong mixing with coefficients of size –rd/(r – d), (e) $\lim_{T\to\infty} T^{-1}E(\sum_{t=1}^{T} U_t\text{-}EU_t)(\sum_{t=1}^{T} U_t\text{-}EU_t)' = V < \infty$ is p.d..

<u>Assumption 2′:</u> Maintain the alternative hypothesis and let $U_t = [u_{t+1}, u_{t+1}z_t, z_t']'$. (a) $E(u_{t+1}| x_{1,t}, u_{t+1-j}\ j \geq 1) = 0$, (b) For some r > 8 U_t is uniformly L^r bounded, (c) For some r > d > 2, U_t is strong mixing with coefficients of size –rd/(r – d), (d) $\lim_{T\to\infty} T^{-1}E(\sum_{t=1}^{T} U_t\text{-}EU_t)(\sum_{t=1}^{T} U_t\text{-}EU_t)' = V < \infty$ is p.d..

Assumption 1 is largely notational but is stated explicitly in order to make the relevant environment clear. We restrict attention to breaks in the intercept of OLS estimated linear regression models. As such we can map the alternative into testing whether or not the scalar $\beta_{3,2}^*$

takes the value zero. Assumptions 2 and 2′ are more substantive but are standard. The primary difference between these is that under the null we require that the subset of predictors z_t be covariance stationary. Under the alternative we do not. We make this distinction explicit because we want to handle environments where lagged dependent variables are used as predictors and hence, when a break occurs, they will fail to be covariance stationary. The moment and mixing conditions are sufficient for application of weak convergence results in Hansen (1992). The assumption that the population forecast errors are martingale differences insures that the asymptotic null distribution is pivotal but is not needed for consistency of the test. Note that we allow for the forecast errors to be conditionally heteroskedastic.

2.1 Chow Test

For the Chow test two linear models, $x_{i,t}'\beta_i^*$, i = 1,2, are each estimated using OLS. We denote the residuals associated with models 1 and 2 as $\hat{v}_{1,t+1} = y_{t+1}-x_{1,t}'\hat{\beta}_{1,T}$ and $\hat{v}_{2,t+1} = y_{t+1}-x_{2,t}'\hat{\beta}_{2,T}$ respectively. The actual statistic takes the form

$$(1) \qquad W = T \times \frac{(T^{-1}\sum_{s=1}^{T}\hat{v}_{1,s+1}^2) - (T^{-1}\sum_{s=1}^{T}\hat{v}_{2,s+1}^2)}{(T^{-1}\sum_{s=1}^{T}\hat{v}_{2,s+1}^2)}.$$

The following Theorem provides the null limiting distribution of the Chow test allowing the potential break location to satisfy limR/T $\in$ [0, 1].

Theorem 2.1: Maintain Assumptions 1 and 2 and let z_1 and z_2 denote independent standard normal variates. (i) if limR/T = $\lambda \in (0,1)$ then $W \to_d [(1-\lambda)^{1/2}z_1 - \lambda^{1/2}z_2]^2$, (ii) if limR/T = 0 then $W \to_d z_1^2$, (iii) if limR/T = 1 then $W \to_d z_2^2$.

Note that in cases (i)-(iii) of Theorem 2.1, the test statistic is asymptotically chi-square(1). Case (i) is the standard case in which both z_1 and z_2 contribute to the limiting distribution but since the quadratic form is reduced rank we retain only 1 degree of freedom. Cases (ii) and (iii) differ in that only z_1 or z_2 contribute to the limiting distribution. Regardless, asymptotically valid critical values are readily obtained.

Cases (ii) and (iii) lead to another potentially useful result. Note that these two cases imply that two Chow tests, one constructed at the beginning of the sample and a second constructed at the end of the sample, are asymptotically independent of one another. This allows for the simple construction of a max-Chow test for structural breaks at either the beginning or end of the sample. In the following let W_1 and W_2 denote Chow tests as in (1) with corresponding potential break point location R_1 and R_2 respectively.

Corollary 2.1: Maintain Assumptions 1 and 2 and let z_1 and z_2 denote independent standard normal variates. If $\lim R_1/T = 0$ and $\lim R_2/T = 1$ then $W_{max} = \max[W_1, W_2] \rightarrow_d \max[z_1^2, z_2^2]$.

As we will see in Theorem 2.2, the ability of these tests to detect alternatives at the ends of the sample depends crucially upon the distance between the potential break point and the actual break point. In the notation of Corollary 2.1, W_2 has little power to detect alternatives at the beginning of the sample while W_1 has little power to detect alternatives at the end of the sample. Clearly the max-Chow test overcomes the problem of testing for breaks when one is unwilling to assume that the hypothesized break is known to have occurred at a particular end of the sample.

Choosing asymptotically valid critical values for the max-Chow test is particularly simple using standard methods. Let $F_z(.)$ denote the c.d.f. associated with a chi-square(1) variate and let θ denote the chosen size of the test. Algebra reveals that the $(1-\theta)$-percentile associated with

W_{max} equals $F_z^{-1}((1-\theta)^{1/2})$. These values are easily approximated using chi-square tables for $\theta \in \{0.10, 0.05, 0.01\}$. Constructing asymptotically valid estimates of the p-value associated with the test are also readily constructed using the approximation $1.0 - F_z(W_{max})^2$.

We now turn attention to the power of Chow tests. As previously noted we only consider power against alternatives that are local to the ends of the sample. We derive our results on the power of these tests in two steps. In the first we provide general propositions that show that the relationship between the location of the potential break and the actual break determines whether or not these tests will detect an alternative that is local to the end of the sample. In the second we specialize these results to a particular parameterization of these locations.

Theorem 2.2: Maintain Assumptions 1 and 2′. (i) If $R \leq T_B$ then $W = O_p(R P_B^2/PT)$, (ii) If $R \geq T_B$ then $W = O_p(P T_B^2/RT)$.

Corollary 2.2: Maintain Assumptions 1 and 2′ and let $\lim R_1/T = 0$ and $\lim R_2/T = 1$. (i) If $R_1 \geq T_B$ then $W_{max} = O_p(P_1 T_B^2/R_1 T)$, (ii) If $R_2 \leq T_B$ then $W_{max} = O_p(R_2 P_B^2/P_2 T)$, (iii) If $R_2 \geq T_B$ and $R_1 \leq T_B$ then $W_{max} = O_p(\max[R_1 P_B^2/P_1 T, P_2 T_B^2/R_2 T])$.

Theorem 2.2 and its corollary shows that consistency of the tests depend crucially upon the relationships among R, P, T_B and P_B. The details of these relationships depend however on the parameterization of the location of potential and actual breaks R and T_B. Consider the parameterization mentioned in the introduction and hence let's focus attention on detecting breaks at the end of the sample. There we considered using $T_B = T - [(1-\lambda)T^b]$ and $R = T - [(1-\lambda)T^a]$ for scalars a,b satisfying $0 < a \leq 1$, $0 < b < 1$ and $0 < \lambda < 1$. Note that by choosing this parameterization we are imposing the constraint that $\lim R/T$ is non-zero. We do so since the power associated with such a potential break point location is uniformly lower than that for all other choices of R satisfying $\lim R/T \in (0,1]$ and hence is uninteresting.

Using this parameterization suppose that $R \leq T_B$ with a equal to 1. This corresponds to case (i) in Theorem 2.2 with $\lim R/T \in (0,1)$. To determine consistency note that

$$R P_B^2 / PT = \frac{T-[(1-\lambda)T]}{[(1-\lambda)T]} \frac{[(1-\lambda)T^b]^2}{T} \approx \lambda(1-\lambda)T^{2b-1}$$

and hence the test is consistent against alternatives for which $1/2 < b < 1$ but is inconsistent against alternatives for which $0 < b \leq 1/2$. This is a somewhat surprising result since the case where $\lim R/T \in (0,1)$ is the standard one within the literature but within that literature it is 'known' (see Dufour, Ghysels and Hall (1994) for a discussion) that the test is inconsistent against alternatives at the ends of the sample. Our results show that result depends crucially upon the parameterization of the actual break point.

Now suppose that we again use the same parameterization but with $a \in (0,1)$. Moreover suppose we use a conservative potential break point location so that $R \leq T_B$ and hence $0 < b \leq a < 1$. This also corresponds to case (i) in Theorem 2.2. Similar arguments to that above reveal that $R P_B^2 / PT \approx (1-\lambda)T^{2b-a}$ and hence the test is consistent against alternatives for which $a/2 < b \leq a$ but is inconsistent against alternatives for which $0 < b \leq a/2$. This result makes clear that for any actual break point characterized by b there exists choices of potential break point parameters a such that the test is consistent. Note however that we also obtain the result that for any potential break point location parameter a there exist alternatives that the test will not detect. Since we have assumed here that $b \leq a$ we obtain the intuitive result that the optimal choice of potential break point location is to equate it to the actual break point location and hence $a = b$.

Suppose again that we are using this parameterization and $a \in (0,1)$. Moreover suppose we use an aggressive potential break point location so that $R \geq T_B$ and hence $0 < a \leq b < 1$. This corresponds to case (ii) in Theorem 2.2. Similar arguments to that above reveal that $PT_B^2/RT \approx (1\text{-}\lambda)T^a$ and hence the test is consistent against all alternatives for which $a \leq b < 1$. It is important to note here that this result essentially requires knowledge of the location of the break and hence consistency is not so surprising. The downside of this result is that the power of the test is increasing in a. That is, as we choose smaller values of a in order to insure consistency we are simultaneously lowering the rate at which the test diverges. Since we have assumed here that $a \leq b$ we once again obtain the intuitive result that the optimal choice of potential break point location is to equate it to the actual break point location and hence $a = b$.

2.2 Predictive Test

Consider the Predictive (Ghysels and Hall, 1990) test for structural change in the intercept of a linear regression. This test is equivalent to a test of zero mean prediction error as discussed in the literature on out of-sample testing.[1] For that reason we are able to apply many existing results from West (1996) and West and McCracken (1998) when discussing the null asymptotics of these tests. Not all results follow from this previous work however. In particular, new null asymptotics are derived here for cases in which $\lim R/T = 0$.

For the Predictive tests only model 1 is estimated. We denote the 1-step ahead forecast error from this model as $\hat{u}_{1,t+1} = y_{t+1} - x_{1,t}'\hat{\beta}_{1,t}$ $t = R,\ldots,T$. We consider three distinct means of constructing the forecasts: the recursive, rolling and fixed schemes. Under the recursive scheme, the regression parameters are reestimated with added data as forecasting moves forward through

[1] The Predictive test is more generally equivalent to a test of efficiency in the literature on out-of-sample testing. Given our simple model, this simplifies to a test of zero mean prediction error.

time and hence for $t = R,\ldots,T$, $\hat{\beta}_{1,t}$ depends upon observations $s = 1,..,t$. Under the rolling scheme, only a fixed window of the past R observations are used and hence for $t = R,\ldots,T$, $\hat{\beta}_{1,t}$ depends upon observations $s = t - R + 1,..,t$. Under the fixed scheme, forecasts are constructed in the same fashion as considered by Ghysels and Hall (1990). That is, $\hat{\beta}_{1,t}$ is estimated only once using observations $s = 1,..,R$ so that $\hat{\beta}_{1,t} = \hat{\beta}_{1,R}$ for $t = R,\ldots,T$.

Below we provide the formulas for the predictive tests. Following suggestions made in West (1996) and West and McCracken (1998) we consider Predictive tests of the form

(2a) $$V_R = \frac{(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2}{(P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2)}$$ Recursive, $0 \le \lim R/T \le 1$,

(2b) $$V_L = \frac{(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2}{(2R/3P)(P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2)}$$ Rolling with $0 \le \lim R/T \le 1/2$

(2c) $$V_L = \frac{(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2}{(1-(P/R)^2/3)(P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2)}$$ Rolling with $1/2 \le \lim R/T \le 1$

(2d) $$V_F = \frac{(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2}{(1+P/R)(P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2)}$$ Fixed (Split Sample)

West (1996) considers the null asymptotics for the recursive scheme when $0 \le \lim R/T \le 1$ while West and McCracken (1998) consider null asymptotics for the fixed and rolling schemes when $0 < \lim R/T \le 1$. In Theorem 2.3 we provide new results for the rolling and fixed schemes when $\lim R/T = 0$. This is important since the previous results indicate that the power of Chow tests to detect breaks at the beginning of the sample is improved if we consider potential break points at the beginning of the sample. The same may hold for Predictive tests.

Theorem 2.3: Maintain Assumptions 1 and 2 and let $\lim R/T = 0$. When the rolling scheme is used make the additional assumption that $\lim P/R^{3/2} = 0$. For the rolling and fixed schemes, $V \to_d$ chi-square(1).

Theorem 2.3 fills a gap in the literature on Predictive tests by showing that even if we let the potential break point be chosen close to the beginning of the sample we can still obtain an asymptotically chi-square test of the null. Since this is not a surprising result we now turn attention to the power of Predictive tests. As before we only consider power against single break alternatives that are local to the ends of the sample. We first derive general results on the power of these tests for arbitrary actual break locations T_B and then we specialize these results to a particular parameterization of these locations.

Theorem 2.4: Maintain Assumptions 1 and 2′ and consider the recursive scheme. (i) If $R \leq T_B$ then $V_R = O_p(T_B^2(\sum_{t=T_B}^{T} t^{-1})^2/P)$, (ii) If $R \geq T_B$ then $V_R = O_p(T_B^2(\sum_{t=R}^{T} t^{-1})^2/P)$.

Theorem 2.5: Maintain Assumptions 1 and 2′ and consider the rolling scheme.

(a) Let $0 \leq \lim R/T \leq 1/2$: (i) If $T_B \leq R$ then $V_L = O_p(\max[P/R,\ T_B^4/R^3])$, (ii) If $T_B > \max[R, P]$ then $V_L = O_p(\max[P/R,\ P_B^2(2R-P_B)^2/R^3])$, (iii) If $R < T_B < P$ then $V_L = O_p(\max[P/R, R])$.

(b) Let $1/2 < \lim R/T \leq 1$: (i) If $T_B \geq R$ then $V_L = O_p(P_B^2(2R-P_B)^2/PR^2)$, (ii) If $T_B < \min[R, P]$ then $V_L = O_p(T_B^4/PR^2)$, (iii) If $P < T_B < R$ then $V_L = O_p(P(2T_B-P)^2/R^2)$.

Theorem 2.6: Maintain Assumptions 1 and 2′ and consider the fixed scheme. (i) If $R \geq T_B$ then $V_F = O_p(P\,T_B^2/RT)$, (ii) If $R \leq T_B$ then $V_F = O_p(R\,P_B^2/PT)$.

The results for each forecasting scheme again clearly indicate that the relationships among R, P, T_B and P_B are crucial for determining whether the test will be consistent and at what rate it will diverge if it is. For the fixed scheme the orders of magnitude match those of the Chow test and as such are easily interpretable. Those for the rolling scheme are distinct but straightforward to calculate. This is not the case for the recursive scheme since closed form (asymptotic) approximations for terms like $T_B^2(\sum_{t=T_B}^{T} t^{-1})^2/P$ and $T_B^2(\sum_{t=R}^{T} t^{-1})^2/P$ are not easily obtained. However, one can show that these are at least of the same order as those associated with the fixed schemes. For example, consider case (i) and let $\lim T_B/T = 1$. Algebra reveals that $T_B^2(\sum_{t=T_B}^{T} t^{-1})^2/P \geq T_B^2(\sum_{t=T_B}^{T} T^{-1})^2/P = T_B^2 P_B^2/PT^2 \approx (1-\lambda)T^{2b-a}$ and hence the test is consistent against alternatives for which $a/2 < b \leq a$ but inconsistent against those for which $0 < b \leq a/2$.

3. Monte Carlo Evidence

In this section we provide Monte Carlo evidence on the usefulness of the Chow and Predictive tests when breaks occur at the ends of the sample. For a range of sample sizes and break point locations we evaluate the finite sample size and power of the tests. For brevity we restrict attention to results for the Chow, max-Chow and recursive Predictive test. Unreported simulations indicate that the fixed Predictive test behaves similar to the Chow test under the null[2] while its power is usually lower than that using the recursive scheme. Results for the rolling Predictive test are subject to large size distortions (e.g. rejections frequencies near 60% for a nominally 5% test!). That the rolling test for zero mean prediction error is subject to large size distortions is also found in West and McCracken (1998).

[2] In the proof of Theorem 2.3 we are able to establish that the fixed Predictive test and the Chow test are asymptotically equivalent in probability under the null.

3.1 Monte Carlo Design

The data-generating process has the representation

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \beta_{0t}^* \\ 0 \end{pmatrix} + \begin{pmatrix} 0.3 & 0.1 \\ 0 & 0.5 \end{pmatrix}\begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} u_{y,t} \\ u_{x,t} \end{pmatrix}$$

for i.i.d. mutually independent standard normal disturbance $u_{y,t}$ and $u_{x,t}$. The intercept β_{0t}^* takes the value 0 prior to the time of the break and takes the value 1 afterwards – an increase of a little less than one standard deviation. In the size experiments the parameter β_{0t}^* takes the value 0 throughout the entire sample of $T \in \{100, 200, 400, 800, 1600, 3200\}$ observations. Starting values for y_t and x_t were drawn from the unconditional distribution. The rejection frequency is estimated based upon 5000 replications.

Throughout the experiments we consider actual and potential break point locations using the parameterization suggested in the text. That is, when a break occurs at the end of the sample, it does so at time $T_B = T - [(1-\lambda)T^b]$ for $b \in \{0.1, 0.2, \ldots, 0.8, 0.9\}$ and $\lambda = 0.85$. The choice of parameter $\lambda = 0.85$ is motivated by suggestions in Andrews (1993) while the choice of parameters b was made simply to span the unit interval. For brevity we only provide Monte Carlo evidence on breaks that occur at the end of the sample.

When the potential break location is at the end of the sample it does so at time $R = T - [(1-\lambda)T^a]$ for $a \in \{0.1, 0.2, \ldots, 0.8, 0.9, 1.0\}$ and $\lambda = 0.85$. Note that although we allow the potential break location to lie on the interior of the sample (i.e. a = 1.0) we do not allow the same for the actual break location. When the potential break location occurs at the beginning of the sample it does so at time $R = [(1-\lambda)T^a]$ for similar values of λ and a. When the max-Chow test is constructed we assume that the choice of potential break locations is symmetric in the sense that $R_1 = T - R_2$ and hence the same value of b can be associated with each of the two potential break points. Note that as a practical matter, we model the potential breaks as indicating that a break occurs between time R and R+1 and hence any dummy variables used in the construction of the test take the value zero at time R and take the value 1 at time R+1.

Table 1 reports the numerical values for the actual and potential break locations associated with the parameterization discussed above. These locations vary with the sample size T but do so more slowly than they would if we were using the standard parameterization $[\lambda T]$ for the actual and potential break points. Note that for a fixed value of T, there is no variation in the location of the actual and potential break point locations for small values of a and b. This, along with the finite sample sizes, implies that there are instances in which the tests cannot be numerically constructed. For instance to construct the recursive Predictive test we require that P is at least equal to two and R is at least equal to 1. Similarly, for the Chow test we need both R and P to be at least equal to one. As we will see in the tables there are times where the tests cannot be constructed and these are denoted "N.A.".

When the tests can be constructed we do so using an OLS estimated linear regression model for the variable y_{t+1}. The linear model always includes an intercept and first lags of both y_{t+1} and x_t as predictors. At times the linear model includes the dummy variable $d_{t,R}$ indicating the location of the potential break as necessary for construction of the relevant test statistic.

3.2 Size Results

Table 2 reports the actual size of 5% nominal tests for each of the three test statistics when the potential break location is at the end of the sample. In the first panel we see that for a fixed value of a, the recursive Predictive tests tend to have higher size distortions at the smaller sample sizes but as the sample size increases the size distortions decrease. For example when a = 0.6 the size of the test varies across 0.306, 0.187, 0.118, 0.088, 0.074, 0.065 as the sample size T increases from 100 to 3200.

In the second and third panels of Table 2 we find that the Chow and max-Chow tests are more reasonably sized than are the recursive Predictive tests. These too are subject to some size distortions but are much less pronounced than those above. For example when a = 0.9 the size of the max-Chow test varies across 0.076, 0.060, 0.056, 0.047, 0.050, 0.055 as the sample size T increases from 100 to 3200.

It is important to emphasize that the Chow and max-Chow tests tend to be reasonably well-sized across all potential break locations for any fixed sample size. This is not the case for the recursive Predictive test. The Predictive test becomes increasingly oversized as the potential break location gets closer to the end of the sample. For example when T = 1600 the size of the test varies across 0.050, 0.053, 0.064, 0.065, 0.074, 0.111, 0.313 as the potential break location parameter a decreases from 1.0 to 0.4. In the same experiment the Chow test remains near 5% regardless of the parameter a.

Table 3 reports the actual size of 5% nominal tests for each of the three test statistics when the potential break location is at the beginning of the sample. In each of the three panels the actual size of the test remains near 5% for all sample sizes and potential break locations.[3] This is to be expected for the Chow and max-Chow tests since the test is constructed in a symmetric fashion to those from Table 2. Since the recursive test is constructed differently, and we now have more out-of-sample observations to construct the test (i.e. P is large), it is not surprising that the test is more reasonably sized.

[3] Note that the third panel of Tables 2 and 3 are identical by construction.

3.3 Power Results

Tables 4 - 6, 7 - 9 and 10 - 12 provide evidence on the power of the three tests when we allow the locations of the potential and actual break points to vary while holding the sample sizes fixed at T = 200, 800, 3200. Tables 4, 7, 10 correspond to the recursive Predictive test, Tables 5, 8, 11 correspond to the Chow test and Tables 6, 9, 12 correspond to the max-Chow test. For each of the Predictive and Chow tests we consider power of the test when the potential break point location is correctly chosen at the end of the sample and incorrectly chosen at the beginning of the sample. Below we discuss several of the most important observations.

<u>Location of Actual Break Relative to Potential Break</u>

For each test and for each choice of potential and actual break location, power of the test increases with the sample size. When the potential break location is correctly chosen so that a = b = 0.5 the power of the recursive Predictive test varies across 0.511, 0.557, 0.800 as T increases from 200 to 3200. Similarly, the power of the Chow test varies across 0.295, 0.505, 0.805 as T increases from 200 to 3200. To conclude that the power of the recursive Predictive test can dominate that of the Chow test is misleading due to the previously discussed size distortions.

Recall that the theory from Section 2 suggests that when the actual and potential break points are the same, the power of the test should increase as the actual break point approaches the interior of the sample. For example, in the first panel of Table 7 the power of the recursive Predictive test varies across 0.498, 0.557, 0.785, 0.972, 0.999, 1.000 as the parameter a (= b) varies from 0.4 to 1.0. In the first panel of Table 8 the power of the Chow test varies across 0.167, 0.276, 0.505, 0.803, 0.978, 0.999, 1.000 as the parameter a varies from 0.3 to 1.0.

Similarly, recall that the theory from Section 2 suggests that the power of the test should increase as the potential break point approaches the location of the actual break point. This holds for all sample sizes and all tests. This implies that for any given row we often see power increase and then decrease in a hump-shaped fashion with the hump occurring when a = b. For example in the first panel of Table 4 when b = 0.8 the power of the recursive Predictive test varies across 0.407, 0.649, 0.835, 0.616, 0.439, 0.469 as the parameter a varies from 1.0 to 0.5. In the first panel of Table 8 when b = 0.5 the power of the Chow test varies across 0.061, 0.091, 0.133, 0.233, 0.405, 0.505, 0.276, 0.166 as the parameter a varies from 1.0 to 0.3.

Since the actual breaks are at the end of the sample the theory predicts that when the potential break is at the beginning of the sample we should observe substantially reduced power. In the second panel of Table 10 when b = 0.8 the power of the recursive Predictive test varies across 0.418, 0.391, 0.371, 0.365, 0.363, 0.364, 0.352, 0.352 as the parameter a varies from 1.0 to 0.3. In the second panel of Table 11 when b = 0.8 the power of the Chow test varies across 0.082, 0.052, 0.046, 0.041, 0.036, 0.040, 0.043 as the parameter a varies from 1.0 to 0.4. In each of these cases the power of the test is reduced relative to when the potential break was correctly chosen at the end of the sample rather than the beginning. The reduction in power is most severe for the Chow test with power essentially equal to the size of the test. This is of course the exact situation for which the max-Chow was designed. In Table 12 when b = 0.8 the power of the max-Chow test varies across 0.955, 1.000, 1.000, 1.000, 0.972, 0.647, 0.258 as the parameter a varies from 1.0 to 0.4.

Comparisons Among the Three Tests

There are some clear relationships among the three test statistics in terms of power. Recall that in each of the power simulations the actual break occurs at the end of the sample. In such an environment we expect that the Chow tests, with potential break points chosen at the end of the sample, will have greater power than the max-Chow tests. Since Chow tests constructed at the beginning of the sample have no power to detect breaks at the end of the sample it seems likely that the power of max-Chow tests will be reduced regardless of the fact that asymptotically appropriate critical values are being used. This is usually the case when the actual and potential break point parameters are large. For example, in the first panel of Table 5 and when b = 0.9 we find that the power of the Chow test varies across 0.827, 0.973, 0.790, 0.526, 0.265, 0.189, 0.110 as a changes from 1.0 to 0.4 while in a similar experiment the power of the max-Chow test varies across 0.742, 0.948, 0.693, 0.420, 0.193, 0.143.

Comparing the power of the Chow and the recursive Predictive test is a bit more complicated. For each of the Chow and max-Chow tests the actual size of the test was always close to the nominal size of 5%. That is not the case for the recursive Predictive test. As the potential break point location gets closer to the end of the sample the size distortions become quite large. In such a situation it is not clear how to evaluate whether one test is more powerful than the other. On the other hand, when the break location is not so close to the end of the sample (a = 1.0, 0.9, 0.8) the size of the Predictive tests are reasonable. In these situations the power of the Chow test dominates that of the Predictive test so long as the actual break point is not too near the end of the sample. In Table 4 when a = 0.9 the power of the Predictive test varies across 0.797, 0.960, 0.766, 0.548, 0.392, 0.436 as b varies from 1.0 to 0.5. In Table 5 the Chow test varies across 0.827, 0.973, 0.790, 0.526, 0.265, 0.189 in the same experiment. If we make the comparison with T = 3200, Tables 10 and 11 indicate that the distinction is moot since each rejects the null 100% of the time for large break parameters a and b.

When the potential break locations are at the beginning of the sample the recursive Predictive test clearly dominates the Chow test so long as the actual break is not too near the end of the sample. That is, in the second panel of Tables 4, 7 and 10 the rows associated with b = 0.8 and

0.9 are uniformly larger than those associated with Tables 5, 8 and 11. Once again the Chow test is not as robust as the recursive Predictive test to large deviations of the potential and actual break locations. As soon as we consider the max-Chow results in Tables 6, 9 and 12 however, we find that it usually the case that the max-Chow dominates the recursive Predictive test.

4. Conclusion

In this paper we provide a set of refined asymptotics for standard Chow and Predictive tests for structural change in the intercept from a linear regression. By treating the locations of the actual and potential break point locations as general integer valued functions we are able to establish the behavior of these tests under the null of no breaks and under a collection of alternatives that allow for breaks to occur "local to the end" of the sample. In particular we establish that the tests are asymptotically chi-square under the null and can be consistent against a range of alternatives. In doing so we also establish the rates at which the tests diverge against these alternatives. Monte Carlo simulations verify the asymptotic results and suggests that the tests can be useful in large samples.

5. Appendix

Theorem 2.1: Maintain Assumptions 1 and 2 and let z_1 and z_2 denote independent standard normal variates. (i) if $\lim R/T = \lambda \in (0,1)$ then $W \to_d [(1-\lambda)^{1/2} z_1 - \lambda^{1/2} z_2]^2$, (ii) if $\lim R/T = 0$ then $W \to_d z_1^2$, (iii) if $\lim R/T = 1$ then $W \to_d z_2^2$.

Proof of Theorem 2.1: It is straightforward to show that $T^{-1}\sum_{t=1}^{T} \hat{v}_{2,t+1}^2 \to_p \sigma^2$ and hence we proceed immediately to the numerator. Note that for $M(T) = -JB_1(T)J' + B_2(T)$ and $F_1 = (T^{-1}\sum_{t=1}^{T} z_t z_t' - (T^{-1}\sum_{t=1}^{T} z_t)(T^{-1}\sum_{t=1}^{T} z_t'))^{-1}$,

$$\sum_{t=1}^{T}(\hat{v}_{1,t+1}^2 - \hat{v}_{2,t+1}^2) = TH_2'(T)M(T)H_2(T)$$

$$= [1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)]^{-1}$$

$$\times \{[(\frac{PR}{T^2})^{1/2}(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1((T^{-1/2}\sum_{t=1}^{T} u_{t+1}z_t) - (T^{-1/2}\sum_{t=1}^{T} u_{t+1})(T^{-1}\sum_{t=1}^{T} z_t))]$$

$$+ (\frac{P}{R})^{1/2}[T^{-1/2}\sum_{t=1}^{T} u_{t+1}] - (\frac{T}{R})^{1/2}[P^{-1/2}\sum_{t=R}^{T} u_{t+1}]\}^2$$

$$= [1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)]^{-1}$$

$$\times \{[(\frac{PR}{T^2})^{1/2}(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1((T^{-1/2}\sum_{t=1}^{T} u_{t+1}z_t) - (T^{-1/2}\sum_{t=1}^{T} u_{t+1})(T^{-1}\sum_{t=1}^{T} z_t))]$$

$$+ [(\frac{P}{T})^{1/2}[R^{-1/2}\sum_{t=1}^{R} u_{t+1}] - (\frac{R}{T})^{1/2}[P^{-1/2}\sum_{t=R}^{T} u_{t+1}]]\}^2.$$

Assumption 2 suffices for each of F_1, $T^{-1/2}\sum_{t=1}^{T} u_{t+1}z_t$, $T^{-1/2}\sum_{t=1}^{T} u_{t+1}$ and $T^{-1}\sum_{t=1}^{T} z_t$ to be $O_p(1)$. Moreover we obtain $P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t = o_p(1)$ since under the null, z_s is covariance stationary. By continuity we then obtain

$$\sum_{t=1}^{T}(\hat{v}_{1,t+1}^2-\hat{v}_{2,t+1}^2) = \{(\frac{P}{T})^{1/2}[R^{-1/2}\sum_{t=1}^{R}u_{t+1}] - (\frac{R}{T})^{1/2}[P^{-1/2}\sum_{t=R}^{T}u_{t+1}]\}^2 + o_p(1).$$

Given Assumption 2, Theorem 3.2 of Hall and Heyde (1980) implies that each of $R^{-1/2}\sum_{t=1}^{R}u_{t+1}$ and $P^{-1/2}\sum_{t=R}^{T}u_{t+1}$ are asymptotically normal with zero mean and variance σ^2. Since the forecast errors are uncorrelated we also know that these two terms are asymptotically independent. Choosing limR/T appropriately we obtain cases (i) - (iii) in the Theorem.

Corollary 2.1: Maintain Assumptions 1 and 2 and let z_1 and z_2 denote independent standard normal variates. If $\lim R_1/T = 0$ and $\lim R_2/T = 1$ then $W_{max} = \max[W_1, W_2] \rightarrow_d \max[z_1^2, z_2^2]$.

Proof of Corollary 2.1: Proof follows directly from the fact that $R_1^{-1/2}\sum_{t=1}^{R_1}u_{t+1}$ and $P_2^{-1/2}\sum_{t=R_2}^{T}u_{t+1}$ are each asymptotically normal and uncorrelated.

Theorem 2.2: Maintain Assumptions 1 and 2′. (i) If $R \geq T_B$ then $W = O_p(PT_B^2/RT)$, (ii) If $R \leq T_B$ then $W = O_p(RP_B^2/PT)$.

Proof of Theorem 2.2: It is straightforward to show that $T^{-1}\sum_{t=1}^{T}\hat{v}_{2,t+1}^2 \rightarrow_p \omega > 0$ and hence we proceed immediately to the numerator. Note that for $F_1 = (T^{-1}\sum_{t=1}^{T}z_tz_t'-(T^{-1}\sum_{t=1}^{T}z_t)(T^{-1}\sum_{t=1}^{T}z_t'))^{-1}$, $M(T) = -JB_1(T)J' + B_2(T)$ and $F_2 = (\frac{T^2}{RP})[1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)'F_1(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)]^{-1}$,

$$\sum_{t=1}^{T}(\hat{v}_{1,t+1}^2-\hat{v}_{2,t+1}^2) = TH_2'(T)M(T)H_2(T) + 2TH_2'(T)(-JB_1(T)J'B_3^{-1}(T)+B_2(T)D_{23}(T))\beta_3^*$$
$$+ T\beta_3^{*'}\begin{pmatrix} 0 & 0 \\ 0 & ((\frac{\min(P,P_B)}{T})F_2^{1/2}-(\frac{P}{T})(\frac{P_B}{T})F_2^{1/2}(P^{-1}\sum_{t=R}^{T}x_{1,t}')B_1(T)(P_B^{-1}\sum_{t=T_B}^{T}x_{1,t}))^2\end{pmatrix}\beta_3^*.$$

That $TH_2'(T)M(T)H_2(T) = O_p(1)$ follows from Theorem 2.1. To show that the second term $2TH_2'(T)(-JB_1(T)J'B_3^{-1}(T)+B_2(T)D_{23}(T))\beta_3^*$ is of lower order than the third term follows from similar arguments to those used to derive the order of the third term. We therefore proceed to deriving the order of the third term.

Recall that under the alternative hypothesis $\beta_{3,2}^* \neq 0$. If we substitute in the definition of F_2 and compute the term $(P^{-1}\sum_{t=R}^{T} x_{1,t}')B_1(T)(P_B^{-1}\sum_{t=T_B}^{T} x_{1,t})$ the third term above can be rewritten as

$$T(\beta_{3,2}^*)^2((\frac{\min(P,P_B)}{T})F_2^{1/2}-(\frac{P}{T})(\frac{P_B}{T})F_2^{1/2}(P^{-1}\sum_{t=R}^{T} x_{1,t}')B_1(T)(P_B^{-1}\sum_{t=T_B}^{T} x_{1,t}))^2$$

$$= (\beta_{3,2}^*)^2 \times [1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)]^{-1} \times$$

$$(\frac{T^3}{RP})[\frac{\min(P,P_B)}{T}-(\frac{PP_B}{T^2})-(\frac{PP_BRT_B}{T^4})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P_B^{-1}\sum_{t=T_B}^{T} z_t - T_B^{-1}\sum_{t=1}^{T_B-1} z_t)]^2.$$

If we now consider the two cases the above term can be rewritten as

$$T(\beta_{3,2}^*)^2((\frac{\min(P,P_B)}{T})F_2^{1/2}-(\frac{P}{T})(\frac{P_B}{T})F_2^{1/2}(P^{-1}\sum_{t=R}^{T} x_{1,t}')B_1(T)(P_B^{-1}\sum_{t=T_B}^{T} x_{1,t}))^2$$

$$= \begin{cases} (\frac{PT_B^2}{RT})(\beta_{3,2}^*)^2[\frac{(1-(\frac{P_BR}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P_B^{-1}\sum_{t=T_B}^{T} z_t - T_B^{-1}\sum_{t=1}^{T_B-1} z_t))^2}{1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)}] & P \leq P_B \\ (\frac{P_B^2R}{PT})(\beta_{3,2}^*)^2[\frac{(1-(\frac{PT_B}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P_B^{-1}\sum_{t=T_B}^{T} z_t - T_B^{-1}\sum_{t=1}^{T_B-1} z_t))^2}{1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)'F_1(P^{-1}\sum_{t=R}^{T} z_t - R^{-1}\sum_{t=1}^{R-1} z_t)}] & P_B \leq P \end{cases}.$$

Assumption 2′ suffices for each of F_1, $P^{-1}\sum_{t=R}^{T} z_t$, $R^{-1}\sum_{t=1}^{R-1} z_t$, $P_B^{-1}\sum_{t=T_B}^{T} z_t$ and $T_B^{-1}\sum_{t=1}^{T_B-1} z_t$ to be $O_p(1)$. By continuity we then know that each of

$$\frac{(1-(\frac{P_BR}{T^2})(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)'F_1(P_B^{-1}\sum_{t=T_B}^{T}z_t-T_B^{-1}\sum_{t=1}^{T_B-1}z_t))^2}{1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)'F_1(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)}$$

and

$$\frac{(1-(\frac{PT_B}{T^2})(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)'F_1(P_B^{-1}\sum_{t=T_B}^{T}z_t-T_B^{-1}\sum_{t=1}^{T_B-1}z_t))^2}{1-(\frac{PR}{T^2})(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)'F_1(P^{-1}\sum_{t=R}^{T}z_t-R^{-1}\sum_{t=1}^{R-1}z_t)}$$

are $O_p(1)$ with strictly positive probability limits. This implies that the order of magnitude of the statistic is determined by the lead terms in the right-hand side of the previous equality and we have the desired result.

Corollary 2.2: Maintain Assumptions 1 and 2′ let $\lim R_1/T = 0$ and $\lim R_2/T = 1$. (i) If $R_1 \geq T_B$ then $W_{max} = O_p(P_1 T_B^2/R_1T)$, (ii) If $R_2 \leq T_B$ then $W_{max} = O_p(R_2 P_B^2/P_2T)$, (iii) If $R_2 > T_B$ and $R_1 < T_B$ then $W_{max} = O_p(\max[R_1 P_B^2/P_1T, P_2 T_B^2/R_2T])$.

Proof of Corollary 2.2: (i) From Theorem 2.2 we know that $W_1 = O_p(P_1 T_B^2/R_1T)$ while $W_2 = O_p(P_2 T_B^2/R_2T)$. The result is immediate since $R_2 > R_1$ implies $P_1/R_1 > P_2/R_2$.

(ii) From Theorem 2.2 w know that $W_1 = O_p(R_1 P_B^2/P_1T)$ while $W_2 = O_p(R_2 P_B^2/P_2T)$. The result is immediate since $R_2 > R_1$ implies $R_1/P_1 < R_2/P_2$.

(iii) The result is trivial given continuity of the max[.,.] function.

Theorem 2.3: Maintain Assumptions 1 and 2 and let $\lim R/T = 0$. When the rolling scheme is used make the additional assumption that $\lim P/R^{3/2} = 0$. For the rolling and fixed schemes, $V \rightarrow_d$ chi-square(1).

Proof of Theorem 2.3: Consider the fixed scheme with limR/T = 0. It is straightforward to show that $P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2 \to_p \sigma^2$ and hence we proceed immediately to the numerator. Since $(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2/(1+P/R) = ((R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2$ we turn our attention to the limiting distribution of $(R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1}$. Adding and subtracting appropriate terms we obtain

$$(R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} = (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}u_{t+1} - (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}x_{1,t}'B_1(R)H_1(R)$$

$$= (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}u_{t+1} - (RP/T)^{1/2}(P^{-1}\sum_{t=R}^{T}x_{1,t}')B_1(R)H_1(R)$$

$$= (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}u_{t+1} - (P/T)^{1/2}(R^{-1/2}\sum_{t=1}^{R-1}u_{t+1})$$

$$+ (P/T)^{1/2}(P^{-1}\sum_{t=R}^{T}z_t\text{-}R^{-1}\sum_{t=1}^{R}z_t)'F_{1,R}((R^{-1/2}\sum_{t=1}^{R}u_{t+1})(R^{-1}\sum_{t=1}^{R}z_t)\text{-}(R^{-1/2}\sum_{t=1}^{R}u_{t+1}z_t))$$

where $F_{1,R} = (R^{-1}\sum_{t=1}^{R-1}z_tz_t'\text{-}(R^{-1}\sum_{t=1}^{R-1}z_t)(R^{-1}\sum_{t=1}^{R-1}z_t'))^{-1}$. Assumption 2 suffices for each of $F_{1,R}$, $R^{-1/2}\sum_{t=1}^{R-1}u_{t+1}z_t$, $R^{-1/2}\sum_{t=1}^{R-1}u_{t+1}$ and $R^{-1}\sum_{t=1}^{R-1}z_t$ to be $O_p(1)$. Moreover we obtain $P^{-1}\sum_{t=R}^{T}z_t$ − $R^{-1}\sum_{t=1}^{R-1}z_t = o_p(1)$ since under the null, z_t is covariance stationary. By continuity we then obtain

$$((\frac{R}{T})^{1/2}P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2 = \{(\frac{P}{T})^{1/2}[R^{-1/2}\sum_{t=1}^{R-1}u_{t+1}] - (\frac{R}{T})^{1/2}[P^{-1/2}\sum_{t=R}^{T}u_{t+1}]\}^2 + o_p(1).$$

The result then follows from Theorem 2.1 since this expansion is identical to that for the Chow test. We therefore obtain the additional result that the fixed Predictive test and the Chow are asymptotically equivalent in probability.

Consider the rolling scheme with limR/T = 0. It is straightforward to show that $P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2$ $\to_p \sigma^2$ and hence we proceed immediately to the numerator. Since $(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2/(2R/3P) =$

$((3/2)^{1/2}R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2$ we turn our attention to the limiting distribution of $R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1}$.

Adding and subtracting appropriate terms we obtain

$$R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} = R^{-1/2}\sum_{t=R}^{T}u_{t+1} - R^{-1/2}\sum_{t=R}^{T}x'_{1,t}B_1(t)H_1(t). \tag{1}$$

Consider the second right hand side term in (1). Note that under the null $(Ex_{1,t}x'_{1,t})^{-1} = B_1$ and $Ex_{1,t} = F$ since z_t is covariance stationary. Using the identities $B_1(t) = [B_1(t) - B_1] + B_1$ and $x_{1,t} = [x_{1,t} - F] + F$ we obtain

$$\begin{aligned} &R^{-1}\sum_{t=R}^{T}x'_{1,t}B_1(t)(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}) \\ &\quad = R^{-3/2}FB_1\sum_{t=R}^{T}(\sum_{s=t-R+1}^{t}h_{1,s+1}) + R^{-1}\sum_{t=R}^{T}(x_{1,t}\text{-}F)'B_1(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}) \qquad (2) \\ &\qquad + R^{-1}\sum_{t=R}^{T}F'[B_1(t)\text{-}B_1](R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}) + R^{-1}\sum_{t=R}^{T}(x_{1,t}\text{-}F)'(B_1(t)\text{-}B_1)(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}). \end{aligned}$$

We will now show that the latter three right-hand side terms in (2) are $o_p(1)$. For both the second and third terms note that by taking absolute values we obtain

$$\begin{aligned} &|R^{-1}\sum_{t=R}^{T}F'[B_1(t)\text{-}B_1](R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1})| \\ &\qquad \le (\frac{P}{R^{3/2}})k^2(|F|)(\sup_t R^{1/2}|B_1(t)\text{-}B_1|)(\sup_t|R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}|) \\ &|R^{-1}\sum_{t=R}^{T}(x_{1,t}\text{-}F)'B_1(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1})| \\ &\qquad \le (\frac{P}{R^{3/2}})k^2(P^{-1}\sum_{t=R}^{T}|x_{1,t}\text{-}F|)(|B_1|)(\sup_t|R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}|). \end{aligned}$$

Assumption 2 is sufficient to show that each of $P^{-1}\sum_{t=R}^{T}|x_{1,t}\text{-}F|$, $\sup_t R^{1/2}|B_1(t)\text{-}B_1|$ and $\sup_t|R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}|$ are $O_p(1)$. Since k^2 is finite, the result follows since $P/R^{3/2}$ is o(1). For the fourth term in (2) note that

$$R^{-1}\sum_{t=R}^{T}(x_{1,t}\text{-}F)'(B_1(t)\text{-}B_1)(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1})$$

$$= (\frac{P}{R^2})^{1/2}[\sum_{t=R}^{T}(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}')\otimes(P^{-1/2}(x_{1,t}\text{-}F))]\text{vec}(B).$$

That $\sum_{t=R}^{T}(R^{-1/2}\sum_{s=t-R+1}^{t}h_{1,s+1}')\otimes(P^{-1/2}(x_{1,t}\text{-}F))$ is Op(1) follows from Assumption 2 and Theorem 3.1 of Hansen (1992). The result follows since P/R^2 is o(1) and B is finite.

Now return to the expression in (1). Note that since $x_{1,t}$ contains 1 in the first element, FB_1 = (-1, 0, 0, …, 0). Since this also implies that the first element of $h_{1,s+1}$ is u_{s+1}, we obtain

$$R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} = R^{-1/2}\sum_{t=R}^{T}u_{t+1} - R^{-3/2}\sum_{t=R}^{T}(\sum_{s=t-R+1}^{t}u_{s+1}) + o_p(1).$$

Now decompose $\sum_{t=R}^{T}(\sum_{s=t-R+1}^{t}u_{s+1})$ into the terms A_1, A_2 and A_3 as in the technical appendix in West (1996) but divide by $R^{3/2}$:

$$R^{-3/2}A_1 = R^{-3/2}[u_1 + 2u_2 + \ldots + Ru_R],$$

$$R^{-3/2}A_2 = R^{-1/2}[u_{R+1} + u_{R+2} + \ldots + u_P],$$

$$R^{-3/2}A_3 = R^{-3/2}[(R\text{-}1)u_{P+1} + (R\text{-}2)u_{P+2} + \ldots + u_T].$$

The term $R^{-3/2}A_2$ is precisely the first (P-R) terms in $R^{-1/2}\sum_{t=R}^{T}u_{t+1}$. Hence $R^{-1/2}\sum_{t=R}^{T}u_{t+1}$ − $R^{-3/2}\sum_{t=R}^{T}(\sum_{s=t-R+1}^{t}u_{s+1})$ equals $R^{-3/2}A_1$ + [$R^{-1/2}\sum_{t=P+1}^{T}u_{t+1} - R^{-3/2}A_3$]. Algebra then reveals that

$$R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} = -R^{-3/2}\sum_{j=1}^{R} ju_j + R^{-3/2}\sum_{j=1}^{R-1} ju_{P+j} + o_p(1).$$

To show that the limiting variance of the sum is (2/3) first note that each of the two terms is uncorrelated with the other. Taking expectations and rearranging terms we find that $\text{Var}(-R^{-3/2}\sum_{j=1}^{R} ju_j) = \sigma^2 R^{-1}\sum_{s=1}^{R}(s/R)^2 = \text{Var}(R^{-3/2}\sum_{j=1}^{R-1} ju_{P+j}) + o(1)$. If we add the two components we obtain $\text{Var}(-R^{-3/2}\sum_{j=1}^{R} ju_j + R^{-3/2}\sum_{j=1}^{R-1} ju_{P+j}) = 2\sigma^2 R^{-1}\sum_{s=1}^{R}(s/R)^2 + o(1)$. Using an argument akin to that in West (1996) we know that $R^{-1}\sum_{s=1}^{R}(s/R)^2 \to \int_0^1 j^2 dj = 1/3$. From this we obtain the desired result.

We must then show that $-R^{-3/2}\sum_{j=1}^{R} ju_j + R^{-3/2}\sum_{j=1}^{R-1} ju_{P+j}$ is asymptotically normal. Define the sequence $Z_{t,T} = -tR^{-3/2}u_{t+1}$ for $1 \le t \le R$, $Z_{t,T} = 0$ for $R+1 \le t \le P$ and $Z_{t,T} = R^{-3/2}(T-t)u_{t+1}$ for $P+1 \le t \le T+1$. Define $\Omega_T \equiv \text{Var}(\sum_{t=1}^{T} Z_{t,T})$ and $X_{T,t} \equiv \Omega_T^{-1/2} Z_{t,T}$. Then Theorem 3.1 of Wooldridge and White (1998) implies that $\sum_{t=1}^{T} X_{T,t}$ is limiting standard normal. Since $\lim\Omega_T = \Omega = (2/3)$ is p.d. we conclude that $R^{-1/2}\sum_{t=R}^{T}\hat{u}_{t+1}$ is limiting normal with asymptotic variance equal to (2/3).

Theorem 2.4: Maintain Assumptions 1 and 2′ and consider the recursive scheme. (i) If $R \le T_B$ then $V_R = O_p(T_B^2(\sum_{t=T_B}^{T} t^{-1})^2/P)$, (ii) If $R \ge T_B$ then $V_R = O_p(T_B^2(\sum_{t=R}^{T} t^{-1})^2/P)$.

Proof of Theorem 2.4: It is straightforward to show that $P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2 \to_p \omega > 0$ and hence we proceed immediately to the numerator. Adding and subtracting terms we obtain the expansion

$$\begin{aligned}
P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} &= P^{-1/2}\sum_{t=R}^{T} u_{t+1} - P^{-1/2}\sum_{t=R}^{T} x_{1,t}'B_1(t)H_1(t) - P^{-1/2}\sum_{t=R}^{T} x_{3,t}'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^* \\
&= P^{-1/2}\sum_{t=R}^{T} u_{t+1} - P^{-1/2}\sum_{t=R}^{T} x_{1,t}'B_1(t)H_1(t) - P^{-1/2}\sum_{t=R}^{T}(x_{3,t}\text{-}Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^* \\
&\quad - P^{-1/2}\sum_{t=R}^{T}(Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*.
\end{aligned}$$

That the first three right-hand side terms above are bounded in probability follows from arguments like those in Theorem 2.3. We now show that the fourth term is the appropriate order.

Recall that $x_{3,t}$ contains the term d_{t,T_B} in the final position. This term takes the value one if t > T_B and zero otherwise. We must therefore consider separately the cases in which R > T_B or R ≤ T_B. Doing so we find that $Ex_{3,t} = (Ex_{1,t}', d_{t,T_B})'$ while

$$JB_1(t)J'B_3^{-1}(t)\text{-}I = \begin{cases} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} & T_B > t \\ \begin{pmatrix} 0 & B_1(t)(t^{-1}\sum_{s=T_B}^{t} x_{1,s}) \\ 0 & -1 \end{pmatrix} & T_B \le t \end{cases}.$$

Algebra then reveals that

$$P^{-1/2}\sum_{t=R}^{T}(Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*$$

$$= \begin{cases} (T_B P^{-1/2}\sum_{t=T_B}^{T} t^{-1})\beta_{3,2}^* \\ \quad -P^{-1/2}\sum_{t=T_B}^{T}(1\text{-}T_B/t)(t^{-1}\sum_{s=1}^{t} z_s\text{-}Ez_t)'F_{1,t}(t^{-1}\sum_{s=1}^{t} z_s\text{-}(t\text{-}T_B)^{-1}\sum_{s=T_B}^{t} z_s)\beta_{3,2}^* & R < T_B \\ (T_B P^{-1/2}\sum_{t=R}^{T} t^{-1})\beta_{3,2}^* \\ \quad -P^{-1/2}\sum_{t=R}^{T}(1\text{-}T_B/t)(t^{-1}\sum_{s=1}^{t} z_s\text{-}Ez_t)'F_{1,t}(t^{-1}\sum_{s=1}^{t} z_s\text{-}(t\text{-}T_B)^{-1}\sum_{s=T_B}^{t} z_s)\beta_{3,2}^* & R \ge T_B \end{cases}.$$

Where $F_{1,t} = (t^{-1}\sum_{s=1}^{t} z_s z_s'\text{-}(t^{-1}\sum_{s=1}^{t} z_s)(t^{-1}\sum_{s=1}^{t} z_s'))^{-1}$. For each of the possible two right-hand side terms the lead term has the highest order. Cases (i) and (ii) follow from squaring those terms.

Theorem 2.5: Maintain Assumptions 1 and 2′ and consider the rolling scheme. (a) Let 0 ≤ limR/T ≤ 1/2: (i) If T_B ≤ R then $V_L = O_p(\max[P/R,\ T_B^4/R^3])$, (ii) If T_B > max[R, P] then $V_L = O_p(\max[P/R,\ P_B^2(2R\text{-}P_B)^2/R^3])$, (iii) If R < T_B < P then $V_L = O_p(\max[P/R, R])$. (b) Let 1/2 < limR/T ≤ 1: (i) If T_B ≥ R then $V_L = O_p(P_B^2(2R\text{-}P_B)^2/PR^2)$, (ii) If T_B < min[R, P] then V_L =

$O_p(T_B^4/PR^2)$, (iii) If $P < T_B < R$ then $V_L = O_p(P(2T_B\text{-}P)^2/R^2)$.

Proof of Theorem 2.5: In either case (a) or (b) it is straightforward to show that $P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2$ $\to_p \omega > 0$ and hence we proceed immediately to the numerator.

(a) It suffices to show that $(R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2$ satisfies cases (i), (ii) and (iii). Adding and subtracting terms we obtain the expansion

$$R^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} = R^{-1/2}\sum_{t=R}^{T}u_{t+1} - R^{-1/2}\sum_{t=R}^{T}x_{1,t}'B_1(t)H_1(t) - R^{-1/2}\sum_{t=R}^{T}x_{3,t}'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*$$

$$= (P/R)^{1/2}P^{-1/2}\sum_{t=R}^{T}u_{t+1} - (P/R)^{1/2}P^{-1/2}\sum_{t=R}^{T}x_{1,t}'B_1(t)H_1(t) \tag{3}$$

$$- (P/R)^{1/2}P^{-1/2}\sum_{t=R}^{T}(x_{3,t}\text{-}Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*$$

$$- R^{-1/2}\sum_{t=R}^{T}(Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*.$$

Arguments like those in Theorem 2.3 suffice to show that $P^{-1/2}\sum_{t=R}^{T}(x_{3,t}\text{-}Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*$, $P^{-1/2}\sum_{t=R}^{T}x_{1,t}'B_1(t)H_1(t)$ and $P^{-1/2}\sum_{t=R}^{T}u_{t+1}$ are $O_p(1)$. It is clear then that the first three right hand side terms in (3) are $O_p((P/R)^{1/2})$.

For the final term note that

$$-R^{-1/2}\sum_{t=R}^{T}(Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*$$

$$= -R^{-1/2}\sum_{t=R}^{T}[Ex_{1,t}'B_1(t)(R^{-1}\sum_{s=t-R+1}^{t}x_{1,s}1(s > T_B)\text{-}1(t > T_B)]\beta_{3,2}^*$$

$$= \begin{cases} -(R^{-1/2}\sum_{t=R}^{R+T_B}(\frac{t-T_B}{R}-1))\beta_{3,2}^* & \\ \quad -R^{-1/2}\sum_{t=R}^{R+T_B}(R^{-1}\sum_{s=t-R+1}^{t} z_s-Ez_t)'F_{1,t}((t-T_B)R^{-2}\sum_{s=t-R+1}^{t} z_s-R^{-1}\sum_{s=T_B}^{t} z_s)\beta_{3,2}^* & T_B \le R \\ -(R^{-1/2}\sum_{t=T_B}^{T}(\frac{t-T_B}{R}-1))\beta_{3,2}^* & \\ \quad -R^{-1/2}\sum_{t=T_B}^{T}(R^{-1}\sum_{s=t-R+1}^{t} z_s-Ez_t)'F_{1,t}((t-T_B)R^{-2}\sum_{s=t-R+1}^{t} z_s-R^{-1}\sum_{s=T_B}^{t} z_s)\beta_{3,2}^* & T_B > \max[R,P] \\ -(R^{-1/2}\sum_{t=T_B}^{T_B+R}(\frac{t-T_B}{R}-1))\beta_{3,2}^* & \\ \quad -R^{-1/2}\sum_{t=T_B}^{T_B+R}(R^{-1}\sum_{s=t-R+1}^{t} z_s-Ez_t)'F_{1,t}((t-T_B)R^{-2}\sum_{s=t-R+1}^{t} z_s-R^{-1}\sum_{s=T_B}^{t} z_s)\beta_{3,2}^* & R < T_B < P \end{cases}$$

where $F_{1,t} = (R^{-1}\sum_{s=t-R+1}^{t} z_s z_s' - (R^{-1}\sum_{s=t-R+1}^{t} z_s)(R^{-1}\sum_{s=t-R+1}^{t} z_s'))^{-1}$. For each of the three cases the lead term has the highest order. Consider the first for which $T_B \le R$. Carrying through the summation we find that $-(R^{-1/2}\sum_{t=R}^{R+T_B}(\frac{t-T_B}{R}-1)) = (T_B^2-T_B)/2R^{3/2}$. For the latter two we find that

$-(R^{-1/2}\sum_{t=T_B}^{T}(\frac{t-T_B}{R}-1)) = (P_B(2R-P_B)-P_B)/2R^{3/2}$ and $-(R^{-1/2}\sum_{t=T_B}^{T_B+R}(\frac{t-T_B}{R}-1)) = (R^2-R)/2R^{3/2}$.

Squaring these terms and keeping in mind that the square of the first three terms in (3) are $O_p(P/R)$ provides the desired result.

(b) Consider the case in which $1/2 \le \lim R/T \le 1$. Since $2/3 \le (1-(P/R)^2/3)^{-1} \le 1$ it suffices to show that $(P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1})^2$ satisfies cases (i), (ii) and (iii). Adding and subtracting terms we obtain the expansion

$$\begin{aligned} P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} &= P^{-1/2}\sum_{t=R}^{T}u_{t+1} - P^{-1/2}\sum_{t=R}^{T}x_{1,t}'B_1(t)H_1(t) - P^{-1/2}\sum_{t=R}^{T}x_{3,t}'(JB_1(t)J'B_3^{-1}(t)-I)\beta_3^* \\ &= P^{-1/2}\sum_{t=R}^{T}u_{t+1} - P^{-1/2}\sum_{t=R}^{T}x_{1,t}'B_1(t)H_1(t) - P^{-1/2}\sum_{t=R}^{T}(x_{3,t}-Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)-I)\beta_3^* \\ &\quad - P^{-1/2}\sum_{t=R}^{T}(Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)-I)\beta_3^*. \end{aligned}$$

Arguments like those in Theorem 2.3 suffice to show that $P^{-1/2}\sum_{t=R}^{T}(x_{3,t}-Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)-I)\beta_3^*$,

$P^{-1/2}\sum_{t=R}^{T} x_{1,t}^{'}B_1(t)H_1(t)$, $P^{-1/2}\sum_{t=R}^{T} u_{t+1}$ and are $O_p(1)$. We must then show that the latter term satisfies (i), (ii) and (iii).

For the final term note that

$$-P^{-1/2}\sum_{t=R}^{T}(Ex_{3,t})'(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*$$

$$= -P^{-1/2}\sum_{t=R}^{T}[Ex_{1,t}^{'}B_1(t)(R^{-1}\sum_{s=t-R+1}^{t}x_{1,s}1(s>T_B)\text{-}1(t>T_B)]\beta_{33}^*$$

$$= \begin{cases} \text{-}(P^{-1/2}\sum_{t=T_B}^{T}(\frac{t\text{-}T_B}{R}-1))\beta_{3,2}^* & \\ \quad \text{-}P^{-1/2}\sum_{t=T_B}^{T}(R^{-1}\sum_{s=t-R+1}^{t}z_s\text{-}Ez_t)'F_{1,t}((t\text{-}T_B)R^{-2}\sum_{s=t-R+1}^{t}z_s\text{-}R^{-1}\sum_{s=T_B}^{t}z_s)\beta_{3,2}^* & T_B \geq R \\ \text{-}(P^{-1/2}\sum_{t=R}^{R+T_B}(\frac{t\text{-}T_B}{R}-1))\beta_{3,2}^* & \\ \quad \text{-}P^{-1/2}\sum_{t=R}^{R+T_B}(R^{-1}\sum_{s=t-R+1}^{t}z_s\text{-}Ez_t)'F_{1,t}((t\text{-}T_B)R^{-2}\sum_{s=t-R+1}^{t}z_s\text{-}R^{-1}\sum_{s=T_B}^{t}z_s)\beta_{3,2}^* & T_B < \min[R,P] \\ \text{-}(P^{-1/2}\sum_{t=R}^{T}(\frac{t\text{-}T_B}{R}-1))\beta_{3,2}^* & \\ \quad \text{-}P^{-1/2}\sum_{t=R}^{T}(R^{-1}\sum_{s=t-R+1}^{t}z_s\text{-}Ez_t)'F_{1,t}((t\text{-}T_B)R^{-2}\sum_{s=t-R+1}^{t}z_s\text{-}R^{-1}\sum_{s=T_B}^{t}z_s)\beta_{3,2}^* & P < T_B < R \end{cases}$$

where $F_{1,t} = (R^{-1}\sum_{s=t-R+1}^{t}z_sz_s^{'}\text{-}(R^{-1}\sum_{s=t-R+1}^{t}z_s)(R^{-1}\sum_{s=t-R+1}^{t}z_s^{'}))^{-1}$. For each of the three cases the lead term has the highest order. Consider the first for which $T_B \geq R$. Carrying through the summation we find that $\text{-}(P^{-1/2}\sum_{t=T_B}^{T}(\frac{t\text{-}T_B}{R}-1)) = (P_B(2R\text{-}P_B)\text{-}P_B)/2RP^{1/2}$. For the latter two we find that $\text{-}(P^{-1/2}\sum_{t=R}^{R+T_B}(\frac{t\text{-}T_B}{R}-1)) = (T_B^2\text{-}T_B)/2RP^{1/2}$ and $\text{-}(P^{-1/2}\sum_{t=R}^{T}(\frac{t\text{-}T_B}{R}-1)) =$ $(P(2T_B\text{-}P)\text{-}P)/2RP^{1/2}$. Squaring these terms provides the desired result.

Theorem 2.6: Maintain Assumptions 1 and 2′ and consider the fixed scheme. (i) If $R \geq T_B$ then $V_F = O_p(P\,T_B^2/RT)$, (ii) If $R \leq T_B$ then $V_F = O_p(R\,P_B^2/PT)$.

Proof of Theorem 2.6: It is straightforward to show that $P^{-1}\sum_{t=R}^{T}\hat{u}_{1,t+1}^2 \to_p \omega > 0$ and hence we proceed immediately to the numerator. Adding and subtracting terms we obtain the expansion

$$(R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}\hat{u}_{1,t+1} = (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}u_{t+1} - (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}x'_{1,t}B_1(t)H_1(t)$$

$$- (R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}x'_{3,t}(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^*.$$

That the first three terms are $O_p(1)$ follows arguments like those in Theorem 2.3. We must therefore show that the third term is the appropriate order.

Since the fixed scheme is being used the third term can be rewritten as

$$(R/T)^{1/2}P^{-1/2}\sum_{t=R}^{T}x'_{3,t}(JB_1(t)J'B_3^{-1}(t)\text{-}I)\beta_3^* = (RP/T)^{1/2}(P^{-1}\sum_{t=R}^{T}x'_{3,t})(JB_1(R)J'B_3^{-1}(R)\text{-}I)\beta_3^*.$$

We must consider separately the cases in which $R > T_B$ or $R \leq T_B$. Doing so we find that

$$(RP/T)^{1/2}(P^{-1}\sum_{t=R}^{T}x'_{3,t})(JB_1(R)J'B_3^{-1}(R)\text{-}I)\beta_3^*$$

$$= \begin{cases} -(R/PT)^{1/2}P_B & T_B > R \\ -(P/RT)^{1/2}T_B & \\ \quad +(RP/T)^{1/2}(1\text{-}T_B/R)(R^{-1}\sum_{t=1}^{R-1}z_t \text{-} P^{-1}\sum_{t=R}^{T}z_t)'F_1(R^{-1}\sum_{t=1}^{R-1}z_t \text{-}(R\text{-}T_B)^{-1}\sum_{t=T_B}^{R-1}z_t) & T_B \leq R \end{cases}$$

where $F_1 = (R^{-1}\sum_{s=1}^{R}z_s z'_s \text{-}(R^{-1}\sum_{s=1}^{R}z_s)(R^{-1}\sum_{s=1}^{R}z'_s))^{-1}$. The lead term in each of the above cases is the higher order term. Cases (i) and (ii) follow from squaring these terms.

6. References

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Table 1: Locations of Actual and Potential Breaks
λ=0.85

End of Sample

T - [(1-λ)T^c]

c\T	100	200	400	800	1600	3200
1	85	170	340	680	1360	2720
0.9	91	183	367	739	1486	2986
0.8	95	190	382	769	1546	3105
0.7	97	194	391	784	1574	3158
0.6	98	197	395	792	1588	3181
0.5	99	198	397	796	1594	3192
0.4	100	199	399	798	1598	3197
0.3	100	200	400	799	1599	3199
0.2	100	200	400	800	1600	3200
0.1	100	200	400	800	1600	3200

Beginning of Sample

[(1-λ)T^c]

c\T	100	200	400	800	1600	3200
1	15	30	60	120	240	480
0.9	9	17	33	61	114	214
0.8	5	10	18	31	54	95
0.7	3	6	9	16	26	42
0.6	2	3	5	8	12	19
0.5	1	2	3	4	6	8
0.4	0	1	1	2	2	3
0.3	0	0	0	1	1	1
0.2	0	0	0	0	0	0
0.1	0	0	0	0	0	0

Table 2: Size of Tests with Potential Break Location at the End
Nominal 5% critical values

Recursive	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
T=100	0.073	0.095	0.131	0.192	0.306	NA	NA	NA	NA	NA
T=200	0.066	0.071	0.092	0.113	0.187	0.304	NA	NA	NA	NA
T=400	0.060	0.062	0.075	0.094	0.118	0.188	NA	NA	NA	NA
T=800	0.050	0.049	0.056	0.064	0.088	0.146	0.295	NA	NA	NA
T=1600	0.050	0.053	0.064	0.065	0.074	0.111	0.313	NA	NA	NA
T=3200	0.047	0.056	0.052	0.052	0.065	0.090	0.201	NA	NA	NA
Chow										
T=100	0.069	0.076	0.064	0.067	0.058	0.053	N.A.	N.A.	N.A.	N.A.
T=200	0.062	0.060	0.062	0.055	0.056	0.052	0.052	N.A.	N.A.	N.A.
T=400	0.058	0.056	0.062	0.059	0.055	0.058	0.049	N.A.	N.A.	N.A.
T=800	0.049	0.047	0.047	0.047	0.052	0.050	0.050	0.047	N.A.	N.A.
T=1600	0.050	0.050	0.060	0.055	0.047	0.051	0.053	0.045	N.A.	N.A.
T=3200	0.048	0.055	0.048	0.048	0.047	0.049	0.056	0.050	N.A.	N.A.
max-Chow										
T=100	0.074	0.076	0.070	0.066	0.062	N.A.	N.A.	N.A.	N.A.	N.A.
T=200	0.060	0.061	0.061	0.055	0.054	0.054	N.A.	N.A.	N.A.	N.A.
T=400	0.056	0.055	0.054	0.060	0.053	0.054	N.A.	N.A.	N.A.	N.A.
T=800	0.054	0.049	0.048	0.049	0.056	0.051	0.049	N.A.	N.A.	N.A.
T=1600	0.051	0.049	0.055	0.052	0.046	0.049	0.050	N.A.	N.A.	N.A.
T=3200	0.045	0.053	0.049	0.051	0.048	0.053	0.053	N.A.	N.A.	N.A.

Table 3: Size of Tests with Potential Break Location at the Beginning
Nominal 5% critical values

Recursive	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
T=100	0.054	0.059	0.056	0.047	0.038	0.032	N.A.	N.A.	N.A.	N.A.
T=200	0.053	0.050	0.054	0.052	0.041	0.036	0.033	N.A.	N.A.	N.A.
T=400	0.052	0.051	0.051	0.054	0.055	0.044	0.041	N.A.	N.A.	N.A.
T=800	0.052	0.050	0.051	0.050	0.050	0.049	0.045	0.043	N.A.	N.A.
T=1600	0.054	0.051	0.053	0.051	0.052	0.053	0.050	0.049	N.A.	N.A.
T=3200	0.050	0.052	0.051	0.052	0.052	0.050	0.049	0.049	N.A.	N.A.
Chow										
T=100	0.065	0.069	0.062	0.061	0.056	N.A.	N.A.	N.A.	N.A.	N.A.
T=200	0.056	0.058	0.058	0.057	0.053	0.052	N.A.	N.A.	N.A.	N.A.
T=400	0.053	0.047	0.054	0.057	0.049	0.052	N.A.	N.A.	N.A.	N.A.
T=800	0.053	0.050	0.051	0.051	0.049	0.046	0.047	N.A.	N.A.	N.A.
T=1600	0.052	0.055	0.053	0.050	0.049	0.049	0.048	N.A.	N.A.	N.A.
T=3200	0.045	0.047	0.051	0.051	0.047	0.049	0.049	N.A.	N.A.	N.A.
max-Chow										
T=100	0.074	0.076	0.070	0.066	0.062	N.A.	N.A.	N.A.	N.A.	N.A.
T=200	0.060	0.061	0.061	0.055	0.054	0.054	N.A.	N.A.	N.A.	N.A.
T=400	0.056	0.055	0.054	0.060	0.053	0.054	N.A.	N.A.	N.A.	N.A.
T=800	0.054	0.049	0.048	0.049	0.056	0.051	0.049	N.A.	N.A.	N.A.
T=1600	0.051	0.049	0.055	0.052	0.046	0.049	0.050	N.A.	N.A.	N.A.
T=3200	0.045	0.053	0.049	0.051	0.048	0.053	0.053	N.A.	N.A.	N.A.

Table 4: Power of Recursive Test for Break at End
T=200, Nominal Size = 5%

Potential Break Correctly Specified at End

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.797	0.960	0.766	0.548	0.392	0.436	NA	NA	NA	NA
0.800	0.407	0.649	0.835	0.616	0.439	0.469	NA	NA	NA	NA
0.700	0.208	0.324	0.510	0.667	0.475	0.489	NA	NA	NA	NA
0.600	0.108	0.145	0.217	0.358	0.499	0.505	NA	NA	NA	NA
0.500	0.087	0.112	0.156	0.237	0.504	0.511	NA	NA	NA	NA
0.400	0.073	0.085	0.114	0.160	0.286	0.518	NA	NA	NA	NA
0.300	0.066	0.070	0.088	0.116	0.191	0.301	NA	NA	NA	NA
0.200	0.066	0.070	0.088	0.116	0.191	0.301	NA	NA	NA	NA

Potential Break Incorrectly Specified at Beginning

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.207	0.194	0.191	0.181	0.157	0.150	0.146	NA	NA	NA
0.800	0.112	0.109	0.106	0.105	0.088	0.082	0.077	NA	NA	NA
0.700	0.079	0.082	0.077	0.075	0.061	0.058	0.054	NA	NA	NA
0.600	0.062	0.062	0.064	0.062	0.050	0.045	0.042	NA	NA	NA
0.500	0.059	0.057	0.058	0.058	0.047	0.043	0.040	NA	NA	NA
0.400	0.057	0.053	0.055	0.054	0.046	0.042	0.036	NA	NA	NA
0.300	0.056	0.053	0.053	0.052	0.043	0.039	0.035	NA	NA	NA
0.200	0.056	0.053	0.053	0.052	0.043	0.039	0.035	NA	NA	NA

Table 5: Power of Chow Test for Break at End
T=200, Nominal Size = 5%

Potential Break Correctly Specified at End

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.827	0.973	0.790	0.526	0.265	0.189	0.110	NA	NA	NA
0.800	0.420	0.706	0.857	0.618	0.329	0.231	0.129	NA	NA	NA
0.700	0.209	0.360	0.573	0.676	0.370	0.262	0.141	NA	NA	NA
0.600	0.111	0.159	0.248	0.373	0.404	0.287	0.152	NA	NA	NA
0.500	0.088	0.118	0.167	0.235	0.412	0.295	0.157	NA	NA	NA
0.400	0.072	0.086	0.108	0.136	0.211	0.295	0.163	NA	NA	NA
0.300	0.063	0.066	0.076	0.076	0.095	0.115	0.165	NA	NA	NA
0.200	0.063	0.066	0.076	0.076	0.095	0.115	0.165	NA	NA	NA

Potential Break Incorrectly Specified at Beginning

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.047	0.040	0.035	0.032	0.036	0.048	NA	NA	NA	NA
0.800	0.046	0.043	0.041	0.040	0.040	0.049	NA	NA	NA	NA
0.700	0.045	0.049	0.045	0.045	0.043	0.049	NA	NA	NA	NA
0.600	0.050	0.051	0.050	0.051	0.048	0.050	NA	NA	NA	NA
0.500	0.051	0.053	0.053	0.053	0.050	0.051	NA	NA	NA	NA
0.400	0.053	0.055	0.055	0.054	0.051	0.051	NA	NA	NA	NA
0.300	0.054	0.058	0.056	0.056	0.053	0.052	NA	NA	NA	NA
0.200	0.054	0.058	0.056	0.056	0.053	0.052	NA	NA	NA	NA

Table 6: Power of max-Chow Test for Break at End
T=200, Nominal Size = 5%

	Potential Break Correctly Specified at End									
Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.742	0.948	0.693	0.420	0.193	0.143	NA	NA	NA	NA
0.800	0.323	0.612	0.798	0.514	0.246	0.175	NA	NA	NA	NA
0.700	0.158	0.284	0.483	0.574	0.283	0.199	NA	NA	NA	NA
0.600	0.091	0.127	0.196	0.290	0.319	0.225	NA	NA	NA	NA
0.500	0.077	0.097	0.133	0.184	0.327	0.233	NA	NA	NA	NA
0.400	0.070	0.077	0.094	0.111	0.166	0.237	NA	NA	NA	NA
0.300	0.063	0.064	0.071	0.068	0.082	0.098	NA	NA	NA	NA
0.200	0.063	0.064	0.071	0.068	0.082	0.098	NA	NA	NA	NA

Table 7: Power of Recursive Test for Break at End
T=800, Nominal Size = 5%

Potential Break Correctly Specified at End

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.999	1.000	0.997	0.922	0.647	0.436	0.425	NA	NA	NA
0.800	0.753	0.962	0.999	0.960	0.724	0.496	0.460	NA	NA	NA
0.700	0.275	0.488	0.780	0.972	0.766	0.530	0.477	NA	NA	NA
0.600	0.105	0.172	0.288	0.515	0.785	0.548	0.488	NA	NA	NA
0.500	0.060	0.089	0.121	0.198	0.374	0.557	0.496	NA	NA	NA
0.400	0.053	0.062	0.073	0.103	0.177	0.347	0.498	NA	NA	NA
0.300	0.051	0.055	0.060	0.079	0.119	0.203	0.498	NA	NA	NA
0.200	0.049	0.053	0.057	0.067	0.085	0.139	0.285	NA	NA	NA

Potential Break Incorrectly Specified at Beginning

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.548	0.516	0.497	0.490	0.483	0.456	0.453	0.451	NA	NA
0.800	0.207	0.194	0.188	0.182	0.180	0.171	0.166	0.163	NA	NA
0.700	0.094	0.092	0.089	0.088	0.086	0.083	0.078	0.076	NA	NA
0.600	0.060	0.057	0.059	0.062	0.060	0.056	0.053	0.050	NA	NA
0.500	0.054	0.051	0.055	0.052	0.056	0.052	0.047	0.044	NA	NA
0.400	0.053	0.049	0.052	0.050	0.052	0.050	0.045	0.042	NA	NA
0.300	0.052	0.050	0.050	0.050	0.051	0.050	0.045	0.043	NA	NA
0.200	0.051	0.050	0.051	0.049	0.051	0.049	0.046	0.043	NA	NA

Table 8: Power of Chow Test for Break at End
T=800, Nominal Size = 5%

Potential Break Correctly Specified at End

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.999	1.000	0.999	0.930	0.643	0.338	0.180	0.118	NA	NA
0.800	0.737	0.973	0.999	0.965	0.736	0.419	0.220	0.138	NA	NA
0.700	0.260	0.525	0.838	0.978	0.777	0.466	0.244	0.153	NA	NA
0.600	0.096	0.189	0.338	0.591	0.803	0.491	0.263	0.160	NA	NA
0.500	0.061	0.091	0.133	0.233	0.405	0.505	0.271	0.164	NA	NA
0.400	0.051	0.063	0.076	0.110	0.175	0.315	0.276	0.166	NA	NA
0.300	0.049	0.056	0.059	0.078	0.104	0.165	0.279	0.167	NA	NA
0.200	0.049	0.049	0.049	0.055	0.064	0.075	0.104	0.169	NA	NA

Potential Break Incorrectly Specified at Beginning

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.089	0.051	0.037	0.034	0.034	0.032	0.043	NA	NA	NA
0.800	0.059	0.043	0.038	0.038	0.041	0.037	0.045	NA	NA	NA
0.700	0.052	0.043	0.043	0.042	0.044	0.043	0.046	NA	NA	NA
0.600	0.052	0.046	0.047	0.047	0.046	0.044	0.047	NA	NA	NA
0.500	0.052	0.048	0.049	0.049	0.048	0.045	0.047	NA	NA	NA
0.400	0.054	0.048	0.050	0.049	0.048	0.045	0.047	NA	NA	NA
0.300	0.054	0.049	0.051	0.049	0.048	0.046	0.047	NA	NA	NA
0.200	0.053	0.050	0.051	0.050	0.049	0.046	0.047	NA	NA	NA

Table 9: Power of max-Chow Test for Break at End
T=800, Nominal Size = 5%

	Potential Break Correctly Specified at End									
Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.997	1.000	0.996	0.878	0.524	0.247	0.129	NA	NA	NA
0.800	0.629	0.947	0.999	0.934	0.638	0.321	0.166	NA	NA	NA
0.700	0.197	0.419	0.762	0.956	0.692	0.367	0.192	NA	NA	NA
0.600	0.081	0.138	0.252	0.496	0.723	0.390	0.202	NA	NA	NA
0.500	0.060	0.073	0.103	0.178	0.325	0.405	0.207	NA	NA	NA
0.400	0.057	0.054	0.067	0.094	0.138	0.239	0.211	NA	NA	NA
0.300	0.056	0.052	0.056	0.068	0.089	0.121	0.210	NA	NA	NA
0.200	0.055	0.050	0.051	0.054	0.064	0.060	0.086	NA	NA	NA

Table 10: Power of Recursive Test for Break at End
T=3200, Nominal Size = 5%

Potential Break Correctly Specified at End

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	1.000	1.000	1.000	1.000	0.964	0.666	0.417	NA	NA	NA
0.800	0.983	1.000	1.000	1.000	0.983	0.743	0.469	NA	NA	NA
0.700	0.471	0.780	0.984	1.000	0.987	0.777	0.494	NA	NA	NA
0.600	0.134	0.246	0.466	0.801	0.988	0.793	0.508	NA	NA	NA
0.500	0.067	0.088	0.138	0.237	0.445	0.800	0.514	NA	NA	NA
0.400	0.049	0.060	0.064	0.089	0.128	0.250	0.517	NA	NA	NA
0.300	0.048	0.057	0.053	0.057	0.079	0.116	0.284	NA	NA	NA
0.200	0.048	0.056	0.051	0.052	0.064	0.091	0.180	NA	NA	NA

Potential Break Incorrectly Specified at Beginning

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.958	0.945	0.936	0.933	0.930	0.929	0.915	0.916	NA	NA
0.800	0.418	0.391	0.371	0.365	0.363	0.364	0.352	0.352	NA	NA
0.700	0.119	0.115	0.111	0.109	0.107	0.109	0.107	0.105	NA	NA
0.600	0.060	0.059	0.061	0.059	0.061	0.060	0.058	0.058	NA	NA
0.500	0.048	0.052	0.052	0.051	0.051	0.050	0.049	0.049	NA	NA
0.400	0.047	0.051	0.050	0.050	0.050	0.050	0.049	0.048	NA	NA
0.300	0.049	0.051	0.050	0.051	0.052	0.049	0.047	0.049	NA	NA
0.200	0.050	0.052	0.051	0.052	0.052	0.050	0.049	0.049	NA	NA

Table 11: Power of Chow Test for Break at End
T=3200, Nominal Size = 5%

Potential Break Correctly Specified at End

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	1.000	1.000	1.000	1.000	0.970	0.653	0.274	0.119	NA	NA
0.800	0.975	1.000	1.000	1.000	0.984	0.745	0.336	0.140	NA	NA
0.700	0.431	0.795	0.991	1.000	0.989	0.782	0.375	0.151	NA	NA
0.600	0.125	0.251	0.514	0.865	0.990	0.797	0.391	0.158	NA	NA
0.500	0.065	0.088	0.151	0.267	0.528	0.805	0.399	0.161	NA	NA
0.400	0.049	0.059	0.068	0.094	0.152	0.281	0.402	0.162	NA	NA
0.300	0.048	0.057	0.052	0.059	0.077	0.102	0.208	0.163	NA	NA
0.200	0.049	0.055	0.050	0.051	0.055	0.060	0.091	0.162	NA	NA

Potential Break Incorrectly Specified at Beginning

Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	0.251	0.115	0.059	0.040	0.028	0.033	0.037	NA	NA	NA
0.800	0.082	0.052	0.046	0.041	0.036	0.040	0.043	NA	NA	NA
0.700	0.048	0.044	0.046	0.046	0.041	0.044	0.046	NA	NA	NA
0.600	0.043	0.046	0.049	0.049	0.044	0.047	0.048	NA	NA	NA
0.500	0.045	0.047	0.050	0.050	0.045	0.048	0.049	NA	NA	NA
0.400	0.046	0.047	0.050	0.051	0.047	0.049	0.049	NA	NA	NA
0.300	0.045	0.047	0.050	0.051	0.047	0.049	0.049	NA	NA	NA
0.200	0.045	0.047	0.050	0.051	0.047	0.048	0.049	NA	NA	NA

Table 12: Power of max-Chow Test for Break at End
T=3200, Nominal Size = 5%

	Potential Break Correctly Specified at End									
Actual Break	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100
0.900	1.000	1.000	1.000	1.000	0.940	0.539	0.196	NA	NA	NA
0.800	0.955	1.000	1.000	1.000	0.972	0.647	0.258	NA	NA	NA
0.700	0.338	0.706	0.982	1.000	0.979	0.694	0.291	NA	NA	NA
0.600	0.095	0.190	0.416	0.793	0.983	0.715	0.307	NA	NA	NA
0.500	0.055	0.075	0.113	0.216	0.434	0.725	0.314	NA	NA	NA
0.400	0.047	0.057	0.062	0.078	0.117	0.224	0.319	NA	NA	NA
0.300	0.045	0.053	0.054	0.057	0.062	0.086	0.159	NA	NA	NA
0.200	0.046	0.052	0.050	0.054	0.053	0.061	0.076	NA	NA	NA