Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

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Working Paper 2010-049B
http://research.stlouisfed.org/wp/2010/2010-049.pdf

November 2010
Revised September 2013

FEDERAL RESERVE BANK OF ST. LOUIS
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P.O. Box 442
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Floor Systems for Implementing Monetary Policy: Some Unpleasant Fiscal Arithmetic*

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September 23, 2013

Abstract

An increasing number of central banks implement monetary policy via a channel system or a floor system. We construct a general equilibrium model to study the properties of these systems. We find that a floor system is weakly optimal *if and only if* the target rate satisfies the Friedman rule. Unfortunately, the optimal floor system requires either transfers from the fiscal authority to the central bank or a reduction in seigniorage payments from the central bank to the government. This is the unpleasant fiscal arithmetic of a floor system. When the central bank faces financing constraints on its interest expense, we show that it is strictly optimal to operate a channel system.

JEL Codes: E52, E58, E59

Key words: monetary policy, floor system, channel system, standing facilities

*The views in this paper are those of the authors and do not represent those of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the FOMC. We would like to thank Cédric Tille and Antoine Martin for their very valuable comments. Corresponding author: Aleksander Berentsen, Tel.: 0041612671205, Email: aleksander.berentsen@unibas.ch.

1 Introduction

Over the past years, a monetary policy framework known as a channel or corridor system has been implemented by several central banks and is being considered by other central banks.[1] In this system, a central bank operates two facilities: a lending facility and a deposit facility. At the lending facility, the central bank stands ready to supply money overnight to financial intermediaries at a given borrowing rate, i_ℓ, against collateral. At the deposit facility, intermediaries can make overnight deposits at the central bank to earn the interest rate $i_d < i_\ell$, where the spread is called the *interest rate corridor* or *channel*. This simple framework immediately raises three questions. First, why provide these facilities? Second, why choose a positive corridor as opposed to a zero corridor? Third, what is the optimal value of i_d? We construct a general equilibrium model of standing facilities to help answer these questions.

Why do we consider a general equilibrium model? The typical answers to the questions above are based on partial equilibrium analysis, which we find incomplete. The usual answer to the first question is that the standing facility provides an outside option for intermediaries who, for whatever reason, were unable to execute desired trades in the money market. In this sense, the central bank is "completing" the money market by providing liquidity insurance to intermediaries. However, whenever insurance is provided, there may be incentive problems that lead to inefficient outcomes. Thus, is it optimal to provide this insurance? The answer to this question requires a general equilibrium model with a well-defined objective for the central bank.

The typical answer to the second question is that a positive corridor gives intermediaries an incentive to trade amongst themselves rather than accessing the standing facilities. By exploiting gains from trade, intermediaries move the market rate, i_m, inside the corridor; i.e., $i_d < i_m < i_\ell$. This allows the central bank to control the money market rate by either changing the width of the corridor or shifting it with the goal of keeping the money market interest rate close to its target.[2]

[1] Channel systems are widely used in practice; see Bernhardsen and Kloster (2010). Versions are, for example, implemented by the Bank of Canada, the Bank of England, the European Central Bank (ECB), the Reserve Bank of Australia, the Swiss National Bank, the Reserve Bank of New Zealand, and the U.S. Federal Reserve System.

[2] In theory, there is no need for direct central bank intervention to control the market rate of interest, since money market participants will never mutually agree to trade at an interest rate that lies outside the interest rate corridor. In practice, central banks still conduct open market operations to adjust the quantity of central bank money in circulation. In "normal" times, they do so to accommodate, for example, seasonal fluctuations in the demand for central bank money. In

A *floor system* is a special case where $i_d = i_m$, which can easily be achieved by setting $i_d = i_m = i_\ell$, i.e., by setting the channel width to zero. This would give the central bank perfect control of the money market rate. Thus, controlling the market rate cannot be the reason for a positive spread. So, is there another reason for doing so?

Finally, in general but more importantly for a floor system, what determines the optimal value of i_d? The typical answer is that i_d is set at the "target" interest rate. But what determines that rate? Are there restrictions that affect the feasible set of target rates? Again, the answers to these questions require a general equilibrium model.

We use a Lagos-Wright monetary model with financial intermediation to study the allocation of money/reserves. In this framework, we assume that some intermediaries are randomly excluded from trading in the money market (i.e., exogenous market segmentation). We then use our model to answer the three questions posed above.

We show that by choosing $i_d = i_m = i_\ell$, i.e., with the use of a floor system, the central bank can effectively eliminate market segmentation and "complete" the money market. We find that this is the optimal policy if the central bank can implement the Friedman rule, which involves paying interest at the deposit facility that compensates for the time cost of holding reserves.

If the Friedman rule cannot be implemented, then it is optimal to do two things. First, set the deposit rate as high as feasible (get as close to the Friedman rule as possible). Second, run a corridor system by setting $i_d < i_\ell$. By increasing the borrowing rate, the central bank "penalizes" intermediaries, who do not have sufficient reserves if they are excluded from the money market. As a result, intermediaries demand more reserves, which increases the real value of money/reserves and welfare. This is a pecuniary general equilibrium effect that is absent in partial equilibrium analysis.

Why might it be infeasible for the central bank to implement the Friedman rule? Under this rule, banks are satiated with reserves and they do not need to borrow from each other or the central bank; yet, they will deposit excess reserves at the central bank to earn the deposit rate. How is this interest expense financed? The central bank can 1) print money, 2) use capital income, 3) receive transfers from the fiscal

"exceptional" times, in response to severe aggregate shocks they do so to restore the functioning of money markets.

authority. Most of the existing analysis ignores this question and assumes that all of these options are sufficient. We argue in this paper that this is not a trivial issue and clearly affects the choice of a floor or corridor system.

Our results take into account the possibility that central banks may be unable, or are unwilling for political reasons, to incur the interest expense required by the optimal floor system. We argue that this possibility is relevant for the following reasons:

(i) Using taxes to finance interest payments to banks may not be politically acceptable, since other areas of government spending may be affected. As Feinman (1993) documents, the Federal Reserve long requested the power to pay interest on reserves only to be denied this on budgetary grounds. To illustrate the political opposition, consider the following Congressional testimony by a U.S. Treasury official on the proposal to pay interest on reserves:

> "As a general matter we are sympathetic to many of the arguments put forth by the proponents, particularly with regards to monetary policy. At the same time, however, we are also mindful of the budgetary costs associated with this proposal which would be significant. The President's budget does not include the use of taxpayer resources for this purpose. At this time, then, the Administration is not prepared to endorse that proposal."[3]

(ii) Interest payments on reserves are quantitatively important. The Federal Reserve's Large Scale Asset Purchases (LSAP) generated $1.5 trillion in reserves at the end of 2012, and they are projected to be over $2.5 trillion if the latest LSAP continue to 2014. Analysis of the Fed's balance sheet by Federal Reserve economists suggest that the interest expense for locking up reserves in the banking system could top $60 billion for a couple of years under a plausible scenario of rising interest rates.[4] In this scenario, the analysis also shows that remittances to the Treasury would be zero for more than five years. To highlight the potential political backlash from such large payments, note that according to Federal Deposit Insurance Corporation

[3] March 13, 2001: Special House Hearing related to H.R. 1009. Statement by Donald V. Hammond, Acting Under Secretary for Domestic Finance, Department of the Treasury. The proposal was not approved.

[4] See Carpenter et al. (2012). For an other recent estimate of the potential costs with probabilities attached to it, see also Christensen et al. (2013)

(FDIC) data, the combined net income of the top 10 U.S. banks in 2010 was less than $55 billion. Furthermore, Federal Reserve H.8 data at the end of 2012 shows that nearly half of all reserves are held by foreign banks, which suggests a transfer of U.S. taxpayer resources to foreign banks in the neighborhood of $30 billion. In the current populist environment confronting U.S. politicians, it is not unreasonable to conjecture that Congress could respond to these large payments to domestic and foreign banks by suspending or eliminating the Fed's power to pay interest on reserves. This would complicate the Fed's strategy for shrinking its balance sheet while attempting to keep inflation under control.

It is important not to confuse the (steady state) results of our model with current short-run policies. In response to the financial crisis, several central banks have moved from a corridor system toward a floor system, at least temporarily (see e.g., Bernhardsen and Kloster, 2010). Short-term interest rates are currently at a record low. With the deposit rate close to zero, the fiscal implications of paying interest on reserves are largely irrelevant. However, once the economy recovers and short-term interest rates rise, the fiscal implications of a floor system will become relevant again, particularly if central banks choose not to drain reserves prior to raising their policy rates.

1.1 Related Literature

Despite the growing use of channel and floor systems to implement monetary policy, only a few theoretical studies on their use exist. The earlier literature on channel systems or aspects of channel systems were conducted in partial equilibrium models.[5] Except for some non-technical discussions (e.g., Goodfriend, 2002, Keister et al., 2008, and Bernhardsen and Kloster, 2010), there are no papers that compare floor versus corridor systems in a general equilibrium model.

General equilibrium models of channel systems are Berentsen and Monnet (2008), Curdia and Woodford (2011), Martin and Monnet (2011), and Chapman et al. (2011), where the latter two build on Berentsen and Monnet (2008). Our model also builds on Berentsen and Monnet (2008), who analyze the optimal interest-rate corridor in a channel system. In Berentsen and Monnet (2008), the central bank requires a real asset as a collateral at its borrowing facility. Due to its liquidity premium, the social

[5] See, e.g., Campbell (1987), Ho and Saunders (1985), Orr and Mellon (1961), Poole (1968), Furfine (2000), Woodford (2001), and Whitesell (2006).

return of the asset is lower than the private return to market participants. From a social point of view, this results in an over-accumulation of the asset if the central bank implements a zero interest-rate spread. It is, therefore, socially optimal to implement a strictly positive interest-rate spread to discourage the wasteful over-accumulation of collateral. In contrast, in our model the collateral is nominal government bonds, and there is no waste involved in producing nominal government bonds.[6] Our result, therefore, that the constrained-efficient monetary policy involves a strictly positive interest-rate spread is due to a mechanism that is very different from the one proposed in Berentsen and Monnet (2008). Furthermore, several aspects of our environment, such as ex-post heterogeneity of money demand, differs substantially from Berentsen and Monnet (2008).

Martin and Monnet (2011) compare the feasible allocations that one can obtain when a central bank implements monetary policy either with a channel system or via open market operations. The focus of our paper is the floor system and the fiscal implications of the optimal floor system. We also have a more complex structure of liquidity shocks, which allows us to study how policy affects the distribution of overnight liquidity in a general equilibrium model. In Chapman et al. (2011) the value of the collateral is uncertain. The focus in their paper is on the optimal haircut policy of a central bank.

Cúrdia and Woodford (2011) study optimal policy in a New Keynesian framework with financial intermediation. They also find that the floor system is optimal, because it eliminates any inefficiencies associated with economizing reserves in the banking system. Unlike our paper, they do not study fiscal restrictions of paying interest on reserves. Furthermore, our paper differs from Cúrdia and Woodford along three additional dimensions. First, we do not have sticky prices. Second, we do not have inefficiencies in the financial intermediation process that give rise to a need for reserves. Our framework operates via a different mechanism – a combination of risk sharing, market segmentation and collateral requirements – none of which are present in Cúrdia and Woodford. Finally, we address the question of whether or not it is optimal for the central bank to run the standing facilities in a way that eliminates the effects of market segmentation. This is not an issue for them because they assume implementing the Friedman rule is fiscally feasible.

[6] Like money, government bonds are essentially pieces of paper that are costless to produce and so there is no social waste in their use.

The structure of our paper is as follows: Section 2 describes the environment. Section 3 characterizes optimal decisions by market participants. Section 4 studies symmetric stationary equilibria. Section 5 identifies the optimal policy and discusses its fiscal implications. Section 6 characterizes the second-best policy. Section 7 contains some extensions of our model, and Section 8 concludes. All proofs are in the Appendix.

2 Environment

Our framework is motivated by the functioning of existing channel systems. For example, as discussed in Berentsen and Monnet (2008), the key features of the ECB's implementation framework and of the euro money market are as follows. First, at the beginning of the day, any outstanding overnight loans at the ECB are settled. Second, the euro money market operates between 7 a.m. and 5 p.m. Third, after the money market has closed, market participants can access the ECB's facilities for an additional 30 minutes. This means that, after the close of the money market, the ECB's lending facility is the only possibility for obtaining overnight liquidity. Also, any late payments received can still be deposited at the deposit facility of the ECB.

To reproduce the above sequence of trading, we assume that in each period three perfectly competitive markets open sequentially (see Figure 1).[7] The first market is a settlement market, where agents can trade money for newly issued bonds and all claims from the previous day are settled. The second market is a money market, where agents can borrow against collateral and lend money to earn the money market interest rate.[8] The third market is a goods market, where agents trade goods for money. At the beginning of the goods market the central bank opens a deposit facility and a borrowing facility. At the deposit facility agents can deposit money to earn the deposit rate, and at the borrowing facility they can get a loan against collateral.

[7] Our environment builds on Berentsen and Monnet (2008) and Lagos and Wright (2005). The framework by Lagos and Wright (2005) is useful because it allows us to introduce heterogeneous preferences while still keeping the distribution of money balances analytically tractable.

[8] We assume that this market is competitive, although in the U.S. it is an over-the-counter market, where banks trade bilaterally and form stable, long-term relationships (see Afonso et al. (2013) for an empirical study). The assumption of competitive markets simplifies the analysis. For an interesting model of pairwise trading in the money market, see Afonso and Lagos (2012).

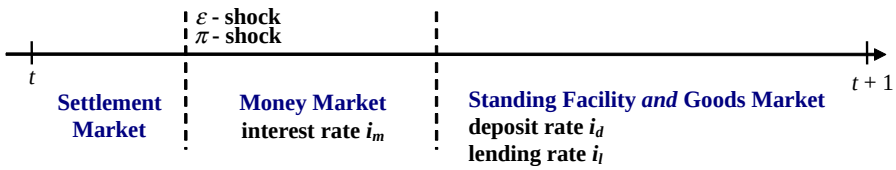

Figure 1: Sequence of markets.

Time is discrete and the economy is populated by infinitely lived agents. There is a generic good that is non-storable and perfectly divisible. Non-storable means that it cannot be carried from one market to the next. There are two types of agents: buyers and sellers. Buyers can only consume and sellers can only produce in the goods market. Both agent types can produce and consume in the settlement market. Each type has measure 1.

In the goods market, a buyer gets utility $\varepsilon u(q)$ from consuming q units of the good, where $u(q) = \log(q)$, and ε is a preference shock that affects the liquidity needs of buyers. The preference shock ε has a continuous distribution $F(\varepsilon)$ with support $(0, \infty]$, is iid across buyers and serially uncorrelated.[9] Sellers incur a utility cost $c(q_s) = q_s$ from producing q_s units. The discount factor across periods is $\beta = (1+r)^{-1} < 1$, where r is the time rate of discount.

In the settlement market, agents have a constant returns to scale production technology, where one unit of the good is produced with one unit of labor generating one unit of disutility. Thus, producing h units of goods implies disutility $-h$. Furthermore, we assume that the utility of consuming x units of goods yields utility x. As in Lagos and Wright (2005), these assumptions yield a degenerate distribution of portfolios at the beginning of the goods market.[10]

The preference shock ε creates random liquidity needs among buyers. They learn it at the beginning of the money market and based on this information they can adjust their money holdings by either accessing the money market or by using the central bank's standing facilities. If the money market rate lies strictly between

[9] See also Berentsen and Rocheteau (2002, 2003) and Lagos and Rocheteau (2005) for a similar characterization of the shocks.

[10] The idiosyncratic preference shocks in the goods market play a role similar to that of random matching in Lagos and Wright (2005). Due to these shocks, buyers spend different amounts of money in the goods market, which yields a distribution of money holdings at the beginning of the settlement market.

the central bank's deposit and borrowing rates, buyers strictly prefer to adjust their money holding in the money market, rather than at the central bank's standing facilities. To generate a role for these facilities, we assume that only a fraction of buyers π can participate in the money market. Those buyers who have no access to the money market can only use the central bank's standing facilities to adjust their money holdings. Finally, we assume that the deposit facility continues to be open during the goods market, which allows the sellers to deposit the proceeds from their sales to earn the deposit rate.

In practice, only qualified financial intermediaries have access to the money market and the central bank's standing facilities. Nevertheless, these intermediaries act on the behalf of their customers: households and firms. We simplify the analysis by assuming that the economy is populated by agents who have direct access to the money market and the central bank's standing facilities. This simplifies the analysis and focuses on the varying liquidity needs of agents in the economy rather than the process of intermediation.

2.1 Money and Bonds

There are two perfectly divisible financial assets: money and one-period, nominal discount government bonds. New money and new bonds are issued in the settlement market. Bonds are payable to the bearer and default-free. One bond pays off one unit of currency in the settlement market of the following period. The central bank is assumed to have a record-keeping technology over bond trades, and bonds are book-keeping entries – no physical object exists. This implies that households are not anonymous to the central bank. Nevertheless, despite having a record-keeping technology over bond trades, the central bank has no record-keeping technology over goods trades.[11]

Private households are anonymous to each other and cannot commit to honor intertemporal promises. Since bonds are intangible objects, they are incapable of being used as media of exchange in the goods market; hence they, are illiquid.[12]

[11] If the central bank had a record-keeping technology over goods trades, it could make this information available to agents in the settlement market. This information could facilitate trade credit in the decentralized market, which would render money inessential.

[12] Claims to collateral cannot be used as medium of exchange, since we assume that agents can perfectly and costlessly counterfeit such claims, which prevents them from being accepted as a means of payment in the goods market (see Lester et al. 2012).

Since households are anonymous and cannot commit, a household's promise in the goods market to deliver bonds to a seller in the settlement market of the following period is not credible.[13]

In summary, agents are anonymous in the goods market and money is the only tangible asset, which is recognizable in the goods market and can be used as a medium of exchange.

2.2 Money Market and Standing Facilities

The money market is perfectly competitive so that the money market interest rate i_m clears the market. Let $\rho_m = 1/(1+i_m)$. We restrict all financial contracts to overnight contracts. An agent who borrows one unit of money in the money market repays $1/\rho_m$ units of money in the settlement market of the following period. Also, an agent who deposits one unit of money receives $1/\rho_m$ units of money in the settlement market of the following period. Finally, borrowers need to post collateral. Our money market is similar as the one in Berentsen et al. (2007). The purpose of the money market is to reallocate money from buyers with a low marginal utility of consumption to those with a high marginal utility. In particular, in this market no private claims are issued that can circulate as payment instruments.[14]

At the beginning of the goods market, the central bank opens a borrowing facility and a deposit facility. At these facilities, claims to a unit of money in the next settlement market are traded. At the deposit facility, the central bank sells claims to a unit of money in the next settlement market. At the borrowing facility, the central bank buys claims to a unit of money in the next settlement market. Therefore, ρ_d is the central bank's ask price for these claims at the deposit facility and ρ_ℓ is the central bank's bid price for these claims at the borrowing facility.

Intuitively, the borrowing facility offers nominal loans ℓ against collateral at interest rate i_ℓ, and the deposit facility promises to pay interest rate i_d on nominal deposits d with $i_\ell \geq i_d$. Let $\rho_d = 1/(1+i_d)$ and $\rho_\ell = 1/(1+i_\ell)$. An agent who

[13] One can show that in our environment illiquid bonds are beneficial. We omit a proof and simply assume it. However, it has been shown previously that it is socially beneficial for bonds to be illiquid; see Kocherlakota (2003), Shi (2008), and Berentsen and Waller (2011). More recent models with illiquid assets include, Lagos and Rocheteau (2008), Lagos (2010b), Lester et al. (2011), and many others.

[14] It can be shown that our money market is equivalent to a narrow banking system, where banks can issue private notes that circulate as payment instruments but are required to hold 100 percent reserves. See Berentsen et al. (2007) for more details.

borrows ℓ units of money from the central bank repays ℓ/ρ_ℓ units of money in the settlement market of the following period. Also, an agent who deposits d units of money at the central bank receives d/ρ_d units of money in the settlement market of the following period.

2.3 Consolidated Government Budget Constraint

Let $\mathcal{S}$ denote the central bank's surplus ($\mathcal{S} > 0$) or deficit ($\mathcal{S} < 0$) at time t. It satisfies

$$\mathcal{S} = M^+ - M + (1/\rho_\ell - 1)\, L - (1/\rho_d - 1)\, D, \tag{1}$$

where M is the stock of money at the beginning of the current-period settlement market and M^+ the stock of money at the beginning of the next-period settlement market.[15] Since in the settlement market total loans, L, are repaid and total deposits, D, are redeemed, the difference $(1/\rho_\ell - 1)\, L - (1/\rho_d - 1)\, D$ is the central bank's revenue from operating the standing facilities.

In Section 7, we amend the central bank's balance sheet (1) along two lines. First, we endow the central bank with a stock of real assets that provides a stream of revenue in each period. Here, we show that our analysis is unaffected if the real return on these assets is not too high. Second, we endow the central bank with a stock of government bonds that pay interest. We show that this is simply another way to transfer tax revenue from the government to the central bank, since the government has to levy taxes to finance the interest payments on the government bonds.

Let $\mathcal{D} = G - T$ denote nominal expenditure by the government, G, minus the government's nominal tax collection, T. If $\mathcal{D} < 0$ ($\mathcal{D} > 0$), the government has a primary surplus (deficit). The government's budget constraint satisfies

$$\mathcal{D} = \rho B^+ - B + \mathcal{S}, \tag{2}$$

where B is the stock of bonds at the beginning of the current-period settlement market, B^+ the stock of bonds at the beginning of the next-period settlement market and $\rho = 1/(1+i)$ the price of bonds in the settlement market, where i denotes the nominal interest rate on government bonds. The government budget constraint simply requires that any primary deficit $\mathcal{D} > 0$ must be financed by either issuing

[15] Throughout the paper, the plus sign is used to denote the next-period variables.

additional debt $\rho B^{+} - B$, central bank surplus $\mathcal{S}$, or both. If $\mathcal{S} < 0$, there is a transfer of funds from the government to the central bank. If $\mathcal{S} > 0$, the transfer is reversed.

The consolidated government budget constraint at time t is given by

$$\mathcal{D} = M^{+} - M + (1/\rho_{\ell} - 1)\, L - (1/\rho_{d} - 1)\, D + \rho B^{+} - B. \tag{3}$$

Equation (3) states that the consolidated deficit must be financed by issuing some combination of money and government bonds as in Sargent and Wallace (1981).[16] In addition, the difference $(1/\rho_{\ell} - 1)\, L - (1/\rho_{d} - 1)\, D$ is the central bank's profit from operating the standing facility.

In what follows, we simplify our analysis by assuming that $G = 0$, which implies that $\mathcal{D} = -T$. Furthermore, we assume that $T = \tau M$ are lump-sum taxes ($T > 0$) or lump-sum subsidies ($T < 0$). This simplification avoids distortionary taxation, which we do not want to be the focus of this paper. Accordingly, if $\mathcal{D} = -\tau M > 0$, the households receive a lump-sum money subsidy from the government; and if $\mathcal{D} = -\tau M < 0$, the households pay a lump-sum tax to the government.

2.4 First-best Allocation

In this section, we characterize the optimal planner (i.e., first-best) allocation. The optimal planner allocation requires neither money nor bonds, since the planner can dictate the consumption and production quantities. We assume without loss in generality that the planner treats all sellers symmetrically. He also treats all buyers experiencing the same preference shock symmetrically. Furthermore, he cares for everyone's utility equally. Given these assumptions, it can be shown that the first-best allocation satisfies

$$q_{\varepsilon}^{*} = \varepsilon \text{ for all } \varepsilon \tag{4}$$

$$q_{s}^{*} = \bar{\varepsilon} \equiv \int_{0}^{\infty} \varepsilon dF(\varepsilon). \tag{5}$$

[16] Sargent and Wallace (1981) study the interactions of monetary and fiscal policy. In particular, they argue that if fiscal policy dominates monetary policy, then monetary policy might not be able to control inflation. We use the same consolidated government budget constraint as they do but our focus is different. We derive the fiscal implications of implementing monetary policy via either a channel system or a floor system.

These are the quantities chosen by a social planner who could dictate production and consumption in the goods market.

3 Household Decisions

In this section, we study the decision problems of buyers and sellers in these markets. For this purpose, P denotes the price of goods in the settlement market and let $\phi \equiv 1/P$. Furthermore, let p be the price of goods in the goods market.

3.1 Settlement Market

$V_S(m, b, \ell, d, z)$ denotes the expected value of entering the settlement market with m units of money, b bonds, ℓ loans from the borrowing facility, d deposits at the deposit facility, and z loans from the money market. $V_M(m, b)$ denotes the expected value from entering the money market with m units of money and b collateral. For notational simplicity, we suppress the dependence of the value function on the time index t.

In the settlement market, the problem of an agent is

$$V_S(m, b, \ell, d, z) = \max_{h,x,m',b'} x - h + V_M(m', b')$$
$$s.t. \quad x + \phi m' + \phi\rho b' = h + \phi m + \phi b + \phi d/\rho_d - \phi\ell/\rho_\ell - \phi z/\rho_m - \phi\tau M,$$

where h is hours worked in the settlement market, x is consumption of the generic good, m' is the amount of money brought into the money market, and b' is the amount of bonds brought into the money market. Using the budget constraint to eliminate $x - h$ in the objective function, one obtains the first-order conditions

$$V_M^{m'} \leq \phi \ (= \text{ if } m' > 0 \ ) \tag{6}$$
$$V_M^{b'} \leq \phi\rho \ (= \text{ if } b' > 0 \ ). \tag{7}$$

$V_M^{m'} \equiv \frac{\partial V_M(m',b')}{\partial m'}$ is the marginal value of taking an additional unit of money into the money market. Since the marginal disutility of working is one, $-\phi$ is the utility cost of acquiring one unit of money in the settlement market. $V_M^{b'} \equiv \frac{\partial V_M(m',b')}{\partial b'}$ is the marginal value of taking additional bonds into the money market. The term

$-\phi\rho$ is the utility cost of acquiring one unit of bonds in the settlement market. The implication of (6) and (7) is that all buyers enter the money market with the same amount of money and the same quantity of bonds. The same is true for sellers, since in equilibrium they will bring no money into the money market.

The envelope conditions are

$$V_S^m = V_S^b = \phi; V_S^d = \phi/\rho_d; V_S^\ell = -\phi/\rho_\ell; V_S^z = -\phi/\rho_m, \tag{8}$$

where V_S^j is the partial derivative of $V_S(m, b, \ell, d, z)$ with respect to $j = m, b, \ell, d, z$.

3.2 Money and Goods Markets

At the beginning of the money market, buyers observe their preference shock ε and learn whether they have access to the money market or not. We call a buyer who has access to the money market an *active* buyer and a buyer who has no access to the money market a *non-active* buyer. The indirect utility function of an $\varepsilon-$buyer in the money market before access is determined is

$$V_M(m, b|\,\varepsilon) = \pi V_A(m, b|\,\varepsilon) + (1-\pi)\, V_N(m, b|\,\varepsilon), \tag{9}$$

where $V_A(m, b|\,\varepsilon)$ is the value of an active buyer at the beginning of the money market and $V_N(m, b|\,\varepsilon)$ is the value of a non-active buyer. Both values are determined further below.

We next consider the problem solved by sellers, and then we study the decisions by active and non-active buyers.

Decisions by sellers Sellers produce goods in the goods market with linear cost $c(q) = q$ and consume in the settlement market, obtaining linear utility $U(x) = x$. It is straightforward to show that sellers are indifferent as to how much they sell in the goods market if

$$p\beta\phi^+/\rho_d = 1, \tag{10}$$

where ϕ^+ is the price of money in the next-period settlement market. Since we focus on a symmetric equilibrium, we assume that all sellers produce the same amount. With regard to bond holdings, it is straightforward to show that, in equilibrium, sellers are indifferent to holding any bonds if the Fisher equation holds and will hold

no bonds if the yield on the bonds does not compensate them for inflation or time discounting. Thus, for brevity of analysis, we assume sellers carry no bonds across periods.

Note that we allow sellers to deposit their proceeds from sales at the deposit facility, which explains the deposit factor ρ_d in (10). Furthermore, it is also clear that they will never acquire money in the settlement market and so for them $m' = 0$.

Decisions by non-active buyers A non-active $\varepsilon-$buyer has access only to the standing facilities of the central bank. The indirect utility function $V_N(m, b|\,\varepsilon)$ of such a buyer is

$$V_N(m, b|\,\varepsilon) = \max_{q_\varepsilon, d_\varepsilon, \ell_\varepsilon} \varepsilon u\left(q_\varepsilon\right) + \beta V_S\left(m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon\right)$$
$$\text{s.t.} \quad m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon \geq 0, \text{ and } \rho_\ell b - \ell_\varepsilon \geq 0,$$

where d_ε is the deposit at the central bank and ℓ_ε is the loan received from the central bank. The first inequality is the buyer's budget constraint. The second inequality is the collateral constraint. Let $\beta\phi^+\lambda_\varepsilon$ denote the Lagrange multiplier for the first inequality and $\beta\phi^+\lambda_\ell$ denote the Lagrange multiplier of the second inequality. Then, using (8) to replace V_S^m, V_S^ℓ and V_S^d, the first-order conditions for q_ε, d_ε, and ℓ_ε can be written as follows:

$$\begin{aligned} \varepsilon u'\left(q_\varepsilon\right) - \beta p \phi^+ \left(1 + \lambda_\varepsilon\right) &= 0 \\ 1/\rho_d - \left(1 + \lambda_\varepsilon\right) &\leq 0 \qquad (= 0 \text{ if } d_\varepsilon > 0) \\ -1/\rho_\ell + \left(1 + \lambda_\varepsilon\right) - \lambda_\ell &\leq 0 \qquad (= 0 \text{ if } \ell_\varepsilon > 0)\,. \end{aligned} \tag{11}$$

Lemma 1 characterizes the optimal borrowing and lending decisions and the quantity of goods obtained by a non-active $\varepsilon-$buyer:

Lemma 1 *There exist critical values ε_d, ε_ℓ, $\varepsilon_{\bar{\ell}}$, with $0 \leq \varepsilon_d \leq \varepsilon_\ell \leq \varepsilon_{\bar{\ell}}$, such that the following is true: if $0 \leq \varepsilon < \varepsilon_d$, a non-active buyer deposits money at the central bank; if $\varepsilon_d \leq \varepsilon \leq \varepsilon_\ell$, he neither borrows nor deposits money; if $\varepsilon_\ell < \varepsilon \leq \varepsilon_{\bar{\ell}}$, he borrows money and the collateral constraint is nonbinding; if $\varepsilon_{\bar{\ell}} \leq \varepsilon$, he borrows money and the collateral constraint is binding. The critical values solve*

$$\varepsilon_d = \beta\phi^+ m/\rho_d,\ \varepsilon_\ell = \beta\phi^+ m/\rho_\ell, \text{ and } \varepsilon_{\bar{\ell}} = \varepsilon_\ell + \beta\phi^+ b. \tag{12}$$

Furthermore, the amount of borrowing and depositing by a non-active buyer with a taste shock ε and the amount of goods purchased by the buyer satisfy:

$$\begin{array}{llll} q_\varepsilon = \varepsilon, & d_\varepsilon = p\left(\varepsilon_d - \varepsilon\right), & \ell_\varepsilon = 0, & \text{if } 0 \leq \varepsilon \leq \varepsilon_d \\ q_\varepsilon = \varepsilon_d, & d_\varepsilon = 0, & \ell_\varepsilon = 0, & \text{if } \varepsilon_d \leq \varepsilon \leq \varepsilon_\ell \\ q_\varepsilon = \varepsilon\rho_\ell/\rho_d, & d_\varepsilon = 0, & \ell_\varepsilon = p\left(\varepsilon\rho_\ell/\rho_d - \varepsilon_d\right), & \text{if } \varepsilon_\ell \leq \varepsilon \leq \varepsilon_{\bar{\ell}}, \\ q_\varepsilon = \varepsilon_{\bar{\ell}}\rho_\ell/\rho_d, & d_\varepsilon = 0, & \ell_\varepsilon = \rho_\ell b, & \text{if } \varepsilon_{\bar{\ell}} \leq \varepsilon. \end{array} \tag{13}$$

The optimal borrowing and lending decisions follow the cut-off rules according to the realization of the taste shock. The cut-off levels, ε_d, ε_ℓ, and $\varepsilon_{\bar{\ell}}$ partition the set of taste shocks into four regions. For shocks lower than ε_d, a buyer deposits money at the deposit facility. For values between ε_d and ε_ℓ, the buyer does not use the standing facilities. For shocks higher than ε_ℓ, the non-active buyer borrows and, finally, the cut-off value $\varepsilon_{\bar{\ell}}$ determines whether his collateral constraint is binding or not.

Decisions by active buyers An active $\varepsilon-$buyer can borrow or lend at the money market rate i_m and use the standing facilities. The indirect utility function $V_A(m, b|\,\varepsilon)$ of such a buyer is

$$V_A(m, b|\,\varepsilon) = \max_{q_\varepsilon, z_\varepsilon, d_\varepsilon, \ell_\varepsilon} \varepsilon u\left(q_\varepsilon\right) + \beta V_S\left(m + \ell_\varepsilon + z_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon, z_\varepsilon\right)$$
$$\text{s.t.} \quad m + z_\varepsilon + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon \geq 0,\ \rho_\ell b - z_\varepsilon - \ell_\varepsilon \geq 0 \text{ and } \rho_m b - z_\varepsilon - \ell_\varepsilon \geq 0.$$

The first inequality is the buyer's budget constraint in the goods market. The second inequality is the collateral constraint associated with the choice of ℓ_ε, and the third inequality is the collateral constraint associated with the choice of z_ε. Let $\beta\phi^+\lambda_\varepsilon^A$ denote the Lagrange multiplier for the first inequality, $\beta\phi^+\lambda_\ell^A$ denote the Lagrange multiplier of the second inequality, and $\beta\phi^+\lambda_z^A$ denote the Lagrange multiplier for the third inequality.

In the above optimization problem, we set $d_\varepsilon = 0$ and $\ell_\varepsilon = 0$ because active buyers never use the standing facilities if $\rho_d > \rho_m > \rho_\ell$. They use the deposit facility if and only if $\rho_d = \rho_m$ and the borrowing facility if and only if $\rho_\ell = \rho_m$. For brevity of our analysis, in the characterization below we ignore these two cases by assuming $\rho_d > \rho_m > \rho_\ell$.

If $\rho_d > \rho_m > \rho_\ell$, we can use (8) to write the first-order conditions for q_ε and z_ε as

follows:

$$\begin{aligned} \varepsilon u'(q_\varepsilon) - \beta p \phi^+ \left(1 + \lambda_\varepsilon^A\right) &= 0 \\ -1/\rho_m + \left(1 + \lambda_\varepsilon^A\right) - \lambda_z^A &= 0. \end{aligned} \tag{14}$$

Lemma 2 characterizes the optimal borrowing and lending decisions and the quantity of goods obtained by an active $\varepsilon-$buyer:

Lemma 2 *There exist critical values ε_z, $\varepsilon_{\bar{z}}$, with $0 \leq \varepsilon_z \leq \varepsilon_{\bar{z}}$, such that the following is true: if $0 \leq \varepsilon < \varepsilon_z$, an active buyer lends money in the money market; if $\varepsilon_z < \varepsilon \leq \varepsilon_{\bar{z}}$, he borrows money and the collateral constraint is nonbinding; if $\varepsilon_{\bar{\ell}} \leq \varepsilon$, he borrows money and the collateral constraint is binding. The critical values in the money market solve*

$$\varepsilon_z = \beta \phi^+ m/\rho_m, \text{ and } \varepsilon_{\bar{z}} = \beta \phi^+ m/\rho_m + \beta \phi^+ b. \tag{15}$$

Furthermore, the amount of borrowing and lending by an active buyer with a taste shock ε and the amount of goods purchased by the buyer satisfy:

$$\begin{aligned} q_\varepsilon &= \varepsilon \rho_m/\rho_d, & z_\varepsilon &= p(\varepsilon - \varepsilon_z), & &\text{if } 0 \leq \varepsilon \leq \varepsilon_z \\ q_\varepsilon &= \varepsilon \rho_m/\rho_d, & z_\varepsilon &= p(\varepsilon - \varepsilon_z), & &\text{if } \varepsilon_z \leq \varepsilon \leq \varepsilon_{\bar{z}}, \\ q_\varepsilon &= \varepsilon_{\bar{z}} \rho_m/\rho_d, & z_\varepsilon &= \rho_m b, & &\text{if } \varepsilon_{\bar{z}} \leq \varepsilon. \end{aligned} \tag{16}$$

Figure 2 illustrates consumption quantities by active and non-active buyers. The black dotted linear curve (the 45–degree line) plots the first-best quantities. Consumption quantities by active buyers are increasing in ε in the interval $\varepsilon \in [0, \varepsilon_{\bar{z}})$ and are flat for $\varepsilon \geq \varepsilon_{\bar{z}}$. Note that initially the slope of the green curve is equal to $\rho_m/\rho_d \leq 1$, which means that the quantities consumed by active buyers are always below the first-best quantities, unless $\rho_m = \rho_d$. In contrast, the non-active buyers consume the efficient quantities in the interval $[0, \varepsilon_d]$. The reason is that, although they face the opportunity cost $1/\rho_d$ when choosing the optimal consumption quantity, the sellers pass over the interest payment they obtain at the deposit facility, which exactly compensates for this; see equation (10).

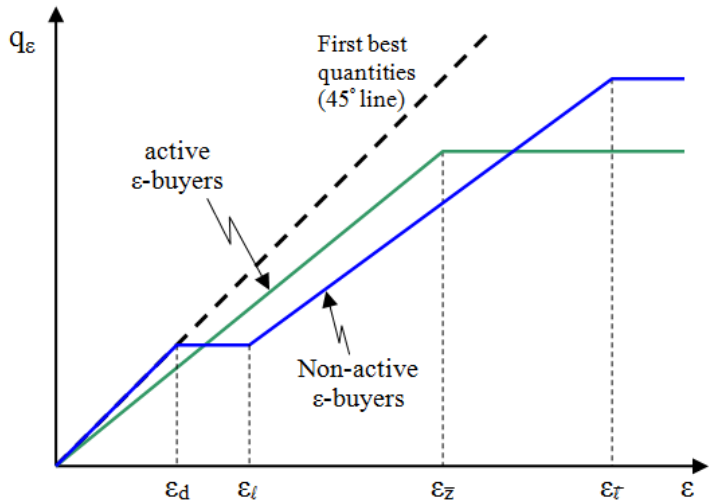

Figure 2: Consumption by active and non-active buyers.

4 Equilibrium

We focus on symmetric stationary equilibria with strictly positive demands for nominal government bonds and money. Such equilibria meet the following requirements: (i) Households' decisions are optimal, given prices; (ii) The decisions are symmetric across all sellers, symmetric across all active buyers with the same preference shock, and symmetric across all non-active buyers with the same preference shock; (iii) All markets clear; (iv) All real quantities are constant across time; (v) The consolidated government budget constraint (3) holds in each period.

Market clearing in the goods market requires

$$q_s - \int_0^\infty q_\varepsilon dF(\varepsilon) = 0, \tag{17}$$

where q_s is aggregate production by sellers in the goods market.

To understand the role of market segmentation and the role of the standing facilities to complete the market, we first consider a laissez-faire money market where

the central bank offers $\rho_d > 1$ and $\rho_\ell = 0$. In this case, no agent uses these facilities. Let ρ_m^u denote the rate that would clear the money market under laissez-faire. From Lemma 2, the laissez-faire supply of money satisfies

$$SM\left(\rho_m^u\right) = \pi \int_0^{\varepsilon_z} p\left(\varepsilon_z - \varepsilon\right) dF\left(\varepsilon\right)$$

and the laissez-faire demand for money satisfies

$$DM\left(\rho_m^u\right) = \pi \int_{\varepsilon_z}^{\varepsilon_{\bar{z}}} p\left(\varepsilon - \varepsilon_z\right) dF\left(\varepsilon\right) + \pi \int_{\varepsilon_{\bar{z}}}^{\infty} \rho_m^u b dF\left(\varepsilon\right),$$

where $\varepsilon_z = \varepsilon_d \frac{\rho_d}{\rho_m^u}$, $\varepsilon_{\bar{z}} = \left(\varepsilon_d \frac{\rho_d}{\rho_m^u}\right)\left(1 + \rho_m^u \mathcal{B}\right)$. Laissez-faire market clearing requires

$$SM\left(\rho_m^u\right) = DM\left(\rho_m^u\right). \tag{18}$$

This is the market rate that would prevail under laissez-faire.

Suppose now $\rho_d \leq 1$ and $\rho_\ell > 0$ and suppose (18) yields $\rho_m^u > \rho_d$; i.e., the deposit rate is higher than the laissez-faire money market rate. In this case, buyers prefer to deposit at the central bank, which reduces the supply of money until $\rho_m^u = \rho_d$. Thus, if $SM\left(\rho_d\right) > DM\left(\rho_d\right)$, we must have $\rho_m = \rho_d$. Along the same lines, suppose (18) yields $\rho_m^u < \rho_\ell$. In this case, buyers prefer to borrow at the central bank, which reduces the demand for money until $\rho_m^u = \rho_\ell$. Thus, if $SM\left(\rho_\ell\right) < DM\left(\rho_\ell\right)$, we must have $\rho_m = \rho_\ell$. Finally, if $\rho_d > \rho_m^u > \rho_\ell$, buyers prefer to trade in the money market and so $\rho_m = \rho_m^u$.

Accordingly, we can formulate the market-clearing condition as follows:

$$\rho_m = \begin{cases} \rho_d & \text{if} \quad DM\left(\rho_d\right) < SM\left(\rho_d\right) \\ \rho_\ell & \text{if} \quad DM\left(\rho_\ell\right) > SM\left(\rho_\ell\right) \\ \rho_m^u & \quad \text{otherwise.} \end{cases} \tag{19}$$

Let $\gamma \equiv M^+/M$ denote the constant gross money growth rate, let $\eta \equiv B^+/B$ denote the constant gross bond growth rate, and let $\mathcal{B} \equiv B/M$ denote the bonds-to-money ratio. We assume there are positive initial stocks of money M_0 and government

bonds B_0.[17]

Lemma 3 *In any stationary equilibrium, the bond-to-money ratio $\mathcal{B}$ has to be constant, and this can be achieved only when the growth rates of money and bonds are equal.*

According to Lemma 3, in any stationary equilibrium the stock of money and the stock of bonds must grow at the same rate. This result follows from the budget constraints of the buyers. By definition, in a stationary equilibrium, all real quantities are constant. Consider, for example, a non-active buyer who, according to Lemma 1, does not use the central bank's standing facilities. His budget constraint satisfies $q_\varepsilon = \beta\phi^+ m/\rho_d$. Symmetry requires that $m = M^+$, which implies that the real stock of money $\phi^+ M^+$ must be constant. Consider, next, a buyer who, according to Lemma 1, is constrained by his bond holdings. His budget constraint satisfies $q_\varepsilon = \left(\beta\phi^+ m + \beta\rho_\ell\phi^+ b\right)/\rho_d$. Then, since $\phi^+ m$ is constant, the real quantity of bonds $\phi^+ b$ must be constant, since in a symmetric equilibrium $b = B^+$. The result that $\phi^+ M^+$ and $\phi^+ B^+$ are constant implies $\gamma = \eta$. Finally, note that the gross inflation rate p^+/p in the goods market is equal to γ. This follows from the seller's first-order condition (10).

The result of Lemma 3 raises an interesting question. Since the growth rate of bonds η is chosen by the government and the growth rate of money γ by the central bank, the question is: Who is in charge? A related issue is discussed in Sargent and Wallace (1981). They show that if fiscal policy is dominant (chosen first), then the central bank may lose control over the inflation rate. In our context, if the government chooses η, then the central bank must follow by setting $\gamma = \eta$. Conversely, if the central bank chooses γ, then the government must choose $\eta = \gamma$. Even though these considerations are interesting, for the optimal policy that we will present below it does not matter which agency is dominant. We, therefore, assume that the government chooses η, which forces the central bank to choose $\gamma = \eta$. It then follows that the remaining policy variables of the central bank are ρ_d and ρ_ℓ.

Proposition 1 *A symmetric stationary equilibrium with a positive demand for money and bonds is a policy (ρ_d, ρ_ℓ) and endogenous variables $(\rho, \rho_m, \varepsilon_d, \varepsilon_\ell, \varepsilon_{\bar{\ell}}, \varepsilon_z, \varepsilon_{\bar{z}})$ satisfy-*

[17] Since the assets are nominal objects, the government and the central bank can start the economy off with one-time injections of cash M_0 and bonds B_0.

ing (19) and

$$\begin{aligned}
\rho_d\eta/\beta &= \pi\left[\int_0^{\varepsilon_{\bar{z}}}(\rho_d/\rho_m)\,dF(\varepsilon)+\int_{\varepsilon_{\bar{z}}}^{\infty}(\rho_d/\rho_m)(\varepsilon/\varepsilon_{\bar{z}})\,dF(\varepsilon)\right] \\
&\quad +(1-\pi)\left[\int_0^{\varepsilon_d}dF(\varepsilon)+\int_{\varepsilon_d}^{\varepsilon_\ell}(\varepsilon/\varepsilon_d)\,dF(\varepsilon)+\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}(\rho_d/\rho_\ell)\,dF(\varepsilon)\right. \\
&\quad \left.+\int_{\varepsilon_{\bar{\ell}}}^{\infty}(\varepsilon/\varepsilon_{\bar{\ell}})(\rho_d/\rho_\ell)\,dF(\varepsilon)\right]
\end{aligned} \tag{20}$$

$$\begin{aligned}
\rho\eta/\beta &= \pi\left[\int_0^{\varepsilon_{\bar{z}}}dF(\varepsilon)+\int_{\varepsilon_{\bar{z}}}^{\infty}(\varepsilon/\varepsilon_{\bar{z}})\,dF(\varepsilon)\right] \\
&\quad +(1-\pi)\left[\int_0^{\varepsilon_{\bar{\ell}}}dF(\varepsilon)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}(\varepsilon/\varepsilon_{\bar{\ell}})\,dF(\varepsilon)\right]
\end{aligned} \tag{21}$$

$$\varepsilon_\ell=\varepsilon_d(\rho_d/\rho_\ell)\,,\ \varepsilon_{\bar{\ell}}=\varepsilon_\ell(1+\rho_\ell\mathcal{B})\,,\varepsilon_z=\varepsilon_d(\rho_d/\rho_m)\,,\ \textit{and}\ \varepsilon_{\bar{z}}=\varepsilon_z(1+\rho_m\mathcal{B})\,. \tag{22}$$

Equation (20) is obtained from the choice of money holdings (6). Equation (21) is obtained from (6) and (7); in any equilibrium with a strictly positive demand for money and bonds, $\rho V_M^m(m,b)=V_M^b(m,b)$. We then use this arbitrage equation to derive (21). Finally, equations (22) are derived from the budget constraints of the buyers.

5 Optimal Policy

Proposition 2 *The optimal policy is to set $\rho_d=\beta/\eta$. This policy implements the first-best allocation. Under the optimal policy, the settlement and money market prices satisfy $\rho=\rho_m=\rho_d$.*

Note that the optimal policy (ρ_d,ρ_ℓ) is not unique, since under the optimal policy ρ_ℓ is irrelevant. The reason is that under the optimal policy the buyers never borrow. Accordingly, any value of $\rho_\ell\leq\rho_d$ is consistent with the optimal policy. Therefore, under the optimal policy, the central bank can operate a floor system or a channel system. They both yield the first-best allocation.

The optimal policy makes holding money costless and therefore satiates money demand as described by the Friedman rule. Note that such a policy means that the settlement market price ρ and the central bank's ask price ρ_d exactly compensate

market participants for their impatience and for inflation. Under this policy, the rate of return on money is the same as the rate of return on government bonds. Hence, they have the same marginal liquidity value, which is zero.[18]

In summary, the optimal monetary policy satisfies the Friedman rule and takes the form of the central bank paying interest on deposits of central bank money. We are clearly not the first to point out that the Friedman rule can take the form of paying interest on deposits. For example, in Section 2.4.1, the textbook of Walsh (2010) states that the Friedman rule can be achieved by paying interest on money. Another example for this result can be found in chapter 6 of the book by Nosal and Rocheteau (2011). Finally, Andolfatto (2010), Lagos (2010a), and Williamson (2012) derive results on the optimality and implementation of the Friedman rule in search theoretical models of money. In Cúrdia and Woodford (2011) the same policy satiates the demand for central bank reserves. The novel results of our paper are presented in Proposition 3 below.

Under the optimal policy, no one borrows from the standard facility, but there are deposits. The question is how are these interest payments on deposits financed?

Corollary 1 *The optimal policy requires that* $\mathcal{D} < 0$.

The optimal policy requires that the government runs a primary surplus ($\mathcal{D} < 0$). Therefore, it must collect taxes and hand them over to the central bank to finance the interest payments on deposits. To see this, from (1) the central bank's surplus is

$$\mathcal{S} = M^{+} - M + (1/\rho_{\ell} - 1)\, L - (1/\rho_{d} - 1)\, D.$$

Under the optimal policy (see the proof of Proposition 2), $L = 0$ and $D = M$. Accordingly, the central bank's surplus is

$$\mathcal{S} = M^{+} - M/\rho_{d}.$$

The remaining question is whether the central bank can attain a positive surplus by printing money. Since in steady state $M^{+}/M = \eta$ and under the optimal policy

[18] Since the first-best quantities are $q_{\varepsilon} = \varepsilon$ with the support of ε being unbounded, the real value of money approaches infinity; i.e., the price level approaches zero. Any finite upper bound would yield a finite, strictly positive price level.

$1/\rho_d = \eta/\beta$, the central bank's surplus is

$$\mathcal{S} = (\eta/\beta)(\beta - 1) M < 0. \tag{23}$$

Under the optimal policy, the central bank makes a deficit, which requires a transfer of funds equal to $(\eta/\beta)(\beta - 1) M$ from the government to the central bank in each period. From the consolidated budget constraint, it then requires that the government has a primary surplus.

One can think of two ways out of this problem. First, the central bank is endowed with a stock of real assets that provide sufficient revenue in each period to cover the losses that occur under the optimal policy. Second, the central bank can be endowed with a stock of government bonds that pay sufficient interest to cover the losses described above. We discuss these possibilities in Section 7.

6 Constrained-optimal Policy

In the previous section, we showed that under the optimal policy the central bank makes a deficit; i.e., $\mathcal{S}^o \equiv \frac{\eta}{\beta}(\beta - 1) M < 0$. This requires a transfer of funds from the government to the central bank in each period. In this section, we assume that the central bank does not receive enough funds to run the optimal policy. Receiving less funds implies that $\mathcal{S} > \mathcal{S}^o$.[19]

Since we find it more intuitive, we phrase the next proposition and our discussion of it in terms of the deposit rate i_d and the loan rate i_ℓ, instead of the ask price ρ_d and the bid price ρ_ℓ. For what follows, let $i_d^* = \eta/\beta - 1$ denote the deposit rate that implements the first-best allocation.

Proposition 3 *If $\mathcal{S} > \mathcal{S}^o$, the constrained-optimal policy is to choose a strictly positive interest rate spread $i_d < i_\ell$ with $i_d < i_d^*$.*

In the proof of Proposition 3, we show that the constrained optimal policy requires that $i_d < i_d^*$ and that increasing i_d is strictly welfare improving. The reason is that paying interest on "idle" money holdings improves economic efficiency (see Berentsen et al. (2007)). Thus, fiscal considerations are the reason why the central bank chooses $i_d < i_d^*$, since without sufficient funds it is not able to set $i_d = i_d^*$.

[19] Note that this constraint includes the case where the central bank makes a surplus ($\mathcal{S} > 0$), as is the case for most central banks.

Note that the constraint $\mathcal{S} > \mathcal{S}^o$ does not mean that the central bank is unable to run a floor system. It can always choose $i_d = i_\ell < i_d^*$. Rather, it means that it is not optimal to do so. By marginally increasing i_ℓ, the central bank makes it relatively more costly to turn bonds into money and hence consumption. This affects the portfolio choice of agents in the settlement market. Most importantly, the demand for money increases; and, since the nominal quantity of money is given, the real value of money increases.[20] For those who are not borrowing-constrained (for non-active buyers with $\varepsilon \in [\varepsilon_d, \varepsilon_{\bar{\ell}}]$) the higher marginal borrowing cost lowers their consumption at the margin. However, starting from $i_d = i_\ell$, this welfare loss is of second order. For those who are borrowing-constrained (for non-active buyers with $\varepsilon > \varepsilon_{\bar{\ell}}$) the marginal higher borrowing cost has no effect on their consumption, yet their higher real balances allow them to consume more. Starting from $i_d = i_\ell$, this welfare gain is of first order, and so the overall effect is strictly positive.

Another way to explain this is as follows. Without central bank intervention, only a fraction π of agents have access to the money market. By setting a floor system with $i_d = i_m = i_\ell$, the central bank completes the market since all agents can trade at rate i_m. This intervention has two effects. First, the central bank provides insurance against the idiosyncratic liquidity shocks ε for the non-active buyers. On the other hand, by insuring these agents, it reduces the demand for money ex-ante and thus decreases its value. Increasing the loan rate i_ℓ marginally removes some of that insurance and increases the demand for money and its value. Thus, the key result of Proposition 3 is that it is not optimal to complete the market, unless the Friedman rule is feasible.

[20] In fact, in the proof of Proposition 3 we show that the real value of money is increasing in i_ℓ. This result is related to Berentsen et al. (2013), who show in a related environment that restricting access to financial markets can be welfare improving. The reason for this result is that in their environment the portfolio choices of agents exhibit a pecuniary externality and that this externality can be so strong that the optimal policy response is to reduce the frequency at which agents can trade in financial markets.

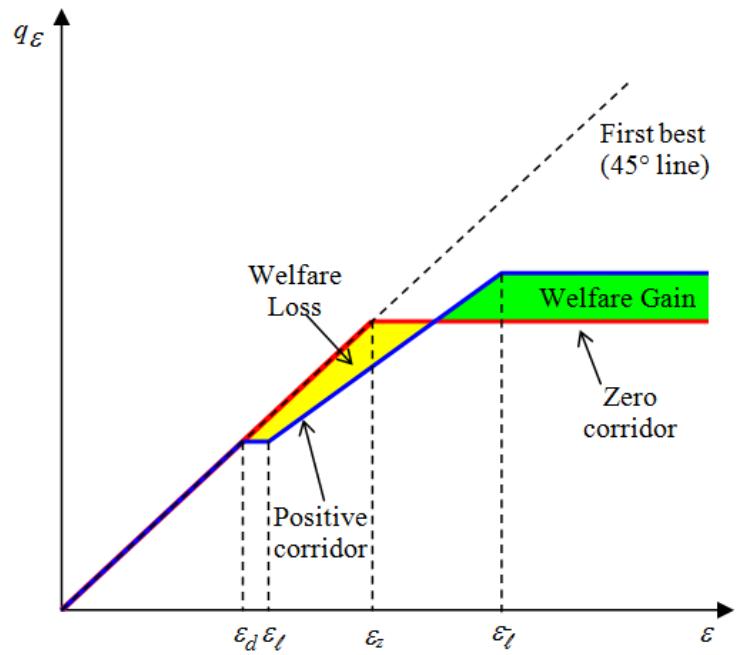

Figure 3: Welfare effects from increasing i_ℓ.

Figure 3 graphically illustrates why increasing the loan rate is welfare improving. The black dotted linear curve (the 45–degree line) plots the first-best consumption quantities. The red curve (labeled *zero corridor*) plots the consumption quantities for non-active buyers when $i_d = i_\ell$. Up to ε_z, they receive the first-best consumption quantities after which the collateral constraint is binding, as indicated by the consumption quantities that are independent of ε. The blue curve (labeled *positive corridor*) plots the quantities for non-active buyers when $i_d < i_\ell$. Up to the critical value ε_d, they consume the first-best quantities; i.e., $q_\varepsilon = \varepsilon$. For $\varepsilon \in [0, \varepsilon_d]$ they deposit any excess money at the deposit facility. For $\varepsilon \in [\varepsilon_d, \varepsilon_\ell]$ they neither deposit nor borrow money. They simply spend all money brought into the period and consume $q_\varepsilon = \varepsilon_d$. For $\varepsilon \in [\varepsilon_\ell, \varepsilon_{\bar{\ell}}]$ they borrow, but their collateral constraints are non-binding. Finally, for $\varepsilon > \varepsilon_{\bar{\ell}}$ the collateral constraints are binding.

As indicated by Figure 3, increasing i_ℓ lowers the consumption of non-active medium ε-buyers and increases the consumption of non-active high-ε buyers. The first effect lowers welfare, while the second increases welfare. In the proof of Proposition 3, we show that, starting from $i_d = i_\ell$, the net gain is always positive.

7 Discussion

We have previously established that under the optimal policy, the central bank makes a deficit, which requires a transfer of funds from the government to the central bank in each period. One can think of two ways out of this problem. First, the central bank is endowed with a stock of real assets that provide sufficient revenue in each period to cover the losses that occur under the optimal policy. In practice, many central banks have such capital and it is often argued that the benefit of being well capitalized is that it helps preserve the independence of a central bank. Second, the central bank is endowed with a stock of government bonds that pay sufficient interest to cover the losses described above.

In what follows we amend the central bank's balance sheet with capital and bonds.

Bonds Let us assume the central bank holds government bonds. Let B_C be the stock of government bonds held by the central bank, and let B be the stock of bonds held by private agents. Then, the total stock of bonds in circulation is $B_G = B_C + B$. The bond-augmented central bank's surplus is therefore

$$\mathcal{S} = M^+ - M + (1/\rho_\ell - 1)\, L - (1/\rho_d - 1)\, D - \rho B_C^+ + B_C.$$

Substituting $\mathcal{S}$ into (2) yields

$$\mathcal{D} = M^+ - M + (1/\rho_\ell - 1)\, L - (1/\rho_d - 1)\, D + \rho B^+ - B$$

which is identical to the consolidated government budget constraint (3). Consequently, the result in Proposition 3 is not affected. What matters for the consolidated budget constraint is the total stock of bonds in circulation and not the stock of bonds in the hands of the central bank.

Endowing the central bank with government bonds is simply a way to hide transfer payments from the government to the central bank. In this case, the government has to levy taxes to finance interest payments on the government bonds, which it then hands over to the central bank. The central bank, then uses these funds to pay interest on reserves. In practice, this means it pays considerable sums to private sector banks, which can be politically problematic, as argued in the introduction.

The literature on paying interest on reserves is largely unconcerned with this point.

For example, Goodfriend (2002, p.5) argues "Suppose a central bank such as the Fed confines its asset purchases mainly to Treasury securities. In that case, interest on the increase in reserves will be self-financing if there is a positive spread between longer term Treasury securities and the rate of interest on reserves. Reserve balances at the central bank paying market interest are like one-day Treasury securities. Hence, interest rate spreads between longer term Treasury securities and overnight deposits at the central bank should exhibit term premia ordinarily reflected in the Treasury yield curve. Therefore, a central bank such as the Fed should be able to self-finance interest on the enlarged demand for reserves in the new regime. In fact, the net interest spread earned on new assets acquired in the interest-and-reserves regime would raise additional revenue for the central bank."

Goodfriend's (2002) argument is a technical one. It states that a central bank can always run the Friedman rule if it is endowed with a sufficiently large stock of bonds or capital. The argument that we make in this paper is not whether it is technically feasible, but rather that it is not sustainable politically, since at the end of the day the private sector has to be taxed to finance the interest on reserves, and these reserves are mainly held by a few large banks, some of them being foreign banks.

Capital Let K be the nominal stock of capital in the central bank's balance sheet and r the rate of return of capital. Then, the capital-augmented central bank's surplus is

$$\mathcal{S} = M^{+} - M + (1/\rho_{\ell} - 1)\, L - (1/\rho_{d} - 1)\, D + rK. \tag{24}$$

Using the last equation, the consolidated government budget constraint (2), is given by

$$\mathcal{D} = M^{+} - M + (1/\rho_{\ell} - 1)\, L - (1/\rho_{d} - 1)\, D + \rho B^{+} - B + rK. \tag{25}$$

Let $\mathcal{K} \equiv K/M$ denote the capital-to-money ratio and $\bar{\mathcal{K}} \equiv (\eta/\beta + \mathcal{B})\,(1-\beta)\,/r > 0$.

Proposition 4 *The optimal policy generates no losses for the central bank if $\mathcal{K} \geq \bar{\mathcal{K}}$.*

According to Proposition 4, the central bank can operate the optimal floor system if it has a sufficiently large capital stock. The income generated by the capital stock is then used to finance the interest payments on reserves. Note that this condition is more likely to be violated if: 1) r is too low, 2) inflation, η, is too high, or 3) the

ratio of bonds to money, $\mathcal{B}$, is too large. Thus, having a significant holding of real assets may still not be sufficient to avoid having to receive fiscal transfers under the optimal floor system.[21]

8 Conclusions

Despite the growing use of channel or floor systems to implement monetary policy, only a few theoretical studies exist. In particular, there are no formal studies that compare the two systems. This paper attempts to close this gap by constructing a general equilibrium model, where a central bank chooses to conduct monetary policy via either a floor system or a channel system. Unlike the existing literature, we explicitly take into account the financial implications of paying interest on deposits.

The following results emerge from our analysis. First, the optimal framework is a floor system *if and only if* the target rate satisfies the Friedman rule. Second, implementing the *optimal* floor system is costly for the central bank. It requires that the central bank either has sufficient capital income to incur the interest expense or receives transfers from the fiscal authority. In either case, fewer resources are available to the government to finance its other priorities, which may lead to a political backlash and restrictions on the central bank's ability to pay interest on reserves. This is the unpleasant fiscal arithmetic of a floor system. Third, if the central bank is constrained by the fiscal authority regarding the size of its interest expense, a channel system is optimal. This last result does not mean that the central bank cannot implement a floor system, since it can always set the loan rate equal to the deposit rate, implying that the money market rate is equal to the deposit rate. Such a floor system, however, is suboptimal and the central bank can always do better by choosing a channel system instead.

In a nutshell, our paper provides a rationale for operating a channel system as opposed to a floor system. Our explanation rests on the idea that central banks may be unable, or are unwilling for political reasons, to incur the interest expense required by the optimal floor system.

[21] Having assets with insufficiently low interest rates is exactly the situation Costa Rica has faced for the past 20 years; its interest-earning assets do not generate enough income to pay the interest on its liabilities. Consequently, the central bank must get annual transfers of revenue from the Treasury.

Appendix

Proof of Lemma 1. We first derive the cut-off values ε_d and ε_ℓ. For this proof, we add a subscript d to the notation of the consumption level of a buyer if the non-active buyer deposits money at the central bank, a subscript ℓ if the buyer takes out a loan and the collateral constraint is nonbinding, a subscript $\bar{\ell}$ if the buyer takes out a loan and the collateral constraint is binding, and a subscript 0 if the buyer does neither.

From (11), the consumption level of a buyer who enters the goods market satisfies

$$q_d(\varepsilon) = \frac{\rho_d \varepsilon}{p\beta\phi^+}, \quad q_\ell(\varepsilon) = \frac{\rho_\ell \varepsilon}{p\beta\phi^+}. \tag{26}$$

A buyer who does not use the deposit facilities will spend all his money on goods, since, if he anticipated that he would have idle cash after the goods trade, it would be optimal to deposit the idle cash in the intermediary, provided $\rho_d < 1$. Thus, consumption of such a buyer is

$$q_0(\varepsilon) = \frac{m}{p}. \tag{27}$$

At $\varepsilon = \varepsilon_d$, the household is indifferent between depositing and not depositing. We can write this indifference condition as

$$\varepsilon_d u(q_d) - \beta\phi^+ [pq_d - (1/\rho_d - 1)\, d] = \varepsilon_d u(q_0) - \beta\phi^+ pq_0.$$

By using (26), (27), and $d = m - pq_d$, we can write the equation further as

$$\varepsilon_d u\left[\frac{\rho_d \varepsilon_d}{\beta\phi^+ m}\right] = \varepsilon_d - \beta\phi^+ m/\rho_d.$$

The unique solution to this equation is $\varepsilon_d = \beta\phi^+ m/\rho_d$, which implies that $\beta\phi^+ m < \varepsilon_d$.

At $\varepsilon = \varepsilon_\ell$, the household is indifferent between borrowing and not borrowing. We can write this indifference condition as

$$\varepsilon_\ell u(q_\ell) - \beta\phi^+ [pq_\ell + (1/\rho_\ell - 1)\, \ell] = \varepsilon_\ell u(q_0) - \beta\phi^+ pq_0.$$

Using (26), (27), and $\ell = pq_\ell - m$, we can write this equation further as

$$\varepsilon_\ell u\left[\frac{\rho_\ell \varepsilon_\ell}{\beta\phi^+ m}\right] = \varepsilon_\ell - \beta\phi^+ m/\rho_\ell.$$

The unique solution to this equation is $\varepsilon_\ell = \beta\phi^+ m/\rho_\ell$. Using the expression for ε_d we get

$$\varepsilon_\ell = \varepsilon_d \left(\rho_d/\rho_\ell\right). \tag{28}$$

We now calculate $\varepsilon_{\bar{\ell}}$. There is a critical buyer who enters the goods market and wants to take out a loan, whose collateral constraint is just binding. From (11), for this buyer we have the following equilibrium conditions: $q_{\bar{\ell}} = \frac{\rho_\ell \varepsilon_{\bar{\ell}}}{\beta\phi^+ p}$ and $pq_{\bar{\ell}} = m + \rho_\ell b$. Eliminating $q_{\bar{\ell}}$ we get

$$\varepsilon_{\bar{\ell}} = \beta\phi^+ m/\rho_\ell + \beta\phi^+ b.$$

Using (28) we get

$$\varepsilon_{\bar{\ell}} = \varepsilon_d \frac{\rho_d}{\rho_\ell}\left(1 + \rho_\ell \frac{b}{m}\right).$$

It is then evident that

$$0 \leq \varepsilon_d \leq \varepsilon_\ell \leq \varepsilon_{\bar{\ell}}.$$

■

Proof of Lemma 2. The proof of Lemma 2 is very similar to the proof of Lemma 1 and is omitted. ■

Proof of Lemma 3. A stationary equilibrium requires that all real quantities are constant and symmetry requires that $m = M^+$ and $b = B^+$. From Lemma 1, there are two critical consumption quantities in our model:

$$\begin{aligned} q_\varepsilon &= \varepsilon_d = \beta\phi^+ M^+/\rho_d && \textit{if } \varepsilon_d \leq \varepsilon \leq \varepsilon_\ell \\ q_\varepsilon &= \varepsilon_{\bar{\ell}}\rho_\ell/\rho_d = \left(\beta\phi^+ M^+ + \beta\rho_\ell\phi^+ B^+\right)/\rho_d && \textit{if } \varepsilon_{\bar{\ell}} \leq \varepsilon. \end{aligned}$$

The first quantity requires that the real stock of money is constant; i.e., $\phi M = \phi^+ M^+$, implying that $\phi/\phi^+ = \gamma$.

Since $\phi^+ M^+$ is constant, the second quantity requires that $\phi^+ B^+$ is constant too; i.e., $\phi B = \phi^+ B^+$. This implies that the stock of bonds has to grow at the same rate as the stock of money. ■

Proof of Proposition 1. The proof involves deriving equations (20), (21), and (22).

Equations (22) are derived in the proof of Lemma 1. To derive equation (20),

differentiate (9) with respect to m to get

$$\begin{aligned} V_M^m(m,b) &= \pi \int_0^\infty \left[\beta V_S^m\left(m+z_\varepsilon+\ell_\varepsilon-pq_\varepsilon-d_\varepsilon,b,\ell_\varepsilon,d_\varepsilon,z_\varepsilon|\,\varepsilon\right)+\beta\phi^+\lambda_\varepsilon^A\right]dF(\varepsilon) \\ &\quad +(1-\pi)\int_0^\infty \left[\beta V_S^m\left(m+z_\varepsilon+\ell_\varepsilon-pq_\varepsilon-d_\varepsilon,b,\ell_\varepsilon,d_\varepsilon,z_\varepsilon|\,\varepsilon\right)+\beta\phi^+\lambda_\varepsilon\right]dF(\varepsilon). \end{aligned}$$

Then, use (8) to replace V_S^m, (11) to replace $\beta\phi^+\lambda_\varepsilon$, and (14) to replace $\beta\phi^+\lambda_\varepsilon^A$ to obtain

$$V_M^m(m,b)=\pi\int_0^\infty \frac{\varepsilon u'(q_\varepsilon)}{p}dF(\varepsilon)+(1-\pi)\int_0^\infty \frac{\varepsilon u'(q_\varepsilon)}{p}dF(\varepsilon). \tag{29}$$

Note that the two integrals are not equal since the quantities q_ε differ for active and non-active buyers. Use the first-order condition (10) to replace p to get

$$V_M^m(m,b)=\left(\beta\phi^+/\rho_d\right)\left[\pi\int_0^\infty \varepsilon u'(q_\varepsilon)\,dF(\varepsilon)+(1-\pi)\int_0^\infty \varepsilon u'(q_\varepsilon)\,dF(\varepsilon)\right].$$

Use (6) to replace $V_M^m(m,b)$ and replace ϕ/ϕ^+ by η to get

$$\frac{\rho_d\eta}{\beta}=\pi\int_0^\infty \varepsilon u'(q_\varepsilon)\,dF(\varepsilon)+(1-\pi)\int_0^\infty \varepsilon u'(q_\varepsilon)\,dF(\varepsilon).$$

Finally, note that $u'(q)=1/q$ and replace the quantities q_ε of the first integral using Lemma 2 and the quantities q_ε of the second integral using Lemma 1 to get (20), which we replicate here:

$$\begin{aligned} \frac{\rho_d\eta}{\beta} &= \pi\left[\int_0^{\varepsilon_{\bar{z}}}\frac{\rho_d}{\rho_m}dF(\varepsilon)+\int_{\varepsilon_{\bar{z}}}^\infty \frac{\varepsilon}{\varepsilon_{\bar{z}}}\frac{\rho_d}{\rho_m}dF(\varepsilon)\right] \\ &\quad +(1-\pi)\left[\int_0^{\varepsilon_d}dF(\varepsilon)+\int_{\varepsilon_d}^{\varepsilon_\ell}\frac{\varepsilon}{\varepsilon_d}dF(\varepsilon)+\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\frac{\rho_d}{\rho_\ell}dF(\varepsilon)+\int_{\varepsilon_{\bar{\ell}}}^\infty \frac{\varepsilon}{\varepsilon_{\bar{\ell}}}\frac{\rho_d}{\rho_\ell}dF(\varepsilon)\right], \end{aligned} \tag{30}$$

where $\varepsilon_\ell=\varepsilon_d\frac{\rho_d}{\rho_\ell}$, $\varepsilon_{\bar{\ell}}=\left(\varepsilon_d\frac{\rho_d}{\rho_\ell}\right)(1+\rho_\ell\mathcal{B})$, $\varepsilon_z=\varepsilon_d\frac{\rho_d}{\rho_m}$, $\varepsilon_{\bar{z}}=\left(\varepsilon_d\frac{\rho_d}{\rho_m}\right)(1+\rho_m\mathcal{B})$. Note

that if $\pi = 0$, then (30) yields ε_d, since no other endogenous variables are contained in (30). To see this, replace ε_ℓ and $\varepsilon_{\bar{\ell}}$ by $\varepsilon_d \frac{\rho_d}{\rho_\ell}$ and $\left(\varepsilon_d \frac{\rho_d}{\rho_\ell}\right)(1 + \rho_\ell \mathcal{B})$, respectively.

To derive (21), note that in any equilibrium with a strictly positive demand for money and bonds, we must have $\rho V_M^m(m,b) = V_M^b(m,b)$. We now use this arbitrage equation to derive (21). We have already derived $V_M^m(m,b)$ in (29). To get $V_M^b(m,b)$ differentiate $V_M(m,b)$ with respect to b and note that $\beta\phi^+\lambda_\ell^A = 0$ to get

$$\begin{aligned} V_M^b(m,b) &= \pi \int_0^\infty \left[\beta V_S^b(m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon | \varepsilon) + \rho_m \beta\phi^+\lambda_z^A\right] dF(\varepsilon) \\ &\quad + (1-\pi) \int_0^\infty \left[\beta V_S^b(m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon | \varepsilon) + \rho_\ell \beta\phi^+\lambda_\ell\right] dF(\varepsilon). \end{aligned}$$

Use (8) to replace V_S^b to get

$$V_M^b(m,b) = \pi\beta\phi^+ \int_0^\infty \left(1 + \rho_m \lambda_z^A\right) dF(\varepsilon) + (1-\pi)\beta\phi^+ \int_0^\infty (1 + \rho_\ell \lambda_\ell)\, dF(\varepsilon).$$

Use (11) to replace λ_ℓ, (14) to replace λ_z^A, and rearrange to get

$$\begin{aligned} V_M^b(m,b) &= \pi \left[\int_0^{\varepsilon_{\bar{z}}} \beta\phi^+ dF(\varepsilon) + \int_{\varepsilon_{\bar{z}}}^\infty \rho_m \frac{\varepsilon u'(q_\varepsilon)}{p} dF(\varepsilon)\right] \\ &\quad + (1-\pi)\left[\int_0^{\varepsilon_{\bar{\ell}}} \beta\phi^+ dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^\infty \rho_\ell \frac{\varepsilon u'(q_\varepsilon)}{p} dF(\varepsilon)\right]. \end{aligned}$$

Use (10) to replace p to get

$$\begin{aligned} V_M^b(m,b) &= \pi \left[\int_0^{\varepsilon_{\bar{z}}} \beta\phi^+ dF(\varepsilon) + \int_{\varepsilon_{\bar{z}}}^\infty \beta\phi^+ (\rho_m/\rho_d)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon)\right] \\ &\quad + (1-\pi)\left[\int_0^{\varepsilon_{\bar{\ell}}} \beta\phi^+ dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^\infty \beta\phi^+ (\rho_\ell/\rho_d)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon)\right]. \end{aligned}$$

Equate $\rho V_M^m(m,b) = V_M^b(m,b)$ and simplify to get

$$\begin{aligned}
& \pi \int_0^\infty \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) + (1-\pi) \int_0^\infty \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) \\
= \; & \pi \left[\int_0^{\varepsilon_{\bar{z}}} (\rho_d/\rho)\, dF(\varepsilon) + \int_{\varepsilon_{\bar{z}}}^\infty (\rho_m/\rho)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) \right] \\
& + (1-\pi) \left[\int_0^{\varepsilon_{\bar{\ell}}} (\rho_d/\rho)\, dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^\infty (\rho_\ell/\rho)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) \right].
\end{aligned}$$

Note that $\pi \int_0^\infty \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) + (1-\pi) \int_0^\infty \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) = \rho_d \eta/\beta$ and rearrange to get

$$\begin{aligned}
\frac{\rho\eta}{\beta} \; = \; & \pi \left[\int_0^{\varepsilon_{\bar{z}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{z}}}^\infty (\rho_m/\rho_d)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) \right] \\
& + (1-\pi) \left[\int_0^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^\infty (\rho_\ell/\rho_d)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) \right].
\end{aligned}$$

Finally, use Lemmas 1 and 2 to get (21), which we replicate here:

$$\frac{\rho\eta}{\beta} = \pi \left[\int_0^{\varepsilon_{\bar{z}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{z}}}^\infty (\varepsilon/\varepsilon_{\bar{z}})\, dF(\varepsilon) \right] + (1-\pi) \left[\int_0^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^\infty (\varepsilon/\varepsilon_{\bar{\ell}})\, dF(\varepsilon) \right].$$

Note that if $\pi = 0$, ρ depends on ε_d only since $\varepsilon_\ell = \varepsilon_d \frac{\rho_d}{\rho_\ell}$ and $\varepsilon_{\bar{\ell}} = \left(\varepsilon_d \frac{\rho_d}{\rho_\ell}\right)(1 + \rho_\ell \mathcal{B})$.
■

Proof of Proposition 2. Set $\rho_d = \beta/\eta$. Then, (20) reduces as follows:

$$\begin{aligned} 1 &= \pi \left[\int_0^{\varepsilon_{\bar{z}}} [\beta/(\eta\rho_m)]\, dF(\varepsilon) + \int_{\varepsilon_{\bar{z}}}^{\infty} [\beta/(\eta\rho_m)] (\varepsilon/\varepsilon_{\bar{z}})\, dF(\varepsilon) \right] \\ &\quad + (1-\pi) \left[\int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} (\varepsilon/\varepsilon_d)\, dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} [\beta/(\eta\rho_\ell)]\, dF(\varepsilon) \right. \\ &\quad \left. + \int_{\varepsilon_{\bar{\ell}}}^{\infty} (\varepsilon/\varepsilon_{\bar{\ell}}) [\beta/(\eta\rho_\ell)]\, dF(\varepsilon) \right]. \end{aligned}$$

This equation can hold if and only if $\varepsilon_d \to \infty$ and $\rho_m \to \beta/\eta$. To see this, note that $[\beta/(\eta\rho_m)] \geq 1$, $[\beta/(\eta\rho_m)](\varepsilon/\varepsilon_{\bar{z}}) \geq 1$ for $\varepsilon \geq \varepsilon_{\bar{z}}$, $(\varepsilon/\varepsilon_d) \geq 1$ for $\varepsilon \geq \varepsilon_d$, $[\beta/(\eta\rho_\ell)] \geq 1$, and $(\varepsilon/\varepsilon_{\bar{\ell}})[\beta/(\eta\rho_\ell)] \geq 1$ for $\varepsilon \geq \varepsilon_{\bar{\ell}}$. Accordingly, the right-hand side is larger or equal to one. Then, $\varepsilon_d \to \infty$ implies $\varepsilon_\ell, \varepsilon_{\bar{\ell}}, \varepsilon_z, \varepsilon_{\bar{z}} \to \infty$ from (22). Accordingly, for $\varepsilon_d \to \infty$ above equation approaches

$$1 = \pi \int_0^{\infty} [\beta/(\eta\rho_m)]\, dF(\varepsilon) + (1-\pi) \int_0^{\infty} dF(\varepsilon) \ ,$$

which implies that $\rho_m \to \beta/\eta$ as $\varepsilon_d \to \infty$. Thus, from Lemma 1, the first-best allocation $q_\varepsilon = \varepsilon$ for all ε is attained. Moreover, from (21), it is clear that the settlement market rate must satisfy $\rho = \beta/\eta$. ■

Proof of Corollary 1. We now show that the optimal policy requires that the government has a primary surplus $\mathcal{D} > 0$. In any equilibrium, the sellers' money holdings satisfy

$$pq_s = M + L - (1-\pi) \int_0^{\varepsilon_d} (M - pq_\varepsilon)\, dF(\varepsilon).$$

The left-hand side is the aggregate money receipts of sellers. The right-hand side is the beginning of period quantity of money, M; plus aggregate lending of money by the central bank, L; minus deposits by non-active buyers at the central bank. These buyers simply deposit any "idle" money to receive interest on it. Furthermore, in any equilibrium aggregate deposits satisfy

$$D = pq_s + (1-\pi) \int_0^{\varepsilon_d} (M - pq_\varepsilon)\, dF(\varepsilon),$$

where pq_s is deposits by sellers. These two equations imply that in any equilibrium, total deposits satisfy

$$D = M + L. \tag{31}$$

From Lemma 1, we know that only non-active buyers with a shock $\varepsilon \geq \varepsilon_\ell$ borrow from the central bank. Thus, aggregate lending is $L = (1-\pi)\int_{\varepsilon_\ell}^{\infty} \ell_\varepsilon dF(\varepsilon)$. From Lemma 1, we also know that $\ell_\varepsilon = p\left[(\rho_\ell/\rho_d)\varepsilon - \varepsilon_d\right]$ if $\varepsilon_\ell \leq \varepsilon \leq \varepsilon_{\bar{\ell}}$, and $\ell_\varepsilon = \rho_\ell b = p\left[(\rho_\ell/\rho_d)\varepsilon_{\bar{\ell}} - \varepsilon_d\right]$ if $\varepsilon \geq \varepsilon_{\bar{\ell}}$. Thus, real aggregate lending is

$$L/p = (1-\pi)\Psi, \tag{32}$$

where

$$\Psi \equiv \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \left[(\rho_\ell/\rho_d)\varepsilon - \varepsilon_d\right] dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \left[(\rho_\ell/\rho_d)\varepsilon_{\bar{\ell}} - \varepsilon_d\right] dF(\varepsilon).$$

Divide both sides of (25) by M to get

$$\frac{\mathcal{D}}{M} = \eta - 1 - \frac{(1/\rho_d - 1)D - (1/\rho_\ell - 1)L}{M} - \mathcal{B}(1-\rho\eta).$$

Eliminating D and L using (31) and (32), respectively, and noting that $M/p = \varepsilon_d$, the last expression can be rewritten as follows:

$$\frac{\mathcal{D}}{M} = \eta - 1/\rho_d - \frac{(1/\rho_d - 1/\rho_\ell)(1-\pi)\Psi}{\varepsilon_d} - \mathcal{B}(1-\rho\eta). \tag{33}$$

Finally, under the optimal policy $\rho_d = \rho$ we have $\rho = \beta/\eta$. Replacing ρ_d and ρ by β/η and noting that $\Psi = 0$ under the optimal policy yields

$$\frac{\mathcal{D}}{M} = (\eta/\beta + \mathcal{B})(\beta - 1) \leq 0.$$

Thus, the optimal policy requires that the government generates a primary surplus.

■

Proof of Proposition 3.

Let $\rho_d^* = \beta/\eta$ denote the deposit factor that implements the first-best allocation. The proof involves showing that if $\mathcal{S} > \mathcal{S}^o$, it is not optimal to operate a floor system with $\rho_d = \rho_\ell > \rho_d^*$, where ρ_d^* is the deposit rate under the optimal policy. The

proof involves two steps. First, we show that in a floor system, where $\rho_d = \rho_\ell$, the constraint $\mathcal{S} > \mathcal{S}^o$ implies $\rho_d = \rho_\ell > \rho_d^*$. Second, we show that if $\rho_d = \rho_\ell > \rho_d^*$, it is optimal to choose a non-zero corridor by increasing the loan rate marginally, i.e., by decreasing ρ_ℓ marginally.

First step. From (1), in any equilibrium the surplus satisfies

$$\mathcal{S} = M^+ - M + (1/\rho_\ell - 1)\, L - (1/\rho_d - 1)\, D.$$

In the proof of Proposition 1, we show that in any equilibrium, $D = M + L$, which allows us to write the previous equation as follows:

$$\mathcal{S} = [\eta - 1/\rho_d + (1/\rho_\ell - 1/\rho_d)\, L/M]\, M,$$

where $M^+/M = \eta$.

Under the optimal policy, we have shown that $\rho_d = \rho_d^*$ and $L = 0$, hence, we get

$$\mathcal{S}^o = (\eta - 1/\rho_d^*)\, M.$$

In any other floor system, we have $\rho_d = \rho_\ell > \rho_d^*$ and, therefore,

$$\mathcal{S} = (\eta - 1/\rho_d)M.$$

This immediately implies that when $\mathcal{S} > \mathcal{S}^o$, then $\rho_d = \rho_\ell > \rho_d^*$.

Second step. Here, we show that for any floor system with $\rho_d = \rho_\ell > \rho_d^*$ it is optimal to deviate and increase the loan rate marginally (i.e., decrease ρ_ℓ marginally). To do so, we calculate $d\mathcal{W}/d\rho_\ell$ and then evaluate it at $\rho_d = \rho_\ell$.

For convenience, let us rewrite the welfare function as follows:

$$\begin{aligned}\mathcal{W} &= \pi\left[\int_0^{\varepsilon_{\bar{z}}}\left[\varepsilon u\left(q_\varepsilon\right)-q_\varepsilon\right]dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{z}}}^{\infty}\left[\varepsilon u\left(q_\varepsilon\right)-q_\varepsilon\right]dF\left(\varepsilon\right)\right] \\ &\quad +\left(1-\pi\right)\left[\int_0^{\varepsilon_d}\left[\varepsilon u\left(q_\varepsilon\right)-q_\varepsilon\right]dF\left(\varepsilon\right)+\int_{\varepsilon_d}^{\varepsilon_\ell}\left[\varepsilon u\left(q_\varepsilon\right)-q_\varepsilon\right]dF\left(\varepsilon\right)\right. \\ &\quad \left.+\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\left[\varepsilon u\left(q_\varepsilon\right)-q_\varepsilon\right]dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u\left(q_\varepsilon\right)-q_\varepsilon\right]dF\left(\varepsilon\right)\right]. \end{aligned} \tag{34}$$

A change of the borrowing rate ρ_ℓ affects $\mathcal{W}$ directly, and indirectly via ε_d and ρ_m. That is,

$$\frac{d\mathcal{W}}{d\rho_\ell}=\frac{\partial\mathcal{W}}{\partial\rho_\ell}+\frac{\partial\mathcal{W}}{\partial\varepsilon_d}\frac{d\varepsilon_d}{d\rho_\ell}+\frac{\partial\mathcal{W}}{\partial\rho_m}\frac{d\rho_m}{d\rho_\ell}. \tag{35}$$

The terms $\frac{\partial\mathcal{W}}{\partial\rho_\ell}$, $\frac{\partial\mathcal{W}}{\partial\varepsilon_d}$, and $\frac{\partial\mathcal{W}}{\partial\rho_m}$ satisfy

$$\frac{\partial\mathcal{W}}{\partial\rho_\ell} = \left(1-\pi\right)\left[\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\left(\frac{\rho_d}{\rho_\ell}-1\right)\frac{\varepsilon}{\rho_d}dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\varepsilon_d\mathcal{B}dF\left(\varepsilon\right)\right] \tag{36}$$

$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial\varepsilon_d} &= \pi\left[\int_{\varepsilon_{\bar{z}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\left(1+\rho_m\mathcal{B}\right)dF\left(\varepsilon\right)\right] \\ &\quad +\left(1-\pi\right)\left[\int_{\varepsilon_d}^{\varepsilon_\ell}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\left(1+\rho_\ell\mathcal{B}\right)dF\left(\varepsilon\right)\right]\end{aligned} \tag{37}$$

$$\frac{\partial\mathcal{W}}{\partial\rho_m} = \pi\left[\int_0^{\varepsilon_{\bar{z}}}\left(\frac{\rho_d}{\rho_m}-1\right)\frac{\varepsilon}{\rho_d}dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{z}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\varepsilon_d\mathcal{B}dF\left(\varepsilon\right)\right]. \tag{38}$$

We get the derivative $\frac{d\varepsilon_d}{d\rho_\ell}$ by taking the total derivative of (20). Let

$$\begin{aligned}\nabla \equiv\ & \pi\left[\int_{\varepsilon_{\bar{z}}}^{\infty}\varepsilon u''(q_\varepsilon)(1+\rho_m\mathcal{B})dF(\varepsilon)\right] \\ & +(1-\pi)\left[\int_{\varepsilon_d}^{\varepsilon_\ell}\varepsilon u''(q_\varepsilon)\,dF(\varepsilon)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon u''(q_\varepsilon)(1+\rho_\ell\mathcal{B})dF(\varepsilon)\right].\end{aligned}$$

From equation (20), we get

$$\begin{aligned}\frac{d\varepsilon_d}{d\rho_\ell} =\ & -\frac{\pi\left[\int_0^{\varepsilon_{\bar{z}}}\varepsilon u''(q_\varepsilon)\frac{\varepsilon}{\rho_d}dF(\varepsilon)+\int_{\varepsilon_{\bar{z}}}^{\infty}\varepsilon u''(q_\varepsilon)\varepsilon_d\mathcal{B}dF(\varepsilon)\right]\frac{d\rho_m}{d\rho_\ell}}{\nabla} \\ & -\frac{(1-\pi)\left[\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\varepsilon u''(q_\varepsilon)\frac{\varepsilon}{\rho_d}dF(\varepsilon)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon u''(q_\varepsilon)\varepsilon_d\mathcal{B}dF(\varepsilon)\right]}{\nabla}.\end{aligned} \tag{39}$$

We now evaluate (36), (37), (38), and (39) at $\rho_\ell = \rho_m = \rho_d$. Note that when $\rho_\ell = \rho_m = \rho_d$, $\varepsilon_\ell = \varepsilon_z = \varepsilon_d$, and $\varepsilon_{\bar{\ell}} = \varepsilon_{\bar{z}} = \varepsilon_d(1+\rho\mathcal{B})$. Moreover, for a given ε the

quantities consumed by an active and a passive buyer are equal. We get

$$\begin{aligned}
\left.\frac{\partial \mathcal{W}}{\partial \rho_\ell}\right|_{\rho_\ell=\rho_m=\rho_d} &= (1-\pi)\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\varepsilon_d \mathcal{B} dF\left(\varepsilon\right) \\
\left.\frac{\partial \mathcal{W}}{\partial \varepsilon_d}\right|_{\rho_\ell=\rho_m=\rho_d} &= \int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\left(1+\rho\mathcal{B}\right)dF\left(\varepsilon\right). \\
\left.\frac{\partial \mathcal{W}}{\partial \rho_m}\right|_{\rho_\ell=\rho_m=\rho_d} &= \pi\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]\varepsilon_d \mathcal{B} dF\left(\varepsilon\right) \\
\left.\frac{d\varepsilon_d}{d\rho_\ell}\right|_{\rho_\ell=\rho_m=\rho_d} &= -\frac{\pi\left[\int_0^{\varepsilon_{\bar{\ell}}}\varepsilon u''\left(q_\varepsilon\right)\frac{\varepsilon}{\rho_d}dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon u''\left(q_\varepsilon\right)\varepsilon_d\mathcal{B}dF\left(\varepsilon\right)\right]\frac{d\rho_m}{d\rho_\ell}}{\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon u''\left(q_\varepsilon\right)\left(1+\rho_d\mathcal{B}\right)dF\left(\varepsilon\right)} \\
&\quad -\frac{(1-\pi)\left[\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\varepsilon u''\left(q_\varepsilon\right)\frac{\varepsilon}{\rho_d}dF\left(\varepsilon\right)+\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon u''\left(q_\varepsilon\right)\varepsilon_d\mathcal{B}dF\left(\varepsilon\right)\right]}{\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon u''\left(q_\varepsilon\right)\left(1+\rho_d\mathcal{B}\right)dF\left(\varepsilon\right)}.
\end{aligned}$$

We can now use the above expressions to rewrite (35) as follows:

$$\left.\frac{d\mathcal{W}}{d\rho_\ell}\right|_{\rho_\ell=\rho_m=\rho_d} = -\frac{\pi\int_0^{\varepsilon_{\bar{z}}}\varepsilon u''\left(q_\varepsilon\right)\frac{\varepsilon}{\rho}dF\left(\varepsilon\right)\frac{d\rho_m}{d\rho_\ell}+(1-\pi)\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\varepsilon u''\left(q_\varepsilon\right)\frac{\varepsilon}{\rho}dF\left(\varepsilon\right)}{\left[\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'\left(q_\varepsilon\right)-1\right]dF\left(\varepsilon\right)\right]^{-1}\int_{\varepsilon_{\bar{z}}}^{\infty}\varepsilon u''\left(q_\varepsilon\right)dF\left(\varepsilon\right)}.$$

The numerator and denominator are negative. Since $\frac{d\rho_m}{d\rho_\ell} \geq 0$ (ρ_m cannot decrease since it is equal to the deposit rate), $\frac{d\mathcal{W}}{d\rho_\ell} < 0$, which implies that decreasing ρ_ℓ (increasing i_ℓ) marginally from $\rho_\ell = \rho_m = \rho_d$ is welfare improving. ■

Proof of Proposition 4. Divide both sides of (25) by M to get

$$\frac{\mathcal{D}}{M} = \eta - 1 + \frac{\left(1/\rho_\ell - 1\right)L - \left(1/\rho_d - 1\right)D}{M} - \mathcal{B}\left(1-\rho\eta\right) + r\mathcal{K}.$$

Eliminating D and L using (31) and (32), respectively, and noting that $M/p = \varepsilon_d$,

the last expression becomes

$$\frac{\mathcal{D}}{M} = \eta - 1/\rho_d - \frac{(1/\rho_d - 1/\rho_\ell)(1-\pi)\Psi}{\varepsilon_d} - \mathcal{B}(1-\rho\eta) + r\mathcal{K}. \tag{40}$$

Optimal policy requires $\rho_d = \rho = \beta/\eta$. Replacing ρ_d and ρ by β/η and noting that $\Psi = 0$ under the optimal policy yields

$$\frac{\mathcal{D}}{M} = (\eta/\beta + \mathcal{B})(\beta - 1) + r\mathcal{K}.$$

Thus, the optimal policy generates no losses for the central bank ($\mathcal{D} > 0$) if and only if $\mathcal{K} \geq \bar{\mathcal{K}}$, where $\bar{\mathcal{K}} \equiv (\eta/\beta + \mathcal{B})(1-\beta)/r$. ■

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