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# Indirect Taxation and the Welfare Effects of Altruism on the Optimal Fiscal Policy*

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**Abstract**

This paper analyzes the welfare effects of altruism on the optimal fiscal policy. The existence of positive bequests links present and future generations in the economy. We show that these altruistic links provide a new role for indirect taxation (consumption and estate taxes) with important welfare implications. We use three different altruistic approaches (warm-glow, dynastic, and family) to illustrate how the presence of bequests in the budget constraint of the donee gives the government the ability to use indirect taxation to mimic lump-sum taxation and to implement the first-best outcome in the long-run. This channel is not present in economies without altruism, such as the infinite-lived consumer economy or the overlapping generations economy, where long-run welfare is suboptimal and indirect taxation is irrelevant.

**Keywords**: optimal taxation, altruism, dynamic general equilibrium.
**J.E.L. classification codes**: H21, H30, E62.

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# 1 Introduction

A very important question in macroeconomics is how the fiscal policy should be determined in the long-run. The integration of public finance into macroeconomics suggests that taxation should be optimally set to maximize society's welfare. Using a traditional macro model with infinite-lived consumers, Judd (1985) and Chamley (1986) show that the optimal capital income tax should be zero in the long-run. However, in an overlapping generations economy, Escolano (1992), Garriga (1999), and Erosa and Gervais (2002) conclude that once we consider intergenerational redistribution, a non-zero capital income tax is not suboptimal. The main differences between both canonical macro models are the assumptions about parental links. The infinite-lived consumer model assumes a perfect link using a homogeneity assumption between present and future generations. The standard overlapping generations model assumes no intergenerational links between cohorts. Therefore, the existence of altruism between generations seems to play an important role in the determination of the optimal policy.

At the aggregate level the importance of altruism is evident, as it is shown in the empirical studies of Gale and Scholz (1994) or Davies and Shorrocks (2000). Nonetheless, one of the most serious difficulties in studying this problem is that the empirical evidence is not conclusive on why individuals leave bequests, and the design of the optimal fiscal policy should depend on it. There exists an extensive literature that proposes and tests several different motives: intended, accidental or unintended, and a mixture of both; see Laitner (1997) for a detailed survey. Unfortunately, none of the theories seem to be conclusive since the results from the empirical tests are ambiguous.

In this paper we explore the connection between altruism and the optimal fiscal policy and its implications for welfare. From a normative perspective, unintended bequests should be taxed at a confiscatory rate since this strategy would allow a Ramsey government to minimize other distortions and to enhance welfare.[1] In the presence of intended bequests a confiscatory estate tax would have severe consequences on the willingness to bequeath and the welfare of the donors. Thus, with altruism a Ramsey government can use estate taxation as an additional instrument to spread the tax burden across different margins.[2] To analyze the aforementioned connection, we consider three formulations of intended bequests. First, we focus the attention to joy-of-giving or warm-glow altruism.[3] In this framework, parents derive utility directly from giving bequests to their offsprings, as in Yaari (1965).[4] Second, we analyze a model with dynastic altruism, where parents derive utility directly from the utility

[1] Kopczuk (2003) shows that a 100 percent estate tax can have an indirect effect that might reduce the labor supply. As a result, a 100 percent tax might not be desirable.

[2] We assume that estate taxes are paid by the donor and not by the donee who receives the bequest. Otherwise the optimal fiscal policy can be trivial in some cases because estate taxation can become an effective lump-sum tax.

[3] In the analysis we use a two period economy for two reasons. The first one is to have comparable results with previous work in the literature. Second, models with more than two periods impose some constraints in the set of fiscal instruments if age-specific taxes are not allowed; see Escolano (1992), Garriga (1999), and Erosa and Gervais (2002). These restrictions usually imply capital income taxes different from zero. Therefore, given that we want to study the pure effects of altruism, the driving forces of the main results should not depend on exogenous restrictions on the set of instruments that the government can use.

[4] A large scale version of this model, where generations live more than two periods, is consistent with the observed wealth distribution (see de Nardi, 2004).

of their offsprings, as in Barro (1974). And, third, we follow Becker (1991) and consider the family altruism, where parents derive utility from the future disposable income of their offsprings. This framework can be interpreted as an intermediate case between warm-glow and dynastic altruism and it is helpful to analyze the consistency of the results.

The existence of positive bequests links present and future generations in the economy. We show that altruistic links give rise to a new role for indirect taxation (consumption and estate taxes) with important welfare implications. The presence of a bequest in the budget constraint of the donee gives the government the ability to use indirect taxation to control the intratemporal allocation of resources and mimic lump-sum taxation. The government can then set the remaining tax rates to eliminate the distortions in the intra and intertemporal decisions and implement a first-best outcome in the long-run. We show that this mechanism, via indirect taxation, operates in all three altruistic approaches (warm-glow, dynastic, and family) in contrast to the infinite-lived consumer economy or the overlapping generations economy, where long-run welfare is suboptimal and indirect taxation is irrelevant.[5] The results for the transition path in the presence of altruism depend on specific assumptions of date 0 taxation.[6] In the economy with warm-glow altruism the choice of the initial consumption and capital income tax allow to reach the first-best path. In the dynastic altruism, the economy is in the first-best path regardless of the government availability to choose initial taxes, whereas in the family altruism the government only needs to be able to choose the indirect tax to be in the first-best path.

Perhaps a striking result is the different outcome that one obtains from solving the dynastic economy and the infinitely-lived consumer economy. We argue that in the later economy there is an implicit homogeneity assumption that prevents the government to use indirect taxation to control the intratemporal allocation of resources. To be more specific, in the dynastic economy there is a distinction between bequests and financial assets. Bequests are not transacted in the market whereas the financial assets are acquired at each period by the current young cohort. Consequently, the government can differentiate the taxation of transfers from the taxation of financial assets. In contrast, in the infinite-lived economy the size of bequests has to be consistent with the size of financial assets, and there is no room to differentiate the tax treatment. This implicit assumption is captured by the fact that the government only faces one implementability constraint (the link is implicit), while in the dynastic economy we have an infinite sequence of implementability constraints (the link is explicit).

It is important to make a few remarks about the paper findings. In general, defining efficiency or first-best allocations in an economy with warm-glow or family altruism is a complicated matter. There are some papers in the literature that provide efficiency concepts in this type of economies. The implicit concept that we have in mind is one where the first-best equilibrium has no unnecessary distortions and where external effects are internal-

[5] Formally, there is one tax that has the same role as debt and, therefore, the optimal fiscal policy is not altered if either the consumption tax or the debt is removed from the set of fiscal instruments available to the government. That is, there is one more tax than number of individual decisions to control.

[6] In infinite horizon economies, the role of initial taxation is critical since it can be used to mimic lump-sum taxation. However, in overlapping generations economies this assumption is not very important since initial taxes can only be used as a lump-sum tax for the initial cohort. In the presence of altruism the choice of initial taxes has a different role, since individuals live a finite number of periods but different cohorts are linked through bequests.

ized.[7] Second, our findings require that bequests are positive; otherwise the economy would behave as a standard overlapping generations economy. That requires making specific assumptions in preferences towards bequests (i.e., Inada conditions) or discount rates. Third, the economies analyzed in the paper ignore uncertainty or the presence of intra-cohort heterogeneity. We think it would be straightforward to generalize the results to include state contingent taxes and government debt. The presence of uncertainty would certainly not eliminate the operating mechanism of altruism, although it could be important for quantitative results. Modeling intragenerational heterogeneity in the context of Ramsey taxation sometimes requires making additional assumptions about the set of tax instruments. These details have been extensively addressed in papers such as Chari and Kehoe (1999), Garriga (1999), or more recently Shin (2005). As usual in the Ramsey taxation literature, time-consistency issues are often ignored (see Martin, 2009, for a recent treatment). We think this dimension is not a severe restriction in the generality of the results, since our focus is the role of indirect taxation to attaining first-best allocations, and we know since Fisher (1980) that the optimal plans of a government that has access to effective lump-sum taxes are time-consistent.

The paper is organized as follows. Section 2 summarizes some known results about the role of indirect taxation in economies without altruism. Section 3 presents the basic results for the warm-glow model. The dynastic altruism and the family altruism optimal fiscal policies are discussed in section 4 and 5, respectively. Finally, section 6 concludes. The proofs of the propositions are in the Appendix.

# 2 Indirect Taxation in Economies without Altruism

In this section we present some standard results that show that in both the standard infinite-lived consumer economy and the overlapping generations economy the presence of indirect taxation is irrelevant for the determination of the optimal fiscal policy when the government has access to a full set of distortionary taxes. These two formulations are used as simple benchmarks to clarify the exposition, and they are not meant to be representative of models as they exist in the literature today. Our contribution is to show that in economies where intended altruism is modelled, indirect taxation is relevant since it has important implications for the path of optimal taxes and welfare.

## 2.1 Infinite-Lived Consumer Economy

We consider a neoclassical production economy with population growing at the rate $n$. Output is produced according to a constant returns to scale technology $f(k_t, l_t)$, where $k_t$ and $l_t$ denote the capital stock and labor, respectively. Any variable $m_t$ is expressed in per capita terms of born at period $t$. The production function $f$ is strictly concave, $C^2$, and satisfies the Inada conditions. At each period capital depreciates at a constant positive rate $\delta$. With competitive markets each input receives its marginal product, i.e., $r_t = f_{k_t} - \delta$ and $w_t = f_{l_t}$, where $r_t$ is the return on capital, $w_t$ is the wage rate, and $f_{m_t}$ is the derivative of $f$ with respect to $m_t$.

[7] See Garriga and Sánchez-Losada (2009) for the analysis of the first-best in a warm-glow economy.

Households are infinite-lived and identical. In each period individuals choose consumption $c_t$, asset holdings $a_{t+1}$, and the allocation of their one unit of time endowment between work $l_t$ and leisure $(1-l_t)$. Formally, each individual solves

$$\max_{\{c_t, a_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \tag{1}$$

$$s.t. \qquad (1+\tau_t^c)c_t + (1+n)a_{t+1} = (1-\tau_t^l)w_t l_t + \left[1 + r_t(1-\tau_t^k)\right] a_t, \quad \forall t, \tag{2}$$

where $\beta \in (0,1)$ is the individual discount factor, and $\tau_t^c$, $\tau_t^k$ and $\tau_t^l$ denote consumption, capital and labor income proportional taxes, respectively. The utility function $U$ is strictly concave, $C^2$, and satisfies the usual Inada conditions. In order to prevent Ponzi schemes, the optimization problem is also subject to the non-binding borrowing constraint $a_{t+1} \geq -A$, where $A$ is a large positive constant. The solution to the household problem yields the standard first-order conditions,

$$\frac{U_{c_t}}{\beta U_{c_{t+1}}} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \frac{\left[1 + r_{t+1}(1-\tau_{t+1}^k)\right]}{(1+n)}, \quad \forall t, \tag{3}$$

$$-\frac{U_{c_t}}{U_{l_t}} = \frac{(1+\tau_t^c)}{(1-\tau_t^l)w_t}, \quad \forall t, \tag{4}$$

and the corresponding transversality condition for asset holdings, where $U_{m_t}$ is the derivative of $U$ with respect to $m_t$.

Let $\pi = \{\tau_{t+1}^k, \tau_{t+1}^c, \tau_t^l, d_{t+1}\}_{t=0}^{\infty}$ be a fiscal policy[8] and the period government budget be defined by

$$g_t + R_t d_t - (1+n)\, d_{t+1} = \tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^l w_t l_t, \tag{5}$$

where $d_t$ and $g_t$ denote government debt and a non-productive government expenditure, respectively, and $R_t$ is the return on government bonds. There is a non-arbitrage condition between the return on government bonds and capital, $R_{t+1} = 1 + r_{t+1}(1-\tau_{t+1}^k)$. The amount of government debt is bounded by a large positive constant to ensure that the government budget constraint is satisfied in present value. Financial assets are allocated either in form of capital or government bonds, so that $a_{t+1} = (k_{t+1} + d_{t+1})$ is satisfied in equilibrium. The economy resource constraint or feasibility constraint is

$$c_t + (1+n)\, k_{t+1} - (1-\delta)k_t + g_t = f(k_t, l_t), \quad \forall t. \tag{6}$$

We consider a government (Ramsey problem) that chooses and commits to a tax policy plan that maximizes society's welfare. In order to solve the government problem, we use the primal approach of optimal taxation proposed by Atkinson and Stiglitz (1980). This approach is based on characterizing the set of allocations that the government can implement for a given fiscal policy $\pi$ given the sequence of government expenditure $\{g_t\}_{t=0}^{\infty}$, the initial taxes $\{\tau_0^k, \tau_0^c\}$, and the initial conditions $a_0 = k_0 + d_0$.[9] We follow Chari and Kehoe (1999)

[8] We will discuss in each case the importance that the government can or not choose some taxes at $t = 0$.

[9] The set of implementable allocations is described by the period resource constraints and the so-called implementability constraints. These constraints capture the effect that changes in the tax policy have on agents decisions and market prices. Thus, the government problem amounts to maximize its objective function over the set of implementable allocations. The implementability constraint (7) directly follows Chari and Kehoe (1999).

and use the primal approach to write the government optimization problem as

$$\max_{\{c_t,k_{t+1},l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

$$s.t. \qquad \sum_{t=0}^{\infty} \beta^t \left(c_t U_{c_t} + l_t U_{l_t}\right) = \frac{U_{c_0}}{1+\tau_0^c}\left[1 + (f_{k_0} - \delta)(1-\tau_0^k)\right](k_0 + d_0), \tag{7}$$

and the resource constraint (6). Writing the Lagrange function and redefining the government period objective function as $W(c_t, l_t) = U + \varphi\left(c_t U_{c_t} + l_t U_{l_t}\right)$, where $\varphi$ is the Lagrange multiplier associated to the implementability constraint (7), the Ramsey optimality conditions are

$$\frac{W_{c_t}}{\beta W_{c_{t+1}}} = \frac{(1-\delta+f_{k_{t+1}})}{(1+n)}, \quad \forall t > 0, \tag{8}$$

$$-\frac{W_{c_t}}{W_{l_t}} = \frac{1}{f_{l_t}}, \quad \forall t > 0, \tag{9}$$

where $W_{m_t}$ is the derivative of $W$ with respect to $m_t$. The optimality conditions at $t = 0$ include additional terms showing the fact that the initial level of capital is given, as the right hand side of (7) shows. The optimal fiscal policy can be implemented by substituting the optimal allocation in the market equilibrium conditions,

$$\tau_{t+1}^k = \frac{(1+n)}{\beta r_{t+1}}\left[\frac{W_{c_t}}{W_{c_{t+1}}} - \frac{(1+\tau_{t+1}^c)}{(1+\tau_t^c)}\frac{U_{c_t}}{U_{c_{t+1}}}\right], \quad \forall t > 0, \tag{10}$$

$$\tau_t^l = 1 - \left[(1+\tau_t^c)\frac{U_{l_t}}{U_{c_t}}\frac{W_{c_t}}{W_{l_t}}\right], \quad \forall t > 0. \tag{11}$$

It is direct to show that given the optimal allocation $\{c_t^*, l_t^*, k_{t+1}^*\}_{t=0}^{\infty}$, there exists an infinite number of fiscal policies $\pi = \{\tau_{t+1}^k, \tau_{t+1}^c, \tau_t^l, d_{t+1}\}_{t=0}^{\infty}$ that satisfy these two equations, i.e., one of the tax instruments is redundant. For example, the choice of a particular consumption tax path $\{\tau_{t+1}^c\}_{t=0}^{\infty}$ only alter the levels of the capital and labor income tax paths $\{\tau_{t+1}^k, \tau_t^l\}_{t=0}^{\infty}$ given by (10) and (11) and the government debt level $\{d_{t+1}\}_{t=0}^{\infty}$ given by (5), but not the allocation implied by the optimal policy and its associated welfare. A non-zero capital income tax $\tau_{t+1}^k \neq 0$ for all $t$ implies a non-constant sequence of consumption taxes $\tau_t^c \neq \tau_{t+1}^c$ for all $t$. Since time varying taxes are usually not observed, it is common to normalize the path of indirect taxes to zero, $\{\tau_{t+1}^c\}_{t=0}^{\infty} = 0$.

It is important to remark that if the government can choose the initial tax on either consumption $\tau_0^c$ or capital $\tau_0^k$, then the first-best path allocation can be implemented, since any initial tax would be chosen such that the implementability constraint (7) is satisfied.[10]

[10] From the sequential budget constraint at date 0 we have

$$c_0 = \frac{(1-\tau_0^l)w_0 l_0}{(1+\tau_0^c)} + \frac{\left[1 + r_0(1-\tau_0^k)\right]a_0 - a_1(1+n)}{(1+\tau_0^c)}.$$

Note that, for example, consumption taxes can be used as a wealth tax on the initial asset endowment. The tax rate arising from the implementability constraint satisfies

$$\tau_0^c = \frac{U_{c_0}\left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)}{\sum_{t=0}^{\infty}\beta^t(c_t U_{c_t} + l_t U_{l_t})} - 1.$$

## 2.2 Overlapping Generations Economy

As a second baseline, we construct an overlapping generations economy where individuals live for two periods.[11] Young generations are endowed with one unit of time which they allocate between work and leisure. Then, they choose consumption $c_{1t}$ and asset holdings $a_t$. Old individuals do not work and consume $c_{2t+1}$. The production structure remains unchanged, and both the goverment budget constraint (5) and the resource constraint (6) are modified such that $c_t = c_{1t} + c_{2t}/(1+n)$. The aggregate level of asset holdings equals the stock of physical capital and government debt at $t+1$, so that $a_t = (k_{t+1} + d_{t+1})(1+n)$.

In this environment, the representative newborn generation in period $t$ solves

$$\max_{\{c_{1t}, c_{2t+1}, l_t\}} U(c_{1t}, l_t) + \beta Z(c_{2t+1}), \tag{12}$$

$$s.t. \qquad (1+\tau_t^c)c_{1t} + a_t = (1-\tau_t^l)w_t l_t, \tag{13}$$

$$(1+\tau_{t+1}^c)c_{2t+1} = a_t\left[1 + r_{t+1}(1-\tau_{t+1}^k)\right], \tag{14}$$

where the utility function $Z$ is strictly concave, $C^2$ and satisfies the usual Inada conditions. We abuse the notation and use the same representation $U$ for the utility of young individuals than in the infinite-lived consumer economy. The optimality conditions are given by

$$\frac{U_{c_{1t}}}{\beta Z_{c_{2t+1}}} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\left[1 + r_{t+1}(1-\tau_{t+1}^k)\right], \quad \forall t, \tag{15}$$

$$-\frac{U_{c_{1t}}}{U_{l_t}} = \frac{(1+\tau_t^c)}{(1-\tau_t^l)w_t}, \quad \forall t, \tag{16}$$

where $Z_{m_t}$ is the derivative of $Z$ with respect to $m_t$. At $t=0$, there exists an initial generation who owns all the assets in the economy and consumes $c_{20} = \left[1 + r_0(1-\tau_0^k)\right]a_{-1}/(1+\tau_0^c)$.

The government problem is a bit more cumbersome since we have an infinite number of generations. A standard way to deal with it is to assign weights to each cohort. Let $\lambda \in (0,1)$ represents the relative weight that the government places between current and future generations. In this case, the Ramsey taxation problem becomes

$$\max_{\{c_{1t}, c_{2t}, k_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \lambda^t [U(c_{1t}, l_t) + \lambda^{-1}\beta Z(c_{2t})], \tag{17}$$

$$s.t. \qquad c_{1t}U_{c_{1t}} + l_t U_{l_t} + \beta c_{2t+1} Z_{c_{2t+1}} = 0, \quad \forall t, \tag{18}$$

the feasibility constraint (6), and the consumption decision of the initial generation (14). The first constraint is the implementability condition of newborn generations, and it is constructed by replacing the first-order conditions of the consumer problem in the budget constraint. The third constraint is the implementability condition of the initial old individual.

Writing the Lagrange function and abusing the notation by redefining the functions as $W(c_{1t}, l_t) = U + \varphi_t(c_{1t}U_{c_{1t}} + l_t U_{l_t})$ and $W(c_{2t+1}) = Z + \varphi_t c_{2t+1} Z_{c_{2t+1}}$, where $\varphi_t$ is the Lagrange multiplier associated to the implementability constraint (18), the Ramsey optimality

[11] A more detailed treatment can be found in Garriga (1999).

conditions are

$$\frac{W_{c_{1t}}}{\lambda W_{c_{1t+1}}} = \frac{(1-\delta+f_{k_{t+1}})}{(1+n)}, \quad \forall t>0, \tag{19}$$

$$\frac{W_{c_{1t}}}{W_{c_{2t}}} = \frac{\beta(1+n)}{\lambda}, \quad \forall t>0, \tag{20}$$

$$-\frac{W_{c_{1t}}}{W_{l_t}} = \frac{1}{f_{l_t}}, \quad \forall t>0. \tag{21}$$

Combining the optimality conditions of the government problem with the first-order conditions of the market equilibrium yields the optimal tax policy,

$$\tau^k_{t+1} = \frac{1}{\beta r_{t+1}}\left[\frac{W_{c_{1t}}}{W_{c_{2t+1}}} - \frac{(1+\tau^c_{t+1})}{(1+\tau^c_t)}\frac{U_{c_{1t}}}{Z_{c_{2t+1}}}\right], \quad \forall t>0, \tag{22}$$

$$\tau^l_t = 1-\left[(1+\tau^c_t)\frac{U_{l_t}}{U_{c_{1t}}}\frac{W_{c_{1t}}}{W_{l_t}}\right], \quad \forall t>0. \tag{23}$$

An inspection of these two equations suggests that this overlapping generations economy also leads to the redundancy of indirect taxation. This is the case even when the optimal capital income tax is not zero in steady state. Moreover, whereas in the infinite-lived consumer economy the choice of initial consumption and capital income taxes $\{\tau^c_0, \tau^k_0\}$ could be used to mimic lump-sum taxation and eliminate all future distortions, in the overlapping generations economy the choice of initial instruments has no role since they cannot be used to mimic lump-sum taxation beyond the initial period.

Next, we argue that in economies with intended altruism the redundancy results do not longer hold. Since different bequest motives imply different formalizations of the government problem (i.e., different sets of implementable allocations), we can only proceed by showing the role of indirect taxation in the three most popular formulations of intended altruism.

# 3 Warm-Glow Altruism Economy

In this section we extend the overlapping generations economy to include warm-glow altruism. In this economy young generations receive a physical bequest $b_t$ from their parents and when they become old, they leave a bequest to their offsprings $(1+n)\,b_{t+1}$, so that each child receives $b_{t+1}$. The warm-glow altruism implies that individuals derive utility from giving bequests to their children, but they do not derive it directly from their children happiness. In this environment, the representative generation in period $t$ solves

$$\max_{\{c_{1t},c_{2t+1},a_{t+1},l_t,b_{t+1}\}} U(c_{1t},l_t)+\beta Q(c_{2t+1},b_{t+1}), \tag{24}$$

$$s.t. \qquad (1+\tau^c_t)c_{1t}+a_{t+1} = (1-\tau^l_t)w_t l_t + b_t, \tag{25}$$

$$(1+\tau^c_{t+1})c_{2t+1}+(1+\tau^b_{t+1})\,(1+n)\,b_{t+1} = a_{t+1}\left[1+r_{t+1}(1-\tau^k_{t+1})\right], \tag{26}$$

where $\tau^b_{t+1}$ is a distortionary estate tax paid by the donor. This particular formalization where the donee receives an after tax transfer prevents estate taxation to become an effective lump-sum tax.[12] Also note that the estate tax is an indirect tax because the bequest

[12] If the donee is taxed, then we should assume that the donor is interested on the net bequest received by the donee.

represents a pure consumption good for the donors. The utility function $Q$ is strictly concave, $C^2$ and satisfies the usual Inada conditions. The first-order conditions for a newborn generation at date $t$ are[13]

$$\frac{U_{c_{1t}}}{\beta Q_{c_{2t+1}}} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\left[1+r_{t+1}(1-\tau_{t+1}^k)\right], \tag{27}$$

$$-\frac{U_{c_{1t}}}{U_{l_t}} = \frac{(1+\tau_t^c)}{(1-\tau_t^l)w_t}, \tag{28}$$

$$\frac{Q_{c_{2t+1}}}{Q_{b_{t+1}}} = \frac{(1+\tau_{t+1}^c)}{(1+n)\left(1+\tau_{t+1}^b\right)}, \tag{29}$$

where $Q_{m_t}$ is the derivative of $Q$ with respect to $m_t$. Note that the ratio of consumption to bequest taxes affects the benefits (marginal utility obtained by the donor of the given bequest) and the costs (marginal utility of the foregone consumption of the donor) from leaving bequests. The government can use these instruments to alter the intergenerational transfers.

At $t=0$ there exists an initial generation who owns all the assets in the economy and solves

$$\max_{\{c_{20},b_0\}} \beta Q(c_{20},b_0), \tag{30}$$

$$s.t. \qquad (1+\tau_0^c)c_{20}+(1+\tau_0^b)(1+n)\,b_0 = \left[1+r_0(1-\tau_0^k)\right](k_0+d_0)(1+n). \tag{31}$$

The government budget constraint needs to be modified to incorporate estate taxation. Let redefine the fiscal policy as $\pi = \{\tau_{t+1}^k, \tau_{t+1}^c, \tau_t^l, \tau_{t+1}^b, d_{t+1}\}_{t=0}^{\infty}$. Then, the government budget constraint is

$$g_t + R_t d_t - (1+n)\,d_{t+1} = \tau_t^c\left(c_{1t}+\frac{c_{2t}}{1+n}\right) + \tau_t^b b_t + \tau_t^k r_t k_t + \tau_t^l w_t l_t. \tag{32}$$

The resource constraint is equation (6) but modified with $c_t = c_{1t} + c_{2t}/(1+n)$. The market clearing condition in the capital market is $a_{t+1} = (k_{t+1}+d_{t+1})(1+n)$.

In an economy with intended bequests, the Ramsey taxation problem has to determine the optimal path for estate taxation. The presence of altruism provides a new role for indirect taxation since each cohort is connected to past and future individuals through altruism. We argue that the government can use indirect taxation to change the intertemporal allocation of resources and attain a fully efficient solution. The Ramsey problem with altruism solves

$$\max_{\{c_{1t},c_{2t},l_t,b_t,k_{t+1},\tau_{t+1}^c\}_{t=0}^{\infty}} \qquad \sum_{t=0}^{\infty}\lambda^t\left[U(c_{1t},l_t)+\lambda^{-1}\beta Q(c_{2t},b_t)\right], \tag{33}$$

$$s.t. \qquad c_{1t}U_{c_{1t}} + l_t U_{l_t} + \beta\left(c_{2t+1}Q_{c_{2t+1}} + b_{t+1}Q_{b_{t+1}}\right) = \frac{b_t U_{c_{1t}}}{(1+\tau_t^c)}, \quad \forall t \geq 0, \tag{34}$$

$$c_{20}Q_{c_{20}} + b_0 Q_{b_0} = \frac{Q_{c_{20}}\left[1+(1-\tau_0^k)(f_{k_0}-\delta)\right](k_0+d_0)(1+n)}{(1+\tau_0^c)}, \tag{35}$$

[13] As in Michel and Pestieau (2004), we exclude non-interior solutions for the leisure decision.

and the resource constraint (6). The construction of the set of implementable allocations can be found in the Appendix.

The implementability constraint for each newborn generation (34) incorporates the bequest received from the older generation. Due to the existence of positive bequests, $b_t > 0$, the government can choose a path of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$ directly from the set of implementable allocations and use indirect taxation as an effective lump-sum tax for each period.

With positive bequests, $b_t > 0$, the implementability constraint can always be satisfied for any feasible allocation making the Lagrange multiplier of the constraint of each newborn generation equal to zero $\forall t \geq 1$. The path of consumption taxes is used to redistribute resources across cohorts over time in a non-distortionary way. This mechanism provides a different role for indirect taxation that is not present in economies without altruism.[14] In the absence of bequests, $b_t = 0$, the model would behave as the overlapping generations economy discussed in Section 2 and indirect taxation would be irrelevant. In our formulation, the positiveness of bequests is guaranteed by the Inada condition $\lim_{b\to 0} Q_b = \infty$. This can be summarized in the next proposition.

**Proposition 1:** *In an economy with warm-glow altruism, when the government can choose the path of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$, the steady state allocation associated to the optimal (second-best) policy coincides with the steady state allocation of the first-best.*

The proposition suggests that the welfare implied from the optimal tax policy converges to the first-best level in the long-run. The welfare level associated to the first-best allocation depends on the particulars of its definition. A key issue in the presence of warm-glow altruism is that the donor neglects the indirect impact of altruism on the donee. However, the treatment of this external effect has important implications in the first-best policy since distortions could be necessary at the margin in light of the trade-off involved between efficiency and the optimal policy induced by handling the external effect. Garriga and Sánchez-Losada (2009) explore these implications and provide a first-best definition where this externalities are corrected.[15] The result from Proposition 1 suggests that the presence of indirect taxation

[14] Note that in this economy the estate tax is also an indirect tax. Using (29) and (34), we we can rewrite the implementability constraint as

$$c_{1t}U_{c_{1t}} + l_t U_{l_t} + \rho\left(c_{2t+1}Q_{c_{2t+1}} + b_{t+1}Q_{b_{t+1}}\right) = \frac{Q_{b_t} b_t U_{c_{1t}}}{Q_{c_{2t}}\left(1+n\right)\left(1+\tau_t^b\right)}, \quad \forall t \geq 0.$$

Therefore, we could use the estate tax instead of the consumption tax.

[15] Their definition of efficiency in an economy with warm-glow altruism raises a number of issues. Since bequests are not part of the resource constraint of the economy, some authors have claimed that they should be infinite and, then, individuals derive a non-bounded utility from giving at no real resource cost. To avoid this problem, a common strategy has been to eliminate any warm-glow (or utility interdependence) in the notion of social optimum. Even though efficient allocations are well-defined in this reduced context, the social planner ignores individual preferences and it does not consider the indirect effect that the donor transfer has on the donee. Garriga and Sánchez-Losada (2009) provide an alternative definition arguing that the previous definitions ignore an implicit constraint in the act of giving: the donor cannot bequeath to the donee more than her existing available resources. Their definition of first-best respects this constraint inherent to the act of giving, but it also allows to redistribute resources and to attain the socially efficient level of altruism.

allows to achieve a first-best steady state allocation where the presence of external effects are internalized. When the government can use some taxes at $t = 0$ in addition to the future path of indirect taxation, the findings from Proposition 1 can be extended to the transition.

**Corollary 1:** *In an economy with warm-glow altruism, if the government can choose $\tau_0^c$ and $\tau_0^k$ in addition to the sequence of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$, then the economy follows the first-best path.*

Inspection of the constraints of the set of implementable allocations suggest that the path of capital accumulation is affected by the initial taxation $\{\tau_0^k, \tau_0^c, \tau_0^b\}$. In contrast with the infinite-lived consumer economy, the choice of $\tau_0^c$ would not be sufficient to ensure a first-best path because this instrument cannot be used by the government to control the initial level of bequest $b_0$. The government needs to choose $\tau_0^k$ in order to control $b_0$. We need both instruments to ensure that (34) and (35) at date 0 are mutually satisfied. Only in this this case the value of $\tau_0^k$ becomes a lump-sum tax to the old generation and the value of $\tau_0^c$ becomes a lump-sum tax to the newborn generation born at $t = 0$, and the path of the economy becomes undistorted. After the choice of both the initial taxes $\{\tau_0^c, \tau_0^k\}$ and the path of optimal consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$, the remainder taxes are set to satisfy the individual first-order conditions evaluated at the first-best allocation, $\{\widehat{c}_{1t}, \widehat{c}_{2t}, \widehat{l}_t, \widehat{b}_t, \widehat{k}_{t+1}\}_{t=0}^{\infty}$. The intuition is very clear by inspecting the consolidated budget constraint. Combining (25) and (26), and dividing by the period consumption tax, we obtain

$$c_{1t} + \frac{(1+\tau_{t+1}^c)}{(1+\tau_t^c)R_{t+1}} c_{2t+1} + \frac{(1+\tau_{t+1}^b)(1+n)}{(1+\tau_t^c)R_{t+1}} b_{t+1} = \frac{(1-\tau_t^l)}{(1+\tau_t^c)} w_t l_t + \frac{b_t}{(1+\tau_t^c)}. \tag{36}$$

In this economy consumption taxes are not redundant due to the presence of bequests. This instrument can be used at every period to mimic lump-sum taxation and correct the tax burden in other margins based on the consumer first-order conditions.

In the initial specification we have assumed that the donor only considers the level of bequest $b_t$, but not the effective value or real bequests $b_t/(1+\tau_t^c)$. Using a warm-glow specification on real bequest implies a second period utility function characterized by $Q(c_{2t+1}, b_{t+1}/(1+\tau_{t+1}^c))$. In this case, the implementability constraint becomes

$$c_{1t} U_{c_{1t}} + l_t U_{l_t} + \beta \left[ c_{2t+1} Q_{c_{2t+1}} + \frac{b_{t+1} Q_{b_{t+1}}}{(1+\tau_{t+1}^c)} \right] = \frac{b_t U_{c_{1t}}}{(1+\tau_t^c)}, \quad \forall t \geq 0, \tag{37}$$

from where it is clear that the result stated in Proposition 1 does not change.

# 4 Dynastic Altruism Economy

In this section we analyze the dynastic altruism, where individuals derive utility from their children well-being, but they do not derive it from giving bequests directly to their children. This framework is useful to compare the results with the infinite-lived consumer economy. In this case, individual preferences are

$$V_t = U(c_{1t}, l_t) + \beta Z(c_{2t+1}) + \gamma V_{t+1}, \tag{38}$$

where $V_{t+1}$ is the utility of their offsprings and $\gamma$ is the altruism factor. In order to ensure that $V_t$ is bounded from above, we assume that $\gamma \in (0,1)$. A newborn generation maximizes (38) subject to (25) and (26). The initial old generation solves a similar problem. Using the envelope theorem, the solution of this optimization problem yields (28) and

$$\frac{U_{c_{1t}}}{\beta Z_{c_{2t+1}}} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\left[1 + r_{t+1}(1-\tau_{t+1}^k)\right], \tag{39}$$

$$\frac{\gamma U_{c_{1t+1}}}{(1+n)\,\beta Z_{c_{2t+1}}(1+\tau_{t+1}^b)} \leq 1, \qquad (= 1 \text{ if } b_{t+1} > 0). \tag{40}$$

In contrast with the warm-glow altruism, in the dynastic altruism the consumption tax equally affects the benefits (marginal utility obtained by the donor from the marginal increase in their children's welfare) and the costs (marginal utility of the foregone consumption of the donor) from leaving bequests and, hence, this tax does not alter the intergenerational transfers. Thus, the government can determine the after-tax bequests without changing the before-tax bequests. Also note that an estate tax paid by the donor has the same distortionary effects as a tax on the inheritance received by the donee. As has been pointed out by Caballé (1988), the threshold level of the altruism factor $\overline{\gamma}$ above (below) which the bequest motive is (is not) operative depends on the fiscal policy. In order to have comparable results with the previous section we assume that the altruism factor $\gamma$ is high enough so that bequests are always operative, $b_{t+1} > 0$.

In this model non-zero capital income taxes are optimal when the government and consumers discount future generations at different rates. Spataro and De Bonis (2005) argue that when the government is more (less) patient than consumers, it is optimal to subsidize (tax) capital even when lump-sum taxes are available. Therefore, a first-best allocation when $\lambda \neq \gamma$ could imply distortions in the savings rate. A simple alternative to avoid this problem is to assume that consumers and government discount future cohorts at the same rate, $\lambda = \gamma$, which we assume. The role of different discount rates has been studied by Farhi and Werning (2006) and Sleet and Yeltekin (2006). For the purpose of our discussion on indirect taxation, this is not a relevant issue since it only matters to the extend that the definition of first-best is affected.

In this economy the allocation associated to the optimal fiscal policy solves

$$\max_{\{c_{1t},c_{2t},l_t,b_t,k_{t+1},\tau_{t+1}^c\}_{t=0}^{\infty}} \qquad \sum_{t=0}^{\infty} \lambda^t \left[U(c_{1t},l_t) + \lambda^{-1}\beta Z(c_{2t})\right], \tag{41}$$

$$s.t. \qquad c_{1t}U_{c_{1t}} + l_t U_{l_t} + \beta c_{2t+1} Z_{c_{2t+1}} + \frac{\lambda b_{t+1} U_{c_{1t+1}}}{(1+\tau_{t+1}^c)} = \frac{b_t U_{c_{1t}}}{(1+\tau_t^c)}, \quad \forall t \geq 0, \tag{42}$$

$$\beta c_{20} Z_{c_{20}} + \frac{\lambda b_0 U_{c_{10}}}{(1+\tau_0^c)} = \frac{\beta Z_{c_{20}}\left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)(1+n)}{(1+\tau_0^c)}, \tag{43}$$

and the resource constraint (6). Note that the assumed government objective function coincides with the case of a government that maximizes the utility function of the initial old. This particular choice eliminates the well-known problem of double counting cohorts in social preferences, i.e., the younger cohort appears first as an independent argument in the social welfare function and, second, through the utility function of the altruistic individual.

In the dynastic economy bequests are exclusively used to transfer resources across generations and only have partial influence on the decision of future cohorts. The government can use indirect taxation to manipulate this intertemporal allocation of resources that links current and future cohorts and attain a first-best path.

**Proposition 2**: *In the dynastic economy, the choice of indirect taxation* $\{\tau_t^c\}_{t=1}^{\infty}$ *allows to attain the first-best path from* $t > 0$ *regardless of the initial tax rates.*

This result contrasts with the findings in the infinite-lived consumer economy where the first-best allocation can only be achieved if the government can choose taxes at $t = 0$, otherwise the first-best allocation can never be attained. The findings from Proposition 2 immediately suggest that the welfare associated to the optimal policies attained in the dynastic economy is higher than in the infinite-lived consumer economy. This is the case even when both economies prescribe zero long-run capital income taxes.

Why both economies can deliver different results? In contrast with the infinite-lived consumer economy, in the dynastic economy there is a distinction between bequests and financial assets. Bequests are not transacted in the market, whereas the financial assets are acquired at each period by the younger cohort. Consequently, the government can differentiate the taxation of transfers from the taxation of financial assets. This differentiation allows to manipulate the intertemporal allocation of resources using indirect taxation on the recipients of the donee. In the infinite-lived consumer economy the size of the bequest has to be consistent with the size of financial assets with no room for differentiation in the tax treatment. To illustrate the argument formally consider the individual budget constraint of a dynastic family where individuals live one period

$$c_{1t} + (1+n)\, b_{t+1} = w_t l_t + b_t(1+r_t), \tag{44}$$

and the budget constraint of an infinite-lived consumer economy,

$$c_{1t} + (1+n)\, a_{t+1} = w_t l_t + a_t(1+r_t), \tag{45}$$

where in order to simplify the argument we have eliminated taxes from the notation. In view of these constraints, it is clear that in the infinite-lived consumer economy we impose ad-hoc homogeneity of individuals by forcing the individuals to save $na_{t+1}$. There is an implicit tax $n$ on savings and the resulting collected quantity is given as a capitalized lump-sum subsidy to the new individuals of this economy. In the dynastic economy, it is the individual decision about bequests $nb_{t+1}$ what causes the homogeneity, but this homogeneity of individuals is only within the new cohort.[16]

Now, let's turn on the two-period dynastic economy. The budget constraint of the older cohort is given by

$$c_{2t+1} + (1+n)\, b_{t+1} = a_{t+1}(1+r_{t+1}). \tag{46}$$

In this economy, individuals give a bequest to their offsprings, but they sell their financial assets to the younger cohort, so that in general $a_{t+1} \neq b_{t+1}$.[17] Notice that this is true even when the older cohort supplies labor. By contrast, in the infinite-lived consumer economy

[16] We owe this intuition to Daniel Cardona.

[17] Note that this rationale does not depend on the magnitude of $n$.

we force parents to bequeath to their children their same financial wealth, $a_{t+1} = b_{t+1}$. It is this ad-hoc homogeneity assumption that eliminates the government ability to use bequests to control intratemporal decisions. This implicit assumption is captured by the fact that the government only faces one implementability constraint, while in the dynastic economy we have an infinite sequence of implementability constraints.[18] This distinction is the crucial element that allows the government to use indirect taxation to implement a first-best outcome in the presence of dynastic altruism and not in the infinite-lived consumer economy.

# 5 Family Altruism Economy

To complete the analysis, we analyze an economy with family altruism where individuals derive utility from their children's disposable income. In this case, individual preferences are

$$U(c_{1t}, l_t) + \beta X(c_{2t+1}, \omega_{t+1}), \tag{47}$$

where $\omega_{t+1} = (1 - \tau^l_{t+1}) w_{t+1} l_{t+1} + b_{t+1}$ is the income of each adult children. The utility function $X$ is strictly concave, $C^2$ and satisfy the usual Inada conditions. A newborn generation maximizes (47) subject to (25) and (26). The initial old generation solves a similar problem. The solution of this optimization problem yields (28) and

$$\frac{U_{c_{1t}}}{\beta X_{c_{2t+1}}} = \frac{(1+\tau^c_t)}{(1+\tau^c_{t+1})} \left[1 + r_{t+1}(1 - \tau^k_{t+1})\right], \tag{48}$$

$$\frac{X_{\omega_{t+1}}(1+\tau^c_{t+1})}{X_{c_{2t+1}}(1+n)(1+\tau^b_{t+1})} \leq 1, \qquad (= 1 \text{ if } b_{t+1} > 0), \tag{49}$$

where $X_{m_t}$ is the derivative of $X$ with respect to $m_t$. Note that consumption taxes affect the size of the bequest in the same way as in the warm-glow altruism. As has been pointed out by Lambrecht et al. (2005), the bequest motive can be non-operative. Since we are interested on the analysis of altruism, we assume that the altruism motive is high enough so that bequests are always operative, $b_{t+1} > 0$.

In this economy the allocation associated to the optimal fiscal policy solves

$$\max_{\{c_{1t}, c_{2t}, l_t, b_t, k_{t+1}, \tau^c_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \lambda^t \left[ U(c_{1t}, l_t) + \lambda^{-1} \beta X \left( c_{2t}, \frac{-l_t U_{l_t}(1+\tau^c_t)}{U_{c_{1t}}} + b_t \right) \right], \tag{50}$$

$$s.t. \qquad c_{1t} U_{c_{1t}} + l_t U_{l_t} + \beta \left( c_{2t+1} X_{c_{2t+1}} + b_{t+1} X_{\omega_{t+1}} \right) = \frac{b_t U_{c_{1t}}}{(1+\tau^c_t)}, \quad \forall t \geq 0, \tag{51}$$

[18] Following Atkinson and Stiglitz (1980), the implementability constraints are the households' present value budget constraint after substituting in the first-order conditions of the consumers' and the firms' problems. Thus, it is not true neither that in the infinite-lived consumer economy the following implementability constraint for every period,

$$c_{1t} U_{c_{1t}} + l_t U_{l_t} + \frac{(k_{t+1} + d_{t+1}) U_{c_{1t}}}{(1+\tau^c_t)} = \frac{(k_t + d_t) U_{c_{1t-1}}}{\beta(1+\tau^c_{t-1})},$$

can be written, since the households' present value budget constraint has not been used, nor that all the individual implementability constraints (42) can be added up, since each generation has her own intertemporal budget constraint that has to be satisfied.

$$c_{20}X_{c_{20}} + b_0 X_{\omega_0} = \frac{X_{c_{20}}\left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)(1+n)}{(1+\tau_0^c)}, \tag{52}$$

and the resource constraint (6). Note that we have substituted for $\tau_t^l$ from (28) into $\omega_t$.

Although the altruism motive is different than the previous examples, the operative mechanism of indirect taxation is the same. Note that the implementability constraint for each newborn generation (51) incorporates the bequest received from the older generation. Due to the existence of positive bequests, $b_t > 0$, the government can choose the path of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$ and bequests $\{b_t\}_{t=1}^{\infty}$ directly from the set of implementable allocations and use them as an effective lump-sum tax to implement an undistorted solution.

The income of each children $\{\omega_t\}_{t=1}^{\infty}$ can be computed by solving the resulting problem where the resource constraint is the only binding constraint.[19]

**Proposition 3:** *In an economy with family altruism, when the government can choose the path of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$, the steady state allocation associated to the optimal (second-best) policy coincides with the steady state allocation of the first-best.*

When consumption taxes at date 0 are a choice variable, the initial consumption tax becomes a lump-sum tax to the first newborn generation at the same time that $\omega_0$ takes the government required value, so that the path of the economy becomes undistorted.[20] This is in contrast with the warm-glow altruism, where two initial taxes are needed.

**Corollary 2:** *In an economy with family altruism, if the government can choose $\tau_0^c$ in addition to the sequence of consumption taxes $\{\tau_t^c\}_{t=1}^{\infty}$, then the economy follows the first-best path.*

# 6 Conclusions

We analyze the welfare effects of altruism on the optimal fiscal policy. The existence of positive bequests links present and future generations in the economy. We show that altruistic links give rise to a new role for indirect taxation (consumption and estate taxes) with important welfare implications since this instrument can be used to attain first-best allocations in the long-run. We argue that the mechanism through which indirect taxation operates is the same in all three altruistic approaches (warm-glow, dynastic, and family). This channel is not present in models without altruism, such as the infinite-lived consumer economy or the overlapping generations economy, where long-run welfare is suboptimal and indirect taxation is irrelevant.

A striking finding is the different outcomes that one obtains from solving the dynastic economy and the infinitely-lived consumer economy. We argue that in the later economy there is an implicit homogeneity assumption that prevents the government to use indirect taxation to control the intratemporal allocation of resources. This implicit assumption is

[19] Note that the external effect of bequests causes the government to introduce distortions in the first-best steady state policy in order to correct this effect.

[20] To be exact, the government needs to be able to choose only one of the initial taxes, $\tau_0^k$, $\tau_0^c$ or $\tau_0^b$.

captured by the fact that the government only faces one implementability constraint (the link is implicit), while in the dynastic economy we have an infinite sequence of implementability constraints (the link is explicit).

# Appendix

**Construction of the set of implementable allocations:** An allocation in the competitive equilibrium $\{c_{1t}, c_{2t}, k_{t+1}, b_t, l_t\}_{t=0}^{\infty}$ satisfies the set of implementable allocations. Moreover, if an allocation is implementable, then we can construct a fiscal policy $\pi$ and competitive prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, such that the allocation together with prices and the policy $\pi$ constitute a competitive equilibrium.

**Proof:** We start by proving the first part of the proposition. Any competitive equilibrium allocation has to satisfy the economy resource constraint. The implementability constraints for the newborn generations (34) can be derived by substituting $\tau_{t+1}^k$, $\tau_t^l$ and $\tau_{t+1}^b$ from (27)-(29) in the individual intertemporal budget constraint. The initial old agent at $t = 0$ has a different implementability constraint (35) because she is endowed with the initial stock of capital and debt. It can be derived using the same procedure.

Now we prove the second part of the proposition. Given an implementable allocation $\{c_{1t}, c_{2t}, k_{t+1}, b_t, l_t\}_{t=0}^{\infty}$, the competitive prices can be backed out using firms' first order conditions. The fiscal policy $\pi = \left\{\tau_{t+1}^k, \tau_{t+1}^c, \tau_t^l, \tau_{t+1}^b, d_{t+1}\right\}_{t=0}^{\infty}$ is recovered from the households' first-order conditions (27)-(29), the implementability constraints (34), and the debt level is found from the market clearing condition in the capital markets. Substituting $U_{c_{1t}}$, $Q_{c_{2t+1}}$, $U_{l_t}$ and $Q_{b_{t+1}}$ from the individual optimal conditions in the implementability constraint we obtain the intertemporal consumer budget constraint. Finally, given the tax on capital income $\tau_{t+1}^k$ and the net interest rate $r_{t+1}$, by arbitrage we find the return on government debt. If the resource constraint and the consumers' budget constraints are satisfied, Walras law ensures that the government budget constraint is also satisfied.

**Proof of Proposition 1:** In order to derive a solution to the Ramsey allocation problem, we redefine the government objective function by introducing the implementability constraint of each generation in. For a newborn generation the government period utility becomes

$$U(c_{1t}, l_t) + \beta Q(c_{2t+1}, b_{t+1})$$
$$+\mu_t \left[ c_{1t} U_{c_{1t}} + l_t U_{l_t} + \beta \left( c_{2t+1} Q_{c_{2t+1}} + b_{t+1} Q_{b_{t+1}} \right) - \frac{b_t U_{c_{1t}}}{(1+\tau_t^c)} \right], \tag{A.1}$$

where $\mu_t$ is the Lagrange multiplier associated to the implementability constraint of a generation born at period $t$. The additional term measures the effect of distortionary taxes on the utility function. The first-order condition of the government problem with respect to $\tau_{t+1}^c$ is

$$\frac{\lambda^{t+1} \mu_{t+1} b_{t+1} U_{c_{1t+1}}}{(1+\tau_{t+1}^c)^2} = 0, \quad \forall t \geq 0. \tag{A.2}$$

It is clear that neither $b_{t+1} = 0$ by the Inada conditions, nor $U_{c_{1t+1}} = 0$ since then $c_{1t+1} \to \infty$, nor $\tau_{t+1}^c \to \infty$. Therefore $\mu_{t+1} = 0$, which means that after choosing the optimal allocation, the government determines the optimal consumption tax such that the restriction is satisfied. Note that it is irrelevant whether the initial consumption tax $\tau_0^c$ is or not given. This instrument cannot be used by the government to control $b_0$, since (34) and (35) have to mutually be satisfied. That can only occur when the government can choose at the same

time $\tau_0^c$ and $\tau_0^k$.

**Proof of Proposition 2:** Redefining the government objective function, the government period utility for a newborn generation becomes

$$U(c_{1t}, l_t) + \beta Z(c_{2t+1}) + \eta_t \left[ c_{1t} U_{c_{1t}} + l_t U_{l_t} + \beta c_{2t+1} Z_{c_{2t+1}} + \frac{\lambda b_{t+1} U_{c_{1t+1}}}{(1+\tau_{t+1}^c)} - \frac{b_t U_{c_{1t}}}{(1+\tau_t^c)} \right], \quad \text{(A.3)}$$

whereas the government period utility for the initial old generation is

$$\beta Z(c_{20}) + \varphi \left[ \beta c_{20} Z_{c_{20}} + \frac{\lambda b_0 U_{c_{10}}}{(1+\tau_0^c)} - \frac{\beta Z_{c_{20}} \left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right] (k_0 + d_0)(1+n)}{(1+\tau_0^c)} \right], \quad \text{(A.4)}$$

where $\eta_t$ and $\varphi$ are the Lagrange multipliers of (42) and (43), respectively. The first-order conditions of the government problem with respect to $\tau_0^c$, $b_0$, $\tau_{t+1}^c$, $b_{t+1}$, $c_{1t}$, $l_t$, $c_{2t+1}$ and $k_{t+1}$ are, respectively,

$$\frac{b_0 U_{c_{10}}}{(1+\tau_0^c)^2} (\eta_0 - \varphi) + \varphi \frac{\beta Z_{c_{20}} \left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right] (k_0 + d_0)(1+n)}{\lambda (1+\tau_0^c)^2} = 0, \quad \text{(A.5)}$$

$$\frac{U_{c_{10}}}{(1+\tau_0^c)} (\eta_0 - \varphi) = 0. \quad \text{(A.6)}$$

$$\frac{\lambda^{t+1} U_{c_{1t+1}} b_{t+1}}{(1+\tau_{t+1}^c)^2} \left(\eta_{t+1} - \eta_t\right) = 0, \quad \forall t \geq 0, \quad \text{(A.7)}$$

$$\frac{\lambda^{t+1} U_{c_{1t+1}}}{(1+\tau_{t+1}^c)} \left(\eta_{t+1} - \eta_t\right) = 0, \quad \forall t \geq 0, \quad \text{(A.8)}$$

$$\lambda^t \left[ U_{c_{1t}} + \eta_t \left( U_{c_{1t}} + c_{1t} U_{c_{1t}c_{1t}} + l_t U_{l_t c_{1t}} - \frac{b_t U_{c_{1t}c_{1t}}}{(1+\tau_t^c)} \right) + \eta_{t-1} \frac{b_t U_{c_{1t}c_{1t}}}{(1+\tau_t^c)} \right] - \zeta_t = 0, \quad \text{(A.9)}$$

$$\lambda^t \left[ U_{l_t} + \eta_t \left( U_{l_t} + l_t U_{l_t l_t} + c_{1t} U_{c_{1t} l_t} - \frac{b_t U_{c_{1t} l_t}}{(1+\tau_t^c)} \right) + \eta_{t-1} \frac{b_t U_{c_{1t} l_t}}{(1+\tau_t^c)} \right] + \zeta_t f_{l_t} = 0, \quad \text{(A.10)}$$

$$\lambda^t \beta \left[ Z_{c_{2t+1}} + \eta_t \left( Z_{c_{2t+1}} + c_{2t+1} Z_{c_{2t+1}c_{2t+1}} \right) \right] - \frac{\zeta_{t+1}}{1+n} = 0, \quad \text{(A.11)}$$

$$-\zeta_t (1+n) + \zeta_{t+1} \left(1 + f_{k_{t+1}} - \delta\right) = 0, \quad \text{(A.12)}$$

where $\zeta_t$ is the Lagrange multiplier associated to the resource constraint. In view of (A.7) and (A.8), the existence of consumption taxes is not essential to the government. Although the role of the initial consumption tax $\tau_0^c$ could seem quite different, since from (A.5) and (A.6) we have $\varphi = 0$, which implies by (A.6) and (A.8) that $\eta_t = 0$ $\forall t \geq 0$, in fact it is not the case and we do not need any initial tax to be in the first-best path from $t = 0$. Combining (A.9), (A.11) and (A.12), and after using (A.8), we have

$$\eta_t = \frac{\left(1 + f_{k_{t+1}} - \delta\right) \beta Z_{c_{2t+1}} - U_{c_{1t}}}{U_{c_{1t}} + c_{1t} U_{c_{1t}c_{1t}} + l_t U_{l_t c_{1t}} - \left(1 + f_{k_{t+1}} - \delta\right) \beta \left( Z_{c_{2t+1}} + c_{2t+1} Z_{c_{2t+1}c_{2t+1}} \right)}. \quad \text{(A.13)}$$

From (39) we have that if in steady state $\tau^k = 0$ then from (A.13) and after (A.8) and (A.6) we are in the first-best path from $t = 0$. If we are not in the first-best path then $\tau^k \neq 0$. In fact, given $\tau_0^c$ and the first-best path, $b_0$ can be adjusted such that (43) is satisfied and $\varphi = 0$ (just choosing the suitable $c_{10}$ the individual chooses the government required $b_0$); $b_1/(1+\tau_1^c)$ can be adjusted such that (42) is satisfied and $\eta_0 = 0$; and so on.

**Proof of Proposition 3:** Redefining the government objective function, the government period utility for a newborn generation becomes

$$U(c_{1t}, l_t) + \beta X(c_{2t+1}, \frac{-l_{t+1}U_{l_{t+1}}(1+\tau_{t+1}^c)}{U_{c_{1t+1}}} + b_{t+1})$$

$$+\psi_t \left[ c_{1t}U_{c_{1t}} + l_t U_{l_t} + \beta \left( c_{2t+1}X_{c_{2t+1}} + b_{t+1}X_{\omega_{t+1}} \right) - \frac{b_t U_{c_{1t}}}{(1+\tau_t^c)} \right], \tag{A.14}$$

whereas the government period utility for the initial old generation is

$$\rho X(c_{20}, \frac{-l_0 U_{l_0}(1+\tau_0^c)}{U_{c_{10}}} + b_0)$$

$$+\theta \left[ c_{20}X_{c_{20}} + b_0 X_{\omega_0} - \frac{X_{c_{20}} \left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)(1+n)}{(1+\tau_0^c)} \right], \tag{A.15}$$

where $\psi_t$ and $\theta$ are the Lagrange multipliers of (51) and (52), respectively. The first-order conditions of the government problem with respect to $\tau_0^c$, $b_0$, $\tau_{t+1}^c$ and $b_{t+1}$ are, respectively,

$$\psi_0 \frac{b_0 U_{c_{10}}}{(1+\tau_0^c)^2} + \lambda^{-1}\theta \frac{X_{c_{20}} \left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)}{(1+\tau_0^c)^2} + \lambda^{-1} \left[ \rho X_{\omega_0} + \theta \left( c_{20}X_{c_{20}\omega_0} \right.\right.$$

$$\left.\left. + b_0 X_{\omega_0\omega_0} - \frac{X_{c_{20}\omega_0} \left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)}{(1+\tau_0^c)} \right) \right] \left( \frac{-l_0 U_{l_0}}{U_{c_{10}}} \right) = 0, \tag{A.16}$$

$$-\psi_0 \frac{U_{c_{10}}}{(1+\tau_0^c)} + \lambda^{-1} \left[ \beta X_{\omega_0} + \theta \left( c_{20}X_{c_{20}\omega_0} + X_{\omega_0} + b_0 X_{\omega_0\omega_0} \right.\right.$$

$$\left.\left. - \frac{X_{c_{20}\omega_0} \left[1 + (1-\tau_0^k)(f_{k_0} - \delta)\right](k_0 + d_0)}{(1+\tau_0^c)} \right) \right] = 0, \tag{A.17}$$

$$\beta \left[ X_{\omega_{t+1}} + \psi_t \left( c_{2t+1}X_{c_{2t+1}\omega_{t+1}} + b_{t+1}X_{\omega_{t+1}\omega_{t+1}} \right) \right] \left( \frac{-l_{t+1}U_{l_{t+1}}}{U_{c_{1t+1}}} \right)$$

$$+\lambda\psi_{t+1} \frac{b_{t+1}U_{c_{1t+1}}}{(1+\tau_{t+1}^c)^2} = 0, \tag{A.18}$$

$$\beta \left[ X_{\omega_{t+1}} + \psi_t \left( c_{2t+1}X_{c_{2t+1}\omega_{t+1}} + X_{\omega_{t+1}} + b_{t+1}X_{\omega_{t+1}\omega_{t+1}} \right) \right] \left( \frac{-l_{t+1}U_{l_{t+1}}}{U_{c_{1t+1}}} \right)$$

$$-\lambda\psi_{t+1} \frac{U_{c_{1t+1}}}{(1+\tau_{t+1}^c)} = 0. \tag{A.19}$$

Combining (A.16) and (A.17), and (A.18) and (A.19), we have, respectively,

$$\theta = \psi_0 \frac{\frac{\lambda U_{c_{10}}}{(1+\tau_0^c)}\left(1 - \frac{b_0 U_{c_{10}}}{l_0 U_{l_0}(1+\tau_0^c)}\right)}{\left(x_{\omega_0} + \frac{U_{c_{10}}}{l_0 U_{l_0}} \frac{X_{c_{20}}\left[1+(1-\tau_0^k)(f_{k_0}-\delta)\right](k_0+d_0)}{(1+\tau_0^c)^2}\right)}, \tag{A.20}$$

$$\psi_t = \psi_{t+1} \frac{\lambda U_{c_{1t+1}}}{\beta X_{\omega_{t+1}}(1+\tau_{t+1}^c)}\left(1 - \frac{b_{t+1} U_{c_{1t+1}}}{l_{t+1} U_{l_{t+1}}(1+\tau_{t+1}^c)}\right). \tag{A.21}$$

Although it seems that another initial tax is needed to be in the first-best path, it is not the case. In fact, given the first-best path, $\tau_0^c$ and $b_0$ can be adjusted such that $\omega_0$ takes the first-best value and (52) is satisfied, so that $\theta = 0$; $\tau_1^c$ and $b_1$ can be adjusted such that $\omega_1$ takes the first-best value and (51) is satisfied, so that $\psi_0 = 0$; and so on. If the initial consumption tax $\tau_0^c$ is given then $\theta \neq 0$, but $\psi_t = 0$ for all $t$.

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