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Optimal Stabilization Policy with Endogenous Firm Entry*

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Abstract

Monetary policy has significant but overlooked effects on entry and exit of firms. We study optimal monetary stabilization policy in a DSGE model with microfounded money demand and endogenous firm entry. Due to a congestion externality affecting firm entry, the optimal policy deviates from the Friedman rule in all states even though all prices are fully flexible. In contrast to previous Ramsey model with flexible price, our calibration exercises suggest that the model can generate a high volatility of the nominal interest rate which is a direct consequence of policy actions to control entry.

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1 Introduction

There is a rapidly growing macroeconomic literature that investigates the role of endogenous firm entry and exit on business cycle fluctuations.[1] Studying entry behavior in a macroeconomy is important for several reasons. First, a key business cycle fact, as shown by Bilbiie, Ghironi and Melitz (2006), is that net firm entry is strongly procyclical and leads GDP.[2] Second, entry creates an extensive output margin that can amplify and propagate aggregate shocks. Moreover, as recently demonstrated by Jaimovich and Floetotto (2008), the amplification effects of shocks via entry are quantitatively important. Third, using a monetary VAR Bergin and Corsetti (2008) document that monetary policy has a significant effect on firm entry.

Based on this line of reasoning, firm entry may be an important factor for monetary policy. This is not just a theoretical concern. For example, the recent housing bubble in the U.S. is widely believed to have been associated with an excessive amount of new construction of houses and retail development suggesting that there was too much entry from a social point of view. As a result, many economists believe that the Federal Reserve should have raised the nominal interest rate to deter entry of builders and developers into the housing and commercial property markets.

The notion that there was "too much entry from a social point of view" suggests that there are entry externalities that individual firms to not take into account with their entry or exit decisions. The main contribution of this paper is to study optimal monetary stabilization policy in a DSGE model, when entry creates 'congestion' that affects the trading opportunities for other firms. This externality causes aggregate output to be inefficient and monetary policy is designed to move output closer to the first-best allocation. Changes in aggregate output can be achieved via the intensive margin (output per producer) or the extensive margin (entry and exit of producers). At first glance, one may think it is irrelevant which margin is used but this is not the case. Suppose the central bank wants output to expand. If producers have convex costs of production, then it would be optimal to increase the number of producers to lower the marginal cost of producing this additional output.

The basic framework is that of Lagos and Wright (2005) where informational frictions

[1] See Jaimovich (2006, 2007), Jaimovich and Floetotto (2008), Bilbiie, Ghironi and Melitz (2006, 2007), Bergin and Corsetti (2008), Kim (2006), Mancini-Griffoli and Elkhoury (2006), Lewis (2008a, 2008b) and Fujiwara (2007). Earlier works on this topic include Chatterjee and Cooper (1993) and Devereaux, Head and Lapham (1996).

[2] They find that the correlation between net firm entry at time t and quarterly real GDP at t+1 is approximately 0.5 using HP filtered data. Using a VAR approach, Bergin and Corsetti (2008) find that the correlation is between 0.53 and 0.73. Using simple detrending methods, Chatterjee and Cooper (1993) estimate it at 0.5.

make money essential as a medium of exchange.[3] We then modify the model in several ways. First, we assume that producers make an entry decision that requires paying a fee every period. Second, we carefully model the existence of a credit market that allows agents to borrow and lend money. Third, we introduce a variety of well-defined shocks, such as productivity and preference shocks, that generate consumption risk for households. Due to the informational frictions that give rise to money, households are unable to perfectly insure themselves against these shocks. This gives rise to a welfare improving role for monetary policy that works by adjusting the nominal interest rate in response to shocks. Optimal policy is determined by choosing a set of state-contingent nominal interest rates to maximize the expected lifetime utility of the agents subject to the constraints of being an equilibrium. Finally, we consider three pricing protocols – competitive pricing, monopoly pricing and price posting – in the market where money is essential as a medium of exchange. This allows us to see how optimal monetary policy changes as a result of the different pricing mechanisms. An important feature of our model is that there is a role for stabilization policy in the absence of sticky prices.

Borrowing from Rocheteau and Wright (2005), we assume that upon entering the market, a producer is able to trade with some probability, which may not be one. In short, he may be shut out of the market despite having paid the entry cost. We then study optimal stabilization under two assumptions regarding this trading probability. In one case, we assume that this trading probability is independent of the number of producers in the market. In the second case, we assume that the probability of trading is decreasing in the number of entering producers. This is intended to capture the idea that as more producers enter congestion occurs making it harder to trade and earn profits.[4]

Our basic results concerning the optimal stabilization policy are as follows. With a fixed probability of trading, the optimal monetary policy is to run the Friedman rule and set the nominal interest rate to zero in all states. This is true for all three pricing protocols. When the trading probability depends on aggregate entry, a congestion externality arises that makes entry inefficiently high. Thus, the central bank finds it optimal to raise interest rates above zero in all states in order to reduce entry even though it lowers average consumption. Once again, this is true for all pricing protocols. In short, the zero lower bound is never a binding

[3] Most stabilization policy analysis has been done using the canonical New Keynesian model, which has sticky prices. In these models, in the absence of nominal rigidities, monetary policy is ineffective since money is neutral. An important contribution of this paper is to show that the informational frictions that give rise to a medium of exchange role for money allow for a welfare improving role of stabilisation policy even when all prices are fully flexible.

[4] This type of matching externality is common in monetary search models. Examples include Shi (1997), Lagos and Rocheteau (2005), Aruoba, Rocheteau and Waller (2007) and Berentsen, Rocheteau and Shi (2007).

constraint in our model. The key to implementing the desired allocation is to manipulate the relative price of goods across markets by choosing state-dependent nominal interest rates.

A final issue that we address with our model is the volatility of the nominal interest rate. The reason for this is as follows. In Ramsey models of the type studied by Chari and Kehoe (1999), for certain preferences, the Friedman rule is optimal for all shocks, so the volatility is zero. Aruoba and Chugh (2007) have shown that even if the Friedman rule is not optimal, the volatility of the nominal interest under the optimal policy is still very small. In contrast, New Keynesian models tend to have the nominal interest rate being too volatile compared to the data. For example, Khan, King and Wolman (2003) show that the optimal policy requires a very volatile interest rate. They explain this finding that with sticky prices the relative price distortion is minimized by setting the expected inflation rate to zero. This leads the optimal nominal interest rate to track the real interest rate which moves with the aggregate shocks. This has led New Keynesian to add a lagged interest rate into the Taylor rule to induce interest rate smoothing.

This dramatic contrast as to how the nominal interest rate should respond to shocks suggests we quantify the volatility of the nominal interest rate in our model as well. In particular, since none of the aforementioned models have entry, it is interesting to know how endogenous entry affects the volatility of the nominal interest rate. Towards this end, we calibrate our model to study the volatility of the nominal interest rate. The key quantitative question we address is the following: If the shock processes in our model are constructed to match the actual behavior of GDP, how does the optimal nominal interest rate behave in our model compared to the data? We find the following. If the shocks are 'demand side' in nature (i.e., preference shocks), then our model generates very little volatility of the nominal interest rate. Alternatively, if productivity shocks are the only shock, then our model generates a volatility of the interest rate that is larger than in the data. These results suggest the following. First, congestion externalities provide a propagation mechanism for productivity shocks to induce substantial interest rate volatility in Ramsey models. Second, a combination of supply and demand shocks are needed to generate the volatility of the nominal interest rate observed in the data.

Our framework for studying stabilization is substantially different than the existing literature on endogenous entry. Jaimovich and Floetotto (2007) use a prototypical real business cycle, hence there is no role for monetary policy. All of the others papers in this area are based on New Keynesian sticky price models while our basic framework displays fully flexible prices. Nevertheless, we show that money is non-neutral and that there is a role for stabilization policy. Furthermore, many of the papers simply look at the effects of monetary shocks – they do not study optimal monetary policy. Of those that do study optimal policy, Bergin

and Corsetti (2008) and Bilbiie, Ghironi and Melitz (2007) do so for a simple class of interest rate rules with a single productivity shock. Bergin and Corsetti (2008) consider a model where entry enlarges the set of goods available to households. The households have a love for variety, so that enlarging the set of goods can have a positive externality on household utility. As mentioned above, we differ from their approach in several dimensions. However, the most important difference is that our entry congestion imposes a negative externality on other market participants. As such, our model captures episodes where there was "too much entry from a social point of view" as suggested by many observers of the recent events in the housing and commercial property markets. Finally, Lewis's (2008b) is also closely related to our work in that she derives the optimal monetary policy using a primal Ramsey approach in a cash-in-advance model. She finds that the Friedman rule is optimal, hence there is no stabilization role for monetary policy. Also, there is also no quantitative analysis in her work. Finally, we address other issues, such as the zero lower nominal bound on interest rates, that these papers do not. We have also studied optimal stabilization in an earlier paper, Berentsen and Waller (2008), but the focus of that paper was the use of price level targeting as a monetary policy strategy to control inflation expectations. Furthermore, we did not study endogenous entry, different pricing protocols or conduct any quantitative analysis.

The paper proceeds as follows. In Section 2 we describe the environment and derive the first-best allocation. In Section 3 we present the agents' decision problems. Section 4 contains the central bank's maximization problem and the optimal monetary policy for each pricing protocol. Section 5 contains our quantitative analysis and Section 6 concludes.

2 The Environment

Time is discrete and continues forever.[5] In each period three perfectly competitive markets open sequentially. The first market is a competitive credit market and the third market is a competitive goods markets. The second market is also a goods market for which we study various market structures. There is a continuum of two types of agents, called households and sellers. They differ in terms of when they produce and consume as follows. All agents can produce and consume a perishable good in the last market. In the second market households can consume but not produce and sellers can produce but not consume. We

[5] The environment combines elements of Lagos and Wright (2005) and Berentsen, Camera and Waller (2007). The Lagos-Wright framework provides a microfoundation for money demand while keeping the distribution of money balances analytically tractable. Berentsen, Camera and Waller (2007) introduce financial intermediation into the Lagos-Wright framework.

assume that all trades in the second market are anonymous ruling out trade credit. Since all agents are anonymous and there is a double coincidence problem, sellers require immediate compensation. So households must pay with money in market 2 generating an essential role for money.

The instantaneous utility of a household at date t is

$$U_t^b = \upsilon(x_t) - y_t + \psi_t u(q_t^b) \tag{1}$$

where x_t is consumption and y_t production in the last market.[6] The quantity q_t^b is a household's consumption in the second market and $\psi \geq 0$ is a preference parameter. We assume $u' > 0$, $u'' < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. Furthermore, we assume the coefficient of relative risk aversion, $R^u \equiv -qu''/u'$, is constant and less than one.[7] In the last market the utility function satisfies $\upsilon' > 0$, $\upsilon'' < 0$, $\upsilon'(0) = \infty$ and there is a x^* such that $\upsilon'(x^*) = 1$.

The instantaneous utility of a seller at date t is

$$U_t^s = \upsilon(x_t) - y_t - (1/\alpha_t)\, c\,(q_t) \tag{2}$$

where x_t is consumption and y_t is production in the last market while q_t is production in the second market. Production disutility satisfies c', $c'', c''' \geq 0$ and $c\,(0) = c'\,(0) = 0$. Denote the elasticity of marginal cost as $R^c \equiv qc''/c'$. The parameter α is a productivity parameter measured in utility terms with higher values of α being associated with higher productivity and thus lower marginal utility costs of production. The discount factor across dates is $\beta = 1/\,(1+r) \in (0,1)$ where r is the time rate of discount.

2.1 Credit market

At the beginning of a period all households receive a preference shock $\psi_t \in \{0, \varepsilon\}$ with $\varepsilon > 0$. The probability that $\psi_t = \varepsilon$ is $1/2$ meaning there is an equal probability that a household wants to consume or not in market 2.[8] We call households that consume buyers and those that do not non-buyers. These preference shocks generate an ex-post inefficiency since non-buyers are holding idle balances while buyers are cash constrained. As in Berentsen, Camera and Waller (2007) this inefficiency generates a welfare improving role for a credit market

[6] As in Lagos and Wright (2005), these assumptions allow us to get a degenerate distribution of money holdings at the beginning of a period. The different utility functions $\upsilon\,(\cdot)$and $u\,(\cdot)$ allow us to impose technical conditions such that in equilibrium all agents produce and consume in the last market.

[7] This restriction on preferences is not necessary for competitive pricing but is needed for interior solutions under monopolistic pricing.

[8] We have also allowed this number to be different than 1/2 and to make it random. However, it added very little to the analytical and quantitative results. Thus we chose 1/2 to simplify notation.

where households can borrow or lend money at the nominal interest rate i.

Whereas goods trade is anonymous, we assume the existence of a record-keeping technology over financial transactions, i.e., financial trading histories are not private information. In all models with credit, default is a serious issue. To focus on optimal stabilization, we simplify the analysis by assuming that some mechanism exists that ensures repayment of loans in the third market.[9] One can show that due to the quasi-linearity of preferences in market 3 there is no gain from multi-period contracts. Furthermore, since the states are revealed prior to contracting, the one-period nominal debt contracts that we consider are optimal.

2.2 Shocks

To study the optimal response to shocks, we assume that α_t and ε_t are stochastic. The random variable α_t has support $[\underline{\alpha}, \overline{\alpha}]$, $0 < \underline{\alpha} < \overline{\alpha} < \infty$, and ε_t has support $[\underline{\varepsilon}, \overline{\varepsilon}]$, $0 < \underline{\varepsilon} < \overline{\varepsilon} < \infty$. Let $\omega_t = (\alpha_t, \varepsilon_t) \in \Phi$ be the state in market 1, where $\Phi = [\underline{\alpha}, \overline{\alpha}] \times [\underline{\varepsilon}, \overline{\varepsilon}]$ is a closed and compact subset on $\boldsymbol{R}_+^2$. We allow for the shocks to be serially correlated. Let $\Omega_t = \{\omega_t, \omega_{t-1}, ...\}$ denote the history of the aggregate state up through period t. For notational simplicity let $dF(\omega_t | \Omega_{t-1}) \equiv dF(\alpha_t | \Omega_{t-1}) dF(\varepsilon_t | \Omega_{t-1})$ denote the conditional density function of ω_t where $dF(\omega_t | \Omega_{t-1}) = f(\omega_t | \Omega_{t-1}) d\omega_t$. For discussion purposes, we label ε_t as a 'demand' shock, while a shock to α_t is referred to as a 'supply' shock.

2.3 Free entry and search frictions

In order to capture the fact that firm entry and exit fluctuates over the business cycle, we assume that entry is endogenous and costly for sellers. At the beginning of every period after observing the shock, sellers have to pay the cost $\kappa > 0$ in terms of disutility to enter the second market.[10] The set of potential firms is denoted $\mathcal{F}$. Let $\mathcal{S} \subseteq \mathcal{F}$ denote the set of sellers that pay the utility cost κ to enter the second market. We assume that the set of potential sellers $\mathcal{F}$ is so large that $\mathcal{S} \subset \mathcal{F}$. Let s denote the measure of $\mathcal{S}$. The set of households is denoted by $\mathcal{H}$ whose size is normalized to 2. Let $\mathcal{B} \subset \mathcal{H}$ denote the set of households with $e = \varepsilon$ (the buyers), where $b = 1$ is the measure of $\mathcal{B}$.

We introduce search frictions along the lines of Rocheteau and Wright (2005) who assume that not all firms that pay the fixed utility cost can trade in market 2. That is, paying κ

[9] In Berentsen, Camera and Waller (2007) we derive the equilibrium when the only punishment for strategic default is exclusion from the financial system in all future periods.

[10] Assuming a fixed utility cost κ is standard in the labor search literature (e.g. Pissarides 2000, Rogerson, Shimer and Wright 2005) or in the money search literature (e.g. Rocheteau and Wright 2005).

means entry into the group $\mathcal{S}$ of firms that try to enter market 2. Only $\tilde{\mathcal{S}} \subseteq \mathcal{S}$ suceed. Denote $\sigma(s)$ the probability of trading in market 2 for a firm that has paid the utility cost. Then, $\sigma(s)\, s$ is the measure of $\tilde{\mathcal{S}}$. We impose the usual assumptions on $\sigma(s)$, namely $\sigma'(s) \leq 0$, $\sigma''(s) \geq 0$, $\sigma(s) \leq 1$, $\sigma(0) = 1$, and $\sigma(\infty) = 0$.[11] Finally, denote $\Sigma^{\sigma} \equiv s\sigma'(s)/\sigma(s) < 0$ as the elasticity of $\sigma(s)$. As is standard in the search literature, we assume this elasticity is constant with $-\Sigma^{\sigma} < 1$.

With $\sigma'(s) < 0$ a firm entering the set $\mathcal{S}$ generates a negative trading externality that the optimal policy must take into account. There are precedents for such macro externalities in the literature. For example, in endogenous entry/search models where the terms of trade are determined by bargaining, there may be too many buyers or sellers relative to the social optimum depending on the bargaining weight. In these models, deviating from the Friedman rule may be optimal to improve the extensive margin.[12] The restriction that $-\Sigma^{\sigma} < 1$ ensures that this congestion externality is not too large.

2.4 Monetary Policy

We assume a central bank exists that controls the supply of fiat currency. We denote the gross growth rate of the money supply as $\gamma(\Omega_t)$, implying $M_t(\Omega_t) = \gamma(\Omega_t) M_{t-1}$ where $M_t(\Omega_t)$ denotes the quantity of money per household in market 3 in period t. We allow the gross growth rate, and thus $M_t(\Omega_t)$, to depend on the entire history of the economy. The central bank implements its policy by providing state contingent lump-sum injections of money to the households. Let $\tau_1(\Omega_t) M_{t-1}$ and $\tau_3(\Omega_t) M_{t-1}$ denote the state contingent cash injections in markets 1 and 3 where $\gamma(\Omega_t) = 1 + \tau_1(\Omega_t) + \tau_3(\Omega_t)$.

The precise sequence of action after the shocks are observed is as follows. First, the monetary injection $\tau_1(\Omega_t) M_{-1}$ occurs. Then, households move to the credit market where non-buyers ($\psi = 0$) lend their idle cash and buyers ($\psi = \varepsilon$) borrow money. Buyers and sellers then move on to market 2 and trade goods. In the third market all financial claims are settled and the central bank injects $\tau_3(\Omega_t) M_{t-1}$ units of money per household.

[11] As argued by Rocheteau and Wright (2005), the probability of trading $\sigma(s)$ has a natural meaning in matching models with bilateral meetings. It's the probability of having a match. In competitive environments it still captures search frictions by assuming that although there is a competitive market not all firms get the chance to trade in this market. It can be derived from the following constant returns to scale matching function $m(b,s) = b^{\theta} s^{1-\theta}$ with $0 < \theta < 1$, where b is the measure of buyers and s the measure of sellers. Then, since $b = 1$, the matching probability of sellers satisfies $\sigma(s) = \min[1, m(b,s)/s] = \min\left[1, s^{-\theta}\right]$.

[12] See Shi (1997), Lagos and Rocheteau (2004), Rocheteau and Wright (2005), Berentsen, Rocheteau and Wright (2007), and Aruoba, Rocheteau and Waller (2007).

2.5 First-best allocation

Our welfare criterion is given by

$$\mathcal{V} = \sum_{t=0}^{\infty} \beta^t \mathcal{W}_t \left(\Omega_{t-1}\right) \tag{3}$$

where

$$\begin{aligned}
\mathcal{W}_t \left(\Omega_{t-1}\right) &= \int_{\Phi}\int_{\mathcal{H}\cup\mathcal{F}} \left\{\upsilon\left[x_t\left(j\right)\right] - y\left(j\right)\right\} dj dF\left(\omega_t \middle| \Omega_{t-1}\right) \\
&\quad - \int_{\Phi}\int_{\mathcal{S}} \kappa dj dF\left(\omega_t \middle| \Omega_{t-1}\right) \\
&\quad + \int_{\Phi} \left\{ \int_{\mathcal{B}} \varepsilon_t u\left[q_t^b\left(j\right)\right] dj - \int_{\tilde{\mathcal{S}}} \left(1/\alpha_t\right) c\left[q_t\left(j\right)\right] dj \right\} dF\left(\omega_t \middle| \Omega_{t-1}\right)
\end{aligned}$$

where for each t, the quantities $q_t^b\left(j\right)_{j\in\mathcal{B}}$ are the consumption quantities of all households with $e = \varepsilon$ (the buyers) and $q_t\left(j\right)_{j\in\tilde{\mathcal{S}}}$ are the production quantities of all firms that pay the entry cost and are able to enter market 2, while $x_t\left(j\right)_{j\in\mathcal{H}\cup\mathcal{F}}$ are the consumption quantities of all household and all firms in market 3 and $y_t\left(j\right)_{j\in\mathcal{H}\cup\mathcal{F}}$ are the production quantities of all households and all firms in market 3.

An efficient allocation is defined as paths for $q_t^b\left(j\right)_{j\in\mathcal{B}}$, $q_t\left(j\right)_{j\in\tilde{\mathcal{S}}}$, $x_t\left(j\right)_{j\in\mathcal{H}\cup\mathcal{F}}$, $y_t\left(j\right)_{j\in\mathcal{H}\cup\mathcal{F}}$, and s_t that maximizes $\mathcal{V}$. In the Appendix we show that the first-best allocation is a symmetric, stationary list

$$\left\{x^*\left(\omega\right), q^{b*}\left(\omega\right), q^*\left(\omega\right), s^*\left(\omega\right)\right\}_{\omega\in\Phi}$$

that is independent of history and satisfies $\upsilon'\left[x^*\left(\omega\right)\right] = 1$, $q^{b*}\left(\omega\right) = \sigma\left[s^*\left(\omega\right)\right] s^*\left(\omega\right) q^*\left(\omega\right)$, and $q^{b*}\left(\omega\right)$ and $s^*\left(\omega\right)$ solve

$$\varepsilon u'\left[q^b\left(\omega\right)\right] = \left(1/\alpha\right) c'\left[q\left(\omega\right)\right] \tag{4}$$

$$\kappa = \sigma\left[s\left(\omega\right)\right]\left(1+\Sigma^{\sigma}\right)\left(1/\alpha\right)\left\{c'\left[q\left(\omega\right)\right] q\left(\omega\right) - c\left[q\left(\omega\right)\right]\right\} \tag{5}$$

where $q\left(\omega\right)$ is a seller's production and $q^b\left(\omega\right)$ a buyer's consumption in market 2, and $x\left(\omega\right)$ is consumption in market 3. Note that $x^*\left(\omega\right)$ is not history dependent – it only depends on the current realization of the aggregate state. The planner faces no intertemporal trade-offs and so he simply chooses the quantities that maximize welfare state by state for all t. This implies that the history of the shock process is irrelevant for the efficient allocation.

We also show in the Appendix that the first-best allocation exists and is unique. Fur-

thermore, comparative statics on (4)-(5) shows that $q^*(\omega)$ is increasing in all of the shocks, while $s^*(\omega)$ is increasing in ε but is ambiguous in α.[13]

Example 1 *To help illustrate how our model works we use a common example throughout the paper. The functional forms are* $\varepsilon u\left[q^b(\omega)\right] = \varepsilon q^b(\omega)^{1-\eta}$, $c\left[q(\omega)\right] = q(\omega)^{\rho}/\rho$, $\sigma(s) = s^{-\theta}$ *with* $\rho > 1 > \theta \geq 0$ *and the entry cost is* $\kappa = 1$. *When* $\theta = 0$ *there is no congestion externality. With these assumptions, the planner's allocation is given by*

$$\begin{aligned} q^*(\omega) &= \left(\tfrac{\theta\rho+\rho}{\rho-1}\right)^{\eta\delta} \alpha^{(\theta+\eta)\delta} * \varepsilon^{\theta\delta} \\ s^*(\omega) &= \left(\tfrac{\rho-1}{\theta\rho+\rho}\right)^{(\rho-1+\eta)\delta} \alpha^{(1-\eta)\delta} * \varepsilon^{\rho\delta}. \end{aligned}$$

where $\delta \equiv \left[(\rho-1+\eta)\theta+\eta\rho\right]^{-1}$. *From this example, we see that* $q^*(\omega)$ *is increasing in both shocks. We also have* $s^*(\omega)$ *increasing in* ε *as well as* α *shocks when* $\eta < 1$. *Thus, the planner wants entry to be procyclical. Note also that as* $\rho \to 1$, *costs become linear, profits go to zero and its optimal to have one seller produce for the entire market since entry is costly.*

Why does the planner want entry to be procyclical? Consider an increase in ε. This implies that households want to consume more and it is optimal to let them consume more. The planner can achieve this higher level of output by increasing the amount of goods produced by each seller, i.e., increase $q(\omega)$, or by having more firms enter and produce, i.e., increase $s(\omega)$. With increasing marginal costs of production, the planner chooses to alleviate higher production costs on each individual firm by having more entry even though it is costly. Hence, the optimal response to an ε shock is to increase both the intensive and extensive margins for output. A similar argument holds for the other shock.

3 Monetary allocation

Let $p_{3t}(\Omega^t)$ be the time t nominal price of goods in market 3 and thus $\phi_t(\Omega_t) \equiv 1/p_{3t}(\Omega_t)$ is the goods price of money. We study equilibria where end-of-period real money balances are history invariant

$$\phi_t(\Omega_t) M_t(\Omega_t) = \phi_{t-1}(\Omega_{t-1}) M_{t-1}(\Omega_{t-1}), \qquad \forall\, \Omega. \tag{6}$$

[13] If the productivity shock affects the entry cost in the same way as the cost function, then $s^*(\omega)$ is increasing in α as well.

We refer to it as a stationary equilibrium. This implies that in a stationary equilibrium $\phi_{t-1}(\Omega_{t-1})/\phi_t(\Omega_t) = \gamma(\Omega_t)$. In what follows, we look at a representative period t and work backwards from the third to the first market to examine the agents' choices. For notational ease, variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1.

3.1 The third market

In the third market households consume x, produce y, and adjust their money balances taking into account cash payments or receipts from the credit market. If a household has net borrowing of ℓ units of money, then he repays $(1+i)\ell$ units of money.

Consider a stationary equilibrium. Let $V_1(m,\Omega,t)$ denote a household's expected lifetime utility at the beginning of market 1 with m money balances and history ω in period t. Let $V_3(m,\Omega,t,\ell)$ denote a household's expected lifetime utility from entering market 3 in period t with m money and ℓ loans with history Ω. For notational simplicity in this section we suppress the dependence of the value functions on time.

Bellman's equation for a household is

$$V_3(m,\Omega,\ell) = \max_{x,y,m_{+1}} \{\upsilon(x) - y + \beta E[V_1(m_{+1},\Omega_{+1})|\,\Omega]\} \tag{7}$$
$$\text{s.t.} \quad x + \phi m_{+1} = y + \phi[m + \tau_3(\Omega) M_{-1}] - \phi(1+i)\ell$$

where m_{+1} is the money taken into period $t+1$ given the history Ω. Rewriting the budget constraint in terms of y and substituting into (7) yields

$$\begin{aligned} V_3(m,\Omega,\ell) &= \phi[m + \tau_3(\Omega) M_{-1} - (1+i)\ell] \\ &\quad + \max_{x,m_{+1}} \{\upsilon(x) - x - \phi m_{+1} + \beta E[V_1(m_{+1},\Omega_{+1})|\,\Omega]\}. \end{aligned}$$

The first-order conditions are $\upsilon'(x^*) = 1$, meaning x^* is constant and

$$-\phi + \beta E[V_1^m(m_{+1},\Omega_{+1})|\,\Omega] = 0 \tag{8}$$

where the superscript denotes the partial derivative with respect to the argument m. Thus, V_1^m is the marginal value of taking an additional unit of money into the first market in period $t+1$. Since the choice of m_{+1} is independent of m, all households enter the following period with the same amount of money.

The envelope conditions are

$$V_3^m(m,\Omega,\ell)=\phi;\quad V_3^\ell(m,\Omega,\ell)=-\phi(1+i). \tag{9}$$

As in Lagos and Wright (2005) the value function is linear in wealth.

Let $W_1(\epsilon,\Omega)$, $\epsilon\in\{0,1\}$, denote a seller's expected lifetime utility at the beginning of market 1 given Ω. If $\epsilon=1$, the seller has paid the entry cost κ and if $\epsilon=0$ he has not. Note that we have also taken into account that sellers bring no money into market 1. Since sellers do not participate in the first market, we have $W_1(\epsilon,\Omega)=W_2(\epsilon,\Omega)$. Let $W_3(m,\Omega)$ denote a sellers's expected lifetime utility from entering market 3 with m units of money given Ω. Bellman's equation for a seller is

$$\begin{aligned} W_3(m,\Omega) &= \max_{x,y}\{\upsilon(x)-y+\beta E[W_1(\epsilon_{+1},\Omega_{+1})|\,\Omega]\} \\ \text{s.t. } x &= y+\phi m. \end{aligned}$$

Rewriting the budget constraint in terms of y and substituting into the objective function yields

$$W_3(m,\Omega)=\phi m+\max_x\{\upsilon(x)-x+\beta E[W_1(\epsilon_{+1},\Omega_{+1})|\,\Omega]\}. \tag{10}$$

The first-order condition is $\upsilon'(x^*)=1$. The envelope condition for a seller is

$$W_3'(m,\Omega)=\phi. \tag{11}$$

As was the case for households, the value function is linear in m.

3.2 The second market

There are 3 types of agents in the second market: buyers (b), non-buyers (o) and sellers (s). Let $V_2(m,\Omega,\ell,j)$ denote the value function of a household of type $j=b,o$. Let q^b and q, respectively, denote the quantities consumed by a buyer and produced by a seller and let p be the nominal price of goods.

Since non-buyers neither consume or produce, the Bellman equation for this household is simply $V_2(m,\Omega,\ell,o)=V_3(m,\Omega,\ell)$. The one for a buyer household is

$$\begin{aligned} V_2(m,\Omega,\ell,b) &= \max_{q^b}\varepsilon u\left(q^b\right)+V_3\left(m-pq^b,\Omega,\ell\right) \\ &\text{s.t. } pq^b\le m. \end{aligned}$$

Using (9) the buyer's first-order condition can be written as

$$\lambda_q = \varepsilon u'\left(q^b\right) - \phi p, \qquad \omega \in \Phi, \tag{12}$$

where λ_q is the multiplier on the buyer's budget constraint. If the budget constraint is not binding, then $\varepsilon u'\left(q^b\right) = \phi p$. If it is binding, then $\varepsilon u'\left(q^b\right) > \phi p$ and the buyer spends all of his money, i.e. $pq^b = m$. In the first case, the buyer equates the marginal rate of substitution between market 2 goods and market 3 goods to the relative price of goods in the two markets.[14] In the latter case, the agent is at a 'corner'.

The marginal value of a loan is the same for all households and so

$$V_2^{\ell}\left(m, \Omega, \ell, j\right) = -\left(1+i\right)\phi, \tag{13}$$

for $j = b, o$. Using the envelope theorem and equations (9) and (12), the marginal values of money for $j = b, o$ are

$$V_2^{m}\left(m, \Omega, \ell, b\right) = \varepsilon u'\left(q^b\right)/p \tag{14}$$

$$V_2^{m}\left(m, \Omega, \ell, o\right) = \phi. \tag{15}$$

We now describe the entry behavior of the sellers in market 2. The Bellman equation for a seller who has paid the entry cost is

$$W_2\left(1, \Omega\right) = \sigma\left(s\right) \max_q \left\{-\left(1/\alpha\right) c\left(q\right) + W_3\left(pq, \Omega\right)\right\} + \left[1 - \sigma\left(s\right)\right] W_3\left(0, \Omega\right) \tag{16}$$

subject to the pricing protocol which we discuss below. The term pq is the money receipts from selling output.

The Bellman equation for a seller who does not pay the entry cost is $W_2\left(0, \Omega\right) = W_3\left(0, \Omega\right)$. At the beginning of the period, sellers observe the current state and the representative seller chooses to enter market 2 with probability $\pi\left(\Omega\right)$ taking as given the entry choices of other sellers. Let $\mathbb{N}$ denote the measure of potential sellers. Then, since we focus on symmetric equilibria, he expects a measure $s\left(\Omega\right) = \Pi\left(\Omega\right)\mathbb{N}$ of sellers entering, where $\Pi\left(\Omega\right)$ is the entering decision of all other sellers. Define

$$\mathcal{D}\left[\Pi\left(\Omega\right)\mathbb{N}\right] \equiv W_2\left(1, \Omega\right) - W_2\left(0, \Omega\right) - \kappa \tag{17}$$

[14] The MRS between the two markets is $\varepsilon u'\left[q^b\left(\omega\right)\right]/v'\left[x\left(\omega\right)\right]$. But from the optimization problem in market 3, $v'\left[x\left(\omega\right)\right] = 1$ for all ω.

Equation (17) is the expected gain from entering the market. The optimal choice of π satisfies

$$\begin{aligned} \pi(\Omega) &= 1 && \text{if } \mathcal{D}[\Pi(\Omega)\mathbb{N}] > 0 \\ \pi(\Omega) &= 0 && \text{if } \mathcal{D}[\Pi(\Omega)\mathbb{N}] < 0 \\ \pi(\Omega) &\in [0,1] && \text{otherwise} \end{aligned}$$

We look for symmetric Nash equilibria where all sellers choose the same entry probability $\pi(\Omega)$. Moreover, the value(s) of $\Pi(\Omega)$ that sustain a symmetric Nash equilibrium are defined as follows:

$$\begin{aligned} \Pi(\Omega) &= 1 && \text{if } \mathcal{D}(\mathbb{N}) \geq 0 \\ \Pi(\Omega) &= 0 && \text{if } \mathcal{D}(0) \leq 0 \\ \mathcal{D}[\Pi(\Omega)\mathbb{N}] &= 0 && \text{otherwise} \end{aligned}$$

Throughout the paper we focus on equilibria where $\mathcal{D}[\Pi(\Omega)\mathbb{N}] = 0$ in all states.[15] Using the expressions for $W_2(1,\Omega)$, $W_2(0,\Omega)$ and (10) we then obtain the free entry condition

$$\kappa = \sigma(s)[\phi pq - (1/\alpha)c(q)]. \tag{18}$$

where the RHS is expected profits. Note that we have suppressed the dependence of s and q on ω for notational convenience. Since the entry cost has to be paid each period, only current profits enters into (18). Free entry requires that expected profits in market 2 equal the entry cost. Revenue after history Ω, measured in utility, is given by ϕpq where p is the nominal price of goods in market 2 and ϕ is the real price of money in the last market, while costs in utility are $-(1/\alpha)c(q)$. Note that $\phi p = p/p_3$ is the relative price of goods across markets 2 and 3.

3.3 The credit market

A household who has m money at the opening of the first market has expected lifetime utility

$$E[V_1(m,\Omega)] = \int_\Phi [V_2(m,\Omega,\ell,b) + V_2(m,\Omega,\ell,o)]\, dF(\omega|\Omega_{-1}). \tag{19}$$

Once trading types are realized, a household of type $j = b, o$ solves

$$\max_\ell V_2(m,\Omega,\ell,b) \;\; s.t. \;\; 0 \leq m.$$

[15] This simply requires that the measure of potential sellers is sufficiently large so that in no state all of them wants to enter. This is a standard assumption in the labor search literature.

The constraint means that money holdings cannot be negative. The first-order condition is

$$V_2^m(m,\Omega,\ell,b)+V_2^\ell(m,\Omega,\ell,b)+\lambda(j)=0$$

where $\lambda(j)$ is the multiplier on the households's non-negativity constraint. It is obvious that households with $\psi=\varepsilon$ will become borrowers while those with $\psi=0$ become lenders. Consequently, we have $\lambda(b)=0$ and $\lambda(o)>0$.

Using (13)-(15), the first-order conditions can be written as

$$\begin{aligned}\varepsilon u'\left(q^b\right) &= \phi p(1+i) &(20)\\ \lambda(o) &= i\phi. &(21)\end{aligned}$$

Using the envelope theorem and equations (12), (20), and (21), the marginal value of money satisfies

$$E\left[V_1^m(m,\Omega)|\,\Omega_{-1}\right]=\int_\Phi\left[\varepsilon u'\left(q^b\right)/p\right]dF(\omega|\,\Omega_{-1}). \tag{22}$$

noting that $\Omega=\{\omega,\Omega_{-1}\}$. Differentiating (22) shows that the value function is concave in m. Use (8) lagged one period to eliminate $E\left[V_1^m(m,\Omega)|\,\Omega_{-1}\right]$ from (22). Then, divide the resulting expression by ϕ_{-1} and rewrite to get

$$1=\beta\int_\Phi\frac{\varepsilon u'\left(q^b\right)}{\gamma(\Omega)\,p\phi}dF(\omega|\,\Omega_{-1}). \tag{23}$$

3.4 Pricing protocols

We now discuss three pricing protocols: competitive pricing, state contingent monopoly pricing and non-state contingent monopoly pricing. We refer to this last pricing protocol as posting. For each pricing protocol, we have

$$q^b=\sigma(s)\,sq \text{ for all } \omega. \tag{24}$$

For competitive pricing this is simply the market clearing condition in market 2.

For monopoly pricing this equation also holds because we assume a matching process that allocates $[\sigma(s)\,s]^{-1}$ buyers to each seller. The benefits of this matching rule are threefold. First, the first-best allocation described in Section 3 is replicated if the monopoly pricing distortion is eliminated. Second, in search-theoretic models of money, bilateral matching creates monopoly power for both buyers and sellers in the bargaining process. This matching rule with monopoly pricing eliminates the monopsony power of the buyer and is consistent

with the pricing frictions in New Keynesian models. Third, the allocation is easily compared to the flexible price allocation since the only difference is the pricing mechanism.[16]

Competitive pricing With price taking, a seller's maximization problem in market 2 is

$$\max_{q} \{-(1/\alpha)\, c(q) + W_3(pq, \Omega)\}$$

Using (9), the first-order condition yields the pricing equation

$$p = \frac{(1/\alpha)\, c'(q)}{\phi}. \tag{25}$$

We can then combine (20) and (25) to get an expression for the interest rate

$$1 + i = \frac{\varepsilon u'[\sigma(s)\, sq]}{(1/\alpha)\, c'(q)}. \tag{26}$$

State contingent monopoly pricing With state contingent monopoly pricing, since seller faces $[\sigma(s)\, s]^{-1}$ buyers, the maximization problem is

$$\max_{q,p} \{-(1/\alpha)\, c(q) + W_3(pq, \Omega)\}$$
$$s.t. \;\; \varepsilon u'[\sigma(s)\, sq] = p\phi(1+i)$$

where the constraint is the buyer's first-order condition for consumption. The solution yields the pricing equation[17]

$$p = \frac{(1/\alpha)\, c'(q)}{\phi(1 - R^u)}. \tag{27}$$

We can then combine (20) and (27) to get an expression for the interest rate

$$1 + i = \frac{(1 - R^u)\, \varepsilon u'[\sigma(s)\, sq]}{(1/\alpha)\, c'(q)}. \tag{28}$$

Non-state contingent monopoly pricing We now assume that sellers must set the price before the realization of the current state, ω. However, they can use the information on the history of the aggregate state up to time $t-1$, Ω_{-1}, in forming their expectations

[16] This assumption is simply made to compare the allocation with monopoly pricing to one with competitive pricing. For this reason, we ignore issues involving $1 < \sigma[s(\Omega)]\, s(\Omega)$.

[17] Given our simple approach to generating monopoly power, the gross markup is given by $(1 - R^u)^{-1}$, which is constant. Since the data suggest the markup is countercyclical, we will clearly be unable to match this stylized fact. Changing the matching function such that the markup depends negatively on entry, as in Jaimovich and Floettoto (2007) would be an interesting extension.

of future profits. They commit to produce whatever is demanded in state ω at the posted price, $p(\Omega_{-1})$. However, upon seeing the shock they can choose to enter and try to sell at the posted price. With this last assumption, no seller will experience negative expected profits in equilibrium.[18] The seller's maximization problem is

$$\max_{p(\Omega_{-1})} \int_{\Phi} \left\{ \begin{array}{c} \pi(\Omega)\sigma[s(\Omega)]\{W_3[p(\Omega_{-1})q(\Omega),\Omega]-(1/\alpha)c[q(\Omega)]\} \\ +\{1-\pi(\Omega)\sigma[s(\Omega)]\}W_3(0,\Omega) \end{array} \right\} dF(\omega|\Omega_{-1})$$
$$s.t.\ \ \varepsilon u'\{\sigma[s(\Omega)]s(\Omega)q(\Omega)\} = p(\Omega_{-1})\phi(\Omega)[1+i(\Omega)] \text{ for all } \omega.$$

where demand in each state satisfies the buyer's first-order condition for consumption, i.e. it satisfies the above constraints. The first-order condition for p yields the pricing equation

$$p(\Omega_{-1}) = \frac{\int_{\Omega} \sigma[s(\Omega)]s(\Omega)q(\Omega)(1/\alpha)c'[q(\Omega)]dF(\omega|\Omega_{-1})}{(1-R^u)\int_{\Omega} \phi(\Omega)\sigma[s(\Omega)]s(\Omega)q(\Omega)dF(\omega|\Omega_{-1})}. \tag{29}$$

where we have taken into account that in a symmetric equilibrium $\pi(\Omega) = s(\Omega)/\mathbb{N}$. Equation (29) then replaces p in (23). We can then combine (20) and (29) to get an expression for the interest rate

$$1+i(\Omega) = \frac{(1-R^u)\varepsilon u'\{\sigma[s(\Omega)]s(\Omega)q(\Omega)\}\int_{\Omega} \phi(\Omega)\sigma[s(\Omega)]s(\Omega)q(\Omega)dF(\omega|\Omega_{-1})}{\phi(\Omega)\int_{\Omega} \sigma[s(\Omega)]s(\Omega)q(\Omega)(1/\alpha)c'[q(\Omega)]dF(\omega|\Omega_{-1})}. \tag{30}$$

4 Optimal stabilization

We now derive the optimal stabilization policy in symmetric stationary monetary equilibrium. To study this problem we pursue the primal approach to the Ramsey problem where the central bank chooses the quantities x_t, y_t, q_t^b, q_t, s_t to maximize (3) subject to the free entry condition (18), the relevant pricing protocol and the resource constraints. In the appendix we show that these quantities can be implemented with history dependent injections $\tau_1(\Omega)$ and $\tau_3(\Omega)$ that satisfy $i_t \geq 0$ and (23). With competitive pricing the pricing protocol is (25), with state-contingent monopoly pricing it is (27), and with non-state contingent monopoly pricing it is (29). It should be clear that in all cases $x_t = x^*$ and y_t is determined by the households budget constraint once all of the other quantities are chosen. Finally,

[18] For this case, we have in mind restaurants who print their menus in advance but upon seeing the state of the economy can choose to open or not.

from (24) q_t^b is determined once we have q_t and s_t. So the central bank's problem reduces to choices of q_t and s_t.

Proposition 1 *Consider the case of competitive pricing. The constrained optimal allocation is stationary and depends only on the current state ω. With $\sigma'[s(\omega)] = 0$, $i(\omega) = 0$, $q(\omega) = q^*(\omega)$ and $s(\omega) = s^*(\omega)$ for all states. With $\sigma'[s(\omega)] < 0$, $i(\omega) > 0$, $q(\omega) < q^*(\omega)$ and $s(\omega) > s^*(\omega)$ for all states.*

The allocation is stationary and only depends on the current state in both cases despite the persistence of the shocks. The reason is that the only equation for which the persistence of the shocks matters is the money demand equation (23). Given its optimal choices $\{q(\omega), s(\omega)\}_{\omega\in\Phi}$ the central bank then chooses $\gamma(\Omega)$ to ensure (23). Thus, any information content provided by the persistence of the shocks is offset by choosing the stochastic inflation rate appropriately.

With $\sigma'[s(\omega)] = 0$, the Friedman rule replicates the first-best allocation. This can be seen by noting that from (20) when $i(\omega) = 0$, $q(\omega) = q^*(\omega)$. Moreover, (39) replicates (5) at $q(\omega) = q^*(\omega)$. The intuition is that with $\sigma'[s(\omega)] = 0$ so there is no congestion externality and the only friction is the cost of holding money across periods. Under the Friedman rule the agents get compensated for these costs and so agents perfectly self-insure against all shocks. Consequently, there are no welfare gains from stabilization policies.[19] Note that Proposition 1 also holds in a model where the number of sellers is exogenously given.

With $\sigma'[s(\omega)] < 0$ the central bank never chooses $i(\omega) = 0$. The reason is the congestion externality. Sellers ignore how their entry lowers the expected profits of other sellers. Consequently, in equilibrium there are too many sellers and the aggregate entry cost $s(\omega)\kappa$ is too high relative to the social optimum. To see why a deviation from the Friedman rule is optimal assume that the central bank sets $i(\omega) = 0$, which generates the efficient quantity $q(\omega) = q^*(\omega)$ in all states. Now consider a reduction in $s(\omega)$ when $i(\omega) = 0$. By marginally reducing $s(\omega)$, $q(\omega)$ is also marginally reduced but the first-order welfare loss from doing so is zero. The reduction in $q(\omega)$ reduces expected profits for firms and thus entry declines. This produces a first-order gain in welfare from reducing $s(\omega)\kappa$. This is achieved by increasing $i(\omega)$ above zero.

Although the argument above does not require $i(\omega) > 0$ for all states, nevertheless it is optimal to do so. The reason is that the central bank wants to smooth consumption across

[19] Ireland (1996) derives a similar result in a model with nominal price stickiness. He finds that at the Friedman rule there is no gain from stabilizing aggregate demand shocks. For the same reason, Khan, King and Wolman (2003) find that with flexible prices the Friedman rule is optimal, although it cannot achieve the first-best allocation because of monopolistic distortion in the price setting.

states. Intuitively, consider two states $\omega, \omega' \in \Phi$ with $i(\omega) = 0$ implying $q(\omega) = q^*(\omega)$ and $i(\omega') > 0$ implying $q(\omega') < q^*(\omega')$. Then, the first-order loss from decreasing $q(\omega)$ is zero while there is a first-order gain from increasing $q(\omega')$. This gain can be accomplished by increasing $i(\omega)$ and lowering $i(\omega')$. Thus, the central bank's optimal policy is to set $i(\omega) > 0$ for all states.

We find this result interesting because it is very reminiscent of the view that the Federal Reserve kept interest rates "too low for too long" from 2003-2005. In short, this argument implies the Fed should have raised interest rates to choke off entry into the housing and commercial property markets. It is important to note that this argument requires some type of congestion externality to make entry inefficient. Entry per se is not enough.

Lastly, the central bank's optimal interest rate policy has $di(\omega)/dq(\omega)$ being of the same sign as $dR^c(\omega)/dq(\omega)$, **where R^c is the elasticity of the marginal cost function.** Thus, for a constant elasticity marginal cost function, we have $di(\omega)/dq(\omega) = 0$ and the central bank perfectly smooths interest rates. The central bank moves the nominal interest rate in a countercyclical fashion when $dR^c(\omega)/dq(\omega) > 0$ and in a pro-cyclical manner when the opposite is true.

Example 2 *Using our assumed functional forms, the central bank's optimal allocation is given by*

$$\begin{aligned} q^c(\omega) &= \left(\tfrac{\theta+\rho}{\theta\rho+\rho}\right)^{\theta\delta} \left(\tfrac{1}{1+\theta}\right)^{\eta\delta} q^*(\omega) < q^*(\omega) \\ s^c(\omega) &= \left(\tfrac{1}{1+\theta}\right)^{(1-\eta)\delta} \left(\tfrac{\theta+\rho}{\rho}\right)^{\rho\delta} s^*(\omega) > s^*(\omega) \\ i^c(\omega) &= \tfrac{\theta(\rho-1)}{\theta+\rho} > 0. \end{aligned}$$

This example illustrates the basic insight of the model. When entry is endogenous, too much entry occurs. To reduce entry, the central bank inflates in order to drive up nominal interest rates. This lowers consumption of market 2 goods and lowers profits for sellers. Expected lower profits reduces entry by sellers. Since $R^c(\omega) = \rho - 1$ for these functional forms, the optimal nominal interest rate is constant across states. Note also that when the entry cost is constant, i.e. when $\theta = 0$, then $q^c(\omega) = q^*(\omega)$, $s^c = s^*$ and $i^c(\omega) = 0$. Again, the central bank wants entry to be procyclical.

We next consider the case of state-contingent monopoly pricing.

Proposition 2 *Consider the case of state contingent monopoly pricing. The constrained optimal allocation is stationary and depends only on the current state ω. With $\sigma'(s) = 0$,*

$i(\omega) = 0$, $q(\omega) < q^*(\omega)$ and $s(\omega) > s^*(\omega)$ for all states. With $\sigma'(s) < 0$, $i(\omega) > 0$, $q(\omega) < q^*(\omega)$ and $s(\omega) > s^*(\omega)$ for all states.

With monopoly pricing and $\sigma'(s) = 0$, the Friedman rule is again optimal. However, the first-best allocation cannot be achieved since the monopoly pricing distortion causes $q(\omega)$ to be inefficiently low and $s(\omega)$ to be inefficiently high in all states

With endogenous entry, once again, due to the entry externality, the central bank pushes up interest rates to reduce profits and thus entry. As with competitive pricing, entry is higher than the social optimum. Also, production is lower than q^* and q^c.

Example 3 *Using our assumed functional forms, the central bank's optimal allocation is given by*

$$\begin{aligned} q^{sm}(\omega) &= \left(\tfrac{\rho-1}{\rho-1+\sigma}\right)^{\eta\delta}(1-\eta)^{(\eta+\theta)\delta} q^c(\omega) < q^c(\omega) < q^*(\omega) \\ s^{sm}(\omega) &= \left(\tfrac{\rho-1+\eta}{\rho-1}\right)^{(\rho-1+\eta)\delta}(1-\sigma)^{(1-\eta)\delta} s^c(\omega) \\ i^{sm}(\omega) &= \tfrac{\theta(\rho-1)}{\theta+\rho} = i^c(\omega) \end{aligned}$$

Note that when the entry cost is constant $\theta = 0$, $q^{sm}(\omega) < q^*(\omega)$, $s^{sm} \neq s^c = s^*$ and $i^{sm}(\omega) = 0$. State-contingent monopoly pricing causes market 2 consumption and entry to be inefficient. Entry can be higher or lower than is socially optimal but it is procyclical.

Finally, we study the case of non-state-contingent monopoly pricing.

Proposition 3 *Consider the case of non-state contingent pricing. In this case, central bank replicates the optimal allocation that occurs under state contingent monopoly pricing.*

Why is the central bank able to replicate the posting allocation? Posting simply imposes a constraint on the behavior of p in market 2. However, the central bank only cares about the *relative* price $\phi(\omega)p(\omega)$ between market 2 and market 3. As long as that is flexible, the central bank can replicate the state-contingent monopoly price allocation.

Implementation In the appendix, we derive implementation schemes for each pricing protocol that supports the desired allocation when $\sigma'[s(\omega)] < 0$. The schemes are not unique since the transfers are nominal injections and the central bank only cares about the relative transfers across states. For illustration, one transfer scheme under competitive

pricing is given by

$$\begin{aligned}
1+\tau_1 &= \beta\int_{\Omega}\frac{\varepsilon u'\left[q^b(\omega)\right]q^b(\omega)}{q^b(\omega_L)c'[q(\omega_L)]q(\omega_L)}f\left(\omega|\,\omega_L,\Omega_{-2}\right)d\omega \\
1+\tau_1\left(\Omega\right) &= \tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\left(1+\tau_1\right) \\
\tau_3\left(\Omega\right) &= \left(1+\tau_1\right)\tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\left\{\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}-1\right\}
\end{aligned}$$

and stochastic inflation rate

$$\gamma\left(\Omega\right)=\left(1+\tau_1\right)\tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}.$$

where $\Omega_{-1}=\{\omega_{-1},\Omega_{-2}\}$. The key thing to note is that the inflation rate is state-dependent and serially correlated. The reason for the serially correlation is as follows. The optimal allocation associated with state ω does not depend on Ω_{-1}. However, Ω_{-1} contains information about the future state ω and this affects agents' demand for real balances at time $t-1$, as is shown by (23). In order to offset any informational value history has on current money demand, the central bank offers a menu of state-contingent transfers that makes the real value of money constant regardless of Ω_{-1}. Note that for general shock processes, the central bank must promise a sequence of transfers for all possible histories. This seems to be an unrealistic implementation policy in practice. However, if the shocks are Markovian, then the central bank's transfers only need to be conditioned on the current and previous state – a much simpler set of transfers to implement.

How does the implementation scheme above work? The injection $1+\tau_1$ pins down the inflation rate in the lowest production state (ε_L, n_L and α_L). In this state, demand for goods is at its lowest level and production costs are at their highest value. Then in states where higher production is desired the central bank alters its injection scheme across markets as demand for goods increases or costs fall in such a way that it generates changes in real balances in market 1.

To make this point more concrete, assume the the only shock is the marginal utility shock ε and it is iid. Then the scheme above reduces to

$$\begin{aligned}
1+\tau_1 &= 1+\tau_1\left(\varepsilon\right)=\beta\int_{\Omega}\frac{\varepsilon u'\left[q^b(\varepsilon)\right]q^b(\varepsilon)}{q^b(\varepsilon_L)c'[q(\varepsilon_L)]q(\varepsilon_L)}f\left(\varepsilon\right)d\varepsilon \\
\tau_3\left(\varepsilon\right) &= \left(1+\tau_1\right)\left\{\tfrac{q^b(\varepsilon_L)c'[q(\varepsilon_L)]}{q^b(\varepsilon)c'[q(\varepsilon)]}-1\right\}
\end{aligned}$$

and stochastic inflation rate

$$\gamma(\varepsilon) = (1+\tau_1)\frac{q^b(\varepsilon_L)c'[q(\varepsilon_L)]}{q^b(\varepsilon)c'[q(\varepsilon)]}.$$

Under this scheme, the central bank injects the same amount of cash in market 1 regardless of the size of the ε shock. However, $q^b(\omega)$ and $q(\omega)$ are increasing in ε. To engineer higher consumption and production as ε increases, the central bank promises to reduce the market 3 injection and thus the inflation rate of market 3 prices. This means money is more valuable in market 3 and raises its price, ϕ. This induces sellers in market 2 to lower their price of goods in market 2, thereby increasing real balances in market 2. Hence, by promising to alter future monetary injections based on today's observed shock, the central bank can alter the real value of money today.

5 Quantitative Analysis

Since our focus is on optimal policy, we want to know whether our model can replicate the behavior of the nominal interest rate in the data. Table 1 reports some statistical properties of GDP, NBI and nominal interest rates during the period 1950-1993. All variables are reported as percentage deviations from an HP trend with smoothing parameter 1600. NBI is new business incorporations.[20] GDP is real GDP and the nominal interest rate is the 3-Month Treasury Bill Secondary Market Rate.

Table 1: Summary Statistics, 1950:1 to 1993:4.

		GDP	**NBI**	**i**
Standard Deviation		0.017	0.058	0.224
Quarterly Autocorrelation		0.816	0.796	0.798
Correlation Matrix	**GDP**	1	0.321	0.473
	NBI		1	0.01
	i			1

NBI is much more volatile than GDP and has a positive correlation with GDP. The nominal interest rate is positively correlated with GDP and is more than 10 times more volatile than

[20] New business incorporations is taken from the Survey of Current Business (November, 1994). The data series starts in 1948 and was discontinued in 1996.

GDP and about four times as volatile as NBI. Finally, there is little correlation between the nominal interest rate and NBI.

It is perhaps surprising that real GDP and the nominal interest rate are positively correlated. However, this is what our model suggests if central banks optimally respond to technology and preference shocks. For example, a positive productivity shock leads to more entry which the central bank might want to temper by raising the nominal interest rate. As a consequence, output, entry and the nominal interest rate increase.

5.1 Parameters and targets

We calibrate the model to match US data choosing the model period to be one quarter with the pricing protocol being state-contingent monopoly pricing. For the calibration, we need to choose values and functional forms for (i) households preferences $\{\varepsilon u(q), \upsilon(x)\}$, (ii) technology $\{(1/\alpha)\, c(q),\, k(s)\}$, and (iv) policy i.

For the utility function in the third market we choose $\upsilon(x) = A \ln(x)$. We set the utility function for goods traded in the market 2 to belong to the CRRA family

$$u(q^b) = (1-\eta)^{-1} \left(q^b\right)^{1-\eta}.$$

We set $c(q)$ to satisfy

$$c(q) = \frac{(\varphi + q)^{1+\rho} - \varphi^{1+\rho}}{1+\rho}.$$

where $\varphi \geq 0$. Finally, we assume that the probability of a sale is $\sigma(s) = s^{-\theta}$ implying

$$k(s) = \kappa/\sigma(s) = \kappa s^{\theta}.$$

With these functional forms, we have to identify the parameters $(A, \eta, \rho, \varphi, \kappa, \theta)$. We use the following data: average mark-up, $\bar{\mu}$, average money demand, $\bar{L}$, average nominal interest rate, $\bar{\imath}$, and the ratio of entry cost to per capita gdp, $\bar{\kappa}$. Over the period from 1950-1993, average annual money demand $L = M/(PY)$ is approximately $\bar{L} = 0.05$, where M is currency demand M_0, and the average nominal interest on the 3-Month Treasury Bill Secondary Market Rate is $\bar{\imath} = 0.05$. As pointed out by Jaimovich and Floetotto (2008), estimates of markups in value added tend be from 1.2 to 1.4, while markups in gross output lie between 1.05 and 1.15. We set the gross output markup to be $\bar{\mu} = 1.1$. Djankov et al (2002) estimates the cost of starting a new business for the US to be $\bar{\kappa} = 0.017$ (as a fraction of GDP per capita in 1999).

We are missing two targets to pin down, θ and A. To fill this gap we simply report the

results for three values of θ, namely $\bar{\theta} = 0.4$, 0.8, and 1.2, and for $A = 1.8$. The reason that we only report results for $A = 1.8$ is that the volatility of the nominal interest rate is little affected by A. Table 2 lists the identification restrictions and the identified values of the parameters. In the Appendix we detail the calibration procedure which uses these restrictions to compute the parameter values listed in Table 2.

Table 2. Calibration and parameter values

Targets	$\bar{\imath}$	$\bar{L}$	$\bar{\mu}$	$\bar{\kappa}$	$\bar{\theta}$	$\bar{A}$
	0.05	0.05	1.1	0.017	0.8	1.8
Parameters	η	ρ	φ	κ	θ	A
	0.82	4.02	1.61	0.019	0.8	1.8

The choice of θ is affecting the volatility of the nominal interest rate because θ represents the congestion externality where a higher θ means more congestion. A change in A affects mainly ρ where an increase in A decreases ρ. The intuition is that the choice of A determines the relative size of market 2 to market 3. An increase in A, ceteris paribus, makes market 3 larger. To compensate for this increase, ρ needs to decrease in order to make market 2 larger. Overall, our various experiments with different values of A had no quantitatively important effects the volatility of the nominal interest rate.

5.2 Model implication for interest rate

To derive the model's prediction for the behavior of the interest rate we need to specify processes for α and ε. To avoid negative values, let $\alpha \equiv \mathrm{e}^a$ and $\varepsilon \equiv \mathrm{e}^e$. For the non-stochastic steady state equilibrium we assume that $a = e = 0$, implying $\alpha = 1$ and $\varepsilon = 1$. We assume that α and ε follow two independent AR(1) processes with persistence parameters ρ_α (ρ_ε) and noise $\nu_{\alpha,t}$ ($\nu_{\varepsilon,t}$) which are distributed $N(0, \sigma_\alpha^2)$ $[N(0, \sigma_\varepsilon^2)]$.

We conduct two experiments. First, we assume that the economy is hit by productivity shocks only. For this purpose, we calibrate the AR(1) process for α such that the model's real gdp matches the autocorrelation and the standard deviation of real GDP in the data. This yields the following process for α_t:[21]

$$\ln \alpha_t = 0.814748 * \ln \alpha_{t-1} + \nu_{\alpha,t} \qquad \sigma_\alpha^2 = 0.0689506 \tag{31}$$

[21] The autocorrelation and the standard deviation of the AR(1) process for α depend on the values of θ and A. However, the effects are small and we therefore only report ρ_a and σ_α for $\theta = 0.8$ and $A = 1.8$.

Our second experiment is to assume that the economy is hit by demand shocks only. In this case, we calibrate the AR(1) process for ε such that the model's real gdp matches the autocorrelation and the standard deviation of real GDP in the data. This yields the following process for ε_t:[22]

$$\ln \varepsilon_t = 0.813505 * \ln \varepsilon_{t-1} + \nu_{\varepsilon,t} \qquad \sigma_\varepsilon^2 = 0.00189196. \tag{32}$$

For each experiment, we then either use (31) or (32) to generate 100,000 observations of α or ε, throwing away the first 10,000 observations. We then use the free entry condition and the central bank's first-order conditions to solve for $s_t(\alpha_t, \varepsilon_t)$ and $q_t(\alpha_t, \varepsilon_t)$.[23] The series for $s_t(\alpha_t, \varepsilon_t)$ is the simulated NBI. We then insert $s_t(\alpha_t, \varepsilon_t)$ and $q_t(\alpha_t, \varepsilon_t)$ into the model's expressions for GDP (derived in the appendix) and the interest rate:

$$GDP_t = 4\left\{(1/\alpha_t)c'\left[q_t(\alpha_t,\varepsilon_t)\right] s_t(\alpha_t,\varepsilon_t)\,\sigma\left[s_t(\alpha_t,\varepsilon_t)\right] q_t(\alpha_t,\varepsilon_t)/(1-\eta) + \hat{A}\right\} \tag{33}$$

$$i_t = \frac{\varepsilon_t\left[s_t(\alpha_t,\varepsilon_t)\,\sigma\left[s_t(\alpha_t,\varepsilon_t)\right] q_t(\alpha_t,\varepsilon_t)\right]^{-\eta}}{(1/\alpha_t)\left[\varphi + q_t(\alpha_t,\varepsilon_t)\right]^{\rho}/(1-\eta)} - 1. \tag{34}$$

This gives us 90,000 simulated observations of GDP, NBI and i. The standard deviations of the simulated data is reported in Table 3 for three values of θ.

Table 3: Results

		GDP	**NBI**	**i**
Data		**0.017**	**0.058**	**0.224**
Model α	$\theta = 0.4$	0.018	0.091	0.476
	$\theta = 0.8$	0.017	0.089	0.519
	$\theta = 1.2$	0.017	0.089	0.565
Model ε	$\theta = 0.4$	0.018	0.09	0.035
	$\theta = 0.8$	0.018	0.09	0.072
	$\theta = 1.2$	0.018	0.09	0.108

The most interesting result of Table 3 is that if productivity shocks were the sole shock, the model predicts that the nominal interest rate should be much more volatile than it is in the data – about *2.5* times more volatile (for $\theta = 0.8$). The opposite is true if demand

[22] Note also that in order to match the volatility of real GDP the standard deviation of the productivity shocks is required to be much larger than the one of the demand shock (see (31) or (32)).

[23] The two equations are found in the Appendix as equations (59) and (60).

shocks were the only ones hitting the economy – the model's nominal interest rate is about *3* times less volatile (for $\theta = 0.8$) than in the data. Note that the volatility of the nominal interest rate is increasing in θ since the congestion externality is increasing in θ (recall that at $\theta = 0$ there is no congestion externality). Finally, simulated NBI is more volatile than in the data. All these results are robust to changes in A.

The intuition for the high volatility of the nominal interest rate in response to productivity shocks and the low volatility in response to demand shocks is as follows. A productivity shock lowers marginal cost which, holding the number of firms constant, increases expected profits. Consequently, more firms attempt to enter market 2. Due to the entry externality, the central bank raises interest rates to dampen demand which reduces profits. This tends to keep aggregate output stable but requires large interest rate movements. In contrast, holding entry constant, a demand shock increases expected profits to a lesser extent since each firm must produce additional output at a higher marginal cost. Since profits increase less, entry responds to a lesser extent, and so the central bank does not need to increase the nominal interest rate as much.[24] Consequently, the central bank is much more aggressive in changing interest rates in response to productivity shocks than to demand shocks.

To check this intuition, we have changed the model so that the productivity shocks affect the entry costs too, i.e., entry costs are now counter-cyclical and given by $(1/\alpha_t)\,\kappa$. This specification does not change the calibrated parameters of the model since in the non-stochastic steady state $\alpha_t = 1$. With this specification, when productivity is high, entry costs are low, which lowers the marginal welfare cost of the entry externality, $(1/\alpha_t)\,\sigma'\left[s\left(\omega_t\right)\right]\kappa$, for any given value of $s\left(\omega_t\right)$. Hence, the central bank is more tolerant of entry and is less willing to raise interest rates to choke off profits and entry. This can be seen in Table 4 by the much higher volatility of firm entry and the lower volatility of the nominal interest rate in the row labelled **Model** $\tilde{\alpha}$ relative to the original specification labelled **Model** α.[25] This results suggest that entry and the structure of entry costs may be an important source of volatility in standard Ramsey problems when search frictions are included.

Table 4: Productivity affects entry cost

[24] Note also that with demand shocks it is optimal to have more firms to satisfy the higher demand for goods. In contrast, with productivity shocks, fewer firms are needed to produce a given quantity of output.

[25] Recall that the volatilities are reported for $\theta = 0.8$ and $A = 1.8$. For $\theta = 0.4$ the volatility of the interest rate drops even below the observed one of 0.224 to 0.198.

	GDP	**NBI**	**i**
Data	**0.017**	**0.058**	**0.224**
Model α	0.017	0.089	0.519
Model $\tilde{\alpha}$	0.018	0.535	0.425

As a robustness check, we have also calibrated the model to M_1 instead of M_0. The basic results are unchanged. The volatility of the nominal interest rate is much larger with shocks to productivity than with demand shocks. In contrast to our baseline calibration, the volatility of the interest rate is too low for both productivity and demand shocks. The interest rate volatility obtained from productivity shocks is roughly one have of the observed volatility. The one obtained with demand shocks is about 20 times smaller.

6 Conclusion

In this paper we have constructed a dynamic stochastic general equilibrium model where money is essential for trade and there is endogenous firm entry. The optimal policy involves having procyclical entry, which matches the stylized facts. If there are congestion externalities from entry, then deviating from the Friedman rule is optimal. This result appears to be consistent with the view that the Fed did not do enough in 2003-2005 to deter excessive amounts of new construction. In the absence of these externalities, implementing a version of the Friedman rule is optimal. Our quantitative exercises are enlightening since they suggest that interest rates should have been much more volatile if productivity shocks were the only source of aggregate uncertainty and much smoother if preference shocks were the main aggregate shock.

There are many extensions of this model that would be interesting to pursue. For example, how would the optimal policy be affected if repayment of loans were endogenous? In particular, does the risk of default alter stabilization? Given the events of 2007-2009, the role of default on stabilization policy appears to be an important issue. Furthermore, we have assumed that the shocks are known to the central bank. An interesting question is what is the optimal policy if the central bank has imperfect information about the nature of the aggregate shocks? How would the existence of inside money affect the equilibrium and optimal policy. For example, would inside money act as an automatic stabilizer, eliminating the need for the central bank to stabilize the economy?

Finally, the state contingent optimal policy of our model requires the central bank observe the current aggregate state. In reality, the central bank might only have imperfect knowledge about the state of the economy. It is therefore of interest to investigate whether simple rules

such as a non-state-contingent interest rate rule can get the economy close to the second best allocation. In preliminary work, we have calculated the welfare gains from the optimal stabilization rule relative to two simple rules: a constant interest rate rule and a policy that yields a constant aggregate output. The welfare loss of following a simple constant interest rate rule is only about 0.00012% of consumption. In contrast, the welfare loss of a policy that yields a constant aggregate output is about 0.55% of consumption. We are planning to explore the welfare implications of optimal stabilization policy in future research.

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Appendix

First-best allocation The planner chooses the allocation

$$A_t \equiv \left[q_t^b(j)_{j\in\mathcal{B}}, q_t(j)_{j\in\tilde{\mathcal{S}}}, x_t(j)_{j\in\mathcal{H}\cup\mathcal{F}}, y_t(j)_{j\in\mathcal{H}\cup\mathcal{F}}\right]$$

and the measures of sellers s_t for each period. The quantities $q_t^b(j)_{j\in\mathcal{B}}$ are the consumption quantities of all households with $e=\varepsilon$ (the buyers) and $q_t(j)_{j\in\tilde{\mathcal{S}}}$ are the production quantities of all firms that pay the entry cost and are able to enter market 2, while $x_t(j)_{j\in\mathcal{H}\cup\mathcal{F}}$ are the consumption quantities of all household and all firms in market 3 and $y_t(j)_{j\in\mathcal{H}\cup\mathcal{F}}$ are the production quantities of all households and firms in market 3. The planner is constrained that the allocation has to be feasible. In the second and third markets, respectively, for each state $\omega\in\Omega$ and each date t, this requires that

$$\int_{\mathcal{B}} q_t^b(j)\,dj \leq \int_{\tilde{\mathcal{S}}} q_t(j)\,dj \tag{35}$$

$$\int_{\mathcal{H}\cup\mathcal{F}} x_t(j)\,dj \leq \int_{\mathcal{H}\cup\mathcal{F}} y_t(j)\,dj. \tag{36}$$

An efficient allocation is defined as paths for $q_t^b(j)_{j\in\mathcal{B}}, q_t(j)_{j\in\tilde{\mathcal{S}}}, x_t(j)_{j\in\mathcal{H}\cup\mathcal{F}}, y_t(j)_{j\in\mathcal{H}\cup\mathcal{F}}$, and s_t that maximize (3) subject to (35) and(36) and an initial aggregate state ω_0. One can easily show that it is optimal to treat all agents of the same type equally. Moreover, using (35) it is straightforward to show that the planner allocation yields

$$\int_{\Phi}\int_{\mathcal{H}\cup\mathcal{F}} [\upsilon(x_t)-y_t]\,dF(\omega_t|\,\Omega_{t-1}) = \int_{\mathcal{H}\cup\mathcal{F}} \upsilon(x^*)-x^*$$

which is not state contingent so we can ignore this term in (3). Accordingly, the Lagrangian of the planner problem is

$$\begin{aligned} L_p &= \int_{\Phi}\left\{\varepsilon_t u\left(q_t^b\right)-\sigma(s_t)\,s_t c(q_t)/\alpha_t\right\} dF(\omega_t|\,\Omega_{t-1}) \\ &\quad -\int_{\Phi} s_t\kappa dF(\omega_t|\,\Omega_{t-1}) + \mu_t\left[\sigma(s_t)\,s_t q_t - n_t q_t^b\right] \end{aligned}$$

The FOCs for this problem after simplifications are

$$\begin{aligned}
0 &= \varepsilon_t u'\left(q_t^b\right) - \mu_t \\
0 &= \mu_t - c\left(q_t\right)/\alpha_t \\
0 &= \sigma\left(s_t\right)\left(1+\Sigma^\sigma\right)\left[c'\left(q_t\right)q_t - c\left(q_t\right)\right]/\alpha_t - \kappa \\
0 &= \sigma\left(s_t\right)s_t q_t - q_t^b
\end{aligned}$$

for all t. It is clear from these FOC that the optimal allocation is independent of Ω_{t-1} and stationary for all $\omega \in \Phi$, implying that, for a given state ω, $x_t = x(\omega)$, $y_t = y(\omega)$, $q_t^b = q^b(\omega)$, $q_t = q(\omega)$ for all t. Furthermore, an interior solution for s_t requires $-\Sigma^\sigma < 1$, which we have assumed.

Define $k[s(\omega)] \equiv \kappa/\sigma[s(\omega)]$ with $k'(s) > 0$. To prove existence and uniqueness of the first-best allocation, we can rearrange (4)-(5) as follows

$$\frac{\varepsilon u'\left\{\sigma\left[s(\omega)\right]s(\omega)\,q(\omega)\right\}}{c'\left[q(\omega)\right]/\alpha} - 1 = 0 \tag{37}$$

$$k\left[s(\omega)\right] - \left(1+\Sigma^\sigma\right)\left\{c'\left[q(\omega)\right]q(\omega) - c\left[q(\omega)\right]\right\}/\alpha = 0 \tag{38}$$

(37) is strictly decreasing function in $[s(\omega), q(\omega)]$ space. It approaches infinity as $s(\omega)$ approaches zero and approaches zero as $s(\omega)$ goes to infinity. If $k'[s(\omega)] > 0$, (38) is strictly increasing in $[s(\omega), q(\omega)]$ space. Morover, at $s(\omega) = 0$ there exists a finite $q(\omega) > 0$ that solves

$$k\left[s(\omega)\right] - \left(1+\Sigma^\sigma\right)(1/\alpha)\left\{c'\left[q(\omega)\right]q(\omega) - c\left[q(\omega)\right]\right\} = 0.$$

Hence, a unique solution $[s^*(\omega), q^*(\omega)]$ exists. If $\sigma[s(\omega)] = \sigma$, (38) is independent of $s(\omega)$ implying that for $\kappa < +\infty$, a unique solution $[s^*(\omega), q^*(\omega)]$ exists.

Proof of Propositions 1. The proof involves three steps. We first derive the solution to the central bank problem. We then demonstrate that the solution satisfies $i(\omega_t) \geq 0$. Finally, we show that there exists a transfer scheme $\tau_1(\Omega)$ and $\tau_3(\Omega)$ that implements the central bank allocation for each Ω and satisfies (23).

First step. The central bank allocation has to satisfy two constraints. The first constraint is the entry condition (18), which holds in each state. The second constraint is the pricing equation (25), which also holds in each state. We can use (18) to eliminate $p_t(\Omega_t)\phi_t(\Omega_t)$ from (25) to get

$$k(s) = \left[c'\left(q_t\right)q_t - c\left(q_t\right)\right]/\alpha_t. \tag{39}$$

The central bank then maximizes (3) subject to (39). Using (36) it is straightforward to show that the optimal policy yields

$$\int_{\Phi}\int_{\mathcal{H}\cup\mathcal{F}}\left[\upsilon\left(x_t\right)-y_t\right]dF\left(\omega_t\middle|\,\Omega_{t-1}\right)=\int_{\mathcal{H}\cup\mathcal{F}}\upsilon\left(x^*\right)-x^*$$

which is not state contingent or dependent on monetary policy so we can ignore this term in (3). Consequently, using (24), the central bank's problem reduces to

$$\begin{aligned}\mathcal{V} &= \sum_{t=0}^{\infty}\beta^t\int_{\Phi}\left\{\varepsilon_t u\left(q_t^b\right)-\sigma\left(s_t\right)s_t c\left(q_t\right)/\alpha_t\right\}dF\left(\omega_t\middle|\,\Omega_{t-1}\right)\\ &\quad-\sum_{t=0}^{\infty}\beta^t\int_{\Phi}\sigma\left(s_t\right)s_t k\left(s_t\right)dF\left(\omega_t\middle|\,\Omega_{t-1}\right)\end{aligned}$$

The Lagrangian is

$$\begin{aligned}L_c &= \sum_{t=0}^{\infty}\beta^t\int_{\Phi}\left\{\varepsilon_t u\left(q_t^b\right)-\sigma\left(s_t\right)s_t c\left(q_t\right)/\alpha_t\right\}dF\left(\omega_t\middle|\,\Omega_{t-1}\right)\\ &- \sum_{t=0}^{\infty}\beta^t\int_{\Phi}\sigma\left(s_t\right)s_t k\left(s_t\right)dF\left(\omega_t\middle|\,\Omega_{t-1}\right)\\ &+ \sum_{t=0}^{\infty}\beta^t\int_{\Phi}\mu_t\left[\sigma\left(s_t\right)s_t q_t-q_t^b\right]dF\left(\omega_t\middle|\,\Omega_{t-1}\right)\\ &+ \sum_{t=0}^{\infty}\beta^t\int_{\Phi}\hat{\lambda}_t\left\{k\left(s_t\right)-\left[c'\left(q_t\right)q_t-c\left(q_t\right)\right]/\alpha_t\right\}dF\left(\omega_t\middle|\,\Omega_{t-1}\right)\end{aligned}$$

where μ_t and $\hat{\lambda}_t\equiv s_t\sigma\left(s_t\right)\lambda_t$ is the time t Lagrangian multiplier for state ω_t. Then for all t the central bank's allocation satisfies

$$0 = \varepsilon_t u'\left(q_t^b\right)-\left(1+\lambda_t R^c\right)c'\left(q_t\right)/\alpha_t \tag{40}$$

$$0 = \varepsilon_t u'\left(q_t^b\right)q_t-c\left(q_t\right)/\alpha_t-k\left(s_t\right)-\left(1+\Sigma^{\sigma}\right)^{-1}\left(1-\lambda_t\right)s_t k'\left(s_t\right) \tag{41}$$

$$0 = k\left(s_t\right)-\left[c'\left(q_t\right)q_t-c\left(q_t\right)\right]/\alpha_t \tag{42}$$

$$0 = q_t^b-\sigma\left(s_t\right)s_t q_t \tag{43}$$

where (40) and (41) are the first-order conditions for q_t and s_t respectively. Note that there are no terms involving past or future values, in (40)-(42) so the allocation is stationary. Hence, as with the planner, the central bank faces no intertemporal trade-offs and so for each aggregate state ω we have $q_t=q\left(\omega\right)$ and $s_t=s\left(\omega\right)$. For notational convenience we now

drop the dependence of q and s on ω with the understanding that they are state dependent.

Use (40) and (42) to write (41) as follows

$$\lambda = \frac{sk'(s)}{R^c(1+\Sigma^\sigma)c'(q)q/\alpha + sk'(s)} \tag{44}$$

Note that $\lambda < 1$. Using (44) to replace λ in (40), q and s satisfy

$$\frac{\varepsilon u'[\sigma(s)sq]}{(1/\alpha)c'(q)} - 1 = \frac{R^c sk'(s)}{R^c(1+\Sigma^\sigma)(1/\alpha)c'(q)q + sk'(s)} \tag{45}$$

$$k(s) = [c'(q)q - c(q)]/\alpha. \tag{46}$$

In (s,q) space, (45) approaches infinity as s approaches zero and it approaches zero as s goes to infinity. If $k'(s) > 0$, (46) is strictly increasing and $q \geq 0$ solves $k - [c'(q)q - c(q)]/\alpha = 0$ at $s = 0$. Hence, a solution (s,q) exists. If $k'(s) = 0$, then $k(s) = k$, (46) is independent of s implying that for $k < +\infty$, a solution (s,q) exists. If (45) is strictly decreasing in (q,s) space, the equilibrium is unique.

Comparing these two expressions to the first-best allocation (37)-(38), it is straightforward to show that $q < q^*$.

We now prove that $s \geq s^*$. Suppose that the CB is constraint to implement s^*. Then, q solves

$$k(s^*) - [c'(q)q - c(q)]/\alpha = 0. \tag{47}$$

Let q^c denote the value of q that solves (47) and let $s^c \equiv s^*$. From (38), it is clear that $q^c < q^*$. Now let the CB choose q and s. Assume - contrary to the claim in the Proposition - that the optimal allocation satisfies $s < s^c \equiv s^*$. Then, from (47), $q < q^c$. It is evident that the allocation (q,s) has lower welfare than (q^c, s^c) since $q < q^c < q^*$ and $s < s^c = s^*$. Thus, in any competitive equilibrium $s \geq s^*$.

Second step. From (26) we have $1 + i = \frac{\varepsilon u'[\sigma(s)sq]}{c'(q)/\alpha}$. Hence, using (45) yields

$$i = \frac{R^c sk'(s)}{R^c(1+\Sigma^\sigma)c'(q)q/\alpha + sk'(s)}$$

Consequently, if $k'(s) > 0$, $i > 0$ in all states. If $k'(s) = 0$, $i(\omega) = 0$ in all states. Note that if $sk'(s)/k(s) = \chi$ and $qc'(q)/c(q) = \upsilon > 1$ are constants, then $R^c = \upsilon - 1$ and the free entry condition reduces to $k(s) = (\upsilon - 1)c(q)/\alpha$. It then follows that i is constant and given by

$$i = \frac{\chi(\upsilon - 1)}{(1+\Sigma^\sigma)\upsilon + \chi}.$$

Third step. We now show that a set of transfers $\tau_1(\Omega)$ and $\tau_3(\Omega)$ exists that implement the CB allocation and satisfy (23). Using (24) and (25), we can write (23) as follows:

$$1 = \beta \int_{\Phi} \left\{ \frac{\alpha \varepsilon u' \left\{ \sigma \left[s(\omega) \right] s(\omega) q(\omega) \right\}}{\gamma(\Omega) c' \left[q(\omega) \right]} \right\} dF(\omega | \Omega_{t-1}). \tag{48}$$

We first consider the case $k'(s) = 0$. In this case, (48) reduces to

$$1 = \beta \int_{\Phi} \frac{1}{\gamma(\Omega)} dF(\omega | \Omega_{t-1}) = \beta \int_{\Phi} \frac{1}{1 + \tau_1(\Omega) + \tau_3(\Omega)} dF(\omega | \Omega_{t-1}).$$

It is clear that any set of transfers $\tau_1(\Omega)$ and $\tau_3(\Omega)$ that satisfies $\gamma(\Omega) = 1 + \tau_1(\Omega) + \tau_3(\Omega) = \beta$ for all Ω implements the central bank allocation.

Consider next the case $k'(s) > 0$. Assume that the transfers are such that the agents have just enough money to buy $q^b(\omega) = \sigma[s(\omega)] s(\omega) q(\omega)$ in each state, i.e. $p(\Omega) q^b(\omega) = M_{-1}[1 + \tau_1(\Omega)]$. From the pricing equation (25) we can write this expression as follows

$$q^b(\omega) c' \left[q(\omega) \right] / \alpha = \phi(\Omega) M_{-1} \left[1 + \tau_1(\Omega) \right]. \tag{49}$$

Let $z \equiv \phi(\Omega) M(\Omega) = \phi(\Omega) M_{-1} [1 + \tau_1(\Omega) + \tau_3(\Omega)]$. Using (49) we get

$$z = q^b(\omega) c' \left[q(\omega) \right] / \alpha + \phi(\Omega) M_{-1} \tau_3(\Omega). \tag{50}$$

We have one degree of freedom for the choice of $\tau_3(\Omega)$. Assume the central bank conditions the transfers on the t and $t-1$ shocks for any Ω_{-2}. We then have $\tau_3(\Omega) = \tau_3(\omega, \omega_{-1}, \Omega_{-2})$. Consider the state $\omega_L = (\alpha_L, \varepsilon_L)$ and set $\tau_3(\omega_L, \omega_L, \Omega_{-2}) = 0$. This pins down the real stock of money $z = q^b(\omega_L) c' [q(\omega_L)] / \alpha_L$. This implies $\gamma(\omega_L, \omega_L, \Omega_{-2}) = 1 + \tau_1(\omega_L, \omega_L, \Omega_{-2})$ which remains to be determined. Return to this later. Now, using the value of z in (50) for $\omega_{-1} = \omega_L$ yields

$$\tau_3(\omega, \omega_L, \Omega_{-2}) = \left[1 + \tau_1(\omega, \omega_L, \Omega_{-2}) \right] \left\{ \tfrac{q^b(\omega_L) c'[q(\omega_L)]/\alpha_L}{q^b(\omega) c'[q(\omega)]/\alpha} - 1 \right\}$$

which gives us the realized money growth rate

$$\gamma(\omega, \omega_L, \Omega_{-2}) = \left[1 + \tau_1(\omega, \omega_L, \Omega_{-2}) \right] \tfrac{q^b(\omega_L) c'[q(\omega_L)]/\alpha_L}{q^b(\omega) c'[q(\omega)]/\alpha}$$

and from the money demand equation we get

$$1 = \beta \int_{\Phi} \tfrac{\varepsilon u' \left[q^b(\omega) \right] q^b(\omega)}{q^b(\omega_L) c'[q(\omega_L)]/\alpha_L} \tfrac{f(\omega | \omega_L, \Omega_{-2})}{1 + \tau_1(\omega, \omega_L)} d\omega. \tag{51}$$

This equation imposes a restriction on the choice of the vector $\{\tau_1(\omega,\omega_L)\}_{\omega\in\Phi}$. One such vector choice is $\tau_1(\omega,\omega_L)=\tau_1$ for all ω. This pins down $\tau_1(\omega_L,\omega_L)$ and requires

$$\tau_1=\beta\int_{\Phi}\frac{\varepsilon u'\left[q^b(\omega)\right]q^b(\omega)}{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}f\left(\omega\middle|\,\omega_L,\Omega_{-2}\right)d\omega-1$$

Now consider an arbitrary ω_{-1}. Again we obtain

$$\tau_3(\omega,\omega_{-1},\Omega_{-2})=\left[1+\tau_1(\omega,\omega_{-1},\Omega_{-2})\right]\left\{\frac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}-1\right\}$$

which gives us the following

$$\gamma(\omega,\omega_{-1},\Omega_{-2})=\left[1+\tau_1(\omega,\omega_{-1},\Omega_{-2})\right]\frac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}$$

and the money demand equation is

$$1=\beta\int_{\Phi}\left\{\frac{\varepsilon u'\left[q^b(\omega)\right]q^b(\omega)}{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}\right\}\frac{f(\omega|\omega_{-1},\Omega_{-2})}{1+\tau_1(\omega,\omega_{-1})}d\omega. \tag{52}$$

Thus, for both (51) and (52) to hold as it does for ω_L we must have

$$1+\tau_1(\omega,\omega_{-1},\Omega_{-2})=\frac{f\left(\omega\middle|\,\omega_{-1},\Omega_{-2}\right)}{f\left(\omega\middle|\,\omega_L,\Omega_{-2}\right)}\left[1+\tau_1(\omega,\omega_L,\Omega_{-2})\right]\quad\forall\omega,\omega_{-1}$$

This pins down every transfer as a function of $\tau_1(\omega,\omega_L,\Omega_{-2})$. Thus, for $\tau_1(\omega,\omega_L,\Omega_{-2})=\tau_1$ we have the transfer scheme

$$\begin{aligned}1+\tau_1(\Omega)&=\frac{f\left(\omega\middle|\,\omega_{-1},\Omega_{-2}\right)}{f\left(\omega\middle|\,\omega_L,\Omega_{-2}\right)}(1+\tau_1)\\ \tau_3(\Omega)&=(1+\tau_1)\frac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\left\{\frac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}-1\right\}\end{aligned}$$

and stochastic inflation rate

$$\gamma(\Omega)=(1+\tau_1)\frac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\frac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}.$$

The remaining endogenous variables are then

$$\phi(\Omega)=\frac{q^b(\omega_L)c'[q(\omega)]/\alpha}{M_{-1}(1+\tau_1)}\frac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\qquad p(\Omega)=M_{-1}\frac{(1+\tau_1)f(\omega|\omega_{-1},\Omega_{-2})}{q^b(\omega)f(\omega|\omega_L,\Omega_{-2})}.$$

■

Proof of Propositions 2. The proof follows along the lines of the proof of Proposition 1.

First step. The central bank allocation has to satisfy two constraints. The first constraint is the entry condition (18), which holds in each state. The second constraint is the pricing equation (27), which also holds in each state. We can use (18) to eliminate $p\phi$ from (27) to get

$$k(s_t) = \left[c'(q_t)\, q_t (1-R^u)^{-1} - c(q_t)\right]/\alpha_t. \tag{53}$$

Notice the appearance of the markup $(1-R^u)^{-1}$, which is absent from (39).

The optimal allocation solves

$$\begin{aligned}
0 &= \frac{\varepsilon_t u'\left(q_t^b\right)}{(1/\alpha_t)\, c'\left[q_t(\omega_t)\right]} - 1 - \lambda_t \frac{R^c + R^u}{1-R^u} && (54)\\
0 &= \varepsilon_t u'\left(q_t^b\right) q_t - c(q_t)/\alpha_t - k(s_t) - (1-\lambda_t)(1+\Sigma^\sigma)^{-1} s_t k'(s_t) && (55)\\
0 &= k(s_t) - \left[c'(q_t)\, q_t (1-R^u)^{-1} - c(q)\right]/\alpha_t && (56)\\
0 &= q_t^b - s_t \sigma(s_t)\, q_t && (57)
\end{aligned}$$

Note that for $R^u = 0$ (54) - (56) and (40)-(42) are identical. Again, because there are no terms involving past or future values, the solution to (54)-(56) is independent of t and Ω so it is therefore stationary. Use (54) and (56) to write (55) as follows

$$\lambda = \frac{R^u (1+\Sigma^\sigma)\, c'(q_t)\, q_t/\alpha_t + (1-R^u)\, s_t k'(s_t)}{(R^c + R^u)(1+\Sigma^\sigma)\, c'(q_t)\, q_t/\alpha_t + (1-R^u)\, s_t k'(s_t)}. \tag{58}$$

Use (58) to replace λ in (54). Then, q and s solve

$$\frac{\varepsilon u'\left\{\left[s\sigma(s)/n\right] q\right\}}{c'(q)/\alpha} - 1 = \left(\tfrac{R^c+R^u}{1-R^u}\right)\left[\tfrac{R^u(1+\Sigma^\sigma)c'(q_t)q_t/\alpha_t+(1-R^u)s_t k'(s_t)}{(R^c+R^u)(1+\Sigma^\sigma)c'(q_t)q_t/\alpha_t+(1-R^u)s_t k'(s_t)}\right] \tag{59}$$

$$k(s) - \left[c'(q)\, q (1-R^u)^{-1} - c(q)\right]/\alpha = 0. \tag{60}$$

Comparing these two expressions to the first-best allocation (37)-(38), it is straightforward to show that $q < q^*$. To establish that $s \geq s^*$ we can replicate the same proof as in the case of competitive pricing above.

Second step. Since (41) and (55) are identical and $\lambda < 1$, we can replicate the proof of step 2 of Proposition 2 one for one.

Third step. We now show that a set of transfers $\tau_1(\Omega)$ and $\tau_3(\Omega)$ exists that implement the CB allocation and satisfy (23). Using (24) and (27), we can write (23) as follows:

$$1 = \beta(1-R^u)\int_\Phi \frac{\varepsilon u'\left\{\sigma\left[s(\omega)\right] s(\omega)\, q(\omega)\right\}}{\gamma(\Omega)(1/\alpha)\, c'\left[q(\omega)\right]} dF\left(\omega|\,\Omega^{-1}\right) \tag{61}$$

We first consider the case $k'(s) = 0$. Since $(1 - R^u)\,\varepsilon u'\{\sigma[s(\omega)]\,s(\omega)\,q(\omega)\} = (1/\alpha)\,c'[q(\omega)]$ under the central bank's allocation, (61) reduces to

$$1 = \beta \int_{\Phi} \frac{1}{\gamma(\Omega)} dF(\omega_t | \Omega_{t-1}) = \beta \int_{\Phi} \frac{1}{1 + \tau_1(\Omega) + \tau_3(\Omega)} dF(\omega_t | \Omega_{t-1}).$$

It is clear that any set of transfers $\tau_1(\Omega)$ and $\tau_3(\Omega)$ that satisfies $\gamma(\Omega) = 1 + \tau_1(\Omega) + \tau_3(\Omega) = \beta$ for all $\omega = \Phi$ implements the central bank allocation.

Consider next the case $k'(s) > 0$. Assume that the transfers are such that the agents have just enough money to buy $q^b(\omega)$ in each state, i.e. $p(\Omega)\,q^b(\omega) = M_{-1}[1 + \tau_1(\Omega)]$. From the pricing equation (27), we can write this expression as follows

$$(1 - R^u)^{-1} q^b(\omega)\,c'[q(\omega)]/\alpha = \phi(\Omega)\,M_{-1}[1 + \tau_1(\Omega)]. \tag{62}$$

Since $z = \phi(\Omega)\,M_{-1}[1 + \tau_1(\Omega) + \tau_3(\Omega)]$, using (49) we get (50). As before, assume the central bank mainly conditions the money growth rate on the shocks at t and $t-1$. Consider the state $\omega_L = (\alpha_L, \varepsilon_L)$ and set $\tau_3(\omega_L, \omega_L) = 0$. Thus, $z = (1 - R^u)^{-1} q^b(\omega_L)\,c'[q(\omega_L)]/\alpha_L$ is the real stock of money. We then have

$$(1 - R^u)^{-1} q^b(\omega_L)\,c'[q(\omega_L)]/\alpha_L = \phi(\Omega)\,M_{-1}[1 + \tau_1(\Omega) + \tau_3(\Omega)].$$

Using this expression and (62) we can obtain $\tau_3(\omega, \omega_L)$ as a function of $\tau_1(\omega, \omega_L)$

$$\tau_3(\omega, \omega_L, \Omega_{-2}) = [1 + \tau_1(\omega, \omega_L, \Omega_{-2})] \left\{ \frac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha} - 1 \right\}. \tag{63}$$

The realized money growth satisfies

$$\gamma(\omega, \omega_L, \Omega_{-2}) = [1 + \tau_1(\omega, \omega_L, \Omega_{-2})] \frac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha} \tag{64}$$

Then, replace $\gamma(\omega, \omega_L, \Omega_{-2})$ in (48) to get

$$1 = \beta(1 - R^u) \int_{\Phi} \frac{q^b(\omega)\varepsilon u'\left[q^b(\omega)\right]}{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L} \frac{f(\omega|\omega_L, \Omega_{-2})}{1 + \tau_1(\omega, \omega_L, \Omega_{-2})} d\omega \tag{65}$$

This equation imposes a restriction on the vector $\{\tau_1(\omega, \omega_L, \Omega_{-2})\}_{\omega \in \Phi}$. However, there are many choices that are consistent with this equation. One particular choice is $\tau_1(\omega, \omega_L, \Omega_{-2}) =$

τ_1. In which case we have the transfer scheme

$$\begin{aligned}\tau_1 &= \beta\left(1-R^u\right)\int_\Phi \tfrac{q^b(\omega)\varepsilon u'\left[q^b(\omega)\right]}{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L} f\left(\omega|\,\omega_L,\Omega_{-2}\right)d\omega-1,\\ \tau_3\left(\omega,\omega_L,\Omega_{-2}\right) &= \left(1+\tau_1\right)\left\{\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}-1\right\}.\end{aligned}$$

Now pick an arbitrary state ω_{-1}. Once again we obtain

$$\begin{aligned}\tau_3\left(\omega,\omega_{-1},\Omega_{-2}\right) &= \left[1+\tau_1\left(\omega,\omega_{-1},\Omega_{-2}\right)\right]\left\{\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}-1\right\}\\ \gamma\left(\omega,\omega_{-1},\Omega_{-2}\right) &= \left[1+\tau_1\left(\omega,\omega_{-1},\Omega_{-2}\right)\right]\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}\end{aligned}$$

and the money demand equation

$$1=\beta\left(1-R^u\right)\int_\Phi \tfrac{q^b(\omega)\varepsilon u'\left[q^b(\omega)\right]}{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}\tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{1+\tau_1(\omega,\omega_{-1},\Omega_{-2})}d\omega \tag{66}$$

Again for the money demand equations (65) and (66) to hold we must have

$$1+\tau_1\left(\omega,\omega_{-1},\Omega_{-2}\right)=\left(1+\tau_1\right)\frac{f\left(\omega|\,\omega_{-1},\Omega_{-2}\right)}{f\left(\omega|\,\omega_L,\Omega_{-2}\right)}$$

which implies the inflation rate is given by

$$\gamma\left(\Omega\right)=\left(1+\tau_1\right)\tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}$$

and the price of money and the price of goods are stochastic and satisfy

$$\phi\left(\Omega\right)=\tfrac{f(\omega|\omega_L,\Omega_{-2})q^b(\omega)c'[q(\omega)]/\alpha}{f(\omega|\omega_{-1},\Omega_{-2})(1-R^u)M_{-1}(1+\tau_1)},\quad p\left(\Omega\right)=M_{-1}\tfrac{(1+\tau_1)f(\omega|\omega_{-1},\Omega_{-2})}{q^b(\omega)f(\omega|\omega_L,\Omega_{-2})}.$$

■

Proof of 3. In this proof we show that it is optimal and feasible to implement the same allocation as for state-contingent monopoly pricing. The central bank allocation has to satisfy the entry condition (18) and the pricing equation (29). Let $\hat{\phi}\left(\Omega_t\right)\equiv\phi\left(\Omega_t\right)p\left(\Omega_{t-1}\right)$, which is the relative price between market 2 and market 3 goods. Since at time t, $p\left(\Omega_{t-1}\right)$ is a predetermined variable, the central bank can affect this relative price by changing $\phi\left(\Omega_t\right)$ via policy. Then rewrite (29) as

$$\int_\Phi \sigma\left(s_t\right)s_tq_t\left[\hat{\phi}_t\left(1-R^u\right)-c'\left(q_t\right)/\alpha_t\right]dF\left(\omega_t|\,\Omega_{t-1}\right)=0.$$

The central bank now chooses q^b, q, s and $\hat{\phi}$ to maximize

$$\begin{aligned} L_p &= \sum_{t=0}^{\infty} \beta^t \int_{\Phi} \left\{ \varepsilon_t u\left(q_t^b\right) - \sigma\left(s_t\right) s_t c\left(q_t\right)/\alpha_t \right\} dF\left(\omega_t \middle| \Omega_{t-1}\right) \\ &- \sum_{t=0}^{\infty} \beta^t \int_{\Phi} s_t k\left(s_t\right) dF\left(\omega_t \middle| \Omega_{t-1}\right) \\ &+ \sum_{t=0}^{\infty} \beta^t \int_{\Phi} \mu_t \left(s_t q_t - q_t^b\right) dF\left(\omega_t \middle| \Omega_{t-1}\right) \\ &+ \sum_{t=0}^{\infty} \beta^t \int_{\Phi} \hat{\lambda}_t \left\{ k\left(s_t\right) - \left[\hat{\phi}_t q_t - c\left(q_t\right)\right]/\alpha_t \right\} dF\left(\omega_t \middle| \Omega_{t-1}\right) \\ &+ \sum_{t=0}^{\infty} \beta^t \theta_t \int_{\Phi} \sigma\left(s_t\right) s_t q_t \left[\hat{\phi}_t\left(1 - R^u\right) - c'\left(q_t\right)/\alpha_t\right] dF\left(\omega_t \middle| \Omega_{t-1}\right) \end{aligned}$$

where $\hat{\lambda}_t = \lambda_t \sigma\left(s_t\right) s_t$ is the Lagrange multiplier for (18) and θ_t is the one for (29). Since we know $s_t q_t = n_t q_t^b$ in equilibrium we can eliminate q_t^b and simply choose q_t, s_t and $\hat{\phi}_t$ for all t. The optimal allocation then solves

The first-order conditions for q_t, s_t and $\hat{\phi}_t$ reduce to

$$0 = \varepsilon_t u'\left[\left(\sigma\left(s_t\right) s_t q_t\right)\right] - c'\left(q_t\right)/\alpha_t - \lambda_t \left(\frac{R^u + R^c}{1 - R^u}\right) c'\left(q_t\right)/\alpha_t \tag{67}$$

$$\lambda_t \left[\hat{\phi}_t - \tfrac{c'(q_t)/\alpha_t}{1-R^u}\right] = \varepsilon_t u'\left[\left(\sigma\left(s_t\right) s_t q_t\right)\right] q_t - c'\left(q_t\right)/\alpha_t - k\left(s_t\right) - s_t k'\left(s_t\right)\left(1 - \lambda_t\right) \tag{68}$$

$$0 = k\left(s_t\right) - \hat{\phi}_t q_t + c\left(q_t\right)/\alpha_t. \tag{69}$$

If the central bank enacts a policy such that the relative price is given by

$$\hat{\phi}\left(\Omega\right) \equiv \phi\left(\Omega\right) p\left(\Omega_{-1}\right) = \frac{c'\left[q\left(\omega\right)\right]/\alpha}{1 - R^u} \tag{70}$$

then (67) - (69) reduce to (54) - (56). Furthermore, this choice also satisfies the pricing equation (29). Consequently, the central bank chooses the same allocation as with state-contingent monopoly pricing.

Implementation: The central bank wants to replicate the state-contingent monopoly pricing allocation. Hence, the optimal quantities come from the solution to that problem. All that remains to be determined is how to implement it with non-state contingent pricing. Consider the case $k'\left(s\right) > 0$. Assume that the transfers are such that the agents have just enough money to buy $q\left(\omega\right)$ in each state. This implies the aggregate money stock must purchase total nominal output in market 2, i.e. $M_{-1}\left[1 + \tau_1\left(\Omega\right)\right] = p\left(\Omega_{-1}\right) q^b\left(\omega\right)$. From (70)

we have

$$\phi(\Omega) M_{-1} [1 + \tau_1(\Omega)] = (1 - R^u)^{-1} q^b(\omega) c'[q(\omega)]/\alpha. \tag{71}$$

We also have $z = \phi(\Omega) M_{-1} [1 + \tau_1(\Omega) + \tau_3(\Omega)]$. Using (71) we get

$$z = (1 - R^u)^{-1} q^b(\omega) c'[q(\omega)]/\alpha + \tau_3(\Omega) \phi(\Omega) M_{-1}. \tag{72}$$

As before, assume the central bank only conditions the money growth rate on the last two shocks for any Ω_{-2}. Denote $\phi(\Omega) = \phi(\omega, \omega_{-1}, \Omega_{-2})$. Consider the state $\omega_L = (\alpha_L, \varepsilon_L)$ and set $\tau_3(\omega_L, \omega_L) = 0$. Thus from (72) we have

$$z = (1 - R^u)^{-1} q^b(\omega_L) c'[q(\omega_L)]/\alpha_L$$

This pins down the real stock of money. It then follows from the buyer's budget constraint that

$$1 + \tau_1(\omega, \omega_L, \Omega_{-2}) = [1 + \tau_1(\omega_L, \omega_L, \Omega_{-2})] \frac{q^b(\omega)}{q^b(\omega_L)}$$

In short, with $p(\Omega_{-1})$ fixed, nominal spending has to rise as $q^b(\omega)$ increases, meaning the nominal injection in market 1 must also rise regardless of what happens in market 3. We can then solve for $\tau_3(\omega, \omega_L)$ as before to obtain (63). Using the expression above in (63) yields

$$\tau_3(\omega, \omega_L, \Omega_{-2}) = [1 + \tau_1(\omega_L, \omega_L, \Omega_{-2})] \left\{ \frac{c'[q(\omega_L)]/\alpha_L}{c'[q(\omega)]/\alpha} - \frac{q^b(\omega)}{q^b(\omega_L)} \right\}$$

so

$$\gamma(\omega, \omega_L, \Omega_{-2}) = [1 + \tau_1(\omega_L, \omega_L, \Omega_{-2})] \frac{c'[q(\omega_L)]/\alpha_L}{c'[q(\omega)]/\alpha}.$$

Using this expression and (70) in (23) we obtain

$$1 = \beta(1 - R^u) \int_\Phi \frac{\varepsilon u'[q^b(\omega)]}{c'[q(\omega_L)]/\alpha_L} \frac{f(\omega|\omega_L, \Omega_{-2})}{1 + \tau_1(\omega_L, \omega_L, \Omega_{-2})} d\omega. \tag{73}$$

This places a restriction on $\tau_1(\omega_L, \omega_L, \Omega_{-2})$ given by

$$\tau_1(\omega_L, \omega_L, \Omega_{-2}) = \beta(1 - R^u) \int_\Phi \frac{\varepsilon u'[q^b(\omega)]}{c'[q(\omega_L)]/\alpha_L} f(\omega|\, \omega_L, \Omega_{-2})\, d\omega - 1.$$

This gives us all of the transfers for $\omega_{-1} = \omega_L$.

Now consider any state ω_{-1}. We get

$$\begin{aligned}\tau_3(\omega,\omega_{-1},\Omega_{-2}) &= [1+\tau_1(\omega,\omega_{-1},\Omega_{-2})]\left[\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}-1\right]\\ \gamma(\omega,\omega_{-1},\Omega_{-2}) &= [1+\tau_1(\omega,\omega_{-1},\Omega_{-2})]\,\tfrac{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}{q^b(\omega)c'[q(\omega)]/\alpha}.\end{aligned}$$

Then from (70) in (23) we get

$$1=\beta(1-R^u)\int_\Phi \tfrac{q^b(\omega)\varepsilon u'\left[q^b(\omega)\right]}{q^b(\omega_L)c'[q(\omega_L)]/\alpha_L}\tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{1+\tau_1(\omega,\omega_{-1},\Omega_{-2})}d\omega \tag{74}$$

In order for (73) and (74) to hold we must have

$$[1+\tau_1(\omega,\omega_{-1},\Omega_{-2})]=[1+\tau_1(\omega_L,\omega_L,\Omega_{-2})]\,\tfrac{q^b(\omega)f(\omega|\omega_{-1},\Omega_{-2})}{q^b(\omega_L)f(\omega|\omega_L,\Omega_{-2})}$$

This pins down $\tau_1(\omega,\omega_{-1})$ as a function of $\tau_1(\omega_L,\omega_L)$ for all ω_{-1}. Thus the implementation scheme

$$\begin{aligned}\tau_1(\omega_L,\omega_L,\Omega_{-2}) &= \beta(1-R^u)\int_\Phi \tfrac{\varepsilon u'\left[q^b(\omega)\right]}{c'[q(\omega_L)]/\alpha_L}f(\omega|\,\omega_L,\Omega_{-2})\,d\omega-1\\ \tau_1(\omega,\omega_{-1},\Omega_{-2}) &= [1+\tau_1(\omega_L,\omega_L,\Omega_{-2})]\,\tfrac{q^b(\omega)f(\omega|\omega_{-1},\Omega_{-2})}{q^b(\omega_L)f(\omega|\omega_L,\Omega_{-2})}-1\\ \tau_3(\omega,\omega_{-1},\Omega_{-2}) &= [1+\tau_1(\omega_L,\omega_L,\Omega_{-2})]\,\tfrac{f(\omega|\omega_{-1},\Omega_{-2})}{f(\omega|\omega_L,\Omega_{-2})}\left[\tfrac{c'[q(\omega_L)]/\alpha_L}{c'[q(\omega)]/\alpha}-\tfrac{q^b(\omega)}{q^b(\omega_L)}\right]\end{aligned}$$

and the subsequent inflation rates

$$\begin{aligned}\gamma(\omega,\omega_L,\Omega_{-2}) &= [1+\tau_1(\omega_L,\omega_L,\Omega_{-2})]\,\frac{c'\left[q(\omega_L)\right]/\alpha_L}{c'\left[q(\omega)\right]/\alpha} \qquad \forall\omega\\ \gamma(\Omega) &= \gamma(\omega,\omega_L,\Omega_{-2})\,\frac{f(\omega|\,\omega_{-1},\Omega_{-2})}{f(\omega|\,\omega_L,\Omega_{-2})}\end{aligned}$$

allow the CB to implement the state-contingent monopoly pricing allocation even though there is price posting. We can then solve for the equilibrium prices

$$\phi(\Omega)=\tfrac{q^b(\omega)c'[q(\omega)]/\alpha}{M_{-1}[1+\tau_1(\omega,\omega_{-1},\Omega_{-2})](1-R^u)} \qquad p(\Omega_{-1})=M_{-1}\tfrac{[1+\tau_1(\omega_L,\omega_L,\Omega_{-2})]f(\omega|\omega_{-1},\Omega_{-2})}{q^b(\omega_L)f(\omega|\omega_L,\Omega_{-2})}.$$

■

Calibration procedure Here we show how we calibrate the parameters $(A,\eta,\rho,\varphi,\kappa,\theta)$. We can calibrate η directly from the mark-up target $\bar{\mu}$ and the money demand target $\bar{L}$ as

follows. The model's mark-up is

$$\mu = \frac{(p_2 y_2)\mu_2 + (p_3 y_3)\mu_3}{p_2 y_2 + p_3 y_3}$$

where μ_j is the mark-up in market j, $p_2 y_2$ is the nominal output in market 2 and $p_3 y_3$ is nominal output in market 3. The mark-up in market 3 is $\mu_3 = 0$, and in market 2 it is $\mu_2 = (p_2/p_3) / [(1/\alpha)c'(q)]$. Then, $\mu_2 = 1/(1-\eta)$ since the seller's first-order condition is $p_2/p_3 = (1/\alpha)c'(q)/(1-\eta)$. Consequently, the mark-up satisfies

$$\mu = \frac{p_2 y_2}{p_2 y_2 + p_3 y_3} \frac{1}{1-\eta}$$

Note that in market 2 in each quarter the total stock of money is spent which implies that $M = p_2 y_2$. Then, since the model's money demand satisfies $L = M/[4(p_2 y_2 + p_3 y_3)] = p_2 y_2/[4(p_2 y_2 + p_3 y_3)]$ we have

$$\eta = 1 - \frac{4\bar{L}}{\bar{\mu}} \tag{75}$$

where we have replaced the model's money demand L with our money demand target $\bar{L}$ and μ with $\bar{\mu}$. So far we have determined the parameter η from (75). We next show how we pin down the remaining parameters $(A, \rho, \varphi, \kappa, \theta)$. We first derive an expression for money demand L. Nominal quarterly output in market 2 can be written as

$$p_2 y_2 = p_2 s\sigma(s) q = p_3 (1/\alpha) c'(q) s\sigma(s) q/(1-\eta).$$

Nominal quarterly output in market 3 is $p_3 \hat{A}$ where $\hat{A} = (2+\wp)A$ since the measure of buyers is normalized to 2, the measure of firms is $\wp$, and both consume $x^* = A$. Without loss in generality, we can pick any $\wp > s$. A different value of $\wp$ only changes A. In what follows we, therefore, are looking for the set of parameters $\left(\hat{A}, \sigma, \rho, \varphi, \kappa, \theta\right)$. Then, aggregate output $p_2 y_2 + p_3 y_3$ per quarter measured in units of market 3 goods is

$$(1/\alpha)c'(q) s\sigma(s) q/(1-\eta) + \hat{A}.$$

Finally, by setting the model's money demand equal to the money demand target we get

$$\bar{L} = \frac{(1/\alpha)s\sigma(s) q c'(q) / (1-\eta)}{4\left[s\sigma(s) q (1/\alpha) c'(q) / (1-\eta) + \hat{A}\right]}. \tag{76}$$

We next derive an expression for the cost of starting a new business as a fraction of per

capita GDP. Since the measure of households is 2, the model's cost of starting a new business as a fraction of per capita GDP is $2\kappa/\left[(1/\alpha)c'(q)\,s\sigma(s)\,q/(1-\eta)+\hat{A}\right]$, and so

$$\bar{\kappa}=\frac{2\kappa}{s\sigma(s)\,q(1/\alpha)c'(q)\,/\,(1-\eta)+\hat{A}} \tag{77}$$

where $\bar{\kappa}=0.017$ is our target.[26] To determine the endogenous quantities q and s we use the entry condition

$$s=\left\{(1/\kappa)\left[(1/\alpha)\,c'(q)\,q\,(1-\eta)^{-1}-(1/\alpha)\,c(q)\right]\right\}^{1/\theta}. \tag{78}$$

and the buyer's first-order condition

$$\frac{(1-\eta)\,\varepsilon u'\left[(s/n)\,q\right]}{(1/\alpha)\,c'(q)}=1+\bar{\imath}. \tag{79}$$

(78) and (79) can be simultaneously solved to yield the functions $\hat{q}=q(\rho,\varphi,\kappa,\theta)$ and $\hat{s}=s(\rho,\varphi,\kappa,\theta)$. Substituting $\hat{q}$ and $\hat{s}$ into the maximization problem of the central bank yields

$$\frac{\varepsilon u'[(\hat{s}/n)\hat{q}]}{(1/\alpha)c'(\hat{q})}-1=\frac{(R^c+\eta)[(1/\alpha)c'(\hat{q})\hat{q}\eta+(1-\eta)\hat{s}k'(\hat{s})]}{(1-\eta)[(1/\alpha)c'(\hat{q})\hat{q}(R^c+\eta)+(1-\eta)\hat{s}k'(\hat{s})]}, \tag{80}$$

into (76) yields

$$\bar{L}=\frac{(1/\alpha)\hat{s}\sigma(\hat{s})\,\hat{q}c'(\hat{q})}{4\left[\hat{s}\sigma(\hat{s})\,\hat{q}(1/\alpha)c'(\hat{q})+(1-\eta)\,\hat{A}\right]}, \tag{81}$$

and into (77) yields

$$\bar{\kappa}=\frac{(1-\eta)\,2\kappa}{\hat{s}\sigma(\hat{s})\,\hat{q}(1/\alpha)c'(\hat{q})+(1-\eta)\,\hat{A}} \tag{82}$$

The equations (80)-(82) can be used to solve for three of the five yet undetermined parameters $\left(\hat{A},\rho,\varphi,\kappa,\theta\right)$. In order to do so we guess values for $\hat{A}$ and θ and then solve (80)-(82) for $\rho\left(\hat{A},\theta\right)$, $\kappa\left(\hat{A},\theta\right)$ and $\varphi\left(\hat{A},\theta\right)$. We then use

$$\left[\hat{A},\eta,\rho\left(\hat{A},\theta\right),\varphi\left(\hat{A},\theta\right),\kappa\left(\hat{A},\theta\right),\theta\right]$$

to simulate the model. The Mathematica files for the calibration available by request.

[26] Note the following. First, $4\left[(1/\alpha)c'(q)\,s\sigma(s)\,q/(1-\eta)+\hat{A}\right]$ is annual GDP. The measure of households is normalized to 1 so $4\left[(1/\alpha)c'(q)\,s\sigma(s)\,q/(1-\eta)+\hat{A}\right]$ is also per capita GDP. Second, the cost of a new business κ has to be paid each quarter. Hence, the annual cost of operating a new business is $4*\kappa$ which explains why the numerator is $(1/\alpha)c'(q)\,s\sigma(s)\,q/(1-\eta)+\hat{A}$.