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Inventories, Liquidity, and the Macroeconomy*

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Abstract

It is widely believed in the literature that inventory fluctuations are destabilizing to the economy. This paper re-assesses this view by developing an analytically-tractable general-equilibrium model of inventory dynamics based on a precautionary stockout-avoidance motive. The model's predictions are broadly consistent with the U.S. business cycle and key features of inventory behavior, including (i) a large inventory stock-to-sales ratio and a small inventory investment-to-sales ratio in the long run, (ii) excess volatility of production relative to sales, (iii) procyclical inventory investment but countercyclical stock-to-sales ratio over the business cycle, and (iv) more volatile input inventories than output inventories. However, contrary to common beliefs, the model predicts that inventories are stabilizing, rather than destabilizing. The volatility of aggregate output could rise by 30% if inventories were eliminated from the economy. Key to this seemingly counter-intuitive result is that a stockout-avoidance motive leads to procyclical liquidity-value of inventories (hence, procyclical relative prices of final goods), which acts as an automatic stabilizer that discourages final sales in a boom and encourages final sales during a recession, thereby reducing the variability of GDP.

Keywords: Inventory, Liquidity, Input-and-Output Inventories, Stockout Avoidance, Countercyclical Stock-to-Sales Ratio, Business Cycle.

JEL codes: E13, E20, E32.

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1 Introduction

An important question in the business cycle literature has been whether inventories are stabilizing or destabilizing to the aggregate economy. Because of the overwhelming empirical evidence indicating that inventory investment is procyclical (consequently, production is more volatile than sales), the consensus view has been that inventory behavior is destabilizing (see, e.g., Blinder 1981, 1986, 1990). But this view may be false.

The belief that inventories are destabilizing is based essentially on a partial-equilibrium argument: by the accounting identity, output equals sales plus inventory investment; therefore, *given sales*, a positive covariance of inventory investment to sales increases the variance of output; hence, inventory behavior is destabilizing. However, in general equilibrium, the volatility of sales is endogenously determined and depends on inventory behavior through price mechanisms. Inventories provide liquidity to demand, and the liquidity value may be procyclical under demand shocks. Namely, agents pay disproportionally higher prices in case of a liquidity shortage. This provides incentives for firms to save through inventory investment instead of capital under a positive interest rate. Hence, as a precautionary saving device, inventories may reduce the volatility of sales more than they increase the volatility of production, so the variance of GDP may be lowered, rather than increased. To sort out the net effect of inventory behavior on the stability of GDP, general equilibrium analyses are essential.

This paper develops an analytically tractable general-equilibrium model of inventories with microfoundations. Inventories exist in the model because of a precautionary stockout-avoidance motive. Under aggregate demand shocks, the model is broadly consistent with the stylized facts of inventory behavior, including: (i) a large stock-to-sales ratio and a small inventory investment-to-GDP ratio in the steady state, (ii) excess volatility of production relative to sales, (iii) more volatile input inventories than output inventories, (iv) procyclical inventory investment but countercyclical inventory-to-sales ratio at the business cycle frequencies, and (v) countercyclical inventory investment at the high frequencies for final consumption goods.

The model is used as a laboratory to assess the contributions of inventory fluctuations to output volatility by counter-factual experiments. It is found that the existence of inventories *reduces* the variance of aggregate output. For example, when the model is calibrated to

match the inventory-to-sales ratio of the U.S. economy, eliminating inventories from the model could increase the variance of output by as much as 30%. This surprising result contradicts the widely held belief that inventories are destabilizing.

Literature, Stylized Facts, and Outline of this Paper. For the postwar period, the stock of finished goods inventories is about 60% of GDP and 90% of aggregate consumption on average. According to National Income and Product Accounts (NIPA), private inventories are three times larger than final sales of domestic business. The change of inventories is extremely volatile and procyclical, making it potentially the single largest contributor to the business cycle. For example, aggregate inventory investment is about 20 times more volatile than GDP and can account for up to 87% of the drop in GNP during the average postwar recession (Blinder and Maccini, 1991). In addition, inventory behavior is so intriguing not only for its magnitude and scale of fluctuations, but also for its paradoxical features. For example, finished goods inventories are procyclical only at the business cycle frequencies, but countercyclical at higher frequencies (Hornstein, 1998; and Wen, 2005a); and despite the large inventory-to-sales ratio, the change of inventory stocks (inventory investment) accounts for less than 1% of GDP on average, suggesting a remarkably low demand for inventory replenishment.

The economy accumulates not only inventories of finished goods, but also inventories of intermediate goods (including raw materials and work-in-process). Intermediate goods inventories behave similarly to finished goods inventories over the business cycle, except they are larger in volume and more volatile. In the manufacturing sector, for example, the average inventory-to-sales ratio for intermediate goods is two times larger than that of finished goods, and input inventory investment can be three times more volatile (Humphreys, Maccini, and Schuh, 2001).[1] Input inventories arise whenever the delivery and usage of input materials differ. Because they provide the linchpin across stages of fabrication and between upstream and downstream firms in the chain of the production process, the dynamic interaction between input and output inventories is emphasized by Humphreys, Maccini, and Schuh (2001) as playing an important role in propagating the business cycle.

Although inventory investment is extremely volatile and strongly procyclical over the business cycle, the ratio of inventory stock to sales is countercyclical. This is puzzling because it suggests that inventory stocks behave sluggishly and fail to keep up with sales in spite of the excess volatility of production over sales. Bils and Kahn (2000) stress the importance of the

[1] Also see Feldstein and Auerbach (1976).

countercyclical inventory-to-sales ratio in understanding the business cycle. According to Bils and Kahn (2000), the countercyclical inventory-to-sales ratio reflects procyclical marginal costs and countercyclical markups – which prevent production from keeping track of sales in booms.

Despite the importance of inventories in economic activities and their potential for understanding the business cycle, general-equilibrium analysis of inventories has been surprisingly rare.[2] The bulk of the inventory literature uses partial-equilibrium models to analyze inventory behavior, and, in the analyses, interactions between input and output inventories are often neglected (Humphreys, Maccini, and Schuh, 2001).[3] Although this literature has improved our knowledge of inventory behavior significantly, partial-equilibrium analysis is not fully satisfactory for addressing certain type of questions because it treats prices, marginal costs, and sales as exogenous. Such a practice fails to take into account the dynamic interactions between supply and demand and the impact of inventories on sales and prices. Consequently, partial-equilibrium analysis may give misleading messages for stabilization policies. There have been attempts in the literature to include inventories in general-equilibrium models; however, this line of general-equilibrium research relies on reduced-form analysis rather than on the microfoundations of inventory behavior. For example, inventories are treated as a factor of production (equivalent to fixed capital) by Kydland and Prescott (1982) and Christiano (1988), whereas they are treated as a source of household utility (equivalent to durable consumption goods) by Kahn, McConnell and Perez-Quiros (2002).[4] In such reduced-form inventory models, the crucial question why firms hold inventories is sidestepped and it is not clear how to appropriately evaluate the importance of inventories for the business cycle in such models, because inventories are by definition essential to the economy.

Inventory investment (as a form of aggregate savings) has a negative real rate of return (e.g., due to storage costs and depreciation) and is thus dominated by capital investment in portfolio choice. Hence, to induce firms to hold inventories in general equilibrium requires frictions that give inventory investment a positive rate of return. In reality, one of the most important and obvious benefits for carrying inventories is liquidity. Output inventories are more liquid in facilitating sales than inputs, and input inventories are more liquid in

[2] Recent exceptions include Fisher and Hornstein (2000), Kahn and Thomas (2007a, 2007b), Kryvtsov and Midrigan (2008), and Wen (2005b).

[3] Important empirical works based on partial-equilibrium analysis include Blanchard (1983), Blinder (1986), Coen-Pirani (2004), Eichenbaum (1989), Haltiwanger and Maccini (1988), Kahn (1992), Ramey (1991), Ramey and West (1999), Wen (2005a), and West (1986), among many others.

[4] The same reduced-form modeling strategies are also taken by Iacoviello, Schiantarelli, and Schuh (2007) in a recent attempt to explain input-and-output inventory dynamics in general equilibrium.

facilitating production than new orders.[5] The existing literature emphasizes two types of frictions to induce inventory holdings as a form of liquidity demand: fixed-cost friction and timing (or information) friction. The traditional (S,s) model of inventories stresses the cost friction.[6] According to the (S,s) theory, firms hold inventories because they face fixed costs of ordering inputs. To economize on these costs, firms choose to order infrequently by carrying inventories. General-equilibrium models based on the (S,s) inventory policy have been developed recently by Fisher and Hornstein (2000) and Khan and Thomas (2007a). This literature shows that the (S,s) inventory theory has the potential to explain aggregate inventory dynamics.

This paper focuses on the timing/information friction. Namely, inventories exist because of a precautionary motive to avoid possible stockouts when stores/firms face demand uncertainty and delivery/production lags (Kahn, 1987).[7] The empirical relevance of the stockout-avoidance motive to understanding inventory behavior and its relation to the business cycle has been re-emphasized recently by Bils (2004), Bils and Kahn (2000), and Coen-Pirani (2004).[8] My strategy is to embed the partial-equilibrium model of Kahn (1987) and Bils and Kahn (2000) into a standard, perfectly competitive, RBC model. Under aggregate demand or supply shocks, the general-equilibrium model is broadly consistent with the stylized facts of inventory behavior aforementioned above.

The main intuition behind the success of the model is as follows. To prevent stockouts, firms produce to meet an optimal target-inventory stock based on the distribution of idiosyncratic demand shocks. Production then moves more than one-for-one with sales so as to replenish inventories on the one hand and prevent anticipated future stockouts on the other hand. This results in procyclical inventory investment. In general equilibrium, the optimal target itself is a decreasing function of the marginal cost of production because the rate of return to liquidity (i.e., inventory investment) depends positively on the endogenous probability of stockouts. A higher marginal cost calls for a higher rate of return to liquidity and a higher probability of stockout. Hence, even with perfect competition and zero markups,

[5] As such, the challenge for modeling inventories in general equilibrium is similar to that of modeling money, which is dominated by interest-bearing assets in the rate of return. This is why the classic money-demand models are closely linked to inventory theories (see, e.g., Baumol, 1952; and Tobin, 1956).

[6] The conventional production-smoothing theory of inventories also focus on the cost friction. Because of increasing marginal costs, firms hold inventories to reduce the volatility of production under demand uncertainty. This theory is well known for its failure in explaining why production is more volatile than sales in the data (see, e.g., Blinder, 1986).

[7] Partial-equilibrium inventory models based on the stockout-avoidance motive have also been developed by Reagan (1982) and Abel (1985).

[8] Works along this line also include Kahn (1992), Brown and Haeglerb (2004), and Wen (2005a), among others.

aggregate inventory stock will fail to keep pace with sales, leading to countercyclical stock-to-sales ratio.[9] The steady-state aggregate inventory-to-sales ratio can be very large without a large variance of aggregate shocks because the measure of firms with positive inventories can be large, depending on the variance of idiosyncratic demand shocks.[10] Also, a large aggregate inventory stock-to-sales ratio is consistent with a small aggregate inventory investment-to-sales ratio if the rate of depreciation of inventories is small, so that the need of replenishment is small in the steady state. Input inventories are more volatile than output inventories because of an endogenous multiplier mechanism that magnifies the impact of aggregate demand shocks from downstream towards upstream industries through an input-output linkage. A one percentage increase in final sales can trigger a more than one-for-one increase in the production of finished goods because of procyclical inventory investment under the optimal target-inventory policy. This leads to an even larger increase in the demand for intermediate goods because of diminishing marginal product. Hence, orders of intermediate goods and input inventory investment have to increase even more under the stockout-avoidance motive. Finally, because finished goods inventories are a better buffer than fixed capital in meeting unexpected consumption demand, they tend to be countercyclical at the very high frequencies (during the impact period of the shocks).

Given the model's success in explaining the inventory behavior of the U.S. economy, it can serve as a laboratory for investigating the key question posed in the beginning of this paper: Are inventories destabilizing to the aggregate economy? Surprisingly, the answer is negative. By eliminating inventories from the model, the volatility of aggregate output is increased, not reduced. This counter-intuitive result originates not only from the endogenous interactions between production, inventory investment, and aggregate demand, but also from an asset-pricing channel pertaining to the endogenous time-varying value of inventory assets under the stockout-avoidance motive in general equilibrium. A precautionary motive for avoiding stockouts induces a procyclical premium on inventory assets that is priced into the final goods, which reflects a procyclical liquidity value of inventories. A procyclical premium means that customers pay higher prices in the time of liquidity (inventory) shortage and lower prices in the time of liquidity abundance. Thus, procyclical liquidity value of inventories acts as an automatic stabilizer to output fluctuations similar to a procyclical income tax: it discourages final demand in booms and encourages it in recessions. This implies that inventories may reduce the variance of final sales more than they increase the variance of

[9] This result is related to the argument of Bils and Kahn (2000).

[10] According to Bils (2004), the probability of stockout at the firm level is very small, about 8%. This suggests a large incentive of holding a large amount of inventories by firms.

production, giving rise to a more stabilized aggregate output.[11]

This paper is closely related to the work of Khan and Thomas (2007a), who addressed similar questions to those of this paper but in a very different general-equilibrium framework. Khan and Thomas' general-equilibrium model is based on the (S,s) inventory theory in which inventories exist because of fixed costs of ordering intermediate goods. Their work has made two important contributions to the literature by showing: (i) a general-equilibrium (S,s) model is able to explain the key features of inventory dynamics aforementioned in this paper; (ii) inventory fluctuations based on nonconvex costs have little impact on the business cycle because they do not substantially raise the volatility of aggregate output. Their explanation as to why procyclical inventory investment increases the variance of the final goods only insignificantly in general equilibrium is based on resource reallocation across sectors in a two-sector model. An economic expansion due to an aggregate TFP shock to the intermediate-goods producing sector causes resources to be diverted from the final-goods sector to the intermediate-goods sector where inventories are produced. Consequently, the final-goods sector does not expand as much as it would otherwise. Or alternatively, since both inventory investment and final sales effectively enter the same aggregate resource constraint, there is a tradeoff between inventory accumulation versus consumption and capital investment under sector-specific TFP shocks. Thus, larger fluctuations in inventory investment are accompanied by smaller fluctuations in the sum of these other activities, implying that the variability of the final goods sector is essentially unaffected by the existence of inventories (Khan and Thomas, 2007a, p1166).

However, the analysis of Khan and Thomas is based only on one of the possible micro-foundations of inventories. It is not immediately clear whether their results and explanations are general enough and applicable to models based on other microfoundations, such as the stockout-avoidance mechanism. My analysis suggests that it is important to develop and investigate alternative general-equilibrium inventory models with different microfoundations. While Khan and Thomas' analysis indicates that inventories destabilize the economy only insignificantly and are thus inessential for understanding the business cycle, my analysis suggests that inventories are important for the business cycle, albeit for the opposite reason: they stabilize rather than destabilize the macroeconomy.[12] Nonetheless, both of our

[11] On the other hand, when the inventory stock-to-sales ratio is procyclical in the model (such as under transitory technology shocks), the liquidity value of inventories becomes countercyclical. In such a case, procyclical inventory investment is destabilizing and can significantly raise the volatility of GDP. The data, however, indicate that the inventory-to-sales ratio is countercyclical (Bils and Kahn, 2000).

[12] Besides the fundamental difference in firms' motives of holding inventories, this paper also differs from Khan and Thomas (2007a) in other aspects. For example, among other things, Thomas and Khan did not

results share one thing in common: the general-equilibrium effect of procyclical inventory investment reduces the variability of final sales (although for fundamentally different reasons). This suggests that, if inventories are indeed as destabilizing in the real world as many people have believed, some unknown form of market structures or distortions not captured by Khan and Thomas and this paper must be important. Finding such frictions remains a major challenge for inventory theory.[13]

The rest of the paper is organized as follows. To gain intuition, section 2 presents a simple benchmark general-equilibrium model of inventories by embedding the partial-equilibrium model of Kahn (1987) and Bils and Kahn (2000) into a standard, perfectly competitive, RBC model. A social planner's version of the model is presented and analyzed. The model offers simple explanations as to why the inventory-to-sales ratio can be countercyclical when inventory investment is strongly procyclical. Section 3 extends the simple model by including both input and output inventories. A decentralized version of the model is presented and the model's dynamic properties under different types of aggregate shocks are studied. Section 4 addresses the central question regarding the (de)stabilizing role of inventories for the aggregate economy. Finally, section 5 concludes the paper with remarks for future research.

2 A Benchmark Model

The model is similar to a standard representative-agent RBC model with Dixit-Stiglitz production technologies. In this model, a final good is allocated between consumption (C) and capital investment (I) and is produced by the Dixit-Stiglitz aggregation function over intermediate goods, $C + I = \left[\int_0^1 \theta(i) y(i)^\rho di\right]^{\frac{1}{\rho}}$, where $\rho \in (0,1)$ pertains to the elasticity of substitution across intermediate goods $y(i)$ and $\theta(i)$ represents idiosyncratic shocks that affect the optimal demand of $y(i)$. The distribution of θ is denoted by the CDF $F(\theta)$. The supply of intermediate good i is denoted by $x(i)$. Without inventories, the resource constraint for intermediate good i is given by $y(i) \leq x(i)$. However, if there is inventory accumulation for good i, the resource constraint is given by $y_t(i) + s_t(i) \leq s_{t-1}(i) + x_t(i)$, where $s_t(i) \geq 0$ denotes the inventory stock of good i carried forward to the next period. For simplicity, a zero rate of depreciation for inventory stocks is assumed for the benchmark model.

consider the dynamic interactions between input and output inventories.

[13] One such possible distortion is that inventories may serve as collateral in a borrowing constrained economy. If credit limits are based on the value of collateralized assets, inventories could significantly destabilize the economy.

Intermediate goods are produced by the technology, $AK^{\alpha}N^{1-\alpha}$, where A represents aggregate technology shocks with the law of motion, $\log A_t = \log A_{t-1} + \varepsilon_{at}$; K the aggregate capital stock and N the aggregate labor. Intermediate goods are homogenous from the viewpoint of the upstream supplier; hence, the aggregate resource constraint for the supply of intermediate goods is $\int x(i)di \leq AK^{\alpha}N^{1-\alpha}$. However, these goods are heterogenous from the viewpoint of the downstream because of the idiosyncratic component in their demand curves, $\theta(i)$, which renders the shadow values of intermediate goods different across i.

To meet the random demand for intermediate good i from the downstream, the amount $x(i)$ must be ordered in advance before $\theta(i)$ is realized in each period. This information lag creates a precautionary stockout-avoidance motive for carrying inventories. The decisions regarding $y(i)$ and $s(i)$ are not subject to this information lag. In addition, aggregate shocks are realized in the beginning of each period before all decisions in the period are made and are orthogonal to idiosyncratic shocks.

A social planner or representative agent in the economy chooses $\{C_t, N_t, K_{t+1}, y_t(i), x_t(i), s_{t+1}(i)\}$ to solve the following program,

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \Theta_t \frac{C_t^{1-\gamma}}{1-\gamma} - a\frac{N_t^{1+\gamma_n}}{1+\gamma_n} \right\}$$

subject to

$$C_t + K_{t+1} - (1-\delta_k)K_t \leq \left[\int_0^1 \theta(i) y(i)^{\rho} di \right]^{\frac{1}{\rho}}, \qquad (\mu\ 1)$$

$$y_t(i) + s_t(i) \leq s_{t-1}(i) + x_t(i), \qquad (\lambda_i\ 2)$$

$$s_t(i) \geq 0, \qquad (\pi_i\ 3)$$

$$\int_0^1 x_t(i)di \leq A_t K_t^{\alpha} N_t^{1-\alpha}; \qquad (v\ 4)$$

where Θ represents aggregate shocks to consumption demand with the law of motion, $\log \Theta_t = \log \Theta_{t-1} + \varepsilon_{\phi t}$.[14]

[14] In general equilibrium models with constant returns to scale, increases in consumption demand due to preference shocks tend to crowd out investment when the shocks are transitory, leading to countercyclical investment. Although allowing for habit formation in consumption can resolve this problem (Wen, 2006), it complicates the model unnecessarily. Hence, I assume permanent preference shocks so as to avoid introducing habit formation. The results are similar under stationary $AR(1)$ preference shocks if habit formation is allowed.

2.1 First-Order Conditions

Denoting $\tilde{Y} \equiv \left[\int_0^1 \theta(i)y(i)^\rho di\right]^{\frac{1}{\rho}}$, $X \equiv A_t K_t^\alpha N_t^{1-\alpha}$, and $\{\mu, \lambda(i), \pi(i), v\}$ as the non-negative Lagrangian multipliers for the constraints (1)-(4), respectively, the first-order conditions with respect to $\{C_t, N_t, K_{t+1}, y_t(i), x_t(i), s_{t+1}(i)\}$ are given, respectively, by

$$\Theta_t C_t^{-\gamma} = \mu_t \tag{5}$$

$$aN_t^{\gamma_n} = v_t(1-\alpha)\frac{X_t}{N_t} \tag{6}$$

$$\mu_t = \beta(1-\delta_k)E_t\mu_{t+1} + \beta\alpha E_t\left(v_{t+1}\frac{X_{t+1}}{K_{t+1}}\right) \tag{7}$$

$$\mu_t \tilde{Y}_t^{1-\rho}\theta_t(i)y_t(i)^{\rho-1} = \lambda_t(i) \tag{8}$$

$$v_t = E_t^i \lambda_t(i) \tag{9}$$

$$\lambda_t(i) = \beta E_t \lambda_{t+1}(i) + \pi_t(i), \tag{10}$$

plus the transversality conditions, $\lim_{T\to\infty}\beta^T E\mu_T K_{T+1} = 0, \lim_{T\to\infty}\beta^T E\lambda_T(i)s_T(i) = 0$, and the complementary slackness condition, $s_t(i)\pi_t(i) = 0$, for all $i \in [0,1]$.

The operator E_t^i in equation (9) denotes expectations based on the information set of period t excluding $\theta_t(i)$. It reflects the information lag in ordering intermediate goods $x(i)$. Without the information lag, equation (9) becomes $v_t = \lambda_t(i)$. Equation (10) then implies $\pi_t(i) = v_t - \beta E_t v_{t+1} > 0$ and $s(i) = 0$ for all i.[15] Hence, it is not optimal to carry inventories when the value of θ is known. Given this, we have $y(i) = x(i), \int y(i)di = X$; and equation (8) implies $\mu_t\tilde{Y}_t = v_t X_t$ and $v_t = \bar{\theta}\mu_t$, where the constant coefficient $\bar{\theta}$ is given by $\bar{\theta} \equiv \left[\int_0^1 \theta(i)^{\frac{1}{1-\rho}}di\right]^{\frac{1-\rho}{\rho}} = \left[\int \theta^{\frac{1}{1-\rho}}dF(\theta)\right]^{\frac{1-\rho}{\rho}}$, by the law of large numbers. Consequently, the first-order conditions (6) and (7) become $aN_t^{\gamma_n} = \mu_t(1-\alpha)\frac{\tilde{Y}_t}{N_t}$ and $\mu_t = \beta E_t\mu_{t+1}\left[\alpha\frac{\tilde{Y}_{t+1}}{K_{t+1}} + 1 - \delta_k\right]$, respectively; and the aggregate resource constraint becomes $C + I = \tilde{Y} = \bar{\theta}AK^\alpha N^{1-\alpha}$. Therefore, without the information lag, the relative price of consumption and investment with respect to output is constant $(\frac{1}{\bar{\theta}})$ and the model is reduced to a standard one-sector

[15] Suppose this is not true and $\pi(i) = 0$ instead; then $v_t = \beta E_t v_{t+1}$, which implies $v_t \to 0$ as time goes to infinity. Since the utility function is strictly increasing, the resource constraint (4) binds with equality in equilibrium, implying $v_t > 0$. This is a contradiction.

RBC model. Obviously, the model is also reduced to a standard one-sector RBC model if there are no idiosyncratic shocks, $\theta(i) = 1$ for all i. In this case, $\bar{\theta} = 1, y(i) = \tilde{Y} = X$, and $C + I = AK^{\alpha}N^{1-\alpha}$. However, with idiosyncratic shocks and the information lag, the model is no longer reducible to a standard one-sector RBC model and inventories will play an important role in aggregate dynamics.

In the above setup, aggregate shocks do not play a role in the existence of inventories.[16] This feature makes the model analytically tractable because the decision rules for inventories can be solved by taking the aggregate variables as given. Then in equilibrium and by the law of large numbers, there is always a positive measure of intermediate-goods firms holding inventories in any time period. Hence, the aggregate inventory stock is strictly positive and the log-linearization technique can be applied to analyzing the model's aggregate dynamics.

2.2 Decision Rules for Inventories

The key to solving for the decision rules in the intermediate goods sector is to determine the optimal stock, $x_t(i) + s_{t-1}(i)$, based on the distribution of θ. The first-order condition for $x(i)$ is given by (9), which suggests that the optimal level of orders depends on the expected shadow value of inventory, $E_t\lambda_t(i)$. Under the law of iterated expectations, we have $E_t\lambda_{t+1}(i) = E_t v_{t+1}$; hence, equations (9) and (10) imply

$$\lambda_t(i) = \beta E_t v_{t+1} + \pi_t(i). \tag{11}$$

The decision rules for the intermediate goods sector are characterized by an optimal cutoff value of the idiosyncratic shock, θ^*, such that the non-negativity constraint (3) on inventory is slack if $\theta(i) \leq \theta^*$, and it binds if $\theta(i) > \theta^*$. Thus, there are two possible cases to consider.

Case A: In the case where $\theta(i) \leq \theta^*$, we have $\pi(i) = 0, s(i) \geq 0$, and $\lambda_t(i) = \beta E_t v_{t+1}$. The resource constraint (2) implies $y(i) \leq x(i) + s_{t-1}(i)$. Since equation (8) implies $y_t(i) = \left[\frac{\mu_t \tilde{Y}_t^{1-\rho}\theta_t(i)}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}}$, we have $\theta(i) \leq [x(i) + s_{t-1}(i)]^{1-\rho}\left[\frac{\beta E_t v_{t+1}}{\mu_t \tilde{Y}_t^{1-\rho}}\right] \equiv \theta^*$, which defines the optimal cutoff value θ^* and the optimal stock as $x(i) + s_{t-1}(i) \equiv \left[\frac{\mu_t \tilde{Y}_t^{1-\rho}\theta^*}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}}$.

Case B: In the case where $\theta(i) > \theta^*$, we have $\pi(i) > 0, s(i) = 0$, and $y(i) = x(i) + s_{t-1}(i) \equiv \left[\frac{\mu_t \tilde{Y}_t^{1-\rho}\theta^*}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}}$. Equation (8) then implies $\lambda_t(i) = \beta E_t v_{t+1}\frac{\theta_t(i)}{\theta^*} > \beta E_t v_{t+1}$.

[16] This is a consequence of the lack of information friction with respect to aggregate shocks. Introducing information frictions at the aggregate level is possible but it may not have significant value added to the results.

Given these two possibilities, equation (9) can be written as

$$v_t = \int_{\theta(i)\leq\theta^*} (\beta E_t v_{t+1})\, dF(\theta) + \int_{\theta(i)>\theta^*} (\beta E_t v_{t+1})\, \frac{\theta_t(i)}{\theta^*} dF(\theta), \tag{12}$$

where the LHS is the marginal cost of inventory, the first term on the RHS is the shadow value of inventory when there is excess supply, and the second term is the shadow value of inventory when there is a stockout. Thus, the optimal cutoff value is determined at the point where the marginal cost equals the expected marginal benefit. Since aggregate variables are independent of idiosyncratic shocks, equation (12) can be written as

$$v_t = \beta E_t v_{t+1} R(\theta_t^*), \tag{13}$$

where $R(\theta^*) \equiv F(\theta^*) + \int_{\theta(i)>\theta^*} \frac{\theta(i)}{\theta^*} dF(\theta) > 1$ measures the rate of returns to liquidity or inventory investment. Notice that the optimal cutoff value θ_t^* is time varying and $\frac{dR(\theta^*)}{d\theta^*} < 0$. The rate of return to inventory investment depends negatively on the cutoff value because a higher cutoff value implies a larger probability of excess supply and a smaller probability of stockout, which lowers the value of inventory. Given aggregate economic conditions, equation (13) solves the optimal cutoff value as $\theta_t^* = R^{-1}(v_t/\beta E v_{t+1})$, which is countercyclical with respect to the current-period marginal cost (v_t) and procyclical with respect to the expected future marginal cost ($E_t v_{t+1}$).

Equation (13) provides the key to understanding why inventory-to-sales ratio is countercyclical under aggregate demand shocks. A rise in sales leads to a rise in the current-period marginal cost of production (v_t) relative to future marginal cost.[17] A higher marginal cost calls for a higher rate of return to liquidity (inventory investment). Since the sale price of goods is higher in the case of stockout, this leads firms to increase the probability of stockout by not increasing inventory investment as much as sales. Hence, inventory stock will fail to keep track with sales, leading to countercyclical stock-to-sales ratio. The same optimal inventory behavior also leads to procyclical liquidity value of inventories, which has important implications for the stability of demand.

The decision rules for the intermediate goods sector are thus given by

$$x_t(i) + s_{t-1}(i) = \tilde{Y}_t \left[\frac{\mu_t \theta_t^*}{\beta E_t v_{t+1}} \right]^{\frac{1}{1-\rho}}, \tag{14}$$

[17] When the shock is persistent, expected future marginal cost will increase as well, but to a less degree. Otherwise the economy will not be stationary.

$$y_t(i) = \tilde{Y}_t \left[\frac{\mu_t}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}} \times \min\left\{\theta_t(i)^{\frac{1}{1-\rho}}, \theta_t^{*\frac{1}{1-\rho}}\right\}, \tag{15}$$

$$s_t(i) = \tilde{Y}_t \left[\frac{\mu_t}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}} \times \max\left\{\theta_t^{*\frac{1}{1-\rho}} - \theta_t(i)^{\frac{1}{1-\rho}}, 0\right\}. \tag{16}$$

The shadow price of inventory i is determined by

$$\lambda_t(i) = \beta E_t v_{t+1} \times \max\left\{1, \frac{\theta(i)}{\theta^*}\right\}, \tag{17}$$

which is downward sticky with respect to the demand shock $\theta(i)$. That is, the price of inventory does not decrease to "clear" the market when demand is low ($\theta \leq \theta^*$). Rather than choosing to sell the good at a price below the shadow value ($\beta E_t v_{t+1}$), firms opt to hold any excess supply as inventories ($s_t(i) > 0$), speculating that demand may be stronger in the future. On the other hand, when demand is high ($\theta > \theta^*$), firms draw down inventories and price rises with θ to clear the market ($\lambda(i) = \beta E_t v_{t+1} \frac{\theta(i)}{\theta^*}$). The optimal cutoff value θ^* determines the probability of stockouts and yields a zero average profit ($E\lambda(i) - v = 0$). The asymmetric price behavior will be averaged out across a large number of firms and will not show up at the aggregate level near the steady state of the model.

Notice that $\frac{\mu_t}{\beta E_t v_{t+1}} = \frac{\mu_t}{v_t}\frac{v_t}{\beta E_t v_{t+1}} = \frac{\mu_t}{v_t} R(\theta_t^*)$. Hence, equation (14) shows that the optimal stock of intermediate good i, $x_t(i) + s_{t-1}(i)$, is determined entirely by four aggregate factors: the level of aggregate output ($\tilde{Y}$), the ratio of marginal utility of aggregate output to the marginal cost of aggregate intermediate good ($\frac{\mu}{v}$), the rate of return to inventory investment (R), and the optimal cutoff value (θ^*). The ratio $\frac{\mu}{v}$ can be interpreted as a pseudo measure of aggregate markup for intermediate goods.[18] Such a decomposition is reminiscent of the decomposition of Bils and Kahn (2000).

2.3 Aggregate Dynamics

Defining the aggregate variables, $Y \equiv \int y(i)di, S \equiv \int s(i)di$, and aggregating the decision rules (14)-(16) under the law of large numbers gives

$$Y_t = \tilde{Y}_t \left[\frac{\mu_t}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}} D(\theta_t^*) \tag{17}$$

[18] The model is equivalent to a perfectly competitive economy, and the true measure of aggregate markup for intermediate goods is $\frac{E\lambda}{v} - 1 = 0$.

$$X_t + S_{t-1} = Y_t \frac{D(\theta_t^*) + H(\theta_t^*)}{D(\theta_t^*)} \tag{18}$$

$$S_t = Y_t \frac{H(\theta_t^*)}{D(\theta_t^*)}, \tag{19}$$

and aggregating the first-order condition (8) (or by using the definition of $\tilde{Y}$) gives

$$v_t = \mu_t R(\theta_t^*) G(\theta_t^*)^{\frac{1-\rho}{\rho}}, \tag{20}$$

where

$$\begin{aligned}
D(\theta^*) &\equiv \int_{\theta(i)\leq\theta^*} \theta(i)^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta(i)>\theta^*} \theta^{*\frac{1}{1-\rho}} dF(\theta) > 0, \qquad (21)\\
H(\theta^*) &\equiv \int_{\theta(i)\leq\theta^*} \left[\theta^{*\frac{1}{1-\rho}} - \theta(i)^{\frac{1}{1-\rho}}\right] dF(\theta) > 0,\\
\theta^{*\frac{1}{1-\rho}} &= D(\theta^*) + H(\theta^*),\\
G(\theta^*) &\equiv \int_{\theta(i)\leq\theta^*} \theta(i)^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta(i)>\theta^*} \theta(i)\theta^{*\frac{\rho}{1-\rho}} dF(\theta) > D(\theta^*).
\end{aligned}$$

The aggregate resource constraint (1) can be written as

$$C_t + K_{t+1} - (1-\delta_k)K_t = P_t \left(A_t K_t^{\alpha} N_t^{1-\alpha} + S_{t-1} - S_t\right), \tag{22}$$

where $P \equiv G(\theta^*)^{\frac{1}{\rho}} D(\theta^*)^{-1}$ measures the relative price of intermediate goods with respect to the final good.

Recall that in a standard RBC model without inventories, $v_t = \mu_t$ in the case of $\theta(i) = 1$ and $v_t = \bar{\theta}\mu_t$ in the case of no information lag. In these cases the pseudo measure of markup ($\frac{\mu_t}{v_t} = 1$ or $\bar{\theta}^{-1}$) and the relative price of intermediate goods ($\frac{\tilde{Y}}{Y} = 1$ or $\bar{\theta}$) are constant. However, when there are inventories, the pseudo markup is given by $R(\theta_t^*)G(\theta_t^*)^{\frac{1-\rho}{\rho}}$ and the relative price is given by $G(\theta_t^*)^{\frac{1}{\rho}} D(\theta_t^*)^{-1}$, which are no longer constant. Thus, inventories bring about important changes to aggregate dynamics and relative price movements.

By equation (13), the optimal cutoff variable θ_t^* is stationary even under permanent shocks. Hence, the aggregate decision rules (18) and (19) indicate that aggregate inventory stock and sales are cointegrated. The decision rules also show that the aggregate stock-to-sales ratio for intermediate goods exceeds one, $\frac{X_t+S_{t-1}}{Y_t} = \frac{D(\theta^*)+H(\theta^*)}{D(\theta^*)} > 1$, and the aggregate

inventory-to-sales ratio is strictly positive, $\frac{S_t}{Y_t} = \frac{H(\theta^*)}{D(\theta^*)} > 0$. Since $\frac{X_t+S_{t-1}}{Y_t} = 1 + \frac{S_t}{Y_t}$, if either one of these ratios is countercyclical, so is the other. These predictions are consistent with the empirical facts.[19] To see the dynamic behavior of $\frac{X_t+S_t}{Y_t}$, notice that $H + D = \theta^{*\frac{1}{1-\rho}}$ and both functions of H and D are increasing in θ^*: $\frac{dH(\theta^*)}{d\theta^*} = \frac{1}{1-\rho}\theta^{*\frac{\rho}{1-\rho}}F(\theta^*) > 0$, and $\frac{dD(\theta^*)}{d\theta^*} = \frac{1}{1-\rho}\theta^{*\frac{\rho}{1-\rho}}\left[1 - F(\theta^*)\right] > 0$, where $F(\theta) \equiv \Pr[\theta \leq \theta^*]$.[20] Given a small change in θ^*, the change in $\frac{X_t+S_{t-1}}{Y_t}$ is given by $\frac{d}{d\theta^*}\left(\theta^{*\frac{1}{1-\rho}}/D\right) = \frac{1}{D^2}\frac{1}{1-\rho}\theta^{*\frac{\rho}{1-\rho}}\left[D - \theta^{*\frac{1}{1-\rho}}(1-F)\right]$, which is positive if $D > \theta^{*\frac{1}{1-\rho}}(1-F)$. This is clearly true because $D = \theta^{*\frac{1}{1-\rho}}(1-F) + \int_{\theta(i)\leq\theta^*}\theta(i)^{\frac{1}{1-\rho}}dF(\theta)$ by (21). Hence, the stock-to-sales ratio comoves with the optimal cutoff variable θ_t^*. By equation (13), θ_t^* is determined completely by movements of marginal costs and is countercyclical with respect to the current-period marginal cost (v_t). Thus, if the marginal cost is procyclical (which is the case under aggregate demand shocks), then the stock-to-sales ratio will be countercyclical.[21]

Assume $\theta(i)$ follows the Pareto distribution, $F(\theta) = 1 - \left(\frac{1}{\theta}\right)^\sigma$, with support $\theta \in (1, \infty)$ and the shape parameter $\sigma > 1$. With this distribution, closed-form solutions for θ^* and the other functions in (21) are available. Combinations of the two parameters, $\{\rho, \sigma\}$, can generate essentially any sensible values for the inventory-to-sales ratio in the steady state. For example, consider $\rho = 0.1$ and $\sigma = 3$, then equation (13) implies $\theta^* = \left[\frac{\beta}{1-\beta}\frac{1}{\sigma-1}\right]^{\frac{1}{\sigma}}$.[22] At a quarterly frequency, if $\beta = 0.99$, then $\theta^* = 3.2$, $\frac{S}{Y} = 1.76$, and $\frac{X+S}{Y} = 2.76$. These numbers suggest that the economy is willing to hold a very large amount of inventories under the stockout-avoidance motive. On the other hand, the ratio of inventory investment-to-sales is given by $\delta\frac{S}{Y}$ in the steady state, which approaches zero if the depreciation rate of inventories (δ) approaches zero. This suggests that a large inventory stock-to-sales ratio is fully consistent with a small inventory investment-to-sales ratio as long as the rate of depreciation is small. These predictions are qualitatively consistent with the U.S. data.

[19] See, e.g., Bils and Kahn (2000) and Kahn (1992).

[20] The function $G(\theta^*)$ also increases with θ^*.

[21] To be more rigorous, the movement in θ_t^* is determined by movements of the growth rate of the marginal cost, $\frac{v_t}{\beta E_t v_{t+1}}$. In a stationary model, changes in expected future marginal cost, $\beta E_t v_{t+1}$, are dominated by changes in the current-period marginal cost, v_t. Hence, it is sufficient to focus on v_t for qualitative analysis.

[22] An interior solution requires $\theta^* > 1$ so that the cutoff value is within the support of the distribution. This conditions requires $1 < \sigma < \frac{1}{1-\beta}$.

Table 1. Parameter Values

α	β	δ_k	δ	γ	γ_n	ρ	σ
0.3	0.99	0.025	0.015	1.0	0.25	0.1	3.0

To be realistic, suppose the depreciation rate of inventories is positive, $\delta = 0.15$. This value of depreciation in conjunction with $\rho = 0.1$ and $\sigma = 3$ implies a stock-to-sales ratio of 2.0 and a 7% probability of stockout, which is comparable to Bils' (2004) estimates of 8% probability of stockout based on firm-level data.[23] The calibrated parameter values are summarized in table 1. The impulse responses of inventory investment and the inventory-to-sales ratio ($\frac{S_t}{Y_t}$) to one-standard-deviation aggregate shocks are graphed in Figure 1. The window on the left shows responses of inventory investment to an aggregate demand shock (circles) and an aggregate technology shock (triangles). The window on the right shows responses of inventory-to-stock ratio to a demand shock and a technology shock, respectively. Under aggregate demand shocks, aggregate inventory investment is procyclical and far more volatile than aggregate output (Y). However, the inventory-to-sales ratio (as well as the total stock-to-sales ratio, $\frac{X_t+S_{t-1}}{Y_t}$) are countercyclical. In the meantime, the pseudo measure of the markup ($\frac{\mu}{v}$) and the rate of return to inventory investment ($R(\theta^*)$) are both procyclical.

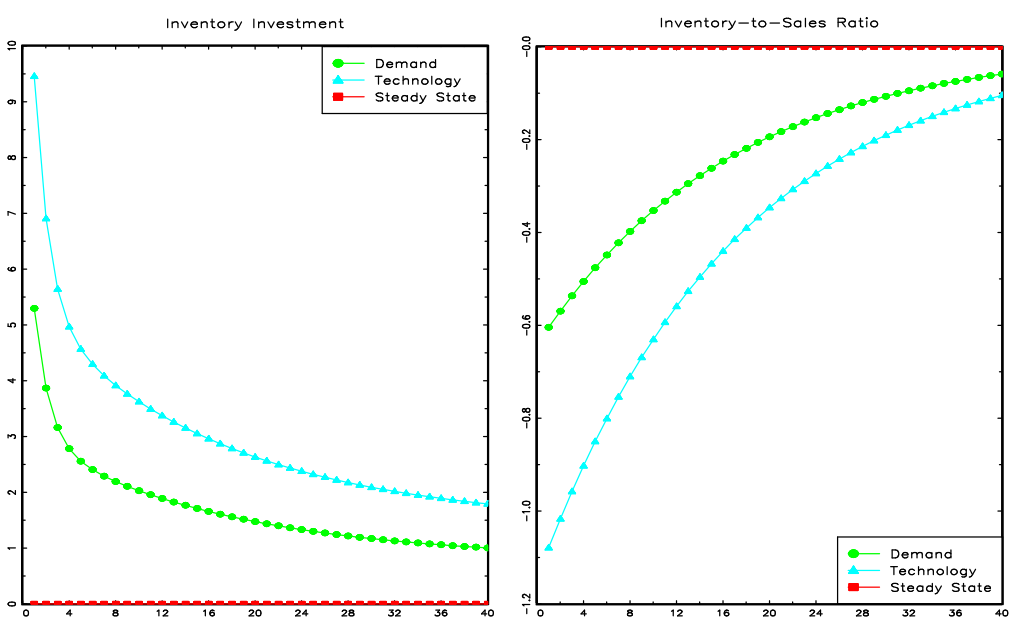

Figure 1. Impulse Responses to Demand & Technology Shocks.

[23] The probability of stockout in the model is given by $\left(\frac{1}{\theta^*}\right)^{\sigma} = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}(\sigma - 1)$, which depends positively on δ. The larger the depreciation rate, the higher the probability of stockout.

Interestingly, the same results can also be obtained by aggregate TFP shocks. In general, the marginal cost (v_t) is countercyclical under technology shocks. This would imply that θ^* as well as the stock-to-sales ratio are procyclical. However, if the shocks are permanent, then the expected future marginal cost can decrease even more than the current marginal cost for a short period of time because of the reinforcement by capacity accumulation, rendering the ratio $\frac{v_t}{\beta E v_{t+1}}$ procyclical. Hence, by equation (13), the cutoff variable θ^* and the stock-to-sales ratio can become countercyclical.

3 The Full Model

This section enriches the benchmark model in several dimensions so as to explain, among other things, two important stylized facts regarding inventory dynamics. First, input inventories are more volatile than output inventories (Humphreys, Maccini, and Schuh 2001); and second, finished goods inventories are countercyclical at the high frequencies (Hornstein, 1998; Wen, 2005a).

3.1 Household

A representative household has preferences over a spectrum of finished goods indexed by $j \in [0, 1]$. From the producer's point of view, these goods are the same (homogenous) because they are produced by the same production technology with the same costs; but they have different colors and yield different utilities to the household. In other words, these goods are not perfect substitutes in the household's utility function. The household purchases these finished goods in different colors in a competitive market and is able to store them in refrigerators if needed (refrigerator j stores good j).[24] The costs for storing goods include the depreciation rate $\delta > 0$ and the discounting of the future. The marginal utility of consumption of good j is subject to idiosyncratic taste shocks, $\theta_1(j)$, with distribution $F(\theta) = \Pr[\theta_1 \leq \theta]$. These taste shocks are not known to the household when orders (purchases) are made.[25] Hence, to cope with the idiosyncratic uncertainty, the household has incentive to store inventories of goods with all colors to avoid stockouts. The problem of the household

[24] Refrigerators in the model are a metaphor for retail stores in the real world. According to Blinder (1981), most of finished goods inventories are held by the retail sector rather than the manufacturing sector.

[25] For example, the household must go shopping in the morning and idiosyncratic taste shocks arrive at noon.

is to solve

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\Theta_t}{1-\gamma} \left[\int_0^1 \theta_{1t}(j) c_t(j)^\rho dj \right]^{\frac{1-\gamma}{\rho}} - a \frac{N_t^{1+\gamma_n}}{1+\gamma_n} \right\}$$

subject to

$$c_t(j) + s_{1t}(j) \leq (1-\delta) s_{1t-1}(j) + y_t(j) \qquad (\lambda_{1i}\ 23)$$

$$s_{1t}(j) \geq 0 \qquad (\pi_{1i}\ 24)$$

$$\int_0^1 y_t(j) dj + W_{t+1} \leq (1+r_t) W_t + w_t N_t + \Pi_t, \qquad (\mu\ 25)$$

where $\delta \in [0,1]$ is the depreciation rate of finished goods inventories (s_1), r is the interest rate on aggregate wealth (W), w is the real wage, and Π is total profit income distributed from firms. The parameters in the utility function satisfy standard restrictions: $\rho \in (0,1), \gamma \geq 0$, and $\gamma_n \geq 0$.

3.2 Firms

Final Goods. Final goods are produced competitively by the technology

$$\tilde{Y} = A K^\alpha \tilde{M}^{1-\alpha}, \qquad (26)$$

where $\tilde{M}$ is a composite of intermediate goods. The price of the composite good is P^m. The firm's problem is to solve

$$\max \left\{ A_t K_t^\alpha \tilde{M}_t^{1-\alpha} - (r_t + \delta_k) K_t - \frac{\xi}{2\bar{K}} (K_t - \bar{K})^2 - P_t^m \tilde{M}_t \right\},$$

where $(r_t + \delta_k)$ is the user's cost of capital with δ_k as the depreciation rate of capital, and $\xi \geq 0$ is the coefficient for a quadratic adjustment cost of capital relative to its steady state ($\bar{K}$).

Intermediate Goods. In this sector a representative firm uses labor to produce intermediate goods $m(i)$. These intermediate goods come with different colors indexed by $i \in [0,1]$. They are used to synthesize the composite good $\tilde{M}$ according to the aggregation technology, $\tilde{M} = \left[\int \theta_2(i) m(i)^\rho di \right]^{\frac{1}{\rho}}$. That is, the marginal revenue product of intermediate goods are subject to idiosyncratic shocks, $\theta_2(i)$, which generate idiosyncratic uncertainty for the demand of intermediate goods of different colors. Assume θ_2 has the same distribution $F(\theta)$.

Intermediate goods are produced by labor under identical linear technologies, $Bn(i)$, where B is a permanent aggregate cost shock to labor's productivity. This shock differs from the TFP shock because it does not directly affect the rate of return to capital investment. The labor market is perfectly competitive and the labor used in producing intermediate good i is a perfect substitute for that used in producing other intermediate goods. However, labor must be determined before the idiosyncratic shocks (θ_2) are realized in each period. Therefore, intermediate goods firms have incentive to keep inventories of work-in-process (s_2) in all colors so as to maximize expected profits. The problem of a representative intermediate goods firm is to solve

$$\max E \sum_{t=0}^{\infty} \beta^t \frac{\mu_{t+1}}{\mu_0} \left\{ P_t^m \left[\int \theta_{2t}(i) m_t(i)^\rho di \right]^{\frac{1}{\rho}} - w_t \int n_t(i) di \right\}$$

subject to

$$m_t(i) + s_{2t}(i) \leq (1-\delta) s_{2t-1}(i) + B_t n_t(i), \qquad (\lambda_{2i}\ 27)$$

$$s_{2t}(i) \geq 0, \qquad (\pi_{2i}\ 28)$$

where μ in the objective function denotes the marginal utility of the final good (i.e., $\frac{\mu_t}{\mu_{t-1}} = 1 + r_t$ is the real interest rate).

3.3 Aggregate Dynamics

Define $C \equiv \int c(i)di, Y = \int y(i)di$, $S_1 \equiv \int s_1(i)di$, $S_2 \equiv \int s_2(i)di$, and $M \equiv \int m(i)di$. The aggregate decision rules for the output inventory sector are given by (see Appendix 1)

$$\mu_t = \Delta_t R(\theta_1^*) G(\theta_1^*)^{\frac{1-\rho}{\rho}} \qquad (45)$$

$$C_t = \tilde{C}_t D(\theta_1^*) G(\theta_1^*)^{-\frac{1}{\rho}} \qquad (46)$$

$$Y_t + (1-\delta) S_{1t-1} = C_t \frac{D(\theta_{1t}^*) + H(\theta_{1t}^*)}{D(\theta_{1t}^*)} \qquad (47)$$

$$S_{1t} = C_t \frac{H(\theta_{1t}^*)}{D(\theta_{1t}^*)}, \qquad (48)$$

where (45) is analogous to (20) and the rest are analogous to (17)-(19).[26] The functions $\{G(\theta), D(\theta), H(\theta)\}$ are the same as those defined in (21). The aggregate decision rules for

[26] Note that (45) can also be written as $\mu = \Delta R(\theta) G(\theta)^{\frac{1-\rho}{\rho}}$, and (17) can also be written as $Y = \tilde{Y} D(\theta) G(\theta)^{-\frac{1}{\rho}}$.

the input inventory sector are similarly given by

$$\frac{w_t}{B_t} = P_t^m R(\theta_2^*) G(\theta_2^*)^{\frac{1-\rho}{\rho}} \tag{49}$$

$$M_t = \tilde{M}_t D(\theta_2^*) G(\theta_2^*)^{-\frac{1}{\rho}} \tag{50}$$

$$B_t N_t + (1-\delta) S_{2t-1} = M_t \frac{D(\theta_{2t}^*) + H(\theta_{2t}^*)}{D(\theta_{2t}^*)} \tag{51}$$

$$S_{2t} = M_t \frac{H(\theta_{2t}^*)}{D(\theta_{2t}^*)}. \tag{52}$$

Substituting out the factor income and aggregate profits, the aggregate resource constraints can be written as

$$C_t + S_{1t} - (1-\delta) S_{1t-1} + K_{t+1} - (1-\delta_k) K_t = A_t K_t^{\alpha} \tilde{M}_t^{1-\alpha} - \frac{\xi}{2\bar{K}} (K_t - \bar{K})^2, \tag{53}$$

$$M_t + S_{2t} - (1-\delta) S_{2t-1} = B_t N_t. \tag{54}$$

For both input and output inventories, the stock-to-sales ratio is determined by the function $\frac{D(\theta^*)+H(\theta^*)}{D(\theta^*)}$, which in turn is a function of the cutoff variable $\theta_t^* \in \{\theta_{1t}^*, \theta_{2t}^*\}$. Thus, the cyclicality of the stock-to-sales ratio in each sector is determined by the movements of marginal cost of inventories in that sector, as in the benchmark model. The aggregate resource constraint in equation (53) suggests that finished goods inventories are a perfect buffer for aggregate consumption and are substitutable for capital investment, whereas the input inventories in (54) are not directly substitutable for either consumption or capital goods. This difference gives rise to different inventory behavior across finished and unfinished goods, especially at the high frequencies.

Structural Parameters. Inventory behavior in the model depends on structural parameters. Although the influence of these parameters on the model are complex and intertwined, their major roles are easy to distinguish. For example, the parameters $\{\rho, \sigma\}$ affect primarily the steady-state stock-to-sales ratio because they influence the variance of sales at the micro level. When ρ is large, there is more substitutability across goods with different colors, making sales of each colored good more volatile for the same distribution of idiosyncratic shocks. The shape parameter σ in the Pareto distribution is negatively associated with the variance of the distribution. Hence, a smaller σ is associated with more volatile sales. Since

a larger variance of sales increases the possibility of stockouts, firms have incentive to keep a larger inventory stock relative to sales for a larger ρ and/or a smaller σ.

The parameters in the utility function $\{\gamma, \gamma_n\}$ affect inventory behavior by primarily affecting the relative strength of the income effect and the substitution effect. For example, the smaller the γ, the more responsive is aggregate consumption to aggregate shocks. In this case, finished goods inventories are more likely to play the role of a buffer stock in the face of consumption changes. Consequently, output inventory investment is more likely to be countercyclical at the high frequencies. On the other hand, larger values of γ or γ_n are more likely to generate negative responses of labor supply to technology shocks because of the increased income effect. Consequently, input inventories are more likely to be countercyclical under TFP shocks.

The adjustment cost parameter, ξ, affects primarily the substitutability between capital investment and inventory investment in finished goods. Hence, as consumption increases under either preference shocks or supply shocks, the effectiveness of buffer-stock roles of capital investment and inventory investment are different. For example, a larger value of ξ tends to attenuate the initial response of capital investment and make finished goods inventory investment more responsive to aggregate shocks on impact. The general dynamic properties of the model can be summarized as follows:

A. Under aggregate demand shocks and with a wide range of parameter values, the model exhibits the following general properties: (i) inventory investment for both finished and intermediate goods is procyclical at the business cycle frequencies; (ii) their respective stock-to-sales ratios are countercyclical; (iii) input inventories are more volatile than output inventories; and (iv) finished goods inventories have a tendency to be countercyclical at high frequencies. By the accounting identity for input and output inventories (production = inventory investment + sales), production/usage is more volatile than sales/orders because inventory investment is procyclical. These predictions are consistent with the data.

B. TFP shocks can generate similar results as those under demand shocks, provided that the substitution effect is strong enough (e.g., $\gamma < 1$). Otherwise, input inventory investment is countercyclical because TFP shocks generate a lower demand for intermediate goods when the income effect dominates. However, regardless of the parameter values, input inventories are less volatile than output inventories, which is inconsistent with the data.

C. Under labor cost shocks, the model's dynamics are very similar to those under preference shocks with a wide range of parameter values. Namely, (i) inventory investment for both finished and intermediate goods are procyclical at the business cycle frequencies;

(ii) their respective stock-to-sales ratios are countercyclical; (iii) input inventories are more volatile than output inventories; and (iv) finished goods inventories have a tendency to be countercyclical at high frequencies. In addition, production is more volatile than sales.

The main intuition behind these results can be analyzed using the aggregate resource equations (53) and (54), which reveal the demand-supply chain of the production process. First, a permanent aggregate preference shock increases the marginal utilities of consumption not only in the present period but also for future periods. This encourages the household to accumulate finished-goods inventories and capital. Such an increase in the demand for wealth raises the shadow price of finished goods and stimulates production; hence, the demand for intermediate goods also increase persistently. This in turn stimulates production of intermediate goods and the accumulation of intermediate-goods inventories. Therefore, a persistent shock to aggregate consumption demand at the downstream can generate synchronized business cycles across sectors. Furthermore, since an increase in the demand of finished goods requires more than a one-for-one increase in intermediate goods because of the diminishing marginal product of intermediate goods in producing the final good, upstream production must increase more than downstream production. This multiplier effect causes input inventory investment to be more volatile than output inventory investment under the stockout-avoidance motive. Finally, increases in demand at all stages of the production process raises the marginal costs of production at each stage, making the stock-to-sales ratio countercyclical for both input and output inventories.

The same type of aggregate fluctuations driven by aggregate demand shocks can also be obtained under permanent cost-push shocks. An increase in B_t increases aggregate supply of intermediate goods as well as input inventories. This reduces the shadow price of intermediate goods and encourages production of the finished goods. More supply of finished goods encourages consumption and accumulation of wealth (including capital and finished goods inventories). Also, because of the diminishing marginal product of the intermediate goods, an increase in intermediate goods can translate only into less than a one-for-one increase in final goods. Hence, output inventory investment is less volatile than input inventory investment. Finally, since the shock is permanent, the decrease in the expected future marginal cost outweighs that of the current marginal cost, leading to countercyclical stock-to-sales ratio in all sectors.

The dynamic effects of TFP shocks are very different from the other two types of shocks. A shock to the TFP serves as a supply-push shock for the final-good sector but a demand-pull shock for the intermediate goods sector. However, the magnitude of the supply-side

effect is larger than that of the demand-side effect. A one-unit increase in intermediate good $\tilde{M}$ under a positive TFP shock is just a one-unit increase in demand for intermediate goods, but it represents more than a one-for-one increase in the supply of finished goods because of the compounded effect from a higher TFP. This explains why input inventory investment is in general less volatile than output inventories under TFP shocks. Also, if the income effect dominates the substitution effect, then a positive shock to TFP leads to a decrease in the demand for intermediate goods, causing input inventory investment to be countercyclical. Hence, the effects of TFP shocks on inventory behavior are more sensitive to structural parameters than those of other shocks.

Finally, since finished goods inventories stored in the refrigerators (i.e., held by retail stores) are a better buffer than capital goods for unexpected increases in consumption needs, finished goods inventories tend to be countercyclical on impact at the high frequencies. On the other hand, since finished goods inventories are substitutable for capital investment, an unexpected rise in the marginal product of capital also tends to crowd out orders of finished goods from the household and reduce inventory investment. Thus, countercyclical final-goods inventory investment at the high frequencies can be generated by both aggregate demand shocks and aggregate supply shocks. This is consistent with the stylized fact documented and analyzed by Wen (2005a).

Calibration and Impulse Responses. The common parameters of the full model are set at the same values as in Table 1. In particular, time period is a quarter, capital's share of income $\alpha = 0.3$, the time-discounting rate $\beta = 0.99$, the inverse labor supply elasticity parameter $\gamma_n = 0.25$ (which corresponds to a log utility function on leisure),[27] the rate of capital depreciation $\delta_k = 0.025$ (which implies the capital stock depreciates about 10% a year), the rate of inventory depreciation $\delta = 0.015$ (which implies a 6% annual rate of depreciation for inventories),[28] the shape parameter $\sigma = 3$ and the substitution parameter $\rho = 0.1$. These values of $\{\beta, \delta, \rho, \sigma\}$ imply an inventory-to-sales ratio of about 1.0 (or a stock-to-sales ratio of 2.0), an inventory investment to GDP ratio of about 1%, and a 7% probability of stockout in the steady state.[29] The adjustment cost parameter is set to $\xi = 0.1$. The risk aversion parameter γ plays an important role in determining the strength of the

[27] With a log function of leisure, $\log(1 - N_t)$, the corresponding elasticity of hours in the log-linearized first-order condition with respect to labor (equation 29) is given by $\frac{N}{1-N}$. Suppose the weekly hours worked are 35, then the fraction of hours worked is given by $N = \frac{35}{7\times 24} = 0.2$, which implies $\frac{N}{1-N} = 0.25$.

[28] Because of wear and tear in use, the capital stock depreciates faster than inventory stocks.

[29] Since the parameters $\{\delta, \rho, \sigma\}$ are assumed to be the same for both input and output inventory sectors, the implied steady-state stock-to-sales ratios and probability of stockout are the same for both sectors.

substitution effect, it is left free for experiments in the impulse response analysis below.

To get a sense of the adjustment cost parameter ξ, we can estimate the adjustment cost as follows. The ratio of the adjustment cost to aggregate output can be written as

$$\frac{\xi}{2}\frac{\bar{K}}{Y_t}\left(\frac{K_t-\bar{K}}{\bar{K}}\right)^2. \tag{55}$$

Assume that the steady-state annual capital-output ratio $\frac{K}{Y} \approx 2$. The estimated variance of the capital stock relative to its HP-filter trend for the manufacturing sector between 1925 and 2002 is roughly $\sigma_k^2 = 0.0013$. Then with $\xi = 0.1$, the steady-state adjustment cost is approximately 0.01% of output a year. Even with $\xi = 5$, it amounts to capital adjustment costs about 0.5% of output. This is a very small number compared with the estimates of Shapiro (1986).[30] Without the adjustment cost, the model can still generate similar inventory dynamics, except the finished goods inventory investment has a higher tendency to be negative on impact. This negative initial response can always be countered by a higher value of γ.

The impulse responses of the model to a one-standard-deviation shock to aggregate demand are graphed in Figure 2. Different values of γ are used in generating Figure 2 in order to illustrate the sensitivity (robustness) of the model to parameter values. Under the shock, aggregate activities – including total output, consumption, capital investment, labor, and inventory investments – all increase and comove. These predictions are robust to the value of γ, except the initial change in output inventories, which may be negative or positive depending on the value of γ. A lower value of γ makes consumption more responsive on impact because of lower risk aversion, which crowds out inventories in the short run. In the longer run, however, finished goods inventories always comove with final sales because of the desire for replenishment. Also, input inventory investment is at least 4 times more volatile than output inventory investment in both the short and long run, and both are significantly more volatile than their respective sales. In the meantime, both output and input inventory-to-sales ratios are countercyclical despite their large volatilities. These predictions are consistent with the data.

[30] Shapiro (1986) estimates the capital investment adjustment costs to be around 0.7% of output for a quarter.

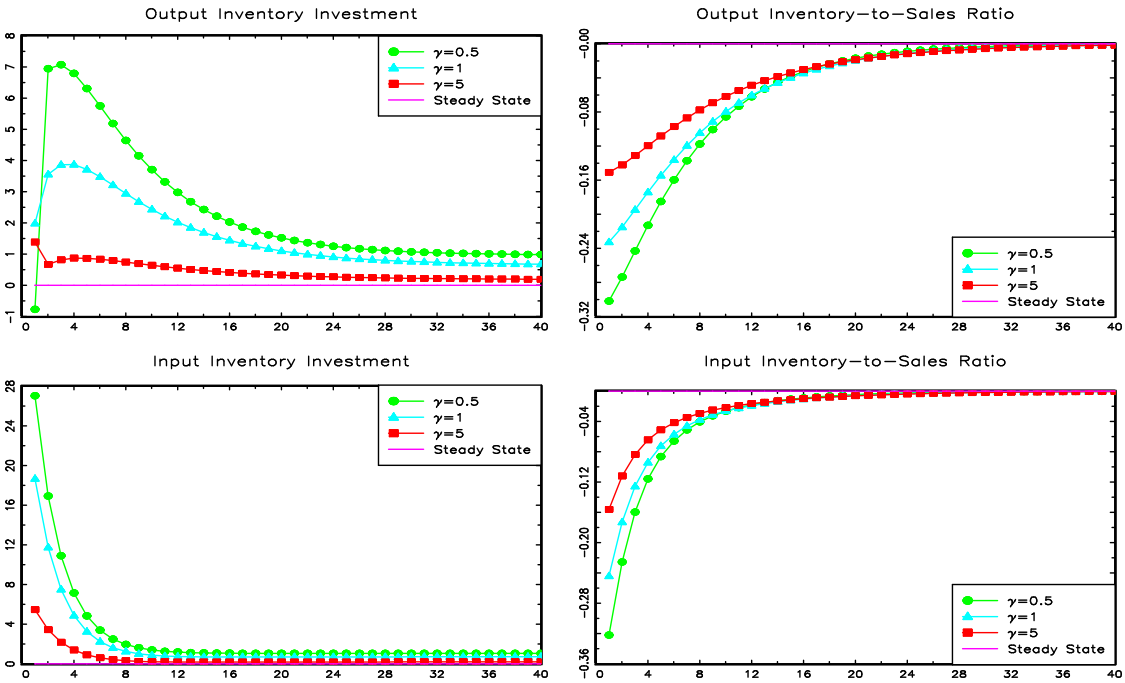

Figure 2. Impulse Responses to Demand Shock.

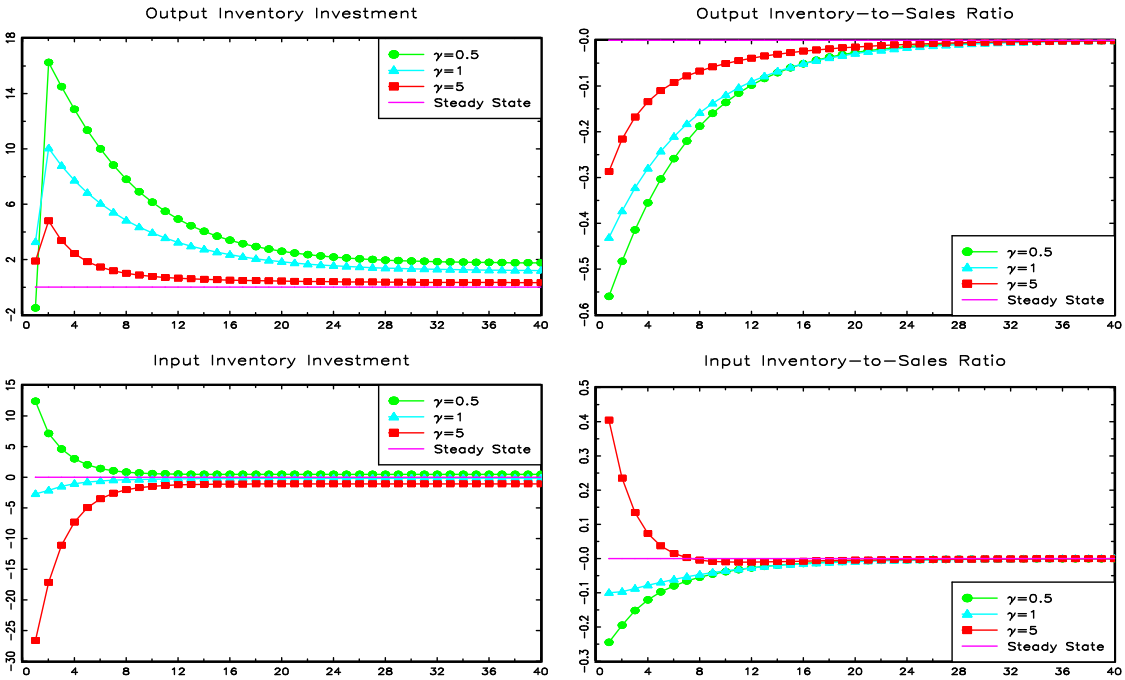

Figure 3. Impulse Responses to TFP Shock.

Under TFP shocks (Figure 3), the predicted inventory dynamics are consistent with the data if γ is sufficiently small (i.e., $\gamma < 1$, e.g., see the lines with circles in Figure 3). In this case, both input and output inventory investment are procyclical and the corresponding

inventory-to-sales ratios are countercyclical. However, if γ is large enough (i.e., $\gamma \geq 1$), input inventory investment becomes countercyclical because a large income effect caused by γ decreases the demand for intermediate goods and input inventories under a positive productivity shock.

The impulse responses of the model to a one-standard-deviation labor cost shock is graphed in Figure 4. The predicted dynamics are nearly identical to those under aggregate demand shocks except more volatile.[31] Namely, inventory investment is procyclical in both input and output sectors; the inventory-to-sales ratio is countercyclical; and input inventory investment is more volatile than output inventory investment. This suggests that inventory behavior in the data, especially the countercyclical stock-to-sales ratio, by itself does not indicate which type of shocks are important in driving the business cycle. This is in contrast to the arguments made by Bils and Kahn (2000).[32]

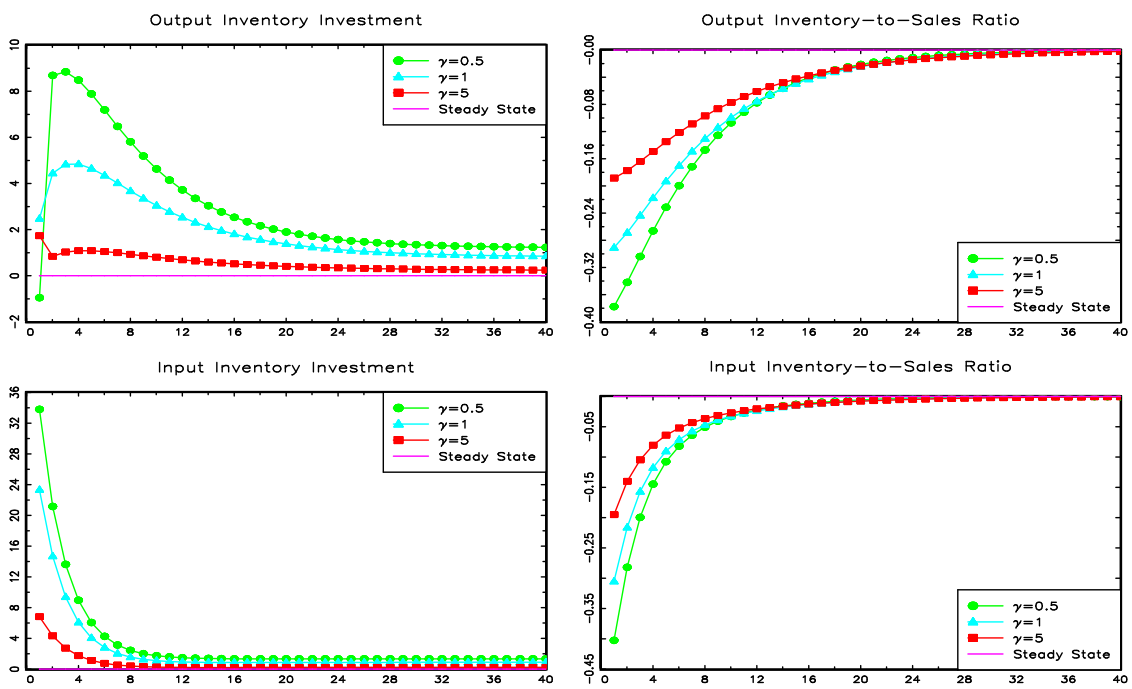

Figure 4. Impulse Responses to Cost Shock.

Matching Data.

The model has no problem matching the long-run ratios of inventory stock to sales and inventory investment to sales by properly choosing the parameter values of $\{\sigma, \rho\}$, as well as

[31] The exception is labor. Labor is much less volatile relative to output under cost shocks than under demand shocks.

[32] Khan and Thomas (2007a) also have similar findings in a general-equilibrium (S,s) model.

matching the other great ratios of the U.S. economy. This section, therefore, focuses instead on the ability of the model to match the second moments of the data.

To ensure consistency between the data and the model in the definition of variables, all variables in the data are transformed into percentage deviations from their respective long-run trends according to the definition, $\hat{X}_t \equiv \log X_t - \log X_t^*$, where the long-run trend (X^*) is defined as the HP trend. This is consistent with the log-linearization solution method of the model. The relationship between a stock variable S and its flow I is defined according to the model as

$$S_t - (1-\delta)S_{t-1} = I_t. \tag{56}$$

Hence, the log-linearized relationship between stock and flow is given by

$$\hat{S}_t - (1-\delta)\hat{S}_{t-1} = \delta \hat{I}_t. \tag{57}$$

Based on this definition, if a flow variable I has both positive and negative entries and cannot be "log-linearized" directly and data on its stock S is not available, then its percentage deviation from trend can be constructed according to relationship (57). For example, to compute percentage changes of aggregate inventory investment in finished goods (I_t), which has non-positive entries sometime, we can first construct the inventory stock variable S_t according to (56) by assuming $\delta = 0.015$. The initial value of S_0 is set such that the imputed stock variable shares a common growth trend with GDP or the stock-to-GDP ratio is stationary over time.[33] The stock variable is then logged and HP filtered, yielding the series $\hat{S}_t$. Using (57), we then obtain $\hat{I}_t$.[34]

Figure 5 shows the aggregate inventory-to-GDP ratio based on the constructed aggregate inventory stock, along with the inventory stock-to-sales ratio in the manufacturing sector. Clearly, the constructed aggregate inventory stock series mimics that of the manufacturing sector very closely over the business cycle. The inventory-to-sales ratio for both types of inventories has exhibited a downward trend since the early 80s, coinciding with the great moderation of the U.S. economy. The average inventory stock-to-GDP ratio is 0.61. This value is 0.92 with respect to aggregate consumption. For the manufacturing sector, the average inventory-to-sales ratio is 1.64 (implying a stock-to-sales ratio of 2.64).

[33] Since the series of inventory stock-to-sales ratio in the manufacturing sector is available, the initial value of S_0 can be further narrowed down by ensuring that the constructed inventory-to-sales ratio of the aggregate finished goods look similar to that of the manufacturing sector. Using this method, the initial value is set at $S_0 = 0.65GDP_0$, where GDP_0 is the initial value of GDP for our U.S. data sample.

[34] The variance of $\hat{I}_t$ based on this construction is sensitive to the value of δ. To make sure that $\delta = 0.015$ does not exaggerate the variance of inventory investment, we have used this procedure to construct the series of log-linearized fixed capital investment under the value $\delta = 0.015$ and found that the variance of fixed investment is not exaggerated compared with the series under direct log-linearization.

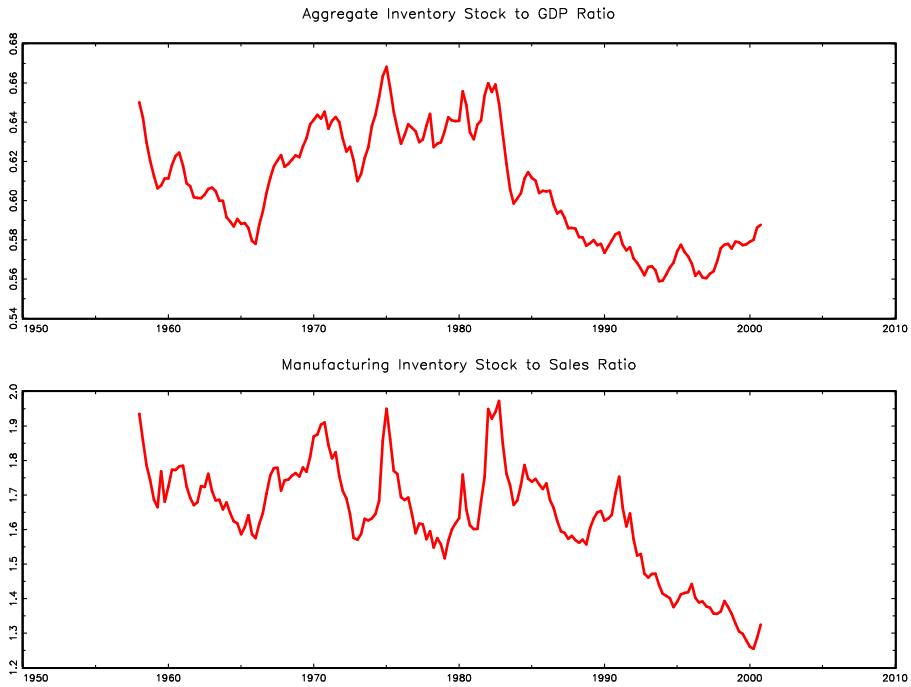

Figure 5. Output and Input Inventory Behavior.

Table 2 reports some selected business cycle statistics of the U.S. economy. All data are measured in billions of 2000 dollars. Aggregate consumption (C), fixed capital investment (dK), and inventory investment (dS_1) are from NIPA tables and they correspond to the final-good sector in the model. Since there is no government and international trade in the model, aggregate production is defined as $Y = C + dK + dS_1$ and aggregate sales is defined as $Y - dS_1$.[35] We use data from the manufacturing sector of the U.S. economy as a proxy that corresponds to the intermediate-good sector of the model, where total manufacturing production is denoted by Z, total sales (shipments) by M, and the inventory stock by S_2 (which includes only inventories of raw materials and work-in-process).[36] Comovements are measured by correlations with sales, as in Khan and Thomas (2007a). Given the extremely high correlation between sales and output, the reported statistics change very little if they are measured instead by correlations with output.

[35] There are no separate data on consumption good inventories and investment good inventories. Hence, the data and the model's final good sector are not a perfect match because in the model there are only consumption goods inventories.

[36] Data on inventory stocks for the manufacturing sector are available from the Bureau of the Census.

Table 2. Business Cycle Statistics (U.S. 1958:1 - 2000:4)

Variables	All Frequencies		8-40 Quarters		2-3 Quarters	
Final Good	*std./y*	*cor./sales*	*std./y*	*cor./sales*	*std./y*	*cor./sales*
Y	1	0.97	1	0.98	1	0.60
C	0.62	0.97	0.60	0.97	0.85	0.94
dK	2.44	0.94	2.44	0.95	2.09	0.70
dS_1	21.6	0.42	17.7	0.62	71.3	-0.36
S_1	0.66	0.35	0.67	0.29	0.57	-0.33
$\frac{S_1}{C}$	0.89	-0.71	0.93	-0.72	1.23	-0.90
Z	1.59	0.57	1.50	0.65	2.72	0.45
Interm. Good	*std./z*	*cor./m*	*std./z*	*cor./m*	*std./z*	*cor./m*
Z	1	0.99	1	0.99	1	0.97
M	0.95	1	0.94	1	0.96	1
dS_2	32.1	0.62	27.5	0.78	72.5	0.22
S_2	1.13	0.32	1.16	0.27	0.58	0.18
$\frac{S_2}{M}$	1.22	-0.48	1.28	-0.49	1.03	-0.83

In Table 2, two classes of statistics of each times series are reported, including standard deviation relative to production (*std./prod*) and correlation relative to sales (*cor./sales*). The HP-filtered data correspond to the "All Frequencies" column,[37] movements isolated by the Band-Pass filter at the business cycle frequencies (8-40 quarters per cycle) correspond to the "8-40 Quarters" column, and those at the high frequencies correspond to the "2-3 Quarters" column. Standard deviations of the final-good sector relative to production ($std./y$) are reported in the upper panel in the first column under each frequency band, and their correlations with total sales in the final-good sector ($cor./(y - ds)$) are reported in the next column under the same frequency band. Similarly, statistics from the intermediate-good sector are reported in the (lower panel) under each frequency band.

Several stylized facts are worth emphasizing in table 2. First, inventory investment is extremely volatile and procyclical over the business cycle. For example, over the 8-40 quarters frequency band, its volatility is 17.7 times that of production in the final-good sector and 27.5 times that of production in the intermediate sector; and its correlation with sales is 0.62 in the final-good sector and 0.78 in the intermediate-good sector. Second, despite this, the inventory stock-to-sales ratio is countercyclical. Its correlation with sales is -0.47 in the

[37] Band-Pass filter with 2-40 quarters window gives nearly identical results.

final-good sector and -0.49 in the other sector.[38] Third, intermediate goods inventories are more than twice as volatile as those for finished goods. To see this, notice that the standard deviation of production in the intermediate-good sector is 1.5 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to the final good production is $27.5 \times 1.5 = 41.25$, which makes it more than twice as large as the volatility of finished goods inventory investment (which is 17.7). Finally, finished goods inventories are countercyclical at high frequencies. For example, their correlation with sales is -0.36 for inventory investment and -0.33 for inventory stock. However, these correlations are positive for intermediate good inventories.

Table 3. Model Predictions under Demand (Technology) Shocks

Var.	All Frequencies		8-40 Quarters		2-3 Quarters	
Final	*std.*/$\tilde{y}$	*corr.*/c	*std.*/$\tilde{y}$	*corr.*/c	*std.*/$\tilde{y}$	*corr.*/c
$\tilde{Y}$	1	0.98 (0.97)	1	0.97 (0.97)	1	0.99 (0.99)
C	0.83 (0.81)	1	0.87 (0.85)	1	0.82 (0.72)	1
dK	1.47 (1.60)	0.82 (0.75)	1.28 (1.37)	0.75 (0.71)	2.28 (2.71)	0.98 (0.98)
dS_1	10.3 (10.9)	0.69 (0.71)	9.61 (10.2)	0.84 (0.86)	11.9 (13.8)	-0.85 (-0.80)
S_1	0.51 (0.52)	0.39 (0.46)	0.65 (0.62)	0.58 (0.59)	0.11 (0.12)	-0.76 (-0.86)
$\frac{S_1}{C}$	0.79 (0.73)	-0.79 (-0.77)	0.73 (0.69)	-0.68 (-0.70)	0.91 (0.83)	-0.99 (-0.99)
Z	1.74 (0.53)	0.93 (0.90)	1.61 (0.51)	0.91 (0.88)	2.16 (0.62)	0.99 (0.99)
Interm.	*std.*/z	*corr.*/m	*std.*/z	*corr.*/m	*std.*/z	*corr.*/m
Z	1	0.97 (0.99)	1	0.98 (0.99)	1	0.99 (1.00)
M	0.82 (0.87)	1	0.88 (0.90)	1	0.68 (0.75)	1
dS_2	17.0 (12.5)	0.66 (0.74)	13.9 (10.5)	0.57 (0.67)	22.3 (17.8)	0.99 (0.99)
S_2	0.57 (0.44)	0.89 (0.82)	0.70 (0.52)	0.92 (0.85)	0.18 (0.14)	0.97 (0.98)
$\frac{S_2}{M}$	0.42 (0.56)	-0.75 (-0.90)	0.36 (0.53)	-0.66 (-0.86)	0.51 (0.61)	-0.99 (-0.99)

Table 3 reports the business cycle statistics predicted by the model (with $\gamma = 0.5$) under demand shocks (where numbers in parentheses are predictions under TFP shocks).[39] The production in the final-good sector is denoted by $\tilde{Y}$, total sales by C, capital investment by dK, inventory investment by dS_1, and inventory stock-to-sales ratio by $\frac{S_1}{C}$. The production in the intermediate-good sector is denoted by Z, sales by M, inventory by S_2, and stock-to-sales ratio by $\frac{S_2}{M}$.

Under aggregate demand shocks, the model is able to qualitatively replicate the stylized

[38] Notice that the stock-to-sales ratio can be countercyclical even when the inventory stock itself is more volatile than sales (see, e.g., the last row in table 1). This could happen if there is a substantial delay in inventory replenishment after a sales shock.

[39] The statistics are based on simulated time series with 2000 observations and are filtered in the same way as for the U.S. data.

facts in table 3. Namely, (i) inventory investment is very volatile and procyclical over the business cycle. Over the 8-40 quarters frequency band, its volatility is about 10 times that of production in the final-good sector and 14 times that of production in the intermediate sector; and it is positively correlated with sales in both sectors (the correlation is 0.84 in the final-good sector and 0.57 in the intermediate-good sector). (ii) The inventory stock-to-sales ratio is countercyclical. Its correlation with sales is -0.68 in the final-good sector and -0.66 in the other sector. (iii) Intermediate goods inventories are more than twice as volatile as those for finished goods. The standard deviation of production in the intermediate-good sector is 1.61 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to the final good production is $14 \times 1.6 = 22$, which makes it more than twice as large as the volatility of finished goods inventory investment (which is 9.61). (vi) Finished goods inventories are countercyclical at high frequencies. For example, their correlation with sales is -0.85 for inventory investment and -0.76 for inventory stock. In the meantime, the respective correlations are positive for intermediate good inventories, as in the data.

The predictions under cost shocks (B_t) are almost identical to those of aggregate demand shocks; hence, they are not reported. The predictions under TFP shocks are also reported in Table 3 (numbers in parentheses). Most of the predictions are consistent with the data, except the volatility of input inventories relative to output inventories. For example, over the 8-40 quarters frequency band, the standard deviation of production in the intermediate-good sector is only 0.51 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to the final-good production is $10.5 \times 0.51 = 5.4$, which makes it only half as large as the volatility of finished-good inventory investment (which is 10.2). The reason is precisely the lack of a multiplier effect of TFP shocks on intermediate-good sector relative to the final-good sector. An increase in TFP raises the final-good production (supply) more than the intermediate-good production (demand). That is, the supply-side effect on final good is the combination of changes in TFP and $\tilde{M}$, whereas the demand-side effect on intermediate goods is only changes in $\tilde{M}$. In addition, for the risk aversion parameter γ large enough, the effect on intermediate-good demand is even negative. This problem does not arise for aggregate demand shocks (which originate from the bottom of the production chain) or aggregate cost shocks to labor or raw materials (which originate upstream).

Finally, notice that the model is qualitatively consistent with the U.S. business cycle

along other dimensions. For example, the model is able to explain the procyclical aggregate consumption, capital investment, and hours across all cyclical frequencies. The model is also able to explain the stylized fact that consumption is less volatile but capital investment is more volatile than GDP at different frequency bands.

4 Inventories and the Business Cycle

Given the model's broad consistence with inventory dynamics of the U.S. economy, it provides a reasonable framework for addressing a key question regarding the relationship between inventories and the business cycle. Namely: Are inventories important for the business cycle? According to Blinder (1981, 1986, 1990), business cycles are to a large degree inventory cycles. A clear message from Blinder is that eliminating inventories could significantly stabilize the economy. However, my general-equilibrium model predicts otherwise. That is, reducing the inventory stock-to-sales ratio or eliminating inventories from the model lead to a higher (not lower) volatility of aggregate output. In other words, inventories are found to be a stabilizer rather than a destabilizer to the economy. Table 4 reports the reduction in the variance of the final-goods supply in the full model ($AK^{\alpha}\tilde{M}^{1-\alpha}$) when the steady-state inventory-to-sales ratio in the model increases (by increasing the variance of the idiosyncratic shocks, θ).[40] Under the stockout-avoidance motive, a higher variance of the idiosyncratic shocks leads to a larger inventory stock and inventory-to-sales ratio. For comparison, a control (RBC) model without inventories (i.e., by setting $s_{1t}(i) = s_{2t}(i) = 0$ for all i and t in the full model) is also reported as a reference point. According to the table, if inventories are eliminated from a world (i.e., model 3) where the inventory-to-sales ratio is 3 (which matches the U.S. data for the private business), the variance of output will increase by about 30%.

Table 4. Contribution of Inventories to Stability

	inventory-sales ratio	variance of $AK^{\alpha}\tilde{M}^{1-\alpha}$	relative variance
Model 1	25	0.19	0.16
Model 2	8	0.55	0.45
Model 3	3	0.84	0.70
Model 4	1	1.06	0.88
Model 5	0.05	1.20	0.99
RBC	0.0	1.21	1.00

[40] Namely, by decreasing the shape parameter of the Pareto distribution, σ. The counterfactual experiments are conducted under preference shocks and the simulated time series (with sample size 2000) are all HP filtered.

The intuition behind this surprising result is that inventories stabilize final demand more than they destabilize production. This stabilizing effect on final demand is rooted in the procyclical liquidity-value of inventories. This procyclical liquidity value is the consequence of the procyclical probability of stockouts, which provides the very incentive for firms to be willing to hold inventories under positive interest rate and the key to explaining the counter-cyclical stock-to-sales ratio. This mechanism can be seen easily from the benchmark model. The aggregate resource constraint in equation (22) indicates that the existence of inventories introduces a time-varying wedge between final demand and aggregate supply. This wedge is captured by the relative price $P_t = G(\theta_t^*)^{\frac{1}{\rho}} D(\theta_t^*)$ in equation (22).[41] Defining $q_t = P_t^{-1}$ as the relative price of consumption goods in terms of inventory goods, equation (22) can be rewritten as $q_t C_t + q_t \left[K_{t+1} - (1-\delta)K_t)\right] + S_t - S_{t-1} = A_t K_t^{\alpha} N_t^{1-\alpha}$. Recall that a countercyclical stock-to-sales ratio requires θ^* to be countercyclical (i.e., the rate of return to inventory investment and the probability of stockout are procyclical, see equation 13). Since the elasticity of q with respect to θ^* is negative (see Appendix 2), q_t is thus procyclical. This implies that consumption (as well as capital investment) is more expensive relative to inventories when the marginal cost is high in a boom period and less expensive when the marginal cost is low in a slump. Thus, the procyclical movements in q_t acts as an automatic stabilizer, which reduces the variability of final demand $(C_t + K_{t+1} - (1-\delta)K_t)$ over the business cycle.

Figure 6 compares impulse responses of the benchmark model (with a high inventory-to-sales ratio of $\frac{S}{Y} = 8$) and those of a control model without inventories ($\frac{S}{Y} = 0$). The top row windows indicate that both consumption (C_t) and capital investment $(K_{t+1} - (1-\delta)K_t)$ have a lower volatility when inventories exist, revealing the stabilizing role of inventories. On the other hand, the left window in the bottom row indicates that labor (N_t) is more volatile when inventories exists, revealing the destabilizing role of inventories (procyclical inventory investment implies more volatile production). However, in net the stabilizing role dominates the destabilizing role because of the tradeoff between the lowered variability of the capital stock and the increased variability of labor in addition to the countercyclical movement in P_t; consequently the variance of final output ($\tilde{Y}_t$) is reduced (the right window in the bottom role).[42]

[41] The same wedge appears in the full model in equations (46) and (50).

[42] The measure of final output is given by $\tilde{Y}_t = P_t \left[A_t K_t^{\alpha} N_t^{1-\alpha}\right]$, which equals aggregate demand $C_t + K_{t+1} - (1-\delta)K_t + P_t(S_t - S_{t-1})$.

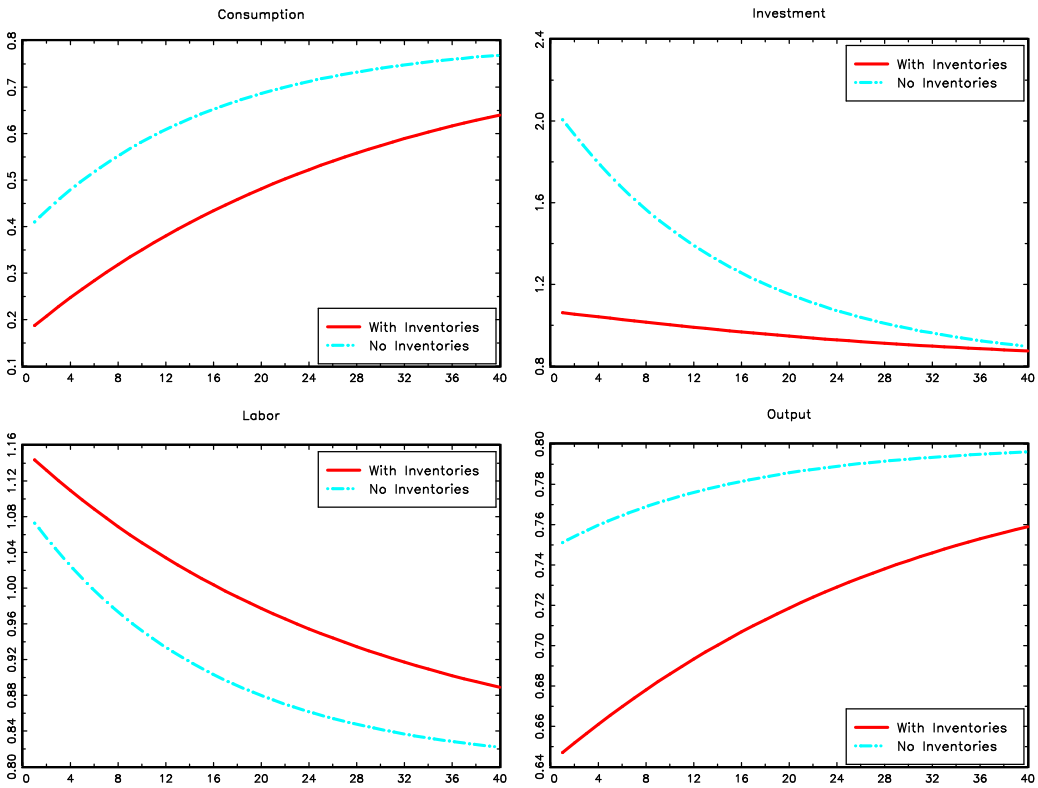

Figure 6. Contributions of Inventories to Stability.

5 Conclusion

This paper has developed an analytically tractable general-equilibrium model of input and output inventories with the stockout-avoidance motive. The model is shown broadly consistent with the stylized inventory behavior of the U.S. economy over the business cycle, such as, among other things, the excess volatility of production relative to sales, procyclical inventory investment and countercyclical inventory-to-sales ratio, more volatile input inventories than output inventories, and countercyclical inventory investment at the high frequencies.[43]

[43] While the model is broadly successful in explaining the key features of the business cycle and inventory behavior, there is still room for further improvements regarding the model's goodness of fit. Most notably, the volatility of inventory investment relative to production in the model is still significantly lower than that of the data. Re-calibrating the structural parameters of the model does not solve this problem completely. Also, the model with a single transitory $AR(1)$ shock is not as successful as that with a single permanent shock in explaining the business cycle and the inventory behavior. For example, under transitory $AR(1)$ demand shocks, although the inventory-to-sales ratio remains countercyclical and inventory investment remains procyclical, capital investment tends to be countercyclical because a sharp rise in consumption tends to crowd out aggregate savings. This is a typical problem of standard RBC models under demand shocks. Allowing for habit formation or introducing increasing returns to scale may resolve this problem (see, e.g., Benhabib and Wen, 2004; and Wen, 2006). Under transitory $AR(1)$ cost-push shocks, although capital investment as well as inventory investment remain procyclical, the inventory stock-to-sales ratio tends to become procyclical because a decrease in the current marginal cost relative to expected future marginal costs drives up the stock-to-sales ratio. Under transitory $AR(1)$ TFP shocks, input inventory investment becomes countercyclical unless the risk aversion parameter γ is further reduced from the benchmark value of

This paper has also uncovered an important general-equilibrium effect of inventories on the stability of the economy: the procyclical asset-pricing value of inventories under the stockout-avoidance motive. On the one hand, inventory behavior is destabilizing because it magnifies the variance of production through procyclical inventory investment; on the other hand, inventory behavior is stabilizing because it reduces the variance of final demand through the time-varying asset-price effects. When the stock-to-sales ratio in the model is countercyclical, the stabilizing effect dominates the destabilizing effect, leading to a more stabilized economy. Without a general-equilibrium analysis based on microfoundations of inventory behavior, such a stabilizing role of inventories is extremely difficult to imagine and detect.

Although the model may have shortcomings because of its extreme simplicity, its analytical tractability makes it easy to introduce inventories into more complicated DSGE models than the one studied in this paper, such as models with borrowing constraints, imperfect competition, firm entry and exit, money and sticky prices, international trade, and so on. Also, the approach can be used to study durable goods inventory behavior, which is another important long-standing puzzle of the business cycle (see, e.g., Feldstein and Auerbach, 1976). Given the sheer magnitude of inventory stocks in the economy and their potentially important role in understanding the business cycle, a business-cycle model without inventories is clearly incomplete and unsatisfactory. General-equilibrium analysis of the business cycle with inventories is still in its infant stage. Hopefully this paper will contribute to further research and development in this area.

General-equilibrium inventory theories are important for macroeconomics not only because inventories are an important component of aggregate fluctuations, but also because such theories can improve our understanding on other macroeconomic issues besides inventories, such as the phenomenon of money demand. The famous Baumol-Tobin model of money demand is based on the (S,s) inventory theory. The general-equilibrium inventory model developed in this paper can be used as an alternative framework for studying money demand under the liquidity preference.

0.5 toward zero. Based on these results, a multiple-shock model with a mixture of permanent and transitory demand and supply shocks may resolve these comovement problems. But this requires careful calibrations of the driving processes and relative variances of different types of shocks.

6 Appendix 1: Solving the Full Model

6.1 First-Order Conditions

Denoting $\{\lambda_1, \pi_1, \mu\}$ as the Lagrangian multipliers of Equations (23)-(25) for the household, respectively, the first-order conditions of the household with respect to $\{N, W, c(j), y(j), s_1(j)\}$ are given, respectively, by

$$aN_t^{\gamma_n} = \mu_t w_t \tag{29}$$

$$\mu_t = \beta E_t \mu_{t+1} (1 + r_{t+1}) \tag{30}$$

$$\Theta_t \tilde{C}_t^{1-\rho-\gamma} \theta_1(j) c(j)^{\rho-1} = \lambda_1(j) \tag{31}$$

$$\mu_t = E\lambda_{1t}(j) \tag{32}$$

$$\lambda_{1t}(j) = \beta(1-\delta)E_t\mu_{t+1} + \pi_{1t}(j), \tag{33}$$

plus the transversality condition $\lim_{T\to\infty} \beta^T E\mu_T W_{T+1} = 0, \lim_{T\to\infty} \beta^T E\lambda_{1T} s_{1T+1} = 0$, and the complementary slackness conditions, $s_{1t}(j)\pi_{1t}(j) = 0$ for all j. Equation (29) determines the optimal labor supply, (30) the optimal wealth accumulation, (31) the optimal level of consumption of color j, (32) the optimal orders of good with color j, and (33) the optimal inventory holdings of color j. Notice that the optimal orders are made before the realization of θ; hence, the household must form expectations regarding the shadow value of the final consumption good.

The first-order conditions for the final good firm with respect to $\left\{K, \tilde{M}\right\}$ are given by

$$r_t + \delta_k + \frac{\xi}{\bar{K}}(K_t - \bar{K}) = \alpha A_1 K_t^{\alpha-1} \tilde{M}_t^{1-\alpha}, \quad P_t^m = (1-\alpha) A_t K_t^{\alpha} \tilde{M}_t^{-\alpha}. \tag{34}$$

Denoting $\{\lambda_2, \pi_2\}$ as the Lagrangian multipliers for Equations (27) and (28), respectively, for the intermediate goods firm, the first order conditions with respect to $\{m(i), n(i), s_2(i)\}$ are given by

$$P_t^m \tilde{M}_t^{1-\rho} \theta_2(i) m(i)^{\rho-1} = \lambda_2(i) \tag{35}$$

$$\frac{w_t}{B_t} = E\lambda_2(i) \tag{36}$$

$$\lambda_{2t} = \beta(1-\delta)E_t \frac{\mu_{t+1}}{\mu_t} \lambda_{2t+1}(i) + \pi_{2t}(i), \tag{37}$$

plus a transversality condition, $\lim_{T\to\infty}\beta^T E\lambda_{2T}s_{2T}=0$, and the complementary slackness conditions, $s_{2t}(i)\pi_t(i)=0$ for all i. Equation (35) determines the optimal usage of the intermediate good with color i, equation (36) the optimal production of the intermediate good i, and (37) the optimal accumulation of inventories of work-in-process for color i.

6.2 Decision Rules of Inventories

The decision rules associated with inventories are derived in a similar manner as in the benchmark model. The decision rules for finished goods inventories are given by

$$\mu_t=\beta(1-\delta)E_t\mu_{t+1}R(\theta^*_{1t}), \tag{38}$$

$$y_t(i)+s_{1t-1}(i)=\tilde{C}_t\left[\frac{\Delta_t\theta^*_{1t}}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{39}$$

$$c_t(i) = \tilde{C}_t\left[\frac{\Delta_t}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}\times\min\left\{\theta_{1t}(i)^{\frac{1}{1-\rho}},\theta_{1t}^{*\frac{1}{1-\rho}}\right\}, \tag{40}$$

$$s_{1t}(i) = \tilde{C}_t\left[\frac{\Delta_t}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}\times\max\left\{\theta_{1t}^{*\frac{1}{1-\rho}}-\theta_{1t}(i)^{\frac{1}{1-\rho}},0\right\}, \tag{41}$$

where $\Delta\equiv\Theta_t\tilde{C}_t^{-\gamma}$ denotes the marginal utility of the composite consumption good $\tilde{C}\equiv\left[\int\theta c(i)^\rho di\right]^{\frac{1}{\rho}}$, and $R(\theta^*_1)$ the rate of return to inventory investment in finished goods. The optimal cutoff value (θ^*_1) in the finished goods industry is determined by equation (38).

The decision rules for input inventories are given by

$$\frac{w_t}{B_t}=\beta(1-\delta)E_t\tilde{\mu}_{t+1}R(\theta^*_{2t}) \tag{42}$$

$$Bn_t(i)+s_{2t-1}(i)=\tilde{M}_t\left[\frac{P_t^m\theta^*_{2t}}{\beta(1-\delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{43}$$

$$m_t(i) = \tilde{M}_t\left[\frac{P_t^m}{\beta(1-\delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}}\times\min\left\{\theta_{2t}^{*\frac{1}{1-\rho}},\theta_{2t}(i)^{\frac{1}{1-\rho}}\right\}, \tag{44}$$

$$s_{2t}(i) = \tilde{M}_t\left[\frac{P_t^m}{\beta(1-\delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}}\times\max\left\{\theta_{2t}^{*\frac{1}{1-\rho}}-\theta_{2t}(i)^{\frac{1}{1-\rho}},0\right\}, \tag{45}$$

where $\tilde{\mu}_{t+1} \equiv \frac{\mu_{t+1}}{\mu_t}\frac{w_{t+1}}{B_{t+1}}$ denotes the next-period marginal cost of labor discounted by the interest rate (the ratio of the marginal utilities of the final good) and $R(\theta_2^*)$ denotes the rate of return to inventory investment in goods-in-process. The optimal cutoff value (θ_2^*) in the input inventory industry is determined by equation (42). Notice that equations (38) and (42) are analogous to equation (13).

Market clearing in the asset and labor markets imply $W_t = K_t$ and $N_t = \int n(i)di$. Aggregating the decision rules (39)-(41) for the finished-good sector and (43)-(45) for the intermediate-good sector under the law of large numbers gives the aggregate decision rules in the main text.

7 Appendix 2: Proof of $\frac{dq}{d\theta^*}\frac{\theta^*}{q} < 0$.

By definition, $q(\theta^*) = D(\theta^*)G(\theta^*)^{-\frac{1}{\rho}}$. Hence, $\frac{dq}{d\theta^*}\frac{\theta^*}{q} = \frac{dD}{d\theta^*}\frac{\theta^*}{D} - \frac{1}{\rho}\frac{dG}{d\theta^*}\frac{\theta^*}{G}$. Under the Pareto distribution, we have $D(\theta^*) = \theta^{*(\frac{1}{1-\rho}-\sigma)} + x(\theta^*)$ and $G(\theta^*) = \theta^{*(\frac{1}{1-\rho}-\sigma)}\frac{\sigma}{\sigma-1} + x(\theta^*)$, where $x(\theta^*) \equiv \int_{\theta(i)\leq\theta^*}\theta(i)^{\frac{1}{1-\rho}}dF = \frac{\sigma}{\sigma-\frac{1}{1-\rho}}\left[1-\theta^{*(\frac{1}{1-\rho}-\sigma)}\right]$. Hence, $\frac{dD}{d\theta^*}\frac{\theta^*}{D} = \frac{\theta^{*(\frac{1}{1-\rho}-\sigma)}}{D(\theta^*)}\frac{1}{1-\rho}$ and $\frac{1}{\rho}\frac{dG}{d\theta^*}\frac{\theta^*}{G} = \frac{\theta^{*(\frac{1}{1-\rho}-\sigma)}}{G(\theta^*)}\frac{1}{1-\rho}\frac{\sigma}{\sigma-1}$. Therefore, $\frac{dq}{d\theta^*}\frac{\theta^*}{q} < 0$ if and only if $G(\theta^*) < \frac{\sigma}{\sigma-1}D(\theta^*)$, which is true because $\sigma > 1$.

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