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Predictability and Comovements:
Evidence from Ireland, the US, and the UK**

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Equity Portfolio Diversification under Time-Varying Predictability and Comovements

Evidence from Ireland, the US, and the UK*

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Abstract

We use multivariate regime switching vector autoregressive models to characterize the time-varying linkages among short-term interest rates (monetary policy) and stock returns in the Irish, the US and UK markets. We find that two regimes, characterized as bear and bull states, are required to characterize the dynamics of returns and short-term rates. This implies that we cannot reject the hypothesis that the regimes driving the markets in the small open economy are largely synchronous with those typical of the major markets. We compute time-varying Sharpe ratios and recursive mean-variance portfolio weights and document that a regime switching framework produces out-of-sample portfolio performance that outperforms simpler models that ignore regimes. Interestingly, the portfolio shares derived under regime switching dynamics implies a fairly low commitment to the Irish market, in spite of its brilliant unconditional risk-return trade-off.

Keywords: multivariate regime switching; Sharpe ratio; time-varying predictability.

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Abstract

We use multivariate regime switching vector autoregressive models to characterize the time-varying linkages among short-term interest rates (monetary policy) and stock returns in the Irish, the US and UK markets. We find that two regimes, characterized as bear and bull states, are required to characterize the dynamics of returns and short-term rates. This implies that we cannot reject the hypothesis that the regimes driving the markets in the small open economy are largely synchronous with those typical of the major markets. We compute time-varying Sharpe ratios and recursive mean-variance portfolio weights and document that a regime switching framework produces out-of-sample portfolio performance that outperforms simpler models that ignore regimes. Interestingly, the portfolio shares derived under regime switching dynamics implies a fairly low commitment to the Irish market, in spite of its brilliant unconditional risk-return trade-off.

1. Introduction

The effect of interest rates or monetary policy on stock returns is of great interest to both macroeconomists and financial economists. Moreover, with increasing financial integration, the transmission of shocks – of monetary origin or not – across markets has become the focus of extensive research. Our paper investigates the relationship between stock returns and short-term interest rates (taken as indicators of the stance of monetary policy) in the context of a small open economy, Ireland, linked to two major international markets, the US and the UK. Ireland seems to offer the ideal case of a small open economy with long-standing political and economic links with both the UK and US.¹ The linkages between such markets is of key importance to an understating and normative prescriptions for international portfolio diversification.

A number of recent papers have brought to the forefront the debate in international finance the fact that correlations could be strongly unstable. For instance, Longin and Solnik (1995) show that correlations between markets increase during periods of high market volatility, with the result that correlations are higher than average exactly in the moment when diversification promises to yield major gains. Such changes in correlations imply that the benefits to portfolio diversification may be rather modest during bear markets (see Butler and Joaquin, 2002). Therefore in this paper we adopt a multivariate Markov switching VAR approach that allows us to accurately model and understand the time-varying nature of the relationship between money and equity markets in a small open economy, Ireland, and the major Anglo-Saxon markets, the UK and the US. Given the importance of time-varying correlations (more generally, the moments of the density of returns and short-term rates) between returns for asset allocation decisions, we examine the implications of accounting for the regime-switching impact of monetary policy on stock returns for the portfolio decisions of an international equity investor.

A vast literature in finance has reported evidence of predictability in stock market returns, mostly in the context of linear, constant-coefficient models, see e.g. Fama and French (1989), Ferson and Harvey (1991), and Goetzmann and Jorion (1993). More recently, some papers have found evidence of regimes in the distribution of returns on individual asset returns or pairs of these (e.g., Guidolin and Timmermann 2005). It is also well known (see e.g. Keim and Stambaugh, 1986) that short-term interest rates are accurate and useful predictors of subsequent, realized excess equity returns. There is evidence that stock returns respond to monetary policy shocks (Bernanke and Kuttner, 2005; Bredin et al., 2007) while Rigobon and Sack (2003) provide evidence on the transmission mechanism between short-term interest rates and equity returns in the US and Ehrmann, Fratzscher and Rigobon (2005) document evidence of linkages between interest rates and equity returns across international markets. Moreover, recent papers by Ang and Bekaert (2002a), Bredin and Hyde (2007) and Guidolin and Timmermann (2008) document that such a relationship between international stock returns and interest rates may contain important nonlinear components that ought to be carefully modeled.

¹For instance, prior to Ireland joining the European Monetary System in 1979, the Irish punt was held at parity with the UK pound sterling. Moreover, the majority of Irish firms are listed on the London stock exchange in addition to Dublin's exchange. Ties with the US have been increasing significantly over the past three decades. By 1994, nearly a quarter of the Irish workforce were employed by US owned firms and in 1999 US foreign direct investment accounted for 90% of capital formation in Ireland.

Our paper has two main goals and – we believe – it advances the existing literature in three ways. Our first and main objective is to characterize within a coherent econometric framework the economic implications of the time-varying nature of the links between monetary policy (i.e., short-term interest rates) and international stock market dynamics. We perform this task using a case-study in which two major economies (hence, sources of real as well as financial shocks) – the UK and the US – are investigated along with one small open economy, Ireland, with strong real and financial ties with the UK and the US. Our notion of economic “value” (as in much recent literature, see e.g., Della Corte et al., 2007) consists of characterizing the effects for the time-varying price of risk (in a mean-variance framework, the Sharpe ratio) and for optimal portfolio choices of our econometric model of time-varying predictability from interest rates to stock returns.

Our second goal consists in documenting that econometric models in the Markov switching class first brought to the attention of applied financial and macro-economists by Hamilton (1991), may offer a convenient and useful framework within which to capture the time-varying and unstable nature of the links between monetary policy and equity markets. In our analysis, useful means that – when they are carefully specified and estimated – models with regimes may offer support to optimal financial and policy decisions. In particular, in our paper we document the out-of-sample performance of simple mean-variance portfolio strategies devised on the basis of our two-state Markov switching model and show (consistently with other papers in the literature, applied to different data sets, see e.g., Guidolin and Timmermann, 2007a) that the phenomena discussed in Sections 2 and 3 are strong enough to lead to improved decisions.

We claim at least three contributions to the extant literature. First, we show that regimes may exist in which monetary policy – even shocks to short-term rates emanating from the US, the country that seems to be exercising an uncontested co-leadership in setting global monetary and financial conditions – has only weak effects on international equity markets. Interestingly, such a regime corresponds to a state of bullish markets, characterized by positive and high excess mean returns. Second, although an expanding literature has been stressing the importance of capturing non-linear dynamics in optimal portfolio choice (see e.g., Detemple et al., 2003, Das and Uppal, 2004, and Guidolin and Timmermann, 2007a) in some cases highlighting the relevance of time-varying predictability from classical instruments (such as the dividend yield or short-term interest rates) to stock returns, ours is to our knowledge the first paper that explicitly characterizes the importance of regimes in the links between monetary policy shocks and equity markets for international portfolio diversification. Third, although the exceptional performance of the Irish stock market (as well its entire economy) starting from the 1990s has drawn considerable media coverage, the financial implications of such a performance for optimal portfolio choices have been under-researched.² It is clear, that if a market consistently offers Sharpe ratios above similar and related markets over time, a near-arbitrage opportunity arises for portfolio managers, while researchers ought to be interested in providing some sort of equilibrium justification for the apparent imbalance. We find that such differences in (average, unconditional) Sharpe ratios across the three equity markets investigated is merely illusory: the Irish performance becomes unfortunately the worst across the three markets while Irish equity returns turn highly correlated with foreign

²See for instance The Economist, “Green is Good”, May 17 1997, issue 8017.

ones exactly when the benefits of diversification ought to pay off, during bear markets.

Our main results may be summarized as follows. We find that a two-state switching VAR(1) model in which (besides the conditional means) covariances and variances also depend on the state (and therefore change over time) is strongly required by the data in order to provide a good fit (as stressed by standard in-sample criteria and tests) and possibly accurate predictions (as revealed by information criteria). The two regimes have an interpretation in terms of bear vs. bull states. Interestingly, while in the bear regime US monetary policy strongly affects all the stock markets with the expected, negative signs and short-term interest rates in other countries have weaker effects, in the bull state monetary policies have ambiguous and rather weak effects on stock markets. Consistently with other results that have appeared in the literature, both volatilities and correlations are above their unconditional, full-sample levels in the bear state, and viceversa in the bull regime.

We then proceed to the recursive calculation of one-month ahead, predicted Sharpe ratios derived under the switching VAR(1) model. We find that the equity Sharpe ratios for the three markets under investigation strongly comove, although correlations remain substantially below one. This suggests that the ISEQ seems to compensate risk in ways that are perfectly consistent with the ratios that are typical of major, developed markets. Also, equity returns correlations systematically below 1 with Sharpe ratios that are similar across markets suggest the existence of enormous potential for international portfolio diversification. When we proceed to compare the evolution of mean-variance portfolio weights induced by the switching VAR and by one natural benchmark (a standard, single-state VAR(1)), differences are striking: the single-state model generates a moderate demand for stocks, 13 percent for Ireland and 16 percent for the S&P 500. A regime switching model implies larger equity weights (on average 40-50%, although rich temporal dynamics can be detected). Interestingly, while single-state models that ignore the presence of regime switching patterns may imply a substantial commitment to the Irish stock market (e.g., 64% in some cases under a VAR(1) model), our baseline model leads to investing only 20-30% of the optimal portfolio in the “Celtic Tiger”.

Finally, a key result is that the recursive, out-of-sample performance of switching VAR portfolios turns out to be systematically superior to the one produced by models that ignore the evidence of regimes. For instance, under moderate risk aversion and a no short sale constraint, a regime switching asset allocation scheme obtains a 1.19 percent per month average performance (higher than 1.06 for a VAR strategy), with an implied Sharpe ratio of 0.68 vs. 0.66 for a single-state VAR(1). Therefore the statistical features that led us to specify and analyse a regime switching VAR seem also to lead to improved out-of-sample predictions and hence to improved portfolio performance. The intuition for these findings is simple. First, the ISEQ index is different from, riskier than, the FTSE and the S&P500 because the mean differentials when going from one state to the other is maximum in the Irish case, e.g., 0.95% per month in the case of Ireland and the UK excess returns vs. 0.31% in the case of the US. Such large jumps in means obviously add to the overall risk/variance of Irish excess returns. Second, a similar phenomenon involves variances, in the sense that the volatility of ISEQ excess returns is the one that depends the most on regimes and this adds to their overall risk. Third, correlations among shocks to excess market returns are systematically higher in bear

states than in bull ones, which implies that diversification pays out the least exactly when it is needed the most. The result of these three concurring effects is that a risk-averse portfolio optimizer will simply bias portfolio weights away from a well-diversified pattern and towards the market with the highest Sharpe ratio, in the case the S&P500.

Two closely related papers are Ang and Bekaert (2002a, AB) and Guidolin and Nicodano (2007, GN). Discussing relationships and differences to these papers also helps highlight the contributions of our paper. AB consider bivariate and trivariate regime models that capture asymmetric correlations in volatile and stable markets and characterize a US investor's optimal asset allocation under power utility. Our focus is distinctly on the dynamic linkages between the market of a small open and major stock markets representative of countries with which the real ties are strong. Moreover, differently from AB, we directly model the regime switching behavior of the vector-autoregressive predictive relationships that link short-term interest rates to excess stock returns. Therefore, while AB's main concern is simply with correlations, our focus is nonlinearities which may simultaneously affect means, variances, as well as covariances. GN show that predictable covariances between means and variances of stock returns within a regime switching framework may have a first-order effect on the composition of equity portfolios. In an international asset menu that includes both European and North American small capitalization stock indices, they find that small cap portfolios become riskier in bear markets, i.e. display negative co-skewness with other stock indices. On the contrary small caps command large optimal weights when the investor ignores variance risk, by incorrectly assuming joint normality of returns and focussing on Sharpe ratios only. The main results of our paper are similar to GN's: we find a portfolio – the ISEQ index – with good risk-return properties that ends up receiving a rather moderate weight on the account of its poor regime-switching induced properties. Apart from differences in the application, also in this case the major difference is on the explicit focus on time-variations in predictive relationships between excess stock returns and short-term interest rates.

The paper has the following structure. Section 2 provides a brief introduction on switching VAR models, on their differences with competing non-linear models, and on the literature linking monetary policy and stock markets. After an introduction to the data employed in the paper, Section 3 reports the main body of empirical results of the paper. Section 4 is devoted to the economic implications of our econometric results, in particular to predicting Sharpe ratios useful in portfolio choice, and to calculating and assessing the recursive out-of-sample performance of portfolio strategies that rely on different statistical models. Section 5 concludes.

2. The Framework of Analysis

As discussed in a number of papers (among the recent examples, see Lobo, 2002, and Bernanke and Kuttner, 2005) monetary policy shocks are transmitted to equity valuations through a number of alternative channels such as: changes in the future, expected cost of capital (if monetary policy affects future, real discount rates); changes in future, expected cash flows due to changes in the strength of composition of the aggregate demand for goods and services; changes in the values and composition of optimal private portfolios (e.g., in the relative weights of bonds vs. equities). The first two channels have been discussed in the empirical literature at least

since the seminal paper by Smirlock and Yawitz (1985). In fact, Rigobon and Sack (2004) stress that much of the transmission of monetary policy comes through the influence of short-term interest rates on other asset prices, as it is the movements in these other asset prices – including stock prices – that determine private borrowing costs and changes in wealth, which in turn importantly influence real economic activity. This implies that not only monetary policy shocks happen to produce such effects on equity prices, but also that this channel is *de facto* crucial to the very effectiveness of monetary policy. In Rigobon and Sack (2004) the main finding is that equity indices display a statistically significant and economically important negative, linear reaction to a tightening monetary policy shock.

However, a novel strand of papers have also reported that monetary policy shock would not always affect stock returns in the same fashion and with constant strenght. For instance, using tick-by-tick data, Fleming and Piazzesi (2005) find that policy surprises have a weaker effect on the price of long-term assets (in their paper, long yields on Treasury notes) when the yield curve is steep than when the yield curve is flat or “inverted”. In fact, their results show that long yields tend to react positively to surprises, but negatively to surprises interacted with slope, which is a rudimentary way to capture non-linear effects. Fleming and Piazzesi’s interpretation is that the yield curve slope is correlated with market participants’ time-varying concerns about inflation, although it is well known that such a slope also correlates to expectations concerning expansions and recessions. In a framework that shares some of our goals, Chen (2007) documents (using both Markov switching and time-varying parameter models) on US stock return (S&P 500) data that monetary policy has larger effects on stock returns in a bear market. Furthermore, Chen shows that a contractionary monetary policy leads to a higher probability of switching to a bear-market regime.

These new results in the literature on stock markets and monetary policy have opened a possible debate on what could be the most suitable econometric framework to adequately describe and predict the links between monetary shocks and equity returns. Although most of the literature has employed so far rather informal linear empirical frameworks in which stock returns are simply regressed over measures of exogeneous monetary policy shocks, more recently a factor-type regression framework has emerged as a common workhorse (see e.g., Ehrmann and Fratzscher, 2006):

$$\begin{aligned}
 \mathbf{x}_t &= \boldsymbol{\alpha}_0 + \mathbf{B}\mathbf{S}_t + \boldsymbol{\Gamma}\boldsymbol{\nu}_t + \sum_{j=1}^q \boldsymbol{\Phi}_j \mathbf{z}_{t-j} + \boldsymbol{\varepsilon}_t \\
 \mathbf{r}_t &= \boldsymbol{\alpha}_1 + \sum_{j=1}^{q_1} \boldsymbol{\Lambda}_j \mathbf{z}_{t-j} + \mathbf{S}_t \\
 \mathbf{r}_t &= \boldsymbol{\alpha}_2 + \sum_{j=1}^{q_2} \boldsymbol{\Upsilon}_j \mathbf{x}_{t-j} + \boldsymbol{\nu}_t.
 \end{aligned} \tag{1}$$

In this model \mathbf{S}_t is a vector of exogenous (orthogonalized) monetary policy shocks, $\boldsymbol{\nu}_t$ is a vector of monetary policy effects caused by equity market shocks, and \mathbf{z}_t is a vector of controls that collects past stock returns, short-term interest rates, seasonal effects, etc. Notice that \mathbf{x}_t collects the excess stock returns under investigation, while \mathbf{r}_t the short-term rates.³ All other matrices and vector variables have obvious interpretation.

³Formally, \mathbf{x}_t is a vector of n (excess) stock index returns, $\mathbf{x}_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$, and \mathbf{r}_t collects l short-term interest rates,

If one simply plugs into the model for excess stock returns the definitions for the shocks \mathbf{S}_t and $\boldsymbol{\nu}_t$ implied by the two remaining (vector) equations, we have:

$$\begin{aligned}\mathbf{x}_t &= \boldsymbol{\alpha}_0 + \mathbf{B} \left(\mathbf{r}_t - \boldsymbol{\alpha}_1 - \sum_{j=1}^{q_1} \boldsymbol{\Lambda}_j \mathbf{z}_{t-j} \right) + \boldsymbol{\Gamma} \left(\mathbf{r}_t - \boldsymbol{\alpha}_2 - \sum_{j=1}^{q_2} \boldsymbol{\Upsilon}_j \mathbf{x}_{t-j} \right) + \sum_{j=1}^q \boldsymbol{\Phi}_j \mathbf{z}_{t-j} + \boldsymbol{\varepsilon}_t \\ &= \boldsymbol{\alpha}_0 - \mathbf{B}\boldsymbol{\alpha}_1 - \boldsymbol{\Gamma}\boldsymbol{\alpha}_2 + (\mathbf{B} + \boldsymbol{\Gamma})\mathbf{r}_t + \sum_{j=1}^q \boldsymbol{\Phi}_j \mathbf{z}_{t-j} - \mathbf{B} \sum_{j=1}^{q_1} \boldsymbol{\Lambda}_j \mathbf{z}_{t-j} - \boldsymbol{\Gamma} \sum_{j=1}^{q_2} \boldsymbol{\Upsilon}_j \mathbf{x}_{t-j} + \boldsymbol{\varepsilon}_t,\end{aligned}$$

which can be re-written as (assuming $q = q_1 = q_2$)

$$\begin{aligned}\mathbf{x}_t - (\mathbf{B} + \boldsymbol{\Gamma})\mathbf{r}_t &= \boldsymbol{\Pi}\mathbf{y}_t = (\boldsymbol{\alpha}_0 - \mathbf{B}\boldsymbol{\alpha}_1 - \boldsymbol{\Gamma}\boldsymbol{\alpha}_2) + \sum_{j=1}^q (\boldsymbol{\Phi}_j - \mathbf{B}\boldsymbol{\Lambda}_j - \boldsymbol{\Gamma}\boldsymbol{\Upsilon}_j) \mathbf{z}_{t-j} + \boldsymbol{\varepsilon}_t \\ &= \boldsymbol{\alpha} + \sum_{j=1}^q \mathbf{C}_j \mathbf{z}_{t-j} + \boldsymbol{\varepsilon}_t\end{aligned}\tag{2}$$

where $\boldsymbol{\alpha} \equiv \boldsymbol{\alpha}_0 - \mathbf{B}\boldsymbol{\alpha}_1 - \boldsymbol{\Gamma}\boldsymbol{\alpha}_2$, $\mathbf{C}_j \equiv \boldsymbol{\Phi}_j - \mathbf{B}\boldsymbol{\Lambda}_j - \boldsymbol{\Gamma}\boldsymbol{\Upsilon}_j$ ($j = 1, \dots, q$) and $\boldsymbol{\Pi}$ is an appropriate matrix that absorbs the contemporaneous effects represented by the coefficient matrices $\mathbf{B} + \boldsymbol{\Gamma}$. One can easily see that a generalized version of such an econometric model is the one in which $\mathbf{y}_t \equiv (\mathbf{x}'_t \mathbf{r}'_t)'$ is represented as a simple, unrestricted VAR(q)

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{j=1}^q \mathbf{A}_j \mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t$$

obtained from (2) by setting $\boldsymbol{\mu} \equiv \boldsymbol{\Pi}^{-1}\boldsymbol{\alpha}$ and $\mathbf{A}_j \equiv \boldsymbol{\Pi}^{-1}\mathbf{C}_j$ ($j = 1, \dots, q$). For instance, Ehrmann, Fratzscher, and Rigobon (2005) have recently used switching VAR models and found that – although stock returns react most strongly to domestic financial shocks – spillovers both within and across asset classes are substantial.

The empirical findings of papers such as Fleming and Piazzesi (2005) and Chen (2007) imply that some doubts should exist on the fact that the coefficients collected in the matrices $\boldsymbol{\mu}$ and $\{\mathbf{A}_j\}_{j=1}^q$ may actually be constant over time. In this paper we extend the existing empirical literature by admitting the possibility that different regimes may exist across which the VAR(q) coefficients (as well as the variances and covariances of the shocks in $\boldsymbol{\varepsilon}_t$) may differ. Our methodology extends the class of multivariate Markov switching models studied by Hamilton (1991) and Krolzig (1997) to investigate the time-varying nature of the relationship between equity returns and short-term interest rates in a small open economy, Ireland and two major markets, the UK and the US. The joint distribution of a vector of n (excess) stock index returns, \mathbf{x}_t , and l short-term interest rates, \mathbf{r}_t , is modeled as a multivariate regime switching process driven by a common discrete state variable, s_t , that takes integer values between 1 and k . We define the $(n + l) \times 1$ vector of state variables $\mathbf{y}_t \equiv (\mathbf{x}'_t \mathbf{r}'_t)'$ and write the switching VAR(q) model as:

$$\mathbf{y}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^q \mathbf{A}_{j,s_t} \mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t\tag{3}$$

$\mathbf{r}_t = (r_{1t}, r_{2t}, \dots, r_{lt})'$.

Here $\boldsymbol{\mu}_{s_t} = (\mu_{1,s_t}, \dots, \mu_{n+l,s_t})'$ is a vector of intercepts in state s_t , \mathbf{A}_{j,s_t} is an $(n+l) \times (n+l)$ matrix of autoregressive coefficients at lag j in state s_t and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \dots, \varepsilon_{n+l,t})' \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{s_t})$ is the vector of return innovations that are assumed to be joint normally distributed with zero mean vector and state-specific, time-varying covariance matrix $\boldsymbol{\Sigma}_{s_t}$. Innovations to returns are thus drawn from a Gaussian mixture distribution that is known to be capable of providing a flexible approximation to a wide class of distributions, see Timmermann (2000). Importantly, it is well known that mixtures of conditionally Gaussian densities can approximate highly non-Gaussian unconditional multivariate distributions rather well. In our application, $n = 3$, $l = 3$ (Ireland, UK, and US), so that we end up modeling a 6×1 vector \mathbf{y}_t .⁴

Moves between states are assumed to be governed by the $k \times k$ transition probability matrix, \mathbf{P} , with generic element p_{ji} defined as

$$p_{ji} \equiv \Pr(s_t = i | s_{t-1} = j), \quad i, j = 1, \dots, k. \quad (4)$$

Each regime is hence the realization of a first-order Markov chain. Our estimates allow s_t to be unobserved and treat it as a latent variable. This feature corresponds to the common observation that although nonstationarities and regime shifts seem to be pervasive, they remain extremely difficult to predict and even pin down once they take place. Notice that when $q \geq 1$, (3) - (4) captures the intuitive idea that it may be that the predictive relationship linking past stock returns and short-term interest rates that may be changing over time as a function of the Markov state variable. Crucially, having $\boldsymbol{\Sigma}_{s_t}$ depend on s_t helps the modeler to capture any heteroskedasticity patterns of variations of (shock) variances across bull and bear markets, besides the now well-documented finding by Longin and Solnik (1995) that correlations would be strongly affected by the market state.

(3) - (4) nests several popular models from the literature as special cases. In the case of a single state, $k = 1$, we obtain a linear vector autoregression (VAR) with predictable mean returns provided that there is at least one lag for which $\mathbf{A}_j \neq 0$. This type of statistical framework has been employed e.g. by Lund and Engsted (1996) and more recently to a problem related to ours by Ehrmann, Fratzscher, and Rigobon (2005) and Ehrmann and Fratzscher (2006). In the absence of significant autoregressive terms ($q = 0$), the discrete-time equivalent of the standard IID Gaussian model adopted by much of the mean-variance based literature obtains.

Switching VAR models are estimated by maximum likelihood. As shown by Hamilton (1993), the relevant algorithms are considerably simplified if (3) is first put in its state-space form. In particular, estimation and inferences are based on the EM (Expectation-Maximization) algorithm proposed by Hamilton (1993), a filter that allows the iterative calculation of the one-step ahead forecast of the state vector

$$\boldsymbol{\xi}_t = [I(s_t = 1) I(s_t = 2) I(s_t = k)]'$$

where $I(s_t = i)$ is a standard indicator variable, given the information set \mathcal{F}_t and the consequent construction

⁴We assume the absence of roots outside the unit circle, thus making the process stationary. Ang and Bekaert (2002b) have recently shown that formally, it is just sufficient for such a condition to be verified in at least one of the k alternative regimes, for covariance stationarity to obtain.

of the log-likelihood function of the data. As for the properties of the resulting ML estimators, under standard regularity conditions (such as identifiability, stability and the fact that the true parameter vector does not fall on the boundaries) Hamilton (1993) and Leroux (1992) have proven consistency and asymptotic normality of the ML estimator $\hat{\boldsymbol{\theta}}$ (which is defined to collect all of the unknown parameters in (3))

$$\sqrt{T} \left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta} \right) \xrightarrow{d} N \left(\mathbf{0}, \mathcal{I}_a(\boldsymbol{\theta})^{-1} \right),$$

where $\mathcal{I}_a(\boldsymbol{\theta})$ is the asymptotic information matrix. In our empirical results we are going to provide standard results based on a ‘sandwich’ sample estimator of $\mathcal{I}_a(\boldsymbol{\theta})$ by which:⁵

$$\widetilde{Var}(\hat{\boldsymbol{\theta}}) = T^{-1} \left[\mathcal{I}_2(\hat{\boldsymbol{\theta}}) \left(\mathcal{I}_1(\hat{\boldsymbol{\theta}}) \right)^{-1} \mathcal{I}_2(\hat{\boldsymbol{\theta}}) \right],$$

where

$$\mathcal{I}_1(\hat{\boldsymbol{\theta}}) \equiv T^{-1} \sum_{t=1}^T \left[\mathbf{h}_t(\hat{\boldsymbol{\theta}}) \right] \left[\mathbf{h}_t(\hat{\boldsymbol{\theta}}) \right]' \quad \mathbf{h}_t(\hat{\boldsymbol{\theta}}) \equiv \frac{\partial \ln p(\mathbf{y}_t | \mathcal{F}_{t-1}; \hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}} \quad \mathcal{I}_2(\hat{\boldsymbol{\theta}}) \equiv -T^{-1} \sum_{t=1}^T \left[\frac{\partial^2 \ln p(\mathbf{y}_t | \mathcal{F}_{t-1}; \hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \right],$$

($p(\mathbf{y}_t | \mathfrak{F}_{t-1}; \tilde{\boldsymbol{\gamma}})$ is the conditional density of the data).

Under a mean squared forecast error (MSFE) criterion, the forecasting algorithms are simple in spite of the nonlinearity of these processes. Considering the process in (3), the function minimizing the MSFE is the standard conditional expectation function. For instance, for a one-step ahead forecast:

$$E[\mathbf{y}_{t+1} | \mathcal{F}_t] = \mathbf{X}_{t+1} \hat{\boldsymbol{\Psi}} \left(\hat{\boldsymbol{\xi}}_{t+1|t} \otimes \boldsymbol{\nu}_{l+q} \right)$$

where $\mathbf{X}_{t+1} = [1 \ \mathbf{y}'_t \dots \mathbf{y}'_{t-p+1}] \otimes \boldsymbol{\nu}_{l+n}$, $\hat{\boldsymbol{\Psi}}$ collects the estimated conditional mean parameters ($\boldsymbol{\mu}_i$ and \mathbf{A}_{ji} , $i = 1, \dots, k$) stacked in appropriate ways, and $\hat{\boldsymbol{\xi}}_{t+1|t}$ is the one-step ahead, predicted latent state vector to be filtered out of the available information set \mathcal{F}_t according to transition equation $\hat{\boldsymbol{\xi}}_{t+1|t} = \hat{\mathbf{P}}' \hat{\boldsymbol{\xi}}_{t|t}$, where also the transition matrix \mathbf{P} has to be estimated.

3. Empirical Results

3.1. The data

We use monthly series on Irish, US, and UK nominal stock returns and short-term interest rates for the period 1978:05-2004:12. In particular, we focus on continuously compounded total (inclusive of dividends and all distributions) returns on the Dublin ISEQ, the US S&P 500, and the UK FTSE 100 stock market indices and Irish and UK money market (the equivalent to federal funds) rates and the US FED funds rate. All data series are obtained from Datastream. Table 1 reports summary statistics for all series under consideration. Consistently with the literature on stock return predictability, we investigate the properties of *excess* equity returns. To make the table easy to read, the statistics refer to monthly percentage returns.

⁵Under the null of no misspecification, $\mathcal{I}_1(\hat{\boldsymbol{\theta}})$ and $\mathcal{I}_2(\hat{\boldsymbol{\theta}})$ should be identical. Since in our paper we do not perform misspecification tests based on the ‘distance’ between $\mathcal{I}_1(\hat{\boldsymbol{\theta}})$ and $\mathcal{I}_2(\hat{\boldsymbol{\theta}})$, we base our inferences on the ‘sandwich’ form.

The three markets display similar median excess returns, in the order of 8-9 percent a year, i.e. values consistent with standard evidence of a high equity premium.⁶ Some structural differences are displayed by the volatility coefficients, a textbook annualized value of 15 percent for the US index, 17 percent for the UK, and more than 18 percent for Irish excess returns. As a result the (median-based, annualized) Sharpe ratios range from 0.49 for the UK to 0.58 for the US (the Irish index is 0.53). However, should we add confidence bands around such values, we would fail to find significant differences among these reward-to-risk ratios, which appear to cluster around a ‘typical’ 0.5 per year value. This feature suggests that in a portfolio logic, an investor might derive substantial benefits from a strategy that diversifies across these three equity portfolios.

However, Panel A of Table 1 also shows that such a simplistic approach may be inappropriate, as the three indices also display asymmetric, left-skewed, and fat-tailed distributions. In particular, excess Irish returns show a large and statistically significant negative skewness (-1.5) and a large kurtosis (12.4) that exceeds the Gaussian benchmark (three) with a negligible p-value. The values of skewness and kurtosis for UK and US excess returns are less impressive, but they still bring to stark rejections (using a standard Jarque-Bera test) of the null hypothesis that each of these univariate series may have a Gaussian unconditional distribution.⁷

Panel B of Table 1 reports summary statistics for short-term interest rates. Means and medians fit classical values of 7-8 percent a year. Irish and US short-term yields are also highly non-normal, positively skewed (1.5 and 0.9) and fat-tailed (kurtosis coefficients are 8.8 and 4.0). However, in this case we obtain for all series evidence that short yields are highly serially correlated (with near-unit root properties) and present strong ARCH effects. However, as argued by many recent papers (e.g. Gray, 1996, and Guidolin and Timmermann, 2007b), such properties are also potentially consistent with the presence of regimes in the (joint) distribution of short-term yields.

Panel C concludes by showing simultaneous correlation coefficients among excess equity returns and short yields. Excess stock returns are generally positively correlated, with coefficients between 0.54 (Ireland-US) and 0.70 (US-UK). Even such a large value implies the existence of substantial international diversification opportunities. A similar remark applies to correlations among interest rates (generally around 0.7). Finally, short-term yields tend to imply simultaneous movements in opposite direction of excess stock returns, although the correlation coefficients are small (generally around -0.1) and weakly significant from a statistical viewpoint.

3.2. *A regime switching vector autoregressive model*

Specification tests are performed for VAR switching models as in (3) when both the number of states k and the number of VAR lags q are allowed to change.⁸ Table 2 reports these results. The null of a single regime

⁶Mean (as opposed to median) excess equity returns are surprisingly low (less than 1 percent) for the UK. This is caused by 2 extreme observations (of -19 and -16 percent) that lie more than 3 standard deviations away from the mean.

⁷The Jarque-Bera statistics are 1301, 594, and 231 for Ireland, UK, and US, respectively. The associated p-values are always essentially zero.

⁸We experiment with a variety of models. Because it is well known that breaks in constant coefficients may create an illusion of vector autoregressive components being needed, we try a variety of MMSI($k, 0$) models, i.e., homoskedastic, with regimes

is decidedly rejected for all models estimated. Even for the worst fitting MMSI(2,0) model (where MMS stands for ‘Multivariate Markov Switching’), the LR statistic takes a value of 441, which is hardly compatible with the null of a single state, even taking into account nuisance parameters issues.⁹ Indeed, the growth in the maximized log-likelihood function and the decline in information criteria is impressive when moving from single-state models (e.g., MMSIA(1,2) which is a simple Gaussian homoskedastic VAR(2)) to two- and three-state models. However, the information criteria appear to be split: while the parsimonious BIC and H-Q criteria select a MMSIAH(2,1) model, i.e.

$$\mathbf{y}_t = [S_t\boldsymbol{\mu}_1 + (1 - S_t)\boldsymbol{\mu}_2] + [S_t\mathbf{A}_1 + (1 - S_t)\mathbf{A}_2]\mathbf{y}_{t-1} + [S_t\boldsymbol{\Omega}_1 + (1 - S_t)\boldsymbol{\Omega}_2]\boldsymbol{\varepsilon}_t,$$

where $S_t = 1$ in state 1 and zero otherwise (i.e., S_t is defined as the first element of $\boldsymbol{\xi}_t$), $\boldsymbol{\Omega}_{s_t}$ is the Choleski factor decomposition of $\boldsymbol{\Sigma}_{s_t}$, the AIC indicates that modeling three different regimes within a MMSIAH(3,2) framework might improve the fit. While the tendency of AIC to select absurdly large nonlinear models has been noticed before in semi-nonparametric contexts (see Fenton and Gallant, 1996), a three-state VAR(2) model with regime-switching covariance matrix implies the need to estimate as many as 303 parameters, which is clearly a serious challenge. To keep the saturation ratio (the ratio between the number of useful observations and the parameters estimated, a non-linear analog to the concept of ‘degrees of freedom’) sufficiently high (15 in the MMSIAH(2,1) vs. only 6 in the MMSIAH(3,2) case) and for consistency with the battery of information criteria results in Table 2, we entertain in what follows a relative simply two-state switching VAR(1) with regime-dependent covariance matrix.¹⁰

Notice that the weak support for richer models with three or four states represents in itself a rather meaningful result: we find no evidence that our small-open economy stock or money markets would command by itself the presence of specific states apt to describe its regime shifts between bear and bull states. As argued in Guidolin and Timmermann (2006), the presence of important asynchronicities among the major, developed market regimes and the patterns of time-variation in smaller markets is bound to lead to the specification of three or even four different states. The fact that a simple two-state, “bull & bear” model is sufficient to capture the nonlinear dynamic properties of \mathbf{y}_t implies that the same underlying state variable seems to characterize many stock and money markets in the world.¹¹

Parameter estimates are reported in Table 3. Panel A presents a benchmark single-state VAR(1) model, in means only, and $q = 0$. Given the widespread evidence of time-varying variances and correlations, we estimate a range of MMSIH($k, 0$) models, i.e. models with regime-dependent constants and covariances matrices but $q = 0$. MMSIAH(k, q) incarnate the presence of switching in both constants and covariance matrices, as well as in VAR coefficient matrices.

⁹In Table 2, tests of linearity are performed using Davies’s (1977) upper bound for the significance level of the LR test under nuisance parameters:

$$\Pr(LR > x) \leq \Pr(\chi_1^2 > x) + \sqrt{2x} \exp\left(-\frac{x}{2}\right) \left[\Gamma\left(\frac{1}{2}\right)\right]^{-1}.$$

where $\Gamma(\cdot)$ is the standard gamma function.

¹⁰Additionally, this model retains the intuitive interpretation of regimes as bear and bull states.

¹¹This finding is consistent with results in Guidolin and Timmermann (2008) that simple two-state, “bull & bear” models can adequately capture the properties of relatively long vectors of equity index returns (e.g. also including Asian, Japanese, and continental European stock returns).

while panel B is devoted to the regime switching estimates. Panel A gives already interesting indications: the US short-term interest rate predicts with the expected sign (negative, as implied by a simple version of Gordon’s dividend discount model) excess stock returns only in the US case; the coefficient is large (in absolute value) and highly significant. The UK rate also (weakly) influences US stock markets, although the sign is difficult to interpret. Irish excess returns display a positive serial correlation coefficient, while the opposite applies to UK excess returns. All short rates are highly serially correlated, although distant from containing a unit root. While US monetary policy influences interest rates in all other countries with the expected sign (positive, see Obstfeld and Rogoff (1995), Canova and De Nicoló (2000) and Kim (2001)), there is evidence that also UK policy impacts Irish short-term rates, consistently with Walsh (1993). Shocks to interest rates are only weakly correlated with excess return shocks.

Panel B of Table 3 presents regime switching estimates. Both states are persistent, with an average duration of 30 months for state 1 and 10 months for state 2. Their interpretation is made easy by computing “within-state” unconditional (monthly) means:

$$E[\mathbf{y}_t|S_t = 1] = [0.39 \ -0.40 \ 0.51 \ 0.33 \ 0.47 \ 0.32]'$$

$$E[\mathbf{y}_t|S_t = 2] = [-0.56 \ 0.56 \ 0.20 \ 1.34 \ 0.95 \ 0.87]'$$

(the first three elements are equity risk premia). It is natural to start the interpretation from the second state, when nominal interest rates are high (double digit, from 10.4 percent in the US to 16.1 percent in Ireland, in annualized terms) and risk premia are low in the US (2.4 percent per year) and in Ireland (negative, -6.7 percent); on the opposite, the UK risk premium is high (6.7 percent). The state probability plots (Figure 1, bear state plot) clearly identify this state with the period 1978-1984, although the state episodically reappears for a few months in the late 1980s, early 1990s, late 1990s, and especially in correspondence to the recent 2001-2002 recession and financial market crises. The fact that in an otherwise bear state, the FTSE risk premium is positive and substantial is well explained by the different reaction of the countries involved to the oil shocks of the late 1970s and early 1980s, and with the real-side effects of the exploitation of the North Sea gas reserves by the UK: for instance, while the sample mean of excess equity returns over 1978:05 - 1984:12 has been -9 and -4 percent (in annualized terms) for the ISEQ and the S&P 500, respectively, the figure turns positive (3.2 percent) for the UK. On the other hand, it is well known that nominal interest rates soared to double-digit heights over this very period, as a reaction to the inflationary pressures caused by the supply-side shocks and the subsequent anti-inflationary stance assumed by the FED.

In this bear state, US monetary policy strongly affects all the stock markets with the expected, negative sign.¹² Interestingly, the corresponding coefficients are rather similar, in the range -4.5 to -5.5 and imply that a one standard deviation expansionary impulse to US short-term rates would cause a stock market reaction that goes from a +1.4 percent in the UK to a +1.8 percent in the US. Also, UK monetary policy responds somewhat to US policy. However, this is not the case for Irish policy, a finding probably explained

¹²The VAR coefficient estimates should be read in the following way: the coefficient illustrates the impact of a change in the variable listed in the corresponding column on the variable listed in the corresponding row.

by the different degree of accommodation of the inflationary phenomenon in the early part of our sample. The UK rate also influences all the stock markets under analysis. Finally, in the bear state interest rates appear to be somewhat volatile, while the pairwise correlations involving excess equity returns are below their unconditional counterparts.

The first state is a regime of low nominal interest rates (between 4 and 6 percent per year) and high risk premia in Ireland and the US (4.7 and 6.1 percent per year, respectively). However, the UK risk premium is negative.¹³ State probability plots confirm that roughly 90% of the period 1985-2004 following the oil shocks is in fact captured by this regime. In this state, international monetary policy has ambiguous effects on stock prices: while in the US the channel work in the traditional way (i.e. higher interest rates cause negative excess stock returns, although with a coefficient that is approximately half the coefficient that is estimated in the second state), Irish excess returns seem to react positively to monetary policy tightening in the US and negatively (and significantly) to tightening in the UK, and once more FTSE excess returns positively respond to domestic increases in the interest rate. In this state, interest rates are more highly serially correlated than they are in unconditional terms and display low volatility.

A number of empirical studies have emphasized the dominant position of the US markets and monetary policies and the potential spillover effects that shifts in interest rates targets, for instance, may have on the Eurozone (including Ireland and the UK). For instance, Ehrmann, Fratzscher, and Rigobon (2005) have recently used switching VAR models and found that – although stock returns react most strongly to domestic financial shocks – spillovers both within and across asset classes are substantial and that US markets remain the main driver of global financial markets. These results are confirmed by Ehrmann and Fratzscher (2006) who analyze data for 50 different stock markets and find that the transmission of global monetary policy shocks to stock returns is particularly strong when the US short-term interest rates react strongly (which is obvious when the shock does originate in the US). Our empirical findings in Tables 3 are entirely consistent with these strands of the empirical literature. Table 3 shows that US monetary policy shocks exert a type of primacy in terms of their effects on stock markets: for instance, out of six possible VAR coefficients measuring the effect of a month t shock to the US short-term rate onto month $t + 1$ excess stock returns (i.e., three stock markets times two regimes), all of them are significant at a 5% test size, and four are highly significant (those from the bull state, when a policy tightening leads to subsequently lower excess equity returns). Clearly, movements in Irish short term rate effects produce weak and insignificant effects, while UK monetary policy has intermediate effects, often statistically significant but economically weaker. Moreover, while the US policy does not seem to respond to UK monetary policy, UK policy does react to US policy and with the expected, positive sign.

¹³This fact is unsurprising: after the oil shocks (i.e. 1985-2004) the sample average of the FTSE-100 excess return has been a meager -0.08 percent per month.

3.3. Diagnostic checks and residual ARCH effects

As shown by Krolzig (1997) standard, residual-based diagnostic checks are made difficult within the MMS class by the fact that in (3) $\boldsymbol{\varepsilon}_t \sim \text{i.i.d. } N(0, \boldsymbol{\Sigma}_{s_t})$ only within a given regime. Since for most times t , the vector of state probabilities $\hat{\boldsymbol{\pi}}_t$ will differ from the unit vectors \mathbf{e}_s ($s = 1, \dots, k$), i.e., in general uncertainty will exist on the nature of the current state, the generalized residuals,

$$\sum_{s=1}^k (\mathbf{e}'_s \hat{\boldsymbol{\pi}}_t) \left(\mathbf{x}_t - \hat{\boldsymbol{\mu}}_s - \hat{\mathbf{A}}_s \mathbf{x}_{t-j} \right),$$

will fail to be either i.i.d. or normally distributed.¹⁴ Therefore standard residual-based tests will fail if focussed around testing the i.i.d. properties of the residuals and will anyway run into difficulties when tests rely on their normality. However, Krolzig (1997) shows that under the assumption of correct specification, one important property ought to pin down at least the one-step ahead forecast errors,

$$\boldsymbol{\eta}_{t+1} \equiv \mathbf{x}_{t+1} - \sum_{s=1}^k (\mathbf{e}'_s \hat{\mathbf{P}} \hat{\boldsymbol{\pi}}_t) \left(\hat{\boldsymbol{\mu}}_s + \hat{\mathbf{A}}_s \mathbf{x}_t \right)$$

(where $\hat{\boldsymbol{\pi}}_t$ is the vector of real-time, filtered state probabilities and $\hat{\boldsymbol{\pi}}'_t \hat{\mathbf{P}} \mathbf{e}_s$ is the one-step ahead prediction of the probability of state $s = 1, \dots, k$): $\{\boldsymbol{\eta}_{t+1}\}$ should define a martingale difference sequence, i.e.

$$E[\boldsymbol{\eta}_{t+1} | \mathfrak{S}_t] = 0.$$

This hypothesis is testable in standard ways, i.e. looking at the ability of elements of the information set at time t (e.g. current excess returns, short-term interest rates, their combinations, etc.) to forecast both elements of $\boldsymbol{\eta}_{t+1}$ as well as their powers (since $E[\boldsymbol{\eta}_{t+1} | \mathfrak{S}_t] = 0$ is more restrictive than $Cov[\boldsymbol{\eta}_{t+1}, Y_t] = 0$, where Y_t is any variable that belongs to \mathfrak{S}_t).

We implement two types of residual-based tests. In each case, we make an effort to provide intuition for what a rejection of the null of the forecast errors being a martingale difference sequence would imply in economic and financial terms. To gain additional insights, we generally apply tests to the each of the elements of $\{\boldsymbol{\eta}_{t+1}\}$ in isolation (i.e. to the univariate series of forecast errors concerning national stock market excess returns and short-term rates). We start by testing whether any lags of returns predict current and future forecast errors. Rejections of the null of zero predictive power, would point to misspecification in the conditional mean function implied by our MMSIAH(2,1) model in particular (but not exclusively) in the VAR order (q). Although we do find some p-values for own- and cross-serial correlations that fall in the range 0.05 - 0.10, in general we obtain at worst weak evidence of appreciable serial correlation structure in the level of one-step ahead forecast errors.¹⁵

¹⁴When $\hat{\boldsymbol{\pi}}_t$ is identified with the vector of smoothed probabilities, the problem is compounded by the fact that the smoothed values are full-sample estimates that by construction overstate the predictive accuracy of the Markov switching model.

¹⁵We also examine the ability of lagged returns of market i and/or short-term yields of country i to predict forecast errors of market/country j , $i \neq j$. We find that there is some linear (cross-) structure only in FTSE100 excess return errors; in particular, $t - 1$ excess S&P returns predict time t FTSE100 forecast errors.

Next, we examine the ability of variables in the information set to predict squared forecast errors. In case of rejections of the no predictability restriction, this test can be interpreted as a test of omitted volatility clustering and ARCH effects in the model. There is borderline evidence of some positive and significant first-order serial correlation in squared forecast errors for UK excess returns and short-term rates only, while both past own and cross-excess returns fail to predict subsequent forecast errors. All in all there is no evidence of a need of specifying ARCH effects on the top of making Σ_{s_t} a function of the state. We also proceed to formally test a regime switching ARCH(1) specification in which

$$\Sigma_t = \Lambda_{0s_t} + \Lambda_{1s_t} \varepsilon_t \varepsilon_t'$$

This specification implies specifying 21 additional parameters, the elements of the matrix Λ_{1s_t} . A LR test resoundingly rejects this specification. Importantly these results are consistent with the evidence in Guidolin and Timmermann (2007a, 2008) that when (international) stock returns and short-term rates are modeled using multivariate regime switching models, the residual evidence for classical multivariate ARCH effects is rather weak.¹⁶

4. Economic Implications

4.1. Time-varying predicted risk premia and second moments

One way in which we can gauge the economic implications of our regime switching VAR is by calculating the one-step ahead predictions of risk premia and volatilities characterizing the three stock markets. These are obviously crucial pieces of information relevant to portfolio managers interested in international portfolio diversification. To this purpose, we proceed to the recursive estimation of our two-state regime switching VAR(1) model over the period 1995:01 - 2004:12. This means that the first estimation uses data for the interval 1978:05 - 1995:01 (i.e. 201 observations), the second for 1978:05 - 1995:02 (202 observations), etc. This recursive updating of the parameter estimates implied by equation (3) captures the expanding learning of an investor who uses the model to characterize the dynamic properties of international equity markets and their linkages to national monetary policies.

Figure 2 shows one-step ahead predicted risk premia, volatilities, and correlations resulting from such a recursive updating process.¹⁷ Clearly, risk premia tend to substantially fluctuate over time, and are sometimes negative, even if only for relatively short periods of time. In particular, two different periods can be isolated: during 1995 - 1998 predicted risk premia are generally positive and scarcely volatile, always falling in the

¹⁶Guidolin and Nicodano (2007) reach similar conclusions and – in an international equity asset allocation problem – show that the qualitative portfolio implications are anyway robust to adopting models in the Dynamic Conditional Correlations-in mean (DCC-M) class, although the latter are usually dominated by the forecasting properties of multi-state regime switching models. Although it is unclear whether their results may extend to our, different asset menu, it is plausible that Markov switching may simply provide an efficient and interpretable way to fit time-varying predictability patterns in international stock returns.

¹⁷While predicted risk-premia are simply calculated as a predicted-probability weighted average of state-specific risk premia, predicted volatilities adjust for possible switches in means between t and $t + 1$ as shown by Timmermann (2000). The same applies to the correlations presented later on.

narrow range 0 - 3 percent per month; on the contrary, over 1999-2003 (and in particular in 2001 and 2002) risk premia appear extremely volatile and often turn negative, with one-month ahead spikes below -5 percent. Additionally, the predicted risk premia tend to move in a largely symmetric fashion across stock markets. Although the plot allows one to detect a few episodic differences, they never exceed 0.3-0.5 percent per month, in absolute value. This implies that in a two-state VAR(1) model, the specific emerging, small open economy features of the Irish stock market fail to be reflected in systematically higher or different risk premia.

The second panel of Figure 2 offers a similar picture for predicted monthly volatilities of excess returns in each of the three markets. Differently from risk premia, volatilities are systematically different across national markets: the ISEQ is always predicted to be most volatile market, with forecasts between 4.3 and 5.8 percent per month while the FTSE 100 and the S&P 500 are more stable with modest fluctuations in the range 4.5 to 5.0 percent and 3.8 - 4.5 percent respectively. Aside from the idiosyncratic drop in ISEQ volatility in 1995, the three volatility series mimic each other.

The last panel of Figure 2 shows the dynamics of the predicted, one-month ahead correlations between the ISEQ and the two major Anglo-Saxon stock markets.¹⁸ The pairwise correlation with the FTSE 100 is systematically higher than the one with the S&P except for brief periods between 2001-2003. Correlations are typically quite high throughout the sample period although they are not significantly higher post 2003 than in early 1995.

4.2. Sharpe-ratio dynamics

Once values for predicted risk premia and volatilities are available, it becomes natural to proceed to recursively calculate one-month ahead predicted Sharpe ratios, which give an indication for the recursive behavior over time of the expected reward-to-risk ratio. In the following we specialize to the viewpoint of a US investor, i.e. compute the predicted Sharpe ratio as:

$$\widehat{SR}_{t+1}^i | \mathfrak{S}_t = \frac{E[x_{t+1}^i + r_t^i | \mathfrak{S}_t] - r_t^{US}}{Var[x_{t+1}^i | \mathfrak{S}_t]}, \quad (5)$$

where $E[x_{t+1}^i + r_t^i | \mathfrak{S}_t]$ and $Var[x_{t+1}^i | \mathfrak{S}_t]$ are computed using $\hat{\boldsymbol{\pi}}_t \in \mathfrak{S}_t$, the vector of recursive state probabilities. $E[x_{t+1}^i + r_t^i | \mathfrak{S}_t] - r_t^{US}$ converts local currency net stock returns into excess returns in the perspective of a US investor, with both r_t^i and r_t^{US} known at time t .¹⁹

Given our finding in Section 4.1 that the predicted risk premia are relatively close to each other over our sample while heterogeneity exists in the dynamics of predicted volatilities, it seems clear that Sharpe ratio forecasts will be mostly driven by the time variation in the latter. However, since (5) comes in the form of a ratio and not of a simpler difference, it is unclear whether heterogeneous volatility dynamics will be sufficient to induce large differences. Figure 3 shows that the ratios fundamentally inherit the dynamic behavior of

¹⁸An equivalent plot concerning the pairwise correlation between FTSE and S&P excess returns is omitted to save space and is available upon request.

¹⁹Notice that $\widehat{SR}_{t+1}^i | \mathfrak{S}_t$ is in fact the ratio of two predicted moments (or functions thereof) and not a direct implication of the two-state model.

predicted expected returns. The period 1995-1998 is characterized by relatively stable and positive Sharpe ratios, while 2000-2003 is marked by high volatility. Also in this case, the Sharpe ratios strongly comove (correlation coefficients range between 0.86 and 0.97), although there is some tendency for the ISEQ first (over 1995-1996) and the S&P 500 later (after 2002) to imply the highest reward-to-risk ratios. These results suggest that the ISEQ seems to compensate risk in ways that are perfectly consistent with the ratios that are typical of major, developed markets. Also, correlations are systematically below 1 while Sharpe ratios are essentially similar across markets suggesting the existence of enormous potential for international portfolio diversification. We test this conjecture in the next section.

4.3. Implications for optimal portfolio decisions

We recursively compute optimal mean-variance portfolio weights and assess the comparative (pseudo) out-of-sample portfolio performance of our two-state regime switching model vs. two common benchmarks, a simple myopic IID model and a single state VAR(1). Assume an investor has preferences described by a simple mean-variance functional:

$$\begin{aligned}
V_t &= E_t[W_{t+1}] - \frac{1}{2}\lambda Var_t[W_{t+1}] \\
W_{t+1} &= \omega_t^{ISEQ}(1 + x_{t+1}^{ISEQ} + r_t^{IRL}) + \omega_t^{FTSE}(1 + x_{t+1}^{FTSE} + r_t^{UK}) + \omega_t^{S\&P}(1 + x_{t+1}^{S\&P} + r_t^{US}) \\
&\quad + (1 - w_t^{ISEQ} - w_t^{FTSE} - w_t^{S\&P})(1 + r_t^{US})
\end{aligned} \tag{6}$$

where λ is interpreted as coefficient of risk aversion that trades-off (conditional) predicted mean and variance of the one-step ahead wealth. At each time t in the sample, the investor maximizes V_t by selecting weights $\omega_t \equiv [\omega_t^{ISEQ} \ \omega_t^{FTSE} \ \omega_t^{S\&P}]'$ when the predicted moments are calculated using some reference statistical model, e.g. our two-state model. Simple algebra shows that:

$$\tilde{\omega}_t = \frac{1}{\lambda} \hat{\Sigma}_t^{-1} [\mathbf{M}_{1:3} \hat{\boldsymbol{\mu}}_t + \mathbf{M}_{1:3} \hat{\mathbf{A}}_t \mathbf{y}_t + (\mathbf{M}_{4:6} \mathbf{y}_t - r_t^{US} \boldsymbol{\iota}_3)],$$

where the time index appended to the matrices $\hat{\Sigma}_t$, $\hat{\boldsymbol{\mu}}_t$, and $\hat{\mathbf{A}}_t$ reflects the possibility that parameters may be a function of the state and hence of current time, r_t^{US} is the US short-term rate, and the \mathbf{M} matrices are “selector” matrices that fish out of the 6×1 vectors produced by the econometric model only the 3×1 vectors useful to compute portfolio weights.²⁰ We solve the problem from a US viewpoint, which explains why (after calculating predicted one month local returns, $\mathbf{M}_{1:3} \hat{\boldsymbol{\mu}}_t + \mathbf{M}_{1:3} \hat{\mathbf{A}}_t \mathbf{y}_t + \mathbf{M}_{4:6} \mathbf{y}_t$) we are subtracting r_t^{US} from predicted mean returns on all markets (i.e., this algebraic operations convert local currency excess returns into excess returns expressed in US dollars). Portfolio weights are calculated recursively using the recursive parameter estimates underlying Sections 4.1-4.2.²¹ (6) is solved both without and with restrictions

²⁰For instance,

$$\mathbf{M}_{1:3} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

i.e. a row-concatenation of an \mathbf{I}_3 identity matrix and a 3×3 matrix of zeros.

²¹For instance, $\tilde{\omega}_{1995:01}$ is based on estimated parameters obtained using data for the interval 1978:05 - 1995:01, etc.

on the admissible range for $\boldsymbol{\omega}_t$; in particular, in what follows we compute and discuss weights that prevent the investor from selling any securities short, i.e. such that $\boldsymbol{\omega}'_t \mathbf{e}_j \in [0, 1] \forall t$ and $j = ISEQ, FTSE, S\&P$.²²

In this section we also extend the calculation of portfolio weights to a natural and yet simple benchmark, a single-state, VAR(1) model,

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Sigma}), \quad (7)$$

in which only risk premia are predictable according to the simple law $E_t[\mathbf{y}_{t+1}] = \mathbf{M}_{1:3}\hat{\boldsymbol{\mu}}_t + \mathbf{M}_{1:3}\hat{\mathbf{A}}_t\mathbf{y}_t + (\mathbf{M}_{4:6}\mathbf{y}_t - r_t^{US}\mathbf{1}_3)$. In this case, variance and covariances are restricted to be constant over time. This is the model employed, for instance, by Campbell and Viceira (1999).

Figure 4 starts by showing plots of recursive, one-month ahead predicted Sharpe ratios for each stock market and under each of the three models investigated. Independently of the stock market under investigation, the plots show the existence of striking differences between simple mean-variance ratios that ignore all kinds of predictability and ratios implied by models that account for predictability. While the former model generates ratios that are small (generally between 0 and 0.15) and that change smoothly over time as a consequence of recursive updating, the predictability models induce substantial variation in Sharpe ratios, which are actually often predicted to be negative. Some differences exist also between regime switching and VAR Sharpe ratios, as the latter tend to be less volatile than the MMS ratios. Moreover, periods can be found in which the VAR and regime switching models imply rather heterogeneous ratios and hence potentially different portfolio implications.

We then proceed to compare the evolution of portfolio weights induced by the three different models in the case $\lambda = 0.5$. Table 4 provides summary statistics. Differences are striking: the single-state VAR(1) model generates only a moderate demand for stocks: 13 percent for Ireland and 16 percent for the S&P 500. The bulk of the portfolio remains invested in the (US) riskless asset (80 percent). Removing short-sale possibilities marginally reduces the equity weights, to approximately 22 percent. Finally, a regime switching model implies larger equity weights. When short-sales are admitted, the investment in stocks is on average 40-50%. Although rich temporal dynamics can be detected and periods exist in which the net weight to all stocks ought to be negative (i.e., $w_t^{ISEQ} + w_t^{FTSE} + w_t^{S\&P} < 0$), in general there is a tendency towards a thorough diversification across the three national stock markets. Table 4 provides a more accurate description by reporting mean values of portfolio weights over our recursive exercise. The mean investment in stocks is 45 percent, with a prevalence of US stocks (30 percent).²³

Table 5 completes the analysis by showing the (pseudo) out-of-sample, one month portfolio performance under different levels of λ and for the two competing models. In particular, we report mean one-month net portfolio return, the lower and upper values of a standard 95% confidence interval (that reflects the volatility of portfolio returns over 1995:01 - 2004:11), and the implied Sharpe ratio that adjusts mean returns to account for risk. The final eight columns of Table 5 report performance measures also for portfolios that

²²When short-sales are restricted, $\tilde{\boldsymbol{\omega}}_t$ has no closed-form solution and is therefore calculated numerically (by grid search).

²³Table 3 also reports summary statistics for recursive portfolio weights under regime switching also for other values of λ , i.e. 0.2, 1, and 2. The evidence of a prevalence of equity investments in the US is consistent with the finding in Guidolin and Timmermann (2006b).

fail to include riskless assets, i.e. pure equity portfolios in which $w_t^{ISEQ} + w_t^{FTSE} + w_t^{S\&P} = 1$.²⁴ The table reports in bold the maximum values of mean portfolio returns and of the Sharpe ratio across models. Mean performance is always superior for all portfolios including the riskless asset, independently of the assumed value for λ , for the two-state regime switching VAR. In most cases, the MMS framework also produces the best possible Sharpe ratios. For instance, when $\lambda = 0.5$ and a no short sale constraint is imposed, a regime switching asset allocation obtains a 1.19 percent per month average performance (higher than 1.06 for a VAR strategy); however, some performance risk should be taken into account, as a 95 percent interval spans $[-0.6, 3.0]$, i.e. covers a negative region. Even adjusting for risk, the Sharpe ratio is 0.68, which is higher than those produced by competing models.²⁵

Table 4 highlights what regime switching is really doing: tilting the balance of risks away from a situation of equilibrium – as evidenced by the fact that the three markets have essentially similar Sharpe ratios, both in the full-sample and in predicted terms (see Figure 3) – to weigh against Irish and UK stocks and in favor of US equities. This is visible in Table 4 when it is clear that while a simple VAR(1) puts a weight between 46 and 64 percent into Irish stocks (and their excellent 0.53 annualized Sharpe ratio), a strategy that accounts for regimes should reduce such a weight to the range 24-26 percent only, i.e., almost exactly half, in favor of UK and especially US equity investments. Table 5 shows that changing the structure of portfolio weights in this way is certainly important: it certainly improves mean portfolio returns, especially if riskless investments and short-sale capabilities are both allowed; it may also benefit the overall risk-return ratio of the portfolio, i.e., deliver higher performance in risk-adjusted terms.

A final question concerns the origins of such differences in portfolio structure and out-of-sample performances. Table 3 and the moment estimates it implies reveal three facts. First, the ISEQ index is different from the FTSE or the S&P500 because the mean differentials when going from one state to the other is maximum: 0.95% per month in the case of Ireland and the UK excess returns vs. 0.31% in the case of the US; 1.01% per month for Irish short-term rates vs. 0.48% for the UK and 0.55% for the US. As shown in Timmermann (2000), such jumps in mean obviously add to the overall risk/variance of a regime switching variable. Second, Figure 2 shows that a similar phenomenon involves the one-step ahead predicted variances, in the sense that the volatility of ISEQ excess returns is the one that depends the most on regimes. Third, Table 3 shows a phenomenon already discussed by Ang and Bekaert (2002a) and Butler and Joaquin (2002): correlations among shocks to excess market returns are systematically higher in bear states than in bull ones, which implies that diversification pays out the least exactly when it is needed the most. The result is that a risk-averse portfolio optimizer will simply bias portfolio weights away from a well-diversified pattern and towards the market with the highest Sharpe ratio, in the case the S&P500.²⁶

²⁴The four columns concerning pure equity portfolios performance are identical across values for λ . The algebra of mean-variance optimization implies that when a riskless asset is available a two-fund separation result applies, such that heterogeneous risk preferences only produce different demands for the riskless asset and a homogeneous risky portfolio.

²⁵However it must be noted that for pure equity allocations, the implied Sharpe ratios are systematically lower, and the single state VAR(1) framework tends to outperform other models.

²⁶Guidolin and Nicodano (2007) frame the discussion of these three effects in terms of the co-skewness and co-kurtosis properties of the international equity portfolios under investigation. Because this paper entertains the simplest but also more intuitive case

5. Conclusions

We have documented that – despite the stellar (+15% a year on average, over the period 1994-1999), “Tiger-like” performance of the Irish stock market during the 1990s – the vector-autoregressive process linking Irish, UK, and US excess equity returns to short-term interest rates is characterized by substantial nonlinearities – in the form of regimes – that make the long-run, overall ‘association’ among the ISEQ, the FTSE 100, and the S&P 500 much higher than commonly thought of. Importantly, not only we have found strong statistical evidence of regimes in the multivariate distribution of the equity returns and short-term interest rates for Ireland, the US, and the UK, but we have also shown that such regimes are economically important insofar as they may improve the out-of-sample performance of a mean-variance portfolio optimization strategy.

The analysis performed in this paper should be taken as “clinical”, case study-type of evidence that the existence of non-linear dynamics in the process of the typical benchmark portfolio returns targeted by international diversification strategies ought to attract careful consideration from money managers around the world. Such non-linear dynamics – incarnated by a simple regime switching VAR in this paper – may imply that a traditional focus on means, variances and especially linear correlations may be dangerously misleading. For instance, despite moderate correlation coefficient estimates between Irish, UK, and US excess returns, we find that state comovements involving the three markets are so relevant to depress the optimal mean-variance weight carried by ISEQ stocks to at most one-quarter of the overall equity portfolio. In this sense, it seems that international bull and bears shared by the more developed US and UK equity markets involve the Dublin’s stock exchange so heavily to greatly reduce the diversification benefits available through indirect equity investments in Ireland. While the academic literature has recently become keenly aware of the possibility of such subtle, non-linear effects (see e.g., Ang and Bekaert, 2002a, Detemple, Garcia, and Rindisbacher, 2003, Guidolin and Timmemann, 2005, 2007a), it remains to be seen how and when corrections for such effects will find their way in the practice of multinational portfolio management.

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Table 1
Summary Statistics

This table reports summary statistics for percentage monthly excess stock returns and short-term (money market) nominal interest rates for Ireland, the United States, and the United Kingdom. The sample period is 1978:05 – 2004:12.

	Mean	Median	Minimum	Maximum	Standard deviation	Skewness	Kurtosis
A. Excess Stock Returns							
ISEQ	0.143	0.816	-39.779	14.378	5.330	-1.492	12.419
FTSE-100	0.007	0.701	-32.742	11.969	4.914	-1.276	9.169
S&P 500	0.216	0.705	-25.241	12.731	4.236	-0.889	6.762
B. Money Market Nominal Interest Rates							
Ireland	0.774	0.732	0.170	3.667	0.464	1.458	8.783
United Kingdom	0.727	0.708	0.250	1.490	0.308	0.357	2.019
United States	0.568	0.504	0.082	1.592	0.317	0.885	3.996
C. Correlation Matrix							
	ISEQ	FTSE-100	S&P 500	Ireland – interest rate	UK – interest rate	US – interest rate	
ISEQ	1.000						
FTSE-100	0.572	1.000					
S&P 500	0.539	0.698	1.000				
Ireland – mon. mkt. int. rate	-0.101	0.044	-0.034	1.000			
UK – money mkt. int. rate	-0.122	-0.019	-0.021	0.680	1.000		
US – money mkt. int. rate	-0.120	-0.012	-0.105	0.653	0.734	1.000	

Table 2

Model Selection of Markov Switching, Conditionally Heteroschedastic VAR Models

The table reports the estimation output for the MMSIAH(k,p) models:

$$y_t = \mu_{s_t} + \sum_{j=1}^q A_{j s_t} y_{t-j} + \Sigma_{s_t} \varepsilon_t$$

where μ_{s_t} is the intercept vector in state s_t , $A_{j s_t}$ is the matrix of autoregressive coefficients associated to lag $j \geq 1$ in state s_t and $\varepsilon_t \sim \text{I.I.D. } N(\mathbf{0}, I_6)$. s_t is governed by an unobservable, discrete, first-order Markov chain that can assume k distinct values (states). Excess stock returns are calculated as the log-difference in the total return indices for ISEQ, FTSE-100, and S&P 500 minus a short-term interest rate. The data are monthly. The sample period is 1978:05 – 2004:12.

Model	Number of parameters	Log-likelihood	LR test for linearity	AIC	BIC	Hannan-Quinn
Base model: MMSIA(1,0)						
MMSI(1,0)	27	-2808.26	NA	17.7203	18.0383	17.8473
MMSIA(1,1)	63	-1881.33	NA	12.1902	12.9338	12.4871
MMSIA(1,2)	99	-1744.38	NA	11.5936	12.7648	12.0614
MMSIA(1,3)	135	-1691.46	NA	11.5234	13.1242	12.1629
Base model: MMSIA(2,0)						
MMSI(2,0)	35	-2587.63	441.2475 (0.000)	16.3914	16.8036	16.5560
MMSIA(2,1)	107	-1528.73	705.2082 (0.000)	10.2553	11.5183	10.7597
MMSIAH(2,1)	128	-1099.27	1564.1296 (0.001)	7.6945	9.2053	8.2978
MMSIAH(2,2)	200	-946.85	1595.0757 (0.001)	7.2129	9.5789	8.3579
Base model: MMSIA(3,0)						
MMSI(3,0)	45	-2480.25	656.0023 (0.000)	15.7828	16.3128	15.9944
MMSIH(3,0)	87	-2193.17	1230.1621 (0.000)	14.2511	15.2756	14.6602
MMSIA(3,1)	153	-1463.83	835.0083 (0.000)	10.1369	11.9427	10.8581
MMSIAH(3,1)	195	-974.77	1813.1214 (0.000)	7.3340	9.6356	8.2532
MMSIAH(3,2)	303	-800.66	1887.4525 (0.000)	6.9412	10.5258	8.3730
Base model: MMSIA(4,0)						
MMSI(4,0)	57	-2431.16	754.1917 (0.000)	15.5510	16.2222	15.8190
MMSIH(4,0)	120	-2027.36	1561.7973 (0.000)	13.4210	14.8341	13.9853
MMSIAH(4,1)	264	-856.12	2050.4344 (0.000)	7.0227	10.1387	8.2671

Table 3

Estimates of Multivariate Regime Switching VAR(1) Model for Excess Stock Returns and Nominal Short-Term Interest Rates

The table reports the estimation output for the MMSIAH(k,p) model:

$$r_t = \mu_{s_t} + A_{s_t} r_{t-1} + \sum_{s_t} \varepsilon_t$$

The data are monthly. The sample period is 1978:05 – 2004:12. The data reported on the diagonals of the correlation matrices are annualized volatilities. Asterisks attached to correlation coefficients refer to covariance estimates.

Panel A – Single State Model						
	ISEQ	FTSE-100	S&P 500	Ireland r	UK r	US r
1. Mean excess return	0.724	-1.051	-0.197	0.016	0.040**	0.007
2. VAR(1) Coefficient						
ISEQ	0.352**	-0.161*	-0.018	0.674	0.300	-2.405
FTSE-100	0.043	-0.300**	-0.001	1.621*	0.962	-1.696
S&P 500	0.034	-0.169**	-0.071	0.672	2.255*	-3.120**
Ireland r	-0.008	0.009**	-0.001	0.670**	0.178**	0.192**
UK r	0.001	-0.002	0.002	0.018	0.827**	0.124**
US r	0.000	0.001	0.001	-0.002	-0.005	0.994**
3. Correlations/Volatilities						
ISEQ	17.515**					
FTSE-100	0.528**	15.609**				
S&P 500	0.487**	0.657**	13.621**			
Ireland r	-0.024	0.063	0.036	0.801**		
UK r	-0.112	-0.161*	-0.026	0.107*	0.363**	
US r	0.028	0.023	-0.015	0.011	0.030	0.190**
Panel B – Two State Model						
	ISEQ	FTSE-100	S&P 500	Ireland r	UK r	US r
1. Mean excess return						
Regime 1 (bear)	0.619**	-0.793**	-0.080**	-0.005*	0.008*	0.001*
Regime 2 (bull)	-0.702	-0.597**	1.946**	1.061**	0.241**	0.034
2. VAR(1) Coefficient						
<i>Regime 1 (bear):</i>						
ISEQ	0.334**	-0.143*	-0.045	1.517*	-3.087**	1.700*
FTSE-100	0.054	-0.317*	-0.088	-0.212*	0.953*	-0.285*
S&P 500	0.043	-0.179*	-0.118*	-0.125	3.110**	-2.656**
Ireland r	-0.000	0.001	-0.000	0.919**	0.043*	0.038*
UK r	0.001	0.001	-0.002	-0.015	0.951**	0.065*
US r	0.000	-0.000	0.001	0.001	-0.026	1.032**
<i>Regime 2 (bull):</i>						
ISEQ	0.303**	-0.153*	0.082	0.556*	3.859**	-4.624**
FTSE-100	0.095	-0.208*	0.150*	0.690*	4.553**	-4.531**
S&P 500	0.045	-0.120	-0.054	-0.403	3.919**	-5.536**
Ireland r	-0.036	0.023	-0.005	0.187*	0.144*	-0.160*
UK r	-0.000	-0.006	0.005	0.001	0.563**	0.200**
US r	0.001	0.004	0.006	-0.019	-0.001	0.988**

* denotes 5% significance, ** significance at 1%.

Table 3 - continued

Estimates of Multivariate Regime Switching VAR(1) Model for Excess Stock Returns and Nominal Short-Term Interest Rates

	ISEQ	FTSE-100	S&P 500	Ireland r	UK r	US r
3. Correlations/Volatilities						
<i>Regime 1 (bear):</i>						
ISEQ	17.904**					
FTSE-100	0.584**	14.771**				
S&P 500	0.552**	0.732**	13.556**			
Ireland r	-0.046	0.026	0.018	0.101**		
UK r	-0.095	-0.044	0.052	0.143*	0.195**	
US r	0.080	0.070	-0.004	0.042	0.034	0.059*
<i>Regime 2 (bull):</i>						
ISEQ	15.528**					
FTSE-100	0.389**	16.507**				
S&P 500	0.305**	0.403**	12.589**			
Ireland r	-0.064	-0.003	-0.098	1.276**		
UK r	-0.128	-0.283*	-0.112	0.108*	0.595**	
US r	-0.025	-0.003	-0.003	-0.050	0.049	0.346**
4. Transition probabilities	Regime 1 (bear/normal)			Regime 2 (bull)		
Regime 1 (bear)		0.967**			0.033	
Regime 2 (bull)		0.100			0.900**	

* denotes 5% significance, ** significance at 1%.

Table 4

Summary Statistics for Recursive Mean-Variance Portfolio Weights

The table reports summary statistics for the weights solving the one-month forward mean-variance portfolio problem:

$$\max_{\mathbf{w}_t} E_t[W_{t+1}] - 1/2 \lambda \text{Var}_t[W_{t+1}],$$

where W_{t+1} is end-of period wealth and λ is a coefficient of (absolute) risk aversion that trades-off mean and variance. The problem is solved recursively over the period 1995:01 – 2004:12 using in each month updated parameter estimates (and when appropriate, filtered state probabilities) obtained over an expanding sample that starts in 1978:05. The table shows means and standard deviations for recursive portfolio weights. For the case of $\lambda = 1/2$, the table also reports summary statistics for portfolio weights obtained under a single-state, the VAR(1) case in which only risk premia are predictable. Finally, the problem is solved from the point of view of a perfectly hedged US investor, i.e. the riskless interest rate is a short-term US yield.

	Statistic	Short Sales Admitted				No Short Sales			
		ISEQ	FTSE 100	S&P 500	Riskless	ISEQ	FTSE 100	S&P 500	Riskless
$\lambda = 0.2$		Two-State VAR(1) Model							
	Mean	0.124	-0.247	0.496	0.627	0.100	0.046	0.333	0.524
	Standard dev.	0.225	0.378	0.322	0.578	0.125	0.119	0.285	0.357
$\lambda = 0.5$		VAR(1) Model							
	Mean	0.129	-0.083	0.157	0.797	0.100	0.006	0.111	0.783
	Standard dev.	0.067	0.100	0.082	0.199	0.063	0.028	0.079	0.145
		Two-State VAR(1) Model							
	Mean	0.118	0.031	0.298	0.553	0.089	0.055	0.230	0.627
	Standard dev.	0.090	0.151	0.129	0.231	0.072	0.099	0.130	0.212
$\lambda = 1$		Two-State VAR(1) Model							
	Mean	0.039	-0.020	0.149	0.831	0.029	0.015	0.114	0.842
	Standard dev.	0.045	0.076	0.064	0.116	0.031	0.039	0.066	0.094
$\lambda = 2$		Two-State VAR(1) Model							
	Mean	0.009	-0.030	0.045	0.976	0.008	0.004	0.029	0.959
	Standard dev.	0.023	0.038	0.032	0.058	0.013	0.013	0.030	0.039

Table 5

Summary Statistics for Recursive Mean-Variance Portfolio Performances

The table reports summary statistics for the 1-month portfolio return based on weights that solve the one-month forward mean-variance portfolio problem:

$$\max_{\mathbf{w}_t} E_t[W_{t+1}] - 1/2 \lambda \text{Var}_t[W_{t+1}],$$

where W_{t+1} is end-of period wealth and λ is the coefficient of (absolute) risk aversion. The problem is solved recursively over the period 1995:01 – 2004:12 using in each month updated parameter estimates (and when appropriate, filtered state probabilities) obtained over an expanding sample that starts in 1978:05. The table shows means and standard deviations for recursive portfolio weights. The problem is solved from the point of view of a perfectly hedged US investor, i.e. the riskless interest rate is a short-term US yield. Boldfaced values for means and Sharpe ratios indicate the best performing model.

Statistic	Unconstrained				No Short-Sales				Pure Equity				Pure Equity, No Short-Sales			
	$\lambda=0.2$	$\lambda=0.5$	$\lambda=1$	$\lambda=2$	$\lambda=0.2$	$\lambda=0.5$	$\lambda=1$	$\lambda=2$	$\lambda=0.2$	$\lambda=0.5$	$\lambda=1$	$\lambda=2$	$\lambda=0.2$	$\lambda=0.5$	$\lambda=1$	$\lambda=2$
	VAR(1) Model															
Mean	1.86	1.26	1.06	0.96	1.37	1.06	0.96	0.91	5.56	5.56	5.56	5.56	2.22	2.22	2.20	2.07
95% l.b.	-3.05	-0.80	-0.11	0.18	-1.79	-0.37	0.04	0.20	-9.13	-9.13	-9.13	-9.13	-4.86	-4.83	-4.81	-4.80
95% u.b.	6.78	3.32	2.22	1.74	4.52	2.50	1.88	1.61	20.2	20.2	20.2	20.2	9.29	9.27	9.21	8.94
Sharpe rat.	0.53	0.67	0.84	0.99	0.50	0.66	0.82	0.96	0.10	0.10	0.10	0.10	0.37	0.37	0.37	0.34
	Two-State VAR(1) Model															
Mean	2.12	1.36	1.11	0.98	1.64	1.19	1.02	0.94	1.87	1.87	1.87	1.87	2.12	2.12	2.12	2.09
95% l.b.	-3.62	-1.03	-0.21	0.14	-2.43	-0.63	-0.05	0.19	-29.7	-29.7	-29.7	-29.7	-4.70	-4.69	-4.67	-4.72
95% u.b.	7.86	3.75	2.43	1.83	5.71	3.00	2.09	1.69	33.5	33.5	33.5	33.5	8.94	8.93	8.91	8.89
Sharpe rat.	0.54	0.66	0.82	0.98	0.53	0.68	0.84	0.98	0.02	0.02	0.02	0.02	0.45	0.46	0.46	0.45

Figure 1

**Smoothed State Probabilities from a Multivariate VAR(1) Two-State Model for
Excess Stock Index Returns and Short-Term Interest Rates**

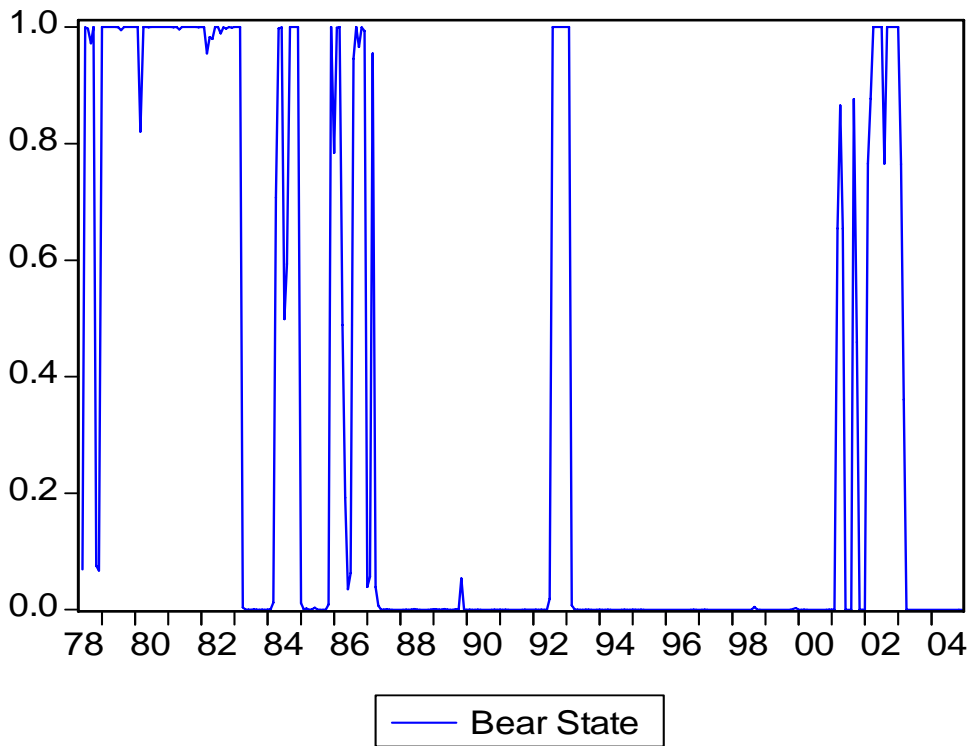
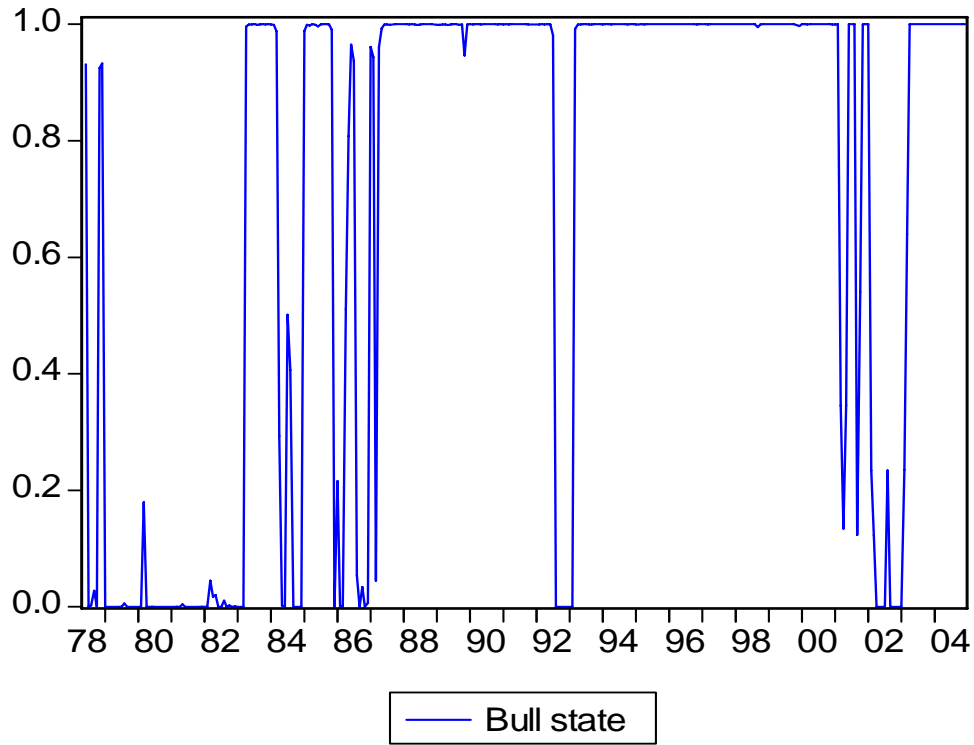


Figure 2

Predicted One-Step Ahead Monthly Risk Premia and Volatilities in a Two-State Model

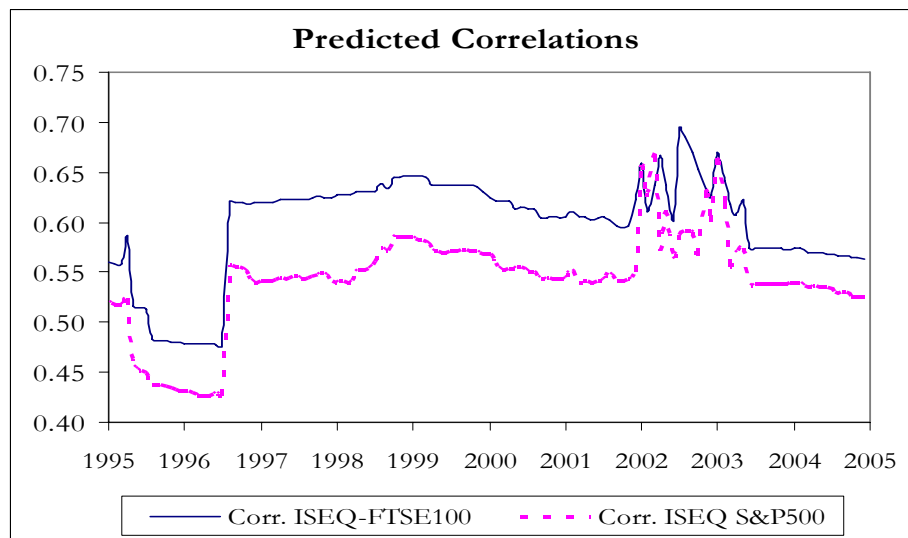
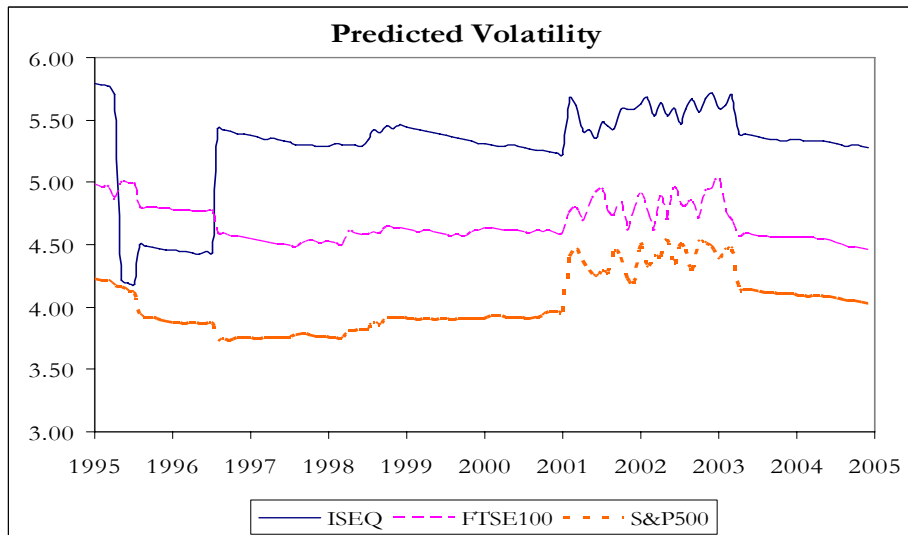
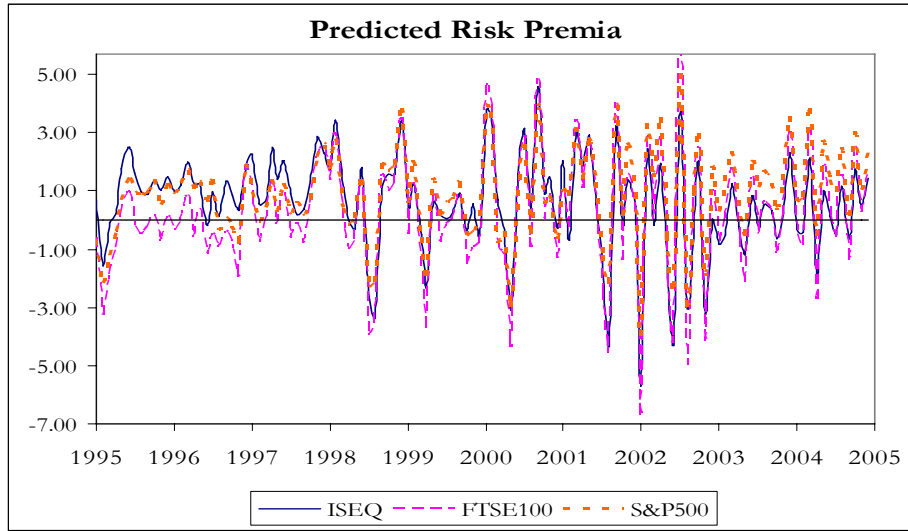


Figure 3

Predicted One-Step Ahead Sharpe Ratios Under a Two-State VAR(1) Model

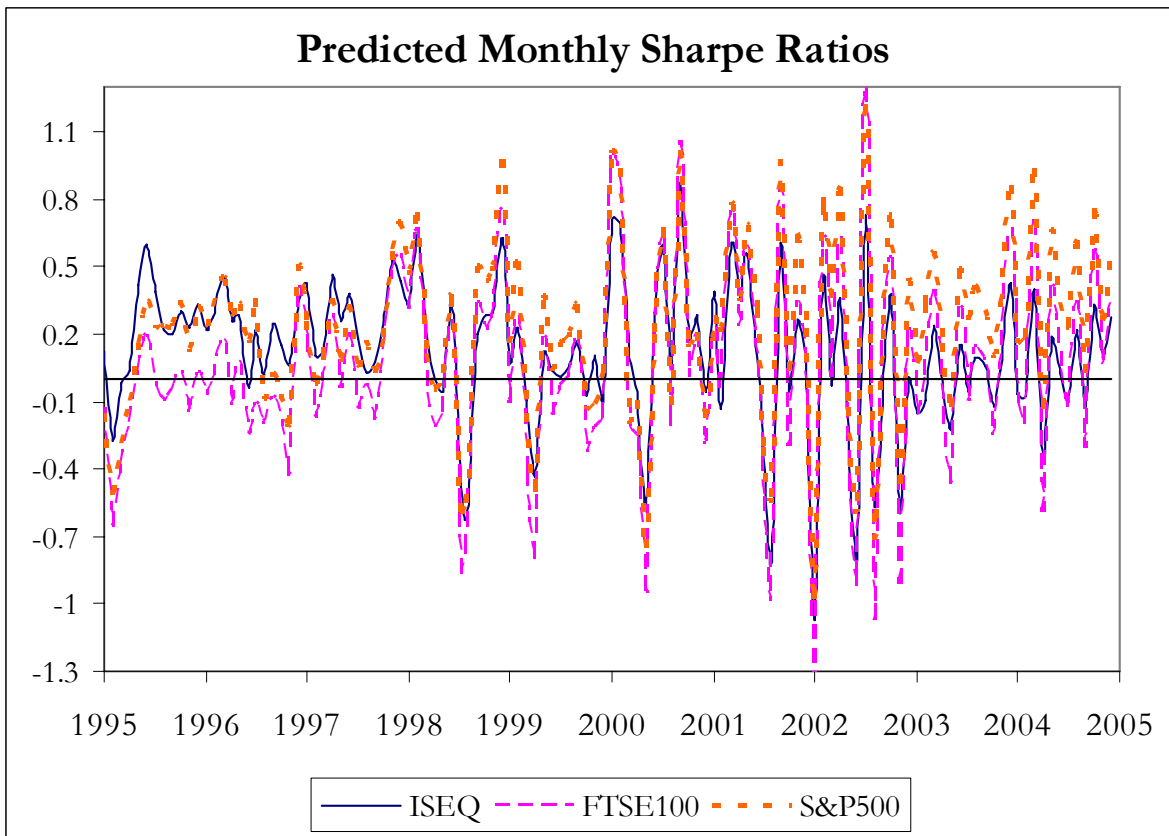


Figure 4

Comparing Predicted One-Step Ahead Sharpe Ratios

