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The Dynamic Relationship Between the Federal Funds Rate and the Treasury Bill Rate: An Empirical Investigation*

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Abstract

This article examines the dynamic relationship between two key US money market interest rates–the federal funds rate and the 3-month Treasury bill rate. Using daily data over the period from 1974 to 1999, we find a long-run relationship between these two rates that is remarkably stable across monetary policy regimes of interest rate and monetary aggregate targeting. Employing a nonlinear asymmetric vector equilibrium correction model, which is novel in this context, we find that most of the adjustment toward the long-run equilibrium occurs through the federal funds rate. In turn, there is strong evidence for the existence of significant asymmetries and nonlinearities in interest rate dynamics that have important implications for the conventional view of interest rate behavior.

JEL classification: E43.
Keywords: interest rates; term structure; equilibrium correction; nonlinear dynamics.

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1 Introduction

Interest rates tend to move together. The nature of the forces that bind interest rates together is not fully understood, however. This paper investigates the relationship between two important short-term interest rates in US financial markets—the overnight federal funds rate and the 3-month Treasury bill (or T-bill) rate. The importance of these two interest rates is widely recognized. The Federal Reserve (Fed) implements monetary policy by targeting the effective federal funds rate: the 3-month T-bill rate is the preeminent default-risk-free rate in the US money market, and is often used by researchers to proxy the risk-free asset whose existence is assumed by much conventional finance theory. Given their importance and visibility, it is not surprising that these interest rates have been studied extensively in economics and finance (see, *inter alios*, Hall, Anderson and Granger, 1992; Taylor, 1993; Rudebusch, 1995, 1998; Anderson, 1997; Enders and Granger, 1998; Stock and Watson, 1999).

A number of authors have argued that the federal funds rate and the T-bill rate move together because they are linked by the expectations hypothesis (EH) (e.g., *inter alios*, Cook and Hahn, 1989; Goodfriend, 1991; Poole, 1991; Rudebusch, 1995, 2001; Woodford, 1999). That is, it is assumed that the T-bill rate is equal to the market's expectation for the federal funds rate over the term of the T-bill rate plus a risk premium.

In the last fifteen years or so, several influential empirical studies have focused on testing the EH of the term structure of interest rates using cointegration and equilibrium correction models (see Engle and Granger, 1987; Stock and Watson, 1988; Campbell and Shiller, 1991; Hall, Anderson and Granger, 1992; Engsted and Tangaard, 1994). Empirical support for the EH is generally weak (e.g., Campbell, Lo and MacKinlay, 1997). This is particularly true when the short-term rate is the effective federal funds rate (see, *inter alios*, Simon, 1990, 1994; Roberds, Runkle and Whiteman, 1996; Thornton, 2000). Nevertheless, regardless of whether the EH holds, this empirical literature has provided evidence that interest rates co-move in the long run.[1]

The objective of the present study is to gain some insights into the dynamic relationship between the federal funds rate and the T-bill rate. Rather than hypothesizing and testing the implications of a particular hypothesis or theory of interest rate behavior, as is frequently done in economics and finance, the approach taken in this paper is agnostic. We investigate the dynamic relationship using a model that relies solely on the fact that the interest rates examined co-move in the long run and, therefore, cointegrate. Cointegration and equilibrium correction modelling techniques are a natural way to investigate the relationship between interest rates: The forces that generate a long-run equilibrium relationship between rates at different maturities imply mean reversion of the spread and the existence of an equilibrium correction model (ECM) that

[1] Note that, while cointegration between interest rates at different maturities is necessary for the EH to hold, this requirement is by no means sufficient. For example, Miron (1991) points out that cointegration between rates at different maturities is consistent with several alternative theories of interest rate behavior, including the EH.

characterizes the dynamic relationship between the rates. As noted by Granger and Swanson (1996, p. 543), while cointegration is "just a property" of the data, the ECM is a potential data generating mechanism. Consequently, identifying a stable ECM between the federal funds rate and the 3-month T-bill rate is an important step toward understanding the dynamic relationship between these two interest rates.

Our modelling strategy takes into account the existence of a recent strand of the empirical literature—discussed in detail below—that suggests interest rate dynamics may be characterized by strong asymmetries and nonlinearities due to factors such as non-zero or asymmetric transactions costs, infrequent trading, or other factors that affect the adjustment of interest rates toward their long-run equilibrium. To date, however, researchers have used dynamic models that allows for either asymmetry or nonlinearity, but not both. We extend the modelling procedures developed by this literature by investigating the dynamic relationship between the federal funds rate and the T-bill rate using an ECM that allows for *both* asymmetries and nonlinearities in a fairly comprehensive fashion. Our model outperforms several alternative econometric models previously employed in this context.

Using daily data on the effective federal funds rate and the 3-month T-bill rate for the sample period from 1974 to 1999, we find a long-run relationship between the federal funds rate and the T-bill rate that is stable over the sample period and across monetary policy regimes. We also find a general nonlinear, asymmetric vector equilibrium correction model that is temporally stable and significantly outperforms the alternative of a linear equilibrium correction model.

Our most unexpected result is that the adjustment toward the long-run equilibrium largely occurs through movements in the federal funds rate rather than the T-bill rate. We note that, *prima facie*, this finding appears to be inconsistent with the conventional EH/monetary policy view that the federal funds rate "anchors" the short end of the US money market. We discuss circumstances that could reconcile the conventional EH/monetary policy interpretation with our results. We undertake several robustness checks designed to investigate these alternative explanations. We conclude that the relationship between the federal funds rate and the T-bill rate appears to be considerably more complex than simple models of monetary policy or the EH suggest.

The remainder of the paper is set out as follows. Section 2 provides an overview of the theoretical considerations and the empirical literature used to motivate our modelling approach. Section 3 describes the data set. Section 4 contains the empirical results from employing conventional unit root and cointegration tests, executing parameter constancy and linearity tests, and estimating asymmetric linear and nonlinear equilibrium correction models. Section 5 reports the results of several robustness checks. The final section briefly summarizes our results and presents the conclusions. The appendix provides technical details of how parameter constancy and linearity tests were performed.

2 Modelling the relationship between the federal funds rate and the T-bill rate

2.1 The expectations hypothesis of the term structure

A number of researchers (e.g., Engle and Granger, 1987; Stock and Watson, 1988; Campbell and Shiller, 1991; Hall, Anderson and Granger, 1992) have noted that cointegration and equilibrium correction techniques are natural ways to test the implications of the EH and, more generally, to model the term structure of interest rates. Although it is not the goal of the present study to test the EH *per se,* it is instructive to show how the approach taken in this paper can be motivated by the EH.

Let $i_{k,t}$ and $f_{k,t}$ be the yield to maturity of a k-period pure discount bond and the forward rate, defined as the contract rate of a one-period pure discount bond bought at time t that matures at time $t+k$. Using the conventional Fisher-Hicks recursive *formulae*, the relationship linking $i_{k,t}$ and $f_{k,t}$ may be described as follows (e.g., Hall, Anderson and Granger, 1992; Campbell, Lo and MacKinlay, 1997):

$$i_{k,t} = \frac{1}{k}\left(\sum_{j=1}^{k} f_{j,t}\right) \qquad \text{for } k = 1, 2, 3, \ldots . \tag{1}$$

Under the conventional assumption that the relationship between forward rates and expected rates may be characterized as $f_{j,t} = E_t\left(i_{k,t+j-1}\right) + \phi_{j,t}$, where $\phi_{j,t}$ is a risk premium required by investors for risk considerations and preferences for liquidity, treating the premia components as time-invariant (as is assumed in the most common and weakest form of the EH), equation (1) can be rewritten as:

$$i_{k,t} - i_{1,t} = \frac{1}{k}\left[\sum_{i=1}^{k-1}\sum_{j=1}^{i} E_t \Delta i_{1,t+j}\right] + \gamma_k, \tag{2}$$

where E is the expectation operator, Δ denotes the first-difference operator, and $\gamma_{k,t} = \frac{1}{k}\sum_{j=1}^{k}\phi_{j,t}$. Equation (2) links yields at different maturities, implying that yields on assets with similar maturities tend to move together.[2] Also, if

[2] The EH of the term structure is often motivated by arbitrage considerations (see, *inter alios*, Hall, Anderson and Granger, 1992) on the ground that departures from the equilibrium relationship between the short-term rate and the long-term rate may imply the possibility of making a riskless profit. Hence, we sometimes use the term arbitrage in this paper. Nevertheless, it should be noted that our use of the term arbitrage is somewhat loose in that it does not exactly match the definition of arbitrage usually employed in finance. Also, note that the absence of arbitrage opportunities does *not necessarily* imply that the EH should hold. As demonstrated by Longstaff (2000), however, all traditional forms of the EH *can* be consistent with the absence of arbitrage if markets are incomplete. (In turn, this implies that the validity of the EH is mainly an empirical issue, meaning that the EH cannot be ruled out on *a priori* theoretical grounds.)

discount yields are integrated stochastic processes of order one, $I(1)$, as reported by a large empirical literature (starting at least from Engle and Granger, 1987; and Stock and Watson, 1988), the right-hand-side of (2) is stationary. This implies that the left-hand-side of (2) is stationary, i.e., $(i_{k,t} - i_{1,t}) \sim I(0)$. It is also easy to see that the EH implies that in long-run equilibrium the rates differ by only a constant risk premium and the cointegrating vector linking the rates is $[1, -1]$.

This framework also has implications for the dynamic structure of the ECM. Specifically, since this framework follows from the Fisher-Hicks formulae that explicitly states that the long-term rate is determined by the market's expectation of the short-term rate over the term of the long-term rate, it implies that the short-term rate is determined independent of the long-term rate. This should be particularly true when the short-term rate is the federal funds rate, which is thought to be directly controlled by the Fed. The market forms expectations of the funds rate based on its understanding of monetary policy. Changes in monetary policy induce changes in the federal funds rate, which in turn move the T-bill rate. The reverse is assumed not to be true, however. That is, changes in the 3-month T-bill rate do not induce changes in the funds rate. If the Fed controls the funds rate, then the T-bill rate may be expected to bear the burden of adjustment to the long-run equilibrium when there is an unexpected change in monetary policy.[3]

2.2 The econometric framework

Our empirical model is essentially the result of generalizing conventional cointegrated vector autoregressions and vector equilibrium correction models to a nonlinear framework. Consider the following p-th order vector autoregression, VAR(p):

$$y_t = \nu + \sum_{i=1}^{p} \Gamma_i y_{t-i} + \varepsilon_t, \tag{3}$$

where the K-dimensional observed time series vector, $y_t = [y_{1t}, y_{2t}, \ldots, y_{Kt}]'$, ν is the vector of intercepts, the Γ_i's are $K \times K$ matrices of parameters, and $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Kt}]'$ is a K-dimensional vector of Gaussian white noise processes with covariance matrix Σ, $\varepsilon_t \sim NID(\mathbf{0}, \Sigma)$. Equation (3) can be rewritten as:

$$\Delta y_t = \nu + \sum_{i=1}^{p-1} \Lambda_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \tag{4}$$

[3] An alternative interpretation is that the EH merely implies that the short-term rate and the long-term rate are determined jointly in equilibrium. Strictly speaking, it may not seem immediately obvious how a change in the long-term rate can occur without a change in the short-term rate, given that—by the Fisher-Hicks formulae—the long-term rate is a weighted average of expected short-term rates. However, this may not be the case if the long-term rate were the 10-year (or the 30-year) Treasury yield. It is well known that policymakers are trying to stabilize inflation and that they may look at long-term Treasury yields as indicators of long-run inflation expectations (e.g., Goodfriend, 1993).

where $\Lambda_i = -\sum_{j=i+1}^{p} \Gamma_j$ for $i = 1, \ldots, p-1$ are matrices of parameters, and $\Pi = \sum_{i=1}^{p} \Gamma_i - \mathbf{I}_K$ is the long-run impact matrix. If y_t is first-difference stationary, i.e., $y_t \sim I(1)$, the rank of Π, $r \leq K-1$ is the number of linearly independent cointegrating vectors and $K-r$ is the number of common trends (e.g., Johansen, 1988, 1991, 1995; Dickey, Jansen and Thornton, 1991). In this framework, $\Pi = \alpha'\beta$ characterizes the long-run equilibrium of the system. The deviation from the long-run equilibrium is measured by the stationary stochastic process $u_t = \alpha' y_t - \beta$ (Granger, 1986; Engle and Granger, 1987).

The present application focuses on a bivariate model comprising the federal funds rate (FF) and the T-bill rate (TB) — i.e., $y_t = [FF_t, TB_t]'$. Hence the series for the deviations from the long-run equilibrium we consider is essentially the cointegrating error term, say u_t, obtained from executing the Johansen (1988, 1991) maximum likelihood cointegration procedure in a VAR(p) involving FF_t, TB_t, and an unrestricted constant term.

As briefly mentioned in the introduction, an interesting strand of this literature has developed recently. This literature allows for asymmetric or nonlinear adjustment toward equilibrium in modelling interest rate movements based on the idea that interest rates arbitrage relationships may be characterized by complex nonlinear dynamics. In particular, Anderson (1997) uses nonlinear equilibrium correction models to study yield movements in the US T-bill market and argues that nonlinear equilibrium correction arises because portfolio adjustment is an "on-off" process, which occurs only when disequilibrium in the bill market is large enough to induce investors to incur the transactions costs associated with buying/selling bills. This, together with heterogeneity of transactions costs, implies that the strength of aggregate equilibrium correction depends on both the distribution of costs and the extent of disequilibrium in the market. Anderson uses smooth transition models with the transition to equilibrium characterized by an exponential function to describe an aggregate adjustment process which is stronger the more distant the market is from equilibrium, but is weak when the market is in the neighborhood of equilibrium.

Enders and Granger (1998) investigate the term structure of interest rates from a different perspective. They employ an asymmetric modelling framework. They also develop critical values to test the null hypothesis of a unit root in the spread between a short-term rate and a long-term rate against the alternative hypothesis of stationarity with asymmetric adjustment toward equilibrium. Within a reasonable range of adjustment parameters, the power of the new tests is shown to be greater than that of the corresponding conventional symmetric unit root tests if the true data generating process is asymmetric. Their results suggest that interest rate movements toward long-run equilibrium may be best captured by an asymmetric process. Enders and Granger also find that the speed of adjustment toward equilibrium is faster for negative deviations from equilibrium than for positive deviations. Their study is primarily methodological. Consequently, Enders and Granger make no attempt to provide an economic interpretation for their finding of an asymmetric adjustment process. Nevertheless, their findings suggest that it may be appropriate to consider a

model of interest rates that explicitly allows for this possibility.[4]

In our empirical model, initially we considered an ECM that allows for asymmetric adjustment to long-run equilibrium in a fashion similar to Enders and Granger (1998). Specifically, we estimated the following two-equation system of linear ECMs, using full information maximum likelihood (FIML):

$$\begin{aligned}\Delta FF_t &= c + \lambda^- u_{t-1}^- + \lambda^+ u_{t-1}^+ + \\ &\quad \textstyle\sum_{i=1}^p \gamma_{1i} \Delta FF_{t-i} + \sum_{i=1}^p \gamma_{2i} \Delta TB_{t-i} + innovations \qquad (5)\end{aligned}$$

$$\begin{aligned}\Delta TB_t &= k + \rho^- u_{t-1}^- + \rho^+ u_{t-1}^+ + \\ &\quad \textstyle\sum_{i=1}^p \theta_{1i} \Delta FF_{t-i} + \sum_{i=1}^p \theta_{2i} \Delta TB_{t-i} + innovations, \qquad (6)\end{aligned}$$

where c and k are constant terms; λ^-, λ^+, ρ^- and ρ^+, are speed-of-adjustment coefficients (equilibrium correction terms); u_{t-1}^- and u_{t-1}^+ denote negative and positive deviations from the long-run equilibrium implied by the long-run relationship between the federal funds rate and the T-bill rate. This model allows for asymmetric responses to lagged deviations from long-run equilibrium in each equation.[5]

Estimates of the asymmetric ECM (5)-(6), however, displayed temporal parameter instability and misspecification, as evidenced by remaining nonlinearity in the residuals. Consequently, we estimated a nonlinear ECM that allows for exponential-type nonlinearity in addition to asymmetric adjustment. Allowing for nonlinearity in the ECM was crucial to obtain parameter stability. It appears that the system (5)-(6) failed because it did not capture adequately the nonlinearity that characterizes the dynamic relationship between the federal funds and T-bill rates. Specifically, we estimated a general two-equation nonlinear, asymmetric ECM in the spirit of the smooth transition regressions suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998):

$$\begin{aligned}\Delta FF_t &= v_1 + \zeta_1^- u_{t-1}^- + \zeta_1^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \mu_{1i} \Delta FF_{t-i} + \sum_{i=1}^p \mu_{2i} \Delta TB_{t-i} + \\ &\quad \left[v_2 + \zeta_2^- u_{t-1}^- + \zeta_2^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \pi_{1i} \Delta FF_{t-i} + \sum_{i=1}^p \pi_{2i} \Delta TB_{t-i}\right] \times \\ &\quad \left\{1 - \exp\left[-\chi \left(u_{t-d} - v_3\right)^2\right]\right\} + innovations \qquad (7)\end{aligned}$$

[4] The work of Enders and Granger (1998) largely focuses on a univariate framework, although the authors report the results from estimating an asymmetric ECM toward the end of their paper.

[5] Formally,

$$\begin{cases} u_t^+ = u_t \quad if \quad u_t \geq 0, \quad u_t = 0 \quad otherwise \\ u_t^- = u_t - u_t^+. \end{cases}$$

Hence, this asymmetric ECM allows a varying strength of attraction to equilibrium where the attractor is assumed to be stronger on one side than on the other, so that u_t^+ may have a different coefficient from u_t^- (see also Granger and Lee, 1989).

$$\begin{aligned}\Delta TB_t &= w_1 + \xi_1^- u_{t-1}^- + \xi_1^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \vartheta_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \vartheta_{2i}\Delta TB_{t-i} + \\ &\quad \left[w_2 + \xi_2^- u_{t-1}^- + \xi_2^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \phi_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \phi_{2i}\Delta TB_{t-i}\right] \times \\ &\quad \left\{1 - \exp\left[-\psi\left(u_{t-d} - w_3\right)^2\right]\right\} + innovations, \end{aligned} \tag{8}$$

where v_1 and w_1 are constant terms; ζ_j^-, ζ_j^+, ξ_j^-, and ξ_j^+ for $j = 1, 2$ are speed-of-adjustment coefficients; u_{t-1}^- and u_{t-1}^+ are as defined above in (5)-(6). The transition functions in the system (7)-(8) are exponential functions. The exponential transition function, say $\Phi(\cdot)$ is bounded between zero and unity, $\Phi : \Re \rightarrow [0, 1]$, has the properties $\Phi[0] = 0$ and $\lim_{x\rightarrow\pm\infty} \Phi[x] = 1$, and is symmetrically inverse–bell shaped around zero. These properties of the exponential function are attractive in the present modelling context because they allow a smooth transition between regimes and faster adjustment of the interest rates examined for larger deviations above and below the equilibrium level. Because the model (7)-(8) allows for different equilibrium correction coefficients depending upon whether the deviation from equilibrium is positive or negative, it explicitly allows for both asymmetries and nonlinearities in a very general, comprehensive fashion. Indeed, the model (7)-(8) nests the linear asymmetric Enders-Granger (1998) model, the symmetric exponential smooth transition model employed by Anderson (1997), and a conventional linear symmetric ECM. The absence of asymmetries and nonlinearities in the dynamics of the interest rates under examination would be exhibited by very similar estimates of the equilibrium correction terms responding to negative and positive deviations from equilibrium (no asymmetry) and/or by the lack of statistical significance of the speed-of-adjustment parameters associated with the nonlinear transition function (no nonlinearity). When this happens, the system (7)-(8) collapses to the conventional linear ECM (4).[6] [7]

[6] Essentially our procedure is an example of a bottom-up procedure and consists of starting with a simple but statistically reliable ECM by imposing linearity and then testing the model against alternatives. In such a procedure, the structure contained in the data is explained by observable variables, therefore being consistent with the general-to-specific approach to econometric modelling. For a comprehensive discussion of the bottom-up procedure in the context of nonlinear and piecewise linear models, see Krolzig (1997).

[7] The class of nonlinear models is infinite. We have chosen to concentrate on the STR formulation primarily because of the above stated properties, its relative simplicity, and the large amount of previous research on the estimation of STR models. Note that Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998) also suggest the logistic function as a plausible transition function for some applications, resulting in a logistic STR or LSTR model. The LSTR model, however, implies asymmetric behavior of the interest rates in question according to whether they are above or below their equilibrium level. Since this behavior is already captured by the differences in the equilibrium correction terms for negative and positive deviations, we expect the LSTR formulation to be somewhat less appropriate in the present context. In addition, the exponential function has been successfully used by Andersen (1997) for modelling T-bill rates and has a natural interpretation in terms of arbitrage, namely that the adjustment toward equilibrium is faster the larger the deviation from equilibrium (i.e., the more profitable the arbitrage opportunity). Nevertheless, in our empirical analysis below, we tested for nonlinearities arising from LSTR formulations as a test of specification of our estimated models and agnostically employ a procedure suggested by Teräsvirta (1994, 1998) in order to choose the most appropriate transition function.

3 Data

The data are daily observations on the US effective federal funds rate (FF) and the US 3-month T-bill rate (TB) over the period from January 1 1974 to December 31 1999. The effective federal funds rate is a weighted average of the rates on federal funds transactions of a group of federal funds brokers who report their transactions daily to the Federal Reserve Bank of New York. Federal funds are deposit balances at Federal Reserve banks that institutions (primarily depositories, e.g. banks and thrifts) lend overnight to each other. These deposit balances are used to satisfy reserve requirements of the Federal Reserve System. Because reserve requirements are binding at the end of the reserve maintenance period, called settlement Wednesday, the funds rate tends to be more volatile on settlement Wednesdays.[8] The federal funds rate time series was adjusted in order to eliminate the effect of the increased volatility on settlement days and during three well-known episodes of high volatility that occurred in 1985 and 1986.[9] The T-bill rate is the daily market closing rate in the secondary market, expressed as a discount quote. Both the federal funds rate and the T-bill rate data are expressed on a 360-day basis.

In our sensitivity analysis, discussed in Section 5, we also use daily time series for the 3-month rate on certificates of deposit (CD rate) for the period from January 1 1974 to December 31 1999, and for the federal funds rate target (FF^{target}) for the period from November 22 1989 to December 31 1999.[10] We also use monthly data on the federal funds and T-bill rates for the period from January 1974 to December 1999. The monthly time series data are expressed as end-of-month daily data.

The time series for FF, TB and CD were taken from the Federal Reserve Bank of St. Louis database, *Federal Reserve Economic Data* (FRED). The main sample period–January 1 1974 to December 31 1999–covers 25 years, a period that should be sufficiently long to capture some of the main features of the unknown stochastic process governing the relationship between the federal funds rate and the T-bill rate. Also, the number of observations, $T = 6,784$, is sufficiently large to be fairly confident of the estimation results. The time series for the adjusted FF and TB are graphed in Figure 1, which makes visually apparent the trending behavior of both time series as well as their co-movement.

[8] Since February 1984 the reserve maintenance period has been two weeks for all institutions. Before 1984 it was one week for most large institutions. For a more detailed discussion of the Federal Reserve's reserve requirements and the microstructure of the federal funds market, see, for example, Taylor (2001). For comprehensive descriptions of the institutional aspects of the federal funds rate market, see Stigum (1990) and Furfine (1999).

[9] Precisely, the adjusted time series for the federal funds rate is the ordinary least squares residual from the regression of the federal funds rate on a dummy which is equal to unity for the three data points corresponding to the outliers mentioned above and zero for all remaining data points, and a dummy variable that equals zero on non-settlement days and 1 on settlement days.

[10] The federal funds rate target can be obtained, for example, from Thornton and Wheelock (2000, Table B1).

4 Empirical analysis

4.1 Preliminary statistics and cointegration analysis

Table 1 presents summary statistics as well as the results of unit root tests for FF and TB.[11] In contrast with the usual view that the term structure of interest rates, which suggests that the yield curve is typically upward sloping, FF averaged about 65 basis points more than TB. One possible explanation is that the markets for federal funds and T-bills are at least partially segmented, i.e., the two rates in question belong to different segments of the US money market (see Campbell, Lo and MacKinlay, 1997). This view, however, is diametrically opposite to the view that the federal funds rate affects all other interest rates at the short end of the term structure because of its key role in the implementation of monetary policy. Another possibility is that Treasury rates are free of default risk, while the funds rate is not. It would appear, however, that only a small fraction of the difference in means can be attributed to the default-risk-free character of the T-bill rate. A more plausible explanation is that the interest on T-bills is exempt from some local and state taxes. This effectively lowers the nominal T-bill rate on average relative to the federal funds rate. Given these tax differences, we think that it seems reasonable to assume that FF and TB belong to the same segment of the US money market, as a large empirical literature has assumed.

The federal funds rate is considerably more variable than the T-bill rate. This is not surprising given the relative size of the federals funds and T-bill markets and the nature of the federal funds market. An examination of the third and fourth moments indicates the existence of both excess skewness and kurtosis, suggesting that the underlying distribution of both FF and TB may be non-normal.

In keeping with a large number of studies of unit root behavior of interest rates, using standard augmented Dickey-Fuller (ADF) test statistics, we were unable to reject the unit root null hypothesis for both FF and TB at conventional nominal levels of significance.[12] Nevertheless, ADF tests on first- and second-differences of the two rates examined indicate that the differenced series are $I(0)$ for each rate, therefore supporting the stylized fact that interest rates are $I(1)$ processes (e.g., Stock and Watson, 1988, 1999).

There is an apparent conflict between conventional economic and finance theory, which often assumes that interest rates are stationary processes,[13] and the mainstream empirical literature on interest rates, which (at least since Engle

[11] In all statistical tests executed in this and subsequent sections, we use a 5 percent nominal significance level, unless otherwise specified.

[12] In each case, the number of lags was chosen such that no residual autocorrelation was evident in the auxiliary regressions. Moreover, using non-augmented Dickey-Fuller tests or augmented Dickey-Fuller tests with any number of lags in the range between 1 and 20 yielded qualitatively equivalent results.

[13] For example, see the vast finance literature assuming a Vasicek (1977) model of interest rates, which is simply a mean-reverting process representable as an Ornstein-Uhlenbeck equation.

and Granger, 1987) either assumes or finds that interest rates are nonstationary processes. We follow the empirical practice because very persistent series with a root at least very close (if not equal) to unity are better approximated by $I(1)$ processes than by stationary ones. Moreover, in such situations, cointegration methods are applicable (see, for example, Stock, 1997 and Lanne, 1999, 2000, and the references therein).

The Monte Carlo results provided by Balke and Fomby (1997) indicate that estimation of the long-run equilibrium using the Johansen (1988, 1991) procedure when the true adjustment toward equilibrium is nonlinear does not yield misleading results in terms of significant loss of power or size distortion. Hence, we use the Johansen maximum likelihood cointegration procedure to test for cointegration between FF and TB and to estimate the equilibrium error from the long-run equilibrium linking these two rates. Panel A of Table 2 reports the maximum eigenvalue and trace statistics from the Johansen procedure in a VAR(5) composed of FF, TB and an unrestricted constant term. The results–with and without Reimers's (1992) adjustment for degrees of freedom–indicate that there is a unique long-run relationship between the federal funds rate and the T-bill rate.

Having established the rank of the long-run impact matrix Π, we re-executed the Johansen cointegration procedure under the restriction that the rank of Π equals unity and the non-binding normalization that the coefficient on FF_t is unity. Panel B of Table 2 then reports the estimated cointegrating vector. The estimated cointegrating relationship appears remarkably stable over time (across different monetary policy regimes), as shown visually by the recursive estimate of the unrestricted parameter on TB, plotted in Figure 2, along with plus/minus twice the corresponding standard errors. The estimated coefficient on TB is correctly signed, but the hypothesis that it is equal to minus unity (the theoretical value implied by the EH) is easily rejected at conventional significance levels. The fact that the cointegrating relationship is consistent with a cointegrating parameter of about -1.2 rather than -1.0 might be seen as further corroborating the idea that it is not *exactly* the EH of the term structure that binds the two rates. The rejection of the hypothesis that the cointegrating parameter in term structure relationships is -1.0 is quite common in studies of the EH using US data (e.g., Bekaert, Hodrick and Marshall, 1997) as well as studies of interest rate convergence in the context of the European Monetary System (e.g. Siklos and Wohar, 1996, 1997; Granger and Siklos, 1999, and the references therein). Nevertheless, the evidence of the existence of a long-run relationship between FF and TB is very strong. The cointegrating residual or estimated equilibrium error, $\widehat{u}_t$, implied by this long-run relationship is plotted in Figure 3, which provides visual evidence that the long-run relationship under investigation is stationary. Noting that $\widehat{u}_t = FF_t - \widehat{\alpha} - \widehat{\beta} TB_t$ has a zero mean (by construction) and essentially measures the deviation from the cointegrating equilibrium relationship, it should be clear that when $\widehat{u}_t > 0$ ($\widehat{u}_t < 0$) the FF rate is essentially too high (low) relative to the equilibrium relationship, while the TB rate is too low (high).

4.2 Asymmetric equilibrium correction models

Preliminary to considering a nonlinear ECM, we estimated the bivariate asymmetric ECM (5)-(6). This model explicitly allows for an asymmetric response of interest rates to negative and positive deviations from long-run equilibrium in a fashion similar to Enders and Granger (1998). We estimated the model by FIML assuming a lag length of five, as suggested by standard information criteria, and employed the conventional general-to-specific procedure to obtain parsimonious equilibrium correction models for each of ΔFF_t and ΔTB_t (see Hendry, Pagan and Sargan, 1984). The resulting models, presented in Table 3, appear to be adequate in terms of approximately white noise residuals, although the adjusted coefficient of determination is not particularly high for the equation for ΔFF and is very low for the equation for ΔTB. Both λ^- and λ^+ are strongly individually significant in the equation for ΔFF. In contrast, only ρ^- is strongly individually significant in the equation for ΔTB. Nevertheless, joint tests of statistical significance of the equilibrium correction terms ($J - ECM$, testing $H_0 : \lambda^- = \lambda^+ = 0$ and $H_0 : \rho^- = \rho^+ = 0$ in equations (5) and (6), respectively) indicate strong rejection of the null hypothesis of no equilibrium correction for both equations. Like Enders and Granger (1998), we find that the speed of adjustment is faster for negative than for positive deviations from equilibrium. The difference in the estimates of the equilibrium correction terms for positive and negative deviations from equilibrium is quite large for both equations, suggesting that the allowance for asymmetric adjustment is very important in this context for each equilibrium correction equation.[14] Moreover, the estimates suggest that most of the adjustment toward long-run equilibrium occurs through the federal funds rate, rather than the T-bill rate.

4.3 Parameter constancy and linearity tests

In order to evaluate the adequateness of the asymmetric equilibrium correction models reported in Table 3, we initially executed some tests of parameter constancy against the alternative of smoothly changing parameters (Lin and Teräsvirta, 1994; see Appendix). The tests results, reported in Panel A of Table 4, indicate a strong rejection of the hypothesis of parameter constancy against the alternative hypothesis of smoothly-changing parameters, suggesting that nonlinear equilibrium correction may be a prerequisite for parameter stability.

Some evidence of general nonlinearity is provided by the RESET test statistics (Ramsey, 1969). The RESET test is a fairly general misspecification test. Under the RESET test statistic, the alternative model involves a higher-order polynomial to represent a different functional form. Under the null hypothesis,

[14] When the equations are estimated imposing the restrictions $\lambda^- = \lambda^+ = \lambda$ and $\rho^- = \rho^+ = \rho$, the estimate of λ is -0.1024 (0.0082) and the estimate of ρ is 0.0046 (0.0021). Figures in parentheses are standard errors. Nevertheless, testing the null hypothesis of no asymmetry in the equilibrium correction, namely $H_0 : \lambda^- = \lambda^+ = \lambda$ and $H_0 : \rho^- = \rho^+ = \rho$ in equations (5) and (6), respectively, yielded rejections of the null with p-values of virtually zero, as reported in Panel B of Table 3.

the statistic is distributed as $\chi^2(g)$ with g equal to the number of higher-order terms in the alternative model.[15] Panel B of Table 4 reports the RESET test statistics applied to the model in Table 3 constructed using squared and cubed fitted values. The results indicate a strong rejection of the null hypothesis of linearity (no misspecification) with p-values of virtually zero.

In addition, we performed tests of linearity against the alternative of smooth transition nonlinearity and followed the Teräsvirta (1994, 1998) decision rule to select the most adequate transition function for modelling nonlinearity in the present context (see Appendix). As shown by the results in Panel C of Table 3, the general linearity test F_L easily rejects the null hypothesis of linearity, especially for small delay parameters. The strongest rejection occurs when $d = 1$ for both equations, suggesting a fast response to disequilibria. Employing the Teräsvirta rule to discriminate between ESTR and LSTR formulations led us to conclude that an ESTR is the most adequate parametric STR formulation (given that F_2 yields the lowest p-value). This finding is consistent with our priors, discussed in Section 2, and with Anderson's (1997) arguments that interest rate arbitrage is an "on-off" process that occurs only when disequilibrium in the interest rate market is large enough to induce investors to incur heterogeneous transactions costs. Thus, the strength of aggregate equilibrium correction depends on the extent of the disequilibrium.

4.4 Nonlinear asymmetric equilibrium correction models

Given the results from the linearity tests, the bivariate system of nonlinear ECM for ΔFF and ΔTB, (7)-(8), was estimated by nonlinear FIML (NFIML) with $\widehat{u}_{t-1}$ as the equilibrium correction term and the transition variable. In estimation, we followed the recommendation of Granger and Teräsvirta (1993) and Teräsvirta (1998) of standardizing the transition parameter by dividing it by the sample variance of the transition variable, $\widehat{\sigma}_u^2$, and using a starting value of unity for the estimation algorithm. We then applied a conventional general-to-specific procedure in order to reach parsimonious empirical models. The parsimonious models were obtained by "testing down" the general system of equilibrium correction models with a lag length of five, imposing exclusion restrictions on the coefficient with the lowest (in absolute size) insignificant t-ratio and re-estimating the system sequentially. We repeated the estimation procedure several times using different alternative sequences and different sets of starting values for the parameters in order to ensure that the results were robust to the specification search rule and that a global optimum was achieved.

The results, reported in Panel A of Table 5, indicate that the dynamics of the interest rates examined is highly nonlinear. The estimated standardized transition parameter appears to be strongly significantly different from zero, in each equation, both on the basis of the individual t-ratios as well as on the basis of the strong rejection of the Skalin's (1998) parametric bootstrap

[15] In constructing the tests we use the F-statistic form. It is well known that in finite samples the actual test size of the F approximation may be closer to the nominal significance level than the actual size of the χ^2 approximation (see Teräsvirta, 1994).

likelihood ratio test (see SLR in Panel B). Since these tests may be construed as tests of nonlinear equilibrium correction, the results indicate strong evidence of nonlinear equilibrium correction for each interest rate.

The estimated speed-of-adjustment parameters imply well-defined and fairly fast estimated transition functions. This is shown in Figure 4, which displays the plots of the estimated transition functions for ΔFF and ΔTB against $\widehat{u}_{t-k} - \widehat{v}_3$ and $\widehat{u}_{t-k} - \widehat{w}_3$, respectively. The limiting case of $\Phi(\cdot) = 1$ is attained in each case. Nevertheless, as the estimated transition functions make clear, the speed of adjustment for ΔFF is larger than for ΔTB. A 20 percent absolute deviation from equilibrium implies a 75 percent movement in the transition function of the ΔTB equation, whereas the estimated transition function for ΔFF would be at its limiting value of unity for the same absolute deviation. The difference in the estimated nonlinear equilibrium corrections is further highlighted by Figure 5, which shows the nonlinear equilibrium corrections implied by our estimates for positive and negative deviations from equilibrium and for both equations in the system (7)-(8). Exponential-type nonlinearity has a small role in the equation for ΔFF, but no role in the equation for ΔTB, given that ${\xi_2}^- = {\xi_2}^+ = 0$. The role of asymmetries in the dynamics toward equilibrium, however, are very strong in both equations. Figure 5 also confirms that the adjustment toward equilibrium occurs largely through movements in the federal funds rate rather than the T-bill rate.

A battery of diagnostic tests are reported in Panel B of Table 5. As indicated by the joint tests of statistical significance of the equilibrium correction terms that are not restricted to zero in the final estimation, the hypothesis of significant equilibrium correction ($J - ECM$, testing $H_0 : {\zeta_2}^- = {\zeta_2}^+ = 0$ and $H_0 : {\xi_1}^- = {\xi_1}^+ = 0$ in equations (7) and (8), respectively) is strongly statistically significant for each equation; however, the equilibrium correction terms associated with the nonlinear transition function enter significantly only in the equation for ΔFF (indeed ${\xi_2}^- = {\xi_2}^+ = 0$ is imposed in the final estimation). A test of the null hypothesis of no asymmetry indicated strong rejection of the null for both equations (see *Asymmetry* in Panel B of Table 5). The residual diagnostics are satisfactory in each case. Eithreim and Teräsvirta's (1996) tests for residual serial correlation were not found to be statistically significant at conventional nominal levels of significance.[16] We also tested for the stability of the model by constructing Lin and Teräsvirta (1994) tests for each nonlinear ECM. The results suggest no structural break, with p-values reasonably larger than the conventional 5 percent, again indicating that nonlinear equilibrium correction is important for parameter stability in this context. For each of the estimated nonlinear models, the null hypothesis of no remaining nonlinearity for values of d ranging from 2 to 20 could not be rejected on the basis of Lagrange multiplier tests (in Table 5 we report only the maximal value of the LM statistic testing for remaining ESTR nonlinearity, $NLES_{MAX}$). Neither could we reject the hypothesis of no remaining nonlinearity of the LSTR variety with values of the

[16] Note that the Ljung-Box statistic does not have its standard asymptotic χ^2 distribution since the residuals come from a nonlinear model (e.g., Eitrheim and Teräsvirta, 1996)–this is probably the case also for the Durbin-Watson statistic.

delay parameter in the range from 1 to 20 ($NLLS_{MAX}$ in Table 5). This procedure therefore confirmed the validity of the choice of $d = 1$.

Although the adjusted coefficient of determination is not particularly high for the equation for ΔFF and is very low for the equation for ΔTB, there is an improvement in terms of goodness of fit relative to the asymmetric linear ECMs reported in Table 3. In order to explicitly compare the goodness of fit of the nonlinear equations (7)-(8) to the linear equations (5)-(6), we calculated the ratio of the $\overline{R}^2$, the residual variance (RV), the Akaike information criterion (AIC), and the Schwartz information criterion (SIC) from each of the estimated nonlinear equations (7)-(8) (reported in Panel A of Table 5) to the corresponding measure for the alternative models in Table 3. The results, reported in Panel C of Table 5, show that for each interest rate the estimated nonlinear asymmetric ECM largely outperforms the best alternative linear model, leading to a substantial reduction in the residual variance (12 and 14 percent for the funds rate and T-bill rate, respectively).

Overall, the nonlinear estimation results indicate the presence of complex nonlinear dynamics in the relationship between the federal funds rate and the T-bill rate, with adjustment toward the long-run equilibrium occurring at a speed which depends both upon the sign of the deviation from the equilibrium and on the absolute size of the deviation itself. The estimated models are in every case statistically well determined, provide good fits to the data and pass a battery of diagnostic tests.

4.5 Interpreting the empirical results

Our empirical results suggest the existence of a unique cointegrating equation characterizing the long-run equilibrium of the federal funds rate and the T-bill rate. The evidence suggests significant nonlinearities and asymmetry in the adjustment toward the long-run equilibrium between the federal funds rate and the T-bill rate. Specifically, the speed of adjustment toward the long-run equilibrium between these rates is a function of both the size and the magnitude of the deviation from equilibrium. These findings accord with an emerging literature on modelling the term structure of interest rates (e.g., Anderson, 1997; Enders and Granger, 1998), suggesting that significant nonlinearities and asymmetries are empirical regularities in the dynamic adjustment of interest rates.

Not all of our results were expected. For example, we were surprised by the remarkable stability of the long-run relationship between the federal funds rate and the T-bill rate in light of the marked changes in Federal Reserve operating procedures that occurred during the sample period.[17] The Fed was explicitly targeting the funds rate from 1974 to October 1979. In October 1979, however, the Fed switched to a nonborrowed reserves operating procedure

[17] One possibility is that changes in monetary policy operating procedures have induced changes in the dynamics of the relationship but not in the long-run equilibrium between the funds rate and the T-bill rate. Also, these changes might be reflected in time-varying risk premia over the sample examined.

in an attempt to reduce inflation by reducing the growth of the M1 monetary aggregate (Meulendyke, 1998). In October 1982 the Fed switched to a borrowed reserves operating procedure. Exactly when the Fed switched from a borrowed reserve operating procedure to an explicit funds rate targeting procedure is contentious. Thornton (1988) presents evidence that the Fed was explicitly targeting the funds rate as early as 1984. Meulendyke (1998), however, suggests the switch was somewhat later, noting that the "informal move away from borrowing reserves operating procedure was speeded by the stock market break on October 19, 1987," when the Federal Open Market Committee (FOMC) found that "a stable relationship between the amount of borrowing and the fund rate did not reemerge."[18] In addition, Hamilton and Jordá (2001) argue that the Fed was explicitly targeting the federal funds rate targeting by late 1989. Consequently, there seems to be general agreement that the Fed has explicitly targeted the funds rate at least since the late 1980s.

In any event, if the process governing the funds rate and its relationship with the T-bill rate reflects the policy considerations of the Fed, one might expect to see changes in the equilibrium relationship between these rates over time. Our results suggest that the long-run relationship between the federal funds rate and the T-bill rate was not affected by changes in the Fed's operating procedure. It should be noted, however, that the simple linear dynamic model displayed parameter instability. A complex nonlinear equilibrium correction model was a prerequisite to achieve parameter stability.

Somewhat more surprising was our finding that the burden of the adjustment toward equilibrium is borne by the federal funds rate. If the Fed controls short-term interest rates by controlling the effective federal funds rate, one might expect that the T-bill rate would adjust to the funds rate, rather than the other way around. Of course, this is not necessarily the case. For example, if the market anticipates changes in the federal funds rate, the T-bill rate will move in advance of the overnight rate. In this case, the data would give the impression that the funds rate is adjusting to the T-bill rate when, in fact, the T-bill rate adjusts to *expected* changes in the federal funds rate. Of course, this explanation applies only when the market correctly anticipates changes in the funds rate. When changes in the funds rate are unexpected, the T-bill rate would adjust to the new level of the funds rate.

The possibility that the finding, that the funds rate bears the burden of the adjustment process, is actually due to the market anticipating changes in the funds rate would seem to be more likely when the Fed is explicitly targeting the funds rate. It is well known that the Fed does not adjust its funds rate target immediately in response to new information. Rather, target adjustments occur at discrete intervals and by relatively large amounts (e.g., Goodfriend, 1991). Consequently, changes in the level of the funds rate should be less continuous when the Fed is targeting the funds rate than when the funds rate is free to respond to market pressures. Moreover, changes might be easier to predict because the funds rate would be directly linked to the policy objectives

[18] Meulendyke (1998, p. 55).

of the Fed. In this respect, it is important to note that there have been a number of procedural changes recently that should have significantly improved the market's ability to anticipate changes in the Fed's federal funds rate target. Since 1994 the Fed has announced target changes immediately upon making them. Before 1994, target changes were not announced: the market had to infer the Fed's actions by observing open market operations and the federal funds rate (e.g., Cook and Hahn, 1989; Rudebusch, 1995; Taylor, 2001). In addition, since 1994, with two exceptions, the Fed has changed the federal funds rate target at regular meetings of the FOMC, which are scheduled far in advance.[19] Prior to that, most target changes were made during the inter-meeting period and at the discretion of the Chairman. Also, since October 1989, the Fed has followed the practice of changing the funds rate target by either 25 or 50 basis points, whereas the previous practice involved making target changes of various amounts.[20] These procedural changes should have improved the market's ability to predict the funds rate.

There is another reason why the relatively rapid adjustment of the funds rate is not necessarily at odds with the view that the Fed controls the structure of short-term rates; it is directly linked to the Fed's funds rate targeting procedure. If the T-bill rate is set more or less equal to the expected path for the funds rate target, the finding that the funds rate bears the burden of the daily adjustment process might merely reflect the fact that the funds rate tends to revert back to the funds rate target when the Fed is explicitly targeting the funds rate. The Fed could play a role in the adjustment process by injecting reserves when the funds rate is above the target and draining reserves when the funds rate is below the target. There is some evidence that the Fed did this during the 1970s (e.g., Cook and Hahn, 1989). Since the early 1980s the Fed has followed the practice of entering the market only once a day, usually before the bulk of federal funds transactions takes place. Consequently, since then the Fed has not actively attempted to offset intra-day deviations of the funds rate from the funds rate target. The Fed can, however, respond to deviations of the funds rate from the funds rate target on the previous day.[21] Alternatively, deviations of the funds rate from the target level might be the result of forecast errors in implementing monetary policy (e.g., Thornton, 2001). Either way, at the daily frequency, the funds rate would tend to revert to the target level and, therefore, the T-bill rate. The reversion of the funds rate to the target would then appear in the data as the funds rate adjusting to the equilibrium relationship with the T-bill rate.

If the relatively fast adjustment of the funds rate is due to the Fed targeting the funds rate, one would expect to see a reduction in the speed at

[19] The exceptions occurred on April 18 1994 and October 15 1998.

[20] There was one exception. On October 15 1994 the Fed raised the funds rate target by 75 basis points.

[21] See Taylor (2001) for a model where the supply of reserves is a function of the spread between the funds rate and the funds rate target on the previous day. See Orphanides (2001) for a critique of this model and Thornton (2001) for evidence of the extent to which the Fed responds to the previous day's spread between the funds rate and the funds rate target.

which the funds rate adjusts to the long-run equilibrium if the Fed's funds rate target is explicitly accounted for. Consequently, as a robustness check, the model is estimated including $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ as a right-hand-side variable to explain movements in the federal funds rate. If the rapid adjustment of the funds rate is due to the Fed's targeting procedure, it seems plausible that $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ should account for a significant fraction of the adjustment of the funds rate.

Also, if more rapid adjustment of the funds rate with daily data is due to the Fed targeting the funds rate, the results should change when lower frequency data are used. Hence, the model is estimated using monthly data over the period 1974-1999 to investigate this possibility.

Finally, despite its wide use in empirical analyses in economics and finance, the 3-month T-bill rate has characteristics that make it somewhat unique (e.g., see Duffee, 1996). Consequently, it is important to test whether the results are robust to the use of other 3-month interest rates. This is done by estimating the model using the daily 3-month CD rate in place of the T-bill rate. The CD rate was selected because large money-market banks finance part of their loan portfolios in the overnight federal funds market and with CDs. Consequently, federal funds and CDs represent alternative sources of funds for these banks.[22]

5 Robustness results

In this section we report three of the several robustness checks carried out in order to evaluate the sensitivity of the empirical results reported in Section 4. In the first model (M1), $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ is included on the right-hand-side of the nonlinear asymmetric ECM (7)-(8).[23] Data on the funds rate target are not available over the entire sample period. In this case the model is estimated over the period November 22, 1989 to December 31, 1999, a period for which there is agreement that the Fed was explicitly targeting the federal funds rate.[24] In the second model (M2), equations (7)-(8) are estimated using monthly data over the period from January 1974 to December 1999. In the third model (M3), the robustness of the findings are investigated by replacing TB_t with CD_t.[25]

[22] A referee suggested that daily data are simply at too low a frequency to pin down the true adjustment process. The T-bill market is very liquid, so that adjustments may occur within minutes or so of a shock. In contrast, the federal funds market is a thinner and less liquid market. Consequently, the adjustment of the funds rate to shocks takes longer. The referee suggests that the the finding, that FF does most of the adjustment, may be partly due to microstructural differences between the two markets. Intra-day examination of the relationship between the federal funds rate and the T-bill rate represents, therefore, an avenue for future research.

[23] In fact, we allowed for longer (up to five) lags of $\Delta\left(FF - FF^{\text{target}}\right)_t$, but lagged values from 2 to 5 were never found to be statistically significant at conventional significance levels.

[24] We chose November 22, 1989 to conform with Hamilton and Jorda (2001).

[25] In testing for cointegration between FF and TB using monthly data (as a preliminary to estimating M2) and between TB and CD (as a preliminary to estimating M3), in each case, the Johansen procedure suggested one significant eigenvalue and, therefore, one cointegrating relationship. Also, the cointegrating relationships were found to be stable over time using recursive estimation methods (results not reported to conserve space but available upon re-

The results of these robustness checks are summarized in Table 6. To conserve space, only the estimated equilibrium correction coefficients and some test statistics are reported. Table 6 reports the equilibrium correction terms and the parameters associated with $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ (namely κ_1, linear, and κ_2, nonlinear, in the equation for ΔFF and the corresponding parameters, ϱ_1 and ϱ_2, in the equation for ΔTB). The linear coefficient on $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ is correctly signed and statistically significant in the funds rate equation. The coefficient is large in absolute value, suggesting that the funds rate tends to adjust rather quickly to deviations from the target rate. More importantly, the equilibrium correction term in the funds rate equation decreases when the funds rate target is explicitly accounted for, suggesting that some of the adjustment process for the funds rate reported in Table 5 was due to the reversion of the funds rate toward the funds rate target. Nevertheless, even allowing for this possibility, the funds rate continues to bear the brunt of the adjustment process. Moreover, the speed of adjustment of the T-bill rate is relatively slow and the coefficient on $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ is insignificantly different from zero.[26] While some of the adjustment of the federal funds rate at the daily frequency appears to be due to reversion of the funds rate to the funds rate target, the conclusion that the federal funds rate bears most of the burden of the adjustment process is not overturned.

The results in the M2 column of Table 6 indicate that the conclusion that the funds rate bears the burden of the adjustment process is also robust to the frequency of the data. The estimated speed of adjustment of the funds rate is more than 20 times faster than that of the T-bill rate when monthly data are used.[27] Hence, consistent with the results reported in the M1 column, the finding that the funds rate bears the burden of the equilibrium adjustment process does not appear to be critically dependent on the high-frequency adjustment of the funds rate to the funds rate target.

Comparing the result of M3 in Table 6 with the results in Table 5 reveals that this finding also does not depend on the choice of the 3-month rate. The funds rate continues to bear the burden of the adjustment process when the CD rate is used instead of the T-bill rate. Indeed, if anything, the estimated speed of adjustment of the funds rate is faster with the CD rate. Whatever accounts for the relatively rapid adjustment of the funds rate, it does not appear to be associated with the default-risk-free characteristic or favorable tax treatment

quest). Nevertheless, as in the case of the relationship between the funds rate and the T-bill rate discussed in Section 4, the asymmetric ECM was unable to fully capture the nonlinearity in the underlying relationship and linearity tests indicated a model of the form (7)-(8).

[26] Allowing for longer lags of $\Delta\left(FF - FF^{\text{target}}\right)_t$ did not produce qualitatively different results.

[27] Also, note that the cointegrating parameter we obtained when applying the Johansen procedure to a VAR comprising the FF rate and the TB rate at monthly frequency was very close to that reported in Table 2, being equal to -1.22. This result corroborates the long-run equilibrium recorded for the model estimated at daily frequency and suggests that the finding of a cointegrating parameter that is not equal to the unity parameter implied by the EH may not be due to the inferential problems caused by the high volatility displayed by the daily time series (see Hall, Martin and Pagan, 1996).

(or other idiosyncratic characteristics, e.g., Duffee, 1996) of T-bills.

6 Conclusion

This article examines the dynamic relationship between two key US money market interest rates, the federal funds rate and the 3-month T-bill rate, employing a very general nonlinear asymmetric vector equilibrium correction model and using daily data over the period 1974-1999. The empirical results provide strong evidence of a cointegrating relationship between the federal funds rate and the T-bill rate that is remarkably stable over regimes of federal funds rate and monetary aggregate targeting. The conventional linear equilibrium correction model is rejected when tested against the nonlinear equilibrium correction model, indicating that the dynamic relationship between these rates is nonlinear. Moreover, consistent with the findings of Enders and Granger (1998), the adjustment to the equilibrium rate structure is asymmetric: the adjustment to negative deviations from the equilibrium rate structure is much faster than that for positive deviations. We know of no convincing theoretical explanation for this result, but this empirical regularity deserves further investigation.

Also, consistent with a number of previous studies, we find that the estimated cointegrating vector is inconsistent with the strict version of the conventional expectation hypothesis of the term structure of interest rates. This is not surprising. Empirical support for the strict version of the expectations hypothesis is generally weak (e.g., Campbell, Lo and MacKinlay, 1997). Indeed, the support is particularly weak when the short-term rate is the federal funds rate (see, *inter alios*, Simon, 1990, 1994; Roberds, Runkle and Whiteman, 1997; and Thornton, 2000).

The more surprising result was the finding that the funds rate adjusts more rapidly than the T-bill rate toward the long-run equilibrium linking these two rates. The conventional view of monetary policy is that the Federal Reserve controls the federal funds rate and all other short-term rates key off of it. In some sense, the federal funds rate is thought to "anchor" the short end of the term structure. Strictly speaking, this paradigm assumes the expectations hypothesis holds, but even if the strict expectations hypothesis does not hold, there is no reason to doubt that markets are forward looking. Hence, it is reasonable to assume that other rates key off of the funds rate, given the apparent ability of the Fed to control the funds rate.

One way to reconcile our finding with the conventional view of monetary policy comes from noting that the funds rate tends to stay close to the Fed's target for the funds rate. If the T-bill rate is set more or less equal to the market's expectation for the Fed's funds rate target, estimates using daily data would suggest that the funds rate is adjusting to the T-bill rate when in fact the funds rate is merely reverting to the funds rate target. Several robustness checks were undertaken to investigate this possibility. Specifically, deviations of the funds rate from the Fed's target on the previous day were included in the model. While the speed of adjustment of the funds rate was somewhat reduced

when deviations of the funds rate from the Fed's target on the previous day were included, the fundamental result that the funds rate bears most of the burden of the adjustment process remained. Also, the funds rate continued to bear the burden of the adjustment process when monthly data were used. Consequently, the finding that it is the funds rate that does most of the adjusting toward equilibrium does not appear to be due to the reversion of the funds rate to the funds rate target. It is also not due to the choice of the 3-month rate, as evidenced by the fact that the results were qualitatively unchanged when the CD rate was used in place of the T-bill rate.[28]

This result from our dynamic model can also be reconciled with the conventional view of the relationship between the federal funds rate and the 3-month T-bill rate (or other short-term US interest rates) if the market is able to make accurate forecasts of the future level of the funds rate. If the market correctly predicts changes in the funds rate, the T-bill rate (and other short-term interest rates) would move in advance of the funds rate. In such a circumstance, it would appear that the funds rate is adjusting to the T-bill rate when in fact the T-bill rate is adjusting to the expected future federal funds rate. There is some evidence, however, that interest rates are relatively difficult to predict even when the Fed is explicitly targeting the funds rate.[29] Nevertheless, this possibility deserves further investigation.

The reader is cautioned that, while the model used here is very general and flexible, the results are based on bivariate interest rate comparisons. The US money market is very complex and involves a wide range of credit instruments. Much more work needs to be done to understand these relationships. Nevertheless, the evidence presented here suggests that the relationship between the federal funds rate and other short-term interest rates is considerably more complex than models of monetary policy or the expectations hypothesis suggest.

[28] While our results may seem somewhat unexpected, they add to some related studies. Garfinkel and Thornton (1995), for example, found that the federal funds rate did not contain information that was not contained in either the overnight repurchase rate or the three-month T-bill rate.

[29] For example, Robertson and Thornton (1997), who investigated the predictive accuracy of a rule based on the federal funds futures rate from October 1988 through August 1997, found that the forecasting rule correctly predicted a target change at the one-month horizon only about one-third of the time. This finding is consistent with Rudebusch (2001), who finds very little predictability of short-term interest rates beyond a one-month horizon. For evidence of the predictability of changes in the Fed's funds rate target at much shorter horizons, see Poole and Rasche (2000) and Kuttner (2000).

Table 1. Preliminary data analysis

Descriptive statistics

	Minimum	Maximum	Mean	Stand. deviation	Skewness	Kurtosis
FF	2.1666	22.0000	7.4979	3.3535	1.3018	1.9114
TB	2.6100	17.1400	6.8391	2.7439	1.2107	1.4272

Augmented Dickey-Fuller (ADF) unit root tests

FF	TB	ΔFF	ΔTB	$\Delta^2 FF$	$\Delta^2 TB$
-2.25	-2.01	-46.24	-33.40	-77.44	-62.98

Notes: FF and TB denote the federal funds rate and the 3-month T-bill rate respectively; Δ is the first-difference operator. Figures reported are annualized; the sample period spans from 1974:1:1 to 1999:12:31, and the number of observations is $6,784$. For ADF tests, the critical value at the five (one) percent level of significance is -2.86 (-3.43) to two decimal places (Dickey and Fuller, 1979; MacKinnon, 1991).

Table 2. Johansen maximum likelihood cointegration procedure

A) Cointegration likelihood ratio tests based on maximum eigenvalue of the stochastic matrix ($\lambda_{\max}$) and on trace eigenvalue of the stochastic matrix (λ_{trace})

H_0	H_1	$\lambda_{\max}$	CV	H_0	H_1	λ_{trace}	CV
$r = 0$	$r = 1$	903.5 (903.2)	14.1	$r = 0$	$r \geq 1$	906.5 (906.2)	15.4
$r \leq 1$	$r = 2$	3.006 (3.005)	3.8	$r \leq 1$	$r = 2$	3.006 (3.005)	3.8

B) Estimated cointegrating vector

FF	TB	*constant*
1.0000	−1.2020	0.7324
[−]	[0.0183]	[0.1352]

Notes: These results were obtained from testing for cointegration in a VAR(5) comprised of FF, TB and an unrestricted constant term. In Panel A, H_0 and H_1 denote the null hypothesis and the alternative hypothesis respectively; figures in parentheses are test statistics adjusted for degrees of freedom (Reimers, 1992); r denotes the number of cointegrating vectors, and CV is the 95 percent critical value (see Osterwald-Lenum, 1992; Johansen, 1995). In Panel B, figures in square brackets are estimated standard errors.

Table 3. Linear asymmetric equilibrium correction models

A) Estimated parsimonious models

	dep. variable: ΔFF_t		dep. variable: ΔTB_t
c	-0.055 (0.007)	k	—
λ^-	-0.256 (0.016)	ρ^-	8.4x10^{-3} (3.3x10^{-3})
λ^+	-0.034 (0.010)	ρ^+	1.6x10^{-3} (2.4x10^{-3})
γ_{11}	-0.319 (0.014)	θ_{11}	0.018 (3.4x10^{-3})
γ_{12}	-0.238 (0.014)	θ_{12}	0.019 (3.2x10^{-3})
γ_{13}	-0.212 (0.013)	θ_{13}	8.7x10^{-3} (2.9x10^{-3})
γ_{14}	-0.253 (0.013)	θ_{14}	—
γ_{15}	0.181 (0.012)	θ_{15}	0.011 (2.7x10^{-3})
γ_{21}	0.223 (0.047)	θ_{21}	0.103 (0.012)
γ_{22}	0.195 (0.046)	θ_{22}	—
γ_{23}	—	θ_{23}	-0.046 (0.012)
γ_{24}	0.113 (0.046)	θ_{24}	—
γ_{25}	-0.099 (0.046)	θ_{25}	0.059 (0.012)

(continued ...)

(... Table 3 continued)

B) Diagnostics

	dep. variable: ΔFF_t	dep. variable: ΔTB_t
$\overline{R}^2$	0.337	0.030
$J-ECM$	{4.1x10^{-65}}	{7.8x10^{-8}}
$Asymmetry$	{5.9x10^{-16}}	{3.7x10^{-10}}
DW	1.960	2.07
$LB(20)$	{0.353}	{0.278}
LR	{0.886}	{0.772}
s	0.198	0.014

Notes: In Panel A, the model estimated is given by the system of equilibrium correction equations (denoted as (5)-(6) in the text):

$$\begin{aligned}\Delta FF_t &= c+\lambda^- u^-_{t-1}+\lambda^+ u^+_{t-1}+ \\ &\quad \textstyle\sum_{i=1}^p \gamma_{1i}\Delta FF_{t-i}+\sum_{i=1}^p \gamma_{2i}\Delta TB_{t-i}+innovations\end{aligned}$$

$$\begin{aligned}\Delta TB_t &= k+\rho^- u^-_{t-1}+\rho^+ u^+_{t-1}+ \\ &\quad \textstyle\sum_{i=1}^p \theta_{1i}\Delta FF_{t-i}+\sum_{i=1}^p \theta_{2i}\Delta TB_{t-i}+innovations,\end{aligned}$$

where c and k are constant terms; λ^-, λ^+, ρ^- and ρ^+ are speed of adjustment coefficients (equilibrium correction terms); u^-_{t-1} and u^+_{t-1} denote negative and positive deviations from the long-run equilibrium implied by the long-run relationship between the federal funds rate and the T-bill rate. The parsimonious models were obtained by estimating the system by FIML and applying a general to specific procedure; figures in parentheses denote estimated standard errors. In Panel B, $\overline{R}^2$ is the adjusted coefficient of determination; $J-ECM$ is a test of joint significance of the two equilibrium correction coefficients in each equation (i.e., $H_0:\lambda^-=\lambda^+=0$ and $H_0:\rho^-=\rho^+=0$), and is distributed as $\chi^2(2)$ under the null; $Asymmetry$ is a test of the null hypothesis of no asymmetric equilibrium correction; DW is the Durbin-Watson statistic; $LB(20)$ is the Ljung-Box test for residual autocorrelation up to order 20, and is distributed as $\chi^2(20)$ under the null; LR is a likelihood ratio test for the vailidity of the restrictions imposed in the final estimation; s is the residual variance. Figures in braces denote p-values.

Table 4. Parameter constancy, misspecification, and linearity tests

A) Lin-Teräsvirta parameter constancy tests

	dep. variable: ΔFF_t	dep. variable: ΔTB_t
LM_{C3}	$4.1\text{x}10^{-12}$	$1.3\text{x}10^{-5}$
LM_{C2}	$4.7\text{x}10^{-81}$	$1.1\text{x}10^{-8}$
LM_{C1}	$3.3\text{x}10^{-63}$	$6.2\text{x}10^{-2}$

B) RESET test statistics

dep. variable: ΔFF	dep. variable: ΔTB
$4.8\text{x}10^{-22}$	$7.5\text{x}10^{-40}$

C) Teräsvirta linearity tests

General linearity test F_L (p=5)

	dep. variable: ΔFF_t	dep. variable: ΔTB_t
$d = 1$	$2.5\text{x}10^{-41}$	$7.4\text{x}10^{-19}$
$d = 2$	$2.1\text{x}10^{-22}$	$3.7\text{x}10^{-10}$
$d = 3$	$6.6\text{x}10^{-26}$	$8.1\text{x}10^{-7}$
$d = 4$	$1.4\text{x}10^{-30}$	$2.8\text{x}10^{-5}$
$d = 5$	$7.2\text{x}10^{-6}$	$3.9\text{x}10^{-3}$
$d = 6$	$1.3\text{x}10^{-21}$	$4.5\text{x}10^{-3}$
$d = 7$	$3.3\text{x}10^{-30}$	$6.4\text{x}10^{-2}$
$d = 8$	$2.3\text{x}10^{-12}$	$2.8\text{x}10^{-3}$
$d = 9$	$1.3\text{x}10^{-5}$	$7.0\text{x}10^{-2}$
$d = 10$	$3.4\text{x}10^{-2}$	$1.2\text{x}10^{-1}$

Teräsvirta rule (p=5; d=1)

	dep. variable: ΔFF_t	dep. variable: ΔTB_t
F_L	$2.5\text{x}10^{-41}$	$1.7\text{x}10^{-10}$
F_3	$7.5\text{x}10^{-8}$	$3.3\text{x}10^{-4}$
F_2	$4.2\text{x}10^{-36}$	$8.5\text{x}10^{-8}$
F_1	$7.3\text{x}10^{-23}$	$9.2\text{x}10^{-3}$

Notes: All test statistics recorded in this table were executed on the parsimonious linear equilibrium correction models reported in Table 3. In Panel A, figures are p-values from Lin-Teräsvirta test statistics of the null hypothesis of parameter constancy against the alternative hypothesis of smooth-transition-type nonconstancy (see the Appendix for details). In Panel B, figures are p-values from RESET test statistics. In Panel C, figures are p-values from Teräsvirta test statistics of the null hypothesis of linearity against the alternative hypothesis of smooth-transition-type nonlinearity using the cointegrating residuals $\widehat{u}_{t-d}$ as the transition variable (see the Appendix for details).

Table 5. Nonlinear asymmetric equilibrium correction models

A) Estimated parsimonious models

	dep. variable: ΔFF_t		dep. variable: ΔTB_t
v_1	—	w_1	—
ζ_1^-	—	ξ_1^-	0.010 (0.004)
ζ_1^+	—	ξ_1^+	3.8x10^{-3} (2.1x10^{-3})
μ_{11}	-0.319 (0.027)	ϑ_{11}	0.102 (0.022)
μ_{12}	-0.201 (0.026)	ϑ_{12}	—
μ_{13}	—	ϑ_{13}	—
μ_{14}	-0.149 (0.019)	ϑ_{14}	—
μ_{15}	0.182 (0.021)	ϑ_{15}	—
μ_{21}	0.213 (0.088)	ϑ_{21}	0.017 (5.9x10^{-3})
μ_{22}	0.215 (0.100)	ϑ_{22}	0.019 (5.2x10^{-3})
μ_{23}	—	ϑ_{23}	—
μ_{24}	—	ϑ_{24}	—
μ_{25}	—	ϑ_{25}	0.011 (4.0x10^{-3})
v_2	-0.054 (0.008)	w_2	—
ζ_2^-	-0.254 (0.024)	ξ_2^-	—
ζ_2^+	-0.035 (0.014)	ξ_2^+	—
π_{11}	0.100 (0.025)	ϕ_{11}	—
π_{12}	0.063 (0.024)	ϕ_{12}	—
π_{13}	-0.212 (0.015)	ϕ_{13}	—
π_{14}	—	ϕ_{14}	—
π_{15}	—	ϕ_{15}	0.190 (0.072)
π_{21}	—	ϕ_{21}	—
π_{22}	—	ϕ_{22}	—
π_{23}	—	ϕ_{23}	0.069 (0.015)
π_{24}	—	ϕ_{24}	—
π_{25}	—	ϕ_{25}	—
χ	0.914 (0.226)	ψ	0.308 (0.093)
v_3	—	w_3	—

(continued ...)

(... Table 5 continued)

B) Diagnostics

	dep. variable: ΔFF_t	dep. variable: ΔTB_t
$\overline{R}^2$	0.395	0.034
$J-ECM$	$\{3.3\text{x}10^{-28}\}$	$\{7.1\text{x}10^{-3}\}$
$Asymmetry$	$\{4.6\text{x}10^{-19}\}$	$\{1.5\text{x}10^{-10}\}$
$Q(1)$	$\{0.872\}$	$\{0.759\}$
$Q(20)$	$\{0.549\}$	$\{0.652\}$
LR	$\{0.475\}$	$\{0.511\}$
$NLES_{MAX}$	$\{0.249\}$	$\{0.673\}$
$NLLS_{MAX}$	$\{0.273\}$	$\{0.442\}$
SLR	$\{2.1\text{x}10^{-26}\}$	$\{1.5\text{x}10^{-3}\}$
LM_{C3}	$\{0.430\}$	$\{0.725\}$
LM_{C2}	$\{0.511\}$	$\{0.459\}$
LM_{C1}	$\{0.362\}$	$\{0.774\}$
s	0.174	0.012

C) Relative goodness of fit: nonlinear versus linear ECM

	dep. variable: ΔFF_t	dep. variable: ΔTB_t
$\overline{R}^2$ ratio	1.17	1.13
RV ratio	0.88	0.86
AIC ratio	1.12	1.18
SIC ratio	1.13	1.20

(continued ...)

(... Table 5 continued)

Notes: In Panel A, the model estimated is given by the system of nonlinear equilibrium correction equations (denoted as (7)-(8) in the text):

$$\begin{aligned}\Delta FF_t &= v_1 + \zeta_1^- u_{t-1}^- + \zeta_1^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \mu_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \mu_{2i}\Delta TB_{t-i} + \\ &\quad \left[v_2 + \zeta_2^- u_{t-1}^- + \zeta_2^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \pi_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \pi_{2i}\Delta TB_{t-i}\right] \times \\ &\quad \left\{1 - \exp\left[-\chi\left(u_{t-d} - v_3\right)^2\right]\right\} + innovations \\ \Delta TB_t &= w_1 + \xi_1^- u_{t-1}^- + \xi_1^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \vartheta_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \vartheta_{2i}\Delta TB_{t-i} + \\ &\quad \left[w_2 + \xi_2^- u_{t-1}^- + \xi_2^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \phi_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \phi_{2i}\Delta TB_{t-i}\right] \times \\ &\quad \left\{1 - \exp\left[-\psi\left(u_{t-d} - w_3\right)^2\right]\right\} + innovations,\end{aligned}$$

where v_1 and w_1 are constant terms; ζ_j^-, ζ_j^+, ξ_j^- and ξ_j^+ for $j = 1, 2$ are speed of adjustment coefficients; u_{t-1}^- and u_{t-1}^+ denote negative and positive deviations from the long-run equilibrium implied by the long-run relationship between the federal funds rate and the T-bill rate. The parsimonious models were obtained by estimating the system of nonlinear equilibrium correction equations by NFIML and applying a general to specific procedure; figures in parentheses denote estimated standard errors. In Panel B, $\overline{R}^2$ is the adjusted coefficient of determination; $J - ECM$ is a test of joint significance of the equilibrium correction coefficients not restricted to zero in the final estimation in each equation (i.e., $H_0 : \zeta_2^- = \zeta_2^+ = 0$ and $H_0 : \xi_1^- = \xi_1^+ = 0$), and is distributed as $\chi^2(2)$ under the null; *Asymmetry* is a test of the null hypothesis of no asymmetric equilibrium correction; $Q(1)$ and $Q(20)$ are Lagrange multiplier test statistics for first-order and up to twentieth-order serial correlation in the residuals respectively, constructed as in Eitrheim and Teräsvirta (1996); LR is a likelihood ratio test for the validity of the restrictions imposed in the final estimation. $NLES_{MAX}$ is the maximal Lagrange multiplier test statistic for no remaining exponential-smooth-transition-type nonlinearity with delay in the range from 2 to 10; $NLLS_{MAX}$ is the maximal Lagrange multiplier test statistic for no remaining logistic-smooth-transition-type nonlinearity with delay in the range from 1 to 12 (Eitrheim and Teräsvirta, 1996). SLR is a parametric bootstrap likelihood ratio test of the null hypothesis that the transition parameters (χ and ψ) are equal to zero, constructed as suggested by Skalin (1998). LM_{C3}, LM_{C2} and LM_{C1} are Lin-Teräsvirta test statistics of the null hypothesis of parameter constancy; s is the residual variance. Figures in braces denote p-values. In Panel C, the $\overline{R}^2$ ratio, the RV (residual variance) ratio, the AIC and the SIC ratios are the ratios of the $\overline{R}^2$, the residual variance, the AIC and the SIC from each of the two estimated nonlinear equations reported in Panel A to the corresponding goodness of fit measures obtained for the alternative linear models reported in Table 3.

Table 6. Robustness: the magnitude of the nonlinear equilibrium correction coefficients

A) Nonlinear ECM for ΔFF (M1, M2 and M3)

	M1	M2	M3
ζ_1^-	—	—	—
ζ_1^+	—	—	—
$\varkappa_1$	-0.742 (0.026)	—	—
ζ_2^-	-0.154 (0.029)	-0.524 (0.159)	-0.323 (0.017)
ζ_2^+	-0.031 (0.017)	-0.201 (0.034)	-0.085 (0.016)
$\varkappa_2$	—	—	—
$J-ECM$	{3.6x10^{-7}}	{9.0x10^{-6}}	{2.1x10^{-81}}
$Asymmetry$	{2.8x10^{-12}}	{4.2x10^{-8}}	{6.8x10^{-22}}
LR	{0.674}	{0.491}	{0.458}

B) Nonlinear ECM for ΔTB (M1 and M2) and ΔCD (M3)

	M1	M2	M3
ξ_1^-	0.012 (0.003)	0.024 (0.010)	0.012 (0.004)
ξ_1^+	3.8x10^{-3} (3.2x10^{-3})	0.007 (0.003)	0.007 (0.001)
ϱ_1	—	—	—
ξ_2^-	—	—	—
ξ_2^+	—	—	—
ϱ_2	—	—	—
$J-ECM$	{9.9x10^{-4}}	{0.035}	{0.011}
$Asymmetry$	{3.1x10^{-9}}	{7.1x10^{-7}}	{2.8x10^{-4}}
LR	{0.481}	{0.442}	{0.530}

(... continued)

(... Table 6 continued)

Notes: The model estimated is a nonlinear asymmetric ECM of the form:

$$\begin{aligned}\Delta Y_t &= v_1 + \zeta_1^- u_{t-1}^- + \zeta_1^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \mu_{1i}\Delta Y_{t-i} + \sum_{i=1}^p \mu_{2i}\Delta X_{t-i} + \\ &\quad \left[v_2 + \zeta_2^- u_{t-1}^- + \zeta_2^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \pi_{1i}\Delta Y_{t-i} + \sum_{i=1}^p \pi_{2i}\Delta X_{t-i}\right] \times \\ &\quad \left\{1 - \exp\left[-\chi\left(u_{t-d} - v_3\right)^2\right]\right\} + innovations\end{aligned}$$

$$\begin{aligned}\Delta X_t &= w_1 + \xi_1^- u_{t-1}^- + \xi_1^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \vartheta_{1i}\Delta Y_{t-i} + \sum_{i=1}^p \vartheta_{2i}\Delta X_{t-i} + \\ &\quad \left[w_2 + \xi_2^- u_{t-1}^- + \xi_2^+ u_{t-1}^+ + \textstyle\sum_{i=1}^p \phi_{1i}\Delta Y_{t-i} + \sum_{i=1}^p \phi_{2i}\Delta X_{t-i}\right] \times \\ &\quad \left\{1 - \exp\left[-\psi\left(u_{t-d} - w_3\right)^2\right]\right\} + innovations,\end{aligned}$$

where Y and X are the two interest rates whose dynamic relationship is being modelled; v_1 and w_1 are constant terms; ζ_j^-, ζ_j^+, ξ_j^- and ξ_j^+ for $j = 1, 2$ are speed-of-adjustment coefficients; u_{t-1}^- and u_{t-1}^+ are negative and positive deviations from the long-run equilibrium involving Y and X (estimated using the Johansen procedure). Y and X are equal to FF and TB in model M1 and M2. M2 is estimated using monthly data (thus M2 is the same as the model (7)-(8) reported in Table 5, except that M2 is estimated using end-of-period monthly data rather than daily data). M1 is estimated using daily data but both ECM equations are augmented by adding $\Delta\left(FF - FF^{\text{target}}\right)_{t-1}$ as a right-hand-side variable both in the linear component of the model (with associated coefficients $\varkappa_1$ and ϱ_1 respectively) and in the nonlinear component of the model (with associated coefficients $\varkappa_2$ and ϱ_2, respectively). In M3, Y and X are FF and CD, and the model is estimated using daily data. The reported estimated coefficients were obtained by NFIML estimation and applying a general to specific procedure; figures in parentheses denote estimated standard errors; figures in braces denote p-values. $J - ECM$ is a test of joint significance of the equilibrium correction coefficients not restricted to zero in the final estimation for each equation, and is distributed as $\chi^2(m)$ (m being the number of restrictions) under the null; *Asymmetry* is a test of the null hypothesis of no asymmetric equilibrium correction; LR is a likelihood ratio test for the validity of the restrictions imposed in the final estimation.

A Appendix: Testing for linearity and for parameter constancy

This appendix describes the procedure employed to test for parameter constancy and for linearity briefly discussed in Section 2 and employed in Section 4.

Assuming that a plausible transition variable is $\widehat{u}_{t-d}$, the appropriate auxiliary regression for the linearity tests against a STR alternative, which is an important preliminary to the specification and estimation of the nonlinear system (7)-(8), is the following:

$$\widehat{v}_t = \boldsymbol{\vartheta}_0'\mathbf{A}_t + \boldsymbol{\vartheta}_1'\mathbf{A}_t\widehat{u}_{t-d} + \boldsymbol{\vartheta}_2'\mathbf{A}_t\widehat{u}_{t-d}^2 + \boldsymbol{\vartheta}_3'\mathbf{A}_t\widehat{u}_{t-d}^3 + innovations, \tag{A1}$$

where $\widehat{v}_t$ is the estimated disturbance retrieved from the parsimonious ECM being tested for linearity (in the present context it is the residual from the each of the models reported in Table 3), and $\mathbf{A}_t$ denotes the vector of explanatory variables in that ECM (see Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998). The general test for linearity against STR is then the ordinary F-test of the null hypothesis:

$$H_{0L} : \boldsymbol{\vartheta}_1' = \boldsymbol{\vartheta}_2' = \boldsymbol{\vartheta}_3' = \mathbf{0} \tag{A2}$$

for $d \in \{1, \ldots, D\}$, where $\mathbf{0}$ is a null vector. If linearity is rejected for more than one value of d, then d is determined as the value $(\widehat{d})$ which minimizes the p-value of the linearity test, and we set $d = \widehat{d}$.

The choice between a LSTR and an ESTR model is based on a sequence of nested tests within (A2). First, the null hypothesis H_{0L} in (A2) must be rejected using an ordinary F-test (F_L). Then the following hypotheses are tested:

$$H_{03} \quad : \quad \boldsymbol{\vartheta}_3' = \mathbf{0} \tag{A3}$$

$$H_{02} \quad : \quad \boldsymbol{\vartheta}_2' \mid \boldsymbol{\vartheta}_3' = \mathbf{0} \tag{A4}$$

$$H_{01} \quad : \quad \boldsymbol{\vartheta}_1' \mid \boldsymbol{\vartheta}_2' = \boldsymbol{\vartheta}_3' = \mathbf{0}. \tag{A5}$$

Again, an F-test is used, with the corresponding test statistics denoted F_3, F_2, and F_1, respectively, and the decision rule is as follows: after rejecting H_{0L} in (A2) and setting $d = \widehat{d}$, the three hypotheses (A3)-(A5) are tested using F-tests; if the test of (A4) has the smallest p-value, an ESTR is chosen, otherwise an LSTR is selected (see Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998). Of course, the linearity testing procedure is applied individually to both parsimonious equilibrium correction equations, namely the equation for ΔFF and the one for ΔTB.

This procedure is then easily extended to test for parameter stability, simply replacing the transition variable $\widehat{u}_{t-d}$ with t and applying the same sequence of tests (see Lin and Teräsvirta, 1994). The test statistics correponding to F_3, F_2, and F_1 are termed LM_{C3}, LM_{C2}, and LM_{C1}, respectively.

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Figure 1: Interest rate data

solid line = FF rate - dashed line = TB rate minus 2.5

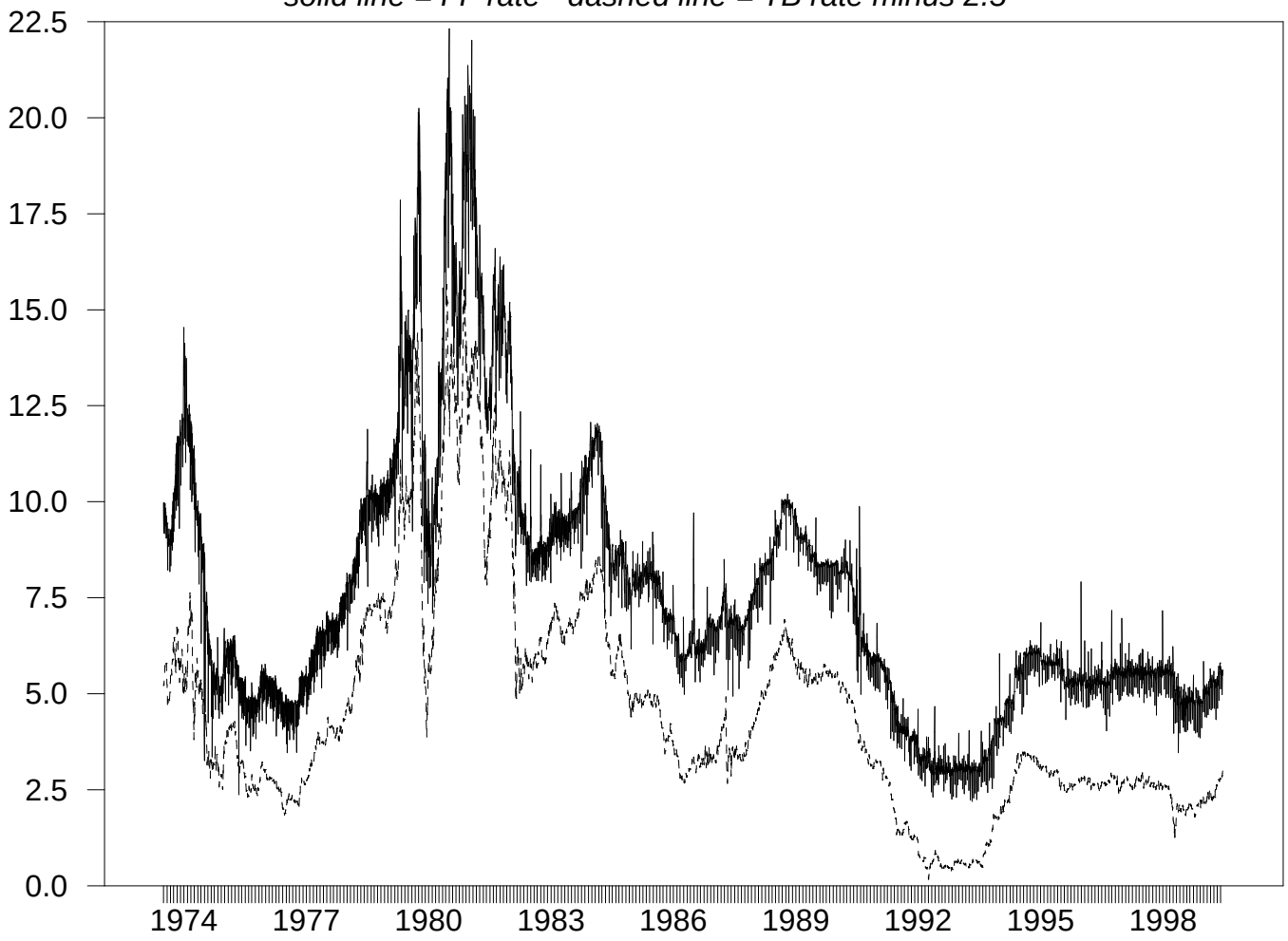


Figure 2: Recursive estimate of the cointegrating parameter

solid line = cointegrating parameter (b) - broken lines = b +/- 2.0 times standard error

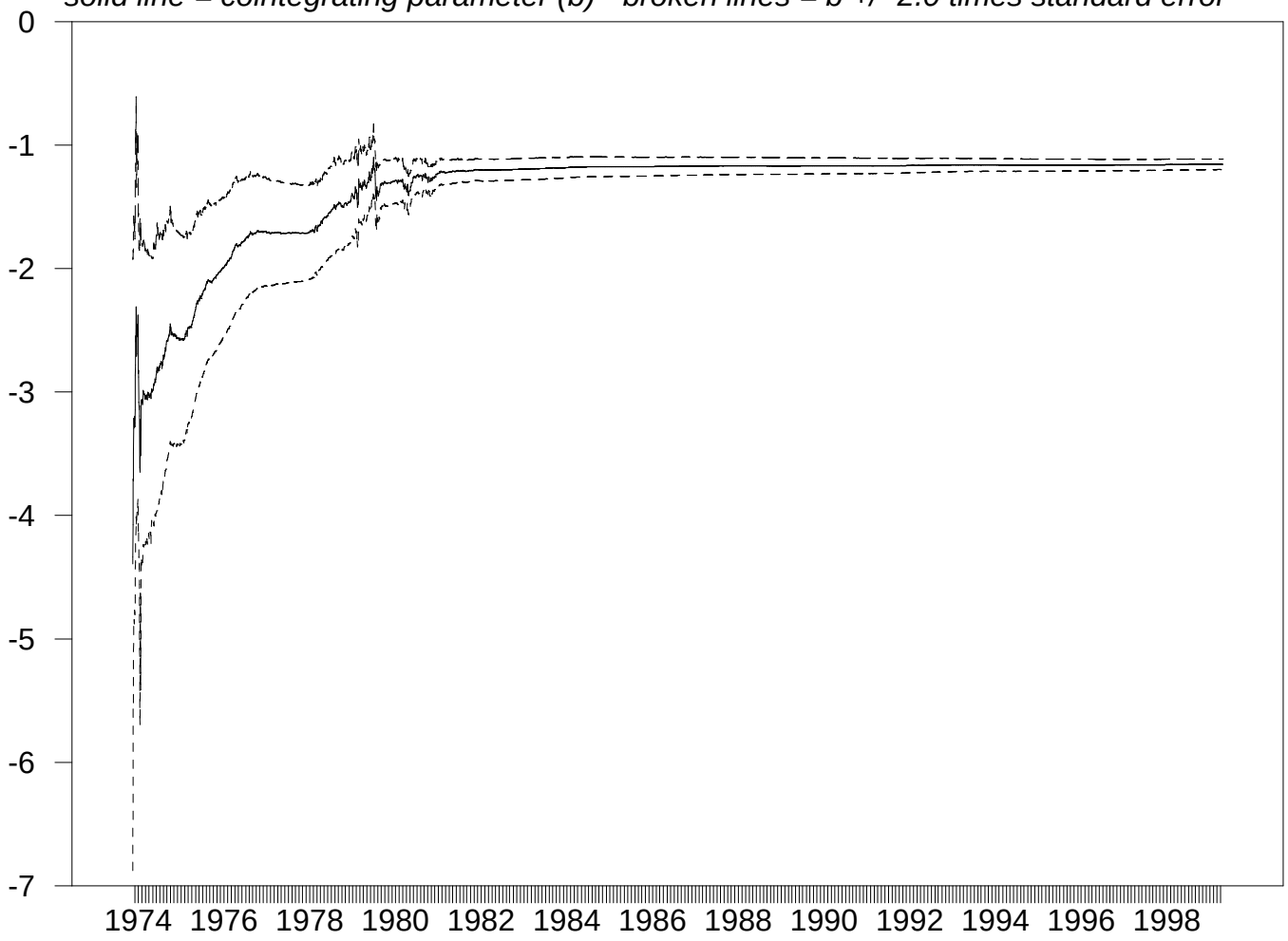


Figure 3: Estimated cointegrating residuals

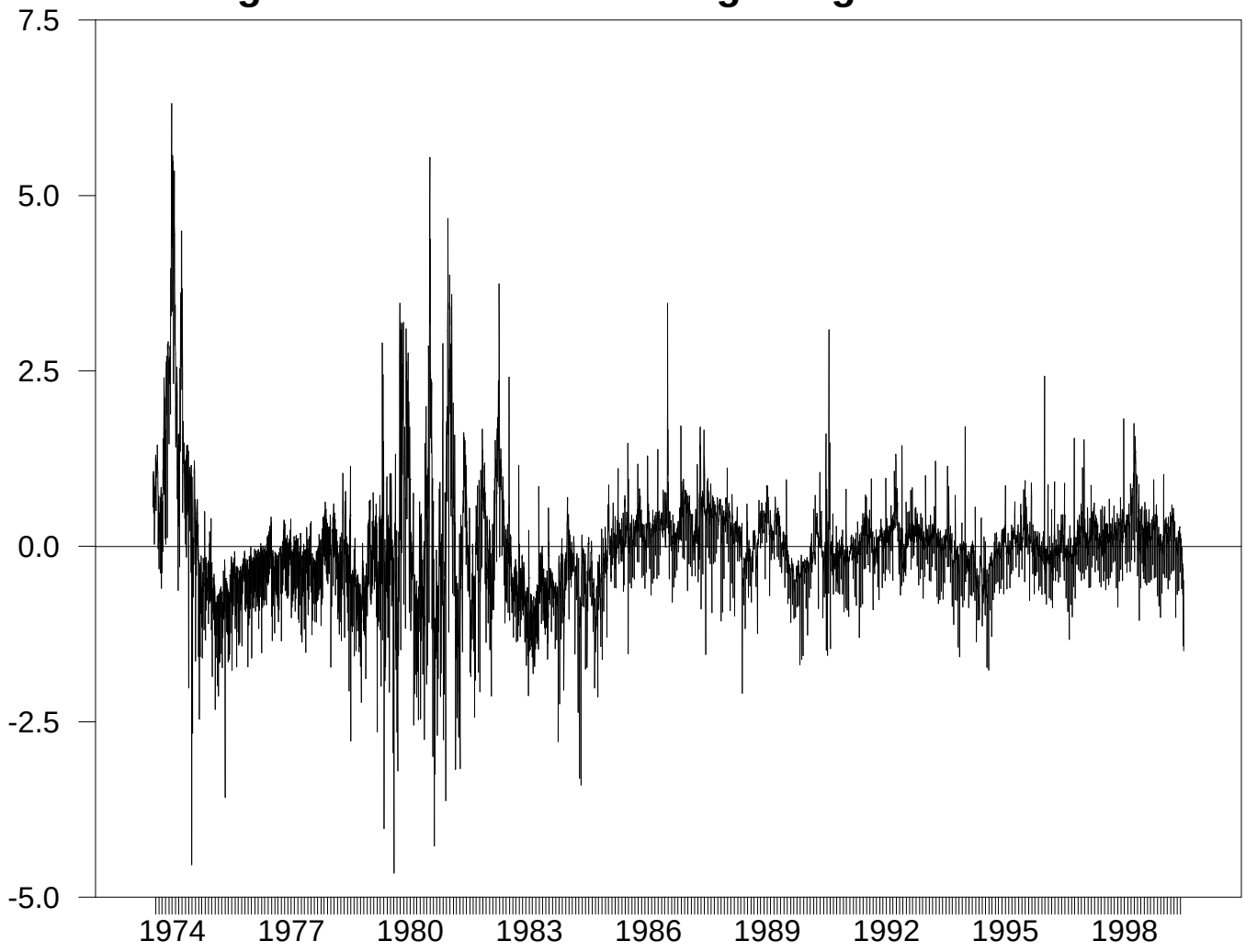


Figure 4: Estimated transition functions

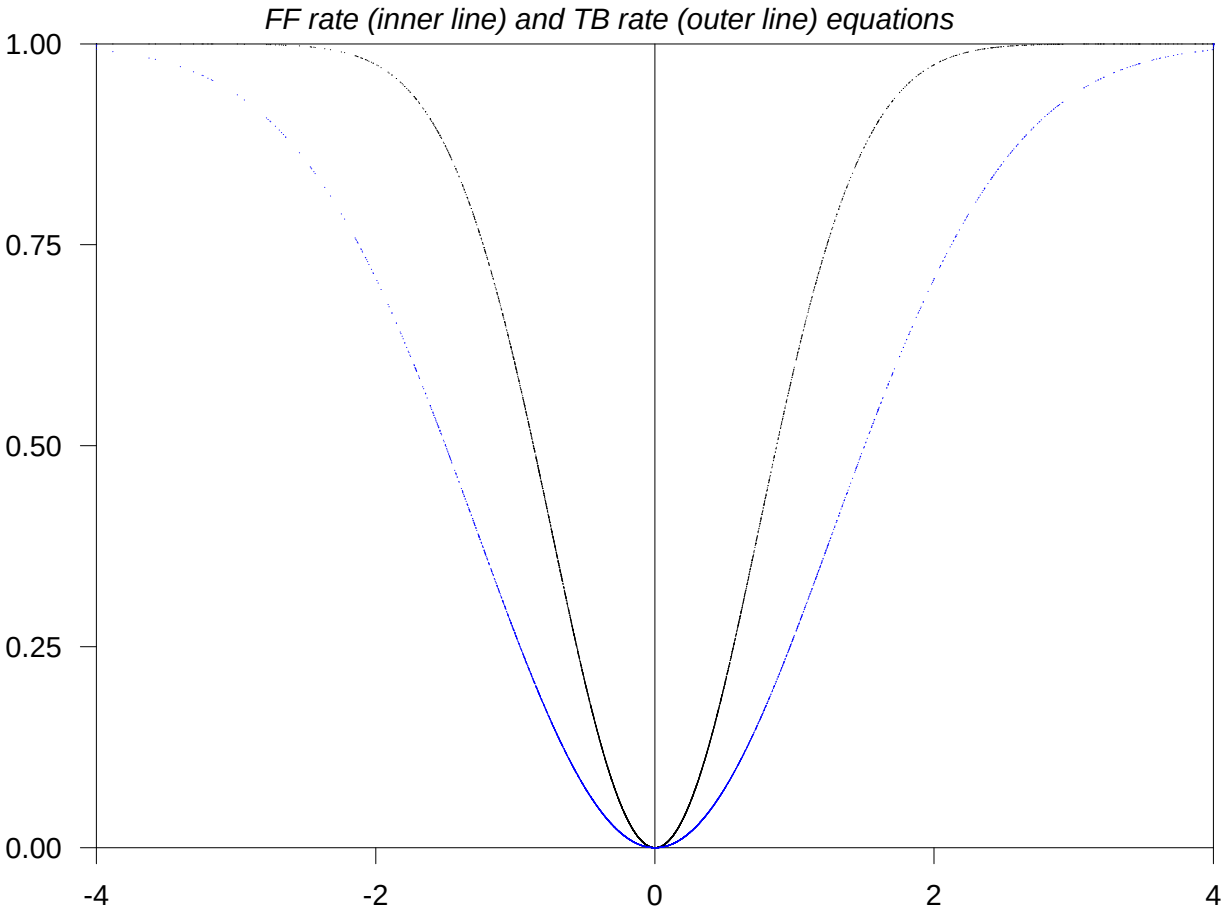
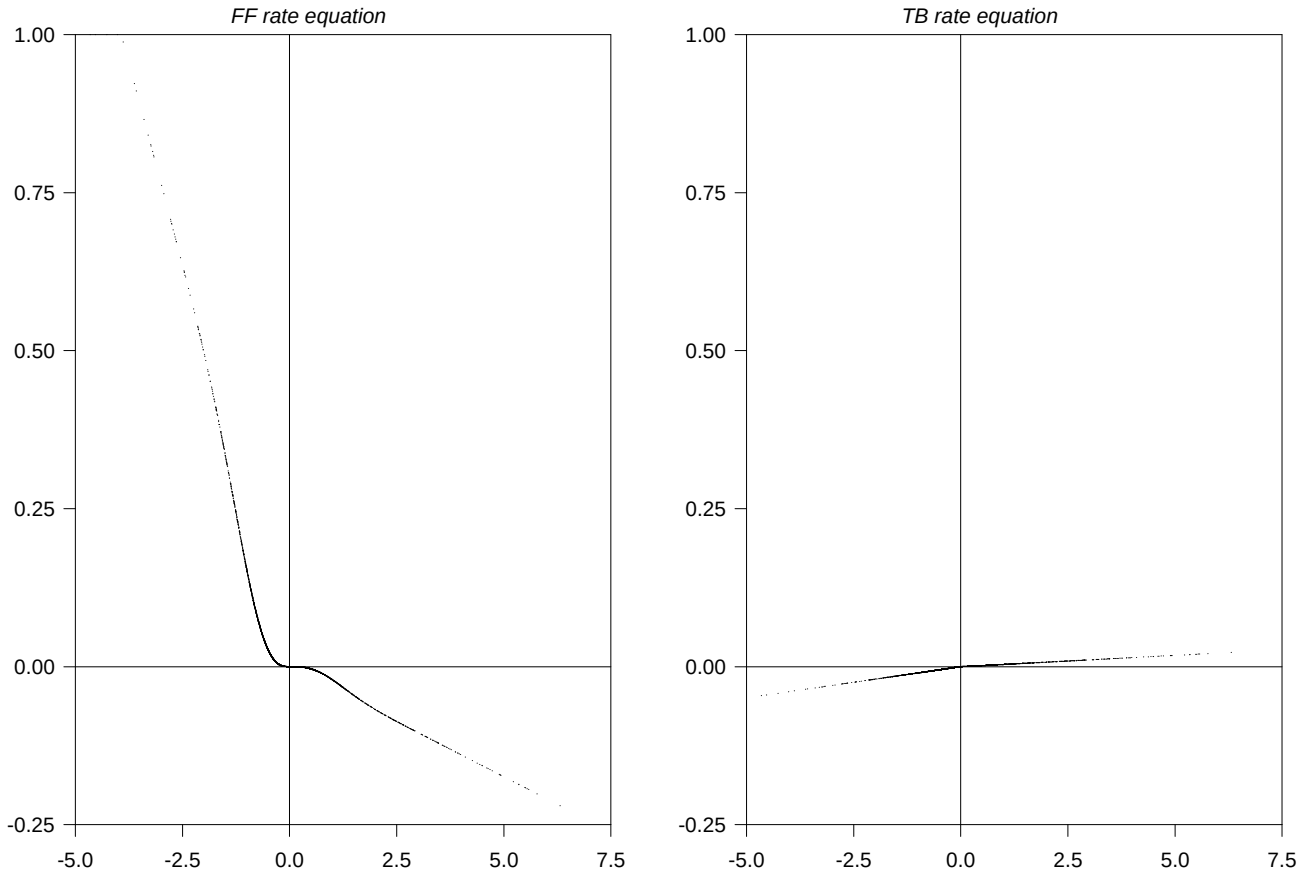


Figure 5

Nonlinear equilibrium correction versus deviation from equilibrium