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Central Bank Design in General Equilibrium

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ABSTRACT. We study the effects of alternative institutional arrangements for the determination of monetary policy in the context of a capital-theoretic, general equilibrium economy. In the absence of an institutional arrangement, there is a continuum of steady state equilibria indexed by rates of inflation ranging from the Friedman rule to a high level. The social optimum is associated with the Friedman rule. We consider three institutional arrangements for determining monetary policy. The first, unconditional majority voting, always leads to a substantial inflation bias. The second, a simple form of bargaining which we interpret as a policy board, generally improves on the unconditional majority voting outcome. Finally, we consider a constitutional rule which always achieves the social optimum. *JEL classification codes: E4, E5, D7.*

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1. INTRODUCTION

1.1. Objectives. In the last twenty years, two areas of research have had a dramatic impact on macroeconomic theory and policy. The first is the stochastic, competitive general equilibrium approach to studying macroeconomic fluctuations advocated by Lucas (1977) and embodied in the equilibrium business cycle paradigm put forward by Kydland and Prescott (1982). This approach advocates the use of stochastic dynamic general equilibrium models to capture the salient features of macroeconomies. It has become the preeminent approach for studying macroeconomics. The appealing feature of this approach is that the underlying structure of the economy is carefully specified from first principles. However, these models often lack a fundamental role for monetary policy, hence discussion of the political economy of monetary policy or central bank design is difficult. The second major area of research is the time inconsistency, monetary policy game literature, first studied by Kydland and Prescott (1977) and elaborated on by Barro and Gordon (1983a). This literature focuses on the strategic interaction of the monetary authority with the private sector and its political principals. The research in this area has had profound effects on the conduct of monetary policy around the world and on the design of the new central banks that have arisen over the last decade. While the policy game models carefully outlined the game-theoretic nature of monetary policy choices and the political economy of monetary policy, the remainder of the specification is typically based on very simple and unrealistic macroeconomic structures, for instance, static, repeated one-period, aggregate supply-aggregate demand models.¹ Despite the large amount of research done in these two areas and the influence they have had, there appears to be a large gap that needs to be bridged. What is needed is a model that has a well-defined dynamic macroeconomic structure but generates interesting political economy issues regarding the conduct of monetary policy and the design of central banks.

We build one such model in this paper, and we use it to study the ways in which different types of social arrangements for deciding on monetary policy influence equilibrium outcomes. We think of these social arrangements as abstract representations of institutions—central banks—with responsibility for monetary policy decisions. We focus the analysis on the question of which types of central banks are likely to lead

¹Dynamic analysis has either focused on reputation strategies as in Barro and Gordon (1983b), or empirically-motivated dynamic relationships have been imposed on the model as in Svensson (1997).

to outcomes involving a bias toward too much inflation, in the sense of stationary equilibria with inflation greater than the social optimum. This question is interesting, in our view, because we observe much more inflation on average across countries worldwide than would be predicted by many standard general equilibrium models with monetary features. We do not attempt to confront the worldwide inflation data directly in this paper, but we do think it provides sufficient motivation for our research and indeed for the large body of research on inflation bias during the last two decades.

Our framework is a relatively standard, general equilibrium growth model with overlapping generations. Because equilibrium growth models have provided a unifying framework for much of macroeconomics over the last decade, it is interesting to learn how stationary equilibria involving inflation bias might arise in such an environment. The model we use is simple but has interesting features, including capital accumulation, partisan elements (with some agents gaining and others losing from changes in inflation), and a well-defined social optimum. Our framework can be viewed as a natural representation of a situation many economists have in mind, in which wage earners benefit from higher inflation while savers are hurt by it.

1.2. Main findings. Our main finding is that central bank design has a profound impact on the nature of the stationary equilibrium in our general equilibrium model.

We begin with a design in which the central bank is set up solely to implement the will of the majority in a democracy. We find that this arrangement is incapable of generating the social optimum, and instead generates a stationary equilibrium with an inefficiently high inflation rate. The key reason for this outcome is that the median voter is unable to commit future voters to low inflation.

We then turn to a design based on bargaining, in which each generation has a representative on a policy board, which has sole responsibility for monetary policy. The board implements monetary policy based on the outcome of a bargaining scheme. We show that a policy board improves upon the outcomes possible under majority voting. In general, however, the policy board cannot achieve the social optimum and so the stationary equilibrium is still characterized by an inflationary bias. We interpret these results as suggesting that central bank designs based on policy boards can mitigate the inflation bias that occurs under majority voting, but cannot eliminate it entirely.

We end by considering a super-majority design which we call a constitutional rule.² This design allows young agents to make decisions on monetary policy, but allows veto power to the older generation. If a veto occurs, then the status quo prevails. Under this arrangement, the young can make decisions on monetary policy today secure in the knowledge that future decisions will not damage them. This design provides a form of commitment on the transfer of political power by making it difficult to force changes in monetary policy which affect some members of the society adversely. The social optimum obtains. We interpret this result as suggesting that a central bank design which commits to and makes it difficult to change (say, by requiring social consensus or a supermajority) fundamental monetary policy goals may go a long way toward achieving a socially preferred outcome.

1.3. Recent related literature. In attempting to unite models of inflation bias with relatively standard growth and business cycle models, we are following a number of authors including Chari, Christiano, and Eichenbaum (1998), Ireland (1996, 1997), Obstfeld (1997) and Rogoff (1995). These authors all use models in which there is a representative household as a basis for discussion, whereas our results depend importantly on heterogeneous households (by age). We think of our model as one where partisan conflict can lead to substantial inflation bias. Versions of the idea of voting and bargaining in overlapping generations contexts can be found in Loewy (1988), Faust (1996), and Azariadis and Galasso (1996). We draw on ideas from these papers in our presentation.

There is some recent research on monetary policy rules along with commitment issues such as Clarida, Gali and Gertler (1999). This literature is characterized by microfounded models with exogenously sticky prices and money demand motivated by money in the utility function of the representative household. The models are often used to study short-run monetary stabilization issues, but typically not questions of long-run inflation outcomes.

We organize the paper as follows. We begin with a presentation of the model and some of its characteristics. We then turn to an analysis of three central bank designs and how they affect stationary equilibrium outcomes in the economy. We provide a brief conclusion.

²We first saw this idea expressed in the context of fiscal transfer schemes by Azariadis and Galasso (1996).

2. THE ENVIRONMENT

2.1. Background. We study a neoclassical growth model. Time is discrete and doubly infinite, such that $t = \dots, -2, -1, 0, 1, 2, \dots$.

There are two assets in the economy, fiat currency and capital. To keep the analysis simple, we impose an arbitrage condition

$$R^h(t) = R^k(t) = R(t) \quad (1)$$

where $R^h(t) = \frac{P(t)}{P(t+1)}$ is the gross real rate of return to fiat currency reserves from time t to time $t + 1$, $P(t)$ is the price of the consumption good at time t (also the aggregate price level), and $R^k(t)$ is the gross rate of return to capital from time t to time $t + 1$. The gross inflation rate is therefore $R(t)^{-1}$.

There is a monetary authority which issues fiat currency according to

$$H(t+1) = \mu(t) H(t), \quad (2)$$

where $\mu(t) \geq 1 \forall t$ and the nominal stock of currency is denoted $H(t) > 0 \forall t$.³ The nature of our analysis is to describe how various institutional arrangements for choosing the $\mu(t)$ sequence affect stationary equilibrium outcomes. The government earns revenue from currency issuance, which is used to purchase goods at market prices. This government consumption leaves the economy and is not rebated to agents.

We now turn to the details of this model.

2.2. Households. At each date t a large number of identical agents, or households, are born. Agents live for three periods and have perfect foresight. The agents do not consume or work in the first period of life, but they do participate in the political systems we set up later in the paper. In the second period of life, the agents supply one unit of labor inelastically to firms, participate in the political system, collect wage income, consume, and save for retirement. In the third period of life, the agents retire, consume their savings, and exit the model. Agents in the last period of life are making no further decisions and so have no incentive to participate in the political system.⁴ We view them as consuming and dying before any political

³To keep the discussion focused, we do not allow $\mu(t) < 1$ which would require currency retirement.

⁴This only happens for the very last generation. In a large model, with, say, 55 period lifetimes, generations 1-54 would participate and the 55th would not. The conflict in a larger model would

decisions are made. Thus we assume the young and the middle-aged are the political players in our system. As we explain in detail below, the young care about the wage they will earn in middle-age, and the middle-aged care about the returns they will earn between middle age and old age.

We denote the size of the labor force at time t by $N(t)$. This labor force consists of the agents in the second period of their lives, and evolves according to

$$N(t+1) = \eta N(t) \quad (3)$$

where $\eta > 1$.

An agent born at time t maximizes

$$V = \ln c_t(t+1) + \beta \ln c_t(t+2) \quad (4)$$

where c represents consumption and $0 < \beta \leq 1$ is the discount factor.⁵ Agents make a decision to save by considering the lifetime budget constraint

$$c_t(t+1) + \frac{c_t(t+2)}{R(t+1)} = w(t+1), \quad (5)$$

where $w(t+1)$ is the real wage at time $t+1$. The first order conditions for this problem imply

$$c_t(t+1) = \left(\frac{1}{1+\beta} \right) w(t+1) \quad (6)$$

and

$$c_t(t+2) = \beta R(t+1) c_t(t+1). \quad (7)$$

The date $t+1$ asset holdings of a household born at time t is then given by

$$a_t(t+1) = w(t+1) - c_t(t+1) = \left(\frac{\beta}{1+\beta} \right) w(t+1). \quad (8)$$

Using our convention (3) for denoting the size of the labor force means there are $N(t+1)$ agents in the labor force who are doing the saving at time $t+1$. Aggregate asset holdings at date t are therefore given by

$$A(t+1) = N(t+1) a_t(t+1) = \frac{N(t+1) \beta w(t+1)}{1+\beta}. \quad (9)$$

be between the young and the old, not just the young and the middle-aged as the discussion of the three period model here suggests.

⁵We use the convention that a subscript denotes the date at which the agent is born, and parentheses represent the real time of the model. In this convention, aggregate variables have no birth date dimension, and therefore have no time subscript (as with the interest rates discussed above).

2.3. Technology. There are a large number of identical competitive firms in the economy, which we analyze as if there was only a single firm. Firms come into existence each period, produce output using capital and labor as inputs to a technology, and go out of business. There is no productivity growth, which will imply that per capita income is constant in a steady state equilibrium of the model. We stress that while we are abstracting from technological change, we think this feature could be incorporated into the analysis (at the cost of some complications) without changing any of the fundamental points.

Firms produce output according to a neoclassical, Cobb-Douglas production function employing labor and capital,

$$Y(t) = K(t)^\alpha N(t)^{1-\alpha}, \quad (10)$$

where $Y(t)$ is aggregate real output, $K(t)$ is the level of capital, and $\alpha \in (0, 1)$ is the capital share. From the production function the net rental rate on capital is $r(t) = \alpha k(t)^{\alpha-1}$ and the wage rate is $w(t) = (1 - \alpha) k(t)^\alpha$, where $k(t) \equiv K(t)/N(t)$ is the capital-labor ratio. Arbitrage requires that $1 + r(t+1) - \delta = R^k(t)$, where δ is the depreciation rate. We assume full depreciation, $\delta = 1$, so that arbitrage requires $r(t+1) = R^k(t) = R(t)$. Thus

$$k(t) = (\alpha^{-1} R(t-1))^{1/(\alpha-1)} \quad (11)$$

and

$$w(t) = (1 - \alpha) (\alpha^{-1} R(t-1))^{\alpha/(\alpha-1)}. \quad (12)$$

2.4. Market clearing. We denote real balances as $H(t)/P(t)$. The market clearing condition for fiat currency is then

$$\frac{H(t)}{P(t)} = A(t) - K(t+1). \quad (13)$$

Equations (2) and (13) determine the competitive equilibria for this economy. These two equations can be combined to yield

$$[A(t+1) - K(t+2)] P(t+1) = \mu(t) [A(t) - K(t+1)] P(t). \quad (14)$$

Given our discussion above, we can write date t aggregate asset holdings as

$$A(t) = N(t) \frac{\beta(1-\alpha)}{1+\beta} \left(\frac{R(t-1)}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}. \quad (15)$$

Using equation (11) and the definition of the capital-labor ratio, we can write aggregate capital as

$$K(t) = N(t) \left(\frac{R(t-1)}{\alpha} \right)^{\frac{1}{\alpha-1}}. \quad (16)$$

Substituting (15) and (16) in (14), using (3), and using the arbitrage condition implies

$$\begin{aligned} \frac{\beta(1-\alpha)}{1+\beta} \left(\frac{R(t)}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} - \eta \left(\frac{R(t+1)}{\alpha} \right)^{\frac{1}{\alpha-1}} = \\ \frac{\mu(t)R(t)}{\eta} \left\{ \frac{\beta(1-\alpha)}{1+\beta} \left(\frac{R(t-1)}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} - \eta \left(\frac{R(t)}{\alpha} \right)^{\frac{1}{\alpha-1}} \right\}. \end{aligned} \quad (17)$$

Equation (17) is a second-order difference equation in $R(t)$, provided we assume a constant $\mu(t) = \mu$ for all t .

2.5. Steady states. Inspection of equation (17) reveals that one steady state occurs at $R(t) = R^* \forall t$, where

$$R^* = \eta\mu^{-1}, \quad (18)$$

provided currency demand is positive at this stationary interest rate. We call this the *monetary steady state*. A second steady state occurs at $R(t) = \bar{R} \forall t$, where \bar{R} causes the left hand side (which is $A - K$) and the term in braces (which is also $A - K$) to equal zero. This occurs when

$$\bar{R} = \frac{\alpha\eta(1+\beta)}{\beta(1-\alpha)}. \quad (19)$$

At this steady state interest rate, since $A - K = 0$, the demand for currency is zero by equation (13). Thus we call this the *nonmonetary steady state*.⁶ The condition that currency demand is positive at the monetary steady state is that $\bar{R} < R^*$. This condition is

$$\frac{\mu\alpha}{1-\alpha} < \frac{\beta}{1+\beta}. \quad (20)$$

We maintain this assumption throughout the remainder of the paper, so that agents voluntarily hold currency at the monetary steady state.⁷

⁶There is also a trivial steady state, in which there is no capital in the economy. We ignore this steady state in the remainder of the paper.

⁷When $\mu = 1$ and there is no discounting, so that $\beta = 1$, this condition reduces to $\alpha < 1/3$. Some authors, such as Auerbach and Kotlikoff (1987), have used calibrated values of $\alpha = 1/4$. But probably our model is too simple to gain insight by quantifying it directly.

When $\mu = 1$ and the monetary steady state exists, $R^* = \eta$, that is, the gross real interest rate equals the gross growth rate of aggregate real output in the economy. This is the golden rule rate of interest, a Pareto optimal outcome. Thus there is a clear social optimum for this economy. This stationary equilibrium involves a constant rate of deflation, since the rate of population growth is positive but the currency stock is constant. The price level will be constant if $\mu = \eta$, and there will be inflation if $\mu > \eta$.

We will assume that the economy always coordinates on the monetary equilibrium, and thus that we always observe the R^* steady state and not the \bar{R} steady state. However, the two steady state interest rates, R^* and \bar{R} , have the interesting property that as μ increases, $R^* \rightarrow \bar{R}$. When $R^* = \bar{R}$, households give up holding currency altogether and the economy is “demonetized.” This occurs at a value of steady state gross money growth given by

$$\bar{\mu} = \frac{\beta(1-\alpha)}{\alpha(1+\beta)} > 1. \quad (21)$$

Thus the available stationary monetary policy choices can be thought of as values of $\mu \in [1, \bar{\mu}]$. For low money growth, namely $\mu = 1$, $R^* = \eta$ and the economy achieves a Pareto optimal outcome. For high money growth, and in particular for $\mu = \bar{\mu}$, $R^* = \bar{R}$ and the economy is dynamically inefficient.

2.6. Local dynamics. Equation (17) can be written as a first-order system. Let $\mu(t) = \mu$, and let

$$X(t) \equiv \frac{\beta(1-\alpha)}{1+\beta} \left(\frac{R(t)}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}. \quad (22)$$

Then the first order system is

$$R(t+1) = \alpha \left[\frac{X(t)}{\eta} - \frac{\mu R(t)}{\eta^2} \left\{ X(t-1) - \eta \left(\frac{R(t)}{\alpha} \right)^{\frac{1}{\alpha-1}} \right\} \right]^{\alpha-1}, \quad (23)$$

$$R(t) = R(t). \quad (24)$$

Define $\varphi(t+1) = [R(t+1), R(t)]'$, and define a function $G(\varphi(t))$ by the right hand side of equations (23) and (24). Then

$$\varphi(t+1) = G(\varphi(t)). \quad (25)$$

We wish to understand how the dynamics of this system relate to the choice of μ . Accordingly, we analyze the local dynamics at the monetary steady state $\varphi = \varphi^* = [R^*, R^*]'$, where $R^* = \eta\mu^{-1}$. The Jacobian matrix $DG(\varphi^*)$ has a characteristic equation which reduces to

$$\lambda^2 + \left[\frac{(\alpha - 1)\beta - \alpha(1 + \beta)\alpha\mu}{(1 + \beta)\alpha\mu} \right] \lambda - \frac{(\alpha - 1)\beta}{(1 + \beta)\mu} = 0. \quad (26)$$

The eigenvalues reduce to $\lambda_1 = \alpha$ and

$$\lambda_2 = \frac{\beta(1 - \alpha)}{(1 + \beta)\alpha\mu}. \quad (27)$$

The eigenvalue λ_2 is real and positive. It will be larger than one under condition (20); that is, so long as currency is valued at the monetary steady state. Since we are maintaining this assumption, we conclude that the eigenvalues are configured as $0 < \lambda_1 < 1 < \lambda_2$.

The local dynamics at a monetary steady state are therefore as follows. There is one initial condition corresponding to the initial level of capital in the economy. This is equivalent to taking $R(t - 1)$ as a given initial condition. Because there is one stable eigenvalue coupled with the one initial condition, equilibrium is determinate. The fact that the stable eigenvalue is real and positive indicates that any adjustment must be monotonic. As an example, suppose we begin at a steady state R^* , and perturb the system by choosing a slightly higher value $\mu' > \mu$, keeping the new higher money growth rate into the indefinite future. The system will transition, monotonically, toward a new steady state $R^{*'} < R^*$. Thus the choice of a slightly higher money growth rate today, and persisting into the future, will lower interest rates today and into the future, until the new steady state interest rate is achieved.

We will use these dynamics to deduce the incentives of agents to choose values for the gross rate of monetary expansion. Since the dynamics we discuss are local, we will refrain from making statements about large changes in $\mu(t)$. Instead, we ask which values of $\mu(t) = \mu \forall t$ can be sustained as steady state political choices under the social institutions we describe below. A candidate value of μ can be sustained as a steady state if, given the local dynamics of equations (23) and (24), no slightly different value $\mu' \neq \mu$ would be chosen by the political system.

2.7. Mechanisms at work. The model we develop above is one in which the economy can be dynamically inefficient in the sense that in a non-monetary steady

state, the agents hold too much capital. Consequently, the real return on capital is very low. Introducing money gives agents an alternative asset to hold in their portfolios. By shifting wealth away from capital to money, agents increase the return on their portfolio, raise the real return on capital and increase steady-state consumption and welfare. Thus, money is valued in the monetary steady state equilibrium. In our economy, an increase in the inflation rate reduces the real return on money, thereby inducing a portfolio shift into capital which increases output and real wages paid to labor. Due to the portfolio allocation, in our monetary equilibrium, inflation generates a Tobin effect. However, the increase in capital is not welfare improving since the economy is already holding too much capital compared to the dynamically efficient level.

2.8. Welfare. We can now deduce some welfare results for this economy. Lifetime utility can be written as

$$V = C_0 + \frac{\alpha(1+\beta)}{\alpha-1} \ln R(t) + \beta \ln R(t+1), \quad (28)$$

where

$$C_0 \equiv (1+\beta) \left[\ln \left(\frac{1-\alpha}{1+\beta} \right) - \frac{\alpha}{\alpha-1} \ln \alpha \right] + \beta \ln \beta. \quad (29)$$

Since $\alpha \in (0, 1)$ we deduce that lifetime utility V is decreasing in $R(t)$, the interest rate that prevails between date t and date $t+1$, and increasing in $R(t+1)$, the interest rate that prevails between date $t+1$ and date $t+2$. This is because the wage that the agent born at date t will earn during middle age, $w(t+1)$, is decreasing in the interest rate $R(t)$, so that a low value of $R(t)$ generates relatively higher wage income for this agent. However the agent will earn returns of $R(t+1)$ on asset holdings between middle age and old age. The agent wants this interest rate to be as high as possible.

In a steady state R^* , lifetime utility becomes

$$V = C_0 + \left(\frac{\alpha + 2\alpha\beta - \beta}{1-\alpha} \right) \ln R^*. \quad (30)$$

which is increasing in R^* provided

$$\beta > \frac{\alpha}{1-2\alpha}. \quad (31)$$

We maintain this condition in the remainder of the paper.⁸ Thus the “social optimum” from the perspective of the highest level of steady state utility occurs at $R^* = \eta$. This social optimum is associated with the lowest gross rate of expansion of the money supply μ , namely $\mu = 1$, and the lowest gross rate of inflation, namely a gross deflation rate of η^{-1} .

2.9. Interpretation. We think these results capture in a stylized model the essence of a situation many economists have in mind when thinking about how inflation might affect economic actors in different ways. The agents in the second period of life receive only wage income. Since higher inflation leads to a higher capital stock due to the Tobin effect, the increase in the capital stock raises the marginal productivity of labor and thus the real wage. A higher real wage allows workers to consume more today and by consumption smoothing, they want more future consumption. Thus they save and accumulate more wealth for old age when the real wage increases. In the third period of life, agents depend on their accumulated wealth plus interest for consumption. The stock of wealth (size of their portfolios) was predetermined in the last period while the interest income on that wealth to finance consumption is received in the current period. A lower inflation rate raises the real interest rate and thus interest income on wealth without reducing the stock of saving (size of their portfolios) thereby increasing consumption in the third period of life. Consequently, young agents want a low real interest rate (high inflation rate) and high capital stock to increase their real wage income in the second period of life, whereas the middle-aged want high real interest rates (low inflation and a low capital stock) to increase their real portfolio income and thus consumption in the third period of life. Thus some tension exists over the desired inflation rate. There is perhaps no more telling way to illustrate this tension than by thinking about the optimal sequence of inflation and interest rates over a typical agent’s lifetime. The optimal inflation sequence for each generation is to have high inflation (low real interest rates) when young and then very low inflation (high real interest rates) when old. Obviously such a policy sequence is not consistent with a steady state equilibrium.

The model has a third aspect which we think many economists have in mind, which is that from the perspective of a social planner or lifetime utility of an agent

⁸This condition simply states that there cannot be “too much” discounting. For a value of $\alpha = 1/4$, this condition evaluates to $\beta > 1/2$.

born at an arbitrary date t , a stationary inflation is distortionary and should be eliminated. Altogether, we think the model provides an interesting laboratory for the study of alternative institutional designs to decide upon an inflation policy.

We now turn to the implications of these welfare results for central bank design.

3. INSTITUTIONAL ARRANGEMENTS

3.1. Overview. We wish to study various central bank designs for our open set of economies. We define a central bank design as a social arrangement for deciding upon a monetary policy that will be in place from date t to date $t+1$.⁹ The monetary policy is simply a choice for $\mu(t) \in [1, \bar{\mu}]$. An essential feature of our framework is that the participants in the economy at date t have no intrinsic ability to commit future participants to values of $\mu(t+s)$, $s = 1, 2, 3, \dots$

We wish to view the situation as follows. In period t , there is a contingent of young agents waiting to begin the working portion of their life at date $t+1$. There is also a generation of middle-aged agents who are working and consuming in period t and who will be old during period $t+1$. The returns from time t to time $t+1$ have yet to be determined. The middle-aged agents have already earned their wage income, as the wage during period t was determined by the previous period's interest rates (level of the capital stock). They have also already decided how much to consume and save (since savings is a fixed fraction of wage income), but they have not yet decided how to allocate their savings between capital and fiat currency. What these middle-aged agents want at this point is just the highest possible interest rate between period t and period $t+1$ so that they can obtain the best possible return on their savings and can thus consume as much as possible during their period of old age. The young agents, who will be middle-aged at date $t+1$, have an immediate concern: What the wage is going to be in their second period of life. Lower interest rates will lead to more capital and higher wages, so that these agents want a low value of $R(t)$, all else equal.

It is just at this point at time t that we envision the next period's middle-aged agents and the next period's old agents following some institutional arrangement in making a decision on monetary policy, the value of $\mu(t)$. (As we discussed in

⁹We abstract from determining how society can commit to certain social arrangements but not to a particular policy outcome over time. This issue is beyond the scope of our analysis. However, we do often observe societies committing to the creation of a central bank even if they cannot commit to a specific inflation policy.

conjunction with the dynamics of equations (23) and (24), a choice for changing the value of $\mu(t)$ relative to its previous value will tend to move interest rates in the opposite direction.) Once the institutional decision on $\mu(t)$ is made, the interest rate is determined, and the agents determine their portfolio allocation. The same process repeats at every date t .

Of course the young agents live for two more periods so that they also care about the returns to saving they receive between middle age and old age, which will be determined by the choice for $\mu(t+1)$. But that has to be decided via the institutional arrangement in the next period.

3.2. Majority voting.

Nature of the arrangement. In this section we assume that there are underlying democratic forces in the economy which cause policy choices to conform to the principle of one agent, one vote.¹⁰ We wish to investigate what outcomes might reasonably be expected under such an arrangement. The young are in the majority because we have assumed that $\eta > 1$. The voting scheme is period by period. An essential feature of this institution is that the median voter cannot commit the next generation to vote for the same monetary policy as is implemented today.

Stationary equilibria. We have noted that the lifetime utility of the time t young is given by

$$V = C_0 + \frac{\alpha(1+\beta)}{\alpha-1} \ln R(t) + \beta \ln R(t+1), \quad (32)$$

The young agents are in the majority today and so can influence $R(t)$ by choice of $\mu(t)$, but will be out of power tomorrow when the next generation will control the median vote. In the absence of a voting commitment technology, the best the current young generation can do is maximize the term

$$\frac{\alpha(1+\beta)}{\alpha-1} \ln R(t) \quad (33)$$

in this expression by choice of $\mu(t)$.

We have assumed that the economy will coordinate on the $R^* = \eta\mu^{-1}$ steady state. The choices for stationary values of μ are $\mu \in [1, \bar{\mu}]$. Because the term (33) is

¹⁰Some recent work on voting in overlapping generations models includes Azariadis and Galasso (1996) and Loewy (1988).

decreasing in $R(t)$, we conclude that the only sustainable steady state value for μ is $\mu = \bar{\mu}$, and hence that $R^* = \bar{R}$. This is a demonetized, high inflation steady state for this economy. To see this, consider a value $\mu \in [1, \bar{\mu})$. This will induce a candidate steady state $R^* = \eta\mu^{-1}$. However, this cannot actually be a steady state since the young agents can choose a value $\mu' > \mu$ which would tend to push $R(t)$ lower and thus increase the term (33) via the dynamics described by equations (23) and (24). Thus the only steady state consistent with the majority voting scheme involves $\mu = \bar{\mu}$ and $R^* = \bar{R}$. We can state this result as:

PROPOSITION 1. Under majority voting, the unique stationary equilibrium outcome is associated with high inflation.

Sustainable plans. Since voting leads to high inflation equilibria which are socially suboptimal, it is reasonable to believe that society will try to adopt some arrangements to reduce the inflation bias. One approach suggested by the literature on sustainable plans would be for the young generation to adopt trigger strategies to enforce low inflation policies. Under such a social convention, if last period's young voters adopted a low inflation policy, today's young voters will adopt the same policy. Otherwise, they will adopt a higher inflation policy to punish the defection. While trigger strategies can improve on the set of majority voting equilibria, there are an infinite number of trigger strategies that can be adopted and thus there is a generational coordination problem as to which trigger strategy should be adopted.

Perhaps more fundamentally, the problem with the majority rule voting institution is that it gives each generation complete power when young and zero power when old. Standard economic intertemporal utility maximization suggests that each generation would benefit from smoothing power; that is, give up some power when young in order to acquire power when old. Trigger strategies do not lead to power smoothing; they simply rely on punishment threats from one young generation to the next to enforce the social convention. Thus it seems reasonable to assume that society will prefer to adopt alternative institutional designs which lead to power smoothing over agents' lifetimes. Such designs make the two generations compromise over current policy. It is to these types of institutional designs that we turn in the next two sections.

3.3. A policy board.

Nature of the arrangement. In this section we study an institutional arrangement based on compromise between young and old agents. We think of this arrangement as an abstract representation of a situation where a congress or parliament delegates authority over monetary policy to a board with appointees representing important segments of society. Recent research by Waller (1989, 1992, 2000) and Faust (1996) suggests that delegating monetary policy to a policy board can improve upon the outcomes under majority rule voting.

The arrangement we study works as follows. Instead of voting at time t when monetary policy is to be decided, the young and middle-aged generations each send a representative to jointly choose $\mu(t) \in [1, \bar{\mu}]$. This choice will influence the value of $R(t)$. However, they do so taking as given what the board has chosen in the past and what it will choose in the future. The same process repeats each period. In this sense, the board is a mechanism for resolving conflict in a given period as opposed to being a mechanism for generating commitment to a policy action in the future.

At time t , the young generation is contemplating lifetime utility is given by

$$V_t = C_0 + \frac{\alpha(1+\beta)}{\alpha-1} \ln R(t) + \beta \ln R(t+1), \quad (34)$$

while the middle-aged households are considering

$$V_{t-1} = C_0 + \frac{\alpha(1+\beta)}{\alpha-1} \ln R(t-1) + \beta \ln R(t). \quad (35)$$

We envision the policy board as only choosing a value for $R(t)$, so that the representatives only consider that part of the utility that is affected by this choice. For the young, this is

$$V_y = \frac{\alpha(1+\beta)}{\alpha-1} \ln R(t), \quad (36)$$

and for the middle-aged, it is

$$V_m = \beta \ln R(t). \quad (37)$$

The policy board is assigned the task of minimizing the loss to each group from compromising on $R(t)$. We define this loss as

$$\mathcal{L} = \theta \left(V_y - \hat{V}_y \right)^2 + (1-\theta) \left(V_m - \hat{V}_m \right)^2 \quad (38)$$

where a hat indicates the optimal value of V_y and V_m , respectively. For the young,

$$\hat{V}_y = \frac{\alpha(1+\beta)}{\alpha-1} \ln \bar{R}. \quad (39)$$

For the middle-aged,

$$\hat{V}_m = \beta \ln R^*. \quad (40)$$

The parameter $\theta \in [0, 1]$ is a weighting parameter that captures a variety of influences on the choice of $\mu(t)$ including population size and bargaining power.¹¹ The first order condition for this problem can be rearranged to yield an optimal choice for $R(t)$ as a result of this policy process. We denote this optimal choice by R^B , which is given by

$$R^B = \exp \left[\frac{\theta \left(\frac{\alpha(1+\beta)}{\alpha-1} \right)^2 \ln \bar{R} + (1-\theta) \beta^2 \ln R^*}{\theta \left(\frac{\alpha(1+\beta)}{\alpha-1} \right)^2 + (1-\theta) \beta^2} \right]. \quad (41)$$

The solution given by (41) depends on the degree of policy power, θ , the discount factor β , and the technology parameter α . We conclude that if $\theta = 1$, so that all of the policy-setting power is given to the young, then $R^B = \bar{R}$. And if $\theta = 0$, so that all of the policy-setting power is given to the middle-aged, then $R^B = R^* = \eta$. Finally, for $\theta \in (0, 1)$, including a balanced board with $\theta = 1/2$, the value of R^B will be intermediate between \bar{R} and $R^* = \eta$. Any value of R^B is associated with a unique value of $\mu \in [1, \bar{\mu}]$, which we will denote μ^B . With regard to the other parameters, if the middle aged discount the future substantially, then the equilibrium interest rate is closer to that preferred by the young. This gives them more capital to work with and a higher wage when middle aged. For very low values of α , the marginal product of capital is very low as is the real return to capital which hurts the portfolio income of the middle aged as they become old. Thus, the compromise choice of R^B moves closer to the preferred outcome of the middle-aged.

We wish to ask what a possible steady state equilibrium would be for the economy under this institutional arrangement. Let us first consider values of $\mu \in [1, \mu^B]$ and the associated steady state interest rates as candidate steady states. These values cannot be consistent with a steady state equilibrium, since the policy board can always choose a slightly larger value $\mu' > \mu$, tending via the dynamics of (23) and (24)

¹¹We could formulate the model in a standard Nash bargaining form whereby the agents choose $R(t)$ to maximize $(V_y - \hat{V}_y)^\theta (V_m - \hat{V}_m)^{1-\theta}$. The solution to the problem yields $\ln R^B = \theta \ln \bar{R} + (1-\theta) \ln R^*$.

to move $R(t)$ closer to R^B , thus achieving a lower loss \mathcal{L} . Similarly, we can consider values of $\mu \in (\mu^B, 1]$ and the associated steady state interest rates as candidate steady states. These values also cannot be consistent with steady state equilibrium, since the policy board can choose a slightly smaller value $\mu' < \mu$, tending to move $R(t)$ closer to R^B and so to again achieve a lower loss \mathcal{L} .

We conclude that the only viable steady state under a policy board is characterized by $\mu = \mu^B$ implying $R^* = R^B$. This stationary equilibrium represents an improvement over the majority voting outcome from the point of view of the social planner who wishes to maximize lifetime steady state utility of a typical household. This is because under majority voting, $R^* = \bar{R}$, and $\bar{R} < R^B$. But the social optimum, $R^* = \eta$, is not achieved under the policy board design except in the extreme case $\theta = 0$.¹²

PROPOSITION 2. A stationary equilibrium under a policy board will generally involve inflation bias. This bias will be smaller than the one associated with an institution characterized by majority voting.

Interpretation. As we have considered it, the purpose of the policy board is to reach a compromise between groups of agents differentially affected by monetary policy choices. The marginal gains to the young from increasing the capital stock and thereby increasing the real wage and consumption they enjoy during middle age are traded off against the highest possible portfolio return for the middle aged as they move toward old age. Naturally, the board reaches a compromise that is intermediate between the desires of the competing groups. Thus, as suggested in the research of Waller (1989, 1992, 2000) and Faust (1996), a policy board can lead to equilibrium outcomes that improve on the majority voting equilibrium. However, this requires that, through some institutional design, the young transfer some power over policy to the older generation.

It is clear from this discussion that if the policy-setting weights could be set as part of the institutional framework, the young would be willing to give all of the power to the middle-aged generation, that is, to set $\theta = 0$. The reason is that from a lifetime

¹²In our environment, lifetime utility would be higher for all generations if all of the bargaining weight were given to the middle-aged agents. This is a consequence of the fact that the social optimum corresponds to the middle-aged agents' most desired outcome. In other versions of the model, it may be the case that the middle-aged agents want an interest rate that exceeds the social optimum (as occurs in Faust (1996)). In that case, giving all of the bargaining weight to the middle-aged agents would not generate the social optimum.

perspective, all generations want the slight deflation associated with $R^* = \eta$. But middle-aged are the only ones who can credibly generate zero inflation. Thus, today's young would be willing to let today's middle-aged generation determine inflation today if they could guarantee themselves that they get to set inflation tomorrow. However, in general it is not clear that a generation's power is constant from period to period. In fact, the worst outcome for the young generation at t is $\theta(t) = 0$ and $\theta(t + 1) = 1$. Consequently, in the absence of an institutional arrangement that locks in bargaining weights, the young may still prefer to adopt an alternative institutional arrangement for determining inflation. In the next section, we consider a simple institutional arrangement that effectively does this.

3.4. Constitutional rule.

Nature of the arrangement. Our final institution is a constitutional rule or supramajority rule voting.¹³ Our abstract constitution works in the following way. To change the existing policy more than a simple majority is needed. Thus, in this situation, despite being the majority, the young can only change the existing policy if a substantial number of middle-aged voters agree with the change. If the young are incapable of generating enough support by the minority, the status quo prevails. In this sense, the constitutional rule is the status quo policy.

In order to model the process of changing the constitutional rule, we adopt the following bargaining process. Each period the young majority makes a take-it-or-leave-it offer to the middle-aged minority of $\mu(t) \in [1, \bar{\mu}]$. If the middle-aged voters reject the offer, the status quo holds until the next period when a new offer is made by the next period's young. If the middle-aged voters accept the offer, it becomes the new status quo. Although simplistic in its structure, we think this voting process captures the essential features of a constitutional rule.

The interesting feature of this arrangement is that the current young can do no worse than today's status quo over their lifetime. Whatever is chosen today can be sustained in the future by rejecting any offers to change the status quo. Thus, the constitutional rule gives them *veto power* over changes in monetary policy when they are middle aged. Thus our abstract constitution provides a form of commitment to

¹³Other recent work on constitutional rules includes Azariadis and Galasso (1996). A similar approach has been taken by Boldrin and Rustichini (2000) in studying social security.

future policy actions that is absent in simple majority voting regimes or the policy board regime.

As we have stressed, lifetime utility from the point of view of the middle-aged at time t is given by

$$V_m = C_0 + \frac{\alpha(1+\beta)}{\alpha-1} \ln R(t-1) + \beta \ln R(t). \quad (42)$$

where the only term in play is $\beta \ln R(t)$. Since this is a strictly increasing function, the middle-aged will accept any offer of higher interest rates (lower inflation). If the young offer higher interest rates, they can in the next period veto any attempt to lower interest rates. Thus the young can lock in any offer they make today.

What is the steady state of a system defined by this process? Let us consider candidate steady states defined by $\mu \in (1, \bar{\mu}]$ and the associated candidate steady state interest rates $R^* = \eta\mu^{-1}$. These values of μ cannot be associated with a steady state for this economy. For any μ in this interval, the young could offer a slightly lower value $\mu' < \mu$ to the middle-aged, which would tend to raise the value of $R(t)$ and thus increase their utility, so that the middle-aged would accept the offer. Furthermore, the young would acquire veto power in middle age, and thus be able to prevent interest rates from falling during their next period of life. Thus the young would guarantee that a higher interest rate remains in place over their entire lifetime, and thus that their lifetime utility is higher. The socially optimal Friedman rule obtains.

PROPOSITION 3. Under the constitutional rule, the social optimum $R^* = \eta$ is the unique stationary equilibrium.

Interpretation. A constitution is a central bank design which gives significant veto power to the minority group, which is the older generation in our framework. So long as the veto power is in place, the young can make a decision concerning today's monetary policy secure in the knowledge that future monetary policy will not result in a lowering of their welfare. This is a form of commitment which allows only the social optimum to be a viable stationary equilibrium. We view the constitutional design as a significant improvement on either the majority voting arrangement or the balanced board arrangement, both of which generally implied stationary equilibria characterized by inflation bias. This result helps explain, for example, why the

Maastricht Treaty made price stability a constitutional objective—it requires a super majority to change the objective rather than a simple majority of voting members.

4. CONCLUSION

We study central bank design in the context of a general equilibrium economy with a financial sector. In our analysis, we take a step toward integrating models of inflation bias with relatively standard general equilibrium growth models. Our model is simple but has the following interesting features: Capital accumulation, conflict between groups, and a clearly defined social optimum.

We study three social arrangements, which we interpret as abstract representations of central banks, for deciding upon a monetary policy. The first is majority voting, with the median voter in the young generation. We find that this design leads to substantial inflation bias. A second social arrangement involves bargaining between agent types, which we interpret as a policy board design. The policy board design can improve markedly on the majority voting outcomes, but cannot achieve the social optimum except in an extreme case.

A third social arrangement is a constitutional rule. Here the problems with majority voting and bargaining are remedied by giving the older, minority generation a veto over proposed policy changes. This acts as a form of commitment, causing the young to choose monetary policy based on lifetime utility, and thus to create a stationary equilibrium at the social optimum. We conclude that central bank designs which enforce commitment by making it difficult to change established long-run monetary policy goals, say by requiring social consensus for such changes, are the most promising if the goal is to attain the most efficient stationary equilibrium.

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