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INTERNATIONAL RISK SHARING AND LOW CROSS-COUNTRY CONSUMPTION CORRELATIONS: ARE THEY REALLY INCONSISTENT?

ABSTRACT

In dynamic equilibrium trade models, the common assumption that asset markets are complete implies that correlations of consumption across countries should be quite high. In contrast, measured consumption correlations tend to be rather low. While some suggest this implies that asset market incompleteness is a fundamental feature determining international trade dynamics, this paper provides an example of a simple model economy in which complete markets can be associated with consumption correlations that are lower than output correlations. Conditions for substitution elasticities associated with this result are derived for a two-country, two-good endowment model with heterogeneous agents.

KEYWORDS: Complete markets, dynamics, risk aversion, risk pooling, trade

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International Risk Sharing and Low Cross-Country Consumption Correlations: Are They Really Inconsistent?

1. Introduction

In equilibrium models of international trade dynamics, it is commonly assumed that asset markets are complete in the Arrow-Debreu sense. This feature seems more realistic than the alternative extreme of no international asset trade, and it provides a tractable framework in which decentralized equilibrium solutions can be found by determining Pareto optimal allocations. For a wide variety of preference and technology specifications, however, the pooling of consumption risk provided by complete asset markets implies a very high cross-country consumption correlation -- much higher than has been found empirically.

In particular, measured consumption correlations tend to be lower than corresponding output correlations. Table 1 illustrates this relationship, comparing various countries' consumption and output correlations with the U.S. (replicating the evidence presented by Backus, Kehoe and Kydland 1992b, 1993).[1] In contrast to this observed empirical regularity, models that assume complete asset markets often predict consumption correlations which are nearly perfect, regardless of the correlations between outputs. Backus, Kehoe and Kydland (1992b) called this implication "the most striking discrepancy"..."between theory and data."

In two-country, one-sector models of international trade dynamics -- such as those of Cantor and Mark (1988), Backus, Kehoe and Kydland (1992b), Baxter and Crucini (1993) -- trade is motivated solely by risk pooling. Hence it is not too surprising that such models imply a high correlation of consumption across countries. Similarly, in two good models in which agents have identical preferences, the existence of a "perfectly pooled equilibrium" (Lucas, 1982) implies perfect correlation of consumption across countries.

Several papers have addressed this issue. Backus, Kehoe and Kydland (1992b) incorporate a variety of features, including non-time separable leisure preferences, time-to-build, and trading frictions, with little effect on the magnitude of the consumption correlation. Devereux, Gregory and Smith (1992) showed that substitutability between consumption and leisure could reduce theoretically generated cross-country consumption correlations; however, their results depended crucially on a particular preference specification, and their simulated consumption correlations were not compared to output correlations.

Kollmann (1990) and Baxter and Crucini (1992) have suggested that asset-market incompleteness might account for the low correlation of consumption across countries. If agents have limited opportunities to pool risk, national consumption levels are tied more closely to domestic output than to world output, so cross-country consumption correlations tend to be lower.

In this paper, I examine whether it is necessary to consider asset market incompleteness in order to explain low cross-country consumption correlations. Previous examinations of complete-market models suggest that it might be. However, the risk pooling in those models represent special cases in which aggregate consumption smoothing is the predominant motive for asset trade.

In the model considered here, a simple form of heterogeneity between agents implies that changes in relative endowments give rise to competing substitution effects across goods and states. If the elasticity of substitution between domestic and imported goods is low relative to the intertemporal or inter-state substitution elasticity, this trade-off can result in low correlations of aggregate consumption across countries. In fact, I show that output correlations *can* exceed consumption correlations in a model with complete markets.

The mechanism underlying these results can be related to the analysis of Feeney and Jones (1994), in which the optimal allocations are shown to depend on two types of risk aversion: aversion to *aggregate* consumption risk and aversion to *compositional* consumption risk. Using this terminology, the condition for low international consumption correlations can be stated as requiring that aversion to compositional risk to be sufficiently stronger than aversion to aggregate consumption risk.

An important feature of the equilibrium dynamics described in this paper is that consumption share allocations are state-contingent. This feature is associated with an important allocative role for asset markets and it implies that an asset structure consisting solely of claims to (constant) shares of the two goods will not support the optimal allocation. It will therefore be useful to compare the heterogeneous-agent model to the "perfectly pooled equilibrium" construct in which identical agents contract to divide goods endowments in constant proportions.[2]

2. A Basic Model

General Environment

The model consists of two countries, each inhabited by an infinitely-lived representative agent. Agents are endowed each period with a stochastic quantity of distinct non-storable consumption goods: The home agent receives an endowment X of good x, while the foreign agent receives an amount Y of good y.

Innovations to the endowments are drawn from a joint log-normal distribution that is symmetric in the sense that $var(X)=var(Y)=\sigma^2$ and $cov(X,Y)=\sigma_{xy}$.[3] In the analysis of dynamics to follow, log-linear approximations are expressed as proportional deviations from a baseline equilibrium, defined by the nonstochastic equilibrium in which endowments take on their (normalized) unconditional expected values, E[X]=E[Y]=1.

Preferences

Both agents are assumed to be expected utility maximizers with preferences for aggregate consumption over time and states of the form

$$V = E_0[\sum_{t=0}^{\infty} \beta^t U(C_t)] = \sum_{t=0}^{\infty} \beta^t \int_{z_t} u(C_t) dF(z_t)$$

where $\beta<1$ is the agents' (common) discount factor and $F(z_t)$ is the distribution function for the state vector $z_t = (X_t\ Y_t)$. The home and foreign agents maximize over distinct aggregate consumption measures C and C^*, which are defined by the aggregator functions:

$$C_t = h(c_{xt}, c_{yt}) \quad and \quad C_t^* = h^*(c_{xt}^*, c_{yt}^*) \ .$$

Preferences are defined in terms of the nesting $U(c_x, c_y)=u[h(c_x, c_y)]$ so as to distinguish between substitution across goods (which depends on the parameters of h and h^*) from substitution across states or time (which depends on u).

Note that agents are assumed to have identical discount factors, β, and aggregate utility functions, $u(\cdot)$, but preferences over the composition of bundles are allowed to differ, $h(\cdot) \neq h^*(\cdot)$.

In particular, the type of heterogeneity considered will be a "taste bias" in favor of either domestic or imported goods.

While the h and h^* functions are presented as preference specifications, they might also be thought of as representing a production stage (possibly home production) in which factors or intermediate goods from each country are combined to produce a final consumption good. Hence they can be interpreted broadly as summarizing elements of both tastes and technology.

The $h(\cdot)$ and $h^*(\cdot)$ functions are assumed to be homothetic and symmetric in the sense that $h(x,y)=h^*(y,x)$ for all (x,y). The elasticity of substitution between X and Y is presumed to be approximately constant for "small" transitory output fluctuations, so the aggregator functions are represented in CES form:

$$h(c_x,c_y) = [\alpha c_x^{(1-\delta)} + (1-\alpha)c_y^{(1-\delta)}]^{1/(1-\delta)}$$

$$h^*(c_x,c_y) = [(1-\alpha)c_x^{*(1-\delta)} + \alpha c_y^{*(1-\delta)}]^{1/(1-\delta)}$$

where the symmetry assumption dictates the way in which the share parameter α enters the specifications. Note that heterogeneity is represented by $\alpha \neq ½$. The substitution parameter, δ $[0 \leq \delta \leq \infty]$, defines the elasticity of substitution between goods as $1/\delta$. The parameter δ is also the coefficient of relative risk aversion with respect to compositional risk (Feeney and Jones, 1994).

The function $u(\cdot)$ is assumed to be of the CRRA form:

$$u(C_t) = C_t^{1-\gamma}/(1-\gamma)$$

where γ is the coefficient of (aggregate) relative risk aversion, and its inverse is the inter-state or intertemporal elasticity of substitution.

Implications

Suppose that contingent-claims trade begins simultaneously with the introduction of uncertainty in period zero. The symmetry assumptions imposed on preferences and the endowment processes insure that the relative goods price under certainty will be one and that the expected value of future endowment streams will be equal. As a result, agents begin asset trade with equal wealth.

When $\alpha=\frac{1}{2}$, identical preferences imply a "constant-share" equilibrium; i.e., optimal allocations will consist of constant shares of the endowments. Because the two agents begin with equal wealth, the optimal contract will allocate to each agent one-half of each endowment in all states. This is the perfectly pooled equilibrium of Lucas (1982). For $\alpha\neq\frac{1}{2}$, the wealth distribution remains constant, but optimal allocation patterns will be more complex.

3. Risk Pooling and Consumption Dynamics

Optimization Problems

The complete-markets equilibrium can be found as the solution to a social planner's problem, or to a decentralized optimization problem with explicit contingent claims trade. Following the latter approach, the optimization problems faced by home and foreign agents can be represented as the maximization of agents' preference functions subject to resource constraints:

$$\sum_{t=0}^{\infty}\int_{z_t}\{q_x(z_t)[X(z_t)-c_x(z_t)]-q_y(z_t)c_y(z_t)\}dz_t=0 \tag{1}$$

for the home agent, and

$$\sum_{t=0}^{\infty}\int_{z_t}\{q_y(z_t)[Y(z_t)-c_y^*(z_t)]-q_x(z_t)c_x^*(z_t)\}dz_t=0 \tag{1*}$$

for the foreign agent. Asset prices $q_x(z_t)$ and $q_x(z_t)$ denote the value of claims to one unit of goods x and y in state z_t relative to the baseline price of good x.

First order conditions for these problems are:

$$\beta^t U_x(c_x(z_t),c_y(z_t))f(z_t) = \lambda \; q_x(z_t) \tag{2a}$$

$$\beta^t U_x^*(c_x^*(z_t),c_y^*(z_t))f(z_t) = \lambda^* q_x(z_t) \tag{2a*}$$

$$\beta^t U_y(c_x(z_t),c_y(z_t))f(z_t) = \lambda \; q_y(z_t) \tag{2b}$$

$$\beta^t U_y^*(c_x^*(z_t),c_y^*(z_t))f(z_t) = \lambda^* q_y(z_t) \tag{2b*}$$

where λ (λ*) represents the home (foreign) marginal utility of wealth. Asset market completeness implies that equations (2) will hold over all states and dates.

Contingent claims prices $q_x(z_t)$ and $q_y(z_t)$ are equalized across countries, implying the following fundamental relationship:

$$\frac{u'(C_t)h_x(c_{xt},c_{yt})}{u'(C_t^*)h_x^*(c_{xt}^*,c_{yt}^*)} = \frac{u'(C_t)h_y(c_{xt},c_{yt})}{u'(C_t^*)h_y^*(c_{xt}^*,c_{yt}^*)} = \varphi \tag{3}$$

In equation (3), $\varphi = \lambda/\lambda^*$ is the (constant) ratio of Lagrange multipliers on the agents' resource constraints (the ratio of the shadow values of wealth). In the solution to a social planners problem, φ would represent the ratio of welfare weights (foreign/home) in the planner's objective function.

Equilibrium also requires that the following aggregate consistency conditions hold:

$$X_t = c_{xt} + c^*_{xt} \tag{4a}$$

$$Y_t = c_{yt} + c^*_{yt} \tag{4b}$$

The equilibrium solution will be a vector of consumptions for each individual state and date, $[c_x(z_t)\ c_y(z_t)\ c^*_x(z_t)\ c^*_y(z_t)]$, which solves equations (3) and (4) for a given wealth distribution, $\varphi=\lambda/\lambda^*$.

With two margins of substitutability and two margins of trade (trade over both goods and states) agents can attain a full Pareto-optimum in which marginal utilities are adjusted independently. Because agents are assumed to begin with equal wealth, $\lambda=\lambda^*$, and the ratio φ is equal to one. Marginal utilities for each good therefore will be equalized across countries.

<u>Approximate Dynamics</u>

The system of equations determining equilibrium is highly nonlinear, so exact solutions are generally unavailable. To examine the dynamic properties of the model, I derive log-linearized versions of the first-order conditions and find approximate solutions in terms of proportional deviations of variables from their baseline certainty-equivalent values, e.g. $\hat{X} = d[\ln(X)]$ or equivalently, $\hat{X} = dX/E[X]$.

To highlight the effects of agent heterogeneity and the associated departures from "perfect pooling", it is useful to find the equilibrium solutions in terms of consumption shares, for example

$s_x = c_x/X$, or $\hat{s}_x = \hat{c}_x - \hat{X}$. Given the assumed functional forms for the $u(\cdot)$ and $h(\cdot)$ functions, these solutions can be expressed as:

$$\hat{s}_x = (1-\theta)\Omega[\hat{X} - \hat{Y}] \tag{5a}$$

$$\hat{s}_x^* = \theta\Omega[\hat{Y} - \hat{X}] \tag{5a*}$$

$$\hat{s}_y = \theta\Omega[\hat{X} - \hat{Y}] \tag{5b}$$

$$\hat{s}_y^* = (1-\theta)\Omega[\hat{Y} - \hat{X}] \tag{5b*}$$

where

$$\Omega = \frac{(1-2\theta)(\delta-\gamma)}{4\theta(1-\theta)(\delta-\gamma)-\delta}$$

The parameter θ is the home agent's consumption share of the x-good in the baseline equilibrium (by symmetry, it is also the foreign country's share of the y-good).[4] The term $(1-2\theta)$ in the numerator of Ω measures the direction of the bias in tastes, and $4\theta(1-\theta)=[1-(1-2\theta)^2]$, which appears in the denominator of Ω, is a measure of the intensity of the bias.

The composite term Ω summarizes the allocational function of asset trade, determining the distribution of relative consumption shares as a function of the realized state. Specifically,

$$(\hat{s}_x - \hat{s}_x^*) = (\hat{s}_y - \hat{s}_y^*) = \Omega(\hat{X} - \hat{Y}).$$

Note that Ω depends on both of the elasticity or risk-aversion parameters, γ and δ. It can therefore be interpreted either as a composite elasticity term or as a measure of the trade-off between aggregate and compositional consumption risk.

A perfectly pooled equilibrium is associated with θ=½. In this case Ω=0, so that consumption shares are constant. For $\theta \neq$½, equations (5) show that shares move in proportion to relative endowment levels and that each agent's share allocations move in the same direction for any given relative endowment deviation. The direction of consumption share deviations depend on the sign of Ω. For example, a relatively high *X*-endowment is associated with an increase in the home agent's consumption shares if Ω>0. The denominator of Ω is always negative:

$$4\theta(1-\theta)(\delta-\gamma)-\delta = -(2\theta-1)^2\delta - 4\theta(1-\theta)\gamma < 0$$

Therefore, Ω will be positive if:

(i.) $\theta > ½$ and $\gamma < \delta$, or

(ii.) $\theta < ½$ and $\gamma > \delta$.

These conditions state that: (i) agents consume more domestic goods than foreign goods and compositional risk aversion is stronger than aggregate risk aversion, or (ii) agents consume more imported goods and the risk aversion coefficients obey an opposite inequality.

Note that Ω equals zero not only when θ=½, but also when $\gamma=\delta$. In this case, preferences over the two goods are separable and a constant-share equilibrium results even though preferences differ.[5] For each of the two cases, asset markets play a limited allocational role: Risk is pooled by distributing constant shares of the two endowments.

Equations (5) can be used to express consumption movements directly as:

$$\hat{c}_x = \hat{X} + (1-\theta)\Omega[\hat{X} - \hat{Y}] \tag{6a}$$

$$\hat{c}_x^* = \hat{X} + \theta\Omega[\hat{Y} - \hat{X}] \tag{6a*}$$

$$\hat{c}_y = \hat{Y} + \theta\Omega[\hat{X} - \hat{Y}] \tag{6b}$$

$$\hat{c}_y^* = \hat{Y} + (1-\theta)\Omega[\hat{Y} - \hat{X}] \tag{6b*}$$

Aggregate consumption levels, C and C^* can then be expressed using linearized forms of the aggregator (h) functions,

$$\hat{C} = \theta\hat{c}_x + (1-\theta)\hat{c}_y \quad and \quad \hat{C}^* = (1-\theta)\hat{c}_x^* + \theta\hat{c}_y^*$$

yielding:

$$\hat{C} = [\theta\hat{X} + (1-\theta)\hat{Y}] + 2\theta(1-\theta)\Omega[\hat{X}-\hat{Y}] \tag{7}$$

$$\hat{C}^* = [(1-\theta)\hat{X} + \theta\hat{Y}] + 2\theta(1-\theta)\Omega[\hat{Y}-\hat{X}] \tag{7*}$$

Each of the expressions for consumption allocations in (6) and (7) include two terms. The first reflects the change in consumption for fixed consumption shares, while the second captures the effect of changes in relative share allocations summarized in equations (5).

When Ω=0, the second terms in expressions (6) and (7) are zero. Individual consumptions of x-goods and y-goods move in direct proportion to world endowments and are therefore perfectly correlated across countries. For θ=½, aggregate consumption levels also will be perfectly correlated. In the constant-share equilibrium for separable preferences ($\delta = \gamma$), relative share differences imply that the aggregate cross-country consumption correlation will be less than perfect.

A Diagrammatic Analysis

Because the equilibrium conditions imply equality of marginal utilities across countries, marginal rates of substitution will also be equal. (Of course, this must be so because equilibrium allocations clear both the asset and goods markets.) Accordingly, all allocations will lie on a conventional goods-market contract curve, and an Edgeworth-box diagram can be a convenient analytical tool for illustrating equilibrium dynamics.

Figure 1 presents diagrams for three values of θ, drawn for the special case in which the *h*-functions are Cobb-Douglas ($\delta=1$). The Cobb-Douglas specification implies that when consumption shares are constant, the relative price of goods moves in equal proportion to relative endowment changes. For the purpose of comparing perfectly-pooled allocations with the more general contingent-claims market allocations, this feature provides a convenient benchmark. Each panel in Figure 1 includes two boxes: The first represents the situation in the baseline case ($X=Y=1$), while the second is drawn for a particular state-realization in which $X>1$ ($\hat{X} > 0$).[6]

The top panel illustrates the case of identical preferences. The contract curve is a straight line connecting the home and foreign origins, O and O*, and the Cobb-Douglas assumption implies that the relative price changes in proportion to the endowment change. The equilibrium allocation in both boxes reflect the perfect pooling nature of this case, with each agent consuming half of the endowment of each good (shown in the left box as point E^{o} and in the right box as E).

The middle panel illustrates a case in which $\theta > ½$. The equilibrium point E reflects the baseline consumption shares θ and $(1-\theta)$. In the right-hand box, point E corresponds to the constant-share equilibrium associated with $\gamma=\delta$. For $\gamma<\delta$ equations (5) imply an increase in the home agents consumption shares, as illustrated by allocation E′. For $\gamma>\delta$, the home agent's consumption shares fall relative to the constant-share allocation (to E″).

The lower panel depicts the $\theta<½$ case. It is essentially a mirror image of the $\theta>½$ case, with $\gamma<\delta$ implying a decline in the home agent's consumption shares (to E′) and $\gamma>\delta$ implying an increase (to E″).

Optimal Allocations and Risk Aversion

The consumption share movements illustrated in the middle and lower panels of Figure 1 depend upon the relative values of the parameters γ and δ, reflecting a tradeoff between substitution across goods and substitution across states or time. This tradeoff can also be expressed in terms of two types of risk aversion: aggregate and compositional risk.

While the relationship between aggregate risk aversion and intertemporal or interstate substitution is fairly standard, the notion of compositional risk aversion is not widely used. As described in Feeney and Jones (1994), however, the concept is directly analogous to the conventional notion of risk aversion. For example, suppose an agent receives an endowment which fluctuates randomly between two bundles, one consisting of more x than y and the other of more y than x. Even if both bundles lie on the same indifference curve, the agent would be willing to pay a premium to consume a convex combination of the two bundles with certainty.

The importance of compositional risk aversion can be demonstrated by considering expected utility. Taking the expected value of a second-order Taylor series expansion of $U(c_x\ c_y)$ around the baseline consumption point,

$$E[U(c_x,c_y)] \approx U(\theta,1-\theta) + \tfrac{1}{2}U_{xx}(\theta,1-\theta)var(c_x) + U_{xy}(\theta,1-\theta)cov(c_x,c_y) + \tfrac{1}{2}U_{yy}(\theta,1-\theta)var(c_y$$

After substituting parameters and expressing second moments in terms of their log-linear approximations [e.g. $var(c_x)=\theta^2 var(\hat{c}_x)$], expected utility (relative to the certainty baseline) can be approximated as:

$$\frac{E[U(c_x,c_y)] - U(\theta,1-\theta)}{(1-\gamma)U(\theta,1-\theta)} \approx -\frac{\gamma}{2}\{\theta^2 var(\hat{c}_x) + 2\theta(1-\theta)cov(\hat{c}_x,\hat{c}_y) + (1-\theta)^2 var(\hat{c}_y)\}$$

$$+ \frac{\delta\theta(1-\theta)}{2}\{2cov(\hat{c}_x,\hat{c}_y) - var(\hat{c}_x) - var(\hat{c}_y)\}$$

$$= -\frac{1}{2}\{\gamma var(\hat{C}) + \delta\theta(1-\theta)var(\hat{c}_y - \hat{c}_x) \qquad (8)$$

From equation (8), it is clear that the welfare implications of international asset trade depend not only on the ability of agents to smooth aggregate consumption variability, but also on their ability to smooth variability in the composition of consumption bundles, as represented here by the (c_y/c_x)-ratio.

To provide a more intuitive explanation of this notion, Figure 2 reproduces the situation in the middle panel of Figure 1, focusing on the home agent's utility. When the two substitution elasticities or risk aversion terms are equal ($\gamma=\delta$), constant consumption shares imply movement from equilibrium E^0 to E, which is associated with a fall in the (c_y/c_x)-ratio -- as indicated by the slope of the ray OE relative to OE^0 -- and an increase in aggregate consumption -- as indicated by the movement from indifference curve U^0 to U.

When agents are relatively more averse to compositional consumption risk than to aggregate consumption risk ($\gamma > \delta$), they are more willing to accommodate large changes in C and relatively less willing to alter the composition of consumption. This situation is illustrated by equilibrium E′, which is associated with a fairly small change in the (c_y/c_x)-ratio [to ray OE′], and a relatively large change in aggregate consumption [to U′]. When the relative magnitudes of risk-aversion terms is reversed, agents are more willing to alter the composition of consumption [a large change in (c_y/c_x) to ray OE″] and less willing to experience changes in aggregate consumption over states [a smaller change in utility from U^0 to U″].

4. Cross-Country Consumption Correlations

Sufficient Conditions: An Example

A simple case that can serve to illustrate sufficient conditions for cross-country consumption correlations to be lower than output correlations is one in which X and Y are uncorrelated. The consumption correlation will be lower than the output correlation if the former is negative; that is, if C and C^* move in opposite directions in response to a change in relative endowments.

Consider a positive x-endowment shock (the case where) $\theta > \frac{1}{2}$. When $\gamma < \delta$ the home country's consumption shares rise relative to the baseline, while the foreign agent's consumption shares fall. The sufficient condition for $corr(C,C^*) < 0$ will be satisfied if the fall in the foreigner's shares is large enough that foreign aggregate consumption falls. For $\theta < \frac{1}{2}$, a similar situation could arise in which the home agent's aggregate consumption falls in response to a positive x-endowment (a complete-markets analog to immizerizing growth).

From equations (7), it can be determined that these patterns will occur when $[1-2\theta\Omega]<0$ or $[1+2(1-\theta)\Omega]<0$. In terms of the underlying parameters,

$$[1 - 2\theta\Omega] < 0 \quad \leftrightarrow \quad \frac{\gamma}{\delta} < \frac{2\theta-1}{2\theta} \tag{9a}$$

or,

$$[1 + 2(1-\theta)\Omega] < 0 \quad \leftrightarrow \quad \frac{\gamma}{\delta} < \frac{1-2\theta}{2(1-\theta)} \tag{9b}$$

Figure 3 uses a representation of the parameter space to illustrate these regions. Two important features of the regions are (i.) the elasticity of substitution across goods must be less than one-half the intertemporal substitution elasticity *and* (ii.) preferences must be skewed away from the case of identical utility.

Necessary and Sufficient Conditions

These conditions are necessary as well as sufficient: That is, allowing for non-zero covariance of X and Y changes the consumption correlation, but does not change the relative magnitudes of the consumption and output correlation. Using equations (7), explicit expressions for $var(\hat{C})=var(\hat{C}^*)$ and $cov(\hat{C},\hat{C}^*)$ can be derived:

$$var(\hat{C}) = var(\hat{C}^*) = (1-K)\,\sigma^2 + K\,\sigma_{xy}$$

$$cov(\hat{C},\hat{C}^*) = K\,\sigma^2 + (1-K)\,\sigma_{xy}$$

where $\sigma^2=var(\hat{X})=var(\hat{Y})$, $\sigma_{xy}=cov(\hat{X},\hat{Y})$, and the parameter K is a function of the underlying share and elasticity parameters: $K=2\theta(1-\theta)[1+2(1-\theta)\Omega][1-2\theta\Omega]$. The cross country consumption correlation can then be expressed as:

$$corr(\hat{C},\hat{C}^*) = \frac{cov(\hat{C},\hat{C}^*)}{var(\hat{C})} = \frac{K + (1-K)\ corr(\hat{X},\hat{Y})}{(1-K) + K\ corr(\hat{X},\hat{Y})} \tag{10}$$

It is straightforward to show from (10) that $corr(\hat{C},\hat{C}^*)<corr(\hat{X},\hat{Y})$ whenever one of conditions (9a) or (9b) hold.

To illustrate the relationship between the consumption and output correlations somewhat more quantitatively, Table 2 reports the consumption correlations associated with a range of elasticity and share parameter values, assuming that the cross-country output correlation is 0.6. Again, the important features distinguishing the cases where $corr(\hat{C},\hat{C}^*)<corr(\hat{X},\hat{Y})$ are that the preferences are not identical ($\theta\neq\frac{1}{2}$) and that aversion to compositional risk significantly exceeds aversion to aggregate risk $\gamma/\delta<<\frac{1}{2}$. Notice that for any relative share distribution, θ, the consumption correlation is monotonically increasing in γ/δ: as aggregate consumption risk becomes more important relative to compositional risk, consumption-smoothing predominates.[7]

Risk Neutrality and the Rybczynski Effect

The example described above can be related to a familiar result from trade theory: the Rybczynski effect. In Figure 2, the point at which ray OE^0 intersects the contract curve represents an upper bound on the rise in the home agent's consumption shares (the fall in the foreign agents' shares), and corresponds to aggregate risk neutrality (γ=0). That is, Ω is maximized at $\Omega = 1/(2\theta-1)$. Note that this solution is independent of δ; In the absence of aggregate risk aversion, compositional risk is fully pooled by asset trade. In this case the (c_x/c_y)-ratio is constant and will be associated with a constant relative goods price.

This situation is analagous to a case where the two countries are "small", facing an exogenous world relative price. Consequently, an increase in the endowment of X leads to an absolute increase in the consumption of the agent who consumes the x-good intensively -- and an absolute decline in the consumption of the other agent.

More generally, this type of model is equivalent to a two-by-two (two factors, two goods) closed-economy general equilibrium production model. Commodities C and C^* are produced using factors X and Y. Optimal allocations and relative price movements are determined by factor intensities and substitutability in production (the h-functions) and preferences (the social planner's weighted average of u and u^*).[8]

5. Discussion and Conclusions

The model in this paper has illustrated a setting in which asset markets play an allocational role which goes beyond simple consumption smoothing. In that context, I have shown that complete asset markets *can* be associated with cross-country consumption correlations that are lower than cross-country output correlations. The conditions for this phenomenon can be described as requiring that the aversion to *compositional* consumption risk be sufficiently stronger than aversion to *aggregate* consumption risk. When this is so, agents use international asset trade to smooth the composition of their consumption bundles, allowing greater variability in aggregate consumption.

A natural question which arises from this analysis is: How realistic are these conditions? Clearly, the homogeneous-agent implication that import shares are equal to one-half is unrealistic. Import shares of national income have averaged only about 15% in the United States, Europe and Japan over the last decade (Backus, Kehoe and Kydland 1992a). The import content of final consumption goods is even lower (Clarida, 1991). The analysis in this paper has modeled these import shares in a very simple way, as parameters of a utility function.

The plausibility of the elasticity restrictions are more difficult to evaluate. The intertemporal elasticity of substitution has been subject to a wide range of estimates. Citing estimates from time-series analyses, most dynamic general equilibrium models are calibrated with a value of γ between 0.5 and 2.0. Hall (1988), and Campbell and Mankiw (1989) found that the intertemporal substitution elasticity is close to zero. However, using a preference specification which is designed to disentangle risk aversion from intertemporal substitutability, Epstein and Zin (1991) found that while the intertemporal elasticity is significantly less than one, the coefficient of relative risk aversion is close to one. In the context examined in this paper, asset trade is used to pool risk across states, with allocations dependent on risk aversion. An appropriate value for γ of between zero and one does not seem completely unreasonable.

Even with a low value for γ, the conditions derived in this paper require a high degree of compositional risk aversion, or a very low elasticity of substitution between domestic and imported goods. There exists a large body of literature measuring import demand and export supply elasticities (e.g., Stern, Francis and Schumacher,1976). This work suggests an elasticity in the range of one to two. However, these studies are highly disaggregated. The empirical counterparts to the consumption and output measures of this model consist of *bundles* of goods: some types of goods may be very close substitutes, other are highly differentiated, some are non-traded, some are subject to distortions, etc. The relevant elasticity of substitution to consider is not a pure preference or technological parameter, but a composite that reflects a number of factors. The appropriate elasticity might be considerably lower than the measured substitutability of categorized, traded goods.

For example, the general condition of low aggregate risk aversion relative to compositional risk aversion is also present in the models of Stockman and Tesar (1995) and Tesar (1993), in which low substitutability between traded goods and nontraded goods drives down the aggregate cross-country consumption correlation.

Interpreting the model in these broad terms, the explanation for the consumption-output correlation described here is not being advanced as the "solution" to this "puzzle". Rather, this paper illustrates a general class of conditions under which complete markets can be associated with low cross-country consumption correlations.

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Notes

1. The importance of this stylized fact has been emphasized by Backus, Kehoe and Kydland (1992b, 1993). Although the relationship between consumption and output correlations may not be as robust as is often suggested [e.g., the results reported in Tesar (1993) and Pakko (1994)], it does characterize the relationship which holds for many bilateral country comparisons (particularly among industrial countries), and is treated in this paper as the relevant criterion for matching theory to data.

2. A perfectly pooled equilibrium is often explicitly assumed in equilibrium exchange rate models [e.g., Svensson (1985), and Stockman and Svensson (1987)] and in analyses of the forward exchange premium [e.g., Hodrick (1989)]. Even in models which literally depart from perfect pooling [e.g., Stockman and Dellas (1989), Stockman and Tesar (1995)] some vestige of perfect pooling is often retained.

3. Any temporal dependence in the endowment distributions will be irrelevant for the analysis of this paper. Trade in contingent claims over all dates and the nonstorability endowments imply a complete time-separability in the maximization problems.

4. The parameter θ is a function of α, δ and the steady state consumption levels; specifically,

$$\theta = \alpha\, c_x^{(1-\delta)} / [\alpha\, c_x^{(1-\delta)} + (1-\alpha)\, c_y^{(1-\delta)}] .$$

5. The sign of Ω is associated with the sign of the cross-derivative of the utility function, U_{xy}. When $\delta=\gamma$, $U_{xy}=0$ and the utility function reduces to the separable form:

$$u[h(c_x,c_y)] = [\alpha\, c_x^{(1-\gamma)} + (1-\alpha)\, c_y^{(1-\gamma)}]/(1-\gamma).$$

6. Figure 1 is similar to diagrams used by Hagiwara (1994) to illustrate terms of trade variability in a model with the same type of heterogeneity considered here. She shows that the terms of trade are more variable for heterogeneity than for identical preferences when $\gamma/\delta>1$.

7. In order to verify that the log-linear approximated solutions do not give misleading results, numerical simulations were conducted using the exact non-linear solutions. Using a standard deviation for X and Y equal to that of U.S. output, the simulated results for consumption correlations yeilded no cases in which there was a statistically significantly difference from the figures reported in Table 2.

8. This type of analogy between social welfare analysis and production activity analysis is explored in Jones (1972).

Country	HP-Filtered Data		First-Differenced Data	
	Consumption	Output	Consumption	Output
Austria	.18	.23	.03	.08
Canada	.58	.72	.35	.48
Germany	.26	.36	.21	.30
Japan	.26	.17	.27	.18
Spain	.10	.25	.10	.16
Switzerland	.35	.37	.11	.16
U.K.	.44	.58	.16	.21

[a]The data in Table 1 are real private consumption expenditures and real gross domestic product, seasonally adjusted. They were obtained from the OECD quartely national accounts tape. The sample periods are as follows: Canada, Japan and the U.K. 1995Q1-1993Q4; Austria 1959Q3-1993Q4; West Germany 1968Q1-1993Q4; France, Spain, and Switzerland 1970Q1-1993Q4.

Table 1: Correlations of Consumption and Output with the U.S.[a]

γ/δ	Domestic Consumption Share (θ)				
	.50	.60	.70	.80	.90
4	1.0000	.9987	.9936	.9791	.9287
2	1.0000	.9948	.9766	.9352	.8408
1	1.0000	.9802	.9231	.8349	.7241
½	1.0000	.9287	.7875	.6741	.6165
⅓	1.0000	.8567	.6575	.5701*	.5662*
¼	1.0000	.7738	.5478*	.5006*	.5382*
⅛	1.0000	.4382*	.2742*	.3284*	.4914*

[a]Entries in Table 1 represent the cross-country consumption correlations for the given parameters. The cross-country output correlation is set to .60. Entries marked with an asterisk (*) are values for which the consumption correlation is lower than the output correlation.

Table 2: Parameterized Model Values For the Cross-Country Consumption Correlation[a]

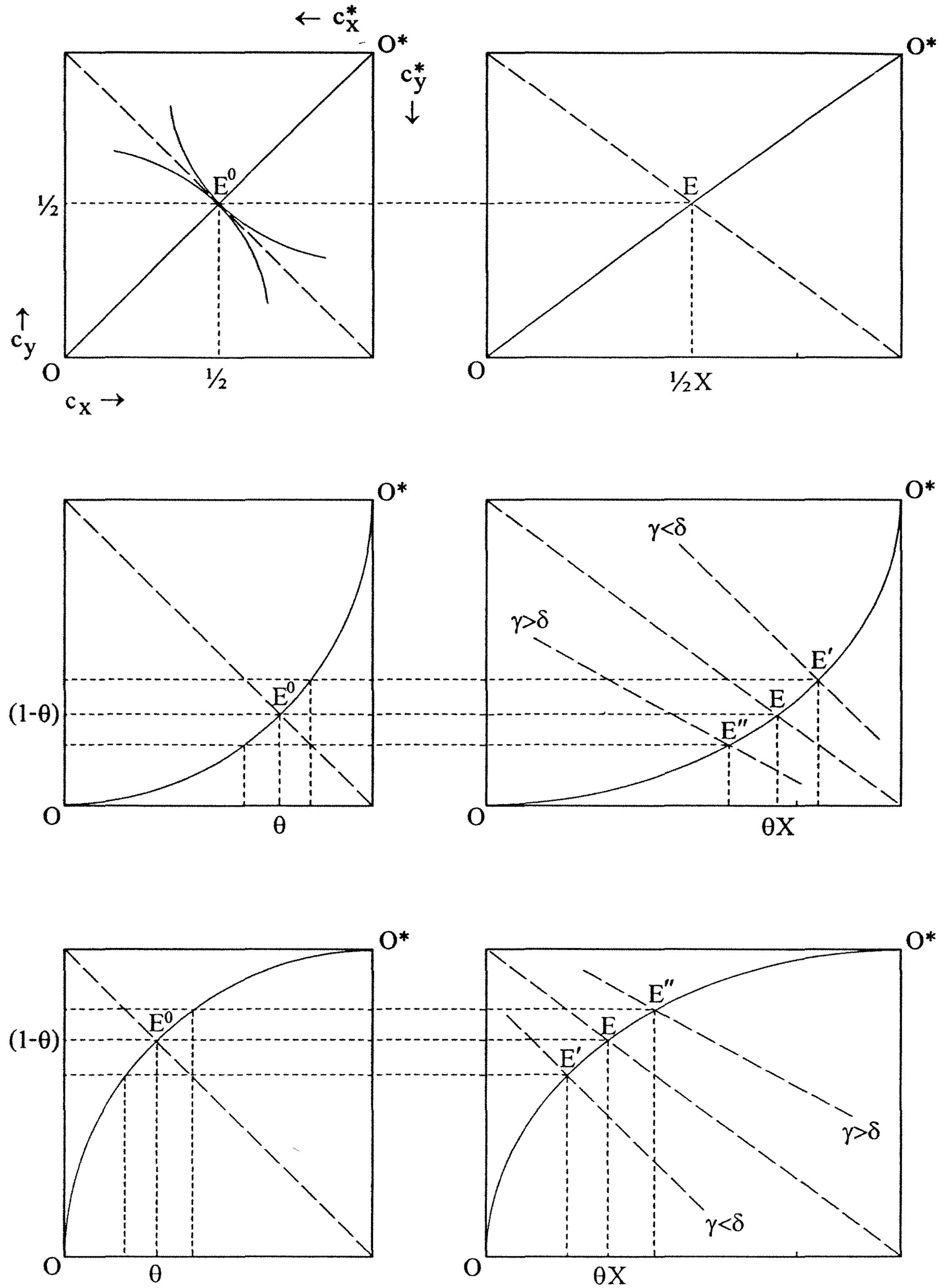

Figure 1: Equilibrium Allocations Under Various Parameter Combinations

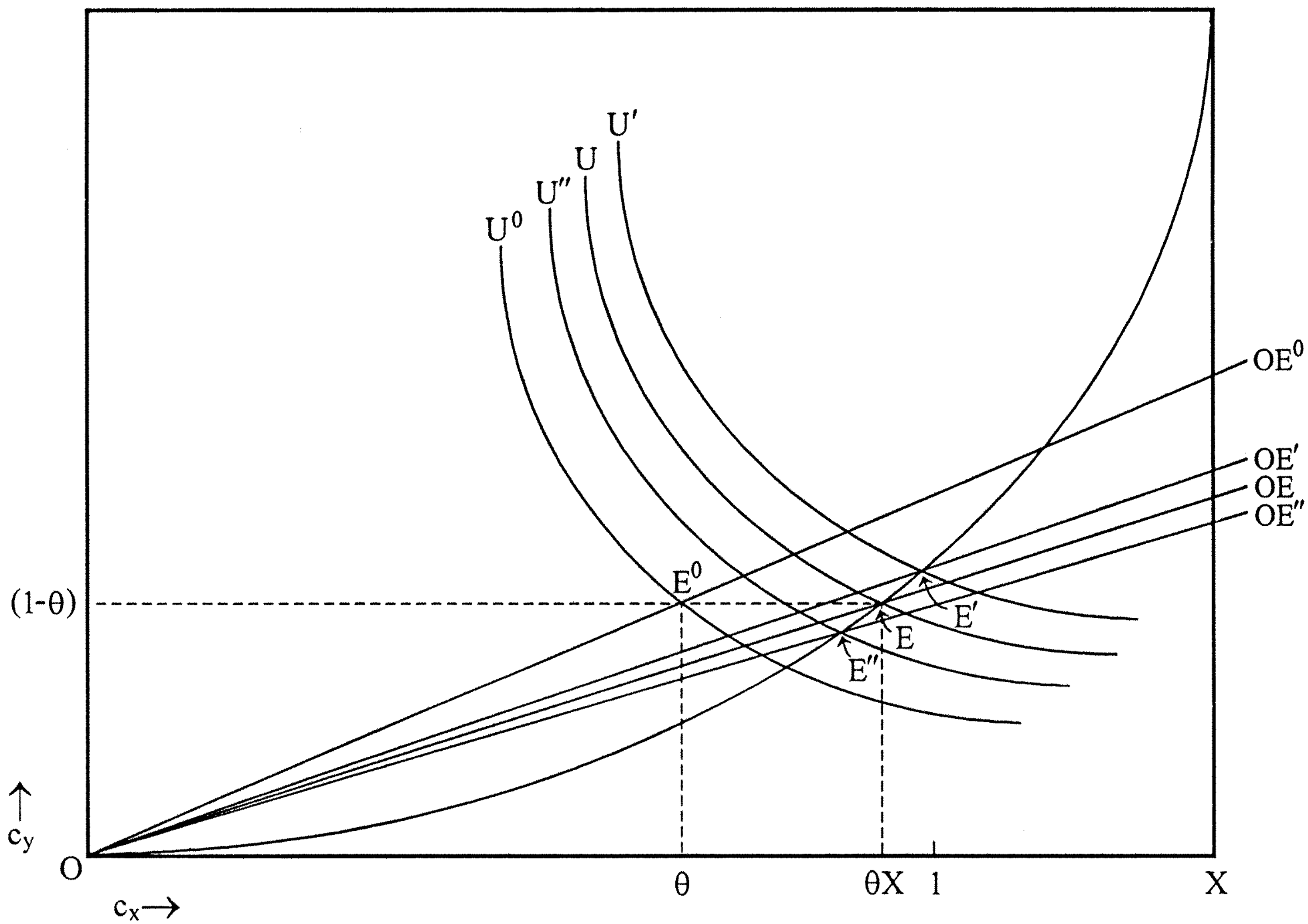

Figure 2. Aggregate Versus Compositional Risk and Equilibrium Allocations

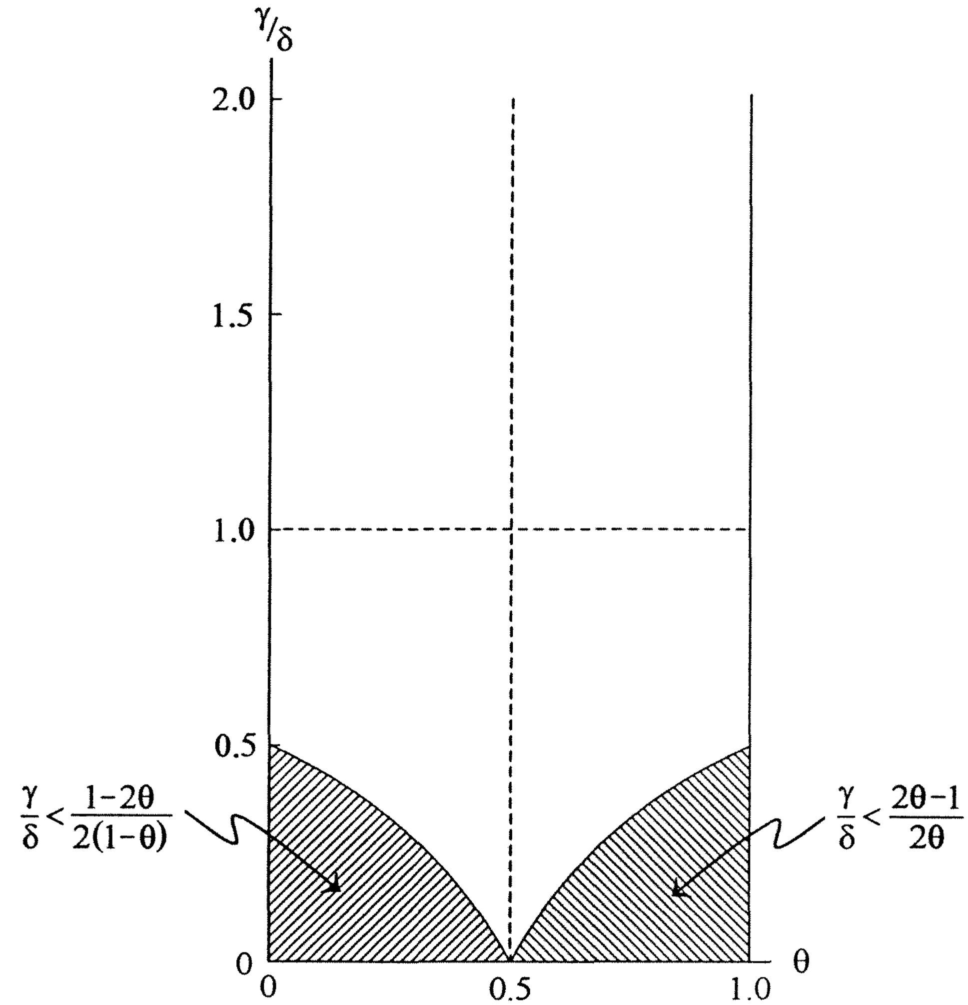

Figure 3. Parameter Values Associated With Corr(C,C*)<Corr(X,Y)