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## **The Andersen-Jordan Equation, Revisited**

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Working Paper 1983-015A  
<http://research.stlouisfed.org/wp/1983/1983-015.pdf>

1983

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THE ANDERSEN-JORDAN EQUATION, REVISITED

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83-015

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The views expressed here do not necessarily reflect those of the Federal Reserve Bank of St. Louis or the Board of Governors of the Federal Reserve System. We would like to thank Keith Carlson, Bill Dewald, and Rik Hafer for helpful comments and Sarah Driver and John Schulte for their research assistance. Any remaining errors are our own.

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## THE ANDERSEN-JORDAN EQUATION, REVISITED

### 1. Introduction

This year marks the fifteenth anniversary of the Andersen-Jordan (1968) equation. Not since the Phillips curve has a piece of empirical macroeconomic research generated such controversy and further research. Andersen and Jordan's conclusions that monetary actions have a significant and lasting impact on nominal income, while fiscal actions have no significant lasting effect has been heralded as strong support for monetarists' propositions by some and as shoddy empiricism by others.

Some critics have argued that the lack of a structural framework may have led to the omission of relevant, non-policy variables from the right-hand-side, resulting in biased parameters. Other critics have contended that the monetary and fiscal variables used were inappropriate and may have distorted the relative importance of monetary and fiscal actions.

Each of the above issues has been considered in some depth elsewhere and, hence, is not considered here.<sup>1/</sup> Instead, we are concerned with two other criticisms of the St. Louis equation: the question of the exogeneity of the right-hand-side variables and

the use of Almon's (1965) polynomial distributed lag (PDL) estimation technique. Since either the inclusion of endogenous right-hand-side variables or the inappropriate choice of lag length or PDL structure can bias estimates of the distributed lag weights, these issues deserve further study.<sup>2/</sup>

The question of exogeneity has received considerable attention, and studies have both confirmed and denied the exogeneity of money. All of the investigations, save a recent study by Hsiao (1981), have given little importance to the specification of the lag structure in their tests and have performed tests with arbitrarily chosen lag lengths. In contrast, the PDL specification of the St. Louis equation has received little attention.<sup>3/</sup>

The outline of this paper is as follows. In section two, the original Andersen-Jordan (A-J) specification is estimated, and the explicit and implicit restrictions of A-J specification are discussed and, to the extent possible, tested. In section three, a procedure for identifying the lag length and polynomial degree of a general PDL model is presented briefly. This procedure is applied to the original A-J data in section four. In section five, the endpoint constraints for the specifications presented in the previous section are tested.

Section six contains tests of exogeneity based on Granger (1969) tests of causality. The conclusions are presented in the final section.

## 2. The Andersen-Jordan Equation

The original specification of the A-J equation was

$$(1) \Delta Y_t = \sum_{j=0}^3 \alpha_j \Delta M_{t-j} + \sum_{i=0}^3 \beta_i \Delta E_{t-i} + \epsilon_t$$

where  $\Delta Y$ ,  $\Delta M$ , and  $\Delta E$  denote first differences of nominal GNP, money (defined as M1) and high-employment government expenditures, respectively, and  $\epsilon_t$  denotes a white noise residual error.<sup>4/</sup>

Ordinary least squares (OLS) and PDL estimates of the A-J equation appear in Table 1. The estimates were made with the original A-J data which cover the period 1952/I - 1968/II.<sup>5/</sup> They assumed that the distributed lags of equation (1) lay on a fourth degree polynomial and imposed the so-called endpoint constraints.<sup>6/</sup> A comparison of the PDL and OLS results reveals some substantial differences in the estimates of the distributed lag weights. This is particularly true for the distributed lag coefficients on money, suggesting that the endpoint constraints (which are imposed to make the equation estimable) may not be supported by the data.<sup>7/</sup> Unfortunately, strictly speaking, this conjecture

cannot be tested.

To see this, consider the following matrix representation of a general distributed lag model

$$(2) \underline{Y} = X \underline{\beta} + \underline{\varepsilon},$$

where  $X$  is a  $T$  by  $(\ell+1)$  matrix of the distributed lags of order  $\ell$  of the regressor variable  $x_t$ ,  $t=1, 2, \dots, T$ , and  $\underline{\beta}$  is a  $(\ell+1)$  by 1 vector of distributed lag coefficients. ( $\underline{Y}$  and  $\underline{\varepsilon}$  have the usual interpretations). It is well-known that the PDL reparameterization of (2) maps  $\underline{\beta}$  into a subspace spanned by polynomial coefficients of order  $p$ . That is,

$$(3) \underline{\beta} = H \underline{\alpha},$$

where  $H$  is a  $(\ell+1)$  by  $(p+1)$  matrix of known coefficients and  $\underline{\alpha}$  is a  $(p+1)$  by 1 vector of polynomial coefficients.<sup>8/</sup> In the usual case,  $p < \ell$  so that the PDL estimates map  $\underline{\beta}$  onto a subspace of  $R^{\ell+1}$ . In this case, however,  $p > \ell$  so that, in the absence of additional restrictions, the PDL model

$$(4) \underline{Y} = XH \underline{\alpha} + \underline{\varepsilon} = Z \underline{\alpha} + \underline{\varepsilon},$$

obtained by combining (2) and (3), is not estimable.

There exists a non-zero  $\underline{\alpha}$  such that  $H \underline{\alpha} = 0$ .

In other words,  $\underline{\alpha}$  spans the null space of  $H$ .<sup>9/</sup>

Estimates of  $\underline{\alpha}$  from (4) cannot be obtained without imposing additional restrictions.

Furthermore, these restrictions must lie in the null

space of H. Restrictions that are orthogonal to the implied restrictions will not suffice. Thus, by selecting  $p > k$ , Andersen and Jordan effectively placed implied restrictions on their model. Their PDL model is estimable because the endpoint constraints are not orthogonal to the implied restrictions.<sup>10/</sup> If the constrained PDL model is tested against OLS, the particular restrictions being tested cannot be identified since either the head or the tail constraints alone are sufficient to obtain OLS estimates of the distributed lag weights. Nevertheless, an F-test of this comparison suggests that these unidentified restrictions cannot be rejected. This result is reported as the first F-test in Table 1.

Another approximate procedure is to test the endpoint constraints as additional restrictions to OLS using restricted least squares (RLS). These tests were performed for the head and tail constraints separately and are reported in Table 1. The results indicate that only the head constraints can be rejected at the 5 percent level. Thus, it appears that A-J may have selected a polynomial restrictions that could bias their results.

While these restrictions may have biased the individual distributed lag weights, they appear to

have had no effect on the policy conclusions. A comparison of the tests of the sums of the distributed lag coefficients shows that the important policy conclusions of the A-J equation are not affected by their PDL specification. Indeed, the sums of the distributed lag coefficients are little affected by the PDL specification.

### 3. The Model Specification

Even though the comparison of the OLS and PDL estimates of the previous section indicates that the policy conclusions of the A-J equation are not dependent upon their unusual choice of a PDL model, it is not clear that the results are independent of the lag length chosen. It is the purpose of this section to outline a procedure for determining both the order of the lag and the polynomial degree. While there are a number of criteria for determining the appropriate values of these parameters, only two, a modified form of a technique suggested by Pagano and Hartley (1981) and Akaike's (1969) final prediction error (FPE) criterion, are considered here. These criteria were chosen because they have performed well in the past.<sup>11/</sup>

These procedures can be illustrated briefly by referring to the general distributed lag model (2).

One begins by choosing a maximum lag length,  $L$ .

Given  $L$ , equation (2) can be rewritten as

$$(2') \underline{Y}_L = X_L \underline{\beta}_L + \underline{\varepsilon}_L.$$

The P-H technique proceeds by decomposing  $X_L$  to

$$(5) X_L = Q_L R_L$$

by the Gram-Schmidt decomposition. Here  $Q_L$  is a matrix whose columns form an orthonormal basis for the column space of  $X_L$ , and  $R_L$  is an upper triangular matrix with positive diagonal elements.<sup>12/</sup> Substituting (5) into (2') yields

$$(6) \underline{Y}_L = Q_L R_L \underline{\beta}_L + \underline{\varepsilon}_L = Q_L \underline{\lambda}_L + \underline{\varepsilon}_L.$$

Given the orthogonality of  $Q_L$ , the least squares estimates of  $\underline{\lambda}_L$  are

$$\hat{\underline{\lambda}}_L = Q_L' \underline{Y}_L.$$

The P-H technique involves choosing a lag length corresponding to the smallest  $j$  such that the hypothesis

$$H_{L-j}: \alpha_{L-j} = 0, \quad j = 0, 1, \dots, L,$$

is rejected using t-statistics from the orthogonal regression.

The FPE criterion is based on a mean square error prediction norm. It attempts to balance the risk due to bias associated with shorter lags against the risk due to the increase in variance associated with longer lags. The criterion is defined as

$$FPE_{L-j} = \frac{T + (L+1-j)}{T - (L+1-j)} \cdot \frac{RSS_{L-j}}{T}, \quad j=0, 1, \dots, L,$$

where  $RSS_{L-j}$  denotes the residual sum of squares associated with the  $L-j$  lag model. The value of  $L-j$  which minimizes this expression is the appropriate lag by the FPE criterion. Minimizing the FPE is equivalent to applying an approximate sequential F-test with varying significance levels.<sup>13/</sup> The  $RSS_{L-j}$  can be calculated easily from the orthogonal regressions of the P-H technique, so that both the P-H t-statistics and the FPE statistics can be obtained in a computationally efficient manner.<sup>14/</sup>

Finally, these procedures can be extended to the choice of polynomial degree. To see this, note that the choice of polynomial degree amounts to choosing the length of the vector  $\underline{\alpha}$  in equation (4). The procedures outlined above apply directly.<sup>15/</sup>

#### 4. Empirical Results

The above technique was applied to the A-J data. Initially,  $L$  was set equal to 12 for each distributed lag variable.<sup>16/</sup> Both the P-H and the FPE criterion selected a lag length of 2 on  $\Delta M$  and 12 on  $\Delta E$ .<sup>17/</sup> The techniques, however, selected different polynomial degrees. The P-H technique selected a first degree polynomial on  $\Delta M$  and a

fifth degree on  $\Delta E$ , while the FPE criterion selected polynomial degrees of 2 and 6 on  $\Delta M$  and  $\Delta E$ , respectively. Since the P-H specification is nested in the FPE specification, it is possible to test the P-H specification relative to that of the FPE. The calculated chi-square statistic was 6.60, significant at the 5 percent level. Thus, the FPE specification is preferred. Nevertheless, because these specifications are so similar and since neither can be rejected against the OLS model with no polynomial restrictions, estimates of both are presented. Both OLS and PDL estimates of the preferred lag specification are presented in Table 2. The PDL estimates do not employ the endpoint constraints.

The tests of the sums of the distributed lag coefficients in Table 2 show that the important policy conclusions of the A-J equation are invariant to both the lag length and the PDL specification. Indeed, the sums of the distributed lag coefficients on  $\Delta M$  from the first two specifications of Table 2 are close to the A-J results. The sums of the coefficients on  $\Delta E$  are larger than the A-J results, but remain statistically insignificant.

### 5. Tests of the Endpoint Constraints

Because Almon argued that the endpoint constraints should be employed routinely, and because they are necessary to make the original A-J specification estimable, these constraints are tested to determine whether they have any effect on the policy conclusions for our specification of the A-J equation. F-statistics for individual and joint endpoint restrictions, for both PDL specifications, are presented in Table 3.

In general, the endpoint restrictions do not fare well. While the head constraints are never rejected, the head and tail constraints together are always rejected. The imposition of the endpoint constraints, however, has no effect on the policy conclusions.<sup>18/</sup>

### 6. Exogeneity Test Results

The purpose of this section is to test the exogeneity of the right-hand-side variables of the St. Louis equation. These tests are performed in the context of Granger (1969) causality, using the Granger specification of the test.<sup>19/</sup>

In order to illustrate this procedure, consider the bivariate Granger specification

$$\Delta M_t = \alpha_0 + \sum_{j=1}^N \delta_j \Delta M_{t-j} + \sum_{i=1}^K \mu_i \Delta Y_{t-i} + u_t,$$

where  $N$  and  $K$  denote the unknown and unspecified orders of the distributed lags on  $\Delta M$  and  $\Delta Y$ , respectively, and  $u_t$  denotes a white noise residual error.<sup>20/</sup> These parameters are determined using the P-H and FPE criteria. If the lag length  $K$  is determined to be greater than zero, then a standard F-test is applied to test the hypothesis  $\mu_1 = \mu_2 = \dots = \mu_K = 0$ .<sup>21/</sup> If this hypothesis cannot be rejected, then  $M$  is exogenous with respect to  $Y$  (i.e.,  $Y$  does not Granger-cause  $M$ ).

The results of these tests are summarized in Table 4. The numbers in parentheses are the orders of the selected lags.<sup>22/</sup> The results indicate that money is exogenous with respect to both income and high-employment expenditures.<sup>23/</sup> Furthermore, high employment expenditures and income appear to be independent series. These results are consistent with those obtained by Hafer (1982).

## 7. Conclusions

Two important issues that have been the foundation for much of the econometric criticism of the A-J equation have been investigated using the original A-J data. The first is that the results

obtained by Andersen and Jordan may be dependent upon their PDL specification. The second is that the right-hand-side variables may not be statistically exogenous with respect to the left-hand-side variable.

We found no empirical support for either criticism. In particular, the general conclusion that monetary actions have a lasting, significant impact on economic activity while fiscal actions do not appears to be independent of the lag length, the polynomial restrictions or the use of the endpoint constraints. Furthermore, neither of the right-hand-side variables was found to be Granger-caused by the left-hand-side variable. Consequently, at least for the A-J data set, there is no evidence consistent with simultaneous equation bias.

We did find, however, a polynomial distributed lag model considerably different than that estimated by Andersen and Jordan. Thus, if Almon's PDL procedure had been more fully understood, and if the model selection procedures employed here had been available in 1968, Andersen and Jordan probably would have estimated a substantially different equation.

## FOOTNOTES

<sup>1/</sup>For a review of these criticisms, see Batten and Thornton (1983a) and the references cited therein.

<sup>2/</sup>See Trivedi and Pagan (1979) or Judge, et. al., (1980).

<sup>3/</sup>See Batten and Thornton (1983a).

<sup>4/</sup>Andersen-Jordan also considered specifications which included high-employment government receipts, either individually or as a surplus. However, our specification tests did not show this variable to be significant in either form. Andersen and Jordan also considered the monetary base as an alternative monetary policy variable. Unfortunately, the data set that we obtained did not contain their monetary base variable. Andersen and Jordan found significant coefficients on  $\Delta(R-E)$  and Schmidt and Waud (1973) found significant coefficients for  $\Delta R$ , but only after imposing the polynomial constraints.

<sup>5/</sup>We would like to thank Keith Carlson for supplying us with the original A-J data. These data were available only in first difference form and exhibited no heteroscedasticity.

<sup>6/</sup>Actually, they state that only the "tail" constraints were imposed. This cannot be true since imposing only the tail constraints would result in estimates of the distributed lag coefficients of (1)

identical with OLS estimates. In conversation, Keith Carlson indicated that both head and tail constraints were employed. Furthermore, the results with both head and tail constraints imposed match the published A-J results to about the third decimal place.

<sup>7/</sup>Schmidt and Waud (1973) have expressed concern over the use of endpoint constraints, and Seaks and Allen (1980) have tested them. To our knowledge, however, no one has commented on the rather unusual PDL specification of the original A-J equation.

<sup>8/</sup>See Judge, et. al., (1980, p. 641).

<sup>9/</sup>See Noble (1969) pp. 119-21.

<sup>10/</sup>The implied null space restriction vector in  $\underline{\alpha}$  space is (0 6 -11 6 -1), while the head and tail endpoint constraints are (-1 1 -1 1 -1) and (1 4 16 64 256), respectively, for each PDL variable included. The simple correlations between the head and tail constraints and the implied constraint are .77 and -.007, respectively. Thus, it is not surprising that when the tail constraints of the A-J specification are imposed on OLS, results very close to those of the Andersen-Jordan specification were obtained.

<sup>11/</sup>See Batten and Thornton (1983a, 1983b) and Hsiao (1981).

12/ If the diagonal elements are chosen to be positive, then  $Q_L$  and  $R_L$  are unique. See Seber (1977).

13/ See Hsiao (1981).

14/ See Batten and Thornton (1983a, 1983b), especially the appendix to (1983a).

15/ It may not be possible, however, to estimate the parameters of equation (4) directly due to the ill-conditioned nature of  $Z$ . See Pagano and Hartley (1981) and Judge, et. al., (1980).

16/ Since the choice of lag length could be affected by the choice of  $L$ , maximum lags of 8 and 12 were employed. Only the results of  $L=12$  are reported here. A complete set of results can be obtained from the authors upon written request.

17/ Since we obtained a lag on  $\Delta E$  equal to  $L$ , we estimated an equation with  $L=16$  for  $\Delta E$  and  $L=8$  for  $\Delta M$ . The selected lag length was unchanged.

18/ The sums and corresponding t-ratios are 5.07 (7.08) and 5.60 (7.66) for  $\Delta M$  and .63 (1.01) and .26 (0.42) for  $\Delta E$  for the P-H and FPE specifications, respectively. The reader should note that the distributed lag weights for  $\Delta M$  are completely determined by the PDL and endpoint constraints in the case of the first-degree polynomial.

instances of importance when the P-H procedure chose a lag length different from that chosen by the FPE criterion. In each of these cases, however, the subsequent F-test yielded results consistent with those reported in Table 4.

<sup>23/</sup>It is interesting to note that the  $FPE_E(2,1) < FPE_E(2,0)$ , yet the F-test cannot reject the null hypothesis. Thus, had we relied solely on the procedure suggested by Hsiao (1981), we would have concluded that income Granger-causes high-employment expenditures.

19/Recent work by Geweke, Meese and Dent (1983) and Guilkey and Salemi (1982) indicates that the Granger test is preferable.

Furthermore, the lack of Granger causality is a necessary but not sufficient condition for statistical exogeneity. The failure to reject the null hypothesis does not eliminate the possibility of "spurious exogeneity." See Jacobs, Leamer and Ward (1979), and Cooley and LeRoy (1982).

20/Tests of Granger causality require the individual time series to be covariance stationary. The sample autocorrelation functions were calculated for each of the series  $\Delta Y$ ,  $\Delta M$  and  $\Delta E$ . The cross-correlations were not significantly different from zero at lags of 4 for  $\Delta M$  and  $\Delta Y$  and at a lag of 8 for  $\Delta E$ . Nevertheless, a time trend was included in all the regression equations.

21/Hsiao (1981) has shown that if  $FPE_M(N, K) < FPE_M(N, 0)$ , then M Granger-causes Y. Thus, he recommends using the above FPE comparison as an operational method of determining Granger causality. This procedure, however, lacks a mechanism for discriminating between significant and insignificant changes in the FPE and, hence, is not used here.

22/The lags reported in Table 4 were determined by the FPE criteria. There were two

Table 1.  
Estimates of the A-J Equation

Lag	OLS		PDL	
	$\frac{\Delta M}{\Delta E}$	$\frac{\Delta E}{\Delta E}$	$\frac{\Delta M}{\Delta E}$	$\frac{\Delta E}{\Delta E}$
0	2.12* (2.87)	0.38 (1.40)	1.53* (2.49)	0.41 (1.59)
1	0.31 (0.32)	0.52 (1.88)	1.57* (3.48)	0.54* (2.81)
2	2.70* (2.69)	0.02 (0.08)	1.44* (3.21)	-0.03 (0.18)
3	0.67 (0.87)	-0.76* (2.95)	1.28* (2.02)	-0.74* (2.96)
Sum	5.80* (7.34)	0.16 (0.52)	5.82* (7.36)	0.18 (0.56)
Constant	2.31* (2.82)		2.28* (2.78)	
$\bar{R}^2$	.61		.61	
S.E.	3.96		3.97	
D.W.	1.75		1.78	
$F_1^1/$	1.09			
F-head <sup>2/</sup>	3.96*			
F-tail <sup>2/</sup>	.95			

\*indicates significance at the 5 percent level.

<sup>1/</sup> Test of the restricted (PDL) model against the unrestricted (OLS) model.

<sup>2/</sup> Test statistic obtained from imposing the head or tail constraints on OLS.

Table 2.

## OLS and PDL Estimates of the "New" A-J Equation

lag	OLS		PDL <sup>1/</sup>		PDL <sup>2/</sup>	
	$\Delta M$	$\Delta E$	$\Delta M$	$\Delta E$	$\Delta M$	$\Delta E$
0	1.51 (1.97)	0.20 (0.65)	1.63*(2.24)	0.25 (0.90)	0.73 (1.42)	0.52 (1.97)
1	0.50 (0.52)	0.59 (1.72)	0.28 (0.31)	0.54*(2.26)	1.69*(7.08)	0.31 (1.59)
2	3.53*(4.55)	** ( ** )	3.69*(4.99)	-0.11 (0.64)	2.65*(5.27)	-0.12 (0.71)
3		-0.52 (1.59)		-0.53*(3.37)		-0.39*(3.05)
4		-0.78*(2.54)		-0.44*(3.16)		-0.35*(2.64)
5		0.33 (1.08)		-0.04 (0.28)		-0.09 (0.75)
6		0.38 (1.14)		0.35*(2.54)		0.23*(2.12)
7		0.22 (0.64)		0.45*(3.42)		0.41*(3.14)
8		0.35 (1.04)		0.19 (1.26)		0.31*(2.17)
9		-0.47 (1.43)		-0.26 (1.53)		-0.07 (0.54)
10		-0.27 (0.78)		-0.56*(3.36)		-0.52*(3.02)
11		-0.52 (1.52)		-0.30 (1.30)		-0.50*(2.55)
12		0.83*(2.57)		0.71*(2.53)		0.90*(3.20)
Sum	5.54*(7.34)	0.34 (0.56)	5.60*(7.66)	0.25 (0.42)	5.07*(7.08)	0.63 (1.01)
Constant	2.85*(3.17)		2.95*(3.40)		2.95*(3.28)	
$\bar{R}^2$	.66		.68		.65	
S.E.	3.58		3.48		3.62	
D.W.	2.03		2.14		2.12	

1/ Second degree polynomial on  $\Delta M$  and sixth degree on  $\Delta E$ .

2/ First degree polynomial on  $\Delta M$  and a fifth degree on  $\Delta E$ .

\* significant at the 5 percent level

\*\* less than .005 in absolute value

Table 3.

F-statistics for Tests of Endpoint Constraints

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	<u>2nd degree on <math>\Delta M</math>, 6th degree on <math>\Delta E</math></u>		
	<u>Head</u>	<u>Tail</u>	<u>Head and Tail</u>
$\Delta M$	2.54	5.88*	4.85*
$\Delta E$	2.73	1.08	3.88*
$\Delta M$ and $\Delta E$			3.65*

  

	<u>1st degree on <math>\Delta M</math>, 5th degree on <math>\Delta E</math></u>		
	<u>Head</u>	<u>Tail</u>	<u>Head and Tail</u>
$\Delta M$	0.07	15.41*	27.59*
$\Delta E$	0.05	16.31*	10.03*
$\Delta M$ and $\Delta E$			17.06*

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\*significant at the 5 percent level.

Table 4.

Results of Granger Causality Tests

<u>Dependent Variable</u>	<u>Independent Variable</u>	<u>F</u>
$\Delta Y(4)$	$\Delta M(5)$	5.51*
$\Delta Y(4)$	$\Delta E(0)$	--
$\Delta M(2)$	$\Delta Y(0)$	--
$\Delta M(2)$	$\Delta E(0)$	--
$\Delta E(2)$	$\Delta Y(1)$	2.26
$\Delta E(2)$	$\Delta M(0)$	--

\*significant at the 5 percent level.

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Table A3:

Likelihood Ratio Test Results

Unconstrained Lag Specification $\Delta M - \Delta E$	Constrained Lag Specification				
	$5 - 12$	$2 - 12$	$\Delta M - \Delta E$ $5 - 4$	$3 - 3$	$2 - 4$
$12 - 12$					
$5 - 12$	--	5.40 (7.82)	21.94* (15.51)	24.72* (19.68)	23.85* (19.68)
$2 - 12$	--	--	N.N.	N.N.	18.45* (15.51)
$5 - 4$	--	N.N.	--	2.78 (7.82)	N.N.

N.N.-indicates that one specification is not nested in the other.  
 \* -significant at the 5 percent level.