# The Maturity Structure of Inside Money

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**Abstract**

In practice, financial liabilities circulate acting as inside money. Suppose that financial liabilities backed by illiquid assets are used by households as inside money to trade in a sequence of decentralized markets. We study how the liabilities' usefulness in facilitating trade depends on the time-structure and the risk-structure of the underlying illiquid assets. We model trade in decentralized markets as subject to search frictions as in Rocheteau and Wright (2005). If the underlying cash flows are too short term, then liabilities can only support early trade. If the underlying cash flows face too much long-term risk, then the liabilities will not support enough early period nor enough late period trade. The optimal maturity structure trades off the costs of asset liquidation and long-term risk to maximize the usefulness of the liabilities in facilitating trade. We examine how the severity of the search frictions, the cost of early reversals, and the riskiness of the long-term real assets impact optimal maturity.

Preliminary and Incomplete

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# 1 Introduction

Liabilities issued by financial intermediaries provide liquidity services to the broader economy by supporting decentralized exchange: those liabilities act as inside money. Typically inside money has shorter maturity and different risk profiles than the liabilities issued by non-intermediaries that do not act use inside money supporting decentralized exchange. How does the usefulness of a liability used as a form of inside money impact the maturity and risk structure chosen by financial intermediaries? We develop a theory to analyze the interplay between maturity structure, risk structure and the liquidity services provided by inside money. Our theory provides implications for the structure of liabilities backed by productive assets. On the margin, the ability to act as inside money changes the risk-return trade-off on the assets—issuers distort productive efficiency by shortening maturity and reducing risk in order to provide greater liquidity services. We also explore the implications of our theory for unbacked liabilities by studying financial intermediaries' ability to engage in maturity transformation.

We study a finite-horizon economy where heterogeneous households trade in frictional decentralized markets following Rocheteau and Wright (2005). As in Kocherlakota (1998), anonymity of households and inability to enforce private credit arrangements leads to household demand for sources of liquidity. As in Cavalcanti and Wallace (1999), that liquidity comes from inside money. Decentralized trade is facilitated by inside money backed by assets with stochastic cash flows. We allow for the possibility of costly early liquidation that changes the time and risk profile of the risky cash flows.

In our model, liquidation of long-term assets has two critical implications for the stochastic process of the cash flows. First, and quite naturally, liquidation transfers cash flows from future periods into the present. Second, liquidation changes the volatility of long-term asset returns. While we impose this relationship exogenously, such a connection between the timing of payments and the long-term riskiness of payments can emerge endogenously in a similar model which incorporates agency frictions. For example, in Calomiris and Kahn (1991), the ability of a bank manager to abscond with assets yields such a relation.

Implicitly, costly early liquidations resemble a shortening of the maturity of the claims

issued by financial intermediaries. We use our model to examine optimal liquidation and maturity policy. When long-term cash flows are sufficiently high in all histories, inside money is valuable enough to support socially efficient trade in both early periods and late periods. In this case, there is no liquidity premium in pricing of inside money. The early period value of inside money is simply the early period cash flow plus the present value of long-term cash flows. Aggregate welfare is independent of the timing of cash flows as long as the present value of the cash flows is unaffected by liquidation.

Instead, when long-term cash flows are low enough in some histories to lead to inefficient trade in the long-term, the pricing of inside money includes a liquidity premium. With enough long-term risk aggregate welfare can be increased by early liquidations that reduce expected present discounted value of long-term cash flows, that reduce the riskiness of the long-term cash flows, and that shorten the maturity of the cash flows.

The result is driven by balancing two opposing forces. First, shortening the maturity is costly for decentralized trading. Reducing future expected cash flows reduces the usefulness of the inside money as a medium of exchange in future periods. Because the value of inside money is forward looking, the reduction in future value implies that a reduction in future cash flows also makes a claim to future cash flows less useful in facilitating current decentralized trade. Indeed, in a model with no risk, the expected cash flow cost of maturity shortening actually provides incentives for financial intermediaries to lengthen the maturity of their claims.

However, shortening the maturity of the asset may be beneficial for decentralized trade if it reduces the riskiness of returns in future periods. When future cash flows are too low to support socially efficient decentralized trade in future periods in all states, then households become more risk-averse towards future cash flow shocks. A Ramsey planner would like to transfer liquidity from states of excess where cash flows are high enough to support efficient trade to states of scarcity where cash flows are not high enough to support efficient trade. As a consequence, a Ramsey planner may be willing to bear the costs of shortening the maturity of cash flows in order to mitigate long-term risk.

One interpretation of our results is that the usefulness of the inside money in facilitating decentralized exchange leads to distortions to productive efficiency. In this sense, our findings

are related to an existing literature on how search frictions lead to distortions on production margins. For example, Lagos and Rocheteau (2008) show that when the socially efficient level of the capital stock is not a sufficient source of liquidity, agents have private incentives to over-accumulate capital. Such over-accumulation of capital provides a role for the government to use monetary policy to induce agents to accumulate the same capital stock a social planner would (see Aruoba et al. (2011) for a quantitative evaluation of the magnitude of these distortions.)

Gu et al. (2013) show that commitment problems endogenously lead to financial intermediaries to make investment decisions and to have their liabilities act as inside money. We show the impact of liabilities acting as inside money on the choice of the underlying assets. Our results are also related to the large literature that studies the impact of liquidity premia on equilibrium asset returns. Following the seminal work of Kiyotaki and Wright (1989) and more recent contributions such as Rocheteau (2011), Lester et al. (2012), and Nosal and Rocheteau (2013), liquidity premia may arise in environments with exogenous asset specific liquidity constraints, informational asymmetries, or asset liquidation costs. In these environments, scarcity of real assets provides incentives of households to hold non-interest bearing assets to facilitate decentralized trade. Implicitly, in our environment, we assume a scarcity of non-interest bearing assets and examine the implications of the use of productive assets as a media of exchange on the maturity and risk structure of the underlying assets.

## 2 The Model

The model has three periods, time 0, time 1 and time 2. Each of time 1 and time 2 are split into two sub-periods, a decentralized market sub-period followed by a centralized market sub-period. Time 0 features only a centralized market sub-period. There are two types of households in the economy, buyers and sellers.

## 2.1 The Environment

**The Information Structure.** At the beginning of period 1 and at the beginning of period 2 a public signal is observed by all. Let $z_t \in \{z_l, z_h\} \equiv Z$ denote the public signal observed at the start of period $t$. The signal determines the underlying cash flows paid by the underlying assets in the economy with $z^1 \equiv (z_1)$ and $z^2 \equiv (z_1, z_2)$ denoting the history of signals at each date.

**The Assets.** The underlying assets in the economy are modeled as a Lucas (1978) tree with the possibility of a liquidation decision. The tree generates stochastic cash flows of $d_1(z^1, L)$ in the first period and $d_2(z^2, L)$ in the second period. The second argument $L$ denotes the liquidation decision, where $0 \leq L \leq 1$ is fraction of the tree liquidated in the firm period: $L = 0$ means that the tree is not liquidated and $L > 0$ means that a positive fraction of the the tree is liquidated in the first period. The liquidation decision can also affect the conditional probability distribution of the signals:

$$\begin{aligned} \gamma(z) &= \text{Prob}\,(z_1 = z) \\ \gamma(z|z_1, L) &= \text{Prob}\,(z_2 = z|z_1, L)\,, \end{aligned} \tag{1}$$

with $\gamma(z^2, L)$ the unconditional distribution of the signals.

We use $p_0(L)$ to denote the initial price of claims on the tree, $p_1(z^1, L)$ to denote the ex-coupon claim price at time 1 so that $p_1(z^1, L) + d_1(z^1, L)$ is the time-1 cum-coupon claim price. Similarly $p_2(z^2, L)$ is the ex-coupon claim price of at time 2 and $p_2(z^2, L) + d_2(z^2, L)$ is the cum-coupon claim price at time 2.

To start with, we will take the liquidation decision as given. In later sections of the paper we endogenize the liquidation decision. To reduce notation we write

$$\{d_t(z^t), p_t(z^t), \gamma(z|z_1)\} \equiv \{d_t(z^t, L), p_t(z^t, L), \gamma(z|z_1, L)\} \tag{2}$$

when we take the liquidation decision as given.

**Households.** There are two types of households: buyers and sellers. We use the superscripts $b$ to denote buyers and $s$ to denote sellers throughout. Let $q_t$ denote goods produced or consumed in the decentralized sub-periods, let $x_t$ denote consumption of goods in the centralized sub-periods, and let $y_t$ denote production of goods in the centralized sub-periods. There is a measure 1 of buyers with period $t$ preferences:

$$U_t^b\left(q_t, x_t, y_t\right) = u(q_t) + \left[v(x_t) - y_t\right], \tag{3}$$

and there is a measure $n$ of sellers with preferences

$$U_t^s\left(q_t, x_t, y_t\right) = -c(q_t) + \left[v(x_t) - y_t\right]. \tag{4}$$

The buyers' utility for decentralized market consumption is $u$, the sellers' disutility cost for production is $c$, and the utility of consuming the centralized market good is $v$. Buyers and sellers have linear disutility of labor in the centralized market, and $\beta$ is the discount rate between periods.[1]

There are gains to trade in decentralized markets. Buyers enjoy utility of $u(q_t)$ from consuming $q_t$ in the decentralized market while sellers have an ability to produce these goods at utility cost $c(q_t)$and do notenjoy utility from consuming them in the decentralized market. The gains from trade are $u(q_t) - c(q_t)$.

**Trading Frictions.** Buyers and sellers face matching fractions in decentralized markets. Specifically, the same number of buyers and sellers match and trade in equilibrium. Let $\alpha(n)$ denote the probability that a buyer meets a seller and therefore $\alpha(n)/n$ is the probability a seller meets a buyer. [2] When a buyer and a seller meet in a decentralized market, they engage in proportional bargaining to determine the terms of trade. We describe the bargaining process and outcomes in more detail in the next section.

[1] Rocheteau and Wright (2005) allow a discount rate of $\beta_d$ between the centralized and decentralized sub-periods. For simplicity, we abstract from between sub-period discounting in our model.

[2] The matching probability satisfies: $\alpha'(n) > 0, \alpha''(n) < 0, \alpha(n) \leq \min\{1, n\}, \alpha(0) = 0, \alpha'(0) = 1, \alpha(\infty) = 1$.

## 2.2 Equilibrium with Fixed Liquidation

**Recursive Definition of the Household's Problem.** We now describe the recursive formulation of the buyers and sellers problems, begining in the period 2 centralized market. In period 2 the history $z^2$ is fully known so the only relevant state for the household is the total number of claims to the tree they hold. Let $a_2$ denote the number of claims that the household owns so that the value of the claims is $a^2 \times \left(p_2(z^2) + d_2(z^2)\right)$. A household of type $i \in \{b, s\}$ solves

$$W_2^i(a_2; z^2) = \max_{x,y} v(x) - y, \tag{5}$$

subject to

$$x \leq y + a_2(p_2(z^2) + d_2(z^2)).$$

The notation reflects that the equilibrium cum-coupon claim price will depend on the history. Since buyers and sellers are symmetric in the centralized market, the decision problem is the same for both types of households.

When buyers and sellers enter the period 2 decentralized market the history $z^2$ is realized. Let $a_2^i$ denote the claims held by a household of type $i \in \{b, s\}$. In a match between a buyer with claims $a_2^b$ and a seller with claims $a_2^s$, the functions $(q_2, m_2)$ denote the terms of trade, where $q_2$ is the amount produced for the buyer and $m_2$ is the amount of valued claims transferred from the buyer to the seller.

We use a proportional bargaining rule to determine the terms of trade. In proportional bargaining, a buyer's surplus from a match is equal to $\eta/(1-\eta)$ times the seller's surplus, with $\eta \in [0, 1]$. In other words, in a match between a buyer and seller with claims $\left(a_2^b, a_2^s\right)$ and in history $z^2$, the terms of trade $(q_2, m_2)$ solve

$$\max_{q_2, m_2} u(q_2) + \left[W_2^b(a_2^b - m_2; z^2) - W_2^b(a_2^b; z^2)\right], \tag{6}$$

subject to

$$u(q_2) + \left[W_2^b(a_2^b - m_2; z^2) - W_2^b(a_2^b; z^2)\right] = \frac{\eta}{1-\eta}\left[-c(q_2) + \left(W_2^s(a_2^s + m_2; z^2\right) - W_2^s(a_2^s; z^2)\right],$$

and

$$m_2 \leq a_2^b.$$

Let $\Omega_2^i$ denote the period 2 distribution over claims held by households of type $i \in \{b, s\}$ at the start of the period 2 decentralized market. Then, in the period 2 decentralized market sub-period, given claims $a_b$ held by a buyer, the buyer's value function is

$$\begin{aligned} V_2^b(a_2^b; z^2) = \alpha(n) \int_{a_2^s} \left\{u[q_2(a_2^b, a_2^s; z^2)] + W_2^b\left[a_b^2 - m_2(a_2^b, a_2^s; z^2); z^2\right]\right\} d\Omega_2^s(a_2^s) \\ + (1 - \alpha(n)) W_2^b(a_2^b; z^2), \end{aligned} \tag{7}$$

and the seller's value function is

$$\begin{aligned} V_2^s(a_2^s; z^2) = \frac{\alpha(n)}{n} \int_{a_2^b} \left\{-c[q_2(a_2^b, a_2^s; z^2)] + W_2^s\left[a_2^s + m_2(a_2^b, a_2^s; z^2); z^2\right]\right\} d\Omega_2^b(a_2^b) \\ + \left(1 - \frac{\alpha(n)}{n}\right) W_2^s(a_2^s; z^2). \end{aligned} \tag{8}$$

Suppose a household of type $i$ enters the period 1 centralized market in history $z^1$ and holding $a_1$ assets that pay $d_1(z^1)$ at time 1 with price $p_1(z^1)$. The value of the portfolio is $a_1 \times \left[d_1(z^1) + p_1(z^1)\right]$. Each household's value function is

$$W_1^i(a_1; z^1) = \max_{x, y, a_2^i} v(x) - y + \beta \sum_{z_2 \in Z} \gamma(z_2|z^1) V_2^i(a_2^i; (z^1, z_2)), \tag{9}$$

subject to

$$x + a_2^i p(z^1) \leq y + a_1\left[p(z^1) + d_1(z^1)\right].$$

Similar to the period 2 decentralized market, in the period 0 decentralized we need only specify a terms of trade mechanism, $q_1(a_1^b, a_1^s; z^1), m_1(a_1^b, a_1^s; z^1)$ and distributions $\Omega_1^i(a_1^i)$ to fully

determine buyers and sellers values. As in the period 1 decentralized market, the terms of trade are determined by proportional bargaining; the terms of trade $(q_1, m_1)$ are the solution to

$$\max_{q_1, m_1} u(q_1) + \left[ W_1^b(a_1^b - m_1; z^1) - W_1^b(a_1^b; z^1) \right], \tag{10}$$

subject to

$$u(q_1) + \left[ W_1^b(a_1^b - m_1; z^1) - W_1^b(a_1^b; z^1) \right] = \frac{\eta}{1-\eta} \left[ -c(q_1) + \left( W_1^s(a_1^s + m_1; z^1 \right) - W_1^s(a_1^s; z^1) \right],$$

and

$$m_1 \leq a_1^b.$$

The value function for a buyer is

$$\begin{aligned} V_1^b(a_1^b; z^1) = \alpha(n) \int_{a_1^s} & \left\{ u[q_1(a_1^b, a_1^s; z^1)] + W_1^b \left[ a_b^1 - m_1(a_1^b, a_1^s; z^1), z^1 \right] \right\} d\Omega_1^s(a_1^s) \\ & + (1 - \alpha(n)) W_1^b(a_1^b, z^1), \end{aligned} \tag{11}$$

and for a seller is

$$\begin{aligned} V_1^s(a_1^s, z^1) = \frac{\alpha(n)}{n} \int_{a_1^b} & \left\{ -c[q_1(a_1^b, a_1^s; z^1)] + W_1^s \left[ a_1^s + m_1(a_1^b, a_1^s; z^1); z^1 \right] \right\} d\Omega_1^b(a_1^b) \\ & + (1 - \frac{\alpha(n)}{n}) W_1^s(a_1^s; z^1). \end{aligned} \tag{12}$$

The value functions for buyers and sellers in the period 0 centralized market are

$$W_0^i(a_0) = \max_{x, y, a_1} v(x) - y + \beta \sum_{z_1 \in Z} \gamma(z_1) V_1^i(a_1^i; z_1), \tag{13}$$

subject to

$$x + a_1^i p_0 \leq y + a_0^i p_0,$$

where $a_0^i$ is type $i$ households initial endowment of claims.

Given a fixed liquidation rule, we define an equilibrium in the standard fashion.

**Definition 1.** *An* equilibrium *is a list of value functions* $\{(W_t^i)_{i=0,1,2}, (V_t^i)_{i=1,2}\}$*, policy functions* $\{(x_t^i, y_t^i, a_t^i)_{i=0,1,2}\}$*, terms of trade,* $\{(q_t, m_t)_{i=0,1}\}$*, and asset prices* $\{p_0, p_1(z_1), p_2(z^2)\}$ *such that*

1. *Given prices and value functions, the policy functions are optimal;*

2. *Given prices and policy functions, the value functions satisfies Equations (5), (9), (8) and (7);*

3. *The terms of trade are the proportional bargain solutions in Equations(6) and (10);*

4. *Goods and asset markets clear:*

$$\begin{aligned}
&x_0^b + nx_0^s = y_0^b + ny_0^s, \\
&\forall z_1, x_1^b(z_1) + nx_1^s(z_1) = y_1^b(z_1) + ny_1^s(z_1) + d_1(z_1), \\
&\forall z^2, x_2^b(z^2) + nx_2^s(z^2) = y_2^b(z^2) + ny_2^s(z^2) + d_2(z^2), \\
&\forall t, z^t, a_t^b(z^t) + na_t^s(z^t) = 1.
\end{aligned} \tag{14}$$

Appendix A provides a characterization of the equilibrium. As in Lagos and Wright (2005), quasi-linearity of preferences ensures that in any centralized market, a household's optimal choice of claims to purchase is independent of the claims they bring into the centralized market. In equilibrium, the distributions of asset holdings for buyers and sellers are therefore degenerate. Following Rocheteau and Wright (2005), we characterize equilibrium in which in each centralized market the buyers purchase all of the claims to the Lucas Tree and use these claims to facilitate trade in the subsequent decentralized market. As a consequence, buyers marginal decision to hold the assets determine the equilibrium asset price in each period and after every history.

In order to describe the equilibrium, let $d^*$ denote the value of cash flows in period 2 which are sufficient to support efficient trade in decentralized markets when each buyer holds 1 unit of the asset:

$$d^* = (1-\eta)u(q^*) + \eta c(q^*), \tag{15}$$

with $q^*$ the efficient level of production, satisfying

$$u'\left(q^*\right) = c'\left(q^*\right). \tag{16}$$

Equilibrium production in the second period is:

$$q_2^{eq}(z^2) = \begin{cases} \left(q \,\middle|\, (1-\eta)u(q) + \eta c(q) = d_2\left(z^2\right)\right), & \text{if } d_2\left(z^2\right) < d^*, \\ q^* & \text{if } d_2\left(z^2\right) \geq d^*, \end{cases} \tag{17}$$

and equilibrium production in the first period is:

$$q_1^{eq}(z^1) = \begin{cases} \left(q \,\middle|\, (1-\eta)u(q) + \eta c(q) = p_1(z^1) + d_1\left(z^1\right)\right), & \text{if } p_1(z_1) + d_1\left(z^2\right) < d^*, \\ q^* & \text{if } p(z_1) + d_1\left(z^1\right) \geq d^*. \end{cases} \tag{18}$$

Production in the second period is constrained when the second period cash-flow is low enough and efficient when the second period cash flow is high enough. Production in the first period is constrained when the first period cum-coupon price is low enough and efficient when the first period cum-coupon price is high enough.

## 2.3 Period 1 Asset Prices

The final period asset price is $p_2\left(z^2\right) = 0$. The first period ex-coupon asset price following a realization of shock $z_1$ is

$$\begin{aligned} p_1\left(z_1\right) = \beta \sum_{z_2 \in Z} \gamma\left(z_2|z_1\right) d_2\left(z_2, z_1\right) \\ + \beta\alpha(n)\eta \sum_{z_2 \in Z} \gamma\left(z_2|z_1\right) d_2\left(z_2, z_1\right) \frac{u'\left(q_2\left(a_2^b, a_2^s; z^2\right)\right) - c'\left(q_2\left(a_2^b, a_2^s; z^2\right)\right)}{(1-\eta)u'\left(q_2\left(a_2^b, a_2^s; z^2\right)\right) + \eta c'\left(q_2\left(a_2^b, a_2^s; z^2\right)\right)}, \end{aligned} \tag{19}$$

with $a_2^b = 1$ and $a_2^s = 0$. The asset price is the asset's discounted expected coupons plus the discounted liquidity premium. The discounted liquidity premium is strictly positive only when decentralized trade is constrained, which occurs when $d_2\left(a_2^b, a_2^s; z^2\right) < d^*$ so that $q_2\left(a_2^b, a_2^s; z^2\right) <$

$q^*$. Equation (19) is familiar models with no decentralized trade and risk-neutral agents in which the asset price is equal to the discounted expected value of the coupons. Equation (19) is also familiar in monetary models where asset prices reflect not only their coupons but also their usefulness in relaxing trading frictions–see Lagos (2010) for example.

# 3 The Ramsey Problem

Up to this point, we take the liquidation decision as given. We now generalize the stochastic process for cash flows and allow them to depend on a liquidation choice. Let $L \in [0, 1]$ denote the liquidation chosen in period 0 to apply to the cash flow process. We then index the Lucas Tree cash flows in each period as a function of the history $z^t$ and the liquidation choice $L$: $d_t(z^t, L)$ denotes the cash flows generated by the Lucas Tree in period $t$ in history $z^t$ given a choice, $L$. Recall that the state variables $z_t$ are binomial, with state space $\{z_l, z_h\}$ ordered with $z_l < z_h$. Liquidation changes the maturity structure and risk structure of the cash flows. Specifically, liquidation increases time 1 cash flows, decreases time 2 cash flows and also changes the riskiness of discounted cash flows. Below, we provide a specific example of a liquidation technology.

We solve the problem of a Ramsey planner who chooses an optimal liquidation amount taking into consideration the impact that liquidation has on cash flows and equilibrium outcomes. The planner chooses a physical maturity structure and then allocates resources to buyers and sellers in centralized and decentralized markets, subject to the decentralized trading frictions. When choosing trade in decentralized markets, the planner is constrained by the proportional bargaining constraints and the asset prices that would emerge in the competitive equilibrium associated with the given liquidation decision.

Before discussing the effects of the liquidation, we note that the equilibrium outcomes discussed in Section 2 are the same as the outcomes chosen by the planner: Conditional on the liquidation choice, the resulting competitive equilibrium attains the same level of ex ante welfare that the planner can attain.

**Lemma 1.** *For a fixed liquidation, welfare in a competitive equilibrium is the same as that obtained in the Ramsey problem.*

Lemma 1 implies that the the value to the planner in each period after each history is the same as the sum of the values of the individual consumers in the competitive equilibrium. Using the characterization in Appendix A, we characterize the welfare obtained by the Ramsey planner using backward induction. Such a characterization helps illustrate how the liquidation decision, and implicitly the maturity structure, impacts the ex ante value the Ramsey planner obtains. In the last centralized market, the planner's value is

$$W_2^P(z^2, L) = (1+n)\bar{v} + d_2(z^2, L), \tag{20}$$

where the superscript $P$ indicates a value of the planner, and $\bar{v}$ is a constant defined in the Appendix. The planner's value in the last period decentralized market is

$$V_2^P(z^2, L) = \alpha(n)\left[u(q_2^P(z^2, L)) - c\left(q_2^P(z^2, L)\right)\right] + W_2^P(z^2, L), \tag{21}$$

where production in the second period is

$$q_2^P(z^2, L) = \begin{cases} \left(q \,\middle|\, (1-\eta)u(q) + \eta c(q) = d_2\left(z^2, L\right)\right), & d_2\left(z^2, L\right) < d^* \\ q^* & \text{else.} \end{cases} \tag{22}$$

Consider the value function in equation (21). Suppose that $d_2\left(z_1, z_l, L\right) < d^* < d_2\left(z_1, z_h, L\right)$. Since decentralized trade is constrained in the history $(z_1, z_l)$, social surplus is lower in that history than social surplus in the history $(z_1, z_h)$. Moreover, since $W_2^P$ is linear in the realized cash flows, $d_2(z^2, L)$, the decentralized market value function $V_2^P$ is concave only if $d_2(z^2, L) < d^*$. In this case, because claims to the Lucas Tree are used to facilitate decentralized trade and that there are histories where the cash flows are not sufficient to support efficient trade, the Ramsey Planner's value function exhibits additional risk aversion relative to the states in which $d_2(z^2, L) > d^*$.

The planner's value function in the centralized market in period 1 is

$$W_1^P(z^1, L) = d_1(z^1, L) + (1+n)\bar{v} + \beta \sum_{z_2 \in Z} \gamma\left(z_2 | z_1\right) V_2^P(z^2, L), \tag{23}$$

and, similar to period 2, in the decentralized market is

$$V_1^P(z^1, L) = \alpha(n)\left[u(q_1^P(z^1, L)) - c\left(q_1^P(z^1, L)\right)\right] + W_1^P(z^1, L). \tag{24}$$

First period production is

$$q_1^P(z^1, L) = \begin{cases} \left(q \,|\, (1-\eta)u(q) + \eta c(q) = p_1(z^1, L) + d_1(z^1, L)\right), & p_1(z^1, L) + d_1(z^1, L) < d^*, \\ q^* & \text{else,} \end{cases} \tag{25}$$

where $p_1(z^1, L)$ satisfies equation (19) after re-writing the Lucas Tree cash flows to depend explicitly on $L$. If $d_2\left(z_1, z_l, L\right) < d^*$, then trade is constrained in period 2 decentralized markets, implying that the liquidity premium component of the asset price is strictly positive in that history. Any change in the period 2 cash flow in history $z^2$ will then impact the liquidity premium and, therefore, asset prices. Changing the cash flows to increase in the asset price through liquidation may prove useful for the planner to relax constraints on decentralized trade in period 1.

The Ramsey planner chooses the liquidation $L$ to solve

$$\max_L W_0^P(L) = \max_L (1+n)\bar{v} + \beta \sum_{z_1} \gamma\left(z_1\right) V_1^P(z_1, L). \tag{26}$$

# 4 A Liquidation Example

We now further specialize the liquidation technology to examine the Planner's optimal choice of liquidation. First, we assume that the probability distribution of the histories is independent of the planner's liquidation choice. Second we assume that $Prob(z_2 = z_1) = 1$ so that all cash-flow risk is resolved in period 0. Third, we assume that liquidation raises period 1 cash flows

in all histories $z_1$ and lowers period 2 cash flows in all histories $z_1$. Specifically, period 1 cash flows are

$$d_1(z_1, L) = \beta\kappa L + d_1(z_1, 0) \tag{27}$$

for $\kappa > 0$ and $d_1(z_1, 0) \geq 0$, and the second period cash flows are:

$$d_2(z_1, L) = \begin{cases} d_2(z_1, 0) - L\left(d_2(z_1, 0) - \frac{d^*}{\beta}\right), & \text{if } d_2(z_1, 0) > \frac{d^*}{\beta}, \\ d_2(z_1, 0) - Ld_2(z_1, 0), & \text{else,} \end{cases} \tag{28}$$

for $d_2(z_1, 0) \geq 0$. Liquidation always increases period 1 cash flows and reduces period 2 cash flows in both states. If there is enough liquidity to support efficient trade after history $z_1$ in the second period with no liquidation, then cash-flows after the liquidation dividends will still support efficient trade in that state. The smaller $\kappa$ is, the costlier the liquidation is. We assume that liquidation always reduces the present value of the cash flows:

**Assumption 1.**

$$\frac{d}{dL}\mathbb{E}\left[d_1(z_1, L) + \beta d_2(z_1, L)\right] < 0, \ \forall L \in [0, 1]. \tag{29}$$

With our choice of liquidation technology, Assumption 1 holds if

$$\kappa + \sum_{z_1} \gamma(z_1) \frac{d}{dL} d_2(z_1, L) < 0, \ \forall L \in [0, 1]. \tag{30}$$

We also restrict the liquidation decision to be independent of the period 1 history. Liquidation reduces period 2 cash flows at potentially different rates depending on the riskiness of period 2 cash flows.

We also assume that in history $z_1 = z_h$ time 2 cash flows are sufficient to support efficient trade in both periods for all liquidation choices.

**Assumption 2.** $d_2(z_h, 0) > \frac{d^*}{\beta}$

Assumption 2 implies that for all $L$, $d_2(z_h, L) \geq d^*$ and therefore supports efficient decentralized trade in period 2 so that period 1 asset prices have no liquidity premium. Moreover, the assumption implies that for all $L$, $p_1(z_h, L) + d_1(z_h, L) \geq d^*$ and therefore supports efficient

decentralized trade in period 1 also. Indeed, Assumption 2 ensures that there is excess liquidity in history $z_t = z_h$. We maintain Assumptions 1 and 2 throughout.

**Ramsey Optimal Liquidation.** We now describe the Planner's trade-offs in the liquidation decision, and show when liquidation is optimal and when liquidation is not optimal.

Incorporating the liquidation decision and the assumption that $z_2 = z_1$ in each history, the planner's value conditional on time 1 information is:

$$V_1^P(z^1, L) = \alpha(n)\left[u(q_1^P(z^1, L)) - c(q_1^P(z^1, L))\right] + d_1(z^1, L) + (1+n)\bar{v} + \beta V_2^P(z^1, L), \tag{31}$$

where $q_1^P(z_1, L)$ satisfies equation (25). To analyze the effect of changes in $L$, we differentiate $V_1^P(z^1, L)$ with respect to $L$:

$$\begin{aligned}\frac{d}{dL}V_1^P(z^1, L) &= \alpha(n)\left[u'\left(q_1^P(z^1, L)\right) - c'\left(q_1^P(z^1, L)\right)\right]\frac{d}{dL}q_1^P(z^1, L) + \frac{d}{dL}d_1(z^1, L) \\ &\quad + \beta\left[\alpha(n)\left[u'\left(q_2^P(z^1, L)\right) - c'\left(q_2^P(z^1, L)\right)\right]\frac{d}{dL}q_2^P(z^1, L) + \frac{d}{dL}d_2(z^1, L)\right].\end{aligned} \tag{32}$$

If the assets are not used in decentralized trade, then we claim that the marginal impact of liquidation, under Assumption 1 is strictly negative for all $L$. If we interpret $\alpha(n)$ as the probability that the inside money is accepted in decentralized trade, then to say that inside money is not used is to assume that $\alpha(n) = 0$. In this case, equation (32) implies that the effect of liquidation satisfies

$$\forall L \in [0,1], \quad \sum_{z_1} \gamma(z_1)\frac{d}{dL}V_1^P(z^1, L) = \frac{d}{dL}\sum_{z_1}\gamma(z_1)\left[d_1(z_1, L) + \beta d_2(z_1, L)\right] < 0 \tag{33}$$

where the inequality follows from Assumption 1.

**Lemma 2.** *If Assumption 1 holds and $\alpha(n) = 0$, then any liquidation is suboptimal.*

When assets are used in decentralized trade, then whether liquidation is useful or not depends on the extent to which liquidity is plentiful or scarce following different histories. To see this, consider first the impact liquidation has in the history $z_1 = z_h$. When Assumption 2 holds,

it is immediate that $(d/dL)\, q_t^P(z^1, L) = 0$ for $t = 1, 2$ since cash-flows are always sufficient to support efficient decentralized trade. Hence, the impact of liquidation in this history is simply

$$\frac{d}{dL} V_1^P(z_h, L) = \frac{d}{dL} \left[ d_1(z_h, L) + \beta d_2\left(z_h, L\right) \right]. \tag{34}$$

Next, consider the impact liquidation has in the history $z_1 = z_l$. Suppose first that liquidity is plentiful in this history also. In other words, suppose that $d_2(z_l, 0) > \frac{d^*}{\beta}$: there is excess liquidity in both states. Then, as in history $z_1 = z_h$, decentralized terms of trade are independent of $L$ ($(d/dL)\, q_t^P(z_l, L) = 0$ for $t = 1, 2$) and the impact of liquidation in this history satisfies

$$\frac{d}{dL} V_1^P(z_l, L) = \frac{d}{dL} \left[ d_1(z_l, L) + \beta d_2\left(z_l, L\right) \right]. \tag{35}$$

As a result, the overall effect of liquidation again satisfies inequality (33) so that liquidation is suboptimal.

**Lemma 3.** *If Assumptions 1 and 2 hold and $d_2(z_l, 0) > d^*/\beta$, then any liquidation is suboptimal.*

Finally, suppose that liquidity is scarce in the history $z_1 = z_l$. In particular, suppose that $d_2(z_l, 0) < d^*$ so that for all $L$, $d_2(z_l, L) < d^*$. Given that there is excess liquidity in history $z_1 = z_h$, from equation (32) and (34), the expected impact of liquidation satisfies

$$\begin{aligned}
&\frac{d}{dL} \mathbb{E}\left[ d_1(z_1, L) + \beta d_2(z_1, L) \right] \\
&+\beta \gamma(z_l) \alpha(n) \left( \left[ u'(q_2^P(z_l, L)) - c'(q_2^P(z_l, L)) \right] \frac{d}{dL} q_2^P(z_l, L) \right) \\
&+\gamma\left(z_l\right) \alpha(n) \left( \left[ u'(q_1^P(z_l, L)) - c'(q_1^P(z_l, L)) \right] \frac{d}{dL} q_1^P(z_l, L) \right).
\end{aligned} \tag{36}$$

Equation (36) shows that the expected impact of liquidation depends on the size of the direct costs of liquidation on the present value of cash flows on the first line, the indirect impact of liquidation on the second period terms of decentralized trade on the second line, and the indirect impact of liquidation on the first period terms of decentralized trade on the third line.

When $d_2(z_l, L) < d^*$, time 2 cash flows are not sufficient to support efficient decentralized

trade in period 2 so that $u'\left(q_2^P(z^1,L)\right) > c'\left(q_2^P(z^1,L)\right)$. Since liquidation reduces time 2 cash flows, liquidation will affect the time 2 terms of trade:

$$\frac{d}{dL}q_2^P(z_l,L) = \frac{1}{(1-\eta)u'\left(q_2^P(z_l,L)\right)+\eta c'\left(q_2^P(z_l,L)\right)}\frac{d}{dL}d_2(z_l,L). \tag{37}$$

In our liquidation example, when $d_2(z_l,0) < d^*$, $(d/dL)\,d_2(z_l,L) = -d_2(z_l,0) < 0$ so that the indirect impact of liquidation on second period terms of trade is negative, of the same order of magnitude as $d_2(z_l,0)$ and converges to 0 as $d_2(z_l,0)$ tends to 0. Thus, the second line in equation (36) converges to 0 as $d_2(z_l,0)$ converges to 0.

Liquidation may also impact the time 1 terms of trade if at a given $L$, $d_1(z_l,L)+p_1(z_l,L) < d^*$ so that

$$\frac{d}{dL}q_1^P(z_l,L) = \frac{1}{(1-\eta)u'\left(q_1^P(z_l,L)\right)+\eta c'\left(q_1^P(z_l,L)\right)}\frac{d}{dL}\left[d_1(z_l,L)+p_1(z_l,L)\right]. \tag{38}$$

We now show that if $d_1(z_l,0)$ and $d_2(z_l,0)$ are both sufficiently small, then some amount of liquidation is optimal. To see this, suppose that $d_1(z_l,0)=0$. We show that $(d/dL)\,V_1^P(z_l,L)$ is strictly positive near $L=0$ when $d_2(z_l,0)$ is sufficiently small. For $L$ near 0, $d_1(z_l,L)+p_1(z_l,L) < d^*$ so that liquidation impacts the time 1 terms of trade. The impact of liquidation on the planner's value function depends on the impact of liquidation on the cum-coupon price of the Lucas Tree:

$$\begin{aligned}
&\frac{d}{dL}\left[d_1(z_l,L)+p_1(z_l,L)\right] \qquad (39)\\
=\;& \beta\kappa - \beta d_2(z_l,0)\\
&-\beta\alpha(n)\eta d_2\left(z_l,0\right)\frac{u'(q_2^P(z_l,L))-c'\left(q_2^P(z_l,L)\right)}{(1-\eta)u'\left(q_2^P(z_l,L)\right)+\eta c'\left(q_2^P(z_l,L)\right)}\\
&-\beta\alpha(n)\eta(1-L)\left[d_2\left(z_l,0\right)\right]^2\frac{u''(q_2^P(z_l,L))c'(q_2^P(z_l,L))-u'(q_2^P(z_l,L))c''(q_2^P(z_l,L))}{\left[(1-\eta)u'(q_2^P(z_l,L))+\eta c'(q_2^P(z_l,L))\right]^3}.
\end{aligned}$$

Recall from equation (19) that the time 1 ex-coupon asset price is simply the discounted cash flow plus the discounted liquidity premium. Therefore, the impact of liquidation on the cum-coupon price of the asset is the marginal impact on discounted cash flows shown in line 2 of

equation (39) plus the marginal impact on the discounted liquidity premium shown in lines 3 and 4 of equation (39).

To the extent that $d_2(z_l, 0) < \kappa$, the impact on cash flows may be strictly positive and improve time 1 terms of decentralized trade. Since liquidation causes the period 2 cash flows to decrease, liquidation induces a downward movement in the liquidity premium – with fewer cash flows in period 2, the asset supports less decentralized trade in period 2 which makes the asset less valuable in period 1. This downward movement is line 3 of equation (39). On the other hand, since more claims against the Lucas Tree are needed to support efficient decentralized trade in period 2, buyers have stronger incentives to acquire these claims in period 1 which induces an upward movement in the liquidity premium and makes the asset more valuable in period 1. This upward movement is line 4 of equation (39). The net effect on the liquidity premium is negative since the upward price movement on line 4 of equation (39) is of second order.

Nonetheless, the overall impact of liquidation on the cum-coupon price of the asset in time 1 converges to $\kappa$ as $d_2(z_l, 0)$ converges to 0. As a consequence, the indirect impact of liquidation on the first period terms of decentralized trade in equation (36) converges to

$$\alpha(n)\eta\gamma(z_l)\beta\kappa\frac{1}{1-\eta} > 0 \tag{40}$$

as $d_2(z_l, 0)$ converges to 0 and $L$ converges to 0. We have shown that when the direct costs of liquidation are small enough and $d_2(z_l, 0)$ is close enough to 0, then the Ramsey planner would always want some strictly positive amount of liquidation.

**Proposition 1.** *Suppose Assumptions 1 and 2 hold. Then, there exists a $\underline{\kappa}$ such that for all $\kappa \geq \underline{\kappa}$ there exists a threshold, $\bar{d}$ such that for all $d_2(z_l, 0) < \bar{d} < \kappa$, $L = 0$ is suboptimal for the Ramsey Planner.*

The two thresholds in the above Proposition ensure that the direct liquidation costs are small – that is, $\kappa$ is large – and that the indirect liquidation costs are small – that is, $d_2(z_l, 0)$ is small.

Equation (36) also makes clear why risk is essential to our argument. In the absence of risk, say, for example, if $\gamma(z_h) = 0$, then even when $d_2(z_l, 0) < d^*$, liquidation is suboptimal. In this case, even when there is no cost in present discounted value to liquidation, so that $\kappa = d_2(z_l, 0)$, $q^P_{1L}(z_l, 0) < 0$. Note that when $\gamma(z_h) = 0$, $\kappa = d_2(z_l, 0)$ is the largest $\kappa$ such that Assumption 1

is satisfied. Consequently, the planner would never liquidate in this case. In fact, the planner would prefer to move assets into the future if possible. We state this result in the following lemma.

**Lemma 4.** *Suppose $\gamma(z_h) = 0$ and $d_2(z_l, 0) < d^*$. Then the planner prefers not to liquidate, or $L = 0$.*

Lemmas 2 to 4 and Proposition 1 identify necessary conditions for the Ramsey planner to choose to liquidate long-term assets and shorten maturity. We find that two conditions are critical. First, risk is essential. In the absence of risk, even if long-term cash flows are not sufficient to support efficient decentralized trade, the planner prefers to lengthen the maturity and avoid liquidation. When there are liquidity states with excess or shortage of liquidity, the planner is willing to bear the costs of maturity shortening in order to reduce risk in expected cash flows and relax trading constraints in decentralized markets. Second, with risk, the costs of liquidation must not be too large. Interesting, we find two different costs of liquidation. The first is the direct cost of reduced expected discounted cash flows. The second cost is the losses associated with reduced decentralized trade in future periods.

# 5 Implementation with Competitive Intermediaries

In this section we examine liquidation decisions which occur in a decentralized environment with a competitive banking sector. Specifically, we introduce a new set of agents, whom we refer to as bankers, who have the capability of committing to a liquidation strategy, $L$, as the Ramsey planner studied in Section 3 above. In this environment, bankers purchase Lucas trees from households, decide on a liquidation strategy, and then issue claims backed by their Lucas trees. To the extent that a different liquidation, or maturity structure is preferred by households over the default liquidation structure (associated with $L = 0$), these bankers are able to improve upon allocations obtained without bankers.

We begin by augmenting our notion of a competitive equilibrium in Section 2 to include period 0 decisions by households and bankers. Given that bankers can commit to a liquidation strategy, each claim offered by a banker is indexed by the particular liquidation strategy chosen

by the banker who issued the claim. We assume that the liquidation choices are perfectly observably by households. Let $\mathcal{L}$ denote the set of liquidation strategies chosen by bankers. Each household in period 0 then decides how much to work, how much to consume of the general good, and how many of each type of claims to purchase in the period 0 centralized market to solve the following maximization problem.

$$\max_{x,y,a(L)} v(x) - y + \beta \mathbb{E} V_1^i((a(L))_{L\in\mathcal{L}}, z_1), \tag{41}$$

subject to the budget constraint

$$x + \sum_{L\in\mathcal{L}} p_0(L)a(L) \leq y + \hat{p}_0 e_0^i \tag{42}$$

for each $i \in \{b, s\}$ where $e_0^i$ denotes a household of type $i$'s endowment of the Lucas Tree and $\hat{p}_0$ is the period 0 price of a Lucas tree..

The value function, $V_1^i(\cdot, z_1)$ is defined as in Section 2 . Here, terms of trade between a buyer and a seller are a function of the total value of the buyer's assets, which is equal to $\sum_{L\in\mathcal{L}} a_0^b(L)[p_1(L, z_1) + d_1(z_1, L)]$.

In period 0, each bank decides how many Lucas trees to purchase, a liquidation strategy to apply to all trees purchased by the banker, and the number of claims to issue. Each banker solves

$$\max_{b^d, A^s, L} -\hat{p}_0 b^d + p_0(L)A^s + (b^d - A^s)\mathbb{E}\sum_t \beta^t d_t(z^t, L), \tag{43}$$

subject to

$$A^s \leq b^d. \tag{44}$$

In period 0, the banker earns revenues $p_0(L)A^s$ from issuing $A^s$ claims at price $p_0(L)$ and pays the cost of purchasing $b^d$ trees, $\hat{p}_0 b^d$. In future periods, the banker enjoys any promised cash flows not promised to the claim holders and discounts at the same rate as households. The banker is constrained to have positive consumption in future periods which yields the constraint.

A *Competitive Equilibrium with Endogenous Liquidation Choices* is defined analogously to to that with exogenous liquidation choices augmented by the period 0 prices of Lucas Trees as well as bankers' period 0 demand for Lucas Trees and supply and demand for bankers' claims such that bankers' choices are optimal and markets for Lucas Trees and bankers' claims clear.

Before describing the differences between competitive outcomes and constrained efficient outcomes, we first provide a brief characterization of the bankers' optimal choices. Because of the quasi-linearity of households' preferences, in an equilibrium with bounded demand for bankers' claims, for any $L$ the price of the banker's liabilities must be larger than $\mathbb{E}\sum_t \beta^t d_t(z^t, L)$. Since the price of the banker's claim is larger than the discounted value of the dividends associated with the banker's assets, the banker's positive consumption constraint binds so that $A^s = b^d$. Hence, the banker's problem can be simplified to

$$\max_{b^d, L} b^d \left[p_0(L) - \hat{p}_0\right]. \tag{45}$$

The banker will therefore choose the liquidation policy to maximize the period 0 price of her liabilities.

Next, we demonstrate that when the Ramsey planner chooses an interior level of liquidation, the planner's allocation cannot be decentralized as a competitive equilibrium without policy. We prove this result by contradiction. Suppose that the planner's allocation is an equilibrium and let $L^* \in (0, 1)$ denote the planner's optimal liquidation choice. In an equilibrium which decentralizes this outcome, each bank purchases 1 unit of the Lucas tree and issues 1 unit of backed claims.

If such an allocation were an equilibrium, then it must be that $L^*$ maximizes the period 0 asset price. Suppose an individual, deviating banker chooses an arbitrary choice of liquidation $\hat{L} \in [0, 1]$ but continues to purchase 1 unit of period 0 Lucas trees and sell 1 unit of claims. We prove that the banker can increase her asset price above $p_0(L^*)$ and therefore make a strictly greater profits than in the conjectured equilibrium.

Since buyers must be indifferent between holding all assets offered in equilibrium, the de-

viating banker's asset price satisfies

$$p_0(\hat{L}) = \mathbb{E}\beta \frac{d}{da} V_1^b(a_0^b(\hat{L}), z_1) \tag{46}$$

with $a_0^b(\hat{L}) = 1$. so that if some buyer holds 1 unit of the deviating banker's claims, the marginal benefit is equal to the price, $p_0(\hat{L})$. We claim that $L^*$ does not solve $\max_{\hat{L}} p_0(\hat{L})$.

The social planner chooses $L$ to solve

$$\max_L \mathbb{E}\beta V_1^b(L, z_1). \tag{47}$$

As we will see, when liquidity premia are strictly positive, maximizing prices may not be equivalent to maximizing welfare.

We focus on the stylized liquidation example from Section 4. In this case, the deviating banker's asset price is

$$\begin{aligned} p_0(\hat{L}) = {} & \beta\gamma(z_h)[d_1(z_h, L) + \beta d_2(z_h, L)] \\ & + \beta\gamma(z_l)\left[p_1(z_l, L) + d_1(z_l, L)\right]\left[1 + \alpha(n)\eta \frac{u'(q_1) - c'(q_1)}{(1-\eta)u'(q_1) + \eta c'(q_1)}\right]. \end{aligned} \tag{48}$$

Of course, there always exists an equilibrium in which each bank purchases one unit of the Lucas Tree, issues 1 unit of claims, and chooses a liquidation strategy $L^{CE}$ which maximizes the price level given in equation (48)). We summarize these results in the following proposition.

**Proposition 2.** *Suppose the constrained efficient liquidation choice, $L^*$ is strictly interior ($L^* \in (0,1)$). Then, liquidation choices in the decentralized equilibrium is strictly lower, $L^{CE} < L^*$ and is constrained inefficient.*

Proof to be Completed.

Proposition 2 illustrates that there is a role for regulative policy when $\alpha(n) > 0$ and the Ramsey planner's optimal choice of $L$ is strictly interior. In this case, in the absence of policy, bankers choose to issue claims which promise too many cash flows in period 2 and too little cash flows in period 1. Inside money issued by banks in the unregulated competitive equilib-

rium feature too much risk in the sense that the variance of expected discounted cash flows is larger than that which the Ramsey planner would select.

Figure 5 illustrates welfare and the price level $p_0(\hat{L})$ for various values of $d_2(z_l, 0) = \epsilon$ for a numerical example. For $\epsilon$ sufficiently small, the Ramsey planner's optimal choice of liquidation is strictly positive. Indeed, in the example $L^* = 1$. For each value, however, the asset price is maximized at a weakly lower level of liquidation. In particular, when the planner chooses an interior level, $L^* \in (0, 1)$, the asset price is maximized at a strictly lower value of $L$.

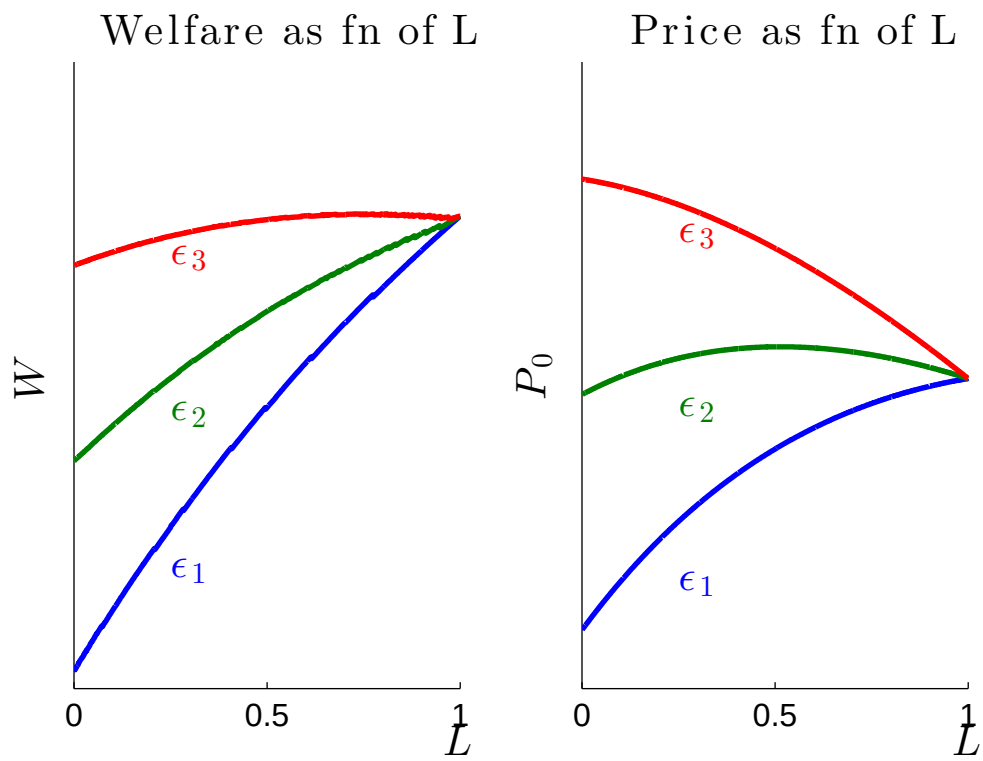


Figure 1: The figure provides numerical examples of the Planner's value function and the Banker's price function, both plotted against amount liquidated. The parameters are: $u(q) = \frac{(q+0.0001)^{1-.2}}{(1-0.2)} - \frac{(0.0001)^{1-.2}}{(1-0.2)}$, $c(q) = q$, $\alpha(n) = n = 0.5$, $\eta = 0.5$, $\gamma(z_l) = \gamma(z_l) = 0.5$, $d_1(z_l, 0) = 0$ $d_2(z_h, 0) = 1.5d^*$ where $d^*$ solves equation (15), and $d_2(z_l, 0) = \epsilon, \epsilon_1 < \epsilon_2 < \epsilon_3$.

These concerns give rise to a role for policy to regulate the maturity decisions of banks. By imposing a liquidation floor, that banks are free to choose $L$ larger than $L^*$, policy can induce banks to select the constrained efficient level of liquidation and, therefore, the constrained efficient maturity structure.

# 6 Conclusions

We develop a theory that links the usefulness of financial intermediaries' liabilities as a medium of exchange to the maturity and risk structure of those liabilities. Shortening the maturity of the liabilities can only increase social surplus if shortening also reduces the riskiness of the long-term cash flows. Our finding provides a novel rationale for why financial intermediaries predominantly issue short maturity liabilities. The difference in maturity structure of financial intermediaries and non-financial firms arises in our model only because liabilities of the financial sector act as inside money. In the our model, liabilities are backed by real assets–there is no maturity mismatch between the assets and liabilities. But even in the absence of roll-over risk, there is a social incentive to shorten maturity and distort productive margins.

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# A Equilibrium Characterization

In this Appendix, we characterize equilibrium outcomes and asset prices with a fixed maturity structure We proceed by backward induction. Clearly, the ex-dividend price of the Lucas tree in the centralized market of period 2 is necessarily zero, or $p_2\left(z^2\right) = 0$. This result implies that the value functions for both buyers and sellers satisfy

$$W_2^i(a, z^2) = ad_2\left(z^2\right) + \bar{v}, \tag{1}$$

where $\bar{v} \equiv \max_x v(x) - x$.

In the decentralized market in period 2, in any match between a buyer and seller, the terms of trade, $q_2(a_2^b, a_2^s, z^2), m_2(a_2^b, a_2^s, z^2)$ are chosen to solve the proportional bargaining problem. Using the form of the value function in equation (1), note that for either a buyer or a seller, and for any amount of shares exchanged, $m$, the net continuation surplus for the consumer is

$$W_2^i(a+m, z^2) - W_2^i(a, z^2) = (a+m)\, d_2\left(z^2\right) + \bar{v} - ad_2\left(z^2\right) - \bar{v} = md_2\left(z^2\right). \tag{2}$$

Requiring buyers to receive total surplus equal to a fraction of the surplus of the seller then is equivalent to requiring that

$$u(q_2) - m_2 d_2(z^2) = \frac{\eta}{1-\eta}\left[-c(q_2) + m_2 d_2\left(z^2\right)\right], \tag{3}$$

or

$$(1-\eta)u(q_2) + \eta c(q_2) = m_2 d_2\left(z^2\right). \tag{4}$$

Hence, for a given amount of production $q_2$, the number of claims to the Lucas tree that must be transferred from the buyer to the seller is

$$m_2 = \frac{(1-\eta)u(q_2) + \eta c(q_2)}{d_2\left(z^2\right)}, \tag{5}$$

Substituting this amount of claims exchanged into the surplus of the buyer, the production

choice is

$$\max_{q_2} \eta \left[ u(q_2) - c(q_2) \right], \tag{6}$$

subject to

$$(1-\eta)u(q_2) + \eta c(q_2) \leq d_2\left(z^2\right) a_2^b. \tag{7}$$

Importantly, $q_2$ and, therefore, $m_2$ is determined independently of $a_2^s$. So the seller's asset holdings have no impact on the terms of trade and we write

$$q_2(a_2^b, a_2^s, z^2) = q_2\left(a_2^b, z^2\right), \text{ and } m_2\left(a_2^b, a_2^s, z^2\right) = m_2(a_2^b, z^2). \tag{8}$$

.

We now determine $q_2$. Recall that $q^*$ satisfies $u'(q^*) = c'(q^*)$. In a match between a buyer and a seller where the buyer has assets $a_2^b$ such that

$$a_2^b \geq \frac{1}{d_2\left(z^2\right)} \left[ (1-\eta)u(q^*) + \eta c(q^*) \right], \tag{9}$$

then $q_2(a_2^b, z^2) = q^*$. Otherwise, the constraint in equation (7) binds so that $q_2$ is determined by equation (7) holding with equality.

The value functions $V_2^b$ and $V_2^s$ therefore are

$$\begin{aligned} V_2^b(a_2^b, z^2) &= \alpha(n)\left[u\left(q_2\left(a_2^b, z^2\right)\right) + W_2^b(a_2^b - m_2\left(a_2^b, z^2\right), z^2\right)\right] + (1-\alpha(n))W_2^b(a_2^b, z^2) \\ &= \alpha\left(n\right)\left[u\left(q_2\left(a_2^b, z^2\right)\right) + W_2^b(a_2^b - m_2\left(a_2^b, z^2\right), z^2\right) - W_2^b(a_2^b, z^2)\right] \\ &\qquad + W_2^b(a_2^b, z^2) \\ &= \alpha(n)\eta\left[u(q_2\left(a_2^b, z^2\right)) - c(q_2\left(a_2^b, z^2\right))\right] + \left[a_2^b d_2\left(z^2\right) + \bar{v}\right], \end{aligned} \tag{10}$$

and

$$V_2^s(a_2^s, z^2) = \frac{\alpha(n)}{n} \int_{a_2^b} \left[ -c\left(q_2\left(a_2^b, z^2\right)\right) + W_2^s(a_2^s + m_2\left(a_2^b, z^2\right), z^2\right) - W_2^s(a_2^s, z^2) \right] d\Omega_2^b(a_2^b)$$
$$+ W_2^s(a_2^s, z^2)$$
$$= \frac{\alpha(n)}{n}(1-\eta) \int_{a_2^b} \left[ u(q_2\left(a_2^b, z^2\right)) - c(q_2\left(a_2^b, z^2\right)) \right] d\Omega_2^b(a_2^b) + \left[ a_2^s d_2\left(z^2\right) + \bar{v} \right]. \quad (11)$$

We determine the value functions and asset price in the period 1 centralized market.

Given the quasi-linearity of preferences in the centralized market, the problem of choosing asset holdings to carry into period 2 is independent of the number and value of the claims the consumer brings into the centralized market. The value function for either type of consumer is

$$W_1^i(a, z_1) = (p_1(z_1) + d_1(z_1))\, a + \bar{v} + \max_{a'} -p_1(z_1)a' + \beta \sum_{z_2} \gamma(z_2|z_1) V_2^i(a', (z_1, z_2)). \quad (12)$$

By construction, the seller's value function $V_2^s$ is linear in $a'$ implying that the seller's optimal choice of $a'$ is bounded only if

$$p_1(z_1) \geq \beta \sum_{z_2} \gamma(z_2|z_1) d_2(z_1, z_2). \quad (13)$$

Inequality (13) holds in equilibrium with strict inequality so that all sellers choose $a_2^s = 0$ for all $z_1$. Next consider the optimal choice of $a'$ for a buyer. Assuming an interior solution, the optimal choice for a buyer satisfies:

$$p_1(z_1) = \beta \sum_{z_2} \gamma\left(z_2|z_1\right) d_2\left(z_1, z_2\right)$$
$$+ \beta\alpha(n)\eta \sum_{z_2} \gamma\left(z_2|z_1\right) \left[ u'(q_2(a', (z_1, z_2)) - c'(q_2(a', (z_1, z_2))) \right] \frac{dq_2(a', (z_1, z_2))}{da'} \quad (14)$$

where

$$\frac{dq_2(a', z^2)}{da'} = \frac{d_2\left(z^2\right)}{(1-\eta)u'(q_2(a', d_2\left(z^2\right))) + \eta c'(q_2(a', d_2\left(z^2\right)))}. \quad (15)$$

Under conditions on preferences and bargaining weights, $V_2^b(a_2^b, z^2)$ is strictly concave for

$a_2^b \leq a^*$ where $a^*$ satisfies inequality (9) with equality. This ensures a unique optimal choice of $a'$ for buyers so that $\Omega_2^b(a_2^b)$ is degenerate. We focus on equilibrium in which $a_2^b = 1$ implying that the asset price is

$$p_1(z_1) = \beta \sum_{z_2} \gamma\left(z_2|z_1\right) d_2\left(z_1, z_2\right) \left[1 + \alpha(n)\eta \frac{u'(q_2(1,(z_1,z_2)) - c'(q_2(1,(z_1,z_2))}{(1-\eta)u'(q_2(1,(z_1,z_2))) + \eta c'(q_2(1,(z_1,z_2)))}\right]. \quad (16)$$

We proceed iteratively to determine the period 1 decentralized market value functions as well as the period 0 centralized market value functions and the asset price $p_0$. It is straightforward to show that the terms of trade are independent of the seller's holdings of claims and satisfy

$$q_1(a_1^b, z_1) = \begin{cases} q^* & \text{if } a_1^b \geq a_1^* = \left[(1-\eta)u(q_1) + \eta c(q_1)\right] / \left(p_1\left(z_1\right) + d_1(z_1)\right) \\ \hat{q}(a_1^b, z_1) & \text{otherwise} \end{cases} \quad (17)$$

where $\hat{q}(a_1^b, z_1)$ is the value of $q$ that satisfies

$$(1-\eta)u(q) + \eta c(q) = \left(p_1\left(z_1\right) + d_1(z_1)\right) a_1^b. \quad (18)$$

Moreover, $m_1(a_1^b, z_1)$ is

$$m_1(a_1^b, z_1) = \frac{(1-\eta)u(q_1\left(a_1^b, z_1\right)) + \eta c(q_1\left(a_1^b, z_1\right))}{(p_1(z_1) + d_1(z_1))}. \quad (19)$$

These terms of trade imply the value functions for buyers and sellers in the period 1 decentralized market are:

$$V_1^b(a_1^b, z_1) = \alpha(n)\eta \left[u(q_1\left(a_1^b, z_1\right)) - c(q_1\left(a_1^b, z_1\right)\right] + W_1^b(a_1^b, z_1), \quad (20)$$

$$V_1^s(a_1^s, z_1) = \frac{\alpha(n)}{n}(1-\eta) \int_{a_1^b} \left[u(q_1\left(a_1^b, z_1\right)) - c(q_1\left(a_1^b, z_1\right)\right] d\Omega_1^b(a_1^b) + W_1^s(a_1^s, z_1). \quad (21)$$

Buyers and sellers problems in the period 0 Centralized Market are

$$W_0^i(a) = p_0 a + \bar{v} + \max_{a'} -p_0 a' + \beta \sum_{z_1} \gamma(s_1) V_1^i(a', z_1). \tag{22}$$

To determine the period 0 asset price, note that the seller's demand for the asset is finite, when

$$p_0 \geq \beta \sum_{s_1} (p_1(z_1) + d_1(z_1)) \tag{23}$$

and at an interior solution for the buyer, we require that

$$\begin{aligned} p_0 = \beta \sum_{s_1} \gamma(s_1) (p_1(z_1) + d_1(z_1)) \\ + \beta \alpha(n) \eta \sum_{s_1} \gamma(s_1) \left[ u'(q_1(a', z_1)) - c'(q_1(a', z_1)) \right] \frac{dq_1(a', z_1)}{da'}, \end{aligned} \tag{24}$$

where for $a' \leq a_1^*$,

$$\frac{dq_1(a', z_1)}{da'} = \frac{(p_1(z_1) + d_1(z_1))}{(1-\eta) u'(q_1(a', z_1)) + \eta c'(q_1(a', z_1)}. \tag{25}$$