

Money and Collateral*

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Abstract

This paper presents a model in which money and collateral are both essential and complement each other as media of exchange. The model has implications for asset prices, output, inflation and monetary policy, both in steady state and along dynamic paths of equilibria.

Keywords: Money, Credit, Collateral, Essentiality

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1 Introduction

The 2008 financial crisis has brought to the fore the role of liquidity, collateral and asset prices for the functioning of the financial system. Whether one subscribes to the account of the events that places center stage the burst of the housing bubble,¹ or the alternative story of the panic-induced run on the repo market,² in any case, liquidity, secured credit and asset price expectations, all appear to have been key elements of the crisis. For an exact understanding of their respective roles not only in turbulent but also normal times, however, a model would be needed in which money, credit and real assets are all *fundamental* features of the exchange process. Unfortunately, such

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¹See Joseph Stiglitz (2009).

²See Gary Gorton (2010) and Robert Lucas and Nancy Stokey (2011).

a model is *not* available. The present paper fills the gap. It will show how collateral and liquidity may complement each other as means to allocate resources among economic agents, in an environment where asset, credit and commodity markets are frictional. The joint, intertwined use of money and collateral will emerge as the *best* trading arrangement among those feasible given the imperfections of the environment, with definite implications for the behavior of output, asset prices and the conduct of monetary policy.³

As a preliminary step, we need to abandon the Arrow-Debreu (AD) frictionless market world, in which there is no need for any trading instrument to transfer resources. The literature has identified two main types of departures from AD: search frictions, capturing uncertainty over the ability to achieve the desired trading outcomes, and informational/commitment frictions, capturing impediments in the ability to enforce intertemporal credit arrangements. Since the seminal work of Nobuhiro Kiyotaki and Randall Wright (1989), these frictions have been explicitly considered in modeling commodity and liquidity markets. More recently, following Darrell Duffie, Nicolae Garleanu and Lasse Pedersen (2005), even asset markets – once considered the ultimate shrine of frictionless trade - have been modeled as frictional. However, even granting that markets are frictional in the aforementioned sense, it is far from obvious whether we get any closer to the explanation why credit and liquidity may both be used to lubricate the functioning of frictional markets. Indeed, in a recent paper, Chao Gu, Fabrizio Mattesini and Wright (2015) have shown that in equilibria where money is valued, credit is inessential – i.e. its use does not improve matters for the agents, and changes in credit conditions are neutral. This occurs in a large variety of environments, where money and credit are competing means of payment, including some in which credit is secured by collateral. Further difficulties are raised by

³A good reason for insisting on the best arrangement, where all assets play a fundamental role, is that, otherwise, the freezing of one of the asset markets, often observed during crises, could be interpreted as irrelevant or even a symptom of improving business conditions.

the presence of multiple assets, with different intrinsic return, since their coexistence seems to fly in the face of the basic principle of arbitrage.

We present a model, based on the Ricardo Lagos and Wright (2005) framework, that features two assets, namely, money, without intrinsic value, and a Lucas tree, with intrinsic value, both of which are held for precautionary reasons, and both of which may turn out to be misallocated after the realization of uncertainty, with the same agents who are in a position to use money for transaction purposes being also the best users of the Lucas tree. Since the agents' human capital needed to generate the returns of the asset are assumed to be non-contractible, contracts contingent on the returns cannot be written, as in Oliver Hart and John Moore (1990). In this context, the problem is to find the best way to convey all the assets into the hands of their best users, given the limitations in the enforcement of contracts and the complete anonymity of the agents. In the absence of well functioning credit markets, the best trading arrangement involves the use of money to acquire the Lucas tree and the use of the latter as collateral to obtain loans of money, which is, in turn, finally used to acquire consumption. In sum, money buys assets, assets borrow money and money buys goods. First, we show that such an arrangement constitutes an equilibrium and characterize it. Second, we consider the feasible alternatives and show that they are socially inferior, leading to a worse allocation for the agents. The intertwined exchange of assets, used in a complementary way, leaves neither money nor the Lucas tree idle, in the hands of an agent who is not its best user. Any other arrangement falls short of this, leaving some asset in the wrong hands.

Hence, the paper shows how money and collateralized credit may both be essential in facilitating the process of exchange, i.e. allow agents to achieve better allocations. The question of the essentiality of money goes back to Frank Hahn (1973) and his criticism of the imposition of a cash in advance constraint on top of an otherwise frictionless general equilibrium model, in which the use of money as a medium of exchange ends up hurting rather than helping traders. Narayana Kocherlakota (1998)

has shown that limitations in the ability of agents to commit themselves to future actions and keep a record of the transactions – the two assumptions being sometimes bundled together under the label "anonymity"- are necessary to generate an essential role for money. The question of the essentiality of multiple trading instruments is still largely open. Gu et al. (2014) have argued that money and credit are (almost) never simultaneously essential. However, they consider only situations in which the two assets are substitute, rather than complementary, means of exchange. Luis Araujo and Braz Camargo (2012) have shown that there is a fundamental tension between money and monitoring-based credit. Our point of view is that the tension between money and collateral-based credit is less acute, since the latter requires only the - less informationally demanding- threat of the loss of collateral to induce debtors to honor their promises. The literature has explored models where both money and credit are used, e.g. Aleksander Berentsen, Gabriele Camera and Chris Waller (2007), and, more specifically, money and collateralized credit, e.g. Shouyong Shi (1996), Leo Ferraris (2010), Ferraris and Makoto Watanabe (2008) among others, but the simultaneous use of the two instruments was assumed rather than derived, and consequently, the question of the coexistence and essentiality of both instruments was not addressed. In fact, even the relatively simpler question why an asset that can serve as collateral does not circulate as a medium of exchange in the first place, has largely been sidestepped.⁴

The paper provides a characterization of both static and dynamic equilibria. The economy behaves in two rather different ways, depending on the availability of the real asset. When the asset is abundant, output and asset prices do not interact and are entirely determined by fundamentals. Economic fluctuations can only be driven by exogenous shocks to fundamentals, as real business cycle theory would predict, and the allocation is efficient unless distorted by monetary intervention. When the asset

⁴Related papers also featuring real assets and money as media of exchange, which do not address coexistence, include Athanasios Geromichalos, Juan Licari and Jose Suarez Lledo (2007) and Lagos and Guillaume Rocheteau (2008).

is scarce, output and asset prices do interact and may be affected by non-fundamental uncertainty, which may lead to fluctuations driven by self-fulfilling expectations, as endogenous business cycle theory would suggest, and asset prices display features reminiscent of Tobin's q (see James Tobin (1969)). The complementarity of money and other assets may therefore matter for the emergence of self-fulfilling economic instability. A related body of literature, inspired by the seminal work of Kiyotaki and Moore (1997),⁵ has addressed the question how asset price fluctuations may amplify economic instability in environments in which money does not play a role or is not essential. Here, instead, instability is endogenously generated, through the self-fulfilling prophecies of the sunspot literature à la David Cass and Karl Shell (1983) and Costas Azariadis (1981). The emergence of sunspot equilibria in models with infinitely lived agents in which financial transactions are restricted has been shown by Michael Woodford (1986). The potentially cyclical behavior of equilibrium in search models has been pointed out by Peter Diamond and Drew Fudenberg (1987), Lagos and Wright (2003) and Ferraris and Watanabe (2011) among others. The novelty, here, consists in the role that the fundamentals and also, notably, monetary policy play as preconditions for the emergence of cycles and sunspots. As regards optimal monetary policy, it involves a zero nominal interest rate as required by the Friedman rule (see Milton Friedman (1969)), but corresponds to no-intervention, which, in some cases, can even achieve the first-best, unlike most of the monetary microfoundation literature, where typically a contraction of the money stock at the rate of time preference is required for optimality.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 examines the equilibrium with money and collateralized credit and contrasts it to the alternative arrangements. Section 4 discusses the main assumptions and concludes. The derivation of the equilibrium conditions and the proofs are in the Appendix.

⁵For instance, Kiyotaki and Mark Gertler (2010) and Vincenzo Quadrini (2011).

2 The Model

Fundamentals The model builds on a version of Lagos and Wright (2005) with competitive markets. Time is discrete and continues forever. Each period is divided into two sub-periods, day and night, in which two goods are produced, traded and consumed by a continuum of mass one of infinitely-lived agents. During the day, agents can trade a perishable consumption good, x , and face randomness in their preferences and production possibilities. With equal probability, an agent may turn out to be in a position to consume but unable to produce, i.e. a buyer, or viceversa, a seller. Consumption yields utility $u(\cdot)$, with $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Production entails a utility cost $c(\cdot)$, with $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$. Usual Inada conditions are also assumed. During the night, agents can produce, trade and consume another perishable good, X , which serves as the numeraire of the economy. In contrast to the first sub-period, there is no randomness in the second sub-period. Agents can consume and produce the night good with linear utility and linear cost of effort. Agents discount future payoffs at a rate $\beta \in (0, 1)$ across periods. For simplicity, there is no discounting between sub-periods.

Exchange of Goods Exchange during the day is anonymous and happens at a competitive price p in units of the good traded at night. The market for the night good is walrasian with price normalized to unity.

Lucas Tree Every period, agents can exchange an asset, a , available in fixed supply A , which yields $R > 0$ units of consumption of the night good per unit of asset during the following period if the agent is a buyer or zero if a seller. The return to a buyer is generated only if the asset remains in the hands of the agent for the entire period. The return cannot be contracted upon and constitutes a private benefit of the owner of the asset. The asset can be traded in two competitive walrasian markets, one open during the day, after the resolution of uncertainty, at a price q , and one at night at

a price ψ . Trade of the asset during the day is subject to anonymity. The asset can be pledged as collateral to obtain credit during the day up to the night value of the asset. The asset can also, potentially, be used as a medium of exchange.

Money An intrinsically worthless, perfectly divisible and storable object called fiat money is available in the economy. Money can be used to trade goods but can also be lent out or borrowed in a competitive market during the day, after the resolution of uncertainty, at a nominal interest rate $i \geq 0$. Due to the agents complete anonymity, loans, l , need to be collateralized with the Lucas tree. A debtor agrees to repay the amount of money borrowed with interest during the night of the same period. Should the debtor fail to repay, the creditors have the right to seize the asset pledged as collateral. Money can always be hidden away, hence, cannot be used as collateral. The value of money in terms of the night good is denoted by ϕ .

Government There is a Government that injects or withdraws money using lump-sum transfers or taxes, τ , distributed to or collected from all agents equally at night. Due to the anonymity of the agents, who can always hide their money holdings, the Government is unable to raise lump-sum taxes. We will assume that asset holdings in general cannot be taxed, hence, $\tau \geq 0$. The total supply of fiat money, M , grows at a constant gross rate γ over time, hence, the evolution of the stock of money is governed by $M_{+1} = \gamma M$. Since the Government needs to satisfy its budget constraint, $\phi M_{+1} = \phi M + \tau$, we have $\tau = \phi M (\gamma - 1) \geq 0$, which implies $\gamma \geq 1$, whenever money has value.

First-best The first-best amount of the day good, x^* , solves $u'(x) = c'(x)$, which equates the marginal benefit of day-time consumption to the marginal cost of production. The first-best allocation for the night good only involves the feasibility condition, due to the linearity of the objective functions. Efficiency requires Lucas trees to be assigned always to buyers, being their best users.

3 Coexistence of Money and Collateral

Money and collateral genuinely coexist not only if there exists an equilibrium in which they are both used as trading instruments, but also if they are both essential for the functioning of the exchange process. A trading instrument is essential for the functioning of the exchange process if the allocation cannot be improved - in terms of the agents' welfare- avoiding its use. Our aim is to prove that the combined use of money and collateral is essential. We proceed as follows. First, we guess a trading arrangement that uses money and collateral in a complementary way and we show that it can be sustained as an equilibrium. Then, we consider the alternative arrangements that are feasible given the imperfections of the environment, and we show that either they cannot be sustained as equilibria or are never superior to the arrangement with money and collateral. The comparison of the amount of day-time consumption and the allocation of the real asset is enough to establish which system is better for the agents.

3.1 Monetary Trade with Collateralized Credit

We construct a symmetric equilibrium with valued money and collateralized credit. The sequence of trades within a period is as follows. During the day, after the realization of uncertainty, first, the buyers acquire the asset from the sellers in a competitive and anonymous market, spending the cash they brought from the previous period. Second, the buyers borrow money from the sellers in a competitive and anonymous market place, using the assets just acquired and those brought from the previous period as collateral. Third, the buyers spend all the money they hold at that point in time to purchase the consumption good in a competitive and anonymous market. No other trade is accepted by any of the agents. During the night, all agents consume and produce the other good, settle their debts and acquire new assets for the following period. The returns of the asset are privately generated by the buyers at this stage.

Individual Behavior We first describe the decision problem of individual agents taking the terms of trade as given, starting with the decisions taken during the day, after the realization of uncertainty, and, then, moving to the decisions taken during the night. The derivations of the optimality conditions can be found in the Appendix.

Day-time. We consider, first, the decision problem of a buyer, then, of a seller. A buyer chooses consumption x^b , asset holdings α^b , loans l^b , to solve

$$V^b(m, a) = \text{Max } u(x^b) + W^b(\tilde{m}^b, \tilde{a}^b),$$

where $W^b(\tilde{m}^b, \tilde{a}^b)$ denotes the value of operating in the night market with holdings \tilde{m}^b for money and \tilde{a}^b for the asset, to be specified below. The constraints, whose non-negative multipliers appear in square brackets, are

$$q\alpha^b \leq \phi m, \quad [\mu] \tag{1}$$

which reflects the purchase of the asset with cash, limited by its initial amount;

$$\phi l^b (1 + i) \leq \psi (a + \alpha^b), \quad [\lambda] \tag{2}$$

which reflects the loan of cash, including the interest payment to be made at night, obtained against the total value of the asset, comprising both the part owned from the previous period and the part just purchased, used as collateral to secure repayment;

$$px^b \leq \phi l^b + \phi m - q\alpha^b, \quad [\delta] \tag{3}$$

which reflects the purchase of the consumption good with the cash just borrowed plus the amount unspent in the asset transaction. Hence, given these transactions, the asset holdings for a buyer at night will be

$$\tilde{m}^b = m - \frac{q}{\phi}\alpha^b - \frac{px^b}{\phi} - [l^b(1+i) - l^b],$$

for money, given by the initial amount net of the amount spent on the asset and consumption, and the interest payment made at night; and

$$\tilde{a}^b = a + \alpha^b,$$

for the asset, given by the initial amount and the amount acquired in the asset transaction. A seller chooses an amount of the good x^s , of the asset α^s and loans l^s , to solve

$$V^s(m, a) = \text{Max} - c(x^s) + W^s(\tilde{m}^s, \tilde{a}^s),$$

where $W^s(\tilde{m}^s, \tilde{a}^s)$ denotes the value of operating in the night market with holdings \tilde{m}^s of money and \tilde{a}^s of the asset, to be specified below. Optimization is subject to two constraints, with their non-negative multipliers in square brackets,

$$q\alpha^s \leq qa, \quad [\zeta] \tag{4}$$

which reflects the sale of the asset, limited by its initial amount;

$$\phi l^s \leq \phi m + q\alpha^s, \quad [\theta] \tag{5}$$

which reflects the monetary loan extended in the current sub-period, limited by the initial cash holdings plus those acquired in the asset transaction. Hence, the asset holdings for a seller at night will be

$$\tilde{m}^s = m + \frac{q}{\phi}\alpha^s + \frac{px^s}{\phi} + [l^s(1+i) - l^s],$$

for money, given by the initial amount, the amount acquired selling the asset and the good, and the interest payment received at night; and

$$\tilde{a}^s = a - \alpha^s,$$

for the asset, where the initial amount is reduced by the amount sold in the asset transaction. For an agent, the expected value of entering any given period, before the realization of uncertainty, is given by

$$V(m, a) = \frac{1}{2}V^b(m, a) + \frac{1}{2}V^s(m, a),$$

since the day begins with assets holdings m and a , and there is equal probability of being of either type.

Night-time. For $j = b, s$, let $R(j)$ be defined as $R(b) = R$, $R(s) = 0$. At the beginning of the night, an agent who was of type $j = b, s$ during the day faces the choice over consumption X^j , effort e^j and money and asset holdings for the future, m_{+1} and a_{+1} , to solve the following problem

$$W^j(\tilde{m}^j, \tilde{a}^j) = \text{Max } X^j - e^j + R(j)\tilde{a}^j + \beta V(m_{+1}, a_{+1}),$$

where $V(m_{+1}, a_{+1})$ denotes the expected value of operating in the following day market with money holdings m_{+1} and asset holdings a_{+1} . The maximization is subject to the budget constraint,

$$X^j + \phi m_{+1} + \psi a_{+1} = e^j + \phi \tilde{m}^j + \psi \tilde{a}^j + \tau,$$

which states that the effort, the real value of current asset holdings and Government transfers can be used to acquire night-time consumption and assets for the future. Substituting from the constraint for $X^j - e^j$ into the objective function, the problem can be reduced to the choice of money and asset holdings for the following period. We have incorporated the idea, which is standard in the Lagos and Wright (2005) framework, that these decisions are the same for all the agents. This is due to the linearity of the night-time payoff, which allows to separate the decisions about future asset holdings from current holdings.

Optimization. The agents' optimization requires that the first order conditions and the complementary slackness conditions for the constraints, stated in the Appendix, hold simultaneously. These conditions give the optimal demand and supply for all the items traded in all the markets, taking prices as given. The prices are, then, determined by the market clearing conditions, which are stated next.

Market Clearing Market clearing for the day-time good requires $x^b = x^s \equiv x$, for the asset during the day $\alpha^b = \alpha^s \equiv \alpha$, for the loans $l^b = l^s \equiv l$, for the asset at night $a = A$, and for money $m = M$. Since the night market for good X clears whenever the other markets do by Walras Law, we omit its market clearing condition.

Money and Collateral Equilibrium In this section, we describe the equilibrium conditions, whose detailed derivation can be found in the Appendix. In order to ensure that both seller and buyers trade the asset during the day, the day-time price of the asset will have to reflect exactly the discounted price at night,

$$q = \frac{\psi}{1+i}. \quad (6)$$

Next, the equilibrium system has two intertemporal optimality conditions governing the accumulation of the two assets. First, the Euler equation for money holdings

$$1 = \beta \frac{\phi_{+1}}{\phi} (1+i_{+1}) \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} \frac{1}{1+i_{+1}} + \frac{R}{2} \frac{1}{\psi_{+1}} + \frac{1}{2} \right], \quad (7)$$

reflecting the benefit of holding an extra unit of money, which can be used during the following day to acquire the asset and the good if held by a buyer or lent out at an interest if held by a seller. Second, there is the intertemporal optimality condition for the Lucas tree, given by the Euler equation

$$1 = \beta \frac{\psi_{+1}}{\psi} \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} \frac{1}{1+i_{+1}} + \frac{R}{2} \frac{1}{\psi_{+1}} + \frac{1}{2} \right], \quad (8)$$

reflecting the benefit of holding an extra unit of the real asset, which can be used to borrow money against its value during the following day and generate a return during the night if held by a buyer, or sold during the following night, if held by a seller. In order to guarantee that both money and the real asset are held simultaneously, the two intertemporal assets accumulation conditions, (7) and (8), should hold simultaneously, implying that the interest rate satisfies the no-arbitrage condition

$$\frac{\phi_{+1}}{\phi} (1+i_{+1}) = \frac{\psi_{+1}}{\psi}. \quad (9)$$

As regards the constraints, except for (2), all other constraints can be shown to bind under all circumstances, in equilibrium. In particular, the constraint (1) is binding, hence, $\phi m = q\alpha$. The binding constraint (4), (6), $m = M$ and $a = A$, together imply $\phi M = \frac{\psi A}{1+i}$, which can be delayed one period to give $\phi_{+1} M_{+1} = \frac{\psi_{+1} A}{1+i_{+1}}$. Dividing the

latter by the former side by side, we obtain

$$\frac{\phi_{+1}}{\phi} \frac{M_{+1}}{M} = \frac{\psi_{+1}}{\psi} \frac{1+i}{1+i_{+1}}, \quad (10)$$

which reflects the assets transformation occurring in the morning. Using $M_{+1} = \gamma M$, equations (9) and (10) together imply that the nominal interest rate is completely controlled by monetary policy,

$$i = \gamma - 1, \quad (11)$$

where $i \geq 0$, since $\gamma \geq 1$. This exact relationship between the nominal interest rate and the growth rate of money supply, emerging from the intertwined use of the two assets that are transformed into each other in the morning, is the hallmark of complementarity.⁶ Using $\phi m = q\alpha$, the constraint (3) can be written as $\phi l = px$. Moreover, $p = c'(x)$ from the assumption of perfect competition. Using $a = A$, the constraint (2), thus, becomes

$$c'(x) x \gamma \leq 2\psi A. \quad (12)$$

The non-negative (shadow) value of liquidity should reflect the marginal net benefit of extra liquidity per unit repayment. The multiplier of (12) can, thus, be written as

$$\lambda = \frac{u'(x)}{c'(x)} \frac{1}{\gamma} - 1. \quad (13)$$

In sum, the equilibrium system reduces to two equations: one of the two equivalent Euler conditions, for instance, equation (8), and the complementary slackness condition for the collateral constraint,

$$\left[\frac{u'(x)}{c'(x)} \frac{1}{\gamma} - 1 \right] [2\psi A - c'(x) x \gamma] = 0, \quad (14)$$

where the two expressions in square brackets in (14) are constrained to be non-negative, the first being the liquidity value, (13), and the second the collateral constraint, (12). Hence, it cannot be the case that the borrowing constraint, reflecting

⁶In other models with nominal and real assets, such as Ferraris and Watanabe (2008), in which the asset transformation is less complete, the relationship between the nominal interest rate and monetary policy is less exact.

the limit imposed on the amount of loans of money a buyer can obtain against the value of the real asset held, is slack and its shadow value is strictly positive. Next, we state our definition of an equilibrium.

Definition 1 *A money and collateral equilibrium (MCE) is a pair (ψ, x) , satisfying (8) and (14). A stationary money and collateral equilibrium (SMCE) is an MCE in which (ψ, x) is time invariant.*

We address first the stationary, then, the dynamic equilibria.

Existence of SMCE At an SMCE, equation (8) can be solved for the price of the Lucas tree to give

$$\psi = \frac{\beta}{1 - \beta - \frac{\beta}{2} \left[\frac{u'(x)}{c'(x)} \frac{1}{\gamma} - 1 \right]} \frac{R}{2}, \quad (15)$$

which includes its fundamental value - the discounted expected return- and a premium for its role as collateral - reflecting the liquidity value, $\lambda \geq 0$. Substituting (15) into (12), the collateral constraint becomes

$$(2 - \beta) c'(x) x \gamma - \beta u'(x) x \leq 2\beta R A. \quad (16)$$

The SMCE is constrained if (16) is binding and unconstrained otherwise, corresponding, by (14), to a liquidity value (13) which is non-negative in the former case and zero in the latter. The SMCE turns out to be constrained or unconstrained depending on how large R , A and β are, relative to $u'(x) x$. To simplify the notation, let $f(x) \equiv u'(x) x$, $g(x) \equiv c'(x) x$ and $\rho \equiv \frac{\beta R}{1 - \beta}$. Assume $f(x)$ monotonic in x .

Proposition 1 *Suppose $f(x^*) \leq \rho A$. An SMCE exists and is unique. i. If $f(0) \leq \rho A$, the SMCE is unconstrained; the asset price equals its fundamental value; ii. if $f(0) > \rho A$, there exists a $\bar{\gamma} \in [1, \infty)$ such that, for $\gamma \leq \bar{\gamma}$ the SMCE is unconstrained and for $\gamma > \bar{\gamma}$ constrained; for $\gamma \leq \bar{\gamma}$, the asset price equals its fundamental value, and, for $\gamma > \bar{\gamma}$, carries a liquidity premium.*

This case corresponds to a situation in which the discounted overall payoff of the asset is sufficiently large to make the first best amount of the good affordable. In this region, the payoff of the asset may be always enough to have an unconstrained equilibrium in all circumstances, or sometimes enough only to guarantee that the equilibrium is unconstrained for low but not for high values of the growth rate of money supply. The other case corresponds to a situation in which the first best allocation cannot be afforded. When the asset payoff is really scarce, the equilibrium is always constrained, otherwise it is sometimes constrained, sometimes unconstrained depending on monetary policy.

Proposition 2 *Suppose $f(x^*) > \rho A$. An SMCE exists and is unique. i. If $f(0) \geq \rho A$, the SMCE is constrained; the asset price carries a liquidity premium; ii. if $f(0) < \rho A$, there exists a value $\hat{\gamma} \in (1, \infty)$ such that, for $\gamma \leq \hat{\gamma}$ the SMCE is constrained and for $\gamma > \hat{\gamma}$ unconstrained; for $\gamma \leq \hat{\gamma}$, the asset price carries a liquidity premium and, for $\gamma > \hat{\gamma}$, equals its fundamental value.*

The two assets, the nominal and real one, are transformed into each other in equilibrium. The buyers turn, first, liquidity into the asset and, then, borrow liquidity back, against the value of the asset. Finally, liquidity is spent on consumption. The only impediment to the smooth working of this scheme, may be the scarcity of the asset, which may limit the amount of liquidity the agents can borrow. When this is not an issue, the equilibrium is unconstrained, the liquidity value (13) is zero, hence, consumption is determined by

$$\frac{u'(x)}{c'(x)} = \gamma, \quad (17)$$

unencumbered by the availability of the asset as a means to obtain loans, in its collateral role; correspondingly, the asset price, as it can be seen substituting (17) into (15), is equal to its fundamental value, namely its discounted expected returns,

$$\psi = \frac{\rho}{2}, \quad (18)$$

absent any premium for its liquidity enhancing role. When the equilibrium is constrained, instead, consumption is determined by the binding collateral constraint (16),

$$(2 - \beta) g(x) \gamma - \beta f(x) = 2(1 - \beta) \rho A, \quad (19)$$

its amount being limited by the availability of the real asset to collateralize monetary loans; on the other hand, the asset price, as it can be seen from the binding (12), is given by

$$\psi = \frac{g(x) \gamma}{2A}, \quad (20)$$

which is above its fundamental value, (18), since it includes the liquidity premium, being (13) positive. A price of the asset above its fundamental value is a symptom of expensive liquidity, hence, of a constrained situation. The two situations are combined into four equilibrium regimes: with high, medium-high, medium-low and low asset payoff relative to the value of consumption. In the first regime, the equilibrium is always unconstrained. In the second, the equilibrium is unconstrained for low values of the nominal interest rate, corresponding to moderate expansionary monetary policies, and constrained otherwise. In the third, low interest rates correspond to a constrained situation and high interest rates to an unconstrained one. In the last regime, the equilibrium is always constrained. The availability of the asset together with its discounted returns and monetary policy determine whether lending is constrained, liquidity expensive and, ultimately, consumption inhibited.

Dynamics Using our notation, we can rewrite (8) as follows

$$g(x_{+1}) \gamma [2\psi - \beta\psi_{+1} - (1 - \beta) \rho] - f(x_{+1}) \beta\psi_{+1} = 0. \quad (21)$$

The complementary slackness condition, (14), can be rewritten as

$$[f(x) - g(x) \gamma] [2A\psi - g(x) \gamma] = 0. \quad (22)$$

The dynamics of the MCE differs in the two cases, when the equilibrium is unconstrained or constrained. We analyze them in turn, looking at the dynamic behavior of the system around the steady state. We also consider sunspot equilibria.

Unconstrained case. When the collateral constraint is not binding, from (22), $f(x) = g(x)\gamma$ must hold. This can be used into (21), to obtain the dynamic equation that governs the evolution of the asset price,

$$\psi_{+1} = \frac{\psi}{\beta} - \frac{R}{2}.$$

Therefore, in this case, consumption is time invariant, while the price of the asset follows a dynamic path governed by a linear difference equation whose unique stationary solution, (18), is unstable, since its eigenvalue is larger than one, $\beta^{-1} > 1$. Hence, in this case, neither dynamic indeterminacy nor cyclical behavior can arise.

Constrained case. When the collateral constraint is binding, $2A\psi = g(x)\gamma$ must hold. This can be used to substitute for the current and future price of the asset into (21), obtaining

$$x = g^{-1} \left(\frac{\beta g(x_{+1})\gamma + 2(1-\beta)\rho A + \beta f(x_{+1})}{2\gamma} \right) \equiv G(x_{+1}),$$

where the function $g(x)$ is invertible, since $g'(x) = c''(x)x + c'(x) > 0$ for all x . Hence, the dynamics of the model is conveniently described by a single backward dynamic equation in which current consumption is a function of future consumption. With standard bifurcation techniques, cycles of period two and of higher order and sunspot equilibria can be shown to exist in this case, when the curvature of the utility function is sufficiently high. Mathematically, the slope of the function $f(x_{+1})$ can be altered changing the relative risk aversion of the utility function, giving rise, in some cases, to an inverse relationship between x and x_{+1} . Economically, the ordinary relationship between current and future consumption can be altered rendering the intertemporal substitution effect, which is controlled by the curvature of the utility function, sufficiently strong. The next Proposition establishes the existence of a local cycle of period two, namely an MCE in which both ψ and x assume two values alternately over time close to the SMCE. The relative risk aversion of the utility function is denoted by ε .

Proposition 3 *There exists a unique critical value $\tilde{\varepsilon} > 1$, such that, when $\varepsilon > \tilde{\varepsilon}$, a stable cycle of period two emerges in a neighborhood of the steady state.*

These cycles are expectations driven. Intuitively, when the agents expect the asset price to be, say, high in the future, they are able to plan to borrow more and finance higher consumption, since a higher price of the asset tends to relax their borrowing constraint. However, a high price of the asset induces a lower demand for it, thus putting a downward pressure on the price, which tends to tighten the borrowing constraint, leading to lower consumption, and so on. Viceversa, when a low asset price is expected.

Self-fulfilling Expectations. The expectations mentioned in the previous paragraph are self-fulfilling. This can be seen considering sunspot events when the collateral constraint is binding. A sunspot, in the tradition of Cass and Shell (1983), is an uncertain event that has no direct effect on economic fundamentals – i.e. preferences, endowments and technologies, but can nevertheless affect economic outcomes through the agents’ expectations about the behavior of other agents, which become self-fulfilling. We consider, here, stationary sunspots of order two, which are the appropriate analogue of the cycles of period two considered before. Suppose that a sunspot event may occur (y) or not (n) following a Markov transition probability matrix,

$$\begin{bmatrix} \pi^y & 1 - \pi^y \\ 1 - \pi^n & \pi^n \end{bmatrix},$$

where π^h , $h \in \{y, n\}$, is the probability that state h will occur in the next period if h has occurred in the current period. Suppose agents believe future asset prices to be perfectly correlated with the stationary sunspot activity. A stationary sunspot equilibrium is a rational expectations equilibrium where such belief is fulfilled. The next Proposition establishes the existence of stationary sunspot equilibria of order two close to the SMCE.

Proposition 4 *When $\varepsilon > \tilde{\varepsilon}$, there are infinitely many local stationary sunspot equilibria of order two in every neighborhood of the SMCE.*

More general types of sunspot equilibria can be shown to exist in this framework, when the constraint is binding. Moreover, exploiting the no-trade equilibrium, which exists always, global cycles and even chaotic trajectories can also be shown to exist, for some values of the risk aversion. On the other hand, when the constraint is not binding, sunspot uncertainty has no bite on the behavior of the agents.

Asset prices, output and inflation. In the unconstrained region, asset prices and output do not interact. When the equilibrium is constrained, output and the asset price comove, as it can be seen from (20), which is increasing in x . Along a dynamic path, therefore, GDP and consumption are positively correlated with movements in asset prices. Equations (9) and (11), holding at any - constrained or unconstrained-MCE, together imply

$$\frac{\phi}{\phi_{+1}} = \gamma \frac{\psi}{\psi_{+1}},$$

which says that consumer and asset price inflation are proportional to each other, with the proportionality factor given by the monetary policy parameter.

Two State Markov Equilibrium So far, we have considered only situations in which the fundamentals are stationary and the economy is either in the unconstrained region or in the constrained one. Even when the economy undergoes oscillations, it does so remaining in the constrained region. We now examine a different situation in which the economy alternates between the constrained and unconstrained regions, depending on the asset return which may be high or low, $R^H > R^L$, uniformly for all buyers. Uncertainty over the return has a Markov structure with a probability σ of remaining next period in the current state, and the complementary probability of switching state. The rest of the model is unchanged. We look for two state Markov equilibria in which the high state is unconstrained and the low state constrained, which we call a *U-C Markov Equilibrium*. Let x^j and ψ^j be the output and price in the two states for $j = H, L$. Since the high state is unconstrained, output is determined by $u'(x^H) = c'(x^H) \gamma$. On the other hand, in the low state, the economy

is constrained, hence, $c'(x^L) x^L \gamma = 2\psi^L A$ holds. The Euler conditions are

$$\psi^j = \frac{\beta}{2} \left\{ \sigma \left[R^j + \frac{u'(x^j) \psi^j}{c'(x^j) \gamma} + \psi^j \right] + (1 - \sigma) \left[R^{j'} + \frac{u'(x^{j'}) \psi^{j'}}{c'(x^{j'}) \gamma} + \psi^{j'} \right] \right\},$$

for $j = H, L$ and $j' \neq j$. The next Proposition shows that such an equilibrium exists.

We assume that $f'(\cdot) < 0$, which holds iff $\varepsilon > 1$.

Proposition 5 *If R^H is sufficiently high and R^L sufficiently low, a U-C Markov Equilibrium exists.*

In this equilibrium, the economy randomly oscillates between a high state, in which the constraint is not binding and output is determined by (17), and a low state, in which output is determined by the binding collateral constraint. The price of the asset also oscillates between a correspondingly low and high value. The oscillations are determined by exogenous shocks to fundamentals, in particular to the asset returns.

Monetary Policy The model has several implications for monetary policy. We analyze, first, optimal monetary policy. Then, away from optimality, we consider how monetary policy may affect asset prices, favor or impede the emergence of cycles and stabilize the economy. Finally, we consider one type of unconventional policy and contractionary monetary policies.

Optimal monetary policy. Consumption is strictly decreasing in γ , in all cases, hence, the optimal monetary policy is constituted always by no-intervention, which occurs when the Government does not alter the stock of money. When there is enough of the asset in the economy, one can make an even stronger claim, namely that no-intervention achieves efficiency, as it can be seen from (17).

Proposition 6 *The optimal monetary policy is $\gamma = 1$. If $f(x^*) \leq \rho A$, $\gamma = 1$ leads to the first-best allocation.*

This somewhat surprising conclusion is driven by the arbitrage requirement between the nominal and real assets that pins the nominal interest rate to the growth

rate of money supply, which, as we have seen above, must hold in any equilibrium regime, and the indifference condition for the borrowers, equating the extra benefit of a loan to its interest cost, which reflects the fact that liquidity is inexpensive in an unconstrained situation. Milton Friedman in an influential essay, Friedman (1969), has advocated the use of what has since been called the Friedman rule, to guarantee that monetary policy is optimally conducted. The Friedman rule involves setting the nominal interest rate to zero, to equate the private opportunity cost of holding fiat money, namely the nominal interest rate, to the social cost of creating it, which can reasonably be taken to be zero. Typically, in the literature, this has been found to correspond to a negative growth rate of money supply, hence, a contraction of its stock over time, and an ensuing deflationary path of prices. In the present environment, the Friedman rule holds in the sense that the optimal monetary policy indeed involves a zero nominal rate of interest, but, from (11), it corresponds to no-intervention, $\gamma = 1$, rather than a contraction of the stock of money. This is due to the complementarity of money and the real asset.

Monetary policy and asset prices. Whenever the economy operates in the unconstrained regions, monetary policy has no effect on asset prices. In the medium-high and medium-low regimes, when the amount of the asset – or, more precisely, the overall discounted value of the asset including its returns relative to the value of consumption- is medium-high or medium-low, monetary policy determines whether the economy is constrained or not. In the medium-high case, a monetary expansion at a high growth rate leads to a constrained situation, in the medium-low case it has the effect of making the economy unconstrained, although always at the cost of reducing output. In the constrained regions, monetary policy affects directly asset prices. Its effect depends on the elasticity of substitution, as controlled by the relative risk aversion of the utility function, ε , which is assumed, for simplicity, constant.

Proposition 7 *i. If the SMCE is unconstrained, ψ is unaffected by γ . ii. If the SMCE is constrained, a higher γ corresponds to a ψ that is higher if $\varepsilon > 1$, lower if*

$\varepsilon < 1$, the same if $\varepsilon = 1$.

In the unconstrained case, the asset price is determined by (18), which does not depend on monetary policy. In the constrained case, instead, the asset price is determined by (20), and, thus, is affected by monetary policy in two ways, as it can be seen from the elasticity of the asset price to changes in monetary policy

$$\frac{\partial \psi}{\partial \gamma} \frac{\gamma}{\psi} = 1 + \frac{g'(x)}{c'(x)} \frac{\partial x}{\partial \gamma} \frac{\gamma}{x},$$

evaluated at steady state. First, there is a direct, positive effect, arising from asset substitution when it is more costly to hold money; second, an indirect, opposite effect via the negative impact of policy on consumption, arising from the complementary role of the two assets in acquiring it. Since the elasticity of consumption to changes in monetary policy depends inversely on the relative risk aversion of the utility function, when this is larger, the negative effect is smaller. Hence, in the constrained region, the model can generate an overall positive or negative effect, depending on the strength of the intertemporal elasticity of substitution. As regards the day-time price of the asset - its *liquidation* price, $q = \frac{\psi}{\gamma}$, it is always decreasing in monetary policy, since only the negative effect is present at the liquidation stage.

Cycles and stabilization. Interestingly, both fundamental and policy conditions may contribute to avoid the emergence of non-fundamental instability. When the asset is very abundant or very scarce, the economy is always unconstrained or constrained independently of monetary policy. On the other hand, in the two intermediate regimes, monetary policy may eliminate the conditions that favor the emergence of cycles and sunspots. Whether monetary policy should be lax or tight to avoid economic instability depends on the availability of the asset. Moreover, even when the economy is already in a constrained region, monetary policy can still affect the cyclical behavior of the economy, as the following Proposition documents.

Proposition 8 *A higher γ corresponds to a lower critical value $\tilde{\varepsilon}$.*

Therefore, monetary policy, in the constrained region can alter the likelihood of the occurrence of cycles and sunspots. In particular, a more expansionary monetary policy is more likely to lead to instability than a less expansionary one.

Unconventional monetary policy. In the region identified by Proposition 2, should the cycle occur at $\gamma = 1$, the Government may try to use alternative instruments to avoid the instability. For instance it could intervene in the night asset market, buying the asset, with the aim of reducing its available amount and, thus, altering its price. This would, in turn, affect the collateral constraint of the agents, and affect the economy. The Government would have to intervene without increasing the growth rate of money, since we know from Proposition 7 that any such increase will make cycles more, rather than less likely. One possibility for the Government would be to acquire the Lucas tree issuing one period bonds at night and, then, hold the asset overnight, to sell it in the day-time market for bonds. These overnight operations could be done indefinitely to alter permanently the asset price and affect the collateral constraint. The next Proposition examines such a policy, under the assumption that there is a cycle at $\gamma = 1$.

Proposition 9 *The asset buying program induces a higher ψ , lower $\tilde{\varepsilon}$ and lower x .*

Hence, if the economy is experiencing instability and the zero lower bound (by (11), $\gamma = 1 \Leftrightarrow i = 0$) has already been reached, an unconventional monetary policy, whereby real assets are acquired by the public authorities from the market, could increase the price of the assets, but with the unfortunate consequence of increasing the likelihood of the occurrence of cycles and sunspots, and reducing output.

Contractionary monetary policy. We have maintained all along the assumption that lump-sum taxation is not feasible since agents are anonymous and can hide their asset holdings. However, since the Lucas tree can be identified and seized by private agents during the night, it would have been more natural to assume that also the Government could seize the real asset for taxation purposes. In this case,

the lower bound on taxation would not be zero, but would be given by the value of the asset at night. However, taxation corresponds to a contractionary monetary policy, $\gamma < 1$, and the nominal interest rate, $i = \gamma - 1$, would become negative. Hence, the equilibrium with monetary loans does not exist when monetary policy is contractionary. Other equilibria, with real credit, where the asset is used to borrow directly the consumption good may exist, but the optimal monetary policy would still be no-intervention. Contractionary monetary policy cannot improve matters. In fact, even if feasible, it could only hamper the functioning of the liquidity market.

3.2 Alternative Trading Systems

The trading scheme analyzed in the previous section uses both money and the Lucas tree in a complementary way to convey all the assets into the hands of their best users, namely the buyers. First, money is spent by the buyers to acquire the Lucas tree from the sellers. Then, the entire value of the Lucas tree, including the part just acquired, is pledged by the buyers to borrow money from the sellers. Finally, the money is spent by the buyers to purchase consumption. In this way, no asset remains idle in the hands of an agent who has no immediate use of it. In this section we examine the possible alternative trading arrangements. We will show that the scheme with money and collateralized credit cannot be beaten, in the sense that the allocation obtained through it is never socially inferior to the allocations obtained through alternative trading systems, and, thus, money and collateralized credit are both essential, complementary means of exchange. When considering alternative schemes, one has to keep in mind that only some arrangements are compatible with the underlying frictions of the environment, which are: complete anonymity of the agents and contractual incompleteness. We will use the same notation adopted before and skip the details of the derivations.

Direct trade and equity The first possibility involves the direct, physical exchange of the Lucas tree by the buyers as a trading instrument. Given that the human capital of the buyer, during any given period, is essential to generate the returns, and the buyers' human capital cannot be credibly pledged and contracted upon, efficiency requires the asset to remain in the hands of the buyers. However, it would be possible to exchange only the shares of the tree rather than the tree itself, which would, then, remain physically in the hands of the buyers for the entire period. Two cases need to be examined: one, in which the shares circulate in the economy after being traded during the day; another, in which they do not circulate. The first case cannot arise due to the assumed anonymity of the agents, since a third party would be unable to identify the physical location of the tree, information which is crucial to generate returns during the following period. The second case is very similar to the main scheme considered in the previous section. Indeed, in our model the exchange of debt with the promise of repayment guaranteed by the right to seize the asset during the night and the exchange of equity with the right to physically obtain the asset during the night are almost equivalent, except that the latter suffers from a renegotiation problem, since the agent holding the property rights to a tree may try to use them at night to obtain part of the returns, threatening to appropriate the asset before the returns are generated. As customary in settings with incomplete contracts, the allocation of property rights is a delicate matter: property rights should be allocated to the agents who are in the position to generate the returns from the investment. Hence, the equity scheme, in the end, is not a viable alternative.

Double collateralization A further possibility involves the buyers borrowing some amount of the asset from the sellers, using their own asset as collateral and, then, pledging the entire value of the assets held at that point to borrow cash from the sellers, and, finally, spend all the cash to acquire consumption. In this case, there would be two borrowing constraints: one for the asset transaction, $q\alpha^b \leq \psi a$, since,

first, the asset would be borrowed against the value of the asset held at the beginning of the period; and a second one for the monetary loan, $\phi l^b (1 + i) \leq \psi (a + \alpha^b)$, since money would be borrowed against the night value of the collateral, including both the amount held initially and the amount borrowed in the first transaction. Finally, the initial amount of money and the amount borrowed would be spent on consumption, giving rise to the constraint, $px^b \leq \phi l^b + \phi m$. This scheme suffers of the problem of overcollateralization. Lenders of the asset and of cash would not trust the borrowers to repay their debts, since the value of the asset owned and pledged by the borrower is smaller than the overall value of the loans obtained, and, thus, in the absence of other forms of punishment, would not lend anything, knowing that the buyers would default on their debts. Hence, this case never arises as an equilibrium.

Mortgage Another possibility involves the buyers borrowing the asset from the sellers against its own value at night, i.e. mortgaging it, giving rise to the (self-financing) constraint $q\alpha^b \leq \psi\alpha^b$, and, then, pledging their own asset as collateral to borrow cash from the sellers, giving rise to the collateral constraint $\phi l^b (1 + i) \leq \psi a$, where money is borrowed against the value of the asset initially held, but not the amount borrowed through the mortgage, due to the overcollateralization problem mentioned above, in order to spend, finally, all the cash to acquire consumption, $px^b \leq \phi l^b + \phi m$. This scheme suffers from the same renegotiation problem considered previously. Since part of the property rights over the tree remain in the hands of the seller, he or she has the incentive to use them at night to try to extract some of the returns from the buyers, thus leading to an inefficient reallocation of the tree. Even if feasible because the buyer is protected from the possibility of renegotiation, the system would lead to lower consumption and output relative to the main scheme, since the agents cannot borrow against the amount of the asset mortgaged.

Non-monetary credit Alternatively, one could consider an arrangement whereby money is used to purchase the asset, but the asset is used as collateral to borrow

directly an amount of the consumption good. This would be a system with money and "real" credit, in which the buyers would be subject to a cash constraint, $q\alpha^b \leq \phi m$, since the asset is acquired through money, and a collateral constraint, $px^b \leq \psi(a + \alpha^b)$, since the asset is used directly to borrow the good, without going through a nominal loan. This case leads to the following equilibrium conditions: an Euler equation for money,

$$\phi = \beta\phi_{+1} \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} + \frac{1}{2} \frac{R}{\psi_{+1}} + \frac{1}{2} \right],$$

since an extra unit of money acquired at night, the following period can be spent to buy the asset which can subsequently be used to obtain consumption on credit and generate the returns if one turns out to be a buyer or spend it at night; an Euler equation for the asset,

$$\psi = \beta\psi_{+1} \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} + \frac{1}{2} \frac{R}{\psi_{+1}} + \frac{1}{2} \right],$$

since an extra unit of the asset acquired at night, the following period can be used as collateral to borrow the good and produce the returns if one turns out to be a buyer or sold out if a seller. Moreover, there is the complementary slackness condition of the collateral constraint. As it can be seen from the Euler conditions, this arrangement is possible only in the knife-edge case with $\frac{\phi_{+1}}{\phi} = \frac{\psi_{+1}}{\psi}$. Moreover, should the return generated by the seller be positive instead of nil, it would never arise.

Idle assets Consider now the possibility that the Lucas tree is not traded in the morning, but simply used to borrow extra money for transaction purposes. The constraints in this case would be: first, $\phi l^b(1+i) \leq \psi a$, where money is borrowed against the value of the asset, and, second, $px^b \leq \phi l^b + \phi m$, since the initial amount of money and the amount borrowed is spent on consumption. This case leads to the following equilibrium conditions: an Euler equation for money,

$$\phi = \beta\phi_{+1} \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} + \frac{1}{2} (1+i_{+1}) \right],$$

since an extra unit of money acquired at night, the following period can be spent on consumption if one turns out to be a buyer or lent out if a seller; an Euler equation for the asset,

$$\psi = \beta\psi_{+1} \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} \frac{1}{1+i_{+1}} + \frac{1}{2} \frac{R}{\psi_{+1}} + \frac{1}{2} \right],$$

since an extra unit of the asset acquired at night, the following period can be used as collateral to borrow money and produce a return if one turns out to be a buyer or sold /lent out if a seller. Moreover, there is the complementary slackness condition of the collateral constraint. This is an arrangement in which the asset serves to collateralize the monetary loans, but money does not serve to acquire the asset, in a sort of half-complementary relationship between the two assets. The main difference with the case of full complementarity, where also money serves to acquire the asset, appears clearly in the Euler equation for money: the return of the asset does not appear there. As a consequence, to ensure that the two Euler conditions hold simultaneously, as required by arbitrage, the nominal interest rate will have to be higher than in the complementary case. This is a symptom of inefficiency. Indeed, solving for the steady state, one finds, that, when the collateral constraint is not binding,

$$\frac{u'(x)}{c'(x)} = \frac{\gamma}{\beta},$$

which is the same as (17), except that here the RHS is multiplied by $\beta^{-1} > 1$, leading to lower consumption, since the LHS is decreasing in x ; and, in the constrained case,

$$(2 - \beta) g(x) \gamma - \beta f(x) = 2\beta(1 - \beta) \rho A,$$

which is the same as (19) except that the RHS is multiplied by $\beta < 1$, implying lower consumption, since the LHS is increasing in x . Moreover, some of the asset remains in the hands of sellers who cannot produce any return with it. Hence, this scheme is dominated by the one considered in the previous part of the paper.

Next, consider a system where the monetary asset remains idle. Such a system, in which the asset borrows asset and money buys consumption, would give rise to

the following constraints for the buyer: first, $q\alpha^b \leq \psi a$, since the asset is borrowed against the night value of the amount of the asset held at the beginning of the period; and, second, $px^b \leq \phi m$, since money is spent on consumption. Since the sellers need to be indifferent between lending the asset or holding on to it, $q = \psi$. This case leads to the following equilibrium conditions: an Euler equation for money,

$$\phi = \beta\phi_{+1} \left[\frac{1}{2} \frac{u'(x_{+1})}{c'(x_{+1})} + \frac{1}{2} \right],$$

since an extra unit of money acquired at night, the following period can be spent to acquire consumption if one turns out to be a buyer or spend it at night; an Euler equation for the asset,

$$\psi = \beta\psi_{+1} \left(\frac{1}{2} \frac{R}{\psi_{+1}} + \frac{1}{2} \right),$$

since an extra unit of the asset acquired at night, the following period can be used to produce the returns if one turns out to be a buyer or sold out if a seller. Solving for the steady state, one finds,

$$\frac{u'(x)}{c'(x)} = \frac{2\gamma - \beta}{\beta},$$

which leads to a level of consumption which is lower even relative to the previous scheme with the mortgage, since $2\gamma - \beta > \gamma$ and the LHS is decreasing in x . This scheme is dominated, since it does not make efficient use of money, leaving some in the wrong hands. In general, any system that implies a combination of the constraints above, does not make an efficient use of the available assets, leaving some in the wrong hands. *A fortiori*, any other system that uses only one of the two assets as trading instruments is dominated by the main scheme considered above, for the same reason.

4 Concluding Comments

We have presented a model in which money and collateralized credit coexist and are fully complementary, with implications for the relation between output and asset prices and the role of monetary policy. The central idea of the paper is that there

are two assets, a nominal one, devoid of intrinsic value, and a real one, with intrinsic value, both of which are held for precautionary reasons, and both of which may turn out to be misallocated after the realization of uncertainty. In the absence of well functioning credit markets, due to the agents anonymity, an arrangement whereby the two are intertwined in a complementary way may be the best option. The credit markets were not functioning well for two reasons: anonymity and incompleteness of contracts. Although the idea seems general, some fairly stark assumptions have been made, to present sharp results. Fortunately, they can be relaxed without altering the gist of the paper. For instance, the assumption that the probability of being either a buyer or a seller is the same can be relaxed without affecting the results. The crucial element is the presence of some uncertainty over who will turn out to be the best user of the assets when they are acquired at night. Equal probability is not essential. The market structure for the day markets does not need to be competitive. We could extend the model to allow for bilateral meetings and bargaining, for instance, or more generally, following Gu et al. (2014), we could simply postulate that the terms of trade in the market for the day good are determined by a non-linear function $p(x)$, with $p(0) = 0$ and $p'(x) > 0$, thus, capturing several potential deviations from the competitive assumption. The same could be done for the asset market and the credit market. The main idea of the paper would survive this extension. The only result that would not survive would be the possibility to reach the first best allocation at no-intervention, since the economy would suffer from an inefficiency driven by the imperfections in the market structure, with their ensuing distortions of the pricing away from the marginal cost. The linearity of the night-time payoff can be relaxed along the lines of Gu et al. (2014). The assumption that the Lucas tree generates a return only if tended by the buyer can also be relaxed, as long as the return to the seller remains smaller than the one accruing to the buyer. Clearly, if the asset was not relatively idle in the hand of the seller, the result would, instead, disappear. In fact, if the best user of the Lucas tree was the seller, then, we would have a model in which

the asset is used directly as a means of exchange by the buyers to acquire money, pretty much as the illiquid bonds in Kocherlakota (2003). More generally, one could imagine an extension of the model, lying in between these two extremes, in which a fraction of the buyers and sellers are able to produce the returns of the asset. The asset does not need to be in fixed supply. A model with reproducible capital could deliver further implications for the impact of monetary policy on capital accumulation, at the cost of some analytical complications. The model could be further enriched, without altering its main message, considering the markets for the asset and for liquidity as intermediated by middlemen and banks respectively.

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5 Appendix

Equilibrium conditions. We state here the equilibrium conditions, beginning with the necessary (and, given the assumptions on fundamentals, sufficient) conditions for the agents' optimality. The first order conditions for a buyer during the day are the following: for the optimal consumption decision, x^b , we have

$$u'(x^b) - p\delta - \frac{p}{\phi}W_m^{b'}(\tilde{m}^b, \tilde{a}^b) = 0, \quad (23)$$

since an extra unit of consumption increases utility but needs to be acquired at price p , spending cash, thus reducing liquidity for the following sub-period; for the decision to acquire the asset, α^b , we have

$$-q\mu + \psi\lambda - q\delta - \frac{q}{\phi}W_m^{b'}(\tilde{m}^b, \tilde{a}^b) + W_a^{b'}(\tilde{m}^b, \tilde{a}^b) = 0, \quad (24)$$

since acquiring an extra unit of the asset tightens the cash constraint, relaxes the borrowing constraint, reduces the amount of cash available to acquire consumption goods, and alters the amount of liquidity and asset holdings, negatively and positively respectively, for the following sub-period; finally, for the loan, l^b , we obtain

$$-\phi(1+i)\lambda + \phi\delta - iW_m^{b'}(\tilde{m}^b, \tilde{a}^b) = 0, \quad (25)$$

since an extra unit of cash borrowed tightens the borrowing constraint, increases the cash available for consumption purposes, and increases the net repayment in the following sub-period. The first order conditions for a seller during the day are the following: for the good, x^s , we have

$$-c'(x^s) + \frac{p}{\phi}W_m^{s'}(\tilde{m}^s, \tilde{a}^s) = 0, \quad (26)$$

since an extra unit produced generates a disutility, but also an inflow of cash that will turn out to be useful in the following sub-period; for the optimal decision to sell the asset, α^s , we have

$$-q\zeta + q\theta + \frac{q}{\phi}W_m^{s'}(\tilde{m}^s, \tilde{a}^s) - W_a^{s'}(\tilde{m}^s, \tilde{a}^s) = 0, \quad (27)$$

since an extra unit of the asset sold on the day market tightens the asset constraint, relaxes the lending constraint and alters the amount of liquidity and asset holdings available for the next sub-period, in a positive and negative way respectively; for the loan, l^s , we obtain

$$-\phi\theta + iW_m^{s'}(\tilde{m}^s, \tilde{a}^s) = 0, \quad (28)$$

since lending an extra unit of money tightens the lending constraint but generates a net interest payment during the following sub-period. The envelope conditions between night and day are

$$V'_m(m, a) = \frac{1}{2} [\phi\mu + \phi\delta + W_m^{b'}(\tilde{m}^b, \tilde{a}^b)] + \frac{1}{2} [\phi\theta + W_m^{s'}(\tilde{m}^s, \tilde{a}^s)], \quad (29)$$

for cash, since an extra unit of cash relaxes the asset constraint and the goods constraint for a buyer, and the lending constraint for a seller, as well as increasing the amount of cash at night for both types; and

$$V'_a(m, a) = \frac{1}{2} [\psi\lambda + W_a^{b'}(\tilde{m}^b, \tilde{a}^b)] + \frac{1}{2} [\psi\zeta + W_a^{s'}(\tilde{m}^s, \tilde{a}^s)], \quad (30)$$

for the Lucas tree, since an extra unit of the asset relaxes the borrowing constraint and generates a return for a buyer, and relaxes the asset-sale constraint for a seller, as well as increasing the amount of the asset at night for both types. The first order conditions for the optimal decision to accumulate cash at night, m_{+1} , is

$$-\phi + \beta V'_m(m_{+1}, a_{+1}) = 0, \quad (31)$$

since an extra unit of cash costs ϕ at night, but generates a benefit in the following period; and for the optimal decision to accumulate the real asset at night, a_{+1} , is

$$-\psi + \beta V'_a(m_{+1}, a_{+1}) = 0, \quad (32)$$

since an extra unit of asset costs ψ at night, but generates a benefit in the following period. The envelope conditions for cash and the asset between day and night are, respectively,

$$W_m^{j'}(\tilde{m}^j, \tilde{a}^j) = \phi, \quad (33)$$

since a unit of cash is worth ϕ units of the good at night; and

$$W_a^{j'}(\tilde{m}^j, \tilde{a}^j) = \psi + R(j), \quad (34)$$

since one unit of the asset is worth ψ units of the good at night and, for a buyer, $j = b$, generates also the return, R . Use (33) and (34) into (29) and (30), then, delay them one period and insert them into (31) and (32), to obtain the Euler equations

$$\phi = \beta\phi_{+1} \left[\frac{1}{2} (\mu_{+1} + \delta_{+1} + 1) + \frac{1}{2} (\theta_{+1} + 1) \right], \quad (35)$$

for cash, equating the current value of the nominal asset to its future value, which reflects the benefit of using money to purchase the Lucas tree and consumption as a buyer or lend it out as a seller; and

$$\psi = \beta\psi_{+1} \left[\frac{1}{2} \frac{R}{\psi_{+1}} + \frac{1}{2} (\lambda_{+1} + 1) + \frac{1}{2} (\zeta_{+1} + 1) \right], \quad (36)$$

for the Lucas tree, equating the current, night-time price of the real asset to its future value, which reflects its expected return and benefit when borrowing money against its value as a buyer or selling it out for money as a seller. In sum, the equilibrium conditions are: the Euler conditions (35) and (36), the first order conditions (23) to (28), the complementary slackness conditions for the constraints, (1), (2), (3) and (4), (5), and the market clearing conditions in the text.

Solution. Combining (23) to (25) and (33) and (34) for $j = b$, we obtain the multipliers

$$\delta = \frac{u'(x^b)}{p} - 1 \geq 0, \quad (37)$$

for the shadow value of consumption, which is given by its marginal utility net of its unit cost, i.e. the price of the good, per unit cost;

$$\lambda = \frac{u'(x^b)}{p} \frac{1}{1+i} - 1 \geq 0, \quad (38)$$

for the shadow value of liquidity, which is given by its marginal benefit in terms of extra consumption net of its cost, i.e. the interest payment, per unit repayment;

$$\mu = \frac{R}{q} + \left[\frac{\psi}{q(1+i)} - 1 \right] \frac{u'(x^b)}{p} \geq 0, \quad (39)$$

for the shadow value of the asset, which is given by its return per unit cost, i.e. its day-time price, and the potential capital gain, evaluated in terms of consumption. Next, consider the sellers. From (26) and (33), we obtain

$$p = c'(x^s), \quad (40)$$

which is the standard condition relating the price and the marginal cost under perfect competition. Combining (27) and (28) and (33) and (34) for $j = s$, we obtain the multipliers

$$\zeta = 1 + i - \frac{\psi}{q} \geq 0, \quad (41)$$

since the seller, in order to be willing to give up the asset during the day, needs to be paid at least its discounted value at night; and

$$\theta = i \geq 0, \quad (42)$$

which says that the shadow value of a loan for a seller is simply the nominal interest rate. Since the buyers are the best users of all the assets during the day, we restrict attention to situations in which $\alpha^s = a$ and $\phi l^s = \phi m + q\alpha^s$, even when (41) and (42) hold at equality. The next Lemma determines the multipliers.

Lemma 1 *The values of the multipliers are: i. $\zeta = 1 + i - \frac{\psi}{q} = 0$; ii. $\mu = \frac{R}{q} > 0$; iii. $\theta = i = \gamma - 1$; iv. $\delta = \frac{u'(x)}{c'(x)} - 1 \geq 0$; v. $\lambda = \frac{u'(x)}{\gamma c'(x)} - 1 \geq 0$.*

Proof of Lemma 1. *i.* From $\phi m + q\alpha^s = \phi l^s$ and $\alpha^s = a$, using $m = M$, we have $\phi M + qa = \phi l^s$. From the complementary slackness condition for (1), $\alpha^b = \alpha^s$, $m = M$ and $\alpha^s = a$, we have $\phi M \geq qa$. From $l^b = l^s$ and $a = A$, we can rewrite these conditions as $\phi M + qA = \phi l^b$ and $\phi M \geq qA$. The constraint $\psi(a + \alpha^b) \geq \phi l^b(1 + i)$

can, thus, be rewritten as $\frac{2\psi A}{\phi M + qA} \geq (1+i)$. Since $\phi M \geq qA$, the largest value of the LHS is $\frac{\psi}{q}$. Therefore, $\frac{\psi}{q} \geq 1+i$. By (41), $1+i \geq \frac{\psi}{q}$. Therefore, $1+i = \frac{\psi}{q}$, i.e. $\zeta = 0$. *ii.* Inserting $1+i = \frac{\psi}{q}$ into (39), we have $\mu = \frac{R}{q} > 0$. *iii.* Since $q = \frac{\psi}{1+i}$ and $\mu > 0$, by the complementary slackness condition for (1), we have $\phi M = \frac{\psi}{1+i}A$. Equations (35) and (36) imply $\frac{\psi}{\psi_{+1}} = \frac{\phi}{\phi_{+1}(1+i_{+1})}$. Therefore, $\frac{\psi}{\psi_{+1}} = \frac{\frac{\psi}{1+i} \frac{A}{M}}{\frac{\psi_{+1}}{1+i_{+1}} \frac{A}{M_{+1}}(1+i_{+1})} \Leftrightarrow 1+i = \frac{M_{+1}}{M} = \gamma$. Hence, from (42), $\theta = i = \gamma - 1$. *iv.* Using (40), (37) and market clearing for x , we have $\delta = \frac{u'(x)}{c'(x)} - 1$. *v.* Using (40), (38), $1+i = \gamma$ and market clearing for x , we have $\lambda = \frac{u'(x)}{\gamma c'(x)} - 1$. ■

Since $\gamma \geq 1$, between δ and λ , it is enough to keep track of the latter. Since the nominal interest rate equates the two Euler conditions, we are left with two identical Euler equations, of which we can choose one, and one complementary slackness condition, for constraint (2). Hence, the equilibrium system reduces to the two equations in the text, (8) and (14). The following Lemma is instrumental in proving the existence and characterization of SMCE. Using (16), define $C(x, \gamma) \equiv 2(1-\beta)\rho A - (2-\beta)\gamma g(x) + \beta f(x) \geq 0$, and $\lambda(x, \gamma) \equiv \frac{f(x)}{\gamma g(x)} - 1 \geq 0$. Thus, (14) becomes $\lambda(x, \gamma) C(x, \gamma) = 0$.

Lemma 2 *At an SMCE, $x \in [0, x^*]$ and*

- i.* $\lambda(x, \gamma) = 0$ and $C(x, \gamma) > 0$, *iff* $f(x) < \rho A$;
- ii.* $\lambda(x, \gamma) = 0$ and $C(x, \gamma) = 0$, *iff* $f(x) = \rho A$;
- iii.* $\lambda(x, \gamma) > 0$ and $C(x, \gamma) = 0$, *iff* $f(x) > \rho A$.

Proof of Lemma 2. Since $\gamma \in [1, \infty)$ and $\frac{u'(x)}{c'(x)} \geq \gamma$, at an SMCE consumption cannot be larger than x^* , hence, we restrict attention to values of $x \in [0, x^*]$. The case in which both the multiplier and the constraint are strictly positive is excluded by the complementary slackness condition. *i.* $f(x) = \gamma g(x)$ and $2(1-\beta)\rho A - (2-\beta)\gamma g(x) + \beta f(x) > 0 \Rightarrow f(x) < \rho A$; *ii.* $f(x) = \gamma g(x)$ and $2(1-\beta)\rho A - (2-\beta)\gamma g(x) + \beta f(x) = 0 \Rightarrow f(x) = \rho A$; *iii.* $f(x) > \gamma g(x)$ and $2(1-\beta)\rho A - (2-\beta)\gamma g(x) + \beta f(x) = 0 \Rightarrow f(x) > \rho A$. Since these are the only possibilities and are mutually exclusive, the reverse implications apply. ■

Next, we prove the Propositions in the text. Let $\varepsilon(x) \equiv -\frac{u''(x)}{u'(x)}x$ and $\eta(x) \equiv \frac{c''(x)}{c'(x)}x$.

Proof of Proposition 1. *i.* Monotonicity of $f(x)$, $f(x^*) \leq \rho A$ and $f(0) \leq \rho A$, guarantee $f(x) \leq \rho$ for all values of $x \in [0, x^*]$. Hence, $\lambda(x, \gamma) = 0 \Leftrightarrow \frac{u'(x)}{c'(x)} = \gamma$ for all values of $x \in [0, x^*]$. There are two possible cases. $C(x, \gamma) > 0$ and $C(x, \gamma) = 0$. The latter holds at $\gamma = 1$ only if $f(x^*) = \rho A$ and, by monotonicity, for values of $\gamma > 1$ only if $f(0) = \rho A$. The function $\lambda(x, \gamma)$ is continuous in both x and γ . By the Inada conditions, $\lambda(0, \gamma) = \infty$, $\lambda(\infty, \gamma) = -1$, $\lambda(x^*, \gamma) = \frac{1}{\gamma} - 1$. By the Intermediate Value Theorem, for any $\gamma \in [1, \infty)$, a value $\tilde{x} \in [0, x^*]$ exists such that $\lambda(\tilde{x}, \gamma) = 0$. Moreover, $\frac{\partial \lambda(x, \gamma)}{\partial x} = -\frac{1}{x} \frac{u'(x)}{\gamma c'(x)} [\varepsilon(x) + \eta(x)] < 0$, hence, \tilde{x} is unique for every γ . From (15), $\psi = \frac{\rho}{2}$. *ii.* Monotonicity of $f(x)$, $f(x^*) \leq \rho A$ and $f(0) > \rho A$, imply that there exists a $\bar{x} \in [0, x^*]$ such that $f(x) \leq \rho A$ for $x \geq \bar{x}$ and $f(x) > \rho A$ for $x < \bar{x}$. Hence, by Lemma 2, $\lambda(x, \gamma) = 0 \Leftrightarrow \frac{u'(x)}{c'(x)} = \gamma$ for $x \geq \bar{x}$ and $\lambda(x, \gamma) > 0 \Leftrightarrow \frac{u'(x)}{c'(x)} > \gamma$ for $x < \bar{x}$. At the same time $C(x, \gamma) > 0$ for $x > \bar{x}$ and $C(x, \gamma) = 0$ for $x \leq \bar{x}$. The latter holds only if $f(x^*) = \rho A$ and, by monotonicity, for values of $\gamma > 1$ only if $f(0) = \rho A$. By the same argument used in the first part of the proof, there exists a unique equilibrium in the unconstrained region. The function $C(x, \gamma)$ is continuous in both arguments. By the Inada conditions, $C(0, \gamma) > 0$, $C(x^*, \gamma) < 0$, $\frac{\partial C(x, \gamma)}{\partial x} < 0$. Hence, there exists a unique equilibrium in the constrained region. The equation $C(\bar{x}, \gamma) = 0$ can be solved to find the unique value $\bar{\gamma} \in [1, \infty)$ that divides the unconstrained and constrained regions. From (15), $\psi = \frac{\rho}{2}$ in the unconstrained region and $\psi > \frac{\rho}{2}$ in the constrained region. ■

Proof of Proposition 2. *i.* Monotonicity of $f(x)$, $f(x^*) > \rho A$ and $f(0) \geq \rho A$, guarantee $f(x) > \rho A$ for all values of $x \in [0, x^*]$, except possibly in the limit $x \rightarrow 0$, where $f(x)$ may be equal to ρ . Hence, $\lambda(x, \gamma) > 0$ and $C(x, \gamma) = 0$ for all x . The properties of $C(x, \gamma)$ have been determined in the proof of Proposition 1. *ii.* The proof mimics the second part of the proof of Proposition 1. ■

Henceforth, denote with \tilde{x} the steady state value of consumption.

Proof of Proposition 3. The function $\varepsilon = \varepsilon(x)$ can be perturbed without affecting \tilde{x} . $G'(\tilde{x}) = \frac{\beta[\gamma g(\tilde{x})(1+\eta(\tilde{x}))+f(\tilde{x})(1-\varepsilon)]}{2\gamma g(\tilde{x})(1+\eta)}$. $G'(\tilde{x}) = -1$ iff $(2 + \beta) \gamma g(\tilde{x})(1 + \eta(\tilde{x})) + \beta f(\tilde{x})(1 - \varepsilon) = 0$, giving the critical value $\tilde{\varepsilon} = 1 + \frac{(2+\beta)(1+\eta(\tilde{x}))\gamma g(\tilde{x})}{\beta f(\tilde{x})}$. By direct computation, $\frac{\partial}{\partial \varepsilon} G'(\tilde{x}) \neq 0$ and $G'''(\tilde{x}) + \frac{3}{2}G''(\tilde{x})^2 < 0$ for $\varepsilon > 1$. By Proposition C.3.1 in Grandmont (2008), a supercritical flip bifurcation occurs at $\tilde{\varepsilon}$, and, thus, a stable cycle of period two exists close to the steady state for $\varepsilon > \tilde{\varepsilon}$. ■

Proof of Proposition 4. Follows from Theorem 2 in Grandmont (1989). ■

Proof of Proposition 5. Define $\Sigma \equiv \sigma(1 - \beta) + \beta(1 - \sigma)$. Using the equations in the text, we can reduce the equilibrium system to one equation in one unknown,

$$F(x^L) \equiv g(x^L) \gamma [2(1 - \beta\sigma) - \beta\Sigma] - \beta\Sigma f(x^L) - 2\beta A [\Sigma R^L + (1 - \sigma) R^H] = 0,$$

with $F(0) = -\infty$, $F(\infty) = +\infty$, $F'(x^L) > 0$. Hence, there exists a unique value that satisfies $F(x^L) = 0$. The rest of the system determines uniquely the other equilibrium values. One is left to check that $c'(x^H) x^H \gamma \leq 2\psi^H A$ and $u'(x^L) \geq c'(x^L) \gamma$, which can be shown to hold provided $f'(\cdot) < 0$, R^H is sufficiently high and R^L low. ■

Proof of Proposition 6. When $\lambda(x, \gamma) = 0$, $\frac{\partial \tilde{x}}{\partial \gamma} = -\frac{\tilde{x}}{\gamma} \frac{1}{\varepsilon(\tilde{x}) + \eta(\tilde{x})} < 0$. When $C(x, \gamma) = 0$, $\frac{\partial \tilde{x}}{\partial \gamma} = -\frac{\tilde{x}}{\gamma} \frac{\beta f(\tilde{x}) + 2(1 - \beta)\rho A}{\beta f(\tilde{x})(\varepsilon(\tilde{x}) + \eta(\tilde{x})) + 2(1 - \beta)\rho A(1 + \eta(\tilde{x}))} < 0$. Hence, in all cases, x is decreasing in γ . By Proposition 1, if $f(x^*) \leq \rho A$, (17) holds at an SMCE, for γ close to 1. Setting $\gamma = 1$, $u'(x) = c'(x)$, which guarantees efficiency for x . All the assets are in the hands of their best users. Hence, the first-best allocation is achieved. ■

Proof of Proposition 7. *i.* (18) is independent of γ .

$$ii. \frac{\partial \psi}{\partial \gamma} = \frac{\psi}{\gamma} \frac{\beta f(\tilde{x})(\varepsilon - 1)}{2A[\beta f(\tilde{x})(\varepsilon(\tilde{x}) + \eta(\tilde{x})) + 2(1 - \beta)\rho A(1 + \eta(\tilde{x}))]} \geq 0 \Leftrightarrow \varepsilon \geq 1. \blacksquare$$

$$\text{Proof of Proposition 8. } \frac{\partial \tilde{\varepsilon}}{\partial \gamma} = \frac{[(2 + \beta)(1 + \eta) + (2 - \beta)(1 - \tilde{\varepsilon})](1 - \tilde{\varepsilon})}{2[\beta f(\tilde{x})(\varepsilon(\tilde{x}) + \eta(\tilde{x})) + 2(1 - \beta)\rho A(1 + \eta(\tilde{x}))]} < 0. \blacksquare$$

Proof of Proposition 9. The scheme in the text is equivalent to a decrease in A . By direct computation, $\frac{\partial \psi}{\partial A} < 0$, $\frac{\partial x}{\partial A} > 0$ and $\frac{\partial \tilde{\varepsilon}}{\partial A} > 0$. ■