Credit Conditions and House Prices*

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Abstract

I develop a new search-theoretic framework to study the effect of credit conditions on real house prices. I incorporate a credit market and a frictional, heterogeneous housing market into a framework that has an explicit role for money. I find that easing of credit conditions, with higher loan-to-value (LTV) ratio and lower mortgage interest rate, pushes up real house prices, and vice versa. I show that changes to credit conditions faced by the marginal buyer affect trading volume and prices in the overall housing market. By modelling explicitly the frictional nature of the housing market, I capture the non-linear dynamics between the LTV ratio and real house prices. I demonstrate that changes to the LTV ratio have a greater effect on real house prices than changes to the mortgage rate and that regulating the LTV ratio is an effective tool for policymakers wanting to mitigate fluctuations in the housing market.

JEL: E44, G21, R31

Keywords: House Prices, Loan-to-value ratio, Borrowing rate, Search and matching

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1 Introduction

Over the past two decades, real house prices soared and then fell rapidly in many developed countries. For instance, the S&P/Case-Shiller Home Price Index shows that U.S. real house prices rose by 86% from trough to peak 1996-2006, followed by a 42% fall from 2006-2012. These large movements in real house prices had substantial implications for households and the real economy. Therefore, it is important to understand what causes these large movements in real house prices, and what tools policymakers can use to mitigate these fluctuations in the housing market.

In this paper I focus on studying the effect that exogenous changes to credit conditions have on real house prices. Prevailing credit conditions affect housing affordability and should therefore affect house prices. The credit variables I focus on are the mortgage interest rate and the loan-to-value (LTV) ratio. With the mortgage rate affecting the cost of borrowing and the size of the LTV ratio determining the amount house buyers need to bring from their own income and savings, both these credit variables affect housing affordability and their levels determine whether households can afford to enter the housing market as buyers. In terms of studying the effect that credit conditions have on real house prices, I analyse whether changes to the credit variables can explain the large movement in real house prices observed in the data. Furthermore, I compare the effect of the two credit variables to determine which is a greater driver of real house prices. Comparing the two credit variables helps determining whether using the conventional tool of controlling the policy rate or adopting the macro-prudential tool of regulating the maximum LTV ratio lending institutions can offer is a more effective tool for policymakers wanting to mitigate movements in the housing market.

Identifying the main sources of volatility in the housing market is important for macroeconomic stability. The empirical literature has emphasised the importance of the supply of credit, and hence the size of the LTV ratio, relative to the mortgage interest rate in affecting real house prices. Mian and Sufi (2009) and Adelino et al (2012), emphasise the positive correlation between the supply of credit and real house prices. Favara and Imbs (2011) demonstrate the causality, with an exogenous increase in the supply of credit causing a rise in real house prices.

Furthermore, Duca et al (2011) point out that U.S. house price models that focus on the borrowing rate as the main credit variable and omit changes to the supply of credit, fail to capture the large rise and fall in real house prices from the mid 2000s. Studying the data,[1] that result is not surprising since the expected negative correlation between the mortgage rate and house prices breaks down in the mid 2000s. Including a measure of supply of credit in terms of theLTV ratio available to the marginal, first-time buyer, improved the fit of their econometric model substantially. Geanakoplos (2010) points out the non-linear relationship between the LTV ratio and real house prices. For a higher LTV ratio agents can leverage themselves more, which has an increasing effect on house price growth.

However, the theoretical literature is yet to capture in detail the stylised facts of the data. In particular, the theoretical literature has had difficulty capturing the large swings in real house prices observed in the data as well as capturing the importance of lending standards and supply of credit relative to the mortgage interest rate. Therefore, as the main contribution of this paper, I develop a new theoretical approach to study credit and house prices. I develop a new search-theoretic framework in which I take seriously modelling the imperfect nature of the housing market. I model explicitly the search and matching frictions faced by housing market participants. I also model the credit constraints faced by potential house buyers and show how changes to credit conditions affect the number of buyers that can enter the housing market. By modelling in a relatively realistic way the process of buying and selling houses, this new framework allows me to capture large swings in real house prices, as a result of changes in credit conditions.

The housing search literature has emphasised the importance of modelling the frictional nature of the housing market, since the level of frictions directly affect prices and sales volume. Petursdottir (2014) demonstrates the importance of modelling explicitly the search and trading frictions faced by potential housing market participants. She finds that these frictions have a direct effect on real house prices and cause prices to become more volatile. Díaz and Jerez (2013) further demonstrate that search and matching frictions in the housing market cause house prices to become more volatile due to the trading delay caused by the frictions. Other

[1] As can be noted from the Federal Reserve Economic Data series on the 30 year fixed mortgage rate

models that use a search-theoretic approach to housing, such as Wheaton (1990), Krainer (2001, 2008), Piazzesi and Schneider (2009) and Ngai and Tenreyro (2013), have been able to explain many of the stylised facts of the housing market that models based on the Walrasian paradigm have failed to capture.[2] These findings emphasise the importance of modelling the frictional nature of the housing market. However, the housing search literature has not studied credit conditions.

In order to study the effect that changes to credit conditions have on real house prices I develop a new search-theoretic framework that combines money, credit and frictional, heterogeneous housing. The model is a novel extension of a New Monetarism alternating markets framework.[3] The framework uses a search-theoretic approach to monetary economics, has an explicit role for money and can capture frictions in the exchange process. The New Monetarism literature has not focused on studying housing markets. Aruoba et al (2011), Petrosky-Nadau and Rocheteau (2013) and He et al (2013) have introduced housing into the framework, however as a frictionless, general, divisible good that is traded in a perfectly competitive market. Furthermore, they have not studied the effect of credit conditions on real house prices.

I incorporate a credit market, in the form of a bank, that is willing to lend a portion of the market value of housing to eligible borrowers. This allows me to study the effect of changes to credit conditions since the bank charges mortgage interest rates and imposes a minimum downpayment. I assume that households differ in their productivity levels and thus income levels. Therefore, many households do not have enough income to be able to buy a house, hence needing access to credit. The downpayment requirement of the bank and the heterogeneity of households in my model, generates co-existence of money and credit, which the New Monetarism literature has had difficulty reconciling as demonstrated by Gu et al (2014). First-time buyers are more likely to be borrowing constrained than repeat buyers. First-time buyers rely solely on their savings as downpayment, whereas repeat buyers can use capital gains and sale proceeds from their previous

[2] For instance, the literature can explain the existence of so-called hot and cold markets. In a hot market, sales volume is high, time on the market is short and real house prices are high. The opposite conditions prevail in a cold market.

[3] An overview of the New Monetarism literature can be found in Nosal and Rocheteau (2011) and Williamson and Wright (2010).

home. Therefore, my focus is on the loan-to-value (LTV) ratio and mortgage rate on offer for the borrowing constrained potential first-time buyer.

Furthermore, I incorporate a frictional, heterogeneous housing market into the model, with housing being an indivisible, durable good. Houses differ with respect to size, location and features, and different types of housing attract different types of buyers. As in the downpayment constraint model of Ortalo-Magné and Rady (2006) I introduce heterogeneity of housing in terms of a property ladder. The property ladder allows the distinction between first-time borrowing constrained buyers, and unconstrained repeat buyers. My results complement the results found by Ortalo-Magné and Rady (2006). They focus on how income shocks faced by borrowing constrained first-time buyers affect prices in the overall housing market, whereas I show that changes to credit conditions faced by borrowing constrained buyers affect the overall housing market.

As is standard in the literature, I find that an increase in the mortgage interest rate has a negative effect on real house prices, and vice versa. Furthermore, I find that an increase in the LTV ratio pushes up real house prices, and vice versa. More importantly, I find that the relationship between the LTV ratio and real house prices is non-linear since real house prices are exponentially increasing in the LTV ratio. This non-linear effect is captured because the frictional nature of the housing market is modelled explicitly. An increase in the LTV ratio allows more households access to the housing market which increases the buyer-to-seller ratio. The increase in the buyer-to-seller ratio allows sellers to increase prices, but at the same time increases buyers' willingness to pay due to better resale conditions.

Importantly, I find that house prices are more sensitive to changes in the LTV ratio than in the mortgage interest rate. This result also holds true when applying my model to U.S. data. I simulate the model using U.S. data on the mortgage interest rate and the LTV ratio, to analyse whether changes to either of the credit variables can generate the large fluctuations in real house prices experienced between 1996 and 2012. The simulation exercise also allows me to compare the effect of each of the variables. I found that changes to the LTV ratio generated much larger movements in real house prices than changes to the mortgage interest rate. In fact, changes to the LTV ratio account for majority of the observed changes in real house prices during that time. Therefore, my research indicates that regulating lending standards and the LTV ratio may give policymakers a greater chance of mitigating fluctuations in real house prices than changing interest rates.

2 Environment

The model environment is a variant of a New Monetarism alternating markets framework.[4] Time is discrete and infinite, and a continuum of heterogeneous households, H, are the only trading agent in the model. Each period households have the potential of participating in trade in frictional decentralised markets as well as a frictionless centralised market. The New Monetarism framework has an explicit role for money because money ameliorates frictions in the trading process. Sellers in the decentralised markets are not willing to lend to buyers, due to limited commitment and lack of record keeping technology. Therefore, sellers require immediate compensation from trade. Money in the model, which can be considered fiat money, is a durable and divisible intrinsically worthless resource, and can be used as a medium of exchange in the decentralised markets. Buyers in the decentralised markets need to carry money into the markets if they want to be able to trade.

In this paper, decentralised market trade takes place with indivisible, heterogeneous, durable goods, called housing.[5] Housing is a heterogeneous good as each type of housing differs with respect to size, location, layout and features. Collectively, these aspects can be said to determine the hedonic value of homes. I capture the heterogeneity of housing in terms of a property ladder, and I model two distinct types of owner-occupied homes. Apartments, which can be considered "starter" homes, are at the bottom of the owner-occupied property ladder, whereas houses, which can be considered "dream" homes, are at the top. Houses

[4] The New Monetarism literature uses a search-theoretic approach to monetary economics. The New Monetarism frameworks are based on strong microfoundations. They have an explicit role for money and capture frictions in the exchange process. Prominent papers from the New Monetarism literature include Kiyotaki and Wright (1989, 1993) and Lagos and Wright (2005). Williamson and Wright (2010) and Nosal and Rocheteau (2011) provide an overview of the New Monetarism literature.

[5] Due to household preferences, housing cannot be used as a medium of exchange.

are larger and contain more and better features than apartments. Therefore, the hedonic value of houses is larger than that of apartments, meaning the fundamental value of houses is higher than the fundamental value of apartments. Housing is a durable good, and owner-occupiers receive utility from housing services of their home. Apartment-owners receive utility v^a from housing services, whereas house-owners receives utility v^h, with $v^h > v^a$ capturing the difference in the fundamental values.

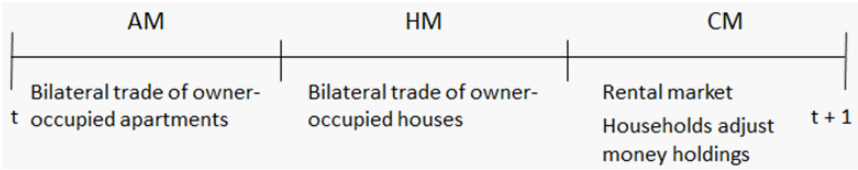
Each type of owner-occupied housing is traded in a distinct decentralised market. The first market to open each period, is a decentralised apartments market, denoted AM. Buyers and sellers in the AM trade bilaterally an existing stock of owner-occupied apartments. The stock of apartments is exogenously fixed at size $\varepsilon^a H$. The portion of households that are apartment-owners is denoted by ε^a. Therefore, the size of the apartment stock equals the number of households that are apartment-owners. The second market to open is a decentralised houses market, denoted HM. Buyers and sellers in the HM trade bilaterally an existing stock of owner-occupied houses. As with the stock of apartments, the stock of houses is exogenously fixed and equal to the number of households that are house-owners, $\varepsilon^h H$.[6] Housing status, preferences and credit availability determine whether households enter none, one or both of the decentralised markets. Housing status also determines whether a household that does enter a decentralised market does so as a buyer or a seller.

The last market to open each period is a frictionless centralised market, denoted CM. All households participate in the CM, where the main purpose is for agents to be able to adjust their money holdings. Households can earn money, m, in the CM by exerting effort y into producing a divisible, perishable, general good, x. One unit of effort, y, produces one unit of the general good, x. Households can also use their money holdings to consume the general good. Households obtain utility $u(x)$ for each unit of x they consume, with $u'(x) > 0$, $u''(x) < 0$ and $u(0) = 0$. Households that intend to enter the AM or HM decentralised markets as buyers in the subsequent period, also have to choose how much money, $\widehat{m}$, to

[6] I do not model production in the apartments or houses markets nor assume a construction sector. I assume that only households that are in the possession of an apartment or a house are able to become sellers in the AM or HM, respectively.

bring with them for trade. Additionally, a Central Bank that controls the supply of fiat money, operates in the CM. The Central Bank injects or withdraws money at rate γ, with $M_{t+1} = \gamma M_t$. The price of money in the CM is ϕ, with $\phi = 1/p$, where p is the price of the general good x. Therefore, intertemporal change in prices is depicted as $p_{t+1}/p_t = \phi_t/\phi_{t+1} = \gamma$. The focus of the model is on steady state equilibrium, in which the growth rate of money, γ, is fixed.[7]

Figure 1: Market timeline



Furthermore, a rental market operates in the CM. A portion ε^R of households do not possess owner-occupied housing. Those households buy housing services from the rental market. The rental stock is owned by Real Estate Investment Trusts (REITs), and supply is always sufficient to meet demand. The rental stock is separate from the owner-occupied housing stock, such that a household-owned apartment or house cannot be rented out. It can be assumed that the households own the REITs. However, since the rental market is a perfectly competitive market, the REITs generate zero profit.

Renters, which are the households buying housing services from the rental market, obtain utility v^R from housing services. As is standard in the literature, I assume that the utility households receive from living in a rental property, is lower than from living in an owner-occupied home. Hence $v^R < v^a < v^h$. This is because owner-occupied housing are generally of greater quality than rental properties. Furthermore, there are greater benefits associated with home-ownership than renting. For instance, home-owners are able to renovate their home and make any desired improvements, whereas renters are more restricted in their actions.

[7] The focus of the paper is to analyse whether modelling credit conditions in a search-theoretic framework can generate the large rise in real house prices experienced in the U.S. from 1996-2006 and the large fall from 2006-2012. Therefore, the focus will be on comparing three different steady state levels, given conditions in 1996, 2006 and 2012.

Figure 1 depicts the market timeline of the model. Households discount across periods with discount factor $\beta \in (0, 1)$, but not across markets within a period.

2.1 Heterogeneous Households and Housing

Trade in the housing markets is utility driven. Therefore, all households strive to move up the property ladder and acquire a house, since living in a house provides the highest utility of housing services. However, households are heterogeneous and differ in their productivity levels, y, and thus income, m. Therefore, not every household can afford to move to the top of the property ladder.

Property Ladder	Household Type	Portion	Housing Utility	Productivity	Income
Top	*House-owner*	ε^h	v^h	y^h	m^h
Middle	*Apartment-owner*	ε^a	v^a	y^a	m^a
Bottom	*Renter*	ε^R	v^R	y^R	m^R

y^R is uniformly distributed on $\left[\underline{y}^R, \overline{y}^R\right]$; m^R is uniformly distributed on $\left[\underline{m}^R, \overline{m}^R\right]$
$v^R < v^a < v^h$, $\overline{y}^R < y^a < y^h$ and $\overline{m}^R < m^a < m^h$

I assume that renters have the lowest productivity level, y^R, and thus the lowest income level, m^R. Renters are potential first-time buyers in the owner-occupied housing market. However, due to renters' low productivity and income levels, their money holdings are not high enough to allow them to buy an owner-occupied home. Therefore, renters are borrowing constrained and need access to credit to become first-time buyers. Changes to access to credit affect the extensive margin in the housing markets. Higher LTV ratio allows households with lower income access to the housing market, since it reduces the required downpayment. Therefore, heterogeneity in terms of renters' income levels is required, in order for changes to credit conditions to have an effect. Hence, I assume that renter's productivity level, y^R, is drawn from a uniform distribution with cdf $F(y)$ and support $\left[\underline{y}^R, \overline{y}^R\right]$. Similarly, income m^R is drawn from a uniform distribution with cdf $G(y)$ and support $\left[\underline{m}^R, \overline{m}^R\right]$. Given access to credit, renters only have sufficient resources to buy an apartment.

Apartment-owners have productivity level y^a, which signifies income level m^a in terms of their money holdings. Apartment-owners have higher productivity and income levels than the highest paid renter.

House-owners are both at the top of the property ladder and the income ladder. They have the highest productivity level, y^h, and thus income, m^h, of all household types. Since the focus of the paper is on the effect that changes to access to credit have on borrowing constrained first-time buyers, I do not need heterogeneity in the income levels of apartment-owners and house-owners since these households are not borrowing constrained, first-time buyers.

Linking housing and income status has precedent in the literature. Life-cycle models such as Ortalo-Magné and Rady (2006) assume that as households age, their income levels increase. They also assume that as households age, they move up the property ladder, due to being less credit constrained because of higher income. Ortalo-Magné and Rady (2006) therefore imply a relationship between housing status and income. Further, surveys conducted by the National Association of Realtors, show that repeat buyers have on average substantially higher income than first-time buyers.[8]

2.2 Credit Market

I assume that every period a credit market in the form of a bank stays open. The bank is willing to lend to potential first-time buyers that have sufficient income.[9] Unlike households sellers, the bank possesses record keeping technology that allows it to keep track of households' trading and financial histories. The bank requires buyers in the AM to use the apartment bought as collateral for their loan. Unlike household sellers, the bank has sufficient technology and resources to seize the collateral in case of a default. Due to the bank's extensive operations, a single

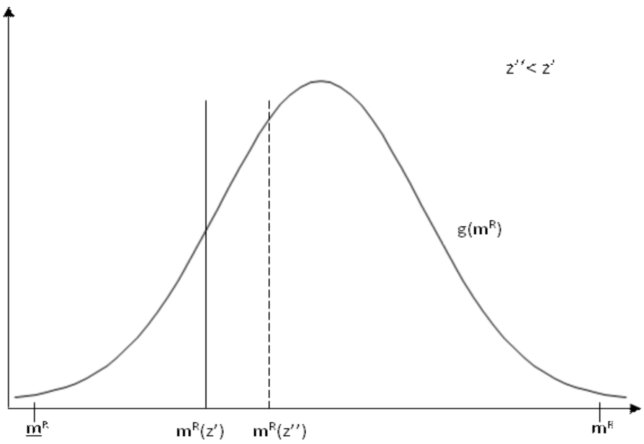
[8] According to the National Association of Realtors, 2013 Profile of Home Buyers and Sellers Survey, the median income of first-time buyers in the U.S. was $64,400 while the median income for repeat buyers was $96,000.

[9] The focus of the paper is on the effect that changes to credit conditions have on the borrowing constrained, first-time buyers and consequently the overall housing market. Therefore, I only model lending to renters, which are the borrowing constrained, potential first-time buyers. Apartment-owners are potential repeat buyers. Hence, their borrowing requirements differ from that of first-time buyers, and as the data shows, they do not need the same level of LTV ratio as first-time buyers.

default will not have a great effect on the bank's balance sheet and profitability. However, due to risk, the bank is only willing to lend a portion z of the market value of the apartment. Hence, $z \in (0, 1)$ represents the LTV ratio the bank is willing to offer. The maximum loan amount buyers in the AM can obtain is $l = zp^a$, where p^a is the market value of the apartment. Borrowers have to pay interest, i, on the loan.

I assume that the credit market is an exogenous market. As such, I do not model how the bank funds its supply of credit. Additionally, I assume that the LTV ratio, z, and the borrowing rate, i, are determined exogenously. The focus of the paper is on the effect of exogenous changes to the LTV ratio and borrowing rate on real house prices. Therefore, the reason behind the change in the credit variables is not essential. Endogenising the credit market is a topic for another paper.[10]

Figure 2: Access to credit for different income levels



The size of the LTV ratio determines whether a renter can get access to enough credit to be able to enter the AM as a buyer. For a higher LTV ratio, a lower-income renter is able to enter the AM due to a lower down-payment requirement. Figure 2, demonstrates the effect of different levels of the LTV ratio on the number

[10] In general the LTV ratio can be thought of as being endogenously determined. More specifically, the status of the housing market, and the level of prices will affect the LTV ratio. It can therefore be assumed that endogenising the LTV ratio would amplify the effect of credit conditions on real house prices.

of renters becoming potential buyers in the AM. For $z' > z''$, more loans are issued when $z = z'$ than when $z'' = z$ due to lower income renters having enough downpayment to enter the AM when $z = z'$.

2.3 Housing Market Entry and Trade

2.3.1 Apartments Market

Renters are potential buyers in the AM, and apartment-owners are potential sellers. Since the decision to trade in the housing markets is utility driven, all renters want to enter the AM as buyers in order to be able to move up the property ladder. However, entry depends on whether a renter can obtain enough credit to cover the price of the apartment, given his income level. I assume that a portion δ of renters are able to get access to credit. Therefore, given Law of large numbers, a portion δ of renters are able to enter the AM as buyers. The portion δ depends on the level of the LTV ratio, z, with $\delta = z$. Therefore, for a higher LTV ratio, more renters are able to enter the AM as buyers, since lower income renters can afford the required downpayment.

I assume that an apartment-owner enters the AM as a seller with probability ρ. If an apartment-owner is hit with the exogenous selling shock ρ then two factors have occurred. Firstly, the apartment-owner has found a house he is able to buy. An apartment-owner is not willing to put his apartment up for sale, unless he knows he has found a house to buy, and thus matched with a seller in the HM. If an apartment-owner sold his apartment without having found a house to buy, he would have to buy housing services from the rental market. Since rental utility is lower than the utility from living in an apartment, an apartment-owner will stay in his flat until he has secured his way up the property ladder. Secondly, the apartment-owner has secured the required resources to be able to move up the property ladder. However, as a part of these resources, the apartment-owner needs the sale proceeds from his existing apartment. Therefore, the apartment-owner cannot move up the property ladder until he has sold his apartment.

The apartments market is a frictional market, in which buyers and sellers are faced with search and matching frictions. Therefore, renters and apartment-owners that have entered the AM as buyers and sellers, respectively, are not guaranteed to be able to trade. I use an aggregate matching function, N, to capture the effect search and matching frictions in the AM have on the equilibrium outcome. The matching function summarises the technology that brings together buyers and sellers in the AM. The matching function $N \equiv N(b^a, s^a)$, represents the number of matches that occur between buyers and sellers in the AM each period, with $b^a = \delta\varepsilon^a H$ representing the measure of buyers, and $s^a = \rho\varepsilon^a H$ represents the measure of sellers. The matching function, N, is increasing in both arguments, strictly concave and exhibits constant returns to scale. The matching function can take on different functional forms as depicted in Petrongolo and Pissarides (2001).

Due to capacity constraints the number of matches can never exceed the short side of the market. However, due to coordination failure and matching frictions, the number of matches can end up being less than the short side of the market. Therefore $N(b^a, s^a) \leq \min\{b^a, s^a\}$.

The matching function can also give an indication of the probability of trade for a buyer and a seller, given the market tightness, θ^a, in the AM. The market tightness, θ^a, is equal to the ratio of buyers to sellers, with $\theta^a = b^a/s^a$. The probability of trade depends on the probability of a match. Therefore, the probability a seller can trade in the AM in a given period is:

$$\pi_s(\theta^a) = \frac{N(b^a, s^a)}{s^a} = N(\theta^a, 1) \tag{1}$$

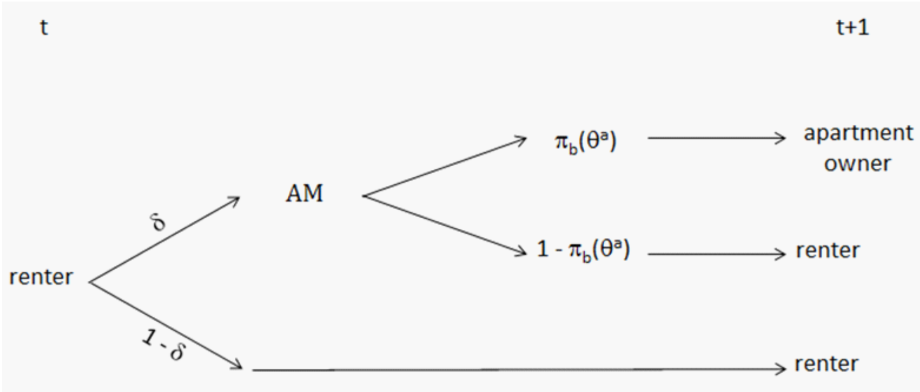
Similarly, the probability a buyer can trade in the AM is given as:

$$\pi_b(\theta^a) = \frac{N(b^a, s^a)}{b^a} = N\left(1, \frac{1}{\theta^a}\right) = \frac{\pi_s(\theta^a)}{\theta^a} \tag{2}$$

I assume that $\pi_s'(\theta^a) > 0$ and $\pi_s''(\theta^a) < 0$. Additionally, $\pi_s(0) = 0$ and $\pi_s(\infty) = 1$. Therefore, as the number of buyers in the AM increases the probability of trade for a seller increases. Similarly, any additional buyer that enters the market will reduce the likelihood of another buyer being matched. This emphasises the effect the level of the LTV ratio has on the extensive margin in the AM, and the probability of trade, since an exogenous increase in the LTV ratio will result in more buyers entering the AM.

Figure 3 demonstrate the transition of a renter in the model. If a renter can get access to enough credit, which occurs with probability δ, he will enter the AM

Figure 3: Renter transition



as a buyer. If that renter overcomes the search and matching frictions in the AM, which happens with probability $\pi_b(\theta^a)$, the renter will be successful at buying an apartment, and enters the following CM as an apartment-owner. However, renters that could not get access to enough credit, or failed to trade after entering the AM as buyers will enter the CM as continued renters.

In equilibrium the number of renters that buy an apartment equals the number of apartment-owners that sold their apartment. Therefore, the flow in and out of apartments is fixed, thus maintaining the portion of apartment-owners, ε^a. However, in order to maintain the portion of renters, ε^R, I assume that new renters enter the economy every period. The number of renters that enter the economy each period equals the number of renters that moved up the property ladder and bought an apartment that period.

2.3.2 Houses Market

Apartment-owners can enter the HM as buyers, and house-owners can enter the HM as sellers. Apartment-owners that were hit with the exogenous selling shock, ρ, at the beginning of the period, will enter the HM as buyers, contingent on selling their apartment in the previous AM, because they need the sale proceeds from the apartment to acquire a house. Due to search and matching frictions in the AM, not every apartment-owner is able to sell. Therefore, the level of frictions in the AM market will affect market activity in the HM. Apartment-owners enter the

HM as buyers with probability $\rho\pi_s(\theta^a)$, where $\pi_s(\theta^a)$ denotes the probability that an apartment-owner is able to trade in the AM market. Therefore, the measure of buyers in the HM is $b^h = \pi_s(\theta^a)\rho\varepsilon^a H$

House-owners that want to exit the economy will enter the HM as sellers. The exiting decision is exogenous in the model. I assume that house-owners exit with probability μ. Therefore, a portion μ of house-owners will enter the HM as sellers.

I assume that any apartment-owner that tries to sell his apartment has already matched with a seller in the HM. Therefore, the measure of sellers in the HM is equal to the number of apartment-owners that entered the AM as sellers. Therefore, $s^h = \mu\varepsilon^h H = \rho\varepsilon^a H$. However, apartment-owners that managed to sell their apartment in the previous AM, are the only apartment-owners that are able to commit to the match they have made with sellers in the HM. Hence, the probability of trade for a seller in the HM is $\theta^h = \pi_s(\theta^a)\rho\varepsilon^a/\mu\varepsilon^h = \pi_s(\theta^a)$, where θ^h also represents the market tightness in the HM. This captures the link between the market tightness of each market. Since apartment-owners have already made a match with a seller in the HM, any buyer that manages to enter the market is guaranteed to be able to trade. Therefore, the probability that buyers in the HM are able to trade is equal to 1.

The number of house-owners stays fixed in equilibrium. That is because every period, the number of apartment-owners that become house-owners equals the number of house-owners that leave the economy, due to selling their house.

Figure 4: Apartment- and house-owner transitions

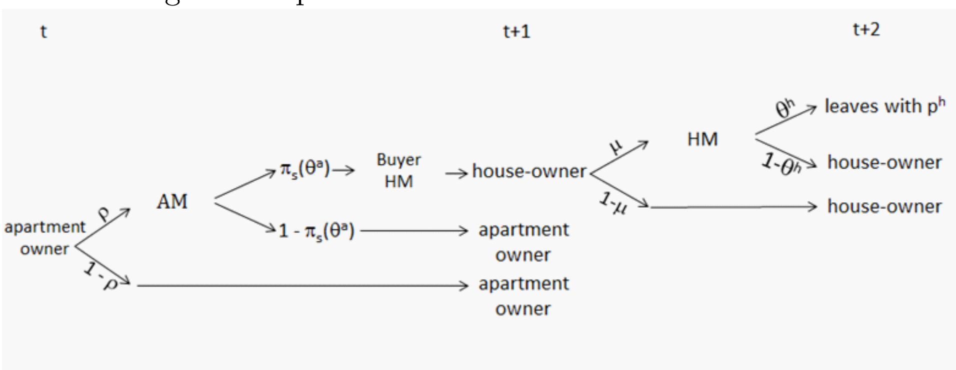


Figure 4 depicts the model transition of apartment-owners and house-owners.

Apartment-owners become sellers in the AM if they are hit with the exogenous moving shock, ρ. The parameter ρ can be thought of as the inverse of the average number of years apartment-owners live in their apartments. A higher ρ means that frictions in the housing market have eased, since apartment-owners stay for a shorter time in their apartment and are able to move up the property ladder quicker. If the apartment-owner is successful at selling his apartment, which occurs with probability $\pi_s(\theta^a)$, he can enter the HM as a buyer, and honour the agreement he had made with the seller he had found in the HM. If an apartment-owner is not hit with the moving shock ρ, or cannot sell his apartment, he will enter the subsequent CM as a continued apartment-owner. A house-owner enters the HM as a seller when he has found a buyer for his house. That occurs with probability μ. However, a house-owner that enter the HM as a seller is only successful at selling his house, if his counterparty of trade was able to sell his apartment in the previous AM.

3 Value Functions

Let W^{HH}, where $HH \in \{R, a, h\}$ denotes whether a household is a renter, apartment-owner or a house-owner, denote the value function for a household entering the CM. Let U_f^a and U_f^h represent the value functions of a household entering the AM and HM, respectively, with $f \in \{b, s\}$ where b denotes a buyer in either of the housing markets and s denotes a seller. A $\frown$ over a variable, signifies the value of the variable for the following period.

3.1 Centralised Market Value Functions

Renters in the CM were unable to buy an apartment in the previous AM, either because they could not enter the market, or because they were unable to trade in the market due to frictions. Renters will therefore enter the CM carrying money holdings m^R, they had hoped to use in the previous AM. Following is the choice faced by renters in the CM:

$$W^R(m^R) = \max_{(x^R, y^R, \phi_t \widehat{m}^R \in [\underline{y}^R, \overline{y}^R])} \left\{ u(x^R) - y^R + v^R + \beta \left[\delta \widehat{U}^a_b(\widehat{m}^R) + (1-\delta)\widehat{W}^R(0) \right] \right\} \tag{3}$$

$$\text{s.t. } x^R + \phi_t p^R + \phi_t \widehat{m}^R = y^R + \phi_t(m^R + T) \tag{4}$$

Renters obtain utility v^R from housing services, pay exogenously given rent, $\phi_t p^R$, and obtain lump-sum transfers, $\phi_t T$, from the Central Bank. Renters are faced with three choices in the CM. How much to consume of a divisible, general good x^R, consumption of x^R will yield utility $u(x^R)$, how much effort, y^R, to exert into producing good x^R and how much real balances, $\phi_t \widehat{m}^R$, to bring into the subsequent AM. Renters' productivity level, y^R, is drawn from a uniform distribution with cdf $F(y)$ and support $[\underline{y}^R, \overline{y}^R]$. Due to different productivity levels, renters have different income levels, m^R, which are drawn from a uniform distribution with cdf $G(m)$. The productivity distribution maps one to one with the distribution of income. Therefore, for any y^R within the support of distribution $F(y)$ the maximum amount of money a renter can bring into the AM is

$$\phi_t \widehat{m}^R = y^R + \phi_t(m^R + T) - x^R - \phi_t p^R \tag{5}$$

Using the quasi-linearity of preferences yields the following unconstrained problem:

$$W^R(m^R) = \max_{(x^R, \widehat{m}^R \in [\underline{y}^R, \overline{y}^R])} \left\{ I + u(x^R) - x^R - \phi_t \widehat{m}^R + \beta \left[\delta \widehat{U}^a_b(\widehat{m}^R) + (1-\delta)\widehat{W}^R(0) \right] \right\} \tag{6}$$

where $I = v^R - \phi_t p^R + \phi_t(m^R + T)$.

Renters enter the AM as buyers in the following period with probability δ in which case they bring real balances $\phi_t \widehat{m}^R$ to use as downpayment. The choice of real balances is determined by

$$\beta\delta \frac{\partial \widehat{U}^a_b(\widehat{m}^R)}{\partial \widehat{m}^R} = \phi_t \tag{7}$$

which shows that all renters with the same productivity level bring the same amount of money into the AM. Therefore, $F(y)$ generates a distribution of the possible money holdings, $G(\widehat{m}^R)$, that can be brought into the AM. This environment, with homogeneous sellers, and homogeneous goods being traded in the AM, satisfies all the properties under which Galenianos and Kircher (2012) demonstrate uniqueness of monetary equilibrium when terms of trade of an indivisible good are determined using price posting with directed search. Therefore, given a unique price, p^a, and a fixed, exogenous LTV ratio, z, the minimum required downpayment brought by buyers in the AM is $\widehat{m}^R = p^a(1-z)$. Hence, the portion of renters that can afford to enter the AM is given by $1 - G(p^a(1-z))$. Given the optimal consumption choice x^{R*} from $u'(x^R) = 1$, any renter with productivity equal or above $\tilde{y}^R$ with

$$\tilde{y}^R = \phi_t p^a(1-z) - \phi_t(m^R + T) + x^{R*} + \phi_t p^R \tag{8}$$

can afford to enter the AM. However, due to inflation money is costly to hold. With a unique p^a and z there is no reason for renters to bring more money holdings than $p^a(1-z)$. Therefore, all renters that enter the AM bring the same amount of real balances to cover the downpayment.

With probability $(1-\delta)$ renters cannot access the AM. Those renters enter the subsequent CM directly, and do not bring real balances with them since they will not trade in a decentralised market that period.

Following is the choice faced by apartment-owners in the CM:

$$W^a(m^a, l) = \max_{x^a, y^a, \widehat{m}^a} \left\{ u(x^a) - y^a + v^a + \beta \left[\rho \widehat{U}^a_s(\widehat{m}^a, \widehat{l}) + (1-\rho)\widehat{W}^a(0, \widehat{l}) \right] \right\} \tag{9}$$

$$\text{s.t. } x^a + \phi_t il + \phi_t \widehat{m}^a = y^a + \phi_t(m^a + T) \tag{10}$$

Apartment-owners bring any unused real balances, $\phi_t m^a$, with them into the CM. Similarly, apartment-owners carry into the CM the loan, l, they took to be able to afford the apartment they own. Furthermore, apartment-owners are faced with the same choices as renters. They choose how much of the general good, x^a, to consume, how much effort, y^a, to exert into production of the general good, and how much money, $\phi_t\widehat{m}^a$, to bring with them into the following period for trade in the HM. The choice of money holdings is determined by $\beta\rho\frac{\partial\widehat{U}^a_s(\widehat{m}^a,\widehat{l})}{\partial\widehat{m}^a} = \phi_t$ showing that all apartment-owners bring the same money holdings into the subsequent period. Apartment-owners receive utility v^a from housing services, and utility $u(x^a)$ from consumption of the general good. They pay interest i of their loan l.[11] Apartment-owners also receive lump-sum transfer $\phi_t T$ from the Central Bank.

Apartment-owners enter the subsequent AM as sellers with probability ρ. If hit with the selling shock, apartment-owners bring real balances $\phi_t\widehat{m}^a$ with them into the following period to use to acquire a house. Apartment-owners are not able to enter the AM as sellers with probability $(1-\rho)$ and enter the following CM directly. Since they will not be buyers in either of the decentralised markets, they do not have use for money, and will not bring real balances with them into the following period.

Due to the quasi-linearity of preferences the budget constraint (10) can be solved in terms of y^a, and plugged into (9) yielding:

$$W^a(m^a,l) = \max_{x^a,\widehat{m}^a}\left\{I' + u(x^a) - x^a - \phi_t il - \phi_t\widehat{m}^a + \beta\left[\rho\widehat{U}^a_s(\widehat{m}^a,\widehat{l}) + (1-\rho)\widehat{W}^a(0,\widehat{l})\right]\right\} \tag{11}$$

where $I' = v^a + \phi_t(m^a + T)$.

In the CM, W^h_N denotes the value function for a new house-owner. New house-owners are households that were apartment-owners at the beginning of the period. However, they were able to overcome all the housing market trading frictions, sell their apartment in the first subperiod, and buy a house in the second subperiod. New house-owners carry into the CM the loan, l, they obtained when buying their apartment. However, new house-owners have sold the apartment that was used as collateral for the loan. Therefore, new house-owners have to pay off the loan, with

[11]The average tenure length for apartment-owners in the U.S. has been around 7-10 years. Assuming a 30 year fixed rate mortgage, during that time apartment-owners are not able to build much equity in terms of paying off the principal. Rather, the largest part of the initial mortgage payments are interest payments. Therefore, in order not to have to keep track of how much each apartment-owner has paid off his principal, I assume as a simplification that the mortgage payments are interest payments only.

interest, in the CM.

New house-owners have reached the top of the property ladder. Therefore, they will not become buyers in either of the decentralised housing markets in future periods. Hence, unlike renters and apartment-owners, house-owners do not bring money holdings into the subsequent period[12]. Instead, their choice comprises how much to consume of the general good, x^h, and thus how much utility $u(x^h)$ they receive, as well as how much effort, y^h, to exert into production of the general good. New house-owners obtain utility v^h from housing services and receive transfer payments, $\phi_t T$. A new house-owner enters the HM as a seller with probability μ. With probability $(1-\mu)$ the new house-owner passes over the decentralised housing markets and goes directly to the subsequent CM as a continued house-owner.

$$W_N^h(0,l) = \max_{x^h, y^h} \left\{ u(x^h) - y^h + v^h + \beta \left[\mu U_s^h(0) + (1-\mu) W^h(0) \right] \right\} \tag{12}$$

$$\text{s.t. } x^h + \phi_t(1+i)l = y^h + \phi_t T \tag{13}$$

The budget constraint (13) for new house-owners can be solved in terms of y^h. Plugging into (12) yields the following unconstrained problem for a new house-owner:

$$W_N^h(0,l) = \max_{x^h} \left\{ u(x^h) - x^h + v^h - \phi_t(1+i)l + \phi_t T + \beta \left[\mu U_s^h(0) + (1-\mu) W^h(0) \right] \right\} \tag{14}$$

The value function for a continued house-owner is denoted as W^h. A continued house-owner is an agent that owned a house at the beginning of the period. Therefore, unlike the new house-owners they do not have a loan, l, they have to pay off in the CM. Other than that, continued house-owners are faced with the same choices and conditions as new house-owners.

$$W^h(0) = \max_{x^h, y^h} \left\{ u(x^h) - y^h + v^h + \beta \left[\mu U_s^h(0) + (1-\mu) W^h(0) \right] \right\} \tag{15}$$

[12] When inflation is present, money is costly to hold. Therefore, house-owners are not willing to bring real balances into subsequent periods since they do not have use for it.

$$\text{s.t.} \quad x^h = y^h + \phi_t T \tag{16}$$

Plugging the budget constraint into the objective function yields:

$$W^h(0) = \max_{x^h} \left\{ u(x^h) - x^h + \phi_t T + v^h + \beta \left[\mu U_s^h(0) + (1-\mu) W^h(0) \right] \right\} \tag{17}$$

3.2 Apartment Market Value Functions

Portion δ of renters are able to gain access to credit, allowing them to enter the AM as buyers. Buyers in the AM bring money holdings m^R into the market as downpayment. Due to search and matching frictions, a renter is able to trade with probability $\pi_b(\theta^a)$ in which case he pays the apartment price, p^a. However, buyers in the AM are faced with a feasibility constraint (19) since they cannot spend more on the apartment, p^a, than the money holdings, m^R, and loan amount, l, they hold. Additionally, buyers are faced with a borrowing constraint (20) since the bank is only willing to lend a portion, z, of the market value of apartments. As is demonstrated in Rocheteau and Wright (2005), when inflation is present and money is costly to hold, the feasibility and borrowing constraints are both binding.

Portion $(1 - \pi_b(\theta^a))$ of buyers in the AM are not successful at buying an apartment. Unsuccessful buyers carry their money holdings, m^R, forward to the subsequent CM where they continue as renters. A buyer's value function in the AM is represented by:

$$U_b^a(m^R) = \pi_b(\theta^a) W^a(m^R + l - p^a, l) + (1 - \pi_b(\theta^a)) W^R(m^R) \tag{18}$$

$$\begin{aligned} \text{s.t.} \quad p^a &= m^R + l && (19) \\ l &= z p^a && (20) \end{aligned}$$

Portion ρ of apartment-owners enter the AM as sellers, carrying money holdings, m^a, intended for use in the subsequent HM. However, due to coordination failure, only a portion $\pi_s(\theta^a)$ of sellers in the AM are able to trade. Apartment-owners that sell their apartment receive the market price, p^a. The successful sellers then proceed to the HM as buyers, carrying forward their money holdings, m^a,and their sale proceeds, p^a.

Apartment-owners do not succeed at selling their apartment with probability $(1 - \pi_s(\theta^a))$. The unsuccessful sellers cannot enter the HM as buyers due to lack of funding.[13] Therefore, they proceed to the subsequent CM as apartment-owners, carrying forward the money holdings, m^a, they had hoped to use to buy a house. A seller's value function in the AM is represented by:

$$U_s^a(m^a, l) = \pi_s(\theta^a)U_b^h(m^a + p^a, l) + (1 - \pi_s(\theta^a))W^a(m^a, l) \tag{21}$$

3.3 House Market Value Functions

Apartment-owners that sold their apartment in the previous AM are buyers in the HM. Therefore, portion $\pi_s(\theta^f)\rho$ of apartment-owners enter the HM as buyers. Apartment-owners do not attempt to sell their apartment unless they have already been matched with a seller in the HM.[14] Therefore, any agent that enters the HM as a buyer is guaranteed to be able to trade. However, the buyer is bound by the feasibility constraint in (23) since he cannot pay more for the house than the sum of his money holdings and sale proceeds from his apartment sale in the previous subperiod. After paying for the house, the house-buyer enters the subsequent CM as a new house-owner. A buyer's value function in HM houses is represented by:

$$U_b^h(m^a, l) = W_N^h(m^a + p^a - p^h, l) \tag{22}$$

$$\text{s.t.} \quad p^h = m^a + p^a \tag{23}$$

House-owners that want to exit the model economy are sellers in the HM. I assume that portion μ of house-owners enter the HM as sellers each period.

[13]Apartment-owners' income, m^a, is not sufficient to cover the price of a house, p^h. They also need the sale proceeds from their existing apartment.

[14]If the apartment is sold before a house is found, the former apartment-owner has to enter the rental market until he can find a house. Utility from rental services are lower than utility from owning an apartment. Therefore, the apartment-owner is better off staying in his apartment until a suitable house is found.

The probability a house-owners sells his house is equal to $\theta^h = \pi_s(\theta^f)\rho\varepsilon^f/\mu\varepsilon^h$. If successful at selling the agent receives the price, p^h, and moves away. With probability $\left(1-\theta^h\right)$ a seller is not able to sell his house, in which case he enters the subsequent CM as a house-owner. A seller's value function in HM houses is represented by:

$$U_s^h(0) = \theta^h \phi_t p^h + (1-\theta^h)W^h(0) \tag{24}$$

3.4 Competitive Search Equilibrium - AM

Terms of trade in the market with apartments are determined by price posting with directed search. Sellers in the AM post in the CM the terms of trade that are to prevail in the following period. Since apartments are an indivisible good, sellers post the price, $\phi_t p^a$, at which they commit to selling their apartment. When choosing the price to post, a seller needs to ensure that the value W^R a buyer obtaines from trading with him, is at least equal to the value, J_b, the buyers can receive from trading with other sellers. J_b can be considered the market value in the AM. Sellers choose a price, p^a, and with the price they post, they also implicitly choose their market tightness, θ^a. More specifically, with the price a seller chooses he simultaneously chooses the number of buyers that will direct their search to him.

Sellers in the AM are faced with the following problem:

$$\max_{p^a,\theta^a} \left\{U_s^a(m^a,l) = \pi_s(\theta^a)W_N^h(m^a+p^a,l) + (1-\pi_s(\theta^a))W^a(m^a,l)\right\} \tag{25}$$

$$\begin{aligned} \text{s.t.} \quad W_{t-1}^R(m^R) &= J_b && (26)\\ p^a &= m^R + l && (27)\\ l &= zp^a && (28) \end{aligned}$$

Plugging in from (6) yields the buyer's value constraint from (26):

$$W_{t-1}^{R}(m^R) = I + u(x^R) - x^R - \phi_{t-1}m^R + \beta\left[\delta U_b^a(m^R) + (1-\delta)W^R(0)\right] = J_b \quad (29)$$

From the feasibility constraint in (27), and the borrowing constraint in (28) $m^R = (1-z)p^a$ and $l = zp^a$. Using (27) and (28) the value constraint from (29) can be rearranged and solved in terms of $\phi_t p^a$ yielding:

$$\phi_t p^a = \frac{\theta^a W^R(0) + \delta\left[W^a(0) - W^R(0)\right] - \theta^a \frac{\overline{J}_b}{\beta}}{\theta^a\left(\frac{\phi_{t-1}/\phi_t}{\beta} - \delta\right)(1-z) + \delta\pi_s(\theta^a)\left(1 - z(1-i)\right)} \quad (30)$$

where $\overline{J}_b = J_b - k$.

Simplifying (25), the seller's problem in the AM becomes:

$$\max_{p^a,\theta^a}\left\{\pi_s(\theta^a)\left[\phi_t p^a + W_N^h(0) - W^a(0)\right]\right\} \quad (31)$$

$$\text{s.t. } \phi_t p^a = \frac{\theta^a W^R(0) + \delta\left[W^a(0) - W^R(0)\right] - \theta^a \frac{\overline{J}_b}{\beta}}{\theta^a\left(\frac{\phi_{t-1}/\phi_t}{\beta} - \delta\right)(1-z) + \delta\pi_s(\theta^a)\left(1 - z(1-i)\right)} \quad (32)$$

Inserting the constraint (32) into the objective function (31), generates an unconstraint problem for the seller in terms of the buyer-seller ratio, θ^a:

$$\max_{\theta^a}\left\{\pi_s(\theta^a)\left[(1-z)\frac{\theta^a W^R(0) + \delta\left[W^f(0) - W^R(0)\right] - \theta^a \frac{\overline{J}_b}{\beta}}{\theta^a\left(\frac{\phi_{t-1}/\phi_t}{\beta} - \delta\right)(1-z) + \delta\pi_s(\theta^a)\left(1 - z(1-i)\right)} + W_N^h(0) - W^a(0)\right]\right\} \quad (33)$$

Solving the first order conditions in terms of θ^a and inserting the values for $\frac{\overline{J}_b}{\beta}$ obtained from the constraint (32), yields the equilibrium price in the AM:

$$\phi_t p^a = \frac{\pi_b(\theta^a)\left[(1-\eta(\theta^a))(1-z)\delta J - \eta(\theta^a)\delta(1-z(1-i))K\right] - \eta(\theta^a)(1-z)(\frac{\gamma}{\beta} - \delta)K}{(1-z)\left[\pi_b(\theta^a)\delta(1-z(1-i)) + \eta(\theta^a)(1-z)(\frac{\gamma}{\beta} - \delta)\right]} \quad (34)$$

with $J = \widehat{W}^a(0) - \widehat{W}^R(0)$, $K = \widehat{W}^h(0) - \widehat{W}^a(0)$ and $\eta(\theta^a) = \frac{\pi_s'(\theta^a)\theta^a}{\pi_s(\theta^a)}$ is the elasticity of the matching function.

Proposition 1 *For given positive values of θ^a, z and δ and all possible values of γ, β, i and the form of the matching function there exists a unique monetary equilibrium price in the AM.*

Studying the price function from (34) a unique price exists for all possible values of β and γ. The parameter β represents the natural real interest rate, r, with $\beta = 1/(1+r)$. A decrease in the real rate, corresponding to an increase in β, pushes up the price in the AM. An increase in inflation γ has a negative effect on the equilibrium price. As Berentsen et al (2007) point out the surplus from trade under competitive search is divided between a buyer and a seller such that it compensates their search efforts. Therefore, the buyer's surplus from trade adjusts endogenously with inflation. The effect of the borrowing rate, i, is demonstrated in section 4.2. To generate a unique price in monetary equilibrium $\pi_b(\theta^a)(1-\eta(\theta^a))(1-z)\delta J > \eta(\theta^a)\delta(1-z(1-i))K - \eta(\theta^a)(1-z)(\frac{\gamma}{\beta}-\delta)K$ has to hold. Given a higher gain from becoming an apartment-owner than becoming a house-owner, the LTV ratio, z, has to be high enough such that enough buyers can enter the AM in order for a unique monetary equilibrium to exist. More specifically, there needs to be enough demand pressure such that the number of buyers exceeds the number of sellers. Therefore, the model is not suitable to study scenarios when there is excess supply of housing. Comparative statics are demonstrated in section 4.

3.5 Competitive Equilibrium - HM

There are no search and matching frictions present in the HM, since I assume that agents do not enter the HM unless they have found counterparty of trade. Therefore, prices cannot be determined using competitive search equilibrium. Instead, to capture the competitive nature of the housing market, I assume that households participating in the HM take price, p^h, as given. Houses in the model are an indivisible good. As such, buyers and sellers in the HM do not choose the quantity they want to trade. Either they trade one house or trade does not take place.

Buyers in the HM bring money holdings, m^a, with them into the market to use for trade. Due to inflation, money is costly to hold. Additionally, potential buyers in the HM are faced with trading frictions and are therefore not guaranteed to be able to trade in the market. Hence, to avoid holding idle real balances, buyers will bring just the sufficient amount to be able to trade in the HM.

Agents in the market for houses are faced with the feasibility constraint $p^h \leq m^a + p^a$. Houses are bigger, have greater features and are overall of greater quality than apartments. Therefore, the fundamental value of houses is higher than that of apartments, which yields $p^h > p^a$. Since buyers and sellers in the indivisible market for houses are price takers, they know prior to entering the market the value of p^h. Additionally, since sellers in the AM post their price, p^a, before the preceding CM closes, buyers in the HM will know the price of apartments before making the decision on how much money m^a to bring into the HM. Since money is costly to hold, buyers will bring an exact amount such that $p^h = m^a + p^a$. Therefore, the equilibrium real price in the HM is:

$$\phi_t p^h = \phi_t m^a + \phi_t p^a \tag{35}$$

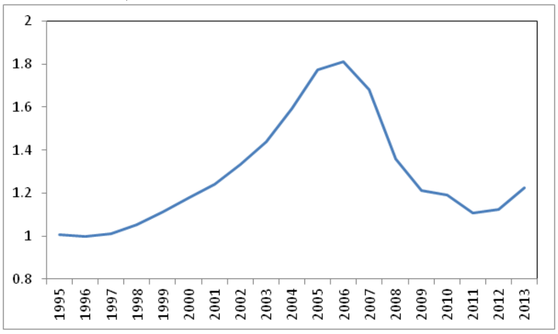
From section 3.1 it was established that all apartment-owners bring the same money holdings, $\phi_t m^a$, into the HM and section 3.4 demonstrates that for given demand pressures there exists a unique equilibrium price, $\phi_t p^a$. Therefore the equilibrium price in (35) is unique. The house price from (35) shows that the price for houses is positively correlated with the price for apartments.

4 Numerical Analysis

In this section I calibrate the model to analyse the effect of credit conditions on real house prices. The price function obtained in (34) is a function of both the LTV ratio, z, and the mortgage rate, i. Therefore, any change to either of the credit variables has a direct effect on the real price of apartments, $\phi_t p^a$. Furthermore, the real price of houses, $\phi_t p^h$, is a function of apartment prices, as can be noted in (35). Thus, changes to credit conditions faced by apartment buyers affect the overall housing market. Therefore, the comparative static results obtained in this section apply both to apartment prices and the price of houses.

Additionally, I conduct a simulation exercise to analyse whether changes to the LTV ratio or the mortgage rate can explain the large real house price movements

Figure 5: S&P/Case-Shiller Real House Price Index



experienced in the U.S. housing market in the past two decades. Figure 5 shows U.S. real house prices according to the S&P/Case-Shiller Home Price Index, the series is indexed to 1 in 1996.[15] From 1996-2006 U.S. real house prices rose by 86% whereas from 2006-2012 prices fell by 42%. I simulate the model given values of the LTV value ratio on offer for first-time buyers in the U.S. in 1996, 2006 and 2012 and given values of the 30 year fixed mortgage rate. I obtain a model prediction of the change in real house prices from 1996 - 2006 and 2006 - 2012, given changes to credit conditions. The simulation exercise also allows comparison between the two credit variables to determine which is the greater driver of real house prices.

4.1 Steady State Values for Calibration

I calibrate the model using estimations from U.S. data. Calibration values for the exogenous variables can be noted in Table 1. The values for the variables that describe the level of trading frictions, are derived using data from the U.S. Census Bureau. According to the data, the U.S. homeownership rate has on average been 64%. Therefore, 64% of households are either apartment- or house-owners. I assume that $\varepsilon^a = 0.4$ and $\varepsilon^h = 0.24$. Households that do not live in owner-occupied homes are renters. Therefore, 36% of households are renters with $\varepsilon^R = 0.36$. Average tenure length signifies the length of time that owner-occupiers stay in their home. Hence, tenure length can be thought of as the time from when an

[15] The data was collected on a quarterly frequency, but has been converted into annual frequency.

apartment-owner bought his apartment, until the time he is able to move up the property ladder. Shorter tenure length signifies easing of trading frictions since apartment-owners are able to move up the property ladder quicker. Tenure length in the U.S. has on average been 9.5 years, which translates into $1/9.5 = 10.5\%$ of apartment-owners attempting to enter the HM as buyers. Therefore, $\rho = 0.105$.

I use the urn-ball matching function to capture the search and matching frictions in the AM. The urn-ball matching function is well suited for studying housing markets, since it depicts how coordination failure among buyers produces matching frictions. The number of matches captured by the urn-ball matching functions is $N = s\left[1 - \left(1 - \frac{1}{s}\right)^b\right]$, which can be approximated as $N = s\left(1 - e^{-\frac{b}{s}}\right)$ for a large population size. Therefore, the probability a seller is able to trade in the AM is $\frac{N}{s} = \pi_s(\theta^a) = \left(1 - e^{-\theta^a}\right)$. Similarly, the probability that a buyer in the AM is able to trade is $\frac{N}{b} = \pi_b(\theta^a) = \left(1 - e^{-\theta^a}\right)/\theta^a$.

Data on mortgage rates is obtained from Federal Reserve Economic Data. Since the model focus is on the steady-state I use data on the 30 year fixed mortgage rate. From 1996 to 2012 the mortgage rate was on average around 6%. Therefore, the model is calibrated at $i = 6\%$.

Table 1		
Variable	**Description**	**Measure**
H	Number of households	$1,000$
ε^R	Portion of households that are renters	0.36
ε^a	Portion of households that are apartment-owners	0.4
ε^h	Portion of households that are house-owners	0.24
ρ	Portion of apartment-owners that enter AM as sellers	0.105
i	Mortgage Rate	0.06
z	Loan-to-Value Ratio	0.9
γ	Inflation rate	1.02
β	Discount factor	0.98
υ^h	Utility from living in a house	1
υ^a	Utility from living in an apartment	0.6
υ^R	Utility from living in a rental property	0.1

Using estimations from Duca et al (2012) on the LTV ratio on offer or first-time buyers indicates an average LTV ratio of 90%. Therefore, $z = 90\%$, which implies a downpayment requirement of 10%. The inflation rate is assumed to be 2%, which complies with the inflation target of the Federal Reserve. Therefore $\gamma = 1.02$. The discount factor is $\beta = 0.98$. For utility values of housing services, I assume that, $v^h = 1$, $v^a = 0.6$ and $v^R = 0.1$. The important factor for utility of housing services is to maintain $v^R < v^a < v^h$.

Changing the calibration values does not change the qualitative results described in this section. However, it does affect the quantitative results slightly. In what follows, I conduct comparative statics by changing the above mentioned exogenous variables, to study their effect on real house prices.

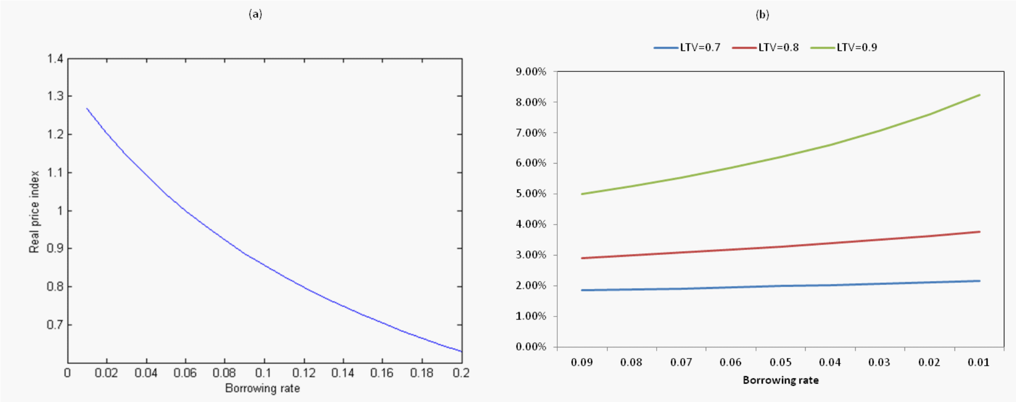
4.2 Mortgage Interest Rate

Solution 2 *An increase in the mortgage interest rate has a negative effect on real house prices, and vice versa. Changes to the mortgage rate, given high levels of the LTV ratio, result in greater house price growth than changes given low levels of the LTV ratio.*

The model generates a negative correlation between the mortgage interest rate and real house prices. Namely, an increase in the mortgage rate results in lower real house prices, and vice versa. This finding is in line with predictions of the standard literature. When housing is debt financed, a higher borrowing rate results in home-owners incurring higher interest payments on their mortgage. Therefore, housing becomes less affordable and buyers are not willing to pay as much to acquire a house. The negative correlation between the mortgage rate and real house prices is depicted in Figure 6(a). The real house price series is indexed to 1 at $i = 6\%$.

Figure 6(b) depicts the percentage change in real house prices given a one percentage point decrease in the mortgage interest rate, for different levels of the LTV ratio. A decrease in the mortgage rate from 6% down to 5% results in a 1.99% increase in real house prices, given a LTV ratio of 70%. However, if the LTV ratio is 90%, that same decrease in the borrowing rate results in a 6.21% increase in real house prices. Therefore, the interaction between the credit variables is non-linear, with higher values of the LTV ratio causing larger movements in real house

Figure 6: Mortgage interest rate - Real house price relationship



prices when the borrowing rate is decreasing. Furthermore, the link between the borrowing rate and real house prices is non-linear, since regardless of the level of the LTV ratio, real house prices are more sensitive to changes at lower levels of the mortgage rate.

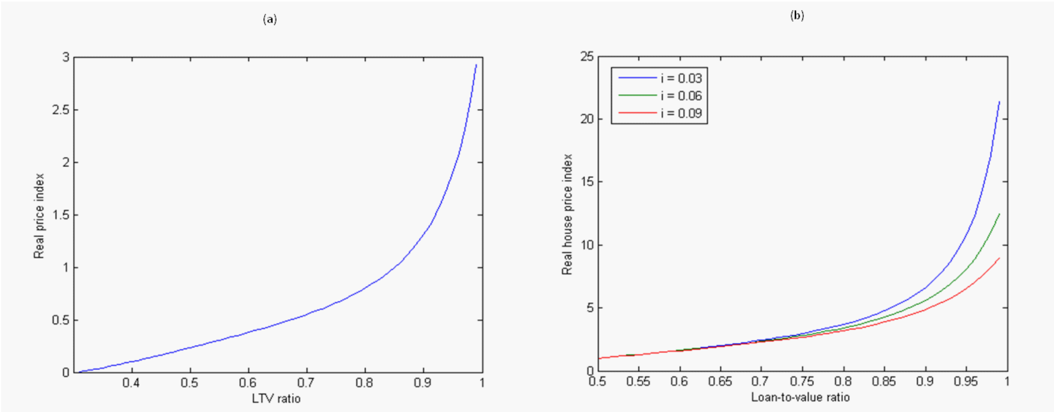
4.3 Loan-to-Value Ratio

Solution 3 *An increase in the LTV ratio pushes up real house prices, and vice versa. The relationship between the LTV ratio and real house prices is non-linear. Changes to the LTV ratio have a greater effect on real house price growth when the change occurs for high levels of the LTV ratio rather than low*

The model generates a positive correlation between the LTV ratio and real house prices, with an increase in the LTV ratio pushing up real house prices, and vice versa. Figure 7 shows the effect of changes to the LTV ratio on real house prices. The real house price series in Figure 7(a) is indexed to 1 at $z = 85\%$. The finding of a positive correlation is intuitive since an increase in the LTV ratio results in a lower required downpayment, which makes housing more affordable.

In addition to the positive correlation, I find that the relationship between the two variables is non-linear, as can be clearly noted in Figure 7. Real house prices are exponentially increasing in the LTV ratio. Therefore, changes to higher levels of the LTV ratio will result in a faster house price growth, than changes to lower levels of the LTV ratio.

Figure 7: LTV Ratio - Real house price relationship



Studying the data gives an indication of the non-linear relationship between the LTV ratio and real house prices, and Geanakoplos (2010) points out that the higher the LTV ratio, the greater the effect will be on real house prices. Fundamentally the LTV ratio has an exponential effect on the maximum price buyers can afford to pay. Given a fixed downpayment, an increase in the LTV ratio from 70% to 80% means that a buyer can afford a house with a 50% higher market value. Similarly, an increase in the LTV ratio from 80% to 90% allows a buyer to buy a house that is 90% more expensive, given a fixed downpayment. However, buyers do adjust the downpayment they bring into the housing market. Therefore, the scale of the price difference in the above example cannot be expected to translate directly into the price level. After calibrating the model, I find that an increase in the LTV ratio from 70% to 80%, and from 80% to 90%, result in a 45% and 64% increase in real house prices, respectively.

Unlike standard linear models, the model I develop in this paper is able to capture the non-linear, exponential effect of the LTV ratio on real house prices by being explicit about modelling search, credit and trading frictions in the housing market. An increase in the LTV ratio results in a lower downpayment requirement which allows more households access into the housing market. This influx of new buyers increases the buyer-to-seller ratio in the housing market. The higher buyer-to-seller ratio allows sellers to post a higher price. It also increases buyers willingness to pay. Bringing more money into the housing market for trade can increase the buyers probability of trade since he can trade with a seller that posts a higher price. Furthermore, since buyers know they will have to sell their house sometime in the future, resale conditions will also affect the price of housing. The increase in the buyer-to-seller ratio also increases buyers willingness to pay due to improved resale conditions. Therefore, in a search and matching model the real price is not only a function of standard fundamental values but also a function of frictions and probability of trade, which amplify the effect that changes to fundamental values have on real house prices.

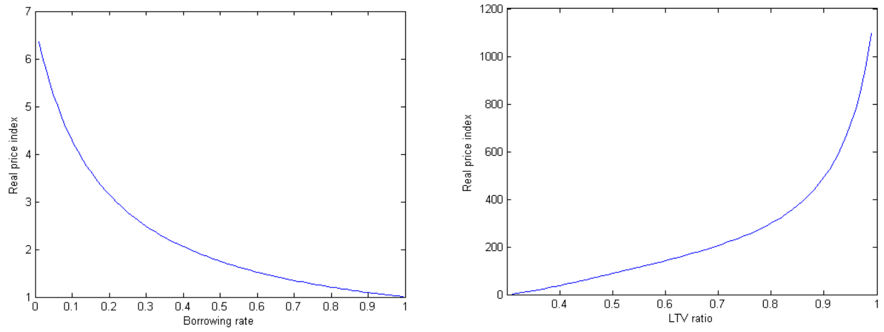
4.3.1 The Effect for Different Levels of the Borrowing Rate

Figure 7(b) shows the interaction effect of the LTV ratio and mortgage rate on real house prices. The real house price series are indexed to 1 at $z = 50\%$. An increase in the LTV ratio results in a larger real house price growth, given a lower level of the prevailing mortgage interest rate. A lower mortgage rate makes housing more affordable which is why real house prices are more sensitive to changes to the LTV ratio when the mortgage rate is low. This also reinforces the result obtained in figure 6(b). Real house prices are more sensitive to changes to the mortgage rate given higher levels of the LTV ratio, but a higher LTV ratio makes housing more affordable. Therefore, when changes to credit conditions occur simultaneously for high levels of the LTV ratio and low levels of the mortgage rate, large movements in real house prices can be expected.

Figure 8 compares the effect of the mortgage rate and the LTV ratio on real house prices to predict which credit variables is the greater driver of real house prices. Figure 8(a) shows the effect of the mortgage interest rate on real house prices. Assuming the mortgage interest rate does not reach higher values than 100%, I index the real house price series to 1 at the lowest achievable house price, which corresponds with the highest value of $i = 100\%$. Figure 8(b) shows the effect of the LTV ratio on real house prices. The real house price series is indexed to 1 at $z = 31\%$, which is the lowest LTV ratio at which equilibrium exists[16]

[16]For low levels of the LTV ratio, the income renters have to use as downpayment and the available credit is not enough to cover the posted price. Given the steady-state values, the model predicts that trade in the overall housing market shuts down for an LTV ratio of $z \leq 30\%$.

Figure 8: Comparison of the effect LTV ratio and borrowing rate have on real house prices



corresponding with the lowest achievable real house price. Comparing the figures it is obvious that the price range given changes to the LTV ratio is much larger than the price range given changes to the mortgage rate. Therefore, it can be concluded that changes to the LTV ratio cause larger movements in real house prices than changes to the mortgage rate. This result can be verified in the simulation exercise conducted in section 4.4.

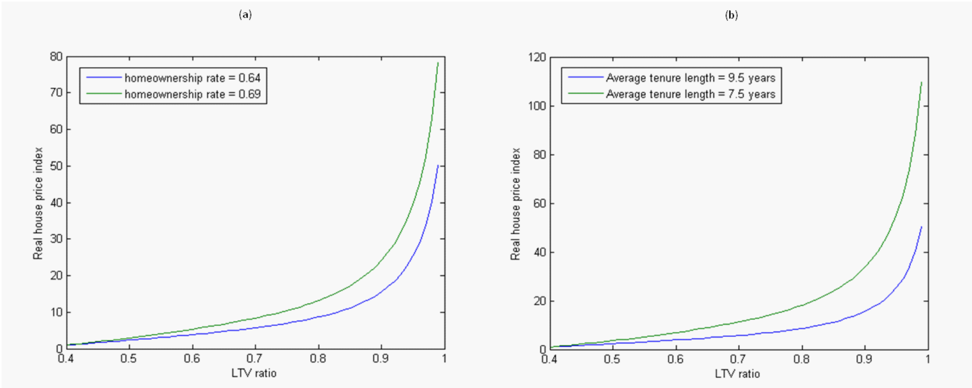
4.3.2 The Effect for Different Levels of Trading Frictions

Trading frictions have a direct effect on real house prices. Easing of trading frictions signifies that more households have gained access to the housing market, and more households are able to move up the property ladder. Therefore, easing of trading frictions is represented by an increase in the homeownership rate, resulting in a decrease in the portion of renters, ε^R, and a subsequent increase in $\varepsilon^a + \varepsilon^h$, and a decrease in average tenure length which is represented by an increase in ρ. Between 1996 and 2006 trading frictions in the U.S. housing market eased. The easing of trading frictions resulted in an increase of the homeownership rate from 64% to 69% and an estimated decrease in average tenure length from 9.5 years down to 7.5 years.[17]

Figure 9 depicts the effect changes to the LTV ratio have on real house prices,

[17] Data obtained from the U.S. Census Bureau

Figure 9: LTV ratio - real house price relationship, for different levels of trading frictions



given different levels of trading frictions. The series are indexed to 1 at $z = 0.4$. The figures clearly show that easing of trading frictions cause real house prices to become more volatile. Real house prices are more sensitive to changes to the LTV ratio for lower levels of trading frictions.

4.3.3 Remarks

The comparative statics conducted in the section demonstrate the non-linear relationship between each of the variables. The model predictions can help explain why the U.S. experienced a large rise and fall in real house prices between the years 1996 and 2012. Changes at the higher end of the LTV ratio and lower end of the mortgage rate, coupled with changes to the level of trading frictions, can be expected to amplify the growth and fall in real house prices. These were the conditions that prevailed in the U.S. from 1996-2012.

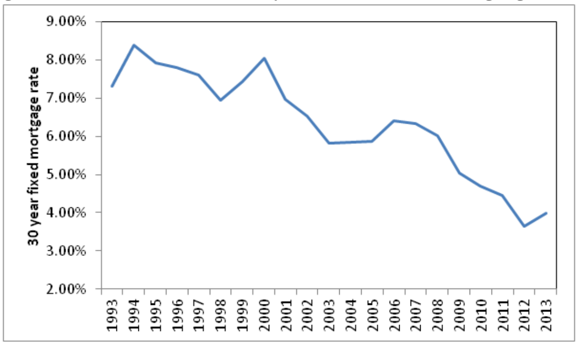
4.4 Quantitative Exercise

In this section I conduct a model simulation, using data on the 30 year fixed mortgage rate and the LTV ratio on offer to first-time buyers, to determine whether observed changes to credit conditions can account for the large increase in real house prices from 1996-2006 and the fall in real house prices from 2006-2012. The simulation exercise also provides the prediction of which credit variable was the greater driver of real house prices during that period. The model equilibrium is a steady-state equilibrium. Therefore, I compare three different steady-states, given credit conditions that prevailed at each time. The beginning of the house price boom in 1996, the house price peak in 2006 and the house price trough in 2012.

4.4.1 The Effect of Credit Conditions on House Prices

The data on the borrowing rate is obtained from the Federal Reserve Economic Data on the 30 year fixed mortgage rate. The 30 year fixed mortgage rate is chosen due to its prevalence in the U.S. mortgage market, as pointed out by Chew et al (2011). The data on the 30 year fixed mortgage rate can be noted in Figure 10.[18]

Figure 10: FRED: 30 year fixed mortgage rate



Given the finding of a negative correlation between the mortgage rate and real house prices, a peak in the mortgage rate should correspond with a trough in real house prices. The 30 year fixed mortgage rate peaked in 1994 at 8.38%. The rate then fell steadily until it bottomed out in 2003 at 5.83%. After staying steady in 2004 and 2005, the mortgage rate rose in 2006 followed by a continued fall until it bottomed out again in 2012 at 3.66%. These are the values I use in the simulation to represent mortgage rate conditions during the observed trough in real house prices in 1996, the peak in 2006 and the new trough in 2012.

[18]The data is collected on a weekly frequency, but averaged to convert it to annual frequency.

Table 2	Simulation Values	
	Mortgage rate	*LTV ratio*
Trough - 1996	8.38%	87%
Peak - 2006	5.83%	94.2%
Trough - 2012	3.66%	90%

Data on the LTV ratio is obtained from Duca et al (2012), which have generated a series on the LTV ratio obtained by first-time buyers in the U.S. housing market. First-time buyers are more likely to be borrowing constrained than repeat buyers. Therefore, changes to the LTV ratio have a greater effect on the behaviour of potential first-time buyers than repeat buyer, as repeat buyers might not choose to obtain the maximum loan available. I use the LTV ratio for first-time buyers, since I want to capture the change in the LTV ratio that was on offer and was obtained by borrowing constrained households.

The LTV ratio rose steadily from 1996 to 2006, except for a dip in 2001. At its trough, the LTV ratio was approximately 87% in the year 1996 while at its peak the LTV ratio was around 94.2% in the year 2006. The Duca et al (2012) series on the LTV ratio ends in the year 2010, when that LTV ratio had fallen to 90%. Given results from surveys on lending standards conducted by the Federal Reserve Board, it can be determined that the LTV ratio on offer continued declining after 2010. However, I use the 2010 value of 90% as the trough value for 2012. Table 2 depicts the simulation values used for each of the credit variables.

Table 3	Data	Model Simulation				
		Mortgage rate		LTV ratio		Both
	% change	*Price Index*	*% change*	*Price Index*	*% change*	% change
1996		1		1		
2006	86%	1.15	14.6%	1.60	60.4%	81.1%
2012	−42%	1.31	14.7%	1.19	−25.9%	−15.4%

The simulation results are demonstrated in Table 3. The first column shows the actual changes in real house prices according to the S&P/Case-Shiller Home Price Index. From 1996-2006 real house prices rose by 86%, whereas from 2006-2012 real house prices fell by 42%. Keeping all other factors constant according to the steady-state values in Table 1, the model generates a 14.6% increase in real house prices given the fall in the mortgage interest rate from 8.38% down to 5.83%. Therefore, the mortgage interest rate can only account for a small part of the actual increase in real house prices from 1996-2006. The model generates a 14.7% increase in real house prices from 2006-2012, given the decrease in the mortgage rate from 5.83% down to 3.66%. However, actual real house prices fell during that period. Therefore, mortgage rates cannot prevent a bust in the housing market. Given the simulation results, changes to the mortgage interest rate are not the main driver of real house prices.

Keeping all other factors constant, an increase in the LTV ratio from 87% to 94.2% results in a 60.4% increase in real house prices. Therefore, majority of the 86% rise in real house prices can be accounted for by the increase in the LTV ratio. Furthermore, the fall in the LTV ratio from 94.2% in 2006 down to 90% results in a 25.9% decrease in real house prices. Therefore, the fall in the LTV ratio can also account for most of the fall in real house prices in the U.S. from 2006 to 2012. These results demonstrate the relative importance of the LTV ratio over the mortgage rate as the main driver of real house prices.

Solution 4 *Changes to the LTV ratio have a greater effect on real house prices than changes to the mortgage interest rate. Majority of the U.S. real house price increase from 1996-2006 and decrease from 2006-2012, can be accounted for by changes to the LTV ratio.*

Studying the joint effect that changes to the LTV ratio and borrowing rate have on real house prices, it can be noted that changes to credit conditions can account for most of the increase in real house prices from 1996 to 2006. The lowering of the borrowing rate combined with the increase of the LTV ratio results in a 81.1% increase in real house prices. However, because of the fall in the mortgage rate from 2006, the joint effect of the borrowing rate and LTV ratio can only account for 15.4% of the fall in real house prices from 2006-2012.

5 Concluding Remarks

I develop a new search-theoretic framework to study the effect of changes to credit conditions on real house prices. The model I construct is the first theoretical framework to incorporate a frictional, heterogeneous housing market, credit market and borrowing constrained heterogeneous households into a framework that has an explicit role for money. By modelling explicitly the restrictions in accessing the owner-occupied housing market, and the search and matching frictions faced by households once they have entered the market, I am able to capture the stylised facts of the data on the effect of credit conditions on house prices.

As is standard in the literature I find that a decrease in the mortgage interest rate pushes up real house prices, and vice versa. Furthermore, I find that an increase in the LTV ratio has a positive effect on real house prices, and vice versa. With a higher LTV ratio, the downpayment constraint eases, allowing households with lower income access to the housing market. By capturing the effect that changes to access to credit have on the buyer-seller ratio in the housing market, I show that real house prices are exponentially increasing in the LTV ratio. This strong non-linear relationship between the LTV ratio and real house prices results in house prices being more sensitive to changes in the LTV ratio than the mortgage interest rate. This also holds true when applying the model to U.S. data, for which I show that changes to lending standards and access to credit can account for most of the changes in U.S. real house prices from 1996-2012.

Comparing the effect of credit variables on real house prices is vital such that policymakers can make informed decisions on the most effective tool to use to mitigate fluctuations in the housing market. Traditionally, central banks use their policy rate as a tool to affect the real economy, including the housing market through the effect that a policy rate change has on the mortgage interest rate. On the other hand, the Reserve Bank of New Zealand has adopted a macro-prudential measure, by regulating the maximum LTV ratio lending institutions can offer to buyers in the housing market. My research provides a tool to compare the effects of the two policies. I show that affecting the mortgage interest rate is not the most effective measure, and changing the interest rate does not prevent a boom or a bust in the housing market. However, regulating the LTV ratio and lending standards is a more direct approach and has the potential to prevent large, unsustainable fluctuations in house prices.

To focus my attention on the effect of exogenous changes to credit conditions on real house prices I have abstracted from many important aspects of credit and housing markets. However, the framework has the potential to be extended in order to capture these factors. For instance, a construction sector can be included to study the effect of construction and the potential over-supply of housing. The rental market can be endogenised in order to study rent-to-price ratio determinants. Furthermore, the credit market can be endogenised to study the reciprocal effects between credit conditions and real house prices.

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