

COSTLY CREDIT AND STICKY PRICES*

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Abstract

This paper constructs models where money and credit are alternative payment instruments, uses them to analyze sluggish nominal prices, and confronts the data. Equilibria entail price dispersion, where sellers set nominal terms they may keep fixed when aggregate conditions change. Buyers can use cash or credit, with the former (latter) subject to inflation (transaction costs). We provide strong analytic results and exact solutions for money demand. The calibrated models match price-change data well, with realistic durations, large average changes, many small and negative changes, and a decreasing hazard. They can also match macro and micro observations on money and credit.

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1 Introduction

This paper has two related goals: (1) construct a framework where money and credit serve as alternative payment instruments; (2) pursue within this setting a theory of endogenously sluggish prices that we can take to the data. It builds on the analysis of price dispersion in frictional goods markets by Burdett and Judd (1983), integrated into the general equilibrium monetary model of Lagos and Wright (2005). This environment generates monetary equilibria with a distribution of prices, and that means sellers can set prices in nominal terms that they may keep fixed when aggregate conditions change. Consumers can use cash or credit, where the former is subject to the inflation tax while the latter involves fixed or variable transaction costs. This makes the choice of payment method nontrivial and, we think, realistic. In particular, buyers tend to use cash (credit) more for small (large) purchases. Also, introducing costly credit allows us to avoid an indeterminacy of equilibrium that plagues some similar models, as we now explain.

To begin, consider Diamond (1971), where each seller (firm) of an indivisible good posts a price p , then buyers (households) sample sellers one at a time until finding one below their reservation price p^* . Clearly, for any seller, the profit maximizing strategy is $p = p^*$. Hence, there is a single price in the market. The Burdett-Judd model makes one change in Diamond's specification: buyers sometimes sample multiple prices simultaneously, and when they do they obviously choose the lowest. This implies there cannot be a single p , nor even a set of sellers with positive measure charging the same p , since that would leave open a profitable deviation to $p - \varepsilon$ (see fn. 8 below for more discussion). In fact, one can compute explicitly the Burdett-Judd distribution $F(p)$, where any p in the support \mathcal{F} yields the same profit: lower-price sellers earn less per unit, but make it up on the volume, by making sales with higher probability.

If one embeds Burdett-Judd in a monetary economy, it makes sense for firms to post prices in dollars. Then, if the money supply M increases, $F(p)$ shifts so

that the real distribution stays the same. Some firms can keep p fixed when M and the average price increase, however, and that means sticky prices according to the usage adopted here, even though sellers are allowed to adjust whenever they like at no cost. For those that stick to their nominal prices, real prices fall, but profits do not since sales increase. This much is similar to Head et al. (2012), but that paper has a technical problem. As discussed below, the combination of indivisible goods and price posting in monetary economies entails an indeterminacy – i.e., a continuum – of stationary equilibria. Hence, the model uses divisible goods, but then another problem pops up: what should firms post? Head et al. assume linear menus: a seller sets p and lets buyers choose any q as long as they pay pq . But there is no reason to think that linear menus maximize profit.

We show that incorporating costly credit also eliminates the indeterminacy, thus allowing us to avoid ad hoc linear pricing. Intuitively, holding more cash reduces the probability of needing to use credit, which delivers a well-behaved money demand function and unique monetary equilibrium. Hence we can revert to the original Diamond-Burdett-Judd specification, with indivisible goods, and get by with fewer “delicate” assumptions.¹ Also, to make a point, changing M is by design neutral. The point is not that neutrality holds in reality; it is that nominal stickiness does not logically imply nonneutrality. While we are not the first to show this, it may be worth re-emphasizing, and in any case there are many other reasons to construct a framework with both money and credit: (1) there is a long tradition of trying to build models along these lines (see fn. 6); (2) they are relevant for applied work because they allow substitution between payment methods as policy changes (Section 8); and (3) they allow us to confront the data in novel ways (Section 7). Given that it is desirable to pursue theories with both money and credit, the framework specified below provides a natural vehicle.

¹We do not take a stand on whether divisible or indivisible goods are more “realistic,” as that depends on the context, but we would certainly argue that indivisibility is an assumption on the physical environment and hence less “delicate” than a restriction on pricing strategies.

In terms of confronting data, Head et al. (2012) calibrate their model to match some features of price-change behavior, and show it also accounts for other facts. We perform a similar but more disciplined exercise, by calibrating to some features of pricing behavior, *plus* observations on the fractions of cash and credit usage, and money demand (the relation between real balances and interest rates). One specification involves a fixed cost of credit. It performs well in terms of money demand and can match the key facts in the pricing data, including long durations, large average price changes, many small changes, many negative changes, a decreasing hazard, and repricing behavior that varies with inflation. It cannot match these plus the shares of cash and credit in micro data, however. Hence, we also consider a proportional cost of credit. That version can simultaneously match the pricing, money demand and payment data.²

Section 2 reviews the literature. Section 3 describes our environment. Sections 4 and 5 consider fixed and variable costs. Section 6 discusses stickiness. Sections 7 and 8 discuss quantitative results and policy implications. Section 9 concludes.

2 Literature

Many sticky-price papers follow Taylor (1980) and Calvo (1983) by letting sellers adjust p only at certain times, or Rotemberg (1982) and Mankiw (1985) by letting them adjust only at a cost. Our sellers can change any time for free, but may choose not to. A few papers push imperfect-information or rational-inattention theories; see Mackowiak and Wiederhold (2009) for references. We are not against these devices, and indeed our formalization can be interpreted as saying sellers may be rationally inattentive to aggregate conditions. However, the focus is on search to exclusion of menu costs, because when Burdett and Menzies (2014) combine them, the analysis is much more difficult, and they find the majority of price dispersion

²To be clear, Head et al. (2012) match the price-change distribution and average duration, but cannot match money demand data at all well, and cannot match the shares of money and credit at all. Trying to be consistent with those data constitutes additional discipline.

in the data (about 70%) is due to search. Moreover, we want to see just how far we get without menu costs.³

The literature on Burdett-Judd pricing is large, including many labor-market applications following Burdett and Mortensen (1998). In monetary economics, prior to Liu (2010), Wang (2011) and Head et al. (2012) putting Burdett-Judd in Lagos and Wright (2005), Head and Kumar (2005) and Head et al. (2010) put it in the related model of Shi (1997). Other theories of price dispersion include Albrecht and Axel (1984) and Diamond (1987), where buyers differ not in terms of what they observe but their intrinsic (e.g., preference) type. A monetary version in Curtis and Wright (2004) delivers a two-point p distribution, for any number of types, which is less useful for our purposes. Also, as in Shi (1995) or Trejos and Wright (1995), and diametrically from us, that paper assumes goods are divisible while money is not, which limits applicability for policy and empirical issues.

Papers by Caplin and Spulber (1987) and Eden (1994) make a point similar to one of our messages, yet are also different in several ways: (1) They do not try to match data, which is big part of this project. (2) We have endogenous price dispersion while Caplin and Spulber proceed by assuming, not deriving, a distribution of prices they take to be uniform. (3) We build on the foundation for monetary economics in Lagos-Wright, which itself builds on Kiyotaki and Wright (1989,1993), Kochelakota (1998), Wallace (2001,2010) and others (see Williamson and Wright 2010, Nosal and Rocheteau 2011 and Lagos et al. 2014 for surveys). We adopt this approach because we think it is best to analyze monetary phenomena in environments that are explicit about the frictions that money-like institutions are meant to ameliorate. Moreover, while similar points can be made in, e.g., cash-in-advance or overlapping-generations models, it is natural here to use monetary theory based on search because it is already a key element of the environment – search is what drives price dispersion and sluggishness.

³Nonmonetary search models with menu costs, where prices are sticky in unit of account as in Benabou (1988,1992) or Diamond (1993), are special cases of Burdett and Menzies (2014).

There is much empirical work on price adjustment. Recently, Campbell and Eden (2014) find in grocery-store data an average duration between price changes of around 10 weeks, but we do not want to focus exclusively on groceries. Going back to Bils and Klenow (2004), in BLS data at least half of prices last less than 4.3 months, or 5.5 months if one excludes sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months, while Nakamura and Steinsson (2008) report 8 to 11 months, excluding substitutions and sales. These papers also find large fractions of small and negative price changes, plus some evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of 6 months for “reference” prices, which are those most often quoted in a quarter (presumably to avoid recording Saturday Night Specials as two changes in a week). Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices from 5.9 months for household appliances to 19.2 for chemicals, and also finds many small changes, with about 2/3 below 2%. More empirical work is surveyed by Klenow and Malin (2010).⁴

One empirical issue emphasized in the literature is that average price changes are fairly big, suggesting high menu costs, but there are also many small changes, suggesting low menu costs. Midrigan (2011) accounts for this by having firms sell multiple products, and paying a cost to change one price lets them change the rest for free. That is nice, but it does not mean one should not consider alternatives. Our model accounts for realistic durations, large average changes, many small changes, and many negative changes. It also delivers a decreasing hazard, and pricing behavior that depends on inflation, both of which are obvious problems for some alternative models. It also naturally generates price dispersion at low or

⁴In conversations with people in the area, there is more or less agreement that these are the stylized facts: (1) Prices change slowly, but exact durations vary across studies. (2) The frequency and size of changes vary across goods. (3) Two sellers changing at the same time usually do not pick the same \hat{p} . (4) Many changes are negative. (5) Hazards decline slightly with duration. (6) There are many small (below 5%) and many big (above 20%) changes. (7) The frequency and size of changes, and fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. Our models are consistent with all these facts.

zero inflation, consistent with the experience (Campbell and Eden 2014). Given this, we think that the approach in this paper should be part of the conversation on sticky prices and their implications.

The paper may also be considered a contribution to pure monetary economics. We deliver exact money demand functions expressing real balances in terms of interest rates, similar to the well-known results of Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966). The economic intuition is also similar, involving a comparison between the opportunity cost of holding cash and the cost of using financial services. However, those earlier papers involve partial-equilibrium analysis, or, more accurately, decision-theoretic analyses of how to manage one's inventory of cash given that it – as opposed to barter, credit or something else – must be used for transactions. While such inventory-theoretic models are still being applied to good effect (e.g., Alvarez and Lippi 2009,2014), and we recognize that this is partly a matter of taste, we like the Lagos-Wright structure because it is tractable, it is easy to integrate with mainstream macroeconomics, and it has proved useful in variety of other applications.⁵

The above-mentioned multiplicity of monetary equilibria in models with indivisible goods and posting occurs in a series of papers spawned by Green and Zhou (1998). Jean et al. (2010) provide citations and further discussion, but here is the idea: If all sellers post p then buyers' best response is to bring $m = p$ dollars to the market as long as p is not too big. If all buyers bring m then sellers' best response is post $p = m$ as long as m is not too small. Hence, any $p = m$ in some range is an equilibrium, and the selection matters for payoffs. Note that this involves a continuum of *stationary* equilibria, very different from the multiplicity of dynamic equilibria in most monetary models. Head et al. (2012) avoid this because, even if

⁵Incorporating price posting with indivisible goods and costly credit is also a further step in understanding this monetary framework, which has become a workhorse in the area. Previous analyses use Nash, Kalai or strategic bargaining, posting with random or directed search, competitive price taking, auctions and pure mechanism design. See the surveys cited above for more discussion and applications in finance, labor, international, growth, etc.

all sellers post p , buyers need not bring $m = p$ when q is divisible, but as we said that leads to other problems. In our alternative approach, even if all sellers post p , buyers need not bring $m = p$ when they have access to credit.

It is also relevant to discuss heterogeneity up front. As is well understood in other applications (e.g., Mortensen and Pissarides 1999), if firms are homogeneous, as they are here, theory does not pin down which one charges which p , but only the distribution $F(p)$. With heterogeneity, however, low cost firms prefer low p since they are more interested in high volume. Still, for any subset of firms with the same marginal cost, it does not matter which one posts which p . Hence, heterogeneity eliminates neither dispersion nor stickiness within sets of similar sellers. This is especially relevant for retail, where the marginal cost is the wholesale price. Even if a few retailers get better deals – e.g., one can imagine that Kmart has a quantity discount – many others face the same wholesale terms. Irrespective of fixed costs, wages etc., these sellers are homogeneous for our purposes.⁶

3 Environment

Each period in discrete time has two subperiods: first there convenes a decentralized market, called BJ for Burdett-Judd; then there is a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms interpreted as retailers with measure 1, and a set of households with measure \bar{b} . Households consume a divisible good x_t and supply labor ℓ_t in AD, while in BJ they consume an indivisible good y_t produced by firms at unit cost $\gamma \geq 0$. As agents are anonymous in the BJ market, they cannot use credit, unless they access a technology to authenticate

⁶As a final note on literature, we alluded above to the venerability of work on money and costly credit. This includes Prescott (1987), Freeman and Huffman (1991), Chatterjee and Corbae (1992), Lacker and Schreft (1996), Freeman and Kydland (2000), Dong (2011), Nosal and Rocheteau (2011) and Lucas and Nicolini (2013). See also Gomis-Porqueras and Sanches (2013), Li and Li (2013), Lotz and Zhang (2013), Araujo and Hu (2014), Gu et al. (2014) and Bethune et al. (2015). None of these consider our applications. It is also worth highlighting that while costly credit resolves technical problems in monetary posting models, price dispersion makes it natural for money and credit to coexist – i.e., sticky prices and costly credit are symbiotically related.

identity and record transactions at a cost. By incurring the cost, households can get BJ goods in exchange for commitments to deliver d_t dollars in the next AD; otherwise cash is required at point of sale.⁷ We consider both a fixed cost of credit δ and a proportional cost τ . To nest these, let the transaction cost in general be $C(d_t) = \delta \mathbf{1}(d_t) + \tau d_t$, where $\mathbf{1}(d_t)$ is an indicator function that is 1 iff $d_t > 0$.

Household utility within a period is $U(x_t) + \mu \mathbf{1}(y_t) - \ell_t$, where $U'(x_t) > 0 > U''(x_t)$, $\mu > \gamma + \delta$ and $\mathbf{1}(y_t)$ is again an indicator function. Let $\beta = 1/(1+r)$, with $r > 0$, be a discount factor between AD today and BJ tomorrow; any discounting between BJ and AD can be subsumed in the notation. We impose $\pi > \beta - 1$, where in stationary equilibrium π is the inflation rate, and the nominal interest rate is given by the Fisher equation $1+i = (1+\pi)(1+r)$. Notice $i > 0$, and the Friedman rule is the limiting case $i \rightarrow 0$. As usual, $1+i$ is the amount of money agents require in the next AD market to give up a dollar in the current AD market, whether or not such trades occur in equilibrium. Let x_t be AD numeraire, and assume it is produced one-for-one with ℓ_t , so the real wage is 1, merely to reduce notation. The AD price of money in numeraire is ϕ_t , and therefore $1/\phi_t$ is the nominal price level.

Firms enter the BJ market for free, but households must pay cost k , which determines participation $b_t \leq \bar{b}$. However, in the baseline model $k = 0$, so $b_t = \bar{b}$. Firms use BJ profits to buy AD goods, over which they have linear utility, but nothing of interest changes if instead they disburse profits to households as dividends. Each firm posts a price taking as given household behavior and the CDF of other firms' prices, $F_t(p)$, with support \mathcal{F}_t . Every household in BJ randomly samples n firms – i.e., sees n independent draws from $F_t(p)$ – with probability $\alpha_n = \alpha_n(b_t)$, depending on the buyer-seller ratio (market tightness). For our purposes it suffices to have $\alpha_1, \alpha_2 > 0$ and $\alpha_n = 0 \forall n \geq 3$, but this can be

⁷While the cost is paid by households, it is not hard to show the allocation is identical if it is instead paid by firms, as in elementary tax-incidence theory. Also, it does not matter if debt due in the frictionless AD market is denominated in dollars or numeraire.

generalized easily enough (e.g., Burdett et al. 2014). In terms of policy, let the money supply per buyer evolve according to $M_{t+1} = (1 + \pi) M_t$, with changes implemented in AD via lump-sum transfers, although the relevant results are the same if instead government uses new money to buy AD goods.

3.1 The Firm Problem

Expected real profit for a firm posting p at date t is

$$\Pi_t(p) = b_t \left[\alpha_1(b_t) + 2\alpha_2(b_t) \hat{F}_t(p_t) \right] (p\phi_t - \gamma), \quad (1)$$

where $\hat{F}_t(p) \equiv 1 - F_t(p)$. Thus, net revenue per unit is $p\phi_t - \gamma$, and the number of units is determined as follows: The probability a household contacts this firm and no other is $\alpha_1(b_t)$. Then the firm makes a sale for sure. The probability a household contacts this firm plus another is $2\alpha_2(b_t)$, as it can happen in two ways, this one first and the other second, or vice versa. Then the firm makes a sale iff it beats the other's price, which occurs with probability $\hat{F}_t(p)$. This is all multiplied by tightness b_t to convert buyer probabilities into seller probabilities.

Profit maximization means every $p \in \mathcal{F}_t$ yields the same profit and no $p \notin \mathcal{F}_t$ yields higher profit. As is standard in this kind of model, $F_t(p)$ is continuous and $\mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$ is an interval.⁸ Taking as given for now \bar{p}_t , and that \bar{p}_t is not so high that buyers reject it, $\forall p \in \mathcal{F}_t$ profit must equal the profit from \bar{p}_t , which is

$$\Pi_t(\bar{p}_t) = b_t \alpha_1(b_t) (\bar{p}_t \phi_t - \gamma). \quad (2)$$

As in Burdett-Judd, equating (1) to (2) and rearranging immediately yields:

Lemma 1 $\forall p \in \mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$

$$F_t(p) = 1 - \frac{\alpha_1(b_t)}{2\alpha_2(b_t)} \frac{\phi_t \bar{p}_t - \phi_t p}{\phi_t p - \gamma}. \quad (3)$$

⁸There cannot be a mass of firms with the same p because any one of them would have a profitable deviation to $p - \varepsilon$, since they lose only ε per unit and make discretely more sales by undercutting others at p . Similarly, if there were a gap between p_1 and $p_2 > p_1$, a firm posting p_1 can deviate to $p_1 + \varepsilon$ and earn more per unit without losing sales. These results are standard in BJ models, although later the analysis is complicated by the possibility of paying with money and the transaction costs of paying without money.

It is easy to check $F'_t(p) > 0$ and $F''_t(p) < 0$ for $\underline{p}_t < p < \bar{p}_t$. Also, since the lower limit satisfies $F(\underline{p}_t) = 0$, (3) implies

$$\underline{p}_t = \frac{\alpha_1(b_t) \phi_t \bar{p}_t + 2\alpha_2(b_t) \gamma}{\phi_t [\alpha_1(b_t) + 2\alpha_2(b_t)]}. \quad (4)$$

Now, to translate from dollars to numeraire, let $q_t = \phi_t p_t$ and write the distribution of real BJ prices as

$$G_t(q) = F_t(\phi_t p) = 1 - \frac{\alpha_1(b_t) \bar{q}_t - q}{2\alpha_2(b_t) q - \gamma}. \quad (5)$$

The support is $\mathcal{G}_t = [\underline{q}_t, \bar{q}_t]$, and we define $\hat{G}_t(q_t) \equiv 1 - G_t(q_t)$.

3.2 The Household Problem

Consider stationary equilibrium, where real variables are constant and nominal variables grow at rate π . This makes it convenient to frame the household problem in real terms. The state variable in AD is net worth, $A = \phi m - \phi d - C(d) + T$, where ϕm and ϕd are real money balances and debt carried over from the previous round of BJ trade, $C(d)$ is the transaction cost of having used credit, and T is a lump sum transfer by which new money is injected. Since preferences are linear in ℓ , we assume with no loss in generality that all obligations are settled in AD and households start BJ debt free. The state variable in BJ is real balances carried into that market, z . The AD and BJ value functions are $W(A)$ and $V(z)$.

Let $W^1(A)$ and $W^0(A)$ be the AD value functions for households that enter and skip the next BJ market, resp. Then $W(A) = \max\{W^1(A), W^0(A)\} = W^1(A)$ as long as some buyers enter. If k is the BJ entry cost, then

$$W(A) = \max_{x, \ell, z} \{\mu(x) - \ell + \beta V(z)\} \text{ st } x = A - k + \ell - (1 + \pi)z.$$

Eliminating ℓ and letting x^* solve $U'(x^*) = 1$, we get

$$W(A) = A + U(x^*) - x^* - k + \beta \max_z O_i(z), \quad (6)$$

where the objective function for the choice of z is $O_i(z) \equiv V(z) - (1 + i)z$, with i again given by the Fisher equation. As in Lagos-Wright, we immediately have:

Lemma 2 $W'(A) = 1$ and the choice of z does not depend on A .

The BJ value function satisfies (see the Appendix for more details):

$$V(z) = W(z + T) + [\alpha_1(b) + \alpha_2(b)] \left[\mu - \mathbb{E}_H q - \delta \hat{H}(z) - \tau(\mathbb{E}_H q - z) \right] \quad (7)$$

where $\hat{H}(q) \equiv 1 - H(q)$, and $H(q)$ is the CDF of transaction prices,

$$H(q) = \frac{\alpha_1(b) G(q) + \alpha_2(b) [1 - \hat{G}(q)^2]}{\alpha_1(b) + \alpha_2(b)}. \quad (8)$$

Notice $H(q)$ differs from the CDF of posted prices $G(q)$, because buyers with multiple draws of q pick the lowest. From (7), the benefit of higher z is that it reduces the expected cost of having to use credit.

For the BJ entry decision, it is easy to see $\Phi \equiv (1 + r)[W^1(A) - W^0(A)]$ is independent of A and satisfies

$$\Phi = [\alpha_1(b) + \alpha_2(b)] \left[\mu - \mathbb{E}_H q - \delta \hat{H}(z) - \tau(\mathbb{E}_H q - z) \right] - \kappa - iz, \quad (9)$$

where $\kappa = k/\beta$. The first term is the expected benefit of participating in BJ, as in (7), while $\kappa + iz$ is the cost. Then

$$b = \bar{b} \Rightarrow \Phi \geq 0; b = 0 \Rightarrow \Phi \leq 0; b \in (0, \bar{b}) \Rightarrow \Phi = 0. \quad (10)$$

3.3 Equilibrium

The above discussion characterizes the behavior of all agents given \bar{q} , which will be determined presently.

Definition 1 A stationary equilibrium is a list $\langle G(q), b, z \rangle$ such that: given $G(q)$, (b, z) solves the household's problem; and given (b, z) , $G(q)$ solves the firms' problem with \bar{q} determined as in Lemma 3 below.

Definition 2 (i) A nonmonetary equilibrium, or NME, has $z = 0$, so all BJ trades use credit. (ii) A mixed monetary equilibrium, or MME, has $0 < z < \bar{q}$, so BJ trades use cash for $q \leq z$ and credit for $q > z$. (iii) A pure monetary equilibrium, or PME, has $z \geq \bar{q}$, so all BJ trades use cash.

It is implicit in these definitions that payoffs are positive, so the BJ market does not shut down; this is always checked in the analysis below. Also, in NME prices must be described in numeraire q , while in MME or PME they can equivalently be described in numeraire or dollars, where $F_t(p) = G(\phi_t p)$ and $\phi_t = z_t/M_t$. Other variables that can be computed include AD consumption x and labor supply ℓ , but we do not need these in what follows.

The next step is to describe \bar{q} , depending on the type of equilibrium. The following is proved in the Appendix:

Lemma 3 (i) NME implies $z = 0 < \bar{q} = (\mu - \delta) / (1 + \tau)$. (ii) MME implies $z < \mu - \delta$ and $\bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$. (iii) PME implies $\bar{q} = z \geq \mu - \delta$.

Furthermore, by virtue of (7) and (5), the following is immediate:

Lemma 4 In MME, $O_i(z)$ is continuous; it is smooth and strictly concave $\forall z \in (\underline{q}, \bar{q})$; and it is linear $\forall z \notin (\underline{q}, \bar{q})$.

Additional results hold in special cases considered below. Also, as a benchmark, let us set $k = 0$ for now, so that $b = \bar{b}$ is fixed and the argument can be omitted from $\alpha_n(b)$; the case $k > 0$ is considered in Section 8.

4 Fixed-Cost Model

Consider first $\tau = 0 < \delta$. Given $\delta < \mu - \gamma$, when $k = 0$ there is a nonmonetary equilibrium with $b = \bar{b}$, where all households participate, and all transactions use credit – basically, the original Burdett-Judd equilibrium. We are more interested in monetary equilibrium. For this, consider the objective function $O_i(z)$. As shown in Figure 1, as a special case of Lemma 4, $O_i(z)$ is linear $\forall z \notin (\underline{q}, \bar{q})$ with slope $O'_i(z) = -i < 0$. Intuitively, a marginal unit of real balances will only affect the expected cost of having to use credit when $z \in \mathcal{G}_t$, and cash is a poor savings vehicle when $i > 0$. It is also easy to derive $O''_i(z) < 0 \forall z \in (\underline{q}, \bar{q})$.

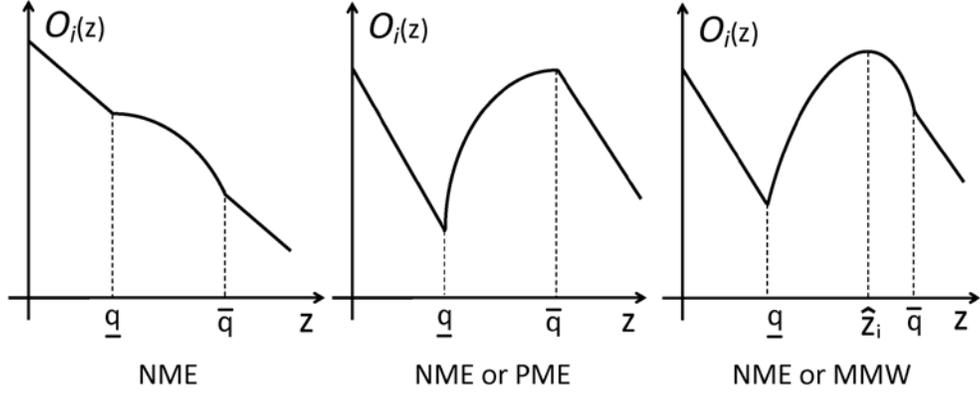


Figure 1: Possible Equilibria with a Fixed Cost

These results imply a unique $z_i = \arg \max_{z \in [\underline{q}, \bar{q}]} O_i(z)$. If $z_i \in (\underline{q}, \bar{q})$, as required for MME, we have the FOC

$$(\alpha_1 + \alpha_2) \delta H'(z_i) = i. \quad (11)$$

To check $z_i \in (\underline{q}, \bar{q})$, let \hat{z}_i be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. If $O_i^+(\underline{q}) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(\underline{q}) > 0$ then we need to check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = z_i$, as in the right panel. As a result of this analysis, with details in the Appendix, we have the following:

Proposition 1 *In the fixed-cost model:*

- (i) $\exists!$ a unique NME;
- (ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{i})$;
- (iii) \exists PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{i}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$;

where the thresholds satisfy $\bar{i} \in (\underline{i}, \infty)$,

$$\underline{i} = \frac{\delta \alpha_1^2}{2\alpha_2(\mu - \delta - \gamma)} \text{ and } \bar{\delta} = \mu - \frac{\gamma(2\alpha_2^2 + 2\alpha_1\alpha_2)}{2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2}.$$

Proposition 1 is illustrated in Figure 2, where one sees that money can be valued only if i is not too high. Note that while there is a continuum of PME

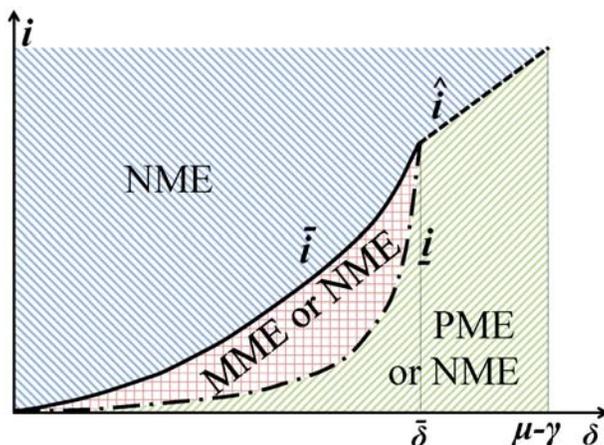


Figure 2: Equilibria in Parameter Space with Fixed Cost

when they exist, Proposition 1 still provides sharp existence conditions. In any case, we are mainly interested in MME, which exists for intermediate values of (δ, i) . Note however that the region for MME appears relatively small, as is made precise in calibration below. In terms of analytics, the simplicity of MME arises because buyers have a unique best response \hat{z}_i to $G(q)$, and sellers' best response conditions hold by construction at $G(q)$ and $\bar{q} = \mu - \delta$, independent of \hat{z}_i .

In MME, inserting $G(q)$ into (11) and rearranging, we get the explicit solution for money demand:

$$\hat{z}_i = \gamma + [\alpha_1^2 \delta (\mu - \delta - \gamma)^2 / 2\alpha_2]^{1/3} i^{-1/3} \quad (12)$$

This expresses real balances in terms of the cube-root of i , reminiscent of Baumol (1952), Tobin (1956), Whalen (1966) or Miller and Orr (1966).⁹ Our result derives from similar economic forces. The usual story behind Baumol-Tobin has an agent sequentially incurring expenses assumed to need currency, with a fixed cost of rebalancing z . The decision rule compares i , the opportunity cost of cash, with the benefit of reducing the number of financial transactions interpretable as trips to the bank. Our buyers make at most one transaction in BJ before rebalancing

⁹Actually, the first two get a square-root rule, while the latter two also get a cube-root. We get something more like a square-root in Section 5.

z , but its size is random, and spending beyond z (accessing credit) is costly. Still, they compare i with the benefit of reducing the use of financial services, again interpretable as trips to the bank, although one might say they now go there to get a loan and not to make a withdrawal.¹⁰

5 Variable Cost

Now consider $\tau > 0 = \delta$. One but not the only interpretation of τ is a proportional tax that can be avoided by using cash (see the calibration discussion below). Regardless of interpretation, variable costs are in some respects easier, make certain results more transparent, and avoid a technical issue that we waited until now to raise. The issue is that in economies with nonconvexities, like fixed costs, it can be desirable to use *lotteries* (Berentsen et al. 2002). Thus, a seller can post: “you get my good for sure if you pay p ; if you pay $\tilde{p} < p$ then you get my good with probability $P = P(\tilde{p})$.” In Section 4, when a buyer with $m = p - \varepsilon$ meets a seller posting p , he pays $p - \varepsilon$ in cash, ε in credit and δ in fixed costs; if ε is small, both parties would prefer to trade using cash only, to avoid δ , and have the good delivered with probability $P < 1$.

Now, one might try to argue that lotteries are infeasible, unrealistic or otherwise unwarranted – but that would be awkward, because ruling out randomized exchange may be considered uncomfortably close to ruling out nonlinear menus, something we criticized above. We cover the fixed-cost case since it has a tradition in the literature (recall fn. 6), but do not want to tackle lotteries in this exercise. To the extent that this is problematic, we now turn to the variable-cost case, which has no such problem. It is also useful to compare the two environments in terms of their analytic money demand functions and their quantitative performance.

¹⁰See Berentsen et al. (2007) or Chiu and Meh (2011) on explicitly adding banking to Lagos-Wright in a way consistent with our application (e.g., Chiu and Meh incorporate a fixed cost of going to the bank). We could do this, too, but prefer to remain agnostic as to the exact nature of credit; trips to the bank are only being used heuristically, just like in Baumol-Tobin.

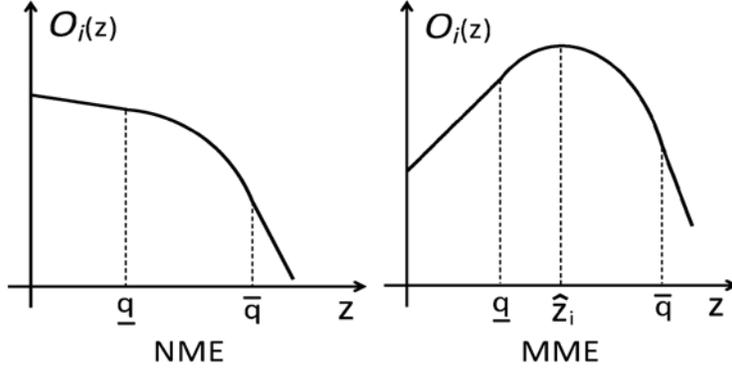


Figure 3: Possible Equilibria with a Variable Cost

The price distribution emerging from the firms' problem is the same as above. In particular, once one calculates

$$\bar{q} = \frac{\mu + z\tau}{1 + \tau} \text{ and } \underline{q} = \frac{\alpha_1(\mu + z\tau) + 2\alpha_2\gamma(1 + \tau)}{(\alpha_1 + 2\alpha_2)(1 + \tau)},$$

it is easy to check that $O_i(z)$ is now, conveniently, differentiable everywhere, including $q = \bar{q}$ and $q = \underline{q}$. As Figure 3 shows, this means there are only two possible types of equilibria, NME and MME. If $i > (\alpha_1 + \alpha_2)\tau$ there is a unique candidate NME, which is an actual NME iff payoffs are positive which holds iff $\tau \leq \mu/\gamma - 1$. If $i < (\alpha_1 + \alpha_2)\tau$ there is a unique candidate MME, which is an actual MME iff payoffs are positive which holds iff

$$\begin{aligned} \Phi &= (\alpha_1 + \alpha_2) [\mu + \tau\hat{z}_i - (1 + \tau) \mathbb{E}_H q] - i\hat{z}_i \\ &= \alpha_2 [\mu + \tau\hat{z}_i - \gamma(1 + \tau)] - i\hat{z}_i \geq 0. \end{aligned}$$

It can be shown that $i\hat{z}_i$ increases and Φ decreases with i . Letting $i^* = i^*(\tau)$ be the i that solves $\Phi = 0$, this proves:

Proposition 2 *In the variable-cost model*

- (i) $\exists!$ a unique NME iff $\tau \leq \mu/\gamma - 1$;
- (ii) $\exists!$ a unique MME iff $i < \min \{ \tau(\alpha_1 + \alpha_2), i^* \}$;
- (iii) \nexists a PME.

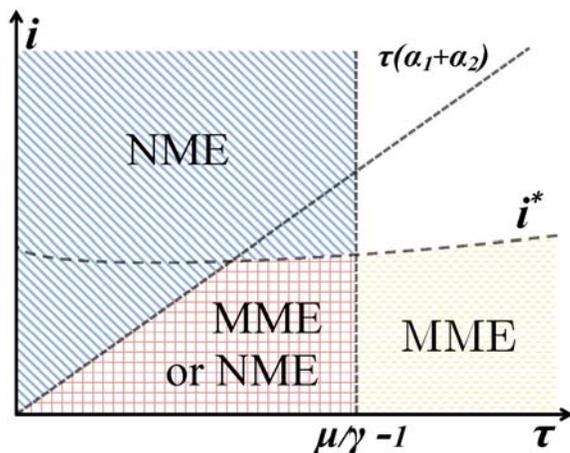


Figure 4: Equilibria in Parameter Space with Variable Cost

In MME, (11) yields a demand function somewhat different from (12),

$$\hat{z}_i = \gamma + \frac{(\mu - \gamma) \left[\tau + (1 + \tau) \sqrt{1 + 4\alpha_2 i / \alpha_1^2 \tau} \right]}{1 + 2\tau + 4\alpha_2 (1 + \tau)^2 i / \alpha_1^2 \tau}. \quad (13)$$

As Figure 4 illustrates, MME exists $\forall \tau$ iff i is not too big. Note that credit is used only if $q > \hat{z}_i$, and the maximum debt $\bar{q} - \hat{z}_i$ increases with i . One can also show $\partial \hat{z}_i / \partial \tau > 0$ and $\partial (\bar{q} - \hat{z}_i) / \partial \tau < 0$. As $\tau \rightarrow \infty$, $\bar{q} \rightarrow \hat{z}_i$ and buyers asymptotically stop using credit. We conclude that this model, like the fixed-cost version, is tractable and delivers a money demand function with natural properties. Plus it avoids completely the issue of lotteries, and as suggested by Figures 2 and 4 it allows MME for a larger set of parameters. As shown below, this leads to better quantitative performance.

6 Sticky Prices

With either a fixed or variable cost, the nominal price distribution $F_t(p)$ is uniquely determined, but individual-firm price dynamics are not. Consider Figure 5, drawn for the calibrated parameters in Section 7. With $\pi > 0$, the density F'_{t+1} is a right shift of F'_t . Firms with $p < \underline{p}_{t+1}$ at t (Region A) must reprice at $t + 1$, because,

while p maximized profit at t , it no longer does so at $t + 1$. But as long as the supports \mathcal{F}_t and \mathcal{F}_{t+1} overlap, there are firms with $p > \underline{p}_{t+1}$ at t (Region B) that can keep the same p at $t + 1$ without reducing profit.

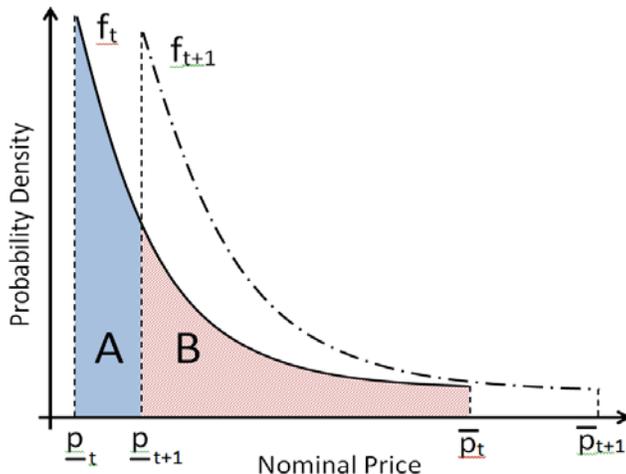


Figure 5: The Nominal Price Density Shifting with Inflation

Since equilibrium puts weak restrictions on an individual firm's pricing strategy, we add a payoff-irrelevant tie-breaking rule: if $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where \hat{p} is a new price; and if $p_t \in \mathcal{F}_{t+1}$ then:

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob } \sigma \\ \hat{p} & \text{with prob } 1 - \sigma \end{cases} \quad (14)$$

Hence, firms that are indifferent stick with probability σ to their current p .¹¹ Then we focus on symmetric equilibrium, where all changers pick a new \hat{p} from the same repricing distribution $R_{t+1}(\hat{p})$. As in Head et al. (2012), given σ , the unique symmetric equilibrium repricing distribution that generates $F_{t+1}(p)$ is:

$$R_{t+1}(p) = \begin{cases} \frac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[F_t(p) - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\underline{p}_{t+1}, \bar{p}_t] \\ \frac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[1 - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases} \quad (15)$$

¹¹In case it is not obvious, this is very different from Calvo pricing, where firms can be *desperate* to change p but are, by fiat, not allowed. Here any firm that wants to can adjust p , but there are some that are indifferent, and they are happy to randomize.

Varying σ generates a wide range of price dynamics, certainly wide enough to line up broadly with the facts, as demonstrated below. Hence, we content ourselves with stationary-symmetric equilibrium given σ , and let data inform us about appropriate values for σ . It is then routine to compute statistics from the model and compare these to the data. While it could be interesting to examine directly the distribution of prices, in the present exercise the concentration is on the distribution of price changes, $(p_{t+1} - p_t)/p_t$, because that is very much the topic of discussion in the research to which we are trying to contribute.¹²

7 Quantitative Results

Given an interest in the dynamics of nominal pricing, it seems appropriate to bring to bear monetary observations. Therefore, we try to match the fractions of money and credit usage in the micro payment data, plus some statistics derived from a standard empirical notion of money demand. As in Lucas (2000), this notion is $L_i = \hat{z}_i/Y$, where $Y = x^* + (\alpha_1 + \alpha_2) \mathbb{E}_H q$ is output aggregated over AD and BJ. If $U(x) = \log(x)$ then $x^* = 1$.¹³ Explicit formulae for L_i and its elasticity η_i are given in the Appendix, and we target these at $\mathbb{E}i$ in the data. Another key statistic is the average BJ markup $\mathbb{E}_G q/\gamma$ (computed here using posted prices, but the results are similar using transaction prices). This is a key statistic because, as is well known, BJ equilibrium can deliver anything from monopoly to marginal-cost pricing as α_1/α_2 varies. Two moments from price-change data are also used: the average absolute change; and the average duration between changes, which pins down σ in the tie-breaking rule.

¹²Moreover, to take $F(p)$ or $G(q)$ to the data one would have to decide which commodities to consider, or somehow aggregate them. It is of course interesting that pricing behavior in the model depends on the utility, cost and search specifications, but this makes direct empirical study of price distributions a challenge deferred to future research.

¹³Obviously $x^* = 1$ is a normalization. Also, generally utility can be written $\log(x) + \mu \mathbf{1}(y) - \psi \ell$, with μ capturing the relative importance of BJ vs AD consumption and ψ the importance of leisure. As in standard business cycle applications, ψ can be set to match average hours in the data $\mathbb{E}\ell$, but as the results below are independent of $\mathbb{E}\ell$ this is ignored.

7.1 Data

We focus on 1988-2004 because the price-change observations are from that period, although in principle information from other periods can also be used to calibrate parameters. For money, the best available data is the M1J series in Lucas and Nicolini (2012) that adjusts M1 for money-market deposit accounts, similar to the way M1S adjusts for sweeps as discussed in Cynamon et al. (2006). Lucas-Nicolini provide an annual series from 1915-2008 and a quarterly series from 1984-2013, and make the case that there is a stable relationship between these and (3-month T-Bill) nominal interest rates. We use their quarterly series, because we prefer a higher frequency, for reasons mentioned below, and because the years better correspond to the price-change sample. In these data the average annualized nominal rate is $\mathbb{E}i = 0.048$, which implies $L_{\mathbb{E}i} = 0.279$ and $\eta_{\mathbb{E}i} = -0.149$.¹⁴

Markup information comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008. In these surveys, at the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target for the gross margin is 1.3, in the middle of these numbers. A gross margin of 1.3 implies a markup 1.39, exactly as in Bethune et al. (2014). While this number is above what macro people used to use (see Lagos and Wright 2005), it is very much consistent with the recent micro data analyzed by Stroebel and Vavra (2015). Moreover, the exact value does not matter too much over a reasonable range, similar to the findings in Aruoba et al. (2011). Although their markup comes from bargaining, not posting, Aruoba et al. suggest an intuition that still applies: any markup above a fairly low number leads to sellers having enough market power to be decidedly different from competitive price takers.

¹⁴In the longer annual sample, $\mathbb{E}i = 0.038$, $L_{\mathbb{E}i} = 0.257$ and $\eta_{\mathbb{E}i} = -0.105$. Using these instead does not affect the results much. We also tried truncating the data in 2004, to better match the pricing sample and eliminate the recent crisis; that did not affect the results much either.

On the fraction of transactions using money and credit, there are various micro data sources. First, in terms of concept, we follow much of the related literature by interpreting monetary transactions broadly to include cash, check and debit card purchases. As rationale, we offer these points: (1) Checks and debit cards use demand deposits, which like currency are quite liquid and pay basically no interest, and it is irrelevant for this theory whether the money is in one’s pocket or one’s checking account. (2) A key feature of credit here is that it allows buyers to pay for BJ goods by working in the *next* AD market, while cash, check and debit purchases require working in the *previous* AD market, which can matter a lot because BJ transactions are random. (3) Importantly, we think, this notion of money in the micro data is consistent with the use of M1J.

Older calibrations of monetary models (Cooley 1995, chapter 7) target 16% for credit purchases, but much more information is now available. In detailed grocery-store data from 2001, Klee (2008) finds credit cards account for 12% of purchases, but that is just for groceries. In 2012 Boston Fed data, discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 22% of purchases in the survey and 17% in the diary sample. In Bank of Canada data, discussed by Arango and Welte (2012), the number is 19%. While there are some differences across studies, the comprehensive Boston Fed and Bank of Canada data are actually fairly close, and suggest a target of 20%. Also note that this number does not change very much over time, where the bigger evolution has been into debit cards, out of checks and to some extent out of currency.¹⁵

For price-change data we mainly use Klenow and Kryvtsov (2008), and benchmark their average duration of 8.6, but alternatives are also considered, since there

¹⁵These numbers are shares of credit transactions by volume. In Canadian data the fraction by value is double, 40%, since as theory predicts credit is used for larger purchases: “Cash [accounts] for 76 per cent of all transactions below \$15, and for 49 per cent of those in the \$15 to \$25 range ... whereas credit cards clearly dominate payments above \$50” (Arango and Welte 2012). However, in Boston Fed data, the fractions by value and volume are basically the same, 16% and 17% in the diary sample, resp. In conversations with analysts there was no clear consensus as to why American and Canadian data differ on value; in any case, we use volume, where they agree.

are differences across and within studies depending on details. Their average absolute price change is 11.3%, above average inflation due to many negative changes. Also, since the Klenow-Kryvtsov data are monthly, the model period is a month, and model-generated money demand data are aggregated to quarterly to line up with Lucas-Nicolini. A month is also natural because it is typically the period during which credit card debt can be paid without interest charges. As discussed below, different observations pin down different parameters, but they are actually determined jointly by minimizing the distance between model and data. While we cannot hit the targets exactly, we get very close except where indicated.

7.2 Findings

Calibration results are in Table 1. Consider first the fixed-cost model, which hits all of the targets except the fraction of credit transactions, because parameters are constrained to stay in the region where MME exists. In particular, trying to set δ to get 20% BJ credit transactions implies MME does not exist for some of values of i in the sample. Hence, we use the smallest δ consistent with MME at the maximum observed $i = 0.085$, which only yields 8% credit transactions. This δ is only about 1.5% of the BJ utility parameter μ . The value of μ comes primarily from matching average real balances. The value of γ , about half of μ , comes primarily from the markup. The probability of sampling one price (two prices) in BJ is $\alpha_1 = 0.18$ ($\alpha_2 = 0.23$), somewhat lower than calibrations in related studies, because our period is a month.¹⁶

	BJ utility μ	BJ cost γ	credit cost δ/τ	$pr(n = 1)$ α_1	$pr(n = 2)$ α_2	tie-break σ
Fix	104.06	57.37	1.643	0.18	0.23	0.90
Var	21.47	11.53	0.063	0.12	0.17	0.90

Table 1: Baseline Calibration

¹⁶As usual a convenient feature of search models is that they can be fit to different frequencies simply by scaling parameters like arrival and discount rates.

Now consider the variable-cost model. In contrast to the fixed-cost model, this one approximates all targets well, including 20% for BJ credit, without compromising existence of MME. Note μ and γ are lower than the fixed-cost case, so BJ goods are now less important relative to AD goods, but γ/μ is very similar. With $\mathbb{E}_H q$ around 15.4, average transaction cost $\tau \mathbb{E}_H q$ is about 0.97. Scaled by BJ utility, $\tau \mathbb{E}_H q / \mu = 0.045$, compared to $\delta / \mu = 0.016$ in the fixed-cost case. To judge whether a value of $\tau = 0.063$ is reasonable, note the average US sales tax is 0.064. Adding fees on credit cards that average around 1.5-2% (without counting small fixed costs per transaction), we are still very much in the ballpark. Also notice α_1 and α_2 are lower than in the fixed-cost model. However, one result that is constant across specifications is the tie-breaking parameter $\sigma = 0.9$, implying that indifferent sellers reprice a mere 10% of the time.

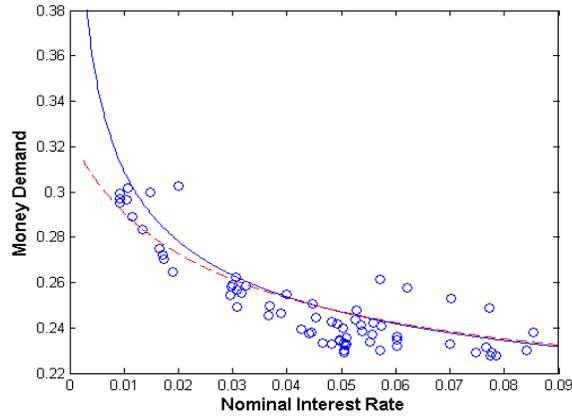


Figure 6: Money Demand for Different Specifications

Figure 6 shows money demand, with the solid curve from the fixed-cost model and the dashed curve from the variable-cost model. The fit is obviously good in both cases, although the curves are somewhat different at low values of i . While this difference could be quite important for other issues – e.g., the welfare cost of inflation – it seems less relevant for our purposes. In terms of the price-change evidence, Figure 7 shows the predicted distributions for the two specifications and

the Klenow-Kryvtsov data. Both models capture the overall shape of the empirical histogram, although the fit is not perfect. We now argue, however, that the models are broadly consistent with several facts deemed important in the literature.

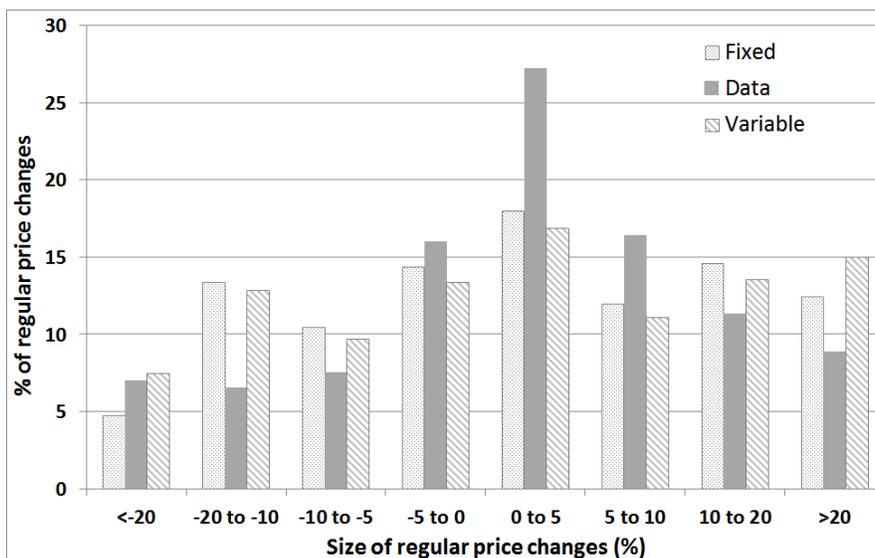


Figure 7: Distribution of Price Changes

In particular, the average absolute change is 11.5% in the fixed-cost model, 12.3% in the variable-cost model and 11.3% in the data. While this is a calibration target, again, we only approximately hit it. Statistics not targeted include the fraction of small changes (below 5% in absolute value), which is 44% in the data, 31% in the fixed- and 30% in the variable-cost model. So, on this, theory is slightly off, but not dramatically so.¹⁷ Similarly, the fraction of big changes (above 20% in absolute value) is 16% in the data, 20% in the fixed- and 23% in the variable-cost model, while the fraction of negative changes is 37% in the data and 43% in both models. So, on these, theory is not too far off. It is not trivial to match these facts in other models. Our setup does fairly well, even without complications, like

¹⁷Eichenbaum et al. (2015) find a fraction of small prices changes lower than other studies, and suggest this is because others did not correctly correct for measurement error. Here we take the Klenow-Kryvtsov numbers at face value, but it is perhaps worth mentioning that their fraction of small changes may be an overestimate.

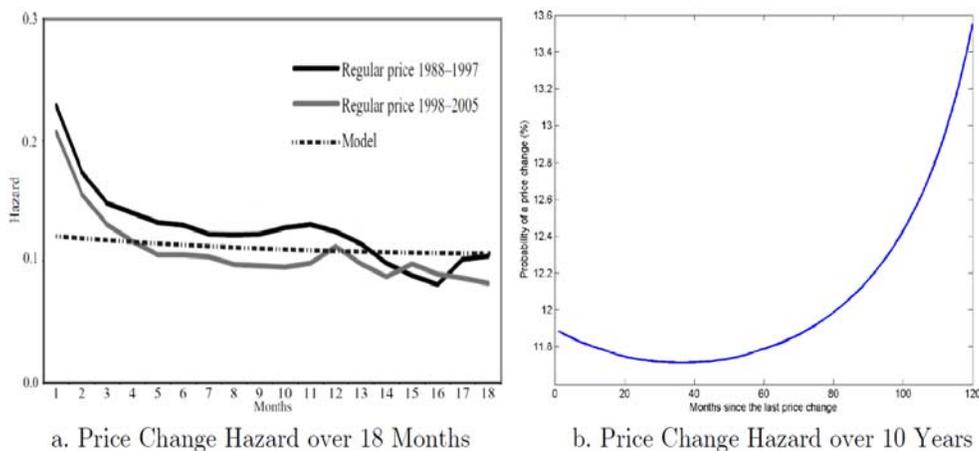


Figure 8: Price Change Hazards

heterogeneity, idiosyncratic shocks or multi-product firms. Basically, a shifting F_t and a tie-breaking rule calibrated to duration generate large average, many small, many big and many negative adjustments.

Figure 8 shows the hazard (probability of changing p as a function of time since the last change). The left panel shows the hazard decreasing over 18 months from the data in Nakamura and Steinsson (2008), who argue this is interesting, and from the model with either a fixed or variable cost since on this they are virtually identical. We do not generate enough action at low durations, evidently, but at least our hazard slopes downward, something Nakamura and Steinsson say is hard to get with other theories. Of course, one should not expect to explain every nuance, as there may be a lot going on in actuality that is not in the model and that could increase adjustments at low durations – e.g., experimentation by sellers trying to learn market conditions. Even without such complications, our hazard decreases initially, before turning up at around 3 years, as shown in the right panel. It is U-shaped over a longer horizon, naturally, as inflation makes any p leave the moving support \mathcal{F}_t eventually. Yet even at 10 years the model hazard is only up to 14%. Hence, some firms can stick to prices for a very long time, easily as long Cecchetti’s (1986) mucilaginous magazines mentioned in Section 2.

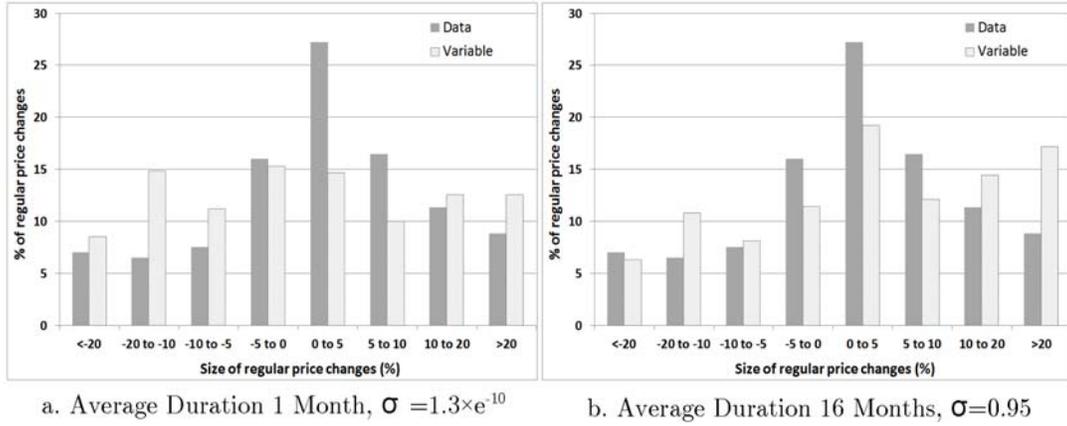


Figure 9: The Effect of Varying Duration

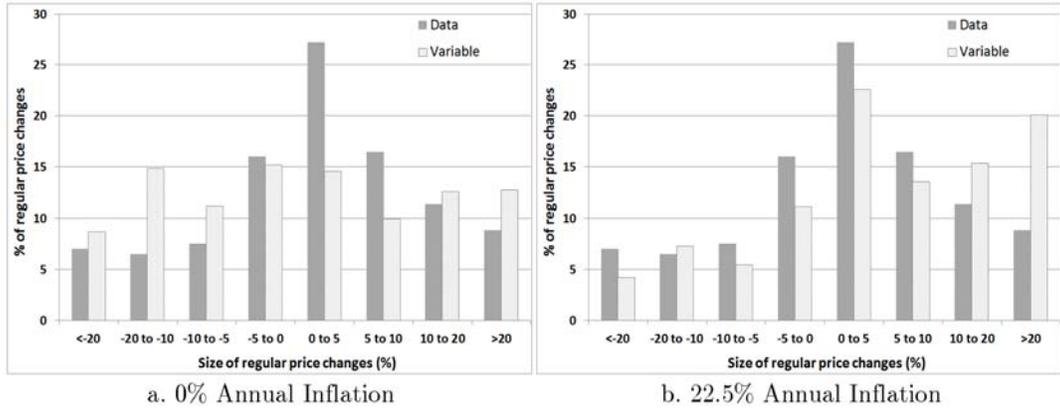


Figure 10: The Effect of Varying Inflation

Figures 9 and 10 show the results of changing duration and inflation in the variable-cost model. The left panel of Figure 9 comes from $\sigma \simeq 0$ and an expected duration of 1 month; the right comes from choosing σ to fit the histogram, implying $\sigma = 0.95$ and an expected duration of 16 months. Clearly, the left panel fits worse and the right better than a benchmark duration of 8.6 months. With too much stickiness, the fit gets very bad: at $\sigma = 0.9999$, e.g., the fraction of negative changes drops to 1.5%, far from the facts. The left panel of Figure 10 sets π to 0, and the right to 22.5% (the maximum consistent with MME). One can check, not surprisingly, that the fraction of negative adjustments decreases while both

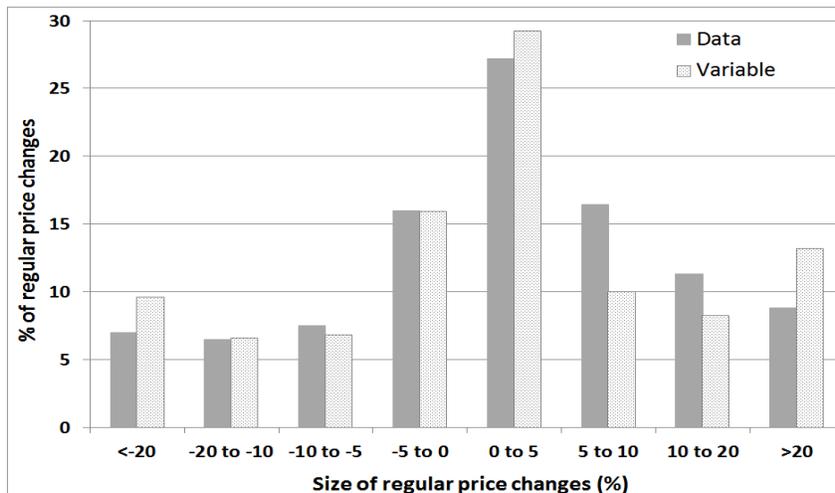


Figure 11: Distribution of Changes Ignoring Money Demand

the frequency and size increase with π . This is consistent with the evidence, as shown by Klenow and Kryvtsov's (2008) regressions of negative adjustments, of size, and of frequency on inflation. Therefore, we emphasize, price adjustments in these models are not invariant to policy, but react to it in realistic ways.

Overall, while the models may miss a few details, we submit that they perform reasonably well. At least, it would be hard to say there is anything especially puzzling about the behavior of price changes in the data – it is pretty much what one should expect from rudimentary search theory. It would be even harder to argue there is anything definitively informative about Mankiw costs or Calvo arrivals in the data, given outcomes generated without such devices. We also emphasize the discipline imposed by macro and micro observations on money and credit, which at least the variable-cost model matches. Ignoring these observations, we can do even better, as shown in Figure 11, which comes from a calibration that does not try to match money demand. The histogram obviously fits well in this picture, but we are way off in terms of money demand. A conclusion is that it is not hard to capture the evidence on sluggish price adjustment, but we do a bit more, simultaneously capturing this plus the evidence on credit and money demand.

8 Participation and Policy

With $k = 0$, all buyers enter the BJ market and $\alpha_n(\bar{b})$ is fixed exogenously. Since y is indivisible, BJ output in each transaction is fixed, too, and AD output is fixed by $U'(x) = 1$. Hence changes in the level of M , or in π and i , have no impact on output: monetary policy is neutral as well as superneutral. However, we now show how to amend the setup following Liu et al. (2011), by letting $k > 0$, so that b adjusts until the marginal entrant is indifferent. This makes BJ output endogenous. First, note that if prices were sticky for Calvo or Mankiw reasons, a one-time unanticipated jump in M can have real effects. This is because at least some firms could not (with Calvo) or would not (with Mankiw) adjust p , and so the nominal distribution $F(p)$ may not change enough to keep the same real distribution $G(q)$. Hence, the real distribution can turn in favor of buyers, increasing b , and thus stimulating output.

By contrast, in our economy jumps in M affect neither $G(q)$ nor b . A policy advisor seeing only a fraction of sellers adjusting p each period, while $\mathbb{E}p$ rises, may conclude that jumps in M would have real effects. That would be wrong (another example of the Lucas critique). Of course, this does not mean all monetary policy is irrelevant. To see this in a simple way, suppose that every period BJ buyers attempt to solicit two price quotes, and succeed in each try with probability $s = s(b)$. Assume $s(0) = 1$, $s(\bar{b}) = 0$, $s'(b) < 0$, and $s''(b) > 0$ to capture the congestion effects standard in search theory. Then $\alpha_1(b) = 2s(b)[1 - s(b)]$ and $\alpha_2(b) = s(b)^2$. Inserting these into (12), we get

$$\hat{z}_i = \gamma + (2\delta)^{1/3} [1 - s(b)]^{2/3} (\mu - \delta - \gamma)^{2/3} i^{-1/3}, \quad (16)$$

defining a relation between \hat{z}_i and b called the RB (real balance) curve. Similarly, (10) defines a relation called the FE (free entry) curve,

$$\kappa = s(b)^2 (\mu - \delta - \gamma) - \frac{\delta [1 - s(b)]^2 (\mu - \delta - \gamma)^2}{(\hat{z}_i - \gamma)^2} + \delta - i\hat{z}_i. \quad (17)$$

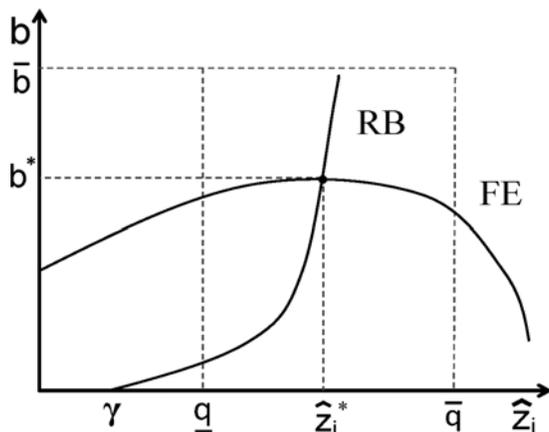


Figure 12: Real Balances and Free Entry

As shown in Figure 12, RB is increasing and convex, with $\hat{z}_i = \gamma$ at $b = 0$, while FE is slopes up (down) to the left (right) of RB, with $b \in (0, \bar{b})$ at $\hat{z}_i = 0$. Hence there is a unique MME at (\hat{z}_i^*, b^*) . An increase in i , which ultimately comes from higher π , given the Fisher equation, shifts both curves toward the origin, so output falls. Monetary policy matters, not because prices are sluggish, but because π is a tax on decentralized exchange. This is a good environment to study such effects, we think, because consumers can endogenously substitute between cash and credit as policy changes. However, the bigger point is that the underlying reason *why* prices are sticky can make a huge difference.¹⁸

9 Conclusion

One contribution of search theorists is to show how certain observations that look anomalous from the perspective of “standard” theory can be more readily understood once frictions are incorporated. An example is price dispersion: deviations from the law of one price, something that cannot occur in frictionless economies,

¹⁸We do not calibrate the model with entry for the following reason: If we specify a matching technology $s(b) = b^\chi$, it introduces a new parameter χ , plus there is entry cost κ , but we also lose two parameters because α_1 and α_2 are now endogenous. Given (χ, κ) leads to the same (α_1, α_2) as direct calibration, the observable results will be identical.

can emerge naturally in search equilibrium. The same is true for price rigidity. It is a weighty puzzle for “standard” theory when sellers let their real prices slide in arbitrary ways by not responding to changes in economic conditions. Yet once one sees the path to price dispersion, stickiness is a small additional step. Theory predicts that letting real prices vary over some range is inconsequential for profit. This is not because of a knife-edge specification where profit functions just happen to be flat; it is because economic forces shape equilibrium so that profit is endogenously flat over an interval.

One component of the project was to exploit that idea to build general equilibrium models with sluggish nominal prices. Others mentioned above previously made related points, including Caplin and Spulber (1987), Eden (1994) and Head et al. (2012). It may even be true that, based on those contributions, it is now well known that sticky prices imply neither nonneutrality nor the existence of menu costs.¹⁹ In any case, we tried go beyond the earlier papers. As insightful and clever as the Caplin-Spulber analysis may be, neither we nor the authors regard it as a serious treatment of monetary theory, and the substantive results are sensitive to details (Caplin and Leahy 1991). Also, neither they nor Eden attempt to match any data. The setup in Head et al. uses similar monetary theory, but is stuck between a rock and a hard place when it comes to either admitting payoff-relevant indeterminacy or imposing the ad hoc restriction of linear menus. Further, while that exercise does a good job matching price-change observations, it cannot simultaneously match this plus the evidence on money and credit.

In addition, perhaps a bigger component of the project was to confront the pricing data in a setting where money and credit coexist. Rigorous monetary

¹⁹In constructive comments, Fernando Alvarez argued it is well known, but certainly this was not always the case. Consider Ball and Mankiw (1994): “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky” and “As a matter of logic, nominal stickiness requires a cost of nominal adjustment.” Also consider Golosov and Lucas (2003): “menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.”

theory is nontrivial, and indeed money is another anomaly for “standard” theory that is better understood when frictions are modeled carefully. Our approach is based on lessons from the literature. We embedded a Burdett-Judd goods market in a Lagos-Wright monetary economy, then added costly credit because it resolves the indeterminacy problem, and, of course, because it is interesting for its own sake. With either a fixed or variable cost of credit, we generated exact money demand functions that fit the data well, and resemble classic (e.g., Baumol-Tobin) results in terms of algebra and in terms of economics. It is in the context of this setting that we address the pricing facts summarized above in fn. 4.

An unanticipated finding was that the model with a fixed cost performed less well than with a variable cost, mainly because it was harder with a fixed cost to satisfy the conditions for MME. Therefore, in calibrating that version, we gave up on matching credit’s share in payment data. To say it differently, if inflation increases much in that formulation, consumers switch entirely to credit, and while some people see this as the wave of the future, at least for now currency is very much in circulation.²⁰ However, we do not take too seriously the prediction of a flight from currency at moderate inflation, even though this certainly might happen at hyper inflation. In reality, there are certain buyers that always use cash and sellers that only take cash for reasons not in the model, and they may continue to do so at moderately high inflation. This suggests that it would be useful to incorporate heterogeneity. While it is important to emphasize the variable-cost model does not have this problem, allowing money and credit to coexist for a larger range of parameters, it still may be interesting to add heterogeneity to that version, too. This is left for future research.

²⁰Cash usage has decreased in some countries, as it like checking gradually gets crowded out by debit cards, but the ratio of currency in circulation to GDP is flat or even increasing in many places, as discussed by Jiang and Shao (2014*a,b*) and references therein. In any case, debit cards, cash and checks are close substitutes for our purposes.

Appendix

Derivation of (7): Consider $\delta > 0 = \tau$ to reduce notation. Then write

$$\begin{aligned} V(z) = & W(z+T) + \alpha_1(b) \int_{\underline{q}}^z (\mu - q) dG_1(q) + \alpha_1(b) \int_z^{\bar{q}} (\mu - q - \delta) dG_1(q) \\ & + \alpha_2(b) \int_{\underline{q}}^z (\mu - q) dG_2(q) + \alpha_2(b) \int_z^{\bar{q}} (\mu - q - \delta) dG_2(q), \end{aligned}$$

where $G_n(q) = 1 - \hat{G}(q)^n$ is the CDF of the lowest of n draws from $G(q)$. The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with $q \leq z$, so only cash is used, times the expected surplus, which is simple because $W'(A) = 1$. The third is the probability of meeting a seller with $q > z$, so credit must be used, which adds cost δ . The last two terms are similar except the buyer meets two sellers. The rest is algebra.

Proof of Lemma 3: (i) In NME, buyers' BJ surplus is $\Sigma = \mu - q - \delta - \tau q$. Note $\Sigma = 0$ at $q = (\mu - \delta) / (1 + \tau)$, so no buyer pays more than this. If $\bar{q} < (\mu - \delta) / (1 + \tau)$ then the highest price seller has profitable deviation toward $(\mu - \delta) / (1 + \tau)$, which increases profit per unit without affecting sales. Hence $\bar{q} = (\mu - \delta) / (1 + \tau)$. (ii) In MME, for $q > z$, as it is near \bar{q} , $\Sigma = \mu - q - \delta - \tau(q - z)$. Note $\Sigma = 0$ at $q = (\mu - \delta + \tau z) / (1 + \tau)$, and repeat the argument for NME to show $\bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$. The definition of MME has $z < \bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$, which reduces to $z < \mu - \delta$. (iii) In PME, given buyers bring z to BJ, they would pay z . Hence $\bar{q} \geq z$, as $\bar{q} < z$ implies the highest price seller has profitable deviation. We also have to be sure there is no profitable deviation to $q > z$, which requires buyers using some credit. The highest such q a buyer would pay solves $\Sigma = \mu - q - \delta - \tau(q - z) = 0$, or $q = (\mu - \delta + \tau z) / (1 + \tau)$. There is no profitable deviation iff $(\mu - \delta + \tau z) / (1 + \tau) \leq z$, which reduces to $z \geq \mu - \delta$. ■

Proof of Proposition 1: (i) With fiat currency $\phi = 0$ is always self-fulfilling, so we simply set $G(q)$ according to (5), corresponding to equilibrium in the original BJ model.

(ii) From Figure 1, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(q) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) is equivalent to $(\alpha_1 + \alpha_2) \delta H^-(\bar{q}) < i$, which holds iff $i > \underline{i}$. Then (b) is equivalent to $(\alpha_1 + \alpha_2) \delta H^+(q) > i$, which holds iff $i < \tilde{i}$ where $\tilde{i} = \delta (\alpha_1 + 2\alpha_2)^3 / 2\alpha_1\alpha_2 (\mu - \delta - \gamma) > \underline{i}$. Also, (c) is equivalent to

$(\alpha_1 + \alpha_2) \delta H(z_i) - iz_i > (\alpha_1 + \alpha_2) \delta H(0)$, which holds iff $\Delta(i) > 0$ where

$$\Delta(i) = -i\gamma + \frac{\delta(\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^{\frac{2}{3}}\delta^{\frac{1}{3}}\alpha_1^{\frac{2}{3}}\alpha_2^{-\frac{1}{3}}(\mu - \delta - \gamma)^{\frac{2}{3}}(2^{-\frac{1}{3}} + 2^{-\frac{4}{3}}).$$

Notice $\Delta(0) > 0 > \Delta(\tilde{i})$ and $\Delta'(i) < 0$. Hence $\exists! \bar{i}$ such that $\Delta(\bar{i}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{i}$. It remains to verify that $\bar{i} > \underline{i}$, so that (a) and (c) are not mutually exclusive. It can be checked that this is true iff $\delta < \bar{\delta}$. Hence a MME exist under the stated conditions. It is unique because $\bar{q} = \mu - \delta$, which pins down $G(q)$, and then $\hat{z}_i = \arg \max_{z \in (q, \bar{q})} O_i(z)$.

(iii) From Figure 1, PME exists iff three conditions hold: (a) $O_i^-(\bar{q}) > 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(\bar{q}) > O_i(0)$. Now (a) holds iff $i < \underline{i}$ and (b) holds iff $i < \tilde{i}$. Condition (c) holds iff $i < \hat{i}$. For $\delta > \bar{\delta}$, it can be checked that $\hat{i} < \underline{i}$ and $\underline{i} < \tilde{i}$, so the binding condition is $i < \hat{i}$. For $\delta < \bar{\delta}$, it is easily checked that $\hat{i} > \underline{i}$, and $\underline{i} < \tilde{i}$, so the binding condition is $i < \underline{i}$. ■

Formulae for Calibration: Consider the fixed-cost model. Inserting \bar{q} and \underline{q} , we reduce the posted-price and transaction-price CDF's to

$$\begin{aligned} G(q) &= 1 - \frac{\alpha_1}{2\alpha_2} \frac{\mu - \delta - q}{q - \gamma}, \\ H(q) &= 1 - \frac{\alpha_1^2 (\mu - \delta - q) (\mu - \delta + q - 2\gamma)}{4\alpha_2 (\alpha_1 + \alpha_2) (q - \gamma)^2}. \end{aligned}$$

The the fraction of monetary transactions and the markup are

$$\begin{aligned} H(\hat{z}_i) &= \frac{(\alpha_1/2 + \alpha_2)^2 - [\alpha_1\alpha_2(\mu - \delta - \gamma)]^{2/3} i^{2/3}}{\alpha_2(\alpha_1 + \alpha_2)(4\delta)^{2/3}} \\ \frac{\mathbb{E}_G q}{\gamma} &= 1 + \frac{\alpha_1(\mu - \delta - \gamma) \log(1 + 2\alpha_2/\alpha_1)}{2\alpha_2\gamma}. \end{aligned}$$

The Lucas-style money demand function and its elasticity are

$$\begin{aligned} L_i &= \frac{\gamma + [\alpha_1^2\delta(\mu - \delta - \gamma)^2/2\alpha_2]^{1/3} i^{-1/3}}{1 + \alpha_1(\mu - \delta) + \alpha_2\gamma} \\ \eta_i &= \frac{-1}{3 + 3\gamma [\alpha_1^2\delta(\mu - \delta - \gamma)^2/2\alpha_2]^{-1/3} i^{1/3}}. \end{aligned}$$

The variable cost case is similar but more algebraically intense. ■

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