

# Adverse Selection and Liquidity in Asset Markets: To pool or not to pool\*

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## Abstract

This paper examines how adverse selection affects asset market outcomes such as liquidity, velocity and prices. To do so, a decentralised asset market with search and bargaining is employed where assets have an essential role in exchange. Under the assumption of private information about quality, the model provides two broad insights. First, information frictions may lead to lower velocity, inefficient output levels and prices. Whether or not this happens depends on the asset's quality which determines whether it suffers from adverse selection or not. Second, a high quality asset might end up with a lower output than the low quality asset. The degree to which this happens depends on whether the high quality asset holder decides to pool or not with her low counterpart. The model informs us that markets transition from a pooling to separating outcome as the quality mix of assets worsens. Besides this, the model also shows how asset prices can change abruptly depending on the state of the economy. In particular, it shows how an inferior asset's price can drop suddenly.

**J.E.L. Classification:** D82, D83, E44

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# 1 Introduction

The global financial crisis of 2007-08 manifested itself through liquidity dry-ups and market declines as financial institutions held back their assets for several reasons, including inter alia, price declines, liquidity spirals, and hoarding of funds (Brunnermeir, 2010). Furthermore, Gorton (2007, 2008, 2009) in his analysis of the crisis, has highlighted the key role played by informational frictions. Gorton (2009) describes how “informationally insensitive” debt can become “informationally sensitive” due to a shock.<sup>1</sup> This paper is an attempt to account for such information asymmetries and its impact through a decentralized asset market where assets have a liquidity role.

Analyzing markets where assets have a liquidity role through monetary models allows one to generate outcomes on prices and velocities endogenously through trading frictions as has been done in many papers including Kiyotaki and Moore (2005), Duffie et al. (2005, 2008), Lagos (2010, 2011) and many others surveyed in Nosal and Rocheteau (2011) and Lagos, Rocheteau and Wright (2015). Such monetary models are particularly useful as it values assets not only for their rates of return but also for their liquidity services (Rocheteau and Wright, 2013).

By adding informational frictions to this class of asset market models, a real world problem can be studied for its effects on asset prices through liquidity. Information frictions may affect liquidity of assets and hence prices. For instance consider this, if the asset holder has private information about quality, then in order to signal that she does not have a ‘lemon’ she might be willing to accept a lower output and velocity in return for a better price. Or, she might in fact choose to not signal her quality at all and get a high output and velocity in return for a lower price. The choice between the two will depend on how many so called ‘lemons’ actually exist in the market and how different her asset is from them. This decision also affects the outcomes of the actual ‘lemons’. In particular when the good quality asset holder decides to pool with them (or not signal) the value of the bad assets rises and falls abruptly when the good decides to signal. Thus, information frictions may lead to lower velocity, inefficient output levels and prices. Whether or not this happens depends on the quality of assets and whether

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<sup>1</sup>Gorton’s usage of this phrase was in terms of information acquisition. I will however argue that this phrase extends to information provision too, as asset holder’s decide whether or not to signal their quality.

it suffers from adverse selection or not. If this is the case then not surprisingly, as agents are better informed, market outcomes improve.

The basic framework is borrowed from Shi (1995) and Trejos and Wright (1995) which features random pairwise meetings with a role for liquid assets that can serve as means of payment or collateral. Along with the aforementioned advantages of the class of such models, I particularly like the fact that such an environment allows to have explicit game-theoretic foundations for exchange and transmission of information. Fiat money is replaced with indivisible and durable assets in fixed supply just like claims to Lucas (1978) trees that give off fruits as dividends. In addition, agents have private information about quality of fruits (or dividend) the tree (or asset) bears. Examples of such assets in the real world could be private equity, a corporate bond or an asset-backed security.

Moreover, asset holdings are assumed to be binary as in the original model. This does make the model tractable but for our purposes it also rules out portfolio diversification, something which we want. The indivisible asset holding however can be restrictive as by itself if does not allow two dimensions of trade and therefore could result in inefficient outcomes such as no-trade or too-much-trade (Berentsen & Rocheteau, 2001). Therefore, as in Berentsen, Molico & Wright, (2002), I allow agents to trade lotteries on their trees, where the valuable asset is withdrawn from circulation in a probabilistic sense, without harming the efficiency of exchange. We will thus be able to look at the model as a signaling game as in Nosal and Wallace (2007). We will also see how an asset holder can signal the quality of her tree through ‘asset retention’ i.e. she might propose to hold on to her asset with a higher probability, something which a bad asset holder does not do.<sup>2</sup> Many results will thus emerge along the intensive margin of trade. The general equilibrium determines the value of assets and shows how an asset can be overvalued (in a sense to be made precise).

It is shown how the market outcome changes from being in a pooling equilibrium to a separating equilibrium, as the proportion of bad assets in the market goes up or the difference in quality between the high and low quality reduces.<sup>3</sup> Whether or not one can interpret the

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<sup>2</sup>This can be related to the Dodd-Frank Act’s 5 % risk retention requirement (Geithner, 2011).

<sup>3</sup>I use the concept of undefeated equilibrium from Mailath, Okuno-Fujiwara and Postlewaite (1993) to discriminate between the two equilibria. See the text and Appendix B for details.

crisis as a situation with a worsening of the quality mix of assets can be open for discussion, but with a slight reinterpretation of Gorton's (2009) phrase as discussed above we can see how a market can go from being informationally insensitive (pooling) to sensitive (separating). When this happens abrupt decline in prices of some assets (the inferior ones) can be also explained.

With regard to the literature, this paper is related to many other papers. A non-exhaustive list is as follows. In random-search and private information, Williamson and Wright (1994) and Berentsen and Rocheteau (2004) have a model with private information on the good but not the asset used in exchange. Velde, Veber & Wright (1999) has a model on commodity money with adverse selection but without lotteries. Nosal and Wallace (2007) and Li and Rocheteau (2011) build a model of counterfeiting using lotteries, but equilibrium refinement used is the Intuitive Criterion which rules out all pooling offers. Rocheteau (2011) has a model with two assets and private information on future dividend flows of one of the assets in a Lagos and Wright (2005) framework but again the equilibrium notion rules out pooling offers.

Papers using a competitive search environment include Guerrieri, Shimer and Wright (2010) where non-asset holders (or goods producers) post contracts and then both types of asset holders decide whether or not to enter the market. The entry decision affects market tightness. However, assets play no role in facilitating exchange and since they look at the extensive margin with price posting through screening the 'asset retention' story is missing. The results they get are similar in terms of how asset's terms of trade are affected as the buyers of assets can successfully screen the two types. They also mention (without using any equilibrium refinement) that pooling outcomes can Pareto dominate the separating in some cases. Chang (2012) uses a similar framework but also includes private information about trading motives rather than just on the quality of assets. Thus she can analyze semi-pooling equilibrium through the interplay of multi-dimensional private information. The impact of such asymmetries is seen along two dimensions: trading price and speed. Delacroix and Shi (2013) also work with competitive search but allow asset holders to post contracts thereby signalling their types; they however have agents that are ex-ante similar who decide whether to be high type or low type.

This paper is also related to the literature focusing on the effect of search frictions in asset markets (Kiyotaki and Wright, 1993; Shi, 1995; Trejos and Wright, 1995), and also the over-

the-counter literature (such as Duffie, Garleanu and Pedersen, 2005). Finally, this paper is related to the long literature on lemons market, which builds on the seminal work of Akerlof (1970). It also borrows from Cho and Kreps (1987); Mailath, Okuno-Fujiwara and Postlewaite (1993) and Farhi and Tirole (2014) for the equilibrium concept in signaling games.

This paper should be seen as contributing to the literature on information and liquidity. The key contribution is to show how asset market outcome transitions from pooling to separating as the quality mix of assets declines. As this change takes place, strong predictions can be made about prices and velocities. In particular, when the quality mix of assets falls below a critical level, some assets experience a sharp fall in prices. Many random-search models with private information by broadly restricting attention to the Cho-Kreps (1978) Intuitive Criterion have ruled out pooling offers from being an equilibrium outcome. By using the undefeated equilibrium concept, which does not rule out all pooling offers, the present model can generate these different outcomes which lends useful insights into the functioning of financial markets. Besides, the model can be extended or applied to other contexts which can be avenues for future research as will be discussed briefly.

The rest of the paper is organized as follows: Section 2 presents the physical environment. The bargaining game is studied in isolation in Section 3 and is embedded in the general equilibrium structure in Section 4. Section 5 concludes. The proofs to all propositions and lemmas are in the appendix.

## 2 Environment

The monetary search framework is based on Shi (1995) and Trejos and Wright (1995) with divisible consumption goods and asset holdings in the set  $\{0, 1\}$ . The formulation of asymmetric information follows Williamson and Wright (1994), with the difference that private information is on the asset holding and not on goods. Finally, the lotteries framework is based on Berentsen, Molico and Wright (2002) where agents are allowed to offer their asset probabilistically in bilateral trades.<sup>4</sup>

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<sup>4</sup>In search theoretic models with indivisible money and complete information, the use of lotteries acts as an imperfect proxy for divisibility of money: it allows larger gains from trade and it eliminates some trade inefficiencies arising from indivisibilities (Berentsen and Rocheteau, 2002).

The time horizon is infinite and is indexed by  $t \geq 0$ . There is a large number of perfectly divisible and perishable goods and a unit measure of agents who specialize in the goods they produce and consume. To generate trade possibilities, assume that agents do not consume their own output. Agents trade in bilateral matches and specialization in production and consumption rules out double-coincidence-of-wants. This specialization does allow single-coincidence matches, however, where one agent wishes to consume the good produced by her partner, but not vice versa. Agents also cannot commit to future actions; there is no enforcement or record-keeping technology hence credit arrangements are infeasible and all trade arrangements must be quid pro quo. Assets are introduced below to play the role of a medium of exchange.

Agents get utility  $u(q)$  from consumption (and dis-utility  $c(q)$  from production) of  $q \in \mathbb{R}_+$  units of the good. Assume that  $u(q)$  and  $c(q)$  are continuously differentiable and strictly increasing. Furthermore,  $u(q) - c(q)$  is strictly concave,  $u(0) = c(0) = 0$ ,  $u'(0) = \infty$  and there exists  $q^* > 0$  such that  $u'(q^*) = c'(q^*)$ . Agents discount the future at rate  $r > 0$ .

In addition to these consumption goods, there are indivisible, durable and storable Lucas (1978) trees in the economy that can be interpreted as private equity, corporate bonds or asset-backed securities. These assets can potentially serve as media of exchange or as collateral.<sup>5</sup> Initially,  $M < 1$  agents are endowed with one unit each of these trees. Trees come in two varieties:  $l$ -type which yields low fruit (or dividend) offering utility flow equal to  $\gamma_l > 0$  to the owner every period and  $h$ -type which yields a higher fruit (or dividend) offering utility flow  $\gamma_h > \gamma_l \geq 0$ . Let  $M_i$  be the measure of agents endowed each with  $i$ -type tree, so that  $M_h + M_l = M$ . Agents with trees are called *buyers* and those without trees *sellers*. Let  $\pi_i = M_i/M$  be the fraction of  $i \in (h, l)$  type buyers.

Only buyers and sellers meet each period. In particular, two tree holders or two sellers never meet. This means that in every period, there are  $M_i$  buyers each with a single tree of type  $i$  and  $N = 1 - M$  sellers with no trees. The probability of single-coincidence match for a tree holder is  $\alpha(N/M)$  i.e. a match where the buyer wants the good that the seller produces. We assume that  $\alpha(N/M)$  is continuously differentiable, strictly increasing and concave. Furthermore,  $\alpha(0) = 0, \alpha(\infty) = 1, \alpha'(0) \leq 1$ . The probability of a single coincidence match for an agent without

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<sup>5</sup>Equivalence of the use of assets as a means of payment or collateral has been described in Lagos (2010)

tree (i.e. a match where the seller wants the good produced by the buyer which is actually unfruitful because even though the seller wants the buyer's good, she cannot buy it as she has nothing to offer in return) is  $\alpha(N/M)M/N$ .<sup>6</sup> Conditional on being matched the seller meets a type- $h$  buyer with probability  $\pi_h = M_h/M$  and a type- $l$  buyer with complementary probability  $\pi_l = M_l/M = 1 - \pi_h$ . In any such meeting, trade may or may not take place.

Terms of trade in bilateral matches are determined by a simple take-it-or-leave-it offer by the buyer to the seller. If the seller accepts the offer, the trade is executed, otherwise the buyer and seller split apart and trade is not executed. The take-it-or-leave-it offer is a simplifying assumption as the model would be quantitatively similar if some other proportional bargaining solution were to be adopted. Moreover, to overcome the indivisibility of assets, this paper borrows from Berentsen, Molico and Wright (2002) and allows buyers to offer lotteries. The offer made specifies a pair  $(q, p)$  where  $q$  is the quantity of the good to be traded and  $p$  is the probability of trading the tree. In other words, the buyer asks the seller to produce  $q$  units of the good in exchange for the tree which the seller would receive with probability  $p$ . Introduction of lotteries creates a notion of prices in an otherwise indivisible asset world and thus allows a deeper analysis of the bargaining game.

Furthermore, it is not easy for sellers to detect the difference between trees of type  $h$  and  $l$ . Following Williamson and Wright (1994), this difficulty is modelled by assuming that, at each meeting, the seller is informed about the type of her partner's tree with probability  $\theta \in [0, 1]$  in which case there is complete information. With the complementary probability  $(1 - \theta)$  the seller is uninformed and there is incomplete information. Thus,  $\theta$  is the degree of completeness of information; it is high if sellers likely know the tree type during the match and vice versa. After matches are terminated, the seller can determine the quality of her tree holdings if trade was executed. A buyer always knows whether a seller can recognize the type of her tree.

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<sup>6</sup>Consider a constant returns to scale matching function  $\mathcal{M}(M, N)$ . The matching probability of a tree holder is then  $\alpha = \mathcal{M}(1, N/M)$  and for an agent without tree it is  $\alpha = \mathcal{M}(M/N, 1)$ . Functional forms that satisfy the assumptions above are  $\alpha(\tau) = 1 - \exp(-x\tau)$  where  $x \in (0, 1]$  or  $\alpha(\tau) = \frac{\tau}{1+\tau}$ ,  $\tau = N/M$ .

### 3 The Bargaining game

In this section the bargaining game between a buyer and a seller is described. The equilibrium of the bargaining game will then be embedded in the general equilibrium model. A buyer is one of two possible types: holding the  $h$ -type tree which occurs with probability  $\pi_h$ , or holding the  $l$ -type tree which occurs with probability  $\pi_l = 1 - \pi_h$ . The buyer makes an offer that the seller can accept or reject. For robustness, this game is analyzed under two alternative equilibrium concepts so the plausible equilibria can be uncovered.

A strategy for the buyer specifies an offer  $(q, p) \in \mathbb{R}_+ \times [0, 1]$  as a function of the buyer's type, where  $q$  is the output that the seller would produce and  $p$  is the transfer of the tree from the buyer to the seller. For now one could interpret the tree as being perfectly divisible and the tree holdings as being normalized to 1. Thus,  $p$  gives a notion of asset liquidity. Later in the general equilibrium model,  $p$  will correspond to the probability of transferring an indivisible unit of the tree. Seller's strategy is an acceptance rule that specifies the set of acceptable offers  $A \subseteq \mathbb{R}_+ \times [0, 1]$ .

Let  $\omega_i$  represent the value of high or low type tree to the holder  $i \in \{h, l\}$  in a period. Here assume that  $\omega_h > \omega_l$ . The precise definition of  $\omega_i$  and this condition will be discussed and verified later in the general equilibrium model where  $\omega_h$  and  $\omega_l$  will be determined endogenously. Payoff to  $i$ -type buyer in a period is:  $[u(q) - p\omega_i] \mathbb{I}_{A(q,p)}$  where  $\mathbb{I}_{A(q,p)}$  is an indicator function equal to 1 if  $(q, p) \in A$ . If an offer is accepted then the buyer realises her utility of consumption,  $u(q)$ , net of the utility she forgoes by transferring  $p$  units of the tree to the seller,  $-p\omega_i$ . The seller's payoff is:  $[-c(q) + p\omega_i] \mathbb{I}_{A(q,p)}$  and it depends on the buyer's type.

The bargaining game is analyzed under two cases to capture the notion of private information in the model. In the general equilibrium model, for any given match (independent of the type of buyer) the seller is informed about the type of tree with probability  $\theta$  and is uninformed with probability  $(1 - \theta)$ . The former is the full information case and the latter is private information where the bargaining game has the structure of a signalling game described below.

### 3.1 Full Information

Consider first, a full or complete information game, where the seller observes the type of tree held by the buyer. The optimal offer  $(q_i^c, p_i^c)$ , where  $c$  stands for complete information, maximizes the surplus of the buyer given her type  $i \in \{h, l\}$  subject to the participation constraint of the seller, i.e.,

$$(q_i^c, p_i^c) = \arg \max_{q, p \in [0, 1]} [u(q) - p\omega_i] \text{ s.t. } -c(q) + p\omega_i \geq 0 \quad (1)$$

The solution to (1) is:

$$q_i^c = \min[q^*, c^{-1}(\omega_i)] \quad (2)$$

$$p_i^c = c(q_i^c)/\omega_i \quad (3)$$

where  $q^*$  solves:  $u'(q^*) = c'(q^*)$ .

In the full information case, the output produced is efficient ( $q^*$ ). However, there is a possibility of low trade ( $q_i^c = c^{-1}(\omega_i) < q^*$ ) when agent's tree holdings falls short (to achieve  $q^*$  the buyer would need  $p > 1$  trees). In any case, this is the first-best outcome that can be achieved setting itself as the benchmark case.

### 3.2 Private Information

Next consider the case when information about the quality of tree is private to its holder. Let  $\lambda(q, p) = \Pr[i = h | (q, p)]$  represent the posterior belief of a seller that the buyer holds a unit of  $h$ -type tree conditional on the offer  $(q, p)$  made. For now consider only pure strategies in a sequential equilibrium - the buyer chooses an offer that maximises her surplus taking as given the acceptance rule of the seller. The seller optimally chooses to reject or accept offers given her posterior belief  $\lambda$ . If  $(q, p)$  corresponds to an equilibrium offer then  $\lambda(q, p)$  is derived from the seller's prior belief according to Bayes' rule. If  $(q, p)$  is an out-of-equilibrium offer then the seller's belief is arbitrary.

For a given belief system  $\lambda$ , seller's acceptance rule is defined as:  $A(\lambda) = \{(q, p) : -c(q) + p\{\lambda(q, p)\omega_h + [1 - \lambda(q, p)]\omega_l\} \geq 0\}$ . In words, for an offer to be acceptable, the seller's dis-utility of

production must be compensated by the expected value of the tree transfer she receives. Assume that a seller accepts any offer that makes her indifferent between accepting and rejecting a trade. The problem of a buyer holding a unit of tree of type  $i \in \{h, l\}$  in an uninformed meeting is:

$$\max_{q,p \in [0,1]} [u(q) - p\omega_i] \mathbb{I}_A(q, p) \quad (4)$$

There could possibly be more than one equilibrium offer that solves the buyer's problem under private information given by (4) i.e. when information about the quality of the tree is private to its holder. Our candidate equilibrium belongs to the class of Perfect Bayesian Equilibrium. The first, is the least-cost separating equilibrium selected by the Intuitive Criterion from Cho and Kreps (1987) and the second is the efficient pooling offer i.e. the one that is preferred by the high type buyers (as it maximizes welfare of high type) among all pooling offers. To discriminate between the two, the one that gives a higher payoff to the high type asset holder is chosen following the concept of undefeated equilibria from Mailath et al. (1993).<sup>7</sup>

### 3.2.1 Least-cost Separating equilibrium

Start by considering a least-cost separating equilibrium offer, where the buyer's offer depends on her type  $i \in \{h, l\}$  thereby signalling her type to the seller. The offer made is the least-cost separating as it maximizes the payoff of the  $h$ -type tree buyer. Such an offer satisfies standard equilibrium refinement concepts such as the Intuitive Criterion by Cho and Kreps (1987).

The least-cost separating equilibrium offer made by an  $l$ -type buyer in an uninformed match solves:

$$(q_l^s, p_l^s) = \arg \max_{q,p \in [0,1]} [u(q) - p\omega_l] \text{ s.t. } -c(q) + p\omega_l \geq 0 \quad (5)$$

The offer made by an  $h$ -type buyer solves:

$$(q_h^s, p_h^s) = \arg \max_{q,p \in [0,1]} [u(q) - p\omega_h] \text{ s.t. } -c(q) + p\omega_h \geq 0, \quad (6)$$

$$u(q) - p\omega_l \leq u(q_l^s) - p_l^s\omega_l \quad (7)$$

The offer made by the  $l$ -type buyer is her complete information offer, as she cannot do better than that. However, the  $h$ -type buyer faces an additional constraint (7) as she has to

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<sup>7</sup>See Appendix B for details on equilibrium selection

ensure that the  $l$ -type buyer has no incentive to imitate the  $h$ -type buyer's offer. In words, given  $\omega_l < \omega_h$ , the surplus obtained by the  $l$ -type buyer under her separating offer (which is her full information offer) is at least as great as the surplus she gets under the  $h$ -type buyer's separating offer. The  $h$ -type buyer makes the separating offer that maximizes her payoff as that is the least costly. Finally, a belief system consistent with the offers (5)-(7) is such that the seller attributes all offers that violate (7) to  $l$ -type buyers, and all other offers to  $h$ -type buyers. The solution to the  $h$ -type buyer's offer is characterized in the following lemma.

**Lemma 1.** (i) *If  $\omega_l > 0$ , then there is a unique solution to (6) - (7) and it is such that*

$$p_h^s = \frac{c(q_h^s)}{\omega_h} \quad (8)$$

$$u(q_h^s) - \frac{\omega_l}{\omega_h} c(q_h^s) = u(q_l^s) - c(q_l^s) \quad (9)$$

Moreover,  $q_h^s < q_l^s$  and  $p_h^s < p_l^s$

(ii) *If  $\omega_l = 0$ , then  $q_h^s = q_l^s = p_h^s = p_l^s = 0$*

*Proof is in the Appendix.*

If  $\omega_l > 0$  then the offer is separating and the  $h$ -type buyer obtains less output than the  $l$ -type buyer ( $q_h^s < q_l^s$ ), but spends her tree with a lower probability ( $p_h^s < p_l^s$ ). If  $\omega_l = 0$  then no output is produced or consumed. This case i.e. where no output is produced or consumed corresponds to the one studied in Nosal & Wallace (2007).

Under the separating equilibrium, the  $h$ -type buyer gets a better price at the cost of illiquidity. To see this, note that price of an asset can be expressed as  $\frac{q_h^s}{p_h^s} = \omega_h > \omega_l = \frac{q_l^s}{p_l^s}$  (for  $c(q) = q$ ). In particular, the  $h$ -type asset circulates with a lower probability than her complete information offer and that of the  $l$ -type ( $p_h^s < p_h^c \leq p_l^s$ ); thus the  $h$ -type buyer signals her quality through asset retention. Finally, a version of Gresham's Law holds since the presence of bad trees drives the good out of the market (or at least relatively so).

### 3.2.2 Efficient Pooling equilibrium

Next consider pooling equilibria, where the offer made by the buyer in an uninformed trade doesn't depend on her type. There can be many pooling equilibria, but let us restrict attention to the efficient or preferred pooling offer by the  $h$ -type buyer i.e. the one that maximizes the payoff for the  $h$ -type buyer among the class of all pooling equilibria. The reason for this restriction will be discussed later when the equilibrium of the bargaining game is defined.

In an uninformed match, the efficient pooling equilibrium offer solves:

$$(q^p, p^p) = \arg \max_{q, p \in [0, 1]} [u(q) - p\omega_h] \text{ s.t. } -c(q) + p(\pi_h\omega_h + \pi_l\omega_l) \geq 0 \quad (10)$$

$(q^p, p^p)$  is such that the surplus of the  $h$ -type buyer is maximized given that the seller's surplus is non-negative. To determine the seller's surplus, expected value of the tree is used as the pooling offer doesn't reveal the type of the tree.

The offer  $(q^p, p^p)$  is part of a pooling equilibrium if the  $l$ -type buyer gets a payoff at least equal to her complete information payoff, i.e.  $u(q^p) - p^p\omega_l \geq u(q_l^c) - p_l^c\omega_l$ . Finally, the belief system that sustains this equilibrium is such that  $\lambda = 0$  for all offers that are strictly preferred to  $(q^p, p^p)$  by  $l$ -type buyers but that would make  $h$ -type buyers worse-off. For all other offers,  $\lambda = \pi_h$ .

From (10) the terms of trade under the pooling equilibrium offer  $(q^p, p^p)$  satisfies:

$$u'(q^p) = \frac{c'(q^p)\omega_h}{\pi_h\omega_h + \pi_l\omega_l} \quad (11)$$

$$p^p = \frac{c(q^p)}{\pi_h\omega_h + \pi_l\omega_l} \quad (12)$$

if  $p^p \leq 1$ , otherwise  $p^p = 1$  and,

$$q^p = c^{-1}(\pi_h\omega_h + \pi_l\omega_l) \quad (13)$$

From (11) and (13),  $q_p < q^*$  for all possible values of  $\omega_h$  and  $\omega_l (< \omega_h)$  whenever  $\pi_l > 0$ . In contrast with the separating equilibrium, the terms of trade in the pooling equilibrium are continuous in  $\pi_l$ , and they approach the complete information outcome as  $\pi_l$  tends to 0.

Under the pooling equilibrium, the  $h$ -type buyer might choose to get a high liquidity in return for a lower price. To see this note that,  $q_h^p = q_l^p = q^p$  and  $p_h^p = p_l^p = p^p$ , so liquidity is high,

but the price for the seller ( $\frac{q^p}{p^p}$ ) =  $\mathbb{E}(\omega)$  reflects the product mix (for  $c(q) = q$ ). Moreover, output is still below the efficient level as  $q^p < q^* \forall \omega_l < \omega_h$ , given  $\pi_l > 0$ . Finally, the  $l$ -type buyer gets a weakly higher surplus than under her complete information offer ( $u(q^p) - p^p \omega_l \geq u(q_l^c) - p_l^c \omega_l$ ).

### 3.2.3 Equilibrium of the Bargaining Game with Private Information

Let  $S_h^s = u(q_h^s) - p_h^s \omega_h$  and  $S_h^p = u(q_h^p) - p_h^p \omega_h$  be the bargaining surplus of  $h$ -type buyer under separating equilibrium and pooling equilibrium respectively.

**Definition 1.** *The equilibrium of the bargaining game under private information specifies buyer  $i$ 's strategy  $(q_i, p_i)$  which is  $(q_i^s, p_i^s)$  in (5)-(7) if  $S_h^s > S_h^p$ , and  $(q^p, p^p)$  in (10) if  $S_h^s \leq S_h^p$ .*

*The seller's belief that sustains this equilibrium is such that  $\lambda = 0$  for all offers that are strictly preferred to  $(q^p, p^p)$  by  $l$ -type and make  $h$ -type worse-off;  $\lambda = 1$  for all offers make  $l$ -type strictly worse-off than  $(q_l^s, p_l^s)$ ; and  $\lambda = \pi_l$  for all other offers.<sup>8</sup>*

The reason to consider the efficient pooling equilibrium offer from the  $h$ -type buyer's perspective is because the equilibrium compares the surplus of the  $h$ -type buyer, and thus she would chose her preferred offer (which is called the efficient pooling offer) given that the  $l$ -type buyer's and the seller's incentives are aligned.

**Proposition 1.** *For given values of  $\omega_l$  and  $\omega_h$ , if  $\omega_l > 0$  then there exists a threshold value of  $\pi_l \in (0, 1)$  denoted by  $\hat{\pi}_l$  such that the equilibrium of the bargaining game is separating iff  $\pi_l > \hat{\pi}_l$  and pooling if  $\pi_l \leq \hat{\pi}_l$ . If  $\omega_l = 0$  then the equilibrium is pooling for all  $\pi_l < 1$ .*

*Proof is in the Appendix.*

On the one hand, if the probability of being an  $l$ -type buyer is high then the equilibrium of the bargaining game is the separating equilibrium. On the other hand, if it is sufficiently infrequent for a buyer to hold  $l$ -type trees then the equilibrium of the bargaining game is pooling. If the  $l$ -type tree has no value then the equilibrium offer is always pooling. Thus, we get the distortion along the intensive margin of trade because, under separating equilibrium

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<sup>8</sup>This is based on the idea of undefeated equilibria by Mailath, Okuno-Fujiwara and Postlewaite (1993). See Appendix B for details

the  $h$ -type trees are less liquid and trade for less than the first-best quantity. Under pooling equilibrium, both types of trees trade for less than the efficient quantity. Thus, the outcome of the one-period bargaining game transitions from pooling to separating as the proportion of low-type trees increases.

## 4 Equilibrium

The bargaining game described above is now embedded in the general equilibrium structure. Let  $V_i$  and  $V_s$  denote the lifetime expected discounted utility of an agent who enters the matching process as a buyer with one unit of Lucas tree of type- $i \in \{h, l\}$  and as a seller respectively. After the match terminates if the seller (buyer) receives (gives up) the asset i.e. if the match was successful and she actually won (lost) the lottery she becomes a buyer (seller) in next period when entering the matching process.

Define  $\beta \equiv (1 + r)^{-1}$  as the agent's discount factor. Denote by  $\omega_i \equiv \beta(V_i - V_s)$ ,  $i \in \{h, l\}$  the present value of the "capital gain" enjoyed by the seller if she receives one unit of type- $i$  tree, or the "capital loss" to the buyer of type- $i$  if she gives up her unit of tree.

Let  $(q_i^u, p_i^u)$  denote the offer of buyer type- $i \in \{h, l\}$  in uninformed matches. The offer is  $(q_i^s, p_i^s)$  given by Lemma 1 (or by (5) for  $l$ -type buyers) if the equilibrium of the bargaining game is separating, or  $(q_i^p, p_i^p)$  given by (11)-(13) if the equilibrium is pooling. The Bellman equation for the value of being a seller satisfies:

$$\begin{aligned}
V_s = & \alpha\theta\pi_h \left[ -c(q_h^c) + p_h^c\beta V_h + (1 - p_h^c)\beta V_s \right] \\
& + \alpha\theta\pi_l \left[ -c(q_l^c) + p_l^c\beta V_l + (1 - p_l^c)\beta V_s \right] \\
& + \alpha(1 - \theta)\pi_h \left[ -c(q_h^u) + p_h^u\beta V_h + (1 - p_h^u)\beta V_s \right] \\
& + \alpha(1 - \theta)\pi_l \left[ -c(q_l^u) + p_l^u\beta V_l + (1 - p_l^u)\beta V_s \right] \\
& + (1 - \alpha)\beta V_s
\end{aligned} \tag{14}$$

In words, with probability  $\alpha\theta$  a seller is in an informed match where she may meet an  $h$  or  $l$ -type

buyer. As per the type she meets, she produces  $q_i^c$  to receive a unit of tree with probability  $p_i^c$  where  $i \in \{h, l\}$ . With probability  $\alpha(1 - \theta)$  a seller is in an uninformed match in which she produces  $q_i^u$  and receives a unit of tree with probability  $p_i^u$ . If the seller receives an  $i$ -type tree she becomes an  $i$ -type buyer from the next period else, she continues as a seller. Finally, with probability  $(1 - \alpha)$  the match is unfruitful (i.e. the seller does not produce the good that the buyer she met consumes) and she continues as a seller.

The Bellman equation for the value of being a buyer of type  $i \in \{h, l\}$ :

$$\begin{aligned}
V_i = & \alpha\theta \left[ u(q_i^c) + (1 - p_i^c)\beta V_i + p_i^c\beta V_s \right] \\
& + \alpha(1 - \theta) \left[ u(q_i^u) + (1 - p_i^u)\beta V_i + p_i^u\beta V_s \right] \\
& + (1 - \alpha)\beta V_i \\
& + \gamma_i
\end{aligned} \tag{15}$$

In words, with probability  $\alpha\theta$  a buyer type- $i$  is in an informed match with a seller whose good she wants. She consumes  $q_i^c$  and delivers her unit of tree with probability  $p_i^c$  where  $i \in \{h, l\}$ . With probability  $\alpha(1 - \theta)$  a buyer is in an uninformed match in which she consumes  $q_i^u$  and delivers her unit of tree with probability  $p_i^u$ . If the buyer delivers the tree she becomes a seller from the next period else, she continues as a buyer. With probability  $(1 - \alpha)$  the match is unfruitful (i.e. she does not want to consume the good that the seller she met produces) and she continues as a buyer. Finally, the buyer of type  $i$  also gets utility flow in the form of the tree's fruits (or dividend flow)  $\gamma_i$  depending on the type of tree she holds.

Throughout the analysis, stationary equilibria are considered where the quantities traded and values of the trees are constant over time.

**Definition 2.** *A symmetric steady-state equilibrium is a list  $\{V_s, V_h, V_l, (q_h^c, p_h^c), (q_l^c, p_l^c), (q_h^u, p_h^u), (q_l^u, p_l^u)\}$  that satisfies:*

(i) *the solution to (2)-(3) in informed matches and to (8)-(9) or (11)-(13) in uninformed matches depending on the equilibrium of the bargaining game given by Definition 1;*

(ii) *the Bellman equations (14)-(15);*

The definition is restricted to symmetric equilibria where buyers of the same type make the same offer. Moreover, whether the equilibrium of the bargaining game is separating or pooling is given under Definition 1.

**Lemma 2.** *In all equilibria,  $V_s = 0$ .*

*Proof is in the Appendix.*

From Lemma 2,  $\omega_i = \beta V_i$ .

#### 4.1 Separating equilibrium

Consider first the separating equilibrium offer (5) - (7) of the bargaining game and conditions under which it prevails as the final equilibrium given Definition 1. Subtract  $\beta V_i$  from both sides of (15) and use  $\omega_i = \beta(V_i - V_s)$ . The Bellman equations for each buyer type,  $i \in \{h, l\}$  under the separating equilibrium offer becomes:

For an  $l$ -type buyer,

$$\omega_l = \frac{\alpha}{r} [u(q_l^c) - c(q_l^c)] + \frac{\gamma l}{r} \quad (16)$$

The separating equilibrium offer in the bargaining game under private information for the  $l$ -type buyer is her complete information offer  $(q_l^c, p_l^c)$ , so it does not matter if she meets an informed or uninformed seller. Thus, the value of  $l$ -type tree is the discounted sum of its value in exchange (which is independent of whether the buyer meets an informed or uninformed seller) and the utility that the tree's fruit yields. The value of the  $l$ -type tree therefore is the same as in the case with no private information.

For an  $h$ -type buyer,

$$\omega_h = \frac{\alpha}{r} \left\{ \theta \left[ u(q_h^c) - c(q_h^c) \right] + (1 - \theta) \left[ u(q_h^s) - c(q_h^s) \right] \right\} + \frac{\gamma h}{r} \quad (17)$$

The  $h$ -type buyer's offer in any meeting with an uninformed seller is the separating equilibrium offer in the bargaining game under private information,  $(q_h^s, p_h^s)$ . Thus, the value of  $h$ -type tree

is the sum of its discounted liquidity value (value in exchange depending on whether the buyer meets an informed or uninformed seller) and discounted dividend return.

Note that the value of the  $h$ -type tree is lower in the separating equilibrium than it would be in the case with no private information. To see this, let  $\theta = 1$  in (17) to get  $\omega_h^c$  and compare that with  $\omega_h^s$  when  $\theta = 0$ . Since  $[u(q_h^s) - c(q_h^s)] < [u(q_h^c) - c(q_h^c)]$ , the latter ( $\omega_h^c$ ) is lower than the former ( $\omega_h^s$ ).

A separating equilibrium can then be defined as a 6-tuple  $(q_i^c, p_i^c, q_h^s, p_h^s, \omega_l, \omega_h)$  that satisfies (2), (3), (8), (9), (16) and (17).

**Proposition 2.** *There exists a unique  $\omega_h > \omega_l$  that solves (16) - (17).*

*Proof is in the Appendix.*

Proposition 2 establishes the existence and uniqueness of  $\omega_h$  and  $\omega_l$  when separating offers are made in matches with uniformed sellers in equilibrium. It also shows that  $\omega_h$  is indeed greater than  $\omega_l$ , as was assumed in the bargaining game. The value of trees (inclusive of the utility from the bargaining surplus and it's fruits) is higher for  $h$ -type trees than  $l$ -type.

Next, consider some comparative statics under the separating equilibrium. Let  $\omega_i(\pi_l, \theta)$  denote the equilibrium value of tree of type  $i \in \{h, l\}$  as a function of exogenous parameters.

**Proposition 3.** *Suppose  $\omega_h > \omega_l$  is true from Proposition 2, then (i)  $\frac{\partial \omega_l}{\partial \pi_l} = 0$  and  $\frac{\partial \omega_h}{\partial \pi_l} = 0$ ; (ii)  $\frac{\partial \omega_l}{\partial \theta} = 0$  and  $\frac{\partial \omega_h}{\partial \theta} > 0$ .*

*Proof is in the Appendix.*

The fraction of  $l$ -type buyers,  $\pi_l$  does not affect the separating equilibrium terms of trade and hence the equilibrium value of the trees. Information frictions,  $\theta$  do not affect the value of low type trees as  $l$ -type tree holders make their complete information offer anyway. However,  $h$ -type trees obtain less output with sellers who can't recognize the type hence their value increases as information frictions get less severe.

To examine how recognizability of trees affects its turnover define the velocity of tree  $i \in \{h, l\}$ ,  $v_i$  as the average probability that the tree changes hands in a bilateral match, i.e.,

$$v_i = \alpha(\theta p_i^c + (1 - \theta)p_i^s) \quad (18)$$

Velocity of the  $l$ -type tree is unaffected by recognizability as  $p_l^c = p_l^s$ . However, since  $p_h^c > p_h^s$  velocity of the  $h$ -type tree increases with  $\theta$ . If sellers can recognize the tree type the  $h$ -type buyers can get better terms of trade through their complete information offers. Also note that in general,  $l$ -type trees have a weakly higher turnover than  $h$ -type trees. This is again intuitive as  $l$ -type trees would be used in exchange more frequently, as they don't face the costs associated with informational frictions unlike the  $h$ -type trees just by their virtue (or vice) of offering low fruit to the holder.

Now turn to normative considerations and define society's welfare as the sum of utilities of all agents in the economy,  $W = NV_s + M_l V_l + M_h V_h$ . Note that  $V_s = 0$  thus,

$$W = (1 + r)[M_h \omega_h + M_l \omega_l] \quad (19)$$

The society's welfare is equal to the aggregate real asset holdings (in the form of the Lucas trees).

**Proposition 4.** *Suppose  $\omega_h > \omega_l$  is true from Proposition 2, then  $\frac{\partial W}{\partial \pi_l} < 0$  and  $\frac{\partial W}{\partial \theta} > 0$ .*

*Proof is in the Appendix.*

As the fraction of  $l$ -type trees increases, society's welfare in a separating equilibrium falls. This is because when fraction of  $l$ -type increases, the proportion of  $h$ -type buyers falls and since the latter type got a higher utility than the former from Lemma 2 society's welfare falls. Note, in this case, the positive effect of higher trading volumes that comes with being an  $l$ -type buyer is outweighed by the positive effect of holding the tree which offers better fruits. Moreover, an increase in recognizability (or an increase in the fraction of informed matches) increases society's welfare by making  $h$ -type trees more valuable.

The separating equilibrium just characterized might not meet the consistency requirement imposed under Definition 1. The surplus enjoyed by the  $h$ -type buyer in a uniformed match under the least-cost separating offer should be greater than what she would obtain at her

preferred or ‘best’ pooling equilibrium offer. Taking as given  $(\omega_l, \omega_h)$  just derived from (16) - (17), the following condition must hold:

$$S_h^s = u(q_h^s) - c(q_h^s) > S_h^p = u(q^p) - \frac{\omega_h c(q^p)}{\pi_h \omega_h + \pi_l \omega_l} \quad (20)$$

where  $(q^p, p^p)$  is the pooling equilibrium offer and solves (11) - (13).

**Proposition 5.** *Given  $\omega_l$  and  $\omega_h$  from (16) and (17), there exists  $\hat{\pi}_l^s$  such that for all  $\pi_l > \hat{\pi}_l^s$  (20) holds.*

*Proof is in the Appendix.*

Given the value of trees  $l$  and  $h$  determined by (16) - (17), Proposition 5 states that, if the supply of  $l$ -type trees is sufficiently high, then the separating equilibrium characterised above prevails as the equilibrium given by Definition 1. Intuitively,  $h$ -type buyers optimally separate from the  $l$ -type buyers only when the market has many trees of the latter type. This makes it worthwhile for them to separate as seller’s expected value of the tree under pooling equilibrium diminishes with many  $l$ -types in the market, as a result better terms of trade have to be offered to the buyer.

Under the separating equilibrium, as shown in Lemma 1, the  $h$ -type tree obtains less output than  $l$ -type and has a lower liquidity in uninformed matches. The  $h$ -type tree thus signals her quality by accepting a lower liquidity and velocity. This in fact is reminiscent of Gresham’s Law: ‘bad’ trees have driven out the ‘good’ ones (or, at least some of them).

## 4.2 Pooling equilibrium

Now the pooling (and semi-pooling or partial pooling) equilibria are considered, where the offer made by the  $h$ -type buyer is imitated by all  $l$ -type buyers (or, some of them in case of semi-pooling) in uninformed matches. Focus on the pooling equilibrium preferred by  $h$ -type buyers (i.e. the efficient pooling equilibrium) for reasons discussed earlier.

So far, attention was restricted to symmetric equilibria where all buyers of the same type made the same offer. This restriction was with no loss in generality under separating equilibria.

However, in the context of pooling equilibria, considering asymmetric equilibria is important if  $u(q^p) - p^p\omega_l \geq u(q_l^c) - p_l^c\omega_l$  is not satisfied; i.e. it is not incentive compatible for all  $l$ -type buyers to pool. In this case, a fraction of  $l$ -type buyers make their complete information offers,  $(q_l^c, p_l^c)$  while the rest offer  $(q^p, p^p)$  that solves:

$$(q^p, p^p) = \arg \max_{q, p \in [0, 1]} [u(q) - p\omega_h] \text{ s.t. } -c(q) + p\mathbb{E}(\omega_i) \geq 0 \quad (21)$$

where the expectation is with respect to the buyer's type  $i \in \{h, l\}$  conditional on the offer made and is derived shortly.<sup>9</sup>

From (21) the terms of trade  $(q^p, p^p)$  satisfies:

$$u'(q^p) = \frac{c'(q^p)\omega_h}{\mathbb{E}(\omega_i)} \quad (22)$$

$$p^p = \frac{c(q^p)}{\mathbb{E}(\omega_i)} \quad (23)$$

if  $p^p \leq 1$ , otherwise  $p^p = 1$  and,

$$q^p = c^{-1}(\mathbb{E}(\omega_i)) \quad (24)$$

Let  $\phi \in [0, 1]$  denote the fraction of  $l$ -type buyers offering  $(q^p, p^p)$  in equilibrium. The remaining  $(1 - \phi)$  fraction make their separating offer which is the complete information offer  $(q_l^c, p_l^c)$ .  $\phi$  solves:

$$\phi \begin{cases} = 1 & \text{if } u(q^p) - p^p\omega_l > u(q_l^c) - c(q_l^c) \\ \in [0, 1] & \text{if } u(q^p) - p^p\omega_l = u(q_l^c) - c(q_l^c) \end{cases} \quad (25)$$

From Bayes' rule,

$$\mathbb{E}(\omega_i) = \frac{\pi_h\omega_h + \phi\pi_l\omega_l}{\pi_h + \phi\pi_l} \quad (26)$$

$\phi = 1$  corresponds to the case where all  $l$ -type buyers make the pooling offers.

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<sup>9</sup>There are many semi-pooling equilibria just like there are many pooling equilibria. For instance, for some parameter values, there is an equilibrium where all  $l$ -type buyers and some  $h$ -type make the same offer while the remaining  $h$ -type make their optimal separating offer. It can be checked that such equilibria do not satisfy the consistency requirements imposed by standard refinement concepts such as the Intuitive Criterion.

In order to obtain the value of the two types of trees in equilibrium, subtract  $\beta V_i$  from both sides of (15) and use  $\omega_i = \beta(V_i - V_s)$ . The Bellman equations for each buyer type,  $i \in \{h, l\}$  under the pooling equilibrium offer becomes:

For an  $l$ -type buyer who pools with the  $h$ -type buyer,

$$\omega_l = \frac{\alpha}{r} \left\{ \theta \left[ u(q_l^c) - c(q_l^c) \right] + (1 - \theta) \left[ u(q^p) - \frac{\omega_l c(q^p)}{\mathbb{E}(\omega)} \right] \right\} + \frac{\gamma l}{r} \quad (27)$$

The  $l$ -type buyer's offer in any meeting with an uninformed seller is the pooling equilibrium offer in the bargaining game under private information,  $(q^p, p^p)$ . Thus, the value of  $l$ -type tree is the sum of its discounted liquidity value (value in exchange depending on whether the buyer meets an informed or uninformed seller) and discounted dividend return.

The value of the  $l$ -type tree is higher in the pooling equilibrium than it would be in the case with no private information. To see this, let  $\theta = 1$  in (27) which implies

$$\omega_l^c = \frac{\alpha}{r} \left[ u(q_l^c) - c(q_l^c) \right] + \frac{\gamma l}{r}$$

and for  $\theta = 0$ ,

$$\omega_l^p = \left[ u(q^p) - \frac{\omega_l c(q^p)}{\mathbb{E}(\omega)} \right] + \frac{\gamma l}{r}$$

And, since  $\left[ u(q^p) - \frac{\omega_l c(q^p)}{\mathbb{E}(\omega)} \right] > [u(q_l^c) - c(q_l^c)]$  for the  $l$ -type buyers to be a part of the pooling offer, the latter ( $\omega_l^p$ ) is higher than the former ( $\omega_l^c$ ).

For an  $h$ -type buyer,

$$\omega_h = \frac{\alpha}{r} \left\{ \theta \left[ u(q_h^c) - c(q_h^c) \right] + (1 - \theta) \left[ u(q^p) - \frac{\omega_h c(q^p)}{\mathbb{E}(\omega)} \right] \right\} + \frac{\gamma h}{r} \quad (28)$$

The  $h$ -type buyer's offer in any meeting with an uninformed seller is the pooling equilibrium offer in the bargaining game under private information,  $(q^p, p^p)$ . Thus, the value of  $h$ -type tree is the sum of its discounted liquidity value (value in exchange depending on whether the buyer meets an informed or uninformed seller) and discounted dividend return.

By a similar analysis as under for the  $l$ -type tree, note that the value of the  $h$ -type tree is lower in the pooling equilibrium than it would be in the case with no private information. To see this, let  $\theta = 1$  in (28) to get  $\omega_h^c$  and compare that with  $\omega_h^p$  when  $\theta = 0$ . Since  $\left[ u(q^p) - \frac{\omega_h c(q^p)}{\mathbb{E}(\omega)} \right] < [u(q_h^c) - c(q_h^c)]$ , the latter ( $\omega_h^p$ ) is lower than the former ( $\omega_h^c$ ).

The pooling equilibrium offer  $(q_p, p_p)$  in the bargaining game under private information given by (22) - (24) is the same by construction for both type buyers. A pooling equilibrium can then be defined as a 7-tuple  $(q_i^c, p_i^c, q^p, p^p, \phi, \omega_l, \omega_h)$  that satisfies (2), (3), (22) or (24), (23), (25), (27) and (28).

**Proposition 6.** *There exists a unique  $\omega_h > \omega_l$  that solves (27) - (28).*

*Proof is in the Appendix.*

Proposition 6 establishes the existence and uniqueness of  $\omega_h$  and  $\omega_l$  when pooling offers are made in matches with uninformed sellers. It also shows that  $\omega_h$  is indeed greater than  $\omega_l$  as was assumed in the bargaining game. The value of trees (inclusive of the utility from the bargaining surplus and it's fruits) is higher for  $h$ -type trees than  $l$ -type.

Despite its simple structure, the pooling equilibrium is hard to study analytically. Comparative statics are investigated through numerical examples in the next section, where a comprehensive analysis is done for the overall results.

Just as in the case of the least-cost separating equilibrium, the pooling equilibrium just characterized might not meet the consistency requirement imposed under Definition 1. Numerical examples show that whenever a semi-pooling equilibrium exists, the least-cost separating equilibrium also exists and prevails as the equilibrium under Definition 1. Thus, only symmetric equilibria are considered i.e. pooling or separating. The pooling equilibrium offer satisfies the consistency requirement if the surplus enjoyed by the  $h$ -type buyer in an uninformed match with the efficient pooling offer is greater than what she would obtain at her least-cost separating equilibrium. Taking as given  $(\omega_l, \omega_h)$  just derived from (27)-(28), the following condition must hold:

$$S_h^s = u(q_h^s) - c(q_h^s) \leq S_h^p = u(q^p) - \frac{\omega_h c(q^p)}{\pi_h \omega_h + \pi_l \omega_l} \quad (29)$$

where  $(q_h^s, p_h^s)$  is the separating equilibrium offer for the  $h$ -type buyer and solves (8)-(9).

$h$ -type buyers optimally pool with the  $l$ -type buyers only when the market has few trees of the latter type. This makes it worthwhile for them to pool and get better velocity and output

despite a low price. As discussed above, this leads to an over-valuation of  $l$ -type trees. Thus,  $h$ -type trees trade at premium and  $l$ -type trees at discount with informed sellers.

### 4.3 Discussion and Applications

Equilibrium regions are represented using a numerical example<sup>10</sup> and are presented in Figure 2. There are three regions, one where separating offers are made, another where pooling offers are made and a small intermediate region where both offers are made. Thus, equilibrium always exists and it is unique for most parameter values; we however cannot rule out multiplicity from the intermediate region where the proportion of  $l$ -type buyers is neither too low nor too high. In all equilibria, trade of both assets always takes place with the terms of trade changing depending on the parameters of the model. A more detailed discussion of the implications of the two types of equilibria on output and liquidity of the asset and society's welfare is done below:

1. *Both assets trade at different rates* (Separating equilibrium): This equilibrium occurs when there are sufficiently many  $l$ -type assets in the market and/or the discrepancy between the two is sufficiently high (and/or search frictions are high).  $h$ -type buyers signal their asset type to sellers by accepting a lower output and their assets end up circulating less than their counterparts. The  $h$ -type assets get a lower surplus in exchange and  $l$ -type assets get their first-best offer. As a result, the  $h$ -type asset is undervalued compared to her first-best.
2. *Both assets trade at the same rate* (Pooling equilibrium): This equilibrium occurs when there are sufficiently few  $l$ -type assets in the market and/or the discrepancy between the two is sufficiently low (and/or search frictions are low).  $h$ -type buyers pool their offers with the  $l$ -type assets and both assets end up circulating at the same rate. The  $h$ -type assets get a lower surplus in exchange but the  $l$ -type assets gets higher than her first-best surplus. As a result, the  $l$ -type assets are overvalued compared to their full information value while the  $h$ -type assets are undervalued in exchange.

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<sup>10</sup>Numerical examples are constructed using  $u(q) = 2\sqrt{q}$ ,  $c(q) = q$  and  $\alpha(\tau) = \frac{\tau}{1+\tau}$ , where  $\tau = N/M$ , so  $\alpha = N$ . Benchmark parametrization is  $r = 0.1, \alpha = 0.6, \theta = 0.5, \pi_l = 0.5, \gamma_l = 0.4, \gamma_h = 1$

These results are summarized in Table 1.

Thus, the market transitions from pooling to separating as the proportion of  $l$ -type assets in the market and/or the discrepancy between the two is rises. Finally, by how much the outcome deviates from the first-best is depends on the level of information frictions. Proposition 3 for separating equilibrium and numerical examples for pooling equilibrium state that the effect is less severe when information frictions reduce; this of course makes sense as the society is in fact closer to the first-best.

|           | Pooling  | Separating  |
|-----------|--|---|
| Velocity* | Equal for both<br>$v_h = v_l = v^p$  | Low for $h$ -type<br>$v_h^s < v_h^c \leq v_l^s = v_l^c$                     |
| Output    | Equal for both<br>$q_h = q_l = q^p < q^*$                                  | Low for $h$ -type<br>$q_h^s < q_l^s = q_l^c \leq q_h^c$                     |
| Surplus   | High for $l$ -type<br>$S_h^p < S_h^c; S_l^p > S_l^c$                       | Low for $h$ -type<br>$S_h^s < S_h^c; S_l^s = S_l^c$                         |
| Price     | $l$ -type overvalued<br>$\omega_h^p < \omega_h^c; \omega_l^p > \omega_l^c$ | $h$ -type undervalued<br>$\omega_h^s < \omega_h^c; \omega_l^s = \omega_l^c$ |

\*Assume  $\theta = 0$ , so velocity:  $v_i = \alpha p_i$

Table 1: Summary of results

Finally, to understand how the asset market outcomes vary with the parameters of the model we should put together the above insights and consider some comparative statics for the general case. From Figures 3 and 4 we see how the velocity, output and value of assets changes as the discrepancy between low and high and the proportion of low type changes.<sup>11</sup>

Consider the first panel of Figure 3 which shows how velocity of the two assets changes. The left figure shows that as the discrepancy increases (as we move from right to left along the horizontal axis) the velocity of the  $h$ -type asset reduces while that of  $l$ -type rises, and both diverge from each other. This is because as the bad asset's quality worsens, the bad quality asset holder does not want to hold on to her asset while the good quality asset becomes more valuable

<sup>11</sup>Numerical example is the same as used earlier, see footnote 10. Also, note that the discrepancy is decreasing from left to right, i.e. as  $\frac{\gamma_l}{\gamma_h}$  increases from 0 (high) to 1 (low)

and is therefore hoarded. The right panel shows that as the proportion of  $l$ -type trees increases, the velocity of  $h$ -type falls and  $l$ -type rises. The changes in both cases are discontinuous at the point when the market transitions from pooling to separating (right to left in the left figure and left to right in the right). Since the velocity is different and in particular lower for the  $h$ -type tree in informed matches, the two are not exactly equal under pooled market outcomes. These results indicate that as the market moves from pooling to separating, the velocities diverge and the good asset is hoarded as the buyers signal their quality (a version of Gresham's law).

Figure 4 shows how the average velocity changes as the two parameters of the model change. The average velocity increases as the market moves from pooled to separated outcomes (discrepancy increases and/or proportion of low types increase). This is because the low types get a high velocity which shifts up the average velocity.

The second panel can be interpreted similar to the first panel of Figure 3. As the market moves from pooling to separating, the output each asset obtains diverges as the good asset is hoarded.

The third panel shows how the value of the two assets change. The first thing to note is that the good asset is always valued higher than the bad except when  $\gamma_l/\gamma_h = 1$  i.e. both assets are the same. The value of the good asset rises as the discrepancy between the two assets falls and rises weakly as the proportion of low types fall (under separating equilibrium the value of assets is independent of the composition of assets). The value of the low-type sees a discontinuous fall as the market moves to separating outcomes, i.e. when the asset gets its actual value (what it would get in informed matches) in trades. This happens as the proportion of bad assets increases and as its quality diverges significantly from the good. Thus, in this sense one can say that the bad assets are overvalued in pooling outcomes.

If one can interpret the decline in quality mix of assets as a situation of financial crisis, then the abrupt decline in prices of some assets (the inferior ones) can be so explained.

This model can also be applied to a historical commodity world context where there are two types of coins with a high and low intrinsic content but with a recognizability problem. This interpretation closely follows Velde, Weber and Wright (1999) but allowing lotteries uncovers new equilibria as discussed. A detailed discussion is beyond the scope of the present paper, but

a brief mention is worth making. Whenever there are two types of coins with different metallic contents in the market, a version of Gresham's law will hold. When there are many light coins (or the content of the two coins are very different or search frictions are high), the heavy coins are in reduced circulation - this is the classic case of 'bad' money driving (some) 'good' money out of circulation. When there are few light coins (or the content of the two coins is not very different or search frictions are low), then heavy coins don't necessarily circulate less but the lighter coins fetch a higher surplus for their holders. This result is consistent with historical experiences as discussed in DeRoover (1949). Thus, the excess light coins is driving out the good coin. One can also extend these results in terms of counterfeiting by considering the case where the intrinsic content of the light coin approaches zero and allowing the possibility of printing/minting counterfeits instead of endowing some agents with such coins.

Other possible applications could go in the direction of understanding the mechanism through which central bank's large scale asset purchase programs can work as unconventional Monetary Policy. In particular, if due to informational asymmetries, an asset which should otherwise be highly valued becomes illiquid, then the central bank can buy back these assets in return for risk-free liquid bonds to inject liquidity into the economy.

## 5 Conclusion

A random-matching or search-based model under private information has been used to study liquidity of assets with different and unknown dividend flows. Adverse selection in asset markets affects liquidity, velocity and prices. The model constructed builds on previous work in the literature, but with some important differences which aided a deeper analysis of adverse selection and liquidity in asset markets. The model featured trades with imperfectly recognizable and indivisible assets in random pairwise meetings. The presence of adverse selection followed from the imperfect information structure and the indivisibility of asset holding. However, lotteries were allowed in exchange i.e. trades were allowed to be probabilistic which allowed an in-depth analysis of bargaining as a signaling game. Finally, a broader equilibrium concept was used which generated some novel results as summarised below.

It has been shown how an asset holder when faced with adverse selection in the market

might either decide to sell her asset at the prevailing price (pool) or hold it back if the price offered is too low (not pool). In particular, agents possessing assets with high future dividend flow might find separating from their lower counterparts too costly and instead decide to pool along with them. And, this decision which depends on the proportion of low type assets and the divergence in dividend flows of the two assets, has economic implications. Output produced in exchange of the high-type asset is below the efficient (or first-best) quantity in all trades as it is illiquid and is undervalued. However, output produced for the low-type assets is efficient (or first-best) when offers are separated. Finally, the society moves away from its first-best outcome as information frictions increase because the value in exchange of the high-type asset falls. The 2007-08 financial crisis exposed the problems associated with liquidity in asset markets; this paper made an attempt to understand how information frictions can help explain liquidity dry-ups, hoarding and price drops in a tractable way.

The model constructed above can also be seen as a baseline model to understand adverse selection and liquidity in different contexts such as commodity money and central bank's large scale asset purchase programs as briefly discussed above. These extensions require more work and would be the subject matter of another paper.

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## A Appendix

### A.1. Proof of Lemma 1

The proof follows by showing that both constraints to the maximization problem (6)-(7) bind. Suppose first that (7) is not binding. Then,  $(q_h^s, p_h^s)$  is just the complete information offer, i.e.,  $q_h^s = \min[q^*, c^{-1}(\omega_h)] \geq q_l^s$  and  $p_h^s = c(q_h^s)/\omega_h > 0$ . In this case, an  $l$ -type buyer who offers  $(q_h^s, p_h^s)$  gets payoff,

$$u(q_h^s) - p_h^s \omega_l = u(q_h^s) - c(q_h^s) + p_h^s (\omega_h - \omega_l) > u(q_l^s) - c(q_l^s)$$

since  $p_h^s > 0$  and  $\omega_h - \omega_l > 0$ . But then (7) is violated (as  $l$ -type buyer has an incentive to copy the  $h$ -type). A contradiction.

Suppose next that the seller's participation constraint in (6) is not binding. Substitute  $u(q) = p\omega_l + u(q_l^s) - p_l^s \omega_l$  at equality, into the buyer's payoff to get

$$\max_{p \in [0,1]} [p(\omega_l - \omega_h) + u(q_l^s) - p_l^s \omega_l]$$

which gives  $p_h^s = 0$  and  $u(q_h^s) = u(q_l^s) - p_l^s \omega_l > 0$  (since  $\omega_l > 0$ ). But then the seller's participation constraint  $-c(q_h^s) \geq 0$  is violated. A contradiction.

So, the constraint in (6) and (7) i.e. both constraints are binding. From the seller's participation constraint we obtain (8). Substitute  $p_h^s$  by its expression given by (8) into (7) at equality to get (9).

In order to establish that there is a unique solution to (6) and (7), notice that the left-hand side of (9) is first increasing and then decreasing in  $q_h^s$ , and it reaches a maximum greater than  $u(q^*) - c(q^*) \geq u(q_l^s) - c(q_l^s)$  (RHS of (9)) for some  $q_h^s > q^*$  solution to  $u'(q_h^s) = \frac{\omega_l}{\omega_h} c'(q_h^s)$ . The equality in (9) might be satisfied with multiple (at most two) solutions. However, only the lowest value for  $q_h^s$  maximizes the payoff of the  $h$ -type buyer. To see this, notice that

$$u(q_h^s) - c(q_h^s) = u(q_h^s) - \frac{\omega_l}{\omega_h} c(q_h^s) - \left(1 - \frac{\omega_l}{\omega_h}\right) c(q_h^s).$$

From (9),

$$u(q_h^s) - c(q_h^s) = u(q_l^s) - c(q_l^s) - \left(1 - \frac{\omega_l}{\omega_h}\right) c(q_h^s).$$

The right-hand side of the equation above is decreasing in  $q_h^s$ ; hence,  $u(q_h^s) - c(q_h^s)$  is maximized at the lowest value of  $q_h^s$  that solves (9).

Next, establish that  $q_h^s < q_l^s$  and  $p_h^s < p_l^s$ . The left-hand side of (9) is increasing in  $q_h^s$  over  $[0, q_l^s]$  from  $0 < u(q_l^s) - c(q_l^s)$  to  $u(q_l^s) - \frac{\omega_l}{\omega_h}c(q_l^s) > u(q_l^s) - c(q_l^s)$  as it reaches it is maximised for some  $q_h^s > q_l^s$ . In other words, at  $q_l^s$  LHS of (9)  $>$  RHS (9). So, there is a unique  $q_h^s \in (0, q_l^s)$  that solves (9) at equality. Thus,  $q_h^s < q_l^s$ . Finally, from (7),  $(p_l^s - p_h^s)\omega_l = u(q_l^s) - u(q_h^s) > 0$ , hence  $p_l^s > p_h^s$ . ■

### A.2. Proof of Proposition 1

This proof proceeds by first establishing independence of  $S_h^s \equiv u(q_h^s) - c(q_h^s)$ , with respect to  $\pi_l$  and then monotonicity of  $S_h^p$  with respect to  $\pi_l$ .

From (6) and (7), the payoff of the  $h$ -type buyer at the separating equilibrium,  $S_h^s \equiv u(q_h^s) - c(q_h^s)$ , is independent of  $\pi_l$ . Whereas, from (10) the payoff of the  $h$ -type under the pooling equilibrium,

$$S_h^p \equiv u(q_h^p) - \frac{\omega_h c(q_h^p)}{(1 - \pi_l)\omega_h + \pi_l\omega_l}$$

is continuous and strictly decreasing with  $\pi_l$ . To see this, note that from (11) and (12)  $q_h^p$  is strictly decreasing in  $\pi_l$ .

Consider first the case when  $\pi_l = 0$ . The problem that determines  $(q_h^s, p_h^s)$ , (6) - (7), corresponds to (10) with an additional binding constraint (7). Now, since the solution of (6) - (7) is such that (7) is binding (see Lemma 1), so  $S_h^p(0) > S_h^s$ .

Next consider  $\pi_l = 1$  and assume  $\omega_l > 0$ . The seller's participation constraint in (10) is identical to the one in the problem of the  $l$ -type buyer, (5). Hence,  $q_h^p \leq q_l^s$  and

$$u(q_h^p) - p^p\omega_l \leq u(q_l^s) - p_l^s\omega_l.$$

Consequently, the incentive compatibility condition (7) holds at  $(q^p, p^p)$ . Now look at the seller's participation constraint in (6). Since,  $-c(q^p) + p^p\omega_l = 0$  and  $p^p > 0$  (since  $\omega_l > 0$ ) so seller's participation constraint in (6) is slack i.e.  $-c(q^p) + p^p\omega_h > 0$ . But, from Lemma 1, the solution to (6) and (7) is such that the constraints hold at equality. Consequently,  $S_h^p(1) < S_h^s$ .

Thus, there is  $\hat{\pi}_l \in (0, 1)$  such that  $S_h^p(\hat{\pi}_l) = S_h^s$ . For all  $\pi_l > \hat{\pi}_l$ ,  $S_h^p(\pi_l) < S_h^s$  and the equilibrium is separating; while for all  $\pi_l \leq \hat{\pi}_l$ ,  $S_h^p(\pi_l) > S_h^s$  and the equilibrium is pooling. ■

### A.3. Proof of Lemma 2

The proof is intuitive as sellers get zero bargaining surplus.

Subtract  $\beta V_s$  from both sides of (14) to get:

$$\begin{aligned}
(1 - \beta)V_s &= \alpha\theta\pi_h \left[ -c(q_h^c) + p_h^c\beta V_h - p_h^c\beta V_s \right] \\
&+ \alpha\theta\pi_l \left[ -c(q_l^c) + p_l^c\beta V_l - p_l^c\beta V_s \right] \\
&+ \alpha(1 - \theta)\pi_h \left[ -c(q_h^u) + p_h^u\beta V_h - p_h^u\beta V_s \right] \\
&+ \alpha(1 - \theta)\pi_l \left[ -c(q_l^u) + p_l^u\beta V_l - p_l^u\beta V_s \right]
\end{aligned}$$

Use  $\omega_i = \beta(V_i - V_s)$  to get:

$$\begin{aligned}
(1 - \beta)V_s &= \alpha\theta\pi_h \left[ -c(q_h) + p_h\omega_h \right] \\
&+ \alpha\theta\pi_l \left[ -c(q_l) + p_l\omega_l \right] \\
&+ \alpha(1 - \theta)\pi_h \left[ -c(q_h^u) + p_h^u\omega_h \right] \\
&+ \alpha(1 - \theta)\pi_l \left[ -c(q_l^u) + p_l^u\omega_l \right]
\end{aligned}$$

Under any take-it-or-leave-it offer by the buyer (complete, separating or pooling) the buyer keeps the seller at zero surplus, which implies that all the terms on the right hand side is zero, hence  $V_s = 0$  for all equilibria. ■

### A.4. Proof of Proposition 2

(i) Start by establishing that a solution  $\omega_h > \omega_l$  to (16) - (17) exists.

At  $\omega_h = \omega_l$ ,  $q_h^s = q_l^s = q_l^c$  from (9). Thus, given that  $\gamma_h > \gamma_l$ , the right-hand side of (17) is greater than right-hand side of (16) i.e.

$$\alpha[u(q_l^c) - c(q_l^c)] + \gamma_h > \alpha[u(q_l^c) - c(q_l^c)] + \gamma_l$$

Consider next,  $\omega_h = \omega_h^c > \omega_l$ , where  $\omega_h^c$  is the solution to  $r\omega_h^c = \alpha[u(q_h^c) - c(q_h^c)] + \gamma_h$  with  $q_h^c = \min[q^*, c^{-1}(\omega_h^c)]$ . Thus, from (16)  $\omega_h^c > \omega_l$  (since  $\gamma_h > \gamma_l$  and  $u(q_l^s) - c(q_l^s) \leq u(q_h^c) - c(q_h^c)$ ).

From the comparison of (1) and (6)-(7),

$$u(q_h^s) - c(q_h^s) \leq u(q_h^c) - c(q_h^c),$$

and with a strict inequality when  $\omega_h > \omega_l$  because the incentive compatibility constraint (7) is binding. Then, the right-hand side of (17) is less than  $\alpha[u(q_h^c) - c(q_h^c)] + \gamma_h = r\omega_h^c$ . By the Intermediate Value Theorem, there is an  $\omega_h \in (\omega_l, \omega_h^c)$ .

(ii) Now to establish uniqueness of  $\omega_h$  (uniqueness of  $\omega_l$  is immediate as  $u(q_l) - c(q_l)$  is concave), rewrite (17) as

$$r\omega_h - \alpha\theta[u(q_h^c) - c(q_h^c)] = \alpha(1 - \theta)[u(q_h^s) - c(q_h^s)] + \gamma_h$$

and show that the left-hand side (LHS) is increasing in  $\omega_h$  (over some range) and right-hand side (RHS) is decreasing.

First, consider LHS. It is convex in  $\omega_h$  as  $u(q_h^c) - c(q_h^c)$  is concave. It is equal to zero when  $\omega_h = 0$  and it reaches a negative minimum at  $\omega_h = c(\bar{q})$  i.e. when  $c^{-1}(\omega_h) < q^*$  and  $q_h^c = c^{-1}(\omega_h) = \bar{q}$  where  $\bar{q}$  solves:

$$\frac{u'(\bar{q})}{c'(\bar{q})} = 1 + \frac{r}{\alpha\theta}$$

and when  $c^{-1}(\omega_h) \geq q^*$  then LHS is equal to  $r\omega_h - \alpha\theta[u(q^*) - c(q^*)]$ . Since it is convex, the LHS is increasing for all  $\omega_h > c(\bar{q})$  and it becomes positive for sufficiently large  $\omega_h$ .

Second, consider RHS. Differentiate (9) to get:

$$\frac{\partial q_h^s}{\partial \omega_h} = -\frac{\omega_l c(q_h^s)}{\omega_h^2 [u'(q_h^s) - \frac{\omega_l}{\omega_h} c'(q_h^s)]} < 0$$

for all  $\omega_h > \omega_l$ . Denominator is positive as  $q_h^s < q^*$ . Thus, the solution  $\omega_h \in (\omega_l, \omega_h^c)$  is unique. ■

#### A.5. Proof of Proposition 3

(i) The right-hand side of (16) and (17) are independent of  $\pi_l$ . Consequently,  $\frac{\partial \omega_i}{\partial \pi_l} = 0$  for  $i \in h, l$ .

(ii) The right-hand side of (16) is independent of  $\theta$ . Consequently,  $\frac{\partial \omega_l}{\partial \theta} = 0$ .

Differentiate (17) with respect to  $\theta$ :

$$r \frac{\partial \omega_h}{\partial \theta} = \alpha \left\{ [u(q_h^c) - c(q_h^c)] + \theta \frac{\partial [u(q_h^c) - c(q_h^c)]}{\partial \omega_h} \frac{\partial \omega_h}{\partial \theta} - [u(q_h^s) - c(q_h^s)] + (1 - \theta) \frac{\partial [u(q_h^s) - c(q_h^s)]}{\partial \omega_h} \frac{\partial \omega_h}{\partial \theta} \right\}$$

$$\frac{\partial \omega_h}{\partial \theta} = \frac{\alpha [u(q_h^c) - c(q_h^c)] - \alpha [u(q_h^s) - c(q_h^s)]}{r - \frac{\partial RHS}{\partial \omega_h}} > 0$$

where RHS is the right-hand side of (17) and  $\frac{\partial RHS}{\partial \omega_h}$  is evaluated at the equilibrium. ■

#### A.6. Proof of Proposition 4

(i) Re-write W from (19) in terms of  $\pi_l$ :

$$W = M(1 + r)[(1 - \pi_l)\omega_h + \pi_l\omega_l]$$

$$\frac{\partial W}{\partial \pi_l} = M(1 + r)[- \omega_h + \omega_l] < 0$$

To see why, note that from Lemma 3,  $\frac{\partial \omega_i}{\partial \pi_l} = 0$  for  $i \in h, l$  and  $\omega_h > \omega_l$ .

(ii) From Lemma 3,  $\frac{\partial \omega_l}{\partial \theta} = 0$  and  $\frac{\partial \omega_h}{\partial \theta} > 0$  thus,

$$\frac{\partial W}{\partial \theta} = (1 + r) \left[ M_h \frac{\partial \omega_l}{\partial \theta} + M_l \frac{\partial \omega_h}{\partial \theta} \right] > 0$$

■

#### A.7. Proof of Proposition 5

The proof is similar to the proof of Proposition 1 with the difference that now  $\omega_l, \omega_h$  are endogenously determined in the general equilibrium framework.

However,  $S_h^s \equiv u(q_h^s) - c(q_h^s)$  is still independent of  $\pi_l$  because (2), (3), (8),(9), (16) and (17) can be solved simultaneously to get  $(q_i^c, p_i^c, q_h^s, p_h^s, \omega_l, \omega_h)$  and they are all independent of  $\pi_l$ . Note that  $(\omega_l, \omega_h)$  are independent of  $\pi_l$ .

As before,  $S_h^p(\pi_l)$  is decreasing in  $\pi_l$ . To see this note that the values of trees we use to determine the pooling terms of trade are given by (16) and (17) which are independent of  $\pi_l$ .

Hence as in the proof for Proposition 1 it can be shown that  $S_h^p(\pi_l)$  is decreasing in  $\pi_l$ . Rest of the proof directly follows from there. ■

#### A.8. Proof of Proposition 6

(i) Following the same structure as in the proof for Proposition 2 start by establishing that a solution  $\omega_h > \omega_l$  to (27) - (28) exists.

At  $\omega_h = \omega_l$ ,  $q^p = q_h^c$  from (21) and (26). Since  $\gamma_h > \gamma_l$ , the right-hand side of (28) is greater than right-hand side of (27) i.e.

$$\alpha[u(q_h^c) - c(q_h^c)] + \gamma_h > \alpha[u(q_h^c) - c(q_h^c)] + \gamma_l$$

Consider next,  $\omega_h = \omega_h^c > \omega_l$ , where  $\omega_h^c$  is the solution to  $r\omega_h^c = \alpha[u(q_h^c) - c(q_h^c)] + \gamma_h$  with  $q_h^c = \min[q^*, c^{-1}(\omega_h^c)]$ . Now from (27),  $\omega_h^c > \omega_l$  (since  $\gamma_h > \gamma_l$  and  $\theta[u(q_l^c) - c(q_l^c)] + (1 - \theta)[u(q^p) - \omega_l/(\omega_l\pi_l + \omega_h\pi_h)c(q^p)] \leq u(q_h^c) - c(q_h^c)$ ).

From the comparison of (1) and (21),

$$u(q^p) - \frac{\omega_h}{\mathbb{E}(\omega_i)}c(q^p) \leq u(q_h^c) - c(q_h^c),$$

and with a strict inequality when  $\omega_h > \omega_l$ . Then, the right-hand side of (28) is less than  $\alpha[u(q_h^c) - c(q_h^c)] + \gamma_h = r\omega_h^c$ . By the Intermediate Value Theorem, there is an  $\omega_h \in (\omega_l, \omega_h^c)$ .

(ii) We use the same method as in the proof of Lemma 2 to show that  $\omega_h$  is unique (uniqueness of  $\omega_l$  can be proved similarly). Re-write (28) as

$$r\omega_h - \alpha\theta[u(q_h^c) - c(q_h^c)] = \alpha(1 - \theta)[u(q_h^p) - \frac{\omega_h c(q_h^p)}{\mathbb{E}(\omega_i)}] + \gamma_h$$

and show that the left-hand side (LHS) is increasing in  $\omega_h$  (over some range) and right-hand side (RHS) is decreasing.

First, consider LHS. It is convex in  $\omega_h$  as  $u(q_h^c) - c(q_h^c)$  is concave. It is equal to zero when  $\omega_h = 0$  and it reaches a negative minimum at  $\omega_h = c(\bar{q})$  i.e. when  $c^{-1}(\omega_h) < q^*$  and  $q_h^c = c^{-1}(\omega_h) = \bar{q}$  where  $\bar{q}$  solves:

$$\frac{u'(\bar{q})}{c'(\bar{q})} = 1 + \frac{r}{\alpha\theta}$$

and when  $c^{-1}(\omega_h) \geq q^*$  then LHS is equal to  $r\omega_h - \alpha\theta[u(q^*) - c(q^*)]$ . Since it is convex, the LHS is increasing for all  $\omega_h > c(\bar{q})$  and it becomes positive for sufficiently large  $\omega_h$ .

Second, consider RHS. Differentiate RHS with respect to  $\omega_h$  to get:

$$-\alpha(1 - \theta)c(q^p) \left[ \frac{\phi\pi_l\omega_l}{(\pi_h + \phi\pi_l)(\mathbb{E}(\omega_i))^2} \right]$$

This is a negative expression and RHS is decreasing in  $\omega_l$ . Thus, the solution  $\omega_h \in (\omega_l, \omega_h^c)$  is unique. ■.

## B Appendix: Equilibrium

A typical signalling game has the following structure (Fudenberg & Tirole, 19–). It has two players: a sender (of the signal) and a receiver. In the above context the sender is the buyer who makes the offer and the receiver is the seller who accepts or rejects the offer. The sender has private information about her type  $i \in I$  which is drawn from some commonly known probability distribution  $\pi$ . After observing her type the sender sends an offer (as her signal)  $a_s \in A_s$  to the receiver. The receiver observes the offer and chooses  $a_r \in A_r$ . Before the game begins, the receiver has prior beliefs about the sender's type. After observing sender's offer, the receiver should update her beliefs,  $\lambda(i|a_s)$  about sender's type depending on the offer made. How the posterior belief of the receiver is formed depends on the equilibrium concept.

In the context of the bargaining game above, sender is the buyer and her offer,  $a_s$  is a pair  $(q, p) \in \mathbb{R}_+^2$  where  $q$  is the output and  $p$  is the probability to hand over the unit of money to the seller. Receiver is the seller and her set of actions,  $a_r$  in our context is  $\{Y, N\}$ . If  $a_r = Y$  then the offer is accepted and if  $a_r = N$  then the offer is rejected.

The payoff of the buyer (sender) given her type  $i$  is  $U^b(i, a_s, a_r) = [u(q) - p\omega_i]\mathbb{I}_Y$ . The payoff of the seller (receiver) is  $U^s(i, a_s, a_r) = [-c(q) + p\omega_i]\mathbb{I}_Y$ . Based on the offer, the seller forms her belief  $\lambda(i|a_s)$  about the type of buyer.

A Perfect Bayesian Equilibrium (PBE) of a signaling game is a strategy profile  $(a_s^*, a_r^*)$  and posterior beliefs  $\lambda(i|a_s)$  such that:

1.  $\forall i \in I, a_s^* \in \arg \max_{a_s} U^b(i, a_s, a_r^*),$
2.  $\forall a_s^*, a_r^* \in \arg \max_{a_r} \sum_i \lambda^*(i|a_s) U^s(i, a_s, a_r),$
3.  $\lambda^*$  satisfies Bayes' rule whenever possible and is unconstrained for out-of-equilibrium offers.

In words, PBE is a set of strategies and beliefs such that, strategies are optimal given beliefs, and the beliefs are obtained from equilibrium strategies and observed actions using Bayes' rule. Note that the concept of PBE defined above is equivalent to sequential equilibrium for the class of signaling games.

The above equilibrium notion can give rise to multiple equilibria, and thus there is a need to refine further to give the model some predictive power. Two classes of equilibria that emerge as candidate equilibria in the signaling game are considered. One is the least-cost separating equilibrium selected by the Intuitive Criterion from Cho and Kreps (1987) and the other is the efficient pooling offer i.e. the one that maximizes welfare of high type asset holders among all pooling offers. To discriminate between the two, the one that gives a higher payoff to the high type asset holder is chosen following the concept of undefeated equilibria from Mailath et al. (1993). Such a refinement has the nice property of being Pareto efficient in the present context. As has been argued by Guerrieri, Shimer and Wright, 2010 (without explicitly using any equilibrium refinement) pooling outcomes can Pareto dominate the separating in some cases.

First, let us establish why Intuitive Criterion from Cho and Kreps (1987) selects the least-cost separating equilibrium among others. Later, we will see how this might be restrictive and not Pareto optimal in some cases. This will lead us to adopt a broader equilibrium definition.

The Cho-Kreps (1987) refinement is based on the idea that out-of-equilibrium actions should never be attributed to a type who would not benefit from it under any circumstances. First, consider the seller's equilibrium actions in our context given that the buyer can be of any type  $i \in I$  where  $I = \{h, l\}$ :

$$\begin{aligned} a_r^*(q, p) &= \{Y\} \text{ if } -c(q) + p\omega_l > 0 \\ &= \{N\} \text{ if } -c(q) + p\omega_h < 0 \\ &= \{Y, N\} \text{ otherwise} \end{aligned}$$

Consider a proposed equilibrium where the payoff of a buyer type- $i$  is denoted by  $U_i^*$ . According to Cho and Kreps (1987, p.202), this proposed equilibrium fails the Intuitive Criterion if there exists an unsent offer  $a'_s$  such that:

1.  $U_l^* > \max_{a_r \in BR(\{h, l\}, a'_s)} U^b(l, a'_s, a_r)$
2.  $U_h^* < \min_{a_r \in BR(\{h\}, a'_s)} U^b(h, a'_s, a_r)$

According to the first requirement, the unmet offer  $a'_s$  would reduce the payoff of the  $l$ -type buyer compared to her equilibrium payoff irrespective of the inference the seller draws from  $a'_s$ . Consequently, the seller should attribute the offer  $a'_s$  to a  $h$ -type buyer. If she does so, the second requirement specifies that the  $h$ -type buyer should obtain a higher utility with  $a'_s$  compared to her equilibrium payoff.

In the context of the bargaining game, the proposed equilibrium (with payoff of a buyer type- $i$  is denoted by  $U_i^*$ ) fails the Intuitive Criterion if there exists an unmet offer  $a'_s$  which is  $(q', p')$  such that the following is true:

1.  $U_l^* > u(q') - p'\omega_l$
2.  $U_h^* < u(q') - p'\omega_h$
3.  $0 \leq -c(q') + p'\omega_h$

In words, the above says that the out-of-equilibrium offer would make the  $h$ -type buyer strictly better off if accepted and it would make the  $l$ -type buyer strictly worse off. Finally, the seller should accept the offer believing that it came from a high-type buyer.

It can be seen that only the least-cost separating offer will be made in equilibrium as discussed below.

- Only least-cost separating among all separating equilibrium (i.e. the one that maximises payoff of the high type), else a deviation will make the high-type buyer better while hurting the low-type.
- No pooling equilibrium, the high-type buyer could make an out-of-equilibrium offer that would make her better-off (ask for a smaller  $q$  in return for smaller transfer  $p$ ) and that would hurt the low-type. By making such an offer the  $h$ -type buyer hopes to convince the seller that she is  $h$ -type.

However, Mailath, Okuno-Fujiwara and Postlewaite (1993) criticized the logical foundation of the Intuitive Criterion which is based on forward induction by arguing that it lacks global consistency. Consider a perfect Bayesian equilibrium of the bargaining game that is pooling.

It has been shown that a high type buyer could make an out-of-equilibrium offer that would make her better-off and that would hurt the low-type. By making such an offer the  $h$ -type buyer hopes to convince the seller that she is  $h$ -type. But if the seller finds the Intuitive Criterion appealing she knows that the  $h$ -type buyer will alter her offer, and hence she should update her belief about the buyer's type if she does receive the equilibrium message. "But if the (seller) does this, in the determination of whether a particular type might benefit from sending some disequilibrium message, the relevant comparison is not with the utility that he would receive in the proposed equilibrium but rather the utility that he would receive given that (the seller) is thinking in this way". (Mailath, Okuno-Fujiwara and Postlewaite, 1993, p.250.) To achieve global consistency, the out-of-equilibrium should itself correspond to an alternative perfect Bayesian equilibrium.

They further argue that besides the question of logic, there is also some doubt about the plausibility of the equilibrium selected by the Intuitive Criterion. In their words, "A crude refinement would be to select the equilibrium that Pareto dominates the other equilibria, if such an equilibrium exists, but this is clearly inconsistent with the intuitive criterion refinement" . (Mailath, Okuno-Fujiwara and Postlewaite, 1993, p.252.) The equilibrium notion used in the paper appeals to such an argument and hence selects equilibrium based on the surplus of the high-type buyer.

An equilibrium is composed of a strategy for buyers,  $a_s$ , that specifies an offer for each type, an acceptance rule for sellers,  $a_r$ , and a belief system for sellers,  $\lambda$ . Let  $\pi(i)$  be the commonly known prior probability of the buyer being type- $i$ . According to Mailath, Okuno-Fujiwara and Postlewaite (1993, p.254, Definition 2) an equilibrium  $(a'_r, a'_s, \lambda')$  defeats  $(a_r, a_s, \lambda)$  if there exists an offer  $a''_s$  such that:

1.  $\forall i \in I, a_s(i) \neq a'_s$  and  $K \equiv \{i \in I | a'_s = a''_s\} \neq \emptyset$
2.  $\forall i \in K, U^b[i, a'_s, a'_r(a'_s)] \geq U^b[i, a_s(i), a_r(a_s(i))]$  and,  $\exists i \in K, U^b[i, a'_s, a'_r(a'_s)] > U^b[i, a_s(i), a_r(a_s(i))]$ .
3.  $\exists i \in K, \lambda(i|a'_s) \neq \rho(i)\pi(i) / \sum_{i'} \rho(i')\pi(i')$  for any  $\rho : I \rightarrow [0, 1]$  satisfying:
  - $i' \in K$  and  $U^b[i', a'_s, a'_r] > U^b[i', a_s, a_r] \implies \rho(i') = 1$  and
  - $i \notin K \implies \rho(i) = 0$

So, for a sequential equilibrium to be defeated there must exist an out-of-equilibrium offer that is used in an alternative sequential equilibrium by a subset  $K$  of buyers' types (requirement 1). For all buyers with types in  $K$ , their payoff at the alternative equilibrium must be greater than the one at the proposed equilibrium with a strict inequality for at least one type (requirement 2). Finally, the belief system in the proposed equilibrium does not update sellers' prior belief conditional on the buyer's type being in  $K$  (requirement 3).

In our context, an equilibrium is a 4-tuple  $\{(q_h, p_h), (q_l, p_l), \mathcal{A}, \lambda\}$  where  $\mathcal{A}$  is the acceptance rule of the seller and other variables are defined before. Consider equilibria where offers are accepted. A proposed equilibrium  $\{(q_h, p_h), (q_l, p_l), \mathcal{A}, \lambda\}$  is defeated by an alternative equilibrium  $\{(q'_h, p'_h), (q'_l, p'_l), \mathcal{A}', \lambda'\}$  if one of the following is true:

1.  $u(q_h) - p_h \omega_h < u(q'_h) - p'_h \omega_h$  if  $(q'_h, p'_h) \neq (q'_l, p'_l)$ .
2.  $u(q_h) - p_h \omega_h < u(q') - p' \omega_h$  and  $u(q_l) - p_l \omega_l \leq u(q') - p' \omega_l$  if  $(q'_h, p'_h) = (q'_l, p'_l) = (q', p')$

In words, thus if the alternative equilibrium is strictly preferred to the proposed one by the high-type when offers of the two types in the alternative equilibrium differ then the proposed equilibrium is defeated. And, in case both types make the same offer in the alternative equilibrium then the high type should strictly prefer the alternative and the low type should be at least as well off in the alternative equilibrium offer as in the proposed one.

Among the class of separating equilibria, the least-cost separating is the only undefeated one. Among all pooling, the efficient pooling equilibrium (or the preferred pooling offer by the high-type) is chosen. To distinguish between the two the one that gives a higher payoff (or surplus) to high-type buyer is chosen.

# Figures

Figure 1: Types of pooling equilibria

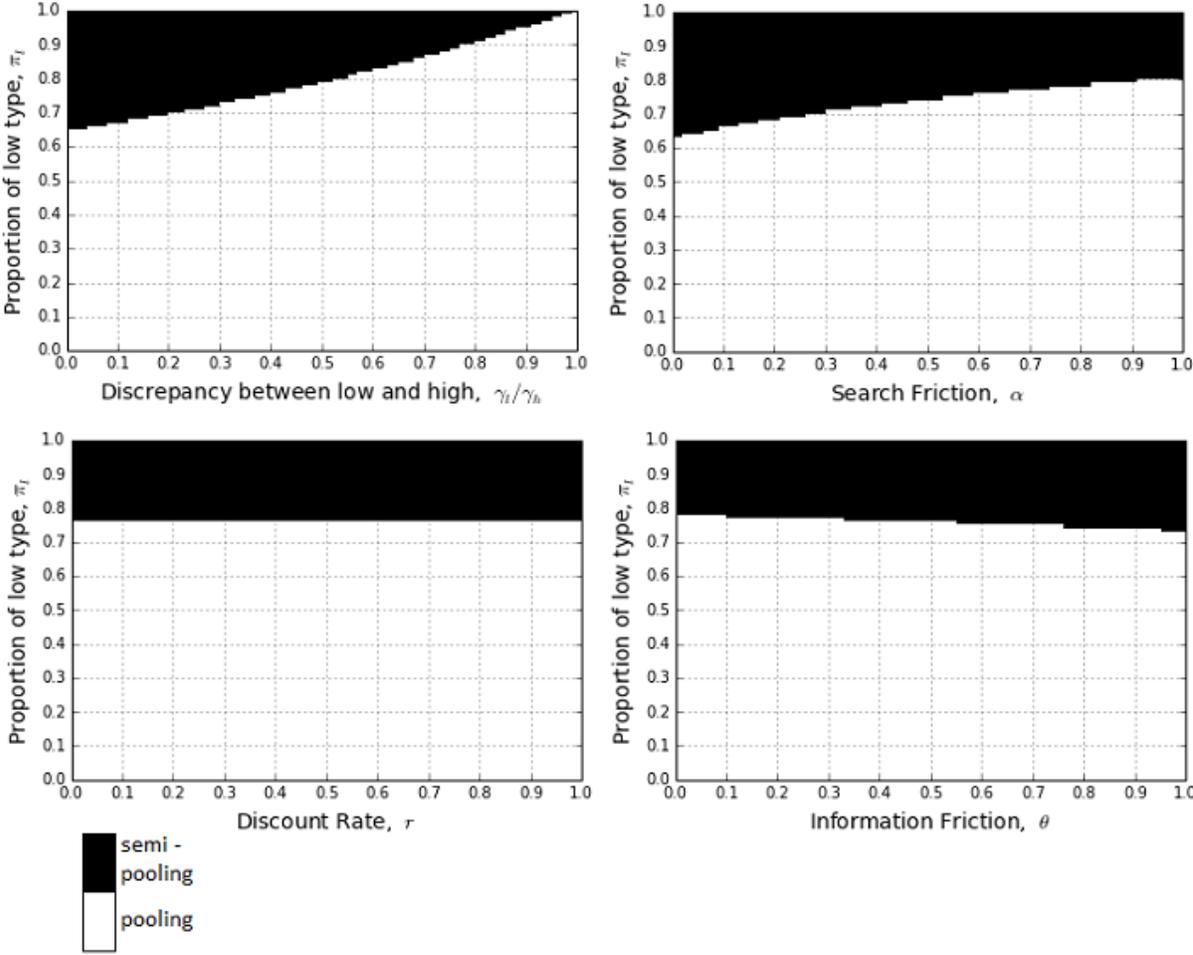
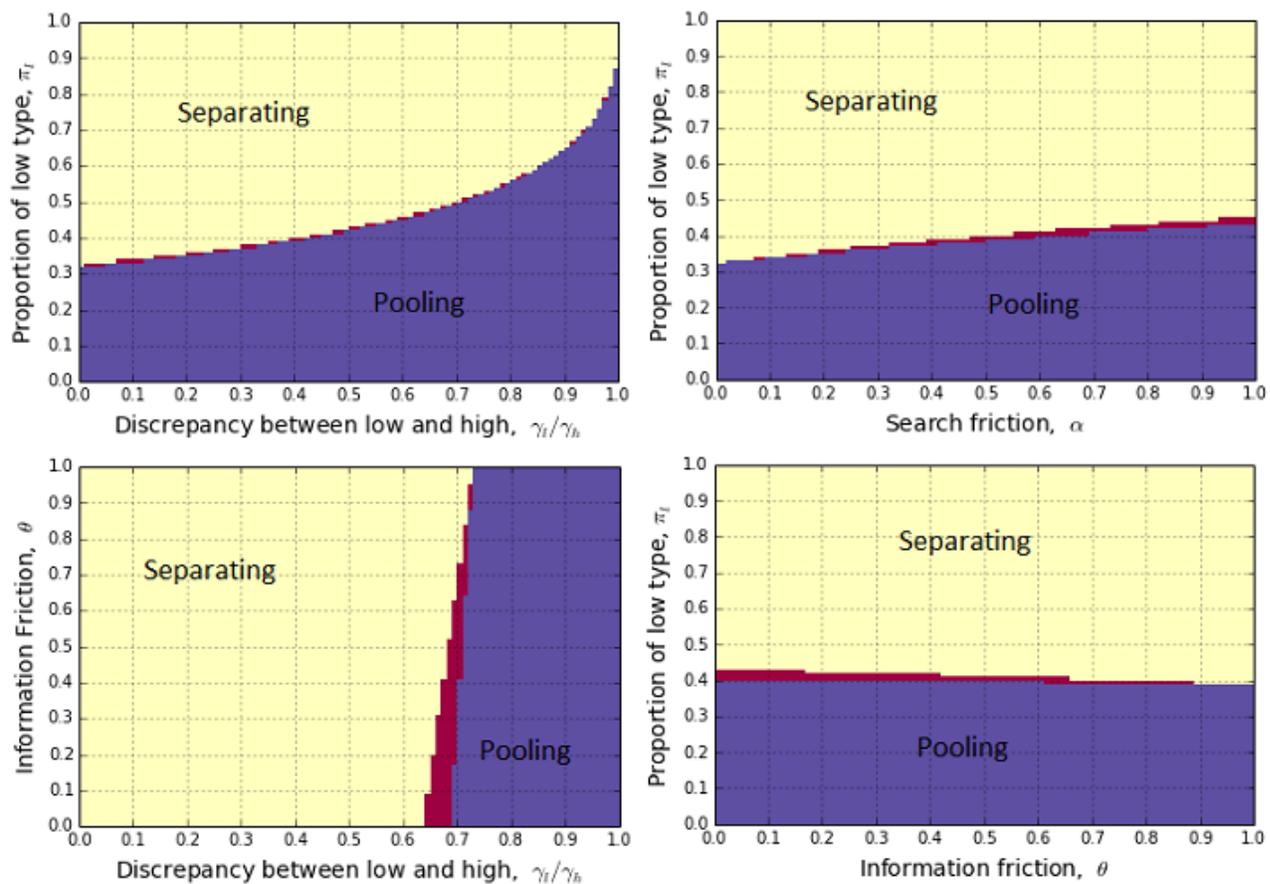


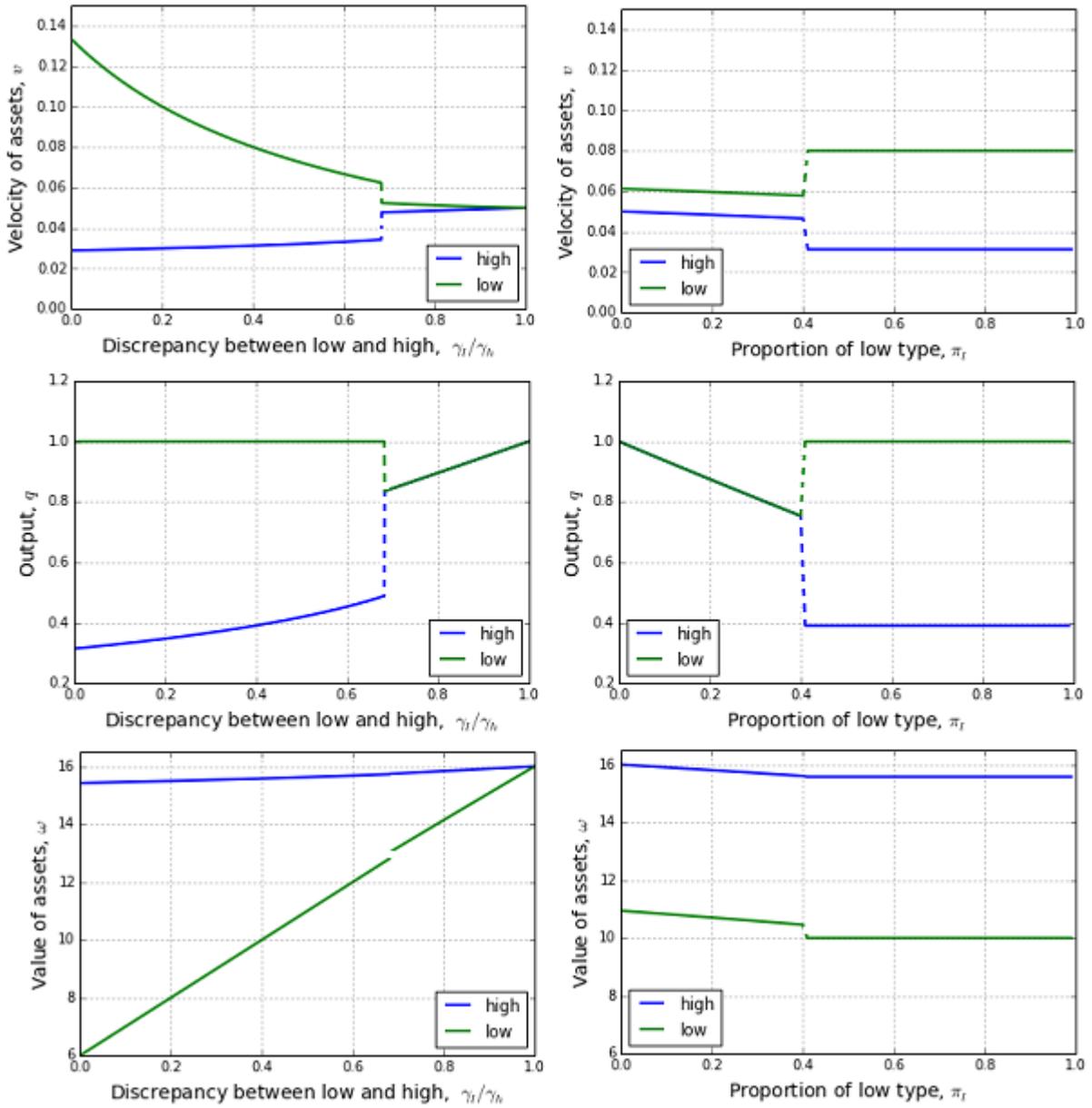
Figure 2: Equilibrium regions



Notes: Both equilibria exist in the intermediate region.

Note: The discrepancy is increasing from right to left, i.e. as  $\frac{\gamma_l}{\gamma_h}$  decreases from 1 (low) to 0 (high)

Figure 3: Comparative statics



Note: The discrepancy is increasing from right to left, i.e. as  $\frac{\gamma_l}{\gamma_h}$  decreases from 1 (low) to 0 (high)

