

# A Simple Model of Monetary Exchange with Sticky and Disperse Prices\*

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## Abstract

We propose a new search-based model of monetary exchange with indivisible assets, based on price posting by sellers, rather than bargaining. The approach generates price dispersion, is consistent with sticky nominal prices, and is well-suited for the study of nonstationary monetary equilibria. The paper also contributes to the literature on general price dispersion, by having buyers constrained by their asset positions, which matters for the number and nature of equilibria. Once some technical results are established, the framework is tractable, and parametric cases can be solved explicitly. In general, we characterize stationary and dynamic equilibria, including sunspot equilibria.

Key words: Search, Money, Sticky Prices, Price Dispersion

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# 1 Introduction

This paper proposes a new candidate for a benchmark search-based model of monetary exchange. To be clear, we mean the benchmark second-generation model. First-generation models like Kiyotaki and Wright (1989,1993) or Aiyagari and Wallace (1992,1992) have indivisible assets and goods, and every trade is a one-for-one swap, so prices are not endogenous. Second-generation papers by Shi (1995) and Trejos and Wright (1995) introduce divisible goods and determine prices (how much one gets for one's money) using bargaining theory.<sup>1</sup> This second-generation approach is quite tractable – e.g., it can easily be taught to undergraduates – but there several issues that we feel are better addressed with our new version, based on price posting, rather than bargaining. Specifically, following Burdett and Judd (1982), our buyers sometimes see one price, and sometimes see more than one price, which leads to competition among sellers that captures as special cases monopoly and competitive pricing, plus much more.

One issue is realism, which is not necessarily a decisive factor, but ought not be dismissed as irrelevant. In actuality, much monetary exchange, including most retail trade, has far more price posting than bargaining. For this reason some commentators are unsympathetic to search-and-bargaining theories of money (e.g., Prescott 2005). We hope they find this version more palatable. Another issue is the following: One of the main applications of the theory is to illustrate rigorously and easily an inherent instability in economies where liquidity considerations play a role by generating equilibria where asset values change

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<sup>1</sup>Third-generation models have divisible assets as well as divisible goods, which has some obvious advantages, but is also much more complicated. Hence, they either rely on computational methods, as in Molico (2006), or on tricks like the large-family specification in Shi (1997) or the alternating-market structure with quasi-linear utility in Lagos and Wright (2005). While those approaches are useful, indivisible-asset models still provide a simple and natural way to illustrate how fiat currency can be valued, how nominal prices are determined, how efficiency is enhanced by money exchange, etc. As argued by Wallace in Altig and Nosal (2013), there are costs and benefits to using second- as opposed to third-generation models, and neither dominates on all dimensions. See Lagos et al. (2015) for a survey of all of these models.

over time purely as self-fulfilling prophecies. However, as explained below, presentations using axiomatic solution concepts have weak strategic foundations in nonstationary situations, and can even be described as tantamount to assuming myopic agents. A few papers use strategic bargaining to avoid that problem, but they are more complicated (Coles and Wright 1998; Ennis 2001). Our approach has fully-rational, strategic, dynamic price setting, yet is not complicated.

A third advantage of the new framework is that, as in other applications of Burdett-Judd, it generates price dispersion. This is consistent with the notion that the law of one price is evidently false in many markets: different sellers often charge different prices for what appears to be the same good. A fourth advantage is that along with price dispersion the model can generate price stickiness, defined for our purposes as some nominal prices failing to respond to changes in the aggregate price level or real economic conditions. Intuitively, some sellers can keep their nominal prices fixed when the average price increases because even though their surplus from a sale falls the probability of a sale increases. While this type of stickiness was previously discussed in third-generation models, those papers rely on technical devices that are not needed here.<sup>∞</sup> Hence, an important point about nominal rigidities can be made here in a relatively clean way.

The paper also contributes to the literature on general price or wage dispersion, including the many labor-market applications following Burdett and Mortensen (1998). Those papers assume buyers or employers have arbitrarily deep pockets – i.e., there is perfectly-transferable utility – while our agents are constrained by their asset positions. This makes a difference. While comparable nonmonetary versions of Burdett-Judd typically have a unique equilibrium, we can have multiple stationary equilibria as well as interesting dynamics. This may have been anticipated by monetary economists, but it is nevertheless useful to verify that some standard results in nonmonetary environments with price dis-

persion, like uniqueness, do not survive when we introduce payment constraints. Moreover, compared to those models, there are some nontrivial technical details to be worked out here – e.g., Lemma 4, which is not surprising, but surprising difficult to prove. Having said that, once the technical results are established, the framework is very tractable. In particular, it is tractable enough that we can generate sunspot equilibria where the price distribution (not just the price level) fluctuates randomly over time, something we have not seen before.

Indivisible-asset models succinctly illustrate several salient points in monetary economics, and have also been used to good effect in intermediation theory (Rubinstein and Wolinsky 1987), banking (Cavalcanti and Wallace 1999), finance (Duffie et al. 2005) and other fields. Hence it seems worth further exploring these environments, even if there are alternatives available with divisible assets. The early second-generation models use symmetric Nash bargaining, while later papers consider generalized Nash, Kalai and strategic bargaining (Coles and Wright 1998; Rupert et al. 2001; Trejos and Wright 2014; Zhu 2014). Others use auctions, abstract mechanism design, and price posting with random or directed search but not Burdett-Judd search (Curtis and Wright 2004; Wallace and Zhu 2007; Zhu and Wallace 2007; Julien et al. 2008). Still others explore different matching processes but not the one we use (Coles 1999; Corbae et al. 2003; Matsui and Shimizu 2005; Julien et al. 2008). Introducing Burdett-Judd pricing and matching is part of an ongoing effort to understand this workhorse model in monetary economics, and search theory, more generally.<sup>2</sup>

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<sup>2</sup>To further differentiate our product, consider the previous sticky price papers mentioned above. There money is neutral by design (related to Caplin and Spulber 1987, Eden 1994 and Golosov and Lucas 2003). By contrast, in second-generation models money is not neutral, clearly, because changes in the supply alter the distribution of liquidity across agents. This captures the venerable idea that the real effects of money hinge on distributional considerations, not revisions in units. Thus, we show how stickiness can also arise without neutrality, but stickiness is not the cause of nonneutrality. Although this may be a minor point, more importantly we also go beyond those analyses by analyzing multiplicity and dynamics. Finally, those papers have a special case of Burdett-Judd, where buyers see either 1 or 2 prices, and we consider the general case where they see any number  $n$  with probability  $\pi_n$ .

## 2 The Model

We begin with a brief review of the standard model, with bargaining, because it shares many features with the new model, and it is easier to say how we deviate from it after describing the original benchmark. A  $[0, 1]$  set of infinitely-lived agents meet and trade in continuous time. For now, they meet bilaterally. There is a set of goods  $\mathcal{G}$ , where each agent  $i$  consumes a subset  $\mathcal{G}^i$  and produces  $g^i \notin \mathcal{G}^i$ . Any  $g \in \mathcal{G}^i$  gives  $i$  the same utility  $u(q)$  from  $q$  units, and has production cost  $c(q)$ . Assume  $u(0) = c(0) = 0$ , and  $\forall q > 0$ ,  $u'(q) > 0$ ,  $c'(q) > 0$ ,  $u''(q) < 0$  and  $c''(q) \geq 0$ . Also, there is a  $\hat{q} > 0$  such that  $u(\hat{q}) = c(\hat{q})$ , and define  $q^* \in (0, \hat{q})$  by  $u'(q^*) = c'(q^*)$ . If  $i$  and  $j$  meet,  $\delta$  is the probability  $i$  produces  $g^i \in \mathcal{G}^j$  and  $j$  produces  $g^j \in \mathcal{G}^i$  – a double coincidence – while  $\sigma$  is the probability  $i$  produces  $g^i \in \mathcal{G}^j$  and  $j$  produces  $g^j \notin \mathcal{G}^i$  – a single coincidence.<sup>3</sup> Without affecting the main results, we set  $\delta = 0$  to preclude barter. Still, for a medium of exchange to be essential we must rule out credit by assuming agents lack commitment and trading histories are private information (Kochelakota 1998).

This implies exchange must be *quid pro quo* and thus assets have a role as media of exchange. This economy has one asset with flow return  $\rho$  measured in utility. If  $\rho > 0$  it can be interpreted as a dividend, or fruit from a tree, as in standard asset-pricing theory; if  $\rho < 0$  it can be a cost of holding the asset; if  $\rho = 0$  the asset can be called fiat money according to standard usage (Wallace 1980). In this class of models, assets are indivisible and can be stored 1 unit at a time, so an individual's state is his asset position  $m \in \{0, 1\}$ . Given a fixed supply  $M \in (0, 1)$ ,  $M$  agents called buyers have  $m = 1$ , while  $1 - M$  agents called sellers have  $m = 0$ . After trade a buyer becomes a seller and *vice versa*. In the standard model, with pure random matching at Poisson rate  $\alpha$ , the probability

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<sup>3</sup>A common example has  $K$  goods and  $K$  types of agents, with type  $k$  consuming good  $k$  and producing good  $k + 1 \bmod K$ . If  $K > 2$  then  $\delta = 0$  and  $\sigma = 1/K$ .

per unit time that a buyer  $i$  meets an appropriate counterparty – i.e., a seller producing  $g \in \mathcal{G}^i$  – is  $\alpha_1 = \alpha(1 - M)\sigma$ . Similarly, the rate at which a seller meets an appropriate counterparty is  $\alpha_0 = \alpha M\sigma$ .

Let  $V_m$  be the payoff or value function for agents with  $m \in \{0, 1\}$ . Then

$$rV_1 = \alpha_1 \mathbb{E}[u(q) + V_0 - V_1] + \rho + \dot{V}_1 \quad (1)$$

$$rV_0 = \alpha_0 \mathbb{E}[V_1 - V_0 - c(q)] + \dot{V}_0, \quad (2)$$

where  $r$  is the discount rate and  $q$  is produced in exchange for assets.<sup>4</sup> In (1), e.g., the flow value  $rV_1$  is the expected surplus from trading away the asset,  $\alpha_1 [u(q) + V_0 - V_1]$ , plus the yield  $\rho$  and a pure capital gain  $\dot{V}_1$ . Let  $\Delta = V_1 - V_0$ , so the buyer's and seller's surpluses from trade are  $u(q) - \Delta$  and  $\Delta - c(q)$ . The model can be closed by determining  $q$  according to some bargaining solution. Consider Kalai's proportional solution, which gives buyers and sellers shares  $\theta_1$  and  $\theta_0 = 1 - \theta_1$  of the total surplus  $u(q) - c(q)$ , and implies  $\Delta = \theta_0 u(q) + \theta_1 c(q)$ . Using this and its time derivative,  $\dot{\Delta} = [\theta_0 u'(q) + \theta_1 c'(q)] \dot{q}$ , after subtracting (1)-(2) we get a differential equation in  $q$ ,

$$[\theta_0 u'(q) + \theta_1 c'(q)] \dot{q} = T(q) - \rho, \quad (3)$$

where  $T(q) \equiv r(\alpha_0 \theta_0 - \alpha_1 \theta_1) [u(q) - c(q)]$ . Paths solving (3) are equilibria as long as  $q$  stays in  $[0, \hat{q}]$ , since  $u(q) \geq c(q)$  iff  $q \leq \hat{q}$ . This is the baseline model of money, or more generally, any asset, circulating as a medium of exchange.<sup>5</sup>

<sup>4</sup>The expectation  $\mathbb{E}$  appears because in principle  $q$  be random, although with bargaining it is not. We write it this way to facilitate comparison with the model below, where  $q$  is random.

<sup>5</sup>Here are some results for comparison to the formulation presented below (see Trejos and Wright 2014 for proofs). For  $\rho = 0$  there is nonmonetary steady state  $q = 0$ , and a unique monetary steady state  $q > 0$  iff  $\theta_1 > (r + \alpha M\sigma) / (r + \alpha\sigma)$ , in which case there are also dynamic equilibria where  $\lim_{t \rightarrow \infty} q = 0$ . For  $\rho \neq 0$  there can be multiple monetary steady states, plus dynamics where  $\lim_{t \rightarrow \infty} q = 0$  or  $\lim_{t \rightarrow \infty} q > 0$ , and there can be sunspot equilibria where  $q$  fluctuates stochastically. In any case, at any point in time there is a single price. This is given by  $p = 1/q$ , in nominal terms, when  $M$  is fiat currency, and it is easy to check  $\partial q / \partial M < 0$  and  $\partial p / \partial M > 0$ . Hence the model displays neither price dispersion nor rigidity. One can prove similar results with generalized Nash bargaining, although the arguments are sometimes more complicated. The survey by Lagos et al. (2015) discusses more results, extensions and applications of this benchmark model.

As mentioned, some people seem antagonistic to bargaining, but at least begrudgingly willing to entertain posting.<sup>6</sup> We are agnostic but sympathetic to the idea that it is interesting to consider alternative mechanisms. However, it is not straightforward to embed posting in the above model. Suppose sellers post  $q$ , or equivalently  $p = 1/q$ . In the baseline specification, that is the same as bargaining with  $\theta_0 = 1$ , and can be regarded as a monetary version of Diamond (1971). In this case, with  $\rho = 0$  the only equilibrium is  $q = 0$ , since no one would produce at cost  $c(q) > 0$  to get fiat currency if it yields no surplus when retraded. This can be overturned by having heterogeneity or directed search, as in Curtis and Wright (2004) or Julien et al. (2008), but those papers imply at most 2 prices in equilibrium, and neither can generate stickiness. Hence, we use Burdett and Judd (1983), which is attractive because it captures other approaches as special cases, has been successfully deployed in other applications, and delivers interesting results.

The key deviation from a more pedestrian usual search process is that a buyer now has some probability of sampling more than one seller at a time. Let  $\alpha_n$  be the rate at which buyers simultaneously sample  $n \in \{1, 2, \dots\}$  sellers – e.g.,  $\alpha$  can be a Poisson arrival rate of ‘catalogues’ containing independent quotes of  $q$  on offer by  $n$  relevant sellers, with  $\Pr(n) = \pi_n$ . We allow  $\pi_0 > 0$ , as a ‘catalogue’ might contain nothing a buyer likes, but of course we assume that  $\pi_0 < 1$ , and that  $n$  has finite mean and variance. Consistency requires connecting buyer and seller arrival rates. Let  $\beta_n$  be the rate at which a seller gets a customer with  $n$  quotes, his own plus  $n - 1$  others. The measure of buyers with  $n$  quotes is  $M\alpha\pi_n$ , and the measure of sellers getting customers with  $n$  quotes is  $(1 - M)\beta_n$ . The

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<sup>6</sup>As Prescott (2005) puts it, “I think the bilateral monopoly problem has been solved. There are stores that compete. I know where the drug store and the supermarket are, and I take their posted prices as given. If some supermarket offers the same quality of services and charges lower prices, I shop at that lower price supermarket.” While this may suggest he would prefer posting with *pure* directed search, Bethune et al. (2015) argue that is going too far.

identity  $(1 - M)\beta_n = M\alpha\pi_n n$  implies  $\beta_n = b\alpha\pi_n n$ , where  $b = M/(1 - M)$  is the buyer/seller ratio, or market tightness.

The formulation is flexible. If  $\pi_n = 0 \forall n > 1$ , e.g., it reduces to Diamond's monopoly model, and if  $\pi_1 = 0$  it looks like Bertrand. Formally we have the following (all proofs are in the Appendix):

**Lemma 1** *In the limit as  $\pi_1 \rightarrow 1$ , there is one price and it is the same as bargaining with  $\theta_0 = 1$  and  $\theta_1 = 0$ . In the limit as  $\pi_1 \rightarrow 0$ , there is one price and it is the same as bargaining with  $\theta_0 = 0$  and  $\theta_1 = 1$ .*

Excepting  $\pi_1 = 1$  or  $\pi_1 = 0$ , there is a distribution of  $q$  with support given by a nondegenerate interval  $\mathcal{Q} = [\underline{q}, \bar{q}]$ , or equivalently a distribution of prices  $p = 1/q$  with support  $\mathcal{P} = [\underline{p}, \bar{p}]$ . Intuitively, we cannot have a single price for the following reason: Suppose all sellers post the same terms. Then a buyer contacting more than one seller is indifferent between them, and this gives sellers an incentive to shade  $q$  up. Hence, a single price cannot be an equilibrium.

**Lemma 2** *If  $\pi_1 > 0$  and  $\pi_n > 1$  for some  $n > 1$ , there are no gaps or mass points in the support  $\mathcal{Q}$ . Moreover, the lower bound  $\underline{q}$  satisfies  $\Delta = u(\underline{q})$ .*

A buyer with  $n > 0$  quotes obviously picks the highest  $q$  or lowest  $p = 1/q$ . Let  $F(q)$  be the CDF of quantity and  $G(p) = 1 - F(1/p)$  the CDF of price. Given  $n$  independent draws from  $F(q)$ , the CDF of the highest  $q$  is  $F(q)^n$ , and similarly for the CDF of the lowest  $p$ . Then for a buyer, the analog of (1) is

$$rV_1 = \sum_{n=1}^{\infty} \alpha_n \int_{\underline{q}}^{\bar{q}} [u(q) - \Delta] dF(q)^n + \rho + \dot{V}_1. \quad (4)$$

For a seller posting  $q$ , the analog of (2) is

$$rV_0(q) = b\alpha \sum_{n=1}^{\infty} \pi_n n F(q)^{n-1} [\Delta - c(q)] + \dot{V}_0(q), \quad (5)$$

since  $\beta_n = b\alpha\pi_n n$  is the rate at which he is in contact with a buyer having  $n$  quotes, whence he gets the sale iff the other  $n - 1$  sellers post less than his  $q$ , which occurs with probability  $F(q)^{n-1}$ . In particular, since the lowest quote  $\underline{q}$  never beats the competition, a seller posting  $\underline{q}$  only sells when  $n = 1$ , and so

$$rV_0(\underline{q}) = b\alpha\pi_1 [\Delta - c(\underline{q})] + \dot{V}_0(\underline{q}). \quad (6)$$

In equilibrium every posted  $q$  entails the same payoff, which means we can equate (5) and (6) to get

$$\sum_{n=1}^{\infty} \pi_n n F(q)^{n-1} = \pi_1 \frac{u(\underline{q}) - c(\underline{q})}{u(\underline{q}) - c(\underline{q})}, \quad (7)$$

using  $\Delta = u(\underline{q})$  by Lemma 2. Having  $F(q)$  satisfy (7) is equivalent to  $V_0(q) = V_0$   $\forall q \in \mathcal{Q}$ . Moreover, since  $F(\bar{q}) = 1$ , (7) implies

$$c(\bar{q}) = \frac{\pi_1 c(\underline{q}) + (\mathbb{E}n - \pi_1)u(\underline{q})}{\mathbb{E}n}. \quad (8)$$

This gives the upper bound as a function of the lower bound, say  $\bar{q} = Q(\underline{q})$ . Notice  $Q(0) = 0$ ,  $Q(\hat{q}) = \hat{q}$  and  $Q'(\underline{q}) > 0$ , which is useful because it implies  $\mathcal{Q} \subset [\underline{q}, \hat{q}]$  as long as  $\underline{q} < \hat{q}$ . The next result says  $F(q)$  is well defined.

**Lemma 3**  $\forall q \in [\underline{q}, \bar{q}]$  (7) yields a unique  $F(q) \in [0, 1]$ , and  $\forall q \in (\underline{q}, \bar{q})$   $F(q)$  is differentiable with

$$F'(q) = \frac{\pi_1 [u(\underline{q}) - c(\underline{q})] c'(q)}{[u(\underline{q}) - c(\underline{q})]^2 \sum_{n=1}^{\infty} \pi_n n(n-1) F(q)^{n-2}}. \quad (9)$$

Conditions (7)-(8) describe  $F(q)$  and  $\bar{q}$  as functions  $q$ ; it remains to determine  $\underline{q}$ . To that end, subtract (4) and (6) to get

$$ru(\underline{q}) = \alpha \sum_{n=1}^{\infty} \pi_n \int_{\underline{q}}^{Q(\underline{q})} [u(q) - u(\underline{q})] dF(q)^n - b\alpha\pi_1 [u(\underline{q}) - c(\underline{q})] + \rho + u'(\underline{q})\dot{\underline{q}}.$$

We can rewrite this as  $u'(\underline{q})\dot{\underline{q}} = T(\underline{q}) - \rho$ , where

$$T(\underline{q}) \equiv \psi u(\underline{q}) - b\alpha\pi_1 c(\underline{q}) - \alpha \sum_{n=1}^{\infty} \pi_n \int_{\underline{q}}^{Q(\underline{q})} u(q) dF(q)^n,$$

and  $\psi = r + \alpha(1 - \pi_0) + b\alpha\pi_1$ . This simplifies to<sup>7</sup>

$$T(\underline{q}) = \psi u(\underline{q}) - b\alpha\pi_1 c(\underline{q}) - \alpha\pi_1 [u(\underline{q}) - c(\underline{q})] \int_{\underline{q}}^{Q(\underline{q})} \frac{u(q)F'(q) dq}{u(\underline{q}) - c(\underline{q})}. \quad (10)$$

A *perfect-foresight monetary equilibrium* is a time path for  $\langle F(q), \underline{q}, \bar{q} \rangle$  satisfying the following conditions:  $\underline{q}$  solves  $u'(\underline{q})\dot{\underline{q}} = T(\underline{q}) - \rho$ ;  $\bar{q}$  solves (8);  $F(q)$  solves (7); and  $\underline{q}$  stays in  $(0, \hat{q}]$ , which ensures  $q \in (0, \hat{q}] \forall q \in [\underline{q}, \bar{q}]$ , at every point in time. From this we also get  $G(p)$  with  $p = 1/q$ . A stationary monetary equilibrium, or *SME*, is one where  $\dot{\underline{q}} = 0$ ; a dynamic monetary equilibrium, or *DME*, is one where this is not the case. There are other possibilities. When  $\rho \leq 0$  there always exists an equilibrium where sellers do not accept assets and buyers dispose of them. This cannot happen if  $\rho > 0$ , but one can show there is equilibrium where buyers hoard (rather than spend) the asset if  $\rho$  is too big. In what follows we typically ignore these no-trade outcomes.<sup>8</sup>

Step 1: If there is no trade, then with  $\rho < 0$  there is disposal and with  $\rho > 0$  there is hoarding.

Step 2: Assume there is no trade. Then, a seller contemplating a one-shot deviation will offer at least  $q^0$  satisfying  $u(q^0) = \rho/r$ , to guarantee the buyer is going to take it. The payoff to the seller will be  $\rho/r - q^0$ . So, for this one-shot deviation to be profitable, we need the existence of a  $q^0$  such that  $u(q^0) \geq \rho/r > q^0$ . This will be the case if and only if  $\rho/r \leq \hat{q}$ . In other words, if  $\rho \leq r\hat{q} < \hat{\rho}$

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<sup>7</sup>To see this, first denote the sum in the previous equation by  $S(\underline{q})$ , and interchange summation and integration to write

$$S(\underline{q}) = \int_{\underline{q}}^{Q(\underline{q})} u(q) \sum_{n=1}^{\infty} \pi_n n F(q)^{n-1} F'(q) dq = \pi_1 [u(\underline{q}) - \underline{q}] \int_{\underline{q}}^{Q(\underline{q})} \frac{u(q)F'(q) dq}{u(\underline{q}) - c(\underline{q})},$$

after using (7). Inserting this back into  $T(\underline{q})$  gives (10).

<sup>8</sup>It will be show that, as usual, for some  $\rho < 0$  an equilibrium with trade coexists with one where where sellers refuse to trade. Similarly, for some  $\rho > 0$  an equilibrium with trade coexists with one where buyers refuse to trade and hoard assets – but this is due to asset indivisibility, and the hoarding equilibrium can be eliminated if we allow agents to use *lotteries* whereby buyers get  $q > 0$  in exchange for a *probability* of handing over the asset (Berentsen et al. 2002). As long as  $\rho < r\hat{q}$ , lotteries can be ignored.

there is no equilibrium with no trade; otherwise, there is.

Step 3: Assume there is trade. When would a one-shot deviation to hoard be profitable? Whenever  $\rho/r$  (hoarding forever) exceeds the gain from continuing to trade:  $u(q) - \Delta$ , or  $ru(q) - r\Delta < \rho$ . But this is exactly where the formula for  $T(q) = \rho$  comes from, so we are really saying that hoarding is an equilibrium when  $\rho$  is too big for  $T(q) = \rho$  to have a root, which is another way of saying that there is no equilibrium with trade when  $\rho > \bar{\rho}$ .

To summarize:

when  $\rho < \underline{\rho}$  the unique equilibrium is disposal of the asset

when  $\underline{\rho} < \rho < 0$  there are two equilibria with trade, and one with disposal

when  $0 < \rho < r\hat{q}$  there is only one equilibrium, with trade

when  $r\hat{q} < \rho < \hat{\rho}$  there are two equilibria, one with trade, and one with hoarding of the asset

when  $\hat{\rho} < \rho < \bar{\rho}$  there are three equilibria, two with trade and one with hoarding

when  $\bar{\rho} < \rho$  there is only one equilibrium, with hoarding

Lemmata 1-3 are extensions of known results for nonmonetary Burdett-Judd models where buyers have deep pockets. The next result, which is a key to characterizing  $\underline{q}$ , is novel compared to those models, for the following reason: while in general the highest price makes buyers just indifferent to trade, when they have deep pockets this is pinned down by primitives, while here it depends on the value of money and that is endogenous. Not merely a technical point, this is what generates multiplicity and interesting dynamics.

**Lemma 4**  $T : [0, \hat{q}] \rightarrow \mathbb{R}$  satisfies  $T(0) = 0$  and  $T(\hat{q}) = [r + \alpha(1 - \pi_0)]\hat{q} > 0$ . If  $u'(0) = \infty$  then  $T'(0) = -\infty$ .

Figure ?? illustrates  $T(\underline{q})$  when there is exactly 1 inflection point where  $T$  changes from convex to concave. While we cannot prove this in general, there was never more than 1 in any example we tried. It is not hard to say what

happens if  $T$  were to wiggle more than this – e.g., instead of saying there is one solution  $\underline{q} > 0$  to  $T(\underline{q}) = 0$  we would say there is generically an odd number of solutions. As it is a routine extension to go from 1 inflection point to many, in the Propositions below, it is a maintained assumption that there is at most 1.<sup>9</sup>

Figure ?? shows three important  $\rho$  values:  $\underline{\rho} = \min_{[0, \hat{q}]} T(\underline{q})$ ;  $\hat{\rho} = T(\hat{q})$ ; and  $\bar{\rho} = \max_{[0, \hat{q}]} T(\underline{q})$ . Clearly,  $T'(0) < 0$  implies  $\underline{\rho} < 0 < \bar{\rho}$ , but it can be that  $\bar{\rho} = \hat{\rho}$ , as in the left panel, or  $\bar{\rho} > \hat{\rho}$ , as in the right panel. If  $\rho < \underline{\rho}$  or  $\rho > \bar{\rho}$  there is no equilibrium with trade. Hence, consider  $\rho \in (\underline{\rho}, \bar{\rho})$ . At the values for  $\rho$  shown in red, in  $(\underline{\rho}, 0)$  in the left panel and  $(\hat{\rho}, \bar{\rho})$  in the right, there are multiple solutions to  $T(\underline{q}) = \rho$ ; for other values of  $\rho$  there is a unique solution. Based on this the following is obvious:

**Proposition 1** *If  $\rho \in [0, \hat{\rho})$  there is a unique SME  $\underline{q}^* > 0$ . If  $\rho \in (\hat{\rho}, \bar{\rho}) \cup (\underline{\rho}, 0)$  there are two SME,  $\underline{q}_L^* \in (0, \hat{q})$  and  $\underline{q}_H^* \in (q_L, \hat{q})$ .*

To characterize DME, simply look at the arrows in Figure ??, pointing left when  $T(\underline{q}) < \rho$  and right when  $T(\underline{q}) > \rho$ . It is just that easy because the path for  $F(q)$  is fully pinned down by the path for  $\underline{q}$ . Formally we have the following:

**Proposition 2** *If  $\rho = 0$  then, in addition to the SME at 0 and  $\underline{q}^* \in (0, \hat{q})$ ,  $\forall \underline{q}_0 \in (0, \underline{q}^*)$  there is a DME starting at  $\underline{q}_0$  with  $\underline{q} \rightarrow 0$  as  $t \rightarrow \infty$ . If  $\rho \in (0, \hat{\rho})$  there is no DME. If  $\rho \in (\hat{\rho}, \bar{\rho})$  then, in addition to the SME  $\underline{q}_L^* \in (0, \hat{q})$  and  $\underline{q}_H^* \in (q_L^*, \hat{q})$ ,  $\forall \underline{q}_0 \in (q_L^*, \hat{q})$  there is a DME starting at  $\underline{q}_0$  with  $\underline{q} \rightarrow \underline{q}_H^*$ . If  $\rho \in (\underline{\rho}, 0)$  then, in addition to SME  $\underline{q}_L^* \in (0, \hat{q})$  and  $\underline{q}_H^* \in (q_L^*, \hat{q})$ ,  $\forall \underline{q}_0 \in (0, \underline{q}_H^*)$  there is a DME starting at  $\underline{q}_0$  with  $\underline{q} \rightarrow \underline{q}_L^*$ .*

As mentioned, one can interpret  $\rho = 0$  as a pure-currency economy, as in the original search-based money models. Then the theory generates dispersion in the

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<sup>9</sup>One case with multiple inflections is shown in Figure ?? below. Also, except for special cases of interest like  $\rho = 0$ , corresponding to fiat money, we typically ignore nongeneric values like  $\rho = \bar{\rho}$ , but describing these cases is a similarly routine exercise.

nominal price  $p = 1/q$ , as well as dynamics with inflation or deflation (falling  $q$  or  $p$ ) as a self-fulfilling prophecy. One can interpret  $\rho > 0$  as a model of an ‘over-the-counter’ financial market, as in Duffie et al. (2005). Then the theory generates asset price dispersion, where prices are generally above their ‘fundamental value’  $\rho/r$ , and can vary over time as a self-fulfilling prophecy, which might be called a ‘bubble’ in standard usage (e.g., Stiglitz 1990). It is all the more obvious prices are above their ‘fundamental value’ when  $\rho < 0$ , but in either case this is due to assets having a liquidity premium.

While the baseline bargaining version does not generate dispersion, it also has belief-based dynamics. However, our results may be more solidly grounded. To make this point, first note that one can always impose as equilibrium conditions cooperative solution concepts like Kalai, Nash, or anything else, but one might worry about the strategic foundations. Consider the standard extensive-form game, where a buyer and seller in a stationary environment make counteroffers of  $q$  until one is accepted, with the time between offers given by  $\delta > 0$ . As shown in Binmore et al. (1986), in the unique subgame-perfect equilibrium the first offer – denoted  $q^\delta$  to indicate dependence on the timing – is accepted, and  $q^\delta \rightarrow q^N$  as  $\delta \rightarrow 0$  where  $q^N$  is the generalized Nash bargaining outcome. This is commonly understood to be an attractive feature of the bargaining approach.<sup>10</sup>

Coles and Wright (1998) and Coles and Muthoo (2003) show that the argument in Binmore et al. applies to environments like the one here *if* we restrict attention to steady state but *not* more generally. Using the same construction as Binmore et al., they show that as  $\delta \rightarrow 0$  we get a path for  $q$  in subgame-perfect

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<sup>10</sup>Similarly, e.g., Dutta (2012) provides strategic foundations for Kalai bargaining. In general, the quest for strategic foundations for axiomatic solutions, goin back to Nash (1953), was dubbed the Nash program by Binmore (1987). As Serrano (2005) puts it in his survey, “Similar to the microfoundations of macroeconomics, which aim to bring closer the two branches of economic theory, the Nash program is an attempt to bridge the gap between the two counterparts of game theory (cooperative and non-cooperative). This is accomplished by investigating non-cooperative procedures that yield cooperative solutions as their equilibrium outcomes.”

equilibrium that satisfies a differential equation having  $q^N$  as a steady state, but  $q \neq q^N$  out of steady state except for special cases, like  $u$  and  $c$  linear, which is not admissible in this model, or  $\theta_1 = 1$ , which is admissible but too special. They also argue that using Nash bargaining out of steady state is equivalent to using the extensive-form game with myopic agents, who negotiate as if economic conditions were constant, even as they change over time. And it matters: one can show that with forward-looking strategic bargaining the equilibrium set can contain limit cycles that do not arise with Nash bargaining.

Again, one can always do dynamics with axiomatic bargaining, but the above discussion should give one pause. Alternatively, one can analyze dynamics with strategic bargaining and rational expectations, but that is complicated (e.g., Coles and Wright 1998; Ennis 2001). Here, with posting, agents are strategic and expectations are rational, plus we get dynamics in terms of a distribution and not just the price level, but the analysis is relatively simple. Section 4 presents still more interesting dynamics; first it is useful to show by way of example how the model works with some parametric specifications.

### 3 Examples

To economize on notation, set  $c(q) = q$  and  $\alpha = 1$ .<sup>11</sup> Now consider first the minimal specification satisfying the conditions in Lemma 2,  $\pi_1 > 0$ ,  $\pi_2 > 0$  and  $\pi_n = 0 \forall n > 2$ , as in some other applications of Burdett-Judd (e.g., Head et al. 2012). In this case (7), which is generally an infinite-dimensional polynomial, solves easily for

$$F(q) = \frac{\pi_1}{2\pi_2} \frac{q - \underline{q}}{u(\underline{q}) - q},$$

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<sup>11</sup>Setting  $c(q) = q$  is wlog since we can measure  $q$  in terms of producer disutility as long as we adjust  $u(q)$ , and setting  $\alpha = 1$  is wlog because we can scale time as long as we adjust  $r$  and  $\rho$ . These normalizations do not appear sooner because it facilitates interpretation to have  $c(q)$  and  $\alpha$  in the results. We also mention that the hard part of Lemma 4 is much easier in the parametric cases presented below.

where  $\bar{q} = [\pi_1 \underline{q} + 2\pi_2 u(\underline{q})] / (\pi_1 + 2\pi_2)$ . Also, (10) reduces to

$$T(\underline{q}) = \psi u(\underline{q}) - b\pi_1 \underline{q} - \frac{\pi_1^2 [u(\underline{q}) - \underline{q}]^2}{2\pi_2} \int_{\underline{q}}^{\bar{q}} \frac{u(q) dq}{[u(\underline{q}) - q]^3}.$$

Figure 1 illustrates this with  $u(q) = q^a$ , with  $a = 0.2$ ,  $\pi_1 = 0.48$ ,  $\pi_2 = 0.36$ ,  $M = 0.3$  and  $r = 0.04$ . In the left panel, with  $\rho = 0.067$ ,  $\underline{q}_L^* = 0.37$  and  $\underline{q}_H^* = 0.66$  both solve  $T(\underline{q}) = \rho$ . The middle panel shows the density  $f(q)$  for the two equilibria, while the right panel shows  $g(p)$ . Notice the two equilibrium densities do not overlap, but they will if we raise  $\rho$  so that  $\underline{q}_L^*$  and  $\underline{q}_H^*$  get closer.

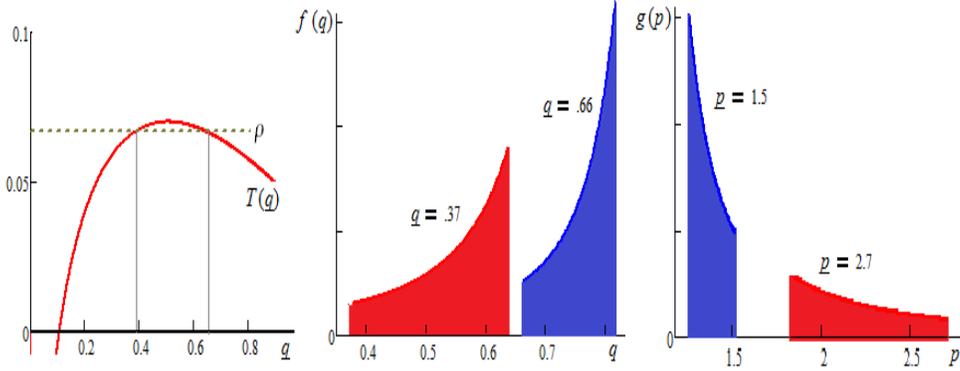


Figure 1: Multiple SME with different densities

For the next example, consider a Poisson distribution,  $\pi_n = e^{-\lambda} \lambda^n / n! \forall n \geq 0$ .

Then (7) becomes

$$\frac{u(\underline{q}) - \underline{q}}{u(\underline{q}) - q} = \sum_{n=1}^{\infty} \frac{\lambda^{n-1} F(q)^{n-1}}{(n-1)!}.$$

From a well-known formula, the RHS is  $e^{\lambda F(q)}$ . Hence, we again get a simple closed-form solution for

$$F(q) = \frac{1}{\lambda} \log \left[ \frac{u(\underline{q}) - \underline{q}}{u(\underline{q}) - q} \right],$$

where  $\bar{q} = e^{-\lambda} \underline{q} + (1 - e^{-\lambda}) u(\underline{q})$ . Now (10) reduces to

$$T(\underline{q}) = \psi u(\underline{q}) - b\lambda e^{-\lambda} \underline{q} - e^{-\lambda} [u(\underline{q}) - \underline{q}] \int_{\underline{q}}^{\bar{q}} \frac{u(q) dq}{[u(\underline{q}) - q]^2},$$

with  $\psi = r + 1 - e^{-\lambda} + b\pi_1$ . Mortensen (2005) uses a Poisson distribution in a version of Head and Kumar (2005), using tools developed in Mortensen (2003), where he shows how to endogenize search effort. That can be done here, too.

Finally, consider a logarithmic distribution,  $\pi_0 = 0$  and  $\pi_n = -\omega^n/n \log(1-\omega)$   $\forall n > 0$ . It is easy to solve for

$$\begin{aligned} F(q) &= \frac{q - \underline{q}}{\omega [u(\underline{q}) - \underline{q}]} \\ T(\underline{q}) &= \psi u(\underline{q}) - b\omega \underline{q} - \int_{\underline{q}}^{\bar{q}} \frac{u(q) dq}{u(\underline{q}) - q}, \end{aligned}$$

where  $\bar{q} = \omega u(\underline{q}) + (1 - \omega)\underline{q}$  and  $\psi = -\log(1 - \omega)(1 + r) + b\omega$ . Since  $F(q)$  is linear,  $q$  and  $p$  are uniformly distributed. In Caplin and Spulber (1987), which is closely related to the discussion in Section 5,  $p$  is assumed to be uniform, with no claim that this is an equilibrium outcome. Our result rationalizes their assumption. More generally, we emphasize that all of these examples are relevant and tractable in applications.

## 4 Extensions

In addition to perfect-foresight equilibria, one can consider sunspot equilibria. Consider in particular a stationary sunspot equilibrium where, even though fundamental are constant,  $q$  fluctuates stochastically between two values,  $q_A$  and  $q_B$ , where wlog we take  $q_B > q_A$ , and  $\varepsilon_S$  is the Poisson arrival rate of a change to  $S' \neq S$  in state  $S \in \{A, B\}$ . Given  $q_S$ , at any point in time, other equilibrium objects are determined as in perfect-foresight equilibrium, although we now write  $F(q|q_S)$  to indicate explicitly that the distribution depends on  $q_S$ .

In state  $S \in \{A, B\}$ , for a buyer

$$rV_1^S = \alpha \sum_{n=1}^{\infty} \pi_n \int_{q_S}^{Q(q_S)} [u(q) - \Delta_S] dF(q|q_S)^n + \rho + \varepsilon_S(V_1^{S'} - V_1^S), \quad (11)$$

which compared to (4) does not have  $\dot{V}_1$  because of stationarity, but does have  $\varepsilon_S(V_1^{S'} - V_1^S)$  because of sunspot switching. Similarly, for a seller posting any  $q$

$$rV_0^S(q) = b\alpha \sum_{n=1}^{\infty} \pi_n n F(q|q_S)^{n-1} [\Delta_S - c(q)] + \varepsilon_S(V_0^{S'} - V_0^S), \quad (12)$$

and, in particular, for a seller posting the lowest  $q_S$

$$rV_0^S(q_S) = b\alpha\pi_1[\Delta_S - c(q_S)] + \varepsilon_S(V_0^{S'} - V_0^S). \quad (13)$$

The procedure used above now yields  $T(q_S) - \rho = \varepsilon_S[u(q_{S'}) - u(q_S)]$ , where  $T$  is the same as (10) above except  $F(q|q_S)$  replaces  $F(q)$ . This reduces to the condition for perfect-foresight SME when  $\varepsilon_S = 0$ , of course. For a proper sunspot equilibrium, or *PSE*, it must hold in both states, with  $\varepsilon_S > 0$  and  $q_B > q_A$ :

$$T(q_A) - \rho = \varepsilon_A[u(q_B) - u(q_A)] \text{ and } T(q_B) - \rho = \varepsilon_B[u(q_A) - u(q_B)]. \quad (14)$$

There are different approaches for establishing the existence of PSE. Following Azariadis (1981), we begin by noting that while it might seem natural to take  $(\varepsilon_A, \varepsilon_B)$  exogenously and solve for  $(q_A, q_B)$  endogenously, doing the opposite serves our purposes equally well because any  $(q_A, q_B, \varepsilon_A, \varepsilon_B)$  solving (14) with  $\varepsilon_S > 0$  and  $q_B > q_A$  is a PSE. Since (14) is linear in  $(\varepsilon_A, \varepsilon_B)$ , it is easy to solve for

$$\varepsilon_A = \frac{T(q_A) - \rho}{u(q_B) - u(q_A)} \text{ and } \varepsilon_B = \frac{\rho - T(q_B)}{u(q_B) - u(q_A)}. \quad (15)$$

Of course we have to check  $\varepsilon_A, \varepsilon_B > 0$ . Since the denominator in both is positive, this is equivalent to  $T(q_A) > \rho > T(q_B)$ . We now show when this can be guaranteed.

Suppose there are multiple SME without sunspots,  $q_L^*$  and  $q_H^* > q_L^*$ . From Proposition 1, this can happen when  $\rho < 0$  or  $\rho > 0$ . Consider first  $\rho < 0$ , as shown in the left panel of Figure ???. There are multiple SME  $\forall \rho \in (\underline{\rho}, 0)$ , and for any  $q_A \in (0, q_L^*)$  and  $q_B \in (q_L^*, q_H^*)$  it is clear that  $T(q_A) > \rho > T(q_B)$ ,

which implies  $\varepsilon_A, \varepsilon_B > 0$ . Thus we have a sunspot equilibrium for any such  $(\underline{q}_A, \underline{q}_B)$ . Similarly consider  $\rho > 0$ , which is not shown in Figure ?? but can be understood from Figure ?. Now there are multiple SME  $\forall \rho \in (\hat{\rho}, \bar{\rho})$ , and for any  $\underline{q}_A \in (\underline{q}_L^*, \underline{q}_H^*)$  and  $\underline{q}_B \in (\underline{q}_H^*, \hat{q})$  it is clear that  $T(\underline{q}_A) > \rho > T(\underline{q}_B)$ , which implies  $\varepsilon_A, \varepsilon_B > 0$ . Thus we again have a sunspot equilibrium for any such  $(\underline{q}_A, \underline{q}_B)$ .

As usual we formalize this below when  $T(\underline{q})$  has a single inflection point, but here we also indicate what happens with multiple inflection points, which is somewhat interesting. In the right panel of Figure ??, with  $\rho < 0$  as drawn there are four SME,  $\underline{q}_j^*$  for  $j = 1, 2, 3, 4$ . Now for any  $\underline{q}_A \in (0, \underline{q}_1^*)$  and  $\underline{q}_B \in (\underline{q}_1^*, \underline{q}_2^*) \cup (\underline{q}_3^*, \underline{q}_4^*)$  we have  $\underline{q}_B > \underline{q}_A$  and  $T(\underline{q}_A) > \rho > T(\underline{q}_B)$ . This is also true for any  $\underline{q}_A \in (\underline{q}_2^*, \underline{q}_3^*)$  and  $\underline{q}_B \in (\underline{q}_3^*, \underline{q}_4^*)$ . Hence in both cases we have PSE. Something similar can be done for  $\rho > 0$ . Given this, we leave the results for any number of inflection points as an exercise, and summarize the above discussion when there is at most one as follows:

**Proposition 3** *If  $\rho \in (\underline{\rho}, 0)$ , in addition to the SME  $\underline{q}_L^*$  and  $\underline{q}_H^*$ , for any  $\underline{q}_A \in (0, \underline{q}_L^*)$  and  $\underline{q}_B \in (\underline{q}_L^*, \underline{q}_H^*)$  there is a PSE where  $\underline{q}$  fluctuates between  $\underline{q}_A$  and  $\underline{q}_B$  with Poisson parameters  $\varepsilon_A, \varepsilon_B > 0$  given by (15). If  $\rho \in (\hat{\rho}, \bar{\rho})$ , in addition to the SME  $\underline{q}_L^*$  and  $\underline{q}_H^*$ , for any  $\underline{q}_A \in (\underline{q}_L^*, \underline{q}_H^*)$  and any  $\underline{q}_B \in (\underline{q}_H^*, \hat{q})$  there is a PSE where  $\underline{q}$  fluctuates between  $\underline{q}_A$  and  $\underline{q}_B$  with  $\varepsilon_A, \varepsilon_B > 0$  given by (15).*

Note that for  $\rho > 0$  the PSE fluctuates around  $\underline{q}_H^*$ , while for  $\rho < 0$  it fluctuates around  $\underline{q}_L^*$  – i.e., in both cases it fluctuates around the SME with  $T'(\underline{q}^*) < 0$ .<sup>12</sup> Also note that in the case of fiat money,  $\rho = 0$ , the method (or the one

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<sup>12</sup>This follows the proof strategy Trejos and Wright (2014) use in bargaining models. A different strategy, closer to what Shi (1995) and Ennis (2001) use, is to consider PSE that fluctuate across two points each in the neighborhood of a different SME. If  $\underline{q}_L^*$  and  $\underline{q}_H^* > \underline{q}_L^*$  exist they solve (14) with  $\varepsilon_A = \varepsilon_B = 0$ . By continuity, for small  $\varepsilon_A, \varepsilon_B > 0$ , there exist  $\underline{q}_A$  and  $\underline{q}_B > \underline{q}_A$  close to  $\underline{q}_L^*$  and  $\underline{q}_H^*$ , resp., solving (14). One can check  $\underline{q}_A < \underline{q}_L^*$  and  $\underline{q}_B < \underline{q}_H^*$  when  $\rho \in (\underline{\rho}, 0)$ , while  $\underline{q}_A > \underline{q}_L^*$  and  $\underline{q}_B > \underline{q}_H^*$  when  $\rho \in (\hat{\rho}, \bar{\rho})$ . Hence, these PSE look qualitatively similar to those implied by Proposition 3, even though, heuristically they fluctuate across

in fn.12) does not work because  $\underline{q}$  cannot fluctuate around 0. But there an equilibrium where  $q_A = \varepsilon_A = 0$  and  $\varepsilon_B = T(\underline{q}_B)/u(\underline{q}_B) > 0 \forall \underline{q}_B \in (0, q^*)$  where  $q^*$  is the unique SME with  $\rho = 0$ . Hence the value of fiat money can crash to 0 probabilistically, but then cannot come back (similar to Kocherlakota 2011). However, one can reinterpret  $\rho \neq 0$ ,

they can work in a version that allows some barter by letting the double-coincidence probability be  $\delta > 0$  instead of  $\delta = 0$ , if we assume as in some other models that agents with  $m = 1$  cannot produce.<sup>13</sup> In this case accepting money has an opportunity cost – the agents loses the ability to barter – which works very much like the storage cost represented by  $\rho < 0$ .

One can also consider shocks to fundamentals. Suppose  $\rho = \rho_S$  is a function of the state  $S \in \{A, B\}$ , where again  $S$  switches according to Poisson parameters  $\varepsilon_A$  and  $\varepsilon_B$ . To simplify, suppose for each  $\rho_S$  there is a unique SME  $q_S^*$ , which is the case for  $\rho_S \in [0, \hat{\rho})$ . For buyers

$$rV_1^S = \alpha \sum_{n=1}^{\infty} \pi_n \int_{\underline{q}_S}^{Q(\underline{q}_S)} [u(q) - \Delta_S] dF_S(q)^n + \rho_S + \varepsilon_S(V_1^{S'} - V_1^S). \quad (16)$$

For sellers the equations are still given by (12)-(13). Similar to the analysis of sunspots, we get

$$T(\underline{q}_A) - \rho_A = \varepsilon_A[u(\underline{q}_B) - u(\underline{q}_A)] \text{ and } T(\underline{q}_B) - \rho_B = \varepsilon_B[u(\underline{q}_B) - u(\underline{q}_A)], \quad (17)$$

the only difference being that now  $\rho_S$  is state dependent. These conditions define two curves depicted in  $(\underline{q}_A, \underline{q}_B)$  space shown in Figure ??.

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points near two SME rather than around one SME. However, the continuity argument only guarantees existence for  $(\underline{q}_A, \underline{q}_B)$  close to  $(\underline{q}_L^*, \underline{q}_H^*)$  and  $(\varepsilon_A, \varepsilon_B)$  close to  $(0, 0)$ , while Proposition 3 can generate PSE for  $(\underline{q}_A, \underline{q}_B)$  far from  $(\underline{q}_L^*, \underline{q}_H^*)$  and  $(\varepsilon_A, \varepsilon_B)$  far from  $(0, 0)$ . In particular, if  $\underline{q}_B - \underline{q}_A$  is small (15) makes  $\varepsilon_A$  and  $\varepsilon_B$  arbitrarily big, while naturally bigger fluctuations occur with lower frequencies.

<sup>13</sup>This is motivated in Aiyagari and Wallace (1992), e.g., by saying that after producing an agent must consume before producing again. See Rupert et al. (2001) for more discussion.

To describe the equilibrium, start with the limiting case where  $\rho_A = \rho_B$ . Then the two curves are symmetric and cross on the 45° line at  $\underline{q}_A^1 = \underline{q}_B^1$ . Now raise  $\rho_A$ . This shifts the blue solid curve to the blue dashed curve. The equilibrium is now given by  $\underline{q}_A^2 > \underline{q}_B^2 > \underline{q}_A^1$ .

As  $\varepsilon_S$  increase, there is an effect in  $\underline{q}_S$ , but it is not necessarily the case that  $\underline{q}_A$  and  $\underline{q}_B$  move towards each other, since  $T$  is not monotonic. But for  $\varepsilon$  small enough we do know that the equilibrium where the parameters cycle around does exist.

Let  $\Gamma = \{\rho, r, M, \alpha\}$ , the parameters of the model. We will consider what happens if there is a process, with arrival rates  $\varepsilon_A, \varepsilon_B$ , between values  $\Gamma_A$  and  $\Gamma_B$ . To simplify matters, pick  $\Gamma_A$  and  $\Gamma_B$  so that under both there exist at least one SME, call them  $q_A^*, q_B^*$ . Wlog we label so  $q_A^* < q_B^*$ . We assume that if  $M$  is the variable that changes, then the agents that would have to shift from sellers to buyers (if  $M$  increases) would have the resulting surplus extracted, and viceversa.

The derivation is similar to the case with sunspots, except that the stochastic term refers to the impact of a real shock on  $\Gamma$ , rather than a shock on expectations.

For  $S = A, B$  and  $S' \neq S$ ,

$$r_S V_1^S = \alpha_S \sum_n \pi_n \int_{\underline{q}_S}^{Q(\underline{q}_S)} [u(q) - \Delta_S] dF_S(q)^n + \rho_S + \varepsilon_S (V_1^{S'} - V_1^S)$$

For a seller posting  $q \in [\underline{q}_S, \bar{q}(\underline{q}_S)]$ , the Bellman equation is

$$r_S V_0^S(q) = b_S \alpha_S \sum_n \pi_n n F_S(q)^{n-1} [\Delta_S - q] + \varepsilon_S (V_0^{S'} - V_0^S),$$

and in particular, at the lowest bound  $\underline{q}_S$ ,

$$r_S V_0^S(\underline{q}_S) = b_S \alpha_S \pi_1 [\Delta_S - \underline{q}_S] + \varepsilon_S (V_0^{S'} - V_0^S).$$

Through the same procedure as before, we arrive to

$$T_S(\underline{q}) = \psi_S u(\underline{q}) - b_S \alpha_S \pi_1 c(\underline{q}) - \alpha_S \pi_1 [u(\underline{q}) - c(\underline{q})] \int_{\underline{q}}^{Q(\underline{q})} \frac{u(q) F'(q | \underline{q}) dq}{u(\underline{q}) - c(\underline{q})}. \quad (18)$$

An Cyclical Stationary Equilibrium (CSE) would correspond to two price distributions, between which the economy oscillates as  $\Gamma$  varies between  $\Gamma_A$  and  $\Gamma_B$ , and that can be characterized by a pair  $\underline{q}_A, \underline{q}_B$  that satisfies

$$\begin{aligned}\varepsilon_A \left[ u(\underline{q}_B) - u(\underline{q}_A) \right] &= T_A(\underline{q}_A) - \rho_A \\ \varepsilon_B \left[ u(\underline{q}_A) - u(\underline{q}_B) \right] &= T_B(\underline{q}_B) - \rho_B\end{aligned}\tag{19}$$

The equations (19) are similar to the one for sunspots, except that now  $\varepsilon$  are not endogenous variables, but parameters that enter into  $T$ .

Define the correspondences

$$\begin{aligned}\sigma_A(\underline{q}_B) &= \left\{ q \mid T_A(q) + \varepsilon_A u(q) - \rho_A = \varepsilon_A u(\underline{q}_B) \right\} \\ \sigma_B(\underline{q}_A) &= \left\{ q \mid T_B(q) + \varepsilon_B u(q) - \rho_B = \varepsilon_B u(\underline{q}_A) \right\}\end{aligned}$$

and the CSEs will be pairs  $\underline{q}_A, \underline{q}_B$  that satisfy  $\underline{q}_A \in \sigma_A(\underline{q}_B)$ ,  $\underline{q}_B \in \sigma_B(\underline{q}_A)$ .

Consider two parameter vectors  $\Gamma_A \neq \Gamma_B$  that each allow for a unique SME. The easiest (but certainly not the only) way to do this is to consider  $\rho_A, \rho_B \in [0, \widehat{\rho}]$ , all other parameters given, in our baseline scenario, and that  $\varepsilon_A = \varepsilon_B = \varepsilon$ .<sup>14</sup> Assuming wlog tht  $\rho_B > \rho_A$ , one can show that there is a unique pair  $\underline{q}_A, \underline{q}_B$  of equilibrium, that  $\underline{q}_A^* < \underline{q}_A < \underline{q}_B < \underline{q}_B^*$ , and that  $\underline{q}_B - \underline{q}_A \rightarrow 0$  as  $\varepsilon \rightarrow \infty$ . To prove this, verify that given uniqueness of SME (that is, there is a unique root for  $\rho = T(q)$ , and at said root  $T'(q) > 0$ ) then the  $\sigma_S$  happen to be increasing functions, that intersect at a point  $\underline{q}_A < \underline{q}_B$ . At any intersection,  $\sigma_B$  happens to be steeper than  $\sigma_A^{-1}$ , which by differentiability means these functions intersect only once. Also, one can show that  $\partial \underline{q}_A / \partial \varepsilon > 0 > \partial \underline{q}_B / \partial \varepsilon$ , and that in the limit

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<sup>14</sup>The simplicity comes from the fact that if the parameter that changes is  $\rho$ , then the functions  $T_A$  and  $T_B$  are the same (and equal to the function  $T$  in the stationary model). But of course, we could do an analogous exercise with the other parameters. This includes  $M$ , that should not be interpreted only as the money supply, because in this model it describes both the amount of assets—a nominal value—and also the value of their aggregate output as well as their distribution across agents—two real values. The model allows for neutrality (if we change the units in which the asset is measured, prices change proportionately), but  $M$  is not the variable to change to verify it.

they converge to the same value. The following picture illustrates the previous results: the dot corresponds to the two SMEs, and the intersection of the two functions  $\sigma_S$  is the cyclical equilibrium.

A corollary of the results in the last paragraph is that for  $\varepsilon_S$  large enough it obtains  $\underline{q}_B \in \left[ \underline{q}_A, Q\left(\underline{q}_A\right) \right]$  so the two price distributions overlap, and the same conclusions about sticky prices apply as before: as the model parameters change, some sellers need to change their prices, but others can choose not to revise them.

A more complex scenario takes place, for instance, if we pick parameters that allow for multiple SMEs; for instance, in a case where  $T'(\hat{q}) < 0$ , pick  $\rho_A, \rho_B \in (\hat{\rho}, \bar{\rho})$ . Then, there are two SMEs for each set of parameters,  $q_S^i, i = 1, 2, S = A, B$ . Then,  $\sigma_S$  are true correspondences (for values of  $q_A$  in a certain range, for instance, there are two values of  $q_B$  that satisfy the second equation in (19)). For very small values of  $\varepsilon$ , we know that there would be four CSEs: in each, as the parameters oscillate between  $\Gamma_A$  and  $\Gamma_B$ , the price distributions will oscillate between one of the  $q_A^i$  and one of the  $q_B^i$  values. Notice that this implies, interestingly, that even if the difference between  $\Gamma_A$  and  $\Gamma_B$  is very small (so the oscillation between  $q_A^1$  and  $q_B^1$ , or between  $q_A^2$  and  $q_B^2$ , could be very small), it may be that the "expectations" component of the oscillation is very big (if, say, the economy oscillates between  $q_A^1$  and  $q_B^2$ ).

The following figure describes the analysis. Notice that in this case we chose  $\rho_A$  and  $\rho_B$  very close together, so that two of the dots indicating SMEs are very close to the 45° line. Furthermore, the analysis of the SME involving the lower  $q_S^1$  for both states (that is, where both intersections happen in the increasing portion of  $T$ ) is exactly the same as before, so one predict that the intersection between the two  $\sigma_S$  near that SME is going to be "down and to the right", that is, towards the 45° line. But around the other three SMEs things are more complicated: we can only predict the direction in which the CSEs are relative to the SMEs for

very low values of  $\varepsilon$  (SW from the NW SME, NW from the NE SME, NE from the SE SME), and the number of CSEs may become larger or smaller. Notice for instance that a small change in parameters moving the correspondence  $\sigma_A$  a little bit up will increase the number of equilibria from four to five, and a bit further up, from five to three.

## 5 Discussion

The effects of parameter changes on SME or DME are simple. An increase in  $r$ , e.g., rotates  $T$  around the origin, so  $\partial \underline{q} / \partial r < 0$  when the SME is unique, or more generally, at any ‘natural’ SME where  $T'(\underline{q}) > 0$  (as usual, with multiple equilibria the comparative statics are reversed at alternate solutions). As  $\bar{q} = Q(\underline{q})$  also falls, the entire distribution shifts left. Similarly, an increase in  $M$  raises  $b$ , which shifts  $T(\underline{q})$  so that  $\partial \underline{q} / \partial M < 0$  in any ‘natural’ SME. Again the distribution shifts to the left, and note that  $\bar{q}$  falls by less than  $\underline{q}$ , as it is easy to check  $\underline{q}Q'(\underline{q})/Q(\underline{q}) < 1$ . Hence, higher  $M$  spreads the support and makes  $F'(q)$  lower at the edges. With fiat money,  $\rho = 0$ , injecting  $M$  makes it harder to find sellers, so agents produce less  $q$  for cash. This nonneutrality has something to do with the restriction  $m \in \{0, 1\}$ , but that seems appropriate, corresponding to a long-standing notion that the real effects of monetary injections depend on changing the distribution of liquidity.<sup>15</sup>

To discuss the implications for pricing behavior, consider Figure 2, where the quantity and price densities are  $f(q, M)$  and  $g(p, M)$ , before and after  $M$  rises to  $M'$ . After  $M$  rises, all sellers that were formerly pricing between  $\underline{p}$  and  $\underline{p}'$  must adjust  $p$  because it is no longer in the equal-payoff support  $\mathcal{P}$ . However, any seller

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<sup>15</sup>As Francis Bacon put it “Money is like muck, not good except it be spread.” Still, as usual, changing the denomination on the asset for all agents holding it is irrelevant, even if changing the measure of agents holding it (spreading the muck) is not.

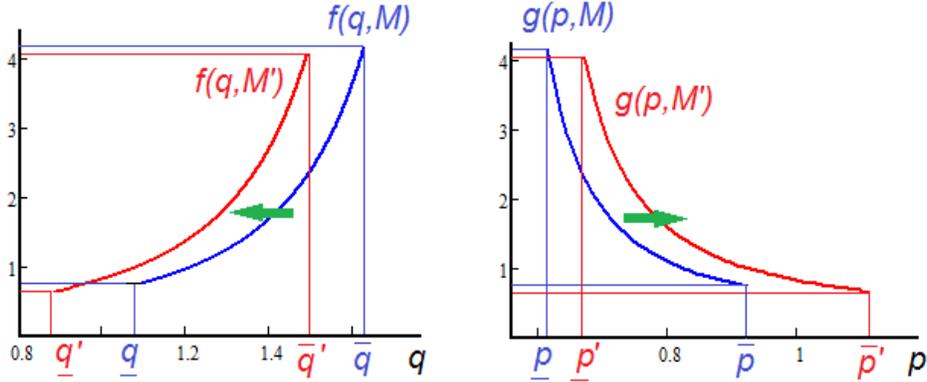


Figure 2: Sticky prices after an increase in  $M$

pricing between  $\underline{p}'$  and  $\bar{p}$  has no incentive to change – his  $p$  can fall relative to the aggregate  $\mathbb{E}p$ , and *by construction* the equilibrium probability of a sale increases by exactly enough to compensate. So prices are sticky in the sense used in the Introduction: some sellers do not reprice when  $\mathbb{E}p$  increases, although of course others do, since how else would  $\mathbb{E}p$  change? Collectively, sellers respond so as to achieve the new SME, but it is *no puzzle* if many individuals stick to their old  $p$  when  $M$  changes. Obviously, nominal prices can be sticky in this sense after changes in utility or technology, too.

Similar remarks apply to DME. With  $\rho = 0$  there are inflationary equilibria where  $\underline{q} \rightarrow 0$  and  $\mathbb{E}p \rightarrow \infty$ , but many sellers stick to posted prices for extended periods, only changing when  $p$  falls out of the shrinking  $\mathcal{P}$ . With  $\rho \neq 0$  there are deflationary equilibria where some sellers stick to their prices while  $\mathbb{E}p$  falls, thus reducing the probability they make a sale, but getting a higher surplus when they do. In the interest of space, we only sketch some additional dynamic implications. First, as in Trejos and Wright (1993), assume  $M$  follows a stochastic process and agents have rational expectations. Then  $\mathbb{E}p$  rises and falls when  $M$  realizations are high or low, but many sellers can stick to the same  $p$  after shocks as long as the supports overlap. Second, as in Shi (1995) or Ennis (2001), there are sunspot equilibria where endogenous variables follow stochastic processes as self-fulfilling

prophecies, and again many sellers can stick to the same  $p$  as  $\mathbb{E}p$  rises and falls.

In sum, there are various reasons for changes in price distributions, including one-time unanticipated movements or different realizations of stochastic processes for  $M$  as well as real factors, and including self-fulfilling prophecies that give rise to deterministic or sunspot dynamics. Prices can be sticky in every case. Once one understands how frictions lead to dispersion, it is immediate how to generate stickiness, with no restrictions on timing or costs of adjustment.<sup>16</sup> For evidence that this is neither trivial nor generally understood, consider Golosov and Lucas (2003): “Menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.” That is incorrect. We are not the first to mention this, and give full credit to the papers mentioned in the Introduction, Caplin and Spulber (1987), Eden (1994) and others. Yet our model is different, and has the virtue of simplicity, as well as relatively solid microfoundations for money, dispersion and rigidity.

## 6 Conclusion

This paper presents a new second-generation search model of monetary exchange. We think this is natural, especially since price posting seems more characteristic of many markets than bargaining. It provides simple, intuitive, results for the effects on steady states of changes in parameters. It also provides a rigorous way to illustrate dynamic equilibria, without some of the technical problems

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<sup>16</sup>The result is robust to various perturbations in the environment. If sellers have heterogeneous costs, e.g., dispersion still obtains, but a seller is only indifferent across  $p$  in a subset of the support. Thus, low-cost sellers prefer low  $p$  and a high probability of a sale, but there is an interval  $\mathcal{P}_L$  in which they are indifferent, and similarly an interval  $\mathcal{P}_H$  for high-cost sellers. Still,  $\mathcal{P} = \mathcal{P}_L \cup \mathcal{P}_H$  looks like the baseline model, and  $p$  can still be sticky in the conditional supports. Heterogeneity does not upset the general argument about stickiness.

that plague bargaining approaches. It also generates price dispersion, which is a realistic feature of many markets. And it generates nominal rigidity, defined as the observation that some sellers keep their prices the same after changes in economic conditions, including changes in the aggregate price level due to changes in  $M$ , or due simply to beliefs.

This is a different position than much of macroeconomics, where nominal rigidity is a primitive, and price dispersion emerges as an outcome due to inflation (e.g., Woodford 2003). We take real frictions as a primitive, and derive dispersion and sticky prices as outcomes. Importantly, dispersion emerges even without inflation, as seems true in the data (Campbell and Eden 2014). We deliver closed-form solutions for special cases used in the literature. Money is nonneutral here, although not because of stickiness. Putting this together with previous results, it can be concluded that sticky prices are neither necessary nor sufficient for nonneutrality, and the fact that some sellers do not change  $p$  over extended periods does not logically require menu costs or related restrictions on changing prices. While many of the components of the analysis can be found elsewhere, putting them together as we did delivers a tractable model of monetary exchange that we think has potential to become the new benchmark in second generation monetary models.

## Appendix: Proofs

In what follows we set  $c(q) = q$ ,  $\alpha = 1$  and  $\pi_0 = 0$  to reduce notation is wlog.

**Lemma 1:** With bargaining, based on (3), we have

$$\theta_1 = 1 \implies \dot{q} = rq - \alpha_1 [u(q) - q] - \rho \quad (20)$$

$$\theta_1 = 0 \implies u'(q) \dot{q} = ru(q) - \alpha_0 [u(q) - q] - \rho \quad (21)$$

where  $\alpha_1$  and  $\alpha_0$  are the effective arrival rates for buyers and sellers. With posting, consider first  $\pi_1 \rightarrow 1$ . Then (8) implies  $\bar{q} = Q(\underline{q}) = \underline{q}$ , so there is a single price. Also, (10) reduces to

$$T(\underline{q}) = (r + \alpha_0) u(\underline{q}) - \alpha_0 \underline{q}$$

where  $\alpha_0$  is the effective arrival rate for a seller. Consequently  $u'(q) \dot{q} = T(\underline{q}) - \rho$ , which is identical to (21).

With posting and  $\pi_1 \rightarrow 0$ , the seller with the lowest  $q$  gets a payoff satisfying  $rV_0(\underline{q}) = \dot{V}_0$ , because there are no buyers with  $n = 1$ , so he never beats the competition. Hence, (5) implies  $c(q) = q = \Delta \forall q$ , and all sellers get 0 surplus. Moreover, since the LHS of (7) must be strictly positive,  $q \rightarrow u(\underline{q})$  as  $\pi_1 \rightarrow 0$ , so again there is a single price. Then (4) implies

$$rV_1 = \alpha_1 [u(\underline{q}) - c\underline{q}] + \rho + \dot{V}_1,$$

where  $\alpha_1$  is the effective arrival rate for a buyer. This combined with  $rV_0(\underline{q}) = \rho_0 + \dot{V}_0$  and  $q = \Delta$  leads to

$$\dot{q} = r\underline{q} - \alpha_1 [u(\underline{q}) - \underline{q}] - \rho,$$

which is identical to (20). ■

**Lemma 2:** If there were a mass point at  $q_1$ , a seller posting  $q_1$  could profitably deviate to  $q_1 + \varepsilon$  for some  $\varepsilon > 0$ , because he would increase the probability of a sale discretely with a small increase in cost. If there were a gap between  $q_1$  and  $q_2 > q_1$ , a seller posting  $q_2$  could profitably deviate to  $q_3 \in (q_1, q_2)$ , since he lowers cost while losing no sales. Finally, if the lowest  $\underline{q}$  does not take the entire surplus from buyers, a seller posting  $\underline{q}$  can profitably deviate to  $\underline{q} - \varepsilon$ . ■

**Lemma 3:** Let  $L(y) = \sum_{n=1}^{\infty} \pi_n n y^{n-1}$ . For a given  $q$ , (7) says that  $F(q) = y$  where  $y$  is the solution to

$$L(y) = \pi_1 \frac{u(\underline{q}) - c(\underline{q})}{u(\underline{q}) - c(\underline{q})}. \quad (22)$$

Notice  $L(0) = \pi_1$  and  $L(1) = \mathbb{E}n$ , and that  $L(y)$  is differentiable with  $L'(y) = \sum_{n=1}^{\infty} \pi_n n(n-1) y^{n-2} > 0$ . Setting  $y = 1$ , (22) reduces to  $q = \bar{q}$ . Setting  $y = 0$ , (7) reduces to  $q = \underline{q}$ . These results imply that  $\forall q \in [\underline{q}, \bar{q}]$  there is a unique  $y \in [0, 1]$  solving (22), and hence a unique number  $F(q)$  in  $[0, 1]$  solving (7). The formula (9) comes from differentiating (7) and rearranging, where the sum in the denominator is well defined because  $n$  has a finite mean and variance. ■

**Lemma 4:** It is obvious that  $T(0) = 0$  and  $T(\hat{q}) = [r + \alpha(1 - \pi_0)]\hat{q} > 0$ , because the integral vanishes at  $\underline{q} = 0$  or  $\underline{q} = \hat{q}$ . For the limit, note from (10) that

$$\begin{aligned} T'(\underline{q}) &= \psi u'(\underline{q}) - b\pi_1 - u[Q(\underline{q})] F'[Q(\underline{q})] [\pi_1 + (\mathbb{E}n - \pi_1) u'(\underline{q})] + \alpha\pi_1 u(\underline{q}) F'(\underline{q}) \\ &\quad + \pi_1 \int_{\underline{q}}^{Q(\underline{q})} u(q) \frac{[u(\underline{q}) - \underline{q}] u'(q) - [u'(q) - 1] [u(q) - \underline{q}]}{[u(q) - \underline{q}]^2} F'(q) dq. \end{aligned}$$

We know there are no mass points in the distribution and  $F(\underline{q})$  is differentiable with  $\lim_{\underline{q} \rightarrow 0} F'(\underline{q}) = \Omega$  for some  $\Omega > 0$ . Hence

$$\begin{aligned} \lim_{\underline{q} \rightarrow 0} T'(\underline{q}) &= \psi \lim_{\underline{q} \rightarrow 0} u'(\underline{q}) - b\pi_1 - \Omega\pi_1 \lim_{\underline{q} \rightarrow 0} u[Q(\underline{q})] \\ &\quad - \Omega(\mathbb{E}n - \pi_1) \lim_{\underline{q} \rightarrow 0} u[Q(\underline{q})] u'(x) + \pi_1 \Omega \lim_{\underline{q} \rightarrow 0} u(\underline{q}) \\ &\quad + \pi_1 \lim_{\underline{q} \rightarrow 0} \int_{\underline{q}}^{Q(\underline{q})} \frac{u(q) [u(\underline{q}) - \underline{q}] u'(q) - [u'(q) - 1] [u(q) - \underline{q}]}{[u(q) - \underline{q}]^2} F'(q) dq \end{aligned}$$

The second and fourth limits on the RHS are trivially 0. The fifth limit on the RHS is 0, because the integral vanishes when the measure of the range goes to 0, even if the integrand goes to  $\infty$  or is indeterminate, by Theorem 5.25 in Zygmund and Wheeden (1977). Using l'Hopital's rule on the third limit, we get

$$\begin{aligned} \lim_{\underline{q} \rightarrow 0} u[Q(\underline{q})] u'(x) &= \lim_{\underline{q} \rightarrow 0} u'[Q(\underline{q})] Q'(\underline{q}) u''(\underline{q}) \\ &= \lim_{\underline{q} \rightarrow 0} u'(\underline{q}) \lim_{\underline{q} \rightarrow 0} u''(\underline{q}) \frac{\pi_1 + (\mathbb{E}n - \pi_1) u'(\underline{q})}{\mathbb{E}n} \end{aligned}$$

Putting things together we have

$$\lim_{\underline{q} \rightarrow 0} T'(\underline{q}) = \lim_{\underline{q} \rightarrow 0} u'(\underline{q}) \left[ \psi + \Omega(\mathbb{E}n - \pi_1) \lim_{\underline{q} \rightarrow 0} u''(\underline{q}) \frac{\pi_1 + (\mathbb{E}n - \pi_1) u'(\underline{q})}{\mathbb{E}n} \right] - b\pi_1.$$

This implies

$$\lim_{\underline{q} \rightarrow 0} \frac{T'(\underline{q})}{u'(\underline{q})} = \psi + \frac{\Omega(\mathbb{E}n - \pi_1) \pi_1}{\mathbb{E}n} \lim_{\underline{q} \rightarrow 0} u''(\underline{q}) + \frac{\Omega(\mathbb{E}n - \pi_1) (\mathbb{E}n - \pi_1)}{\mathbb{E}n} \lim_{\underline{q} \rightarrow 0} u'(\underline{q}) u''(\underline{q})$$

The last two terms are negative, and at least the last one is  $-\infty$ . Therefore, it must be the case that  $\lim_{\underline{q} \rightarrow 0} T'(\underline{q}) = -\infty$ . ■

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