Finance and Misallocation:
Evidence from Plant-Level Data*

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Abstract

We study a model of establishment dynamics in which entrepreneurs face a financing constraint. We ask: does the model, when parameterized to match salient features of plant-level data, predict large aggregate TFP losses from misallocation? Our answer is: No. We estimate financing frictions that are fairly large: in our economy half of the establishments face binding borrowing constraints and an implicit external finance premium of 55% on average. Efficient establishments are, nonetheless, able to quickly accumulate internal funds and grow out of their borrowing constraint. Parameterizations of the model under which this process of internal accumulation is hindered can, in principle, cause very large TFP losses. Such parameterizations are, however, at odds with important features of plant-level data: the variability and persistence of plant-level output, as well as differences in the return to capital and output growth rates across young and old plants.

Keywords: Productivity. Misallocation. Finance Frictions

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1. Introduction

Differences in GDP per capita across countries are large and to a large extent accounted for by differences in total factor productivity. A key question in economic development is thus: What accounts for the large cross-country differences in total factor productivity? One hypothesis that has received much attention recently[1] is that of establishment-level *misallocation*. According to this hypothesis, poor countries are poor not only because individual establishments are less efficient, but also because those establishments that are efficient and should operate at a higher scale are unable to do so.

A number of theories have been proposed to explain why poorer countries face a more severe misallocation problem: distortionary government policies[2], frictions that distort factor mobility[3], credit frictions[4] or lack of insurance against the risk associated with entrepreneurial activity[5].

We study, in this paper, the role of credit constraints in generating aggregate TFP losses. There is strong evidence that financial markets are much less developed in poorer countries. Figure 1 illustrates this by showing a scatterplot of TFP versus a measure of how developed financial markets are: the ratio of external finance (private credit and stock market capitalization) to GDP for a sample of countries for 1996[6]. The question is then, to what extent does this correlation reflect the distortionary effect of financing frictions on TFP?

Existing quantitative studies find an important causal role for finance in accounting for TFP. For example, Jeong and Townsend (2006) attribute 70% of Thailand's TFP growth from the 70s to the 90s to an improvement of the financial sector. Amaral and Quintin (2005), Buera, Kaboski and Shin (2009), Moll (2010), Greenwood, Sanchez and Wang (2010) provide quantitative estimates of the effect of finance on misallocation. The TFP losses that these studies report are staggering: TFP would double if one were to increase access to external finance in poor countries to levels similar to those in developed countries like US. For example, 80% of the TFP gap between US and Mexico and 50% of the gap between US and Colombia

[1] Bartelsman and Doms (2000) and Tybout (2000) review the evidence; Restuccia-Rogerson (2008), Hsieh and Klenow (2008), Bartelsman, Haltiwanger and Scarpetta (2008) are several important recent contributions.

[2] Restuccia and Rogerson (2008), Hsieh and Klenow (2008), Guner,Ventura, Xu (2008).

[3] Hopenhayn and Rogerson (1993), Lagos (2006).

[4] Banerjee and Duflo (2005), Jeong and Townsend (2006), Buera, Kaboski and Shin (2010), Greenwood, Sanchez and Wang (2010).

[5] Banerjee and Duflo (2005), Angeletos (2008), Castro, Clementi, MacDonald (2009).

[6] As Moll (2010) does, we use the TFP data from Caselli (2005) and the data on finance from Beck, Demirguc-Kunt and Levine (2000).

is accounted for by finance frictions alone, according to these studies.

Our goal in this paper is to use micro-level data in order to revisit these conclusions. We study, through the lens of a model of establishment dynamics with financing frictions, plant-level data from manufacturing plants in Korea and Colombia. We choose these two countries as they provide us with relatively high quality micro-level data, but also because the two are at opposite ends of the finance spectrum. Korea is a country with relatively well-functioning credit markets with an external finance to GDP ratio equal to 120%, while Colombia has relatively poor credit markets and an external finance to GDP ratio of around 30%.

We require that our model accounts not only for the size distribution of establishments, as existing quantitative studies do, but also a number of additional salient features of the micro data: the variability and persistence of output at the plant level, as well as the difference in returns to capital and output growth rates for young and old plants. We find that, when we parameterize our model to account for these additional facts, it predicts much smaller TFP losses from misallocation than existing studies have found: the model predicts that the TFP gains from moving from an environment with no external borrowing to the level of external borrowing observed in the US are at most 4-5%.

This is not an impossibility result: we present parametrizations of the model that can easily generate very large TFP losses, similar to those reported in existing studies. We show however that all of these parameterizations miss important features of the plant-level data.

The economy we study is a model of industry dynamics as in Hopenhayn (1992). A continuum of entrepreneurs differ in the efficiency with which they can operate a plant. Efficiency fluctuates over time, thus giving rise to micro-level dynamics and the need for external credit to finance expansions. We assume, given the evidence in Moskowitz and Vissing-Jorgensen (2002), that entrepreneurial risk is not diversified and that dividends from the establishment are the only source of income for entrepreneurs. Plant owners can save using a one-period risk-free security, but the amount they can borrow is subject to a collateral constraint, as in Evans and Jovanovic (1989).

We study two versions of the model. In the first version there is no entry and exit. In this environment the key parameter that determines the relationship between finance and TFP is the standard deviation of shocks to an entrepreneur's productivity. The larger the shocks are, the greater the need for external borrowing to finance expansions, and hence the greater the losses from the borrowing constraint. Since variation in productivity is the sole source of variation in output in this version of the model, we pin down the size of productivity shocks by requiring the model to match moments of the distribution of output growth rates among establishments. We find that there is simply too little time-series variability in output in the data (in both Korea and Colombia) for productivity shocks to distort allocations much.

We then allow entry and exit into entrepreneurship by introducing a occupational choice: agents must decide whether to work or become entrepreneurs. Productive agents that have sufficient funds to operate at a large enough scale enter entrepreneurship, while the rest work. In addition, we assume a constant death hazard each period. Agents that die lose all their assets and are replaced by young agents that receive an endowment that is potentially correlated with their productivity. Allowing for a constant death hazard (exogenous exit) is necessary in order to allow the model to account for the fact that some very large establishments shut down in the data. Moreover, without exogenous exit, establishment exit and entry plays little role since only marginal entrepreneurs with low productivity switch occupations and these account for too small a share of aggregate output for this margin to be quantitatively important.

In this second environment another key parameter that determines the size of aggregate TFP losses is the extent to which a newly born agent's endowment is correlated with its ability as an entrpreneur. If all newborn agents have the same endowment, then productive agents join entrepreneurship but are initially very constrained. In such an environment TFP losses from misallocation are quite large. We show, however, that the predictions of such a model are grossly at odds with the characteristics of young vs. old plants in the data. In particular, young plants grow much faster in the model than in the data[7] compared to old plants and have much greater returns to capital (as measured by the average product of capital) than old plants. Hence, the model generates large TFP losses for the wrong reasons, by implying that young plants are much more severely brorowing constrained than they are in the data.

The counterfactual predictions above can be easily remedied by allowing a newly born agent's initial endowment to be correlated with its productivity (for example due to seed funding by venture capitalists of the high-potential entrepreneurs). When we choose the correlation between the initial endowment and productivity to match the differences in output growth rates and rates of return to capital among young and old plants in the data,

[7]This is a typical predictions of this class of models. See for example Cooley and Quadrini (2001).

we find once again fairly small TFP losses (5%) from misallocation. Importantly, most of these losses arise due to misallocation of capital across existing plants, not due to distortions in the occupational choice.

2. Model

The economy is inhabited by a continuum of entrepreneurs, each of whom has access to a technology that produces output using inputs of capital and labor. Production is subject to decreasing returns to scale. All entrepreneurs produce a homogenous good and operate in a perfectly competitive envronment. Because our focus is on aggregate TFP losses in the ergodic steady-state of a small open economy with no aggregate uncertainty, we conduct the analysis of this section in a partial equilibrium setup. The general equilibrium extension is relevant and pursued in Section 5 when we study the model with an occupational choice.

A. Environment

Let i index an individual entrepreneur. Such an entrepreneur has an objective given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{it}\right)$$

where C_{it} is consumption. We assume $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$, with $\gamma > 0$. The entrepreneur has access to a production technology given by:

$$Y_{it} = F\left(L_{it,} K_{it}\right) = A_{it}\left(L_{it}^{\alpha} K_{it}^{1-\alpha}\right)^{\eta}$$

where Y_{it} is output, L_{it} is the amount of labor it hires, K_{it} is the capital stock and A_{it} is the entrepreneur's productivity. The parameter $\eta \in (0,1)$ is the span-of-control parameter and governs he degree of returns to scale[8]. The share of labor in production is governed by α. We assume that the log of productivity, $a = \log(A)$, follows a continuous-state Markov process with transition density

$$\Pr\left(a_{it+1} = a' | a_{it} = a\right) = \pi\left(a' | a\right)$$

[8]Clearly, this formulation can be alternatively interpreted as arising from an enviornment in which monopolistically competitive firms face a constant elasticity demand function. Under this alternative interpretation Y represents revenue and $\eta = 1 - 1/\theta$ where θ is the demand elasticity.

We next describe the assumptions we make regarding the financial side of the model. To maintain comparability with earlier work, we follow a setup similar to that in Buera, Kaboski and Shin (2010) in which all debt is intra-period and agents cannot borrow intertemporally in order to smooth consumption. We show below, in a robustness section, that our results are robust to allowing for intertemporal borrowing.

Let B_{it} denote an entrepreneur's assets at the end of period $t-1$. These assets are deposited with a financial intermediary and pay a risk-free interest rate r. At the beginning of period t the entrepreneur must hire workers and install capital. The key assumption we make is that factor payments must be made at the beginning of period t, before production takes place. Letting W denote the wage rate, the entrepreneur must spend a total of

$$WL_{it} + K_{it}$$

at the beginning of period t. We assume that the entrepreneur finances this expenditure by borrowing from financial interemediaries, also at an interest rate r. The amount the entrepreneur can borrow is limited, however, by a collateral constraint:

$$WL_{it} + K_{it} \leq \lambda B_{it}$$

where, recall, B is the amount of funds the entrepreneur has deposited with the bank and λ is a parameter that governes the strength of the borrowing constraint. On one hand, if $\lambda = 1$, the entrepreneur cannot borrow externally. On the other hand, if $\lambda = \infty$, the entrepreneur faces no within-period borrowing constraints. We refer to this economy as the frictionless economy. We depart slightly from Buera, Kaboski and Shin (2010) by assuming that labor expenditures must be paid prior to production taking place. This assumption ensures that both the capital and labor allocations are distorted by financing frictions and so magnifies the effects of the borrowing constraints. Finally, we define debt as

$$D_{it} = WL_{it} + K_{it} - B_{it}.$$

At the end of period t production takes place and the entrepreneur receives Y_{it} +

$(1-\delta) K_{it}$, the output and the undepreciated portion of its capital stock. The entrepreneur then decides how much to consume, $C_{it,}$ and how much to save, $B_{it+1} \geq 0$, subject to its budget constraint:

$$C_{it} + B_{it+1} = Y_{it} + (1-\delta) K_{it} + (1+r) [B_{it} - WL_{it} - K_{it}]$$

The budget constraint says the the amount available for consumption and saving is equal to output and undepreciated capital, net of the debt payments of the entrepreneur.

In this economy, if the entrepreneur is sufficiently patient, she quickly accumulates assets in order to avoid hitting the borrowing constraint. To allow finance frictions to play a role, we must preclude entrepreneurs from accumulating assets. We do so here by assuming that the rate of time preference, β, is low relative to the rate at which agents can save, r. In particular, we assume $\beta (1+r) < 1$.

B. Recursive Formulation and Decision Rules

We next discuss the decision rules in this environment. Since all debt is intratemporal, the entrepreneur's problem of how much capital and labor to hire is static. We can thus first solve the entrepreneur's profit maximization problem and then its consumption-savings decision. The profit maximization problem reduces to:

$$\max A_{it} \left(L_{it}^{\alpha} K_{it}^{1-\alpha} \right)^{\eta} - (1+r) WL_{it} - (r+\delta) K_{it}$$

subject to

$$WL_{it} + K_{it} \leq \lambda B_{it}$$

The solution to this problem is straightforward:

$$\begin{aligned} F_L (L_{it}, K_{it}) &= (1+\tilde{r}_{it}) W \\ F_K (L_{it}, K_{it}) &= \tilde{r}_{it} + \delta \end{aligned}$$

where

$$\tilde{r}_{it} = r + \mu_{it}$$

is the entrepreneur's shadow cost of funds (effective interest rate) and μ_{it} is the multiplier on the borrowing constraint. Notice that optimal factor choices, as well as output and profits, are all proportional to $(A_{it})^{\frac{1}{1-\eta}}$. For example:

$$\begin{aligned} K_{it} &= \alpha^{\frac{\alpha\eta}{1-\eta}}(1-\alpha)^{\frac{1-\alpha\eta}{1-\eta}}(\tilde{r}_{it}+\delta)^{-\frac{1-\alpha\eta}{1-\eta}}\left[(1+\tilde{r}_{it})W\right]^{-\frac{\alpha\eta}{1-\eta}}(A_{it})^{\frac{1}{1-\eta}} \\ L_{it} &= \alpha^{\frac{1-(1-\alpha)\eta}{1-\eta}}(1-\alpha)^{\frac{(1-\alpha)\eta}{1-\eta}}(\tilde{r}_{it}+\delta)^{-\frac{(1-\alpha)\eta}{1-\eta}}\left[(1+\tilde{r}_{it})W\right]^{-\frac{1-(1-\alpha)\eta}{1-\eta}}(A_{it})^{\frac{1}{1-\eta}} \end{aligned}$$

Finally, notice the source of aggregate productivity losses in this economy. To the extent to which financing frictions induce dispersion in the entrepreneur's shadow cost of funds, $\tilde{r}_{it}$, the marginal products of capital and labor are not equalized. Moreover, since $r+\delta<1$, most of these aggregate productivity losses are due to capital misallocation: any given amount of dispersion in the internal cost of funds generates must greater dispersion in the user cost of capital, $\tilde{r}_{it}+\delta$, then in the cost of labor.

Since quantities are proportional to $(A_{it})^{\frac{1}{1-\eta}}$, it is useful to rescale all variables by this object. Let lower-case letters denote rescaled variables. For example

$$y_{it} = Y_{it}/A_{it}^{\frac{1}{1-\eta}} = f(k_{it}, l_{it}) = \left(l_{it}^{\alpha}k_{it}^{1-\alpha}\right)$$

Let

$$\pi(b) = \max_{k,l:wl+k\leq b}\left(l^{\alpha}k^{1-\alpha}\right) - (1+r)Wl - (r+\delta)k$$

where $b = B/A^{\frac{1}{1-\eta}}$ are the entrepreneur's rescaled assets. This formulation makes it clear that the firm's rescaled profits are a function of the (rescaled) asset holdings of the entrepreneur. This homogeneity simplifies the dynamic program of the entrepreneur considerably.

We can then write the entrepreneur's dynamic program as:

$$V(b,a) = \max_{b'\geq 0}\frac{c^{1-\gamma}}{1-\gamma} + \beta\int \exp\left(\frac{1-\gamma}{1-\eta}(a'-a)\right)V\left(\frac{b'}{\exp\left(\frac{1}{1-\eta}(a'-a)\right)}, a'\right)\pi(a'|a)\,da' \tag{1}$$

where

$$c = (1+r)b + \pi(b) - b'.$$

The first-order condition is:

$$c^{-\gamma} = \beta \int (1 + r + \mu') \exp\left(\frac{-\gamma}{1-\eta}(a' - a)\right) c'^{-\gamma} \pi (a'|a)\, da'$$

where, recall, μ is the multiplier on the within-period borrowing constraint. Notice that the constraint that asset holdings across periods are non-negative does not bind here because of the nature of the within-period borrowing constraint which precludes entrepreneurs with negative wealth from borrowing. Hence, a standard Euler equation, modified to reflect the effect of the borrowing constraint on the return to assets, characterizes the consumption-savings decision in this economy.

Note that the entrepreneur's productivity, a, enters this rescaled formulation of the problem only through the effect it has on the growth rate of productivity, $a' - a$. Changes in productivity have two effects here: they alter the rate at which the entrepreneur's discounts the future, as well as the rescaled amount of assets available next period.

Figure 2 summarizes the optimal decision rules[9] by showing the relationship between asset holdings, b, and the shadow cost of funds, $\tilde{r}$, as well as savings, b'. We contrast the decision rules in our economy with those in an economy with no borrowing constraints, i.e., in which $\lambda = \infty$. Clearly, the greater the agent's assets are, the less is its reliance on external funds and the lower the shadow cost of funds. Sufficiently rich entrepreneurs have an shadow cost of funds equal to the risk-free rate, r. In contrast, poor entrepreneurs face a high shadow cost of funds. This is shown in Panel A of Figure 2.

Panel B shows the entrepreneur's savings decision: its savings, b', expressed relative to its initial asset holdings, b. In both the economy with and without financing frictions rich entrepreneurs dissave, $b'/b < 1$, since $\beta (1 + r) < 1$ and entrepreneurs are impatient. Poor entrepreneurs, in contrast, accumulate assets, $b'/b > 1$, and do so at relatively higher values of b in the economy with borrowing constraints.

If any individual entrepreneur's productivity is constant over time, the decision rules in Figure 2 imply that the distribution of assets, b, across entrepreneurs would collapse over time to a mass point at which $b'/b = 1$. This is true regardless of the underlying amount of dispersion in productivity across entrepreneurs. Whether or not entrepreneurs are borrowing

[9] We use projection methods and Gaussian quadrature to compute the solution to the entepreneur's problem.

constrained at that point would then be irrelevant for the size of aggregate productivity losses: in such an economy the marginal product of capital and labor would be equal and the efficient allocations of capital and labor across productive units would be obtained (though of course in an equilibrium setting other margins, e.g., the consumption-savings choice, would be distorted by the borrowing constraint).[10]

Changes in productivity, in contrast, induce dispersion in the shadow cost of funds, since, as (1) shows, these act like shocks to any given entrepreneur's rescaled asset holdings. Greater productivity shocks induce a greater decline in the entrepreneur's rescaled assets, a greater increase in the internal cost of funds and therefore the marginal product of capital and labor. Hence, finance frictions can generate large dispersion in the marginal product of capital and labor – and therefore large aggregate productivity losses, only if changes in productivity are sufficiently large. We ask whether this is indeed the case in our empirical analysis below.

To further understand the workings of the model, Figure 3 shows the impulse response to a temporary increase in the entrepreneur's productivity, a. The evolution of the entrepreneur's productivity is shown in Panel A of the figure: we assume a mean-reverting AR(1) process in this particular example. The increase in productivity erodes the entrepreneur's rescaled assets, thus raising its shadow cost of funds, as shown in Panel B of the figure. Since the borrowing constraint binds, the entrepreneur cannot raise its stock of capital[11] sufficiently relative to what would be optimal absent financial frictions. Hence, as Panel C illustrates, its stock of capital increases gradually, much more so than in the frictioness version of the model (with $\lambda = \infty$). In fact, the capital stock drops immediately in the aftermath of the shock, as the entrepreneur directs funds to finance expenditure on labor. Finally, Panel D shows the evolution of the entrepreneur's actual (not rescaled) assets. Since the entrpreneur is more productive, its output and therefore assets increase quickly. This allows the entrepreneur to eventually overcome the borrowing constraint and eventually hire the optimal stock of capital and labor. Eventually, as the productivity shock dies out, the entrepreneur decumulates assets and once again becomes constrained.

To summarize, increases in an entrepreneur's productivity give rise to a tightening of the borrowing constraint. Finance frictions act here much like physical adjustment costs and

[10]See Banerjee and Moll (2009) who formalize this idea.

[11]The response of labor is similar and not reported here.

slow down the respone of capital and labor to productivity shocks. The difference between finance frictions and physical adjustment costs is that the former imply an asymmetric response to positive and negative productivity shocks. Entrepreneurs can respond more easily to negative productivity shocks since these make it optimal to sell capital and labor and thus relax the collateral constraint.

The fact that more productive entrepreneurs in our model are more severely constrained may seem counter-intuitive, especially in light of the results of Kiyotaki and Moore (1997). The difference between our setup and that of Kiyotaki and Moore is that they study the response of the model economy to an *aggregate* productivity shock. An aggregate productivity shock in their model increases the price of capital and thus relaxes the borrowing constraint. This latter effect is absent here because we consider *idiosyncratic* productivity shocks that have no effect on prices.

C. TFP losses from misallocation

We next describe how we compute total factor productivity and the losses from misallocation in our model economy. Consider the problem of allocating the aggregate stock of capital $K = \int_0^1 K_i di$ and labor, $L = \int_0^1 L_i di$ in this economy so as to maximize total output:

$$\max_{K_i, L_i} Y = \int_0^1 A_i \left(L_i^{\alpha} K_i^{1-\alpha} \right)^{\eta} di$$

$$\text{s.t.} \quad K = \int_0^1 K_i di \text{ and } L = \int_0^1 L_i di$$

Clearly, the solution to this problem requires that the returns to factors are equal across entrepreneurs and that the allocations of capital and labor satisfy:

$$L_i = \frac{A_i^{\frac{1}{1-\eta}}}{\sum A_i^{\frac{1}{1-\eta}}} L \text{ and } K_i = \frac{A_i^{\frac{1}{1-\eta}}}{\sum A_i^{\frac{1}{1-\eta}}} K.$$

Then aggregate output is equal to

$$Y = A \left(L^{\alpha} K^{1-\alpha} \right)^{\eta}$$

and the efficient level of total factor productivity satisfies:

$$A = \frac{Y}{\left(L^{\alpha}K^{1-\alpha}\right)^{\eta}},$$

where

$$A = \left(\int_0^1 A_i^{\frac{1}{1-\eta}}\right)^{1-\eta}.$$

It is useful to contrast this expression to that obtained assuming that capital and labor are indendent of an entrepreneur's productivity:

$$K_i/K = L_i/L = 1$$

This is a useful benchmark since we have shown above that borrowing constraints act much like an adjustment cost that prevent entrepreneurs from quickly raising capital and labor to respond to positive productivity shocks. The economy with constant capital and labor shares provides thus an upper bound on the size of aggregate TFP losses that prevent capital and labor reallocation across firms. Clearly, in this economy, which we refer to as the 'worst-case' scenario, aggregate productivity is given by

$$A^W = \int A_i di$$

Consider finally the economy with borrowing frictions. Now the optimality conditions are:

$$f_l\left(l,k\right) = w\left(1+\tilde{r}\right) \text{ and } f_k\left(l,k\right) = \tilde{r}+\delta$$

and the marginal products of capital and labor are no longer equal across entrepreneurs. The labor and capital allocations satisfy

$$L_i = \omega^L\left(\tilde{r}_i\right)\frac{A_i^{\frac{1}{1-\eta}}}{\sum A_i^{\frac{1}{1-\eta}}}L \text{ and } K_i = \omega_i^K\left(\tilde{r}_i\right)\frac{A_i^{\frac{1}{1-\eta}}}{\sum A_i^{\frac{1}{1-\eta}}}K.$$

where the wedges $\omega^L(\tilde{r}_i)$ and $\omega^K(\tilde{r}_i)$ are decreasing in the cost of internal funds, $\tilde{r}_i$:

$$\begin{aligned}\omega^L(\tilde{r}_i) &= \omega_i^L = (\tilde{r}_i+\delta)^{-\frac{(1-\alpha)\eta}{1-\eta}}\left[(1+\tilde{r}_i)\right]^{-\frac{1-(1-\alpha)\eta}{1-\eta}} \\ \omega^K(\tilde{r}_i) &= \omega_i^K = (\tilde{r}_i+\delta)^{-\frac{1-\alpha\eta}{1-\eta}}\left[(1+\tilde{r}_i)\right]^{-\frac{\alpha\eta}{1-\eta}}\end{aligned}$$

We can then still write

$$Y = A^c\left(L^\alpha K^{1-\alpha}\right)^\eta,$$

where the productivity level, A^c, is a function of the distribution of $\tilde{r}_i$ and entrepreneurial productivity A_i, as well as production function parameters:

$$A^F = \frac{\int_0^1\left(\frac{A_i}{(\omega_i^L)^\alpha(\omega_i^K)^{1-\alpha}}\right)^{\frac{1}{1-\eta}}di}{\left[\int_0^1(\omega_i^L)^{\frac{(1-\alpha)\eta-1}{1-\eta}}(\omega_i^K)^{\frac{-(1-\alpha)\eta}{1-\eta}}A_i^{\frac{1}{1-\eta}}di\right]^{\alpha\eta}\left[\int_0^1(\omega_i^K)^{\frac{\alpha\eta-1}{1-\eta}}(\omega_i^L)^{\frac{-\alpha\eta}{1-\eta}}A_i^{\frac{1}{1-\eta}}di\right]^{(1-\alpha)\eta}}$$

To get some intuition for what determines the size of the aggregate productivity losses, let us set the share of labor, α, to 0. In this case this expression simplifies considerably:

$$A^c = \frac{\int_0^1(\tilde{r}_i+\delta)^{-\frac{\eta}{1-\eta}}A_i^{\frac{1}{1-\eta}}}{\left(\int_0^1(\tilde{r}_i+\delta)^{-\frac{1}{1-\eta}}A_i^{\frac{1}{1-\eta}}\right)^\eta}.$$

To further build some intuition, assume that $\tilde{r}_i+\delta$ is related to a_{it} according to:

$$\tilde{r}_i+\delta = c\exp(\xi a_i)$$

where γ determines the sensitivity of the marginal product of capital to productivity shocks. On one extreme, if $\gamma = 1$, then the marginal product of capital increases one-for-one with productivity: the stock of capital is therefore independent of the entrepreneur's productivity, as in the 'worst-case' scenario above. On the other extreme, if $\gamma = 0$, then the unconstrained allocations are achieved.

Asumming that a_i are normally distributed with variance σ^2, aggregate productivity losses from misallocation in this example are equal to:

$$\log(A) - \log(A^c) = \xi^2 \frac{\eta}{1-\eta} \frac{\sigma^2}{2}.$$

The size of the productivity losses depends on the extent to which the internal cost of funds varies with productivity, ξ, the amount of dispersion in productivity, σ^2, as well as the degree of returns to scale, η. The first two parameters, ξ and σ_a^2, determine the extent of dispersion in the marginal product of capital across entrepreneurs. In contrast, a greater span-of-control parameter, η, makes it easier to transfer capital to the more productive entrepreneurs, therefore magnifying the losses from finance frictions which prevent reallocation. Our goal in the next section is to quantify and measure the size of these effects in the plant-level data.

3. Data

We next discuss the source of the plant-level data we use and the strategy we employ to pin down values for the key parameters of the model. We then study the model's implications for the relationship between aggregate productivity and the economy's debt-to-GDP ratio.

A. Data Description

We use data for two countries, the more financially developed South Korea, as well as the less financially developed Colombia, a country in which the external finance to GDP ratio is one-fourth of that in South Korea. We next describe each of the two datasets in part.

Korea

The data we use are from the Korean Annual Manufacturing Survey, which is collected by the Korean National Statistical Office. The survey is conducted every year from 1991 to 1998, except for the year of the Industrial Census (1993) for which we supplement the data using the Census data (which covers all establishments). The survey covers all manufacturing plants with five or more workers.

The survey reports information about each plant's total revenue, number of employees, total wage bill, payments for intermediate inputs (materials), as well as energy use. The survey also reports the book value of a plant's capital stock, as well as purchases, retirement/sales, and depreciation for land, buildings, machinery and equipment. This information allows us to construct a measure of plant-level capital using a perpetual inventory method, using the reported book value of capital to initialize each series and augmenting each year's series to include purchases net of depreciation and retirements[12]. We follow earlier work and focus on buildings, machinery and equipment as our measure of capital stock. Finally, we augment each plant's stock of capital to include the amount it leases. We define labor expenditure as wage and benefit payment to workers. The intermediate inputs include raw materials, water, and fuel. All series are real.

We drop observations that are clearly an outcome of coding errors: observations with negative values for revenue, expenditure of labor and intermediate inputs, and book value of capital. Our sample consists of $591,665$ plant-year observations over an eight year period from 1991 to 1998. We mostly focus on the 1991-1996 period, the years before the financial crisis, and study the last two years of the crisis in the final section of this paper.

We augment the data using information from the Bank of Korea Financial Statement Analysis on the aggregate debt-to-value added ratio in Korean manufacturing. The Financial Statement Analysis is a survey of all large firms as well as a stratified random sample of smaller firms. The aggregate debt-to-sales ratio of firms in this dataset is equal to 0.50, implying a debt-to-GDP ratio equal to 1.2 (this number is very close to that reported in Beck et. al (2000) for this period).

Colombia

The data are from the Colombian Industrial Survey and covers the years 1981 to 1991. The Survey collects data on all establishments with more than 10 workers. The survey provides information on the book value, purchases, sales, and depreciation of capital. This allows us to construct measures of capital stock in a similar fashion as for the Korean data described above. We measure labor expenditure as wage and benefit payments. Intermediate inputs include energy, raw materials, and various other industrial expenditures (such as fuels and lubricants). All series are real.

After excluding observations that are an obvious outcome of coding error using the same criteria as in Korean data, we are left with $71,330$ plant-year observations for 1981 to 1991. Finally, Beck et. al. (2000)) report that the external debt to GDP ratio in Colombia

[12]See e.g. Caballero et al. (1995).

is equal to 0.30 in this period, thus much smaller than that in Korea.

B. Establishment-level facts

We next describe several features of the plant-level data. These are not unique to the particular countries we study: may of these have been documented in earlier work[13]. We present these features here in order to guide our quantitative analysis below. Since the economy we study assumes no entry and exit, we focus now on a balanced panel of plants in both countries that are continuously in sample throughout the years for which we have data available. Roughly 32000 plants are fit this criterion in Korea and 5000 plants in Colombia. We later show that the facts we document below are very similar when we study the entire (unbalanced) panel of plants.

Since the process for productivity is what primarily determines the size of aggregate productivity losses, we focus on features of the data that allow us to pin down this process. Although we do not directly observe an individual's plant productivity, we note that, since productivity is the sole source of variation in output in our model, we can identify its process by requiring the model to account for the distribution of output and its growth rate across plant in our data. This is the approach taken in most of the quantitative studies of models of establishment dynamics. Below we conduct a robustness check by computing a process for productivity directly using data on capital and labor.

The measure of output in the data that most closely corresponds to that in our model is value added, i.e, revenue net of expenditure on intermediate inputs, since we have abstracted from the latter in the theory. We thus report salient features of the data on value-added in the two manufacturing panels we study.[14]

Distribution of output growth

Panel A of Table 1 shows that the data is characterized by large dispersion in output growth rates. The standard deviation of changes in the log of value added from one year to another is equal to 0.54 in Korea and 0.49 in Colombia. In addition to dispersion, we report a number of higher-order moments of the growth rate of output. First, notice that the distributions show excess kurtosis (fat tails): the kurtosis of growth rates is equal to 13 in Korea and 21 in Colombia. For comparison, the kurtosis of a Gaussian is equal to 3.

[13]See for example Rossi-Hansberg and Wright (2007).

[14]In a previous version of the paper, Midrigan and Xu (2009) we explicily model intermediate inputs and study revenue, rather than value-added data. In that paper we report a very similar set of results.

Thus, a small number of plants experience very large increases or declines in their output. Another way to gauge the thickness of the tails (the kurtosis itself is very sensitive to outlier) is to compare the interquartile range of the distribution to the standard deviation. The former is, by definition, unaffected by the shape of the tails, while the standard deviation is. Notice in Table 1 that the interquartile range is smaller than the standard deviation in both datasets: 0.49 in Korea and 0.36 in Colombia. For comparison, the interquartile range of a Gaussian is about 1.3 times larger than its standard deviation. Finally, Panel A of the table reports the difference between the top and bottom 1st, 5th and 10th percentiles of the distribution of growth rates: clearly, some plants experience very large changes in their output. For example, the difference between the 99th and 1st percentile is equal to 2.96 in both countries, thus about 5.5 times the standard deviation for Korea and about 8.2 times the standard deviation for Colombia. For comparison, this difference is equal to 4.7 standard deviations for a Gaussian.

Persistence and scale dependent growth

Panel B reports the correlation of a given plant's log output with its lags at horizons of one to five years. Note that the first-order autocorrelation is equal to 0.93 for Korea and 0.96 for Colombia. Hence, despite a fair amount of variability in output growth rates, output across plants is fairly persistent. That these autocorrelations are below unity suggests that output tends to mean-revert, or that growth rates are negatively correlated with the size of the establishment. To see this, note that the first-order autocorrelations reported in the Table imply that the coefficient of a regression of output growth rates on lagged output:

$$y_{it} - y_{it-1} = (\rho - 1)\, y_{it-1} + \varepsilon_{it}$$

is equal to $\rho - 1 = -0.07$ (0.93 - 1) for Korea and $\rho = -0.04$ for Colombia. Larger plants therefore grow slower: doubling output tends to decrease a plant's growth rate by about 7% in Korea and 4% in Colombia.

The table also reports higher-order autocorrelations. An important feature of the data is that these decay slowly with the horizon, so that output is much more persistent than suggested by the first-order autocorrelation. For example, the autocorrelations at lags 1 through 5 for Korea are equal to 0.93, 0.91, 0.89, 0.88 and 0.86. In contrast, an AR(1) process that decays geometrically with serial correlation parameter 0.93 would imply a much lower fifth-order autocorrelation: $0.93^5 = 0.69$. A similar pattern holds for Colombia: the fifth-order autocorrelation is equal to 0.89.

Size distribution of establishments

The final feature of the data we document is the size distribution of establishments. Panel C of Table 1 shows that output is heavily concentrated in a few large establishments: the largest 1% of establishments account for 57% (30%) of all manufacturing value added in Korea and Colombia, respectively. Similarly, the largest 20% of establishments account for 91% (88%) of all value added.

Cross-country comparison

The statistics reported above suggest that the size distributions of Korean and Colombian plants are fairly similar: in both countries there is a lot of dispersion in plant-level growth rates, lots of persistence in output and output is concentrated in the highest plants. Perhaps the only noticeable difference is that output is slightly less concentrated and more persistent in Colombia than it is in Korea. This difference may reflect, however, differences in the sampling criteria in these two datasets. While the Korean survey includes data on all plants with more than 5 workers, the Colombian data includes only plants with more than 10 workers. To see the role of these sampling differences, the last column of Table 1 reports statistics for a truncated sample of Korean plants with more than 10 workers, i.e., the same criterion used for Colombia. Notice that the Korean numbers change very little when we eliminate the roughly 5000 plants that have fewer than 10 workers, thus suggesting that the differences between Colombian and Korean datasets do not reflect sampling differences.

4. Quantitative Analysis

Recall that our question is, what is the effect of finance frictions on aggregate productivity? To answer this question, we next study a quantitative version of the model parameterized to fit the salient plant-level facts described above. We next discuss the strategy we use to pin down the model's parameters.

A. Parameterization

We group parameters into two categories. The first category includes parameters that are difficult to identify using our data. These include preference and production function parameters. We assign these values that are common in existing work. We show below that our results are robust to perturbations of these parameter values. The second category includes parameters that determine the process for productivity at the micro-level, as well as the size of the financing frictions, which are the key determinants of the size of aggregate productivity losses in our model. We pin down values for these parameters by requiring that the model accounts for the salient features of the data discussed above.

Assigned Parameters

The period is one year. We set the intertemporal elasticity of substitution, governed by γ, equal to 1. We set the interest rate equal to 4% per year, $r = 0.04$. The discount factor, β, determines the extent to which entrepreneurs accumulate assets and hence their ability to grow out of the borrowing constraint. We follow Buera, Kaboski and Shin (2010) and use $\beta = 0.92$, implying that entrepreneurs are fairly impatient. We assign production function parameters that are standard in existing work: capital depreciates at a rate $\delta = 0.06$, the span-of-control parameter is equal to $\eta = 0.85$, as in Atkeson and Kehoe (2005)[15] and a share of labor equal to $\alpha = 2/3$. The latter choice allows the model to match the expenditure share of labor in value added in our data. In a robustness section below, we evaluate how changing some of these parameters affects the answer to our question.

Calibrated Parameters

The rest of the parameters are jointly pinned down by the requirement that the model accounts for the plant-level facts. We use an indirect inference approach to estimate these parameter values, by choosing parameter values that minimize the distance between a number of plant-level moments in the model and in the data.

Since we would like our model to simultaneously account for a number of features of the data, we assume a somewhat more complex process for entrepreneurial productivity. In particular, we assume that entrepreneurial productivity is the sum of three components:

$$a_{it} = Z_i + z_{it} + \tilde{a}_{it}$$

where Z_i is a fixed (permanent) productivity component. We assume that $\exp(Z_i)$ is dis-

[15]Other values of η used in recent work include 0.75 (Bloom 2009), 0.79 (Buera, Kaboski and Shin (2010), 0.82 (Bachmann, Caballero and Engel 2010) and 0.90 (Khan and Thomas 2008).

tributed according to a Pareto with an upper bound H and a shape parameter, μ :

$$\Pr\left[\exp\left(Z_i\right) \leq x\right] = \frac{1 - x^{-\alpha}}{1 - H^{-\alpha}}.$$

We think of Z_i as capturing 'entrepreneurial' ability. The other two components of productivity are time-varying. Here

$$z_{it} \sim iid, N\left(0, \sigma_z^2\right)$$

is a transitory component of productivity, and

$$\tilde{a}_{it} = \rho \tilde{a}_{it-1} + \varepsilon_{it},$$

is a persistent, but variable productivity component. We illustrate below the role of each of these three components and show that all three are important in allowing the model to account for the plant-level facts.

Since we have documented that the distribution of output growth rates shows excess kurtosis, we allow for fat-tailed shocks to the variable productivity component, u_{it}. In particular, we assume that the shocks, ε_{it} are drawn from a mixture of Normals:

$$\varepsilon_{it} \sim \begin{cases} N\left(0, \sigma_{1,\varepsilon}^2\right) \text{ with prob. } 1 - \kappa \\ N\left(0, \sigma_{2,\varepsilon}^2\right) \text{ with prob. } \kappa \end{cases}$$

where $\sigma_{2,\varepsilon}^2 > \sigma_{1,\varepsilon}^2$ and κ determines the probability with which shocks are drawn from the more dispersed distribution. Intuitively, fat-tailed shocks have the potential to amplify the size of aggregate TFP losses since they imply that a small fraction of firms experience very large increases in productivity, thus amplifying their need for external borrowing.

A common strategy in quantitative evaluations of the effect of finance frictions on aggregate TFP (see e.g. Buera, Kaboski and Shin (2010) and Moll (2010)) is to calibrate the model to data from a relatively undistorted economy (usually the U.S.) and then trace out the effect of varying the collateral constraint, λ, on aggregate productivity. We follow a similar strategy here. In particular, we pin down key parameters of the model by requiring that the model accounts for the establishment-level facts in the relatively more financially developed Korea. We then hold all other parameters constant and compute the effect of varying λ on aggregate productivity and other implications of the model.

Objective Function

The moments we use are the 16 moments that characterize the plant-level facts in Table 1 for Korea. We have 8 parameters to calibrate: $\theta = \{\lambda, \rho, \sigma_{1,\varepsilon}, \sigma_{2,\varepsilon}, \kappa, \sigma_z, \mu, H\}$. To pin down these parameters, we use an indirect inference approach. Let Γ^d denote the 16×1 vector of moments in the data. Let $\Gamma(\theta)$ denote the vector of moments in the model. Let W denote the inverse of the variance-covariance matrix of the data moments, computed by bootstrapping repeated samples of the data with replacement. We pin down θ by minimizing the following objective:

$$\min_{\theta} \left[\Gamma(\theta) - \Gamma^d\right]' W \left[\Gamma(\theta) - \Gamma^d\right] \tag{2}$$

Finally, we compute standard errors for our estimates of θ using

$$V = \frac{1}{N}\left[\frac{\partial \Gamma(\theta)}{\partial \theta'} W \frac{\partial \Gamma(\theta)}{\partial \theta}\right]^{-1},$$

where N is the number of plants in the data and $\frac{\partial \Gamma(\theta)}{\partial \theta}$ is the gradient of the vector of moments with respect to the parameters. We find this calculation useful as it allows us to gauge the extent to which the moments we use are informative about the parameter values we attempt to identify.

Table 2 (column labeled Benchmark) presents the parameter values that minimize the objective (2). The collateral constraint, λ, is equal to 2.3, implying a leverage ratio D/B equal to 1.3. The serial correlation of the persistent productivity component, ρ, is equal to 0.73, the standard deviation of shocks to this component is equal to $\sigma_{1,\varepsilon} = 0.07$ and $\sigma_{2,\varepsilon} = 0.32$ and firms draw from the more volatile distribution $\kappa = 0.065$ of the time. Finally, the transitory productivity component is also fairly volatile, $\sigma_z = 0.07$, while the permanent component has a shape parameter $\mu = 3.89$ and an upper bound $H = 4.66$. Notice in Table 2 that all of these parameters are very precisely estimated, suggesting that the moments we have used are indeed very sensitive to changes in these parameter values.

Before we proceed, it is useful to decompose the importance of each of these components of productivity. The parameter values we have calibrated imply that the transitory productivity component, z, accounts for only 4% of the cross-sectional variance of productivity, the persistent component, $\tilde{a}$, accounts for 26%, and the permanent component, Z, accounts for 70%. As for changes, 2/3 of the variance of changes in productivity are accounted for by changes in the persistent component and 1/3 are accounted for by transitory shocks. Intuitively, the model requires a fairly volatile transitory component in order to account for the relatively low first-order serial correlation of output in the data simultaneously with the slow rate at which the autocorrelation declines over time. Similarly, the permanent component, Z, must be fairly dispersed in order to allow the model to reconcile the concentration of output among the largest plants together with the fairly strong scale dependence of output growth rates.

Table 3 reports how the model does at matching the moments in the data. The fit is very good, reflecting the rich process for productivity we have assumed. The model accounts well for the distribution of output growth rates, the pattern of serial correlation and the high concentration of output among the largest plants. The root mean square error, computed as

$$RMSE = \left(\frac{1}{16} \sum_{i=1}^{16} \left(\ln \left(\Gamma_i^d \right) - \ln \Gamma_i \left(\theta \right) \right)^2 \right)^{\frac{1}{2}}$$

is equal to 1.9% and most of this is accounted for by a slight mismatch in the concentration statistics in the model and data.

Constrained models

We next attempt to gauge the role of each of the components of productivity we have assumed. In particular we estimate constrained version of the above model, by first eliminating the transitory shock, z_{it}, and then eliminating both the transitory and permanent component, z_{it} and Z_i. We re-calibrate these two other economies using the same strategy and same vector of moments discussed above. Table 2 reports the parameter values that best fit the data and Table 3 reports how the models do at matching the salient establishment-level facts.

Notice that the economy without iid shocks, z_{it}, does a fairly good job at accounting for the moments of the data. In this economy the autocorrelation decays a bit too quickly, compared to the data, but the fit of the model is excellent otherwise: the root mean square error is equal to 4.4%.

In contrast, eliminating the permanent productivity component, Z_i, worsens the model's fit considerably. Since there is too much scale dependence in growth, large plants do not stay large for long and the model misses both the size distribution of plants in the data, as well as the autocorrelation pattern. Absent a permanent productivity component, there is a tradeoff between matching the dispersion of the growth rates on one hand, and the large concentration of plants in the data on the other hand. The root mean square error in this version of the model is much higher and equal to 23.5%.

B. Model Predictions

We next discuss the model's predictions about the extent to which entepreneurs are constrained and the size of aggregate productivity losses it generates.

Size of financing frictions

Recall that the shadow cost of funds (the effective interest rate at which an entrepreneur borrows) is equal to

$$\tilde{r}_{it} = r + \mu_{it}$$

where r is the risk-free rate and μ_{it} is the multiplier on the borrowing constrained. Clearly, since $\tilde{r}_{it}$ determines an entrepreneur's user cost of capital and labor, dispersion in $\tilde{r}_{it}$ is the sole source of aggregate TFP losses in this economy. We first report some measures of the extent to which $\tilde{r}_{it}$ is dispersed in the ergodic steady-state of our economy. Note, in Panel A of Table 4 that 55% of entrepreneurs are financially constrained, and the median premium $(\tilde{r}_{it} - r)$ they face, if constrained, is equal to 0.022, thus a 55% premium over the risk-free rate. The interquartile range of the premium is 0.026, the 90th percentile of the premium is 0.057, while the 99th percentile is equal to 0.147. Thus a small fraction of firms are constrained. As anticipated, productive entrepreneurs are more likely to be constrained, since they are the ones who need to increase their stock of capital and labor and may not have sufficient funds to do so. We illustrate this in Figure 4. We group entrepreneurs into percentile of the a distribution and, for each percentile, compute the average shadow cost of funds across entrepreneurs in that category. Figure 4 shows that very unproductive entrepreneurs face a shadow cost of funds equal to 0.04, the risk-free rate, while the very productive entrepreneurs face a shadow cost of funds equal to 0.08, thus an 100% premium.

TFP losses from misallocation

Table 4 reports the size of aggregate TFP losses in our Benchmark economy, computed as described above. These amount to 3.62%, a fairly large number relative to, say, those in Hopenhayn and Rogerson (1993), but small when compared to the 30% TFP gap between Korea and the US in 1996.

To understand the size of these TFP losses and how these vary with the degree of financial development, we next conduct a number of experiments in which we vary λ and hold all other parameters constant. As discussed above, this is a typical thought experiment in recent quantitative studies that attempt to isolate the role of financing constraints. We study the model's implications for several values of $\lambda : \lambda = \{50, 1.2, 1\}$, each chosen to match a debt-to-GDP ratio equal to $\{2.3, 0.3, 0\}$, as in the U.S., Colombia and in an economy with no external borrowing.

As Table 4 shows, changing the debt-to-GDP ratio has a noticeable impact on establishment-level statistics. While very few (1.3%) entrepreneurs are constrained in the 'U.S.' calibration, the majority (82%) are constrained in the 'Colombia' calibration, or in the economy with no external borrowing (87%). Moreover, financing frictions have an important role on the pattern of establishment-level dynamics: the standard deviation of changes in log-output is equal to 0.77 in the 'U.S.' calibration, and thus almost twice as high than in an economy with little external borrowing (0.39 in 'Colombia' and 0.36 in an economy with no external borrowing). Notice also that finance frictions are a source of fat-tails in the distribution of output growth rates. While kurtosis is fairly low (6.4) in the 'U.S.' calibration, it increases to almost 30 in an economy with little external finance. Finally, financing frictions make output more persistent: the serial correlation of output is equal to 0.89 in the 'U.S.' calibration and increases to 0.97 in an economy with little external finance. Intuitively, finance frictions impart persistence to output since entrepreneurs react to good productivity shocks gradually by slowly accumulating internal funds and growing out of the borrowing constraint.

These predictions of the model regarding the effect of finance frictions on establishment-level dynamics are consistent with the pattern we have documented in the micro-data. Recall that Colombian plants experience less volatile output growth rates (the standard deviation is 0.49 vs. 0.54 in Korea, while the interquartile range is 0.36 vs. 0.49 in Korea), more persistent output (the serial correlation of output is 0.96 vs. 0.93 in Colombia) and a more fat-tailed distribution of output growth rates (the kurtosis of changes in output is equal to

12.9 in Korea and 20.8 in Colombia).

Table 4 also reports the answer to our key question: what is the size of TFP losses induced by financing frictions? The table shows that, unlike establishment-level micro moments, aggregate TFP losses vary fairly little across these different experiments. The "U.S." economy implies TFP losses of 1.26%, the "Colombia" economy predicts TFP losses of 5.24%, while an economy with no external borrowing has TFP losses equal to 5.28%. The model thus accounts for a small fraction (4% vs. 60%) of the aggregate TFP differences between financially developed countries like U.S. and poor countries with low TFP and little external finance.

Why are the TFP losses small?

To see why the TFP losses are small here, recall that absent changes in productivity, the ergodic distribution of entrepreneurs would collapse to a mass point at $b_i = \bar{b}$, implying that all entrepreneurs would face the same shadow cost of funds. This would imply that the marginal product of capital and labor would be equalized across entrepreneurs and there would be no aggregate TFP losses from misallocation. Finance frictions can thus generate TFP losses only in an economy where the variable component of productivity, $\tilde{a}_{it} + z_{it}$, is sufficiently dispersed. It turns out, however, that this is not the case in the calibration of our model consistent with the micro data. To see this, we compute the worst-case TFP losses, i.e., those in an economy in which capital and labor do not vary with $\tilde{a}_{it} + z_{it}$. Recall that these worst-case TFP losses are equal to

$$\Delta \ln TFP = \frac{1}{1-\eta} \ln \left(\int_0^1 \exp \left(\frac{1}{1-\eta} (a_i + z_i) \right) di \right) - \ln \int \exp (a_i + z_i) \, di.$$

Table 4 shows that this statistic is equal to 7.23% in our economy, thus suggesting that there is too little dispersion in the variable productivity component so that even the most extreme form of adjustment costs cannot distort aggregate TFP too much. Relative to this benchmark, finance frictions in our model are quite potent, since they generate a substantial proportion of what the TFP losses would be absent any adjustment of factors of production to productivity shocks: roughly half (3.62/7.23) in Korea and roughly three-quarters (5.24/7.23) in Colombia.

To summarize, our model predicts that the TFP losses from financing frictions are fairly small, on the order of 5% even for economies with little external finance, thus much too small to account for the cross-country dispersion in TFP. The reason losses are small here is that productivity shocks must be fairly small in order for the model to account for the dispersion in output growth rates in the data. Small shocks imply that any form of adjustment costs (of which finance frictions are a special case) cannot do much damage in this environment.

C. Counterfactual experiments

We next conduct several counterfactual experiments in order to illustrate how ignoring key features of the plant-level data can lead to the conclusion that TFP losses from misallocation are, in fact, much larger in this class of models. We report the results of these experiments in Table 5. For comparison, Panels I and II report the predictions of our Benchmark economy with a permanent productivity component, Z. As shown above, this model, whether or not we allow for transitory productivity shocks, z, accounts for the plant-level facts very well and also predicts small TFP losses from misallocation.

Consider next the consequence of ignoring data on the dispersion of output growth rates. We eliminate the permanent productivity component, as well as the transitory one (since the latter plays a minor role), and assume productivity follows a simple AR(1) process:

$$a_{it} = \rho a_{it-1} + \varepsilon_{it},$$

where ε_{it} is an iid, normal random variable with variance $\sigma_{1,\varepsilon}^2$. We choose the two parameters characterizing this process, ρ and $\sigma_{1,\varepsilon}^2$, to match a) the serial correlation of output in the data of 0.93, as well as b) statistics that characterize the degree of concentration of the size distribution of firms.

Panel III of Table 5 shows that this version of the model requires much more volatile productivity shocks in order to fit the size distribution of firms. Intuitively, since we have eliminated the permanent component of productivity, there is too much mean-reversion, implying that rich entrepreneurs shrink fastest and cannot achieve the large concentration observed in the data without resorting to large productivity shocks. Such shocks imply, however, that output growth rates are much more volatile than they are in the data: the standard deviation is 1.05 vs. 0.54 in the data, and the interquartile range is 1.22 vs. 0.49 in the data. Moreover, this version of the model predicts too little autocorrelation in an entrepreneur's output at horizons longer than a year. Since the autocorrelation decays geometrically here, the 5th-order serial correlation is much smaller in the model (0.69) than it is in the data. Clearly, this counterfactual is greatly at odds with the micro data.

Notice also that the model now predicts substantially larger TFP losses from misallocation: 10.5% for Korea and 18.1% for Colombia, reflecting the large shocks to productivity that entrepreneurs cannot easily react to. These losses are approximately 3-4 times greater than in the Benchmark economy. However, the model generates these losses for the wrong reasons, by implying changes in output from one year to another that are much greater than in the data.

In a recent paper Moll (2009) has argued that plant-level productivity is much less persistent in the data than what we have assumed here. He estimates a serial correlation parameter ($\rho = 0.80$) that is much lower than what we have used here ($\rho = 0.92$) and argues that the resulting TFP losses are much greater when the serial correlation of productivity is low. Intuitively, he argues, persistent shocks allow entrepreneurs to accumulate assets and overcome financial constraints quickly.

The difference between our estimates of ρ reflect differences in methodology. While Moll (2009) computes a Solow residual measure of plant productivity, we require that the model accounts for the serial correlation of output in the data. Given the difficulty of measuring productivity and the uncertainty regarding the value of ρ, we ask whether our results are indeed sensitive to the value of this parameter. We do so by assigning ρ a value equal to 0.8 and recalibrating $\sigma^2_{1,\varepsilon}$ to allow the model to match the size distribution of establishments in the data.

Panel IV of Table 5 reports the results of this experiment. Since now there is much faster mean reversion in productivity, even greater shocks to productivity ($\sigma_{1,\varepsilon} = 0.40$) are required to account for the size distribution of establishments in the data. Because shocks are more volatile, TFP losses are even greater now: 18% for Korea and almost 30% for Colombia. Once again, however, the model generates the large TFP losses for the wrong reasons, by implying much too volatile plant-level dynamics: the standard deviation of output growth rates is equal to 2.17, thus four times greater than in the data.

Notice also that a lower value of ρ, on its own, does not generate greater TFP losses, if one were to hold constant the standard deviation of shocks to productivity (rather than the unconditional variance of productivity, as Moll 2009 does, or as we have done in the previous example). To see this, we set $\rho = 0.8$, but now keep the standard deviation of shocks equal to 0.21, the value in economy III with more persistent productivity, $\rho = 0.92$. We report the results of this experiment in Panel V of Table 5. Notice that now TFP losses are, in fact, smaller than in the economy with more persistent shocks (6.6% vs. 10.5% earlier for, say, Korea). Thus, holding constant the standard deviation of shocks, more persistent productivity actually amplifies TFP losses. Intuitively, if the shocks are equally sized, a more permanent shock lasts for a larger number of periods, and since it takes a while for the entrepreneur to grow out of the borrowing constraint (see Figure 3), misallocation persists. In contrast, a more transitory shock reverts quicker to the mean, thus imply a more short-lived increase in the entrepreneur's marginal product of capital.

D. Robustness [to be completed]

We next gauge the robustness of our results to varying several of the parameters that are difficult to identify using our plant-level data and that we have simply assigned values to.

Span-of-control, η

Discount factor, β

Elasticity of substitution between capital and labor

An economy with intertemporal borrowing [to be completed]

We have assumed until now that all loans are intra-period. Although this assumption considerably simplifies the problem, none of our results hinge on this particular timing protocol. To see this, we next study an extension of the model with intertemporal borrowing. As earlier, to allow finance frictions to play a role, we assume that the entrepreneur must pay its labor and capital expenditure uprfront, before production takes place, but after repaying its outstanding debt, D_t. Output is thus received with a one-period delay, at the beginning of $t + 1$.

Formally, let B_{it} denote the entrpreneur's net worth at the beginning of period t, that is, its assets net of liabilities. Net worth evolves according to:

$$B_{it+1} = Y_{it} + (1 - \delta) K_{it} - D_{it}$$

where D_{it} is the amount the entrepreneur owes at date t. Given its date-t net worth and labor and capital choices, the entrepreneur's consumption is given by

$$C_{it} = B_{it} - wL_{it} - K_{it} + \frac{D_{it+1}}{1+r}$$

Finally, assume a collateral constraint similar to that in Kiyotaki and Moore (1997) so that the firm cannot borrow more than a fraction λ of its next period's assets:

$$D_{it+1} \leq \lambda \left[Y_{it} + (1-\delta) K_{it} \right] = \lambda \left[B_{it+1} + D_{it+1} \right].$$

We can once again rescale the entrepreneur's problem by noting that the optimal choices of capital and labor are proportional to $A_{it}^{\frac{1}{1-\eta}}$. We can then write the problem recursively as:

$$V(b,a) = \max_{l,k,d'} \frac{c^{1-\gamma}}{1-\gamma} + \beta \int \exp\left(\frac{1-\gamma}{1-\eta} (a'-a) \right) V(b',a') \pi(a'|a)\, da',$$

where

$$c = b + \frac{d'}{1+r} - k - wl,$$

and the laws of motion for the states are:

$$b' = \frac{f(k,l) + (1-\delta) k - d'}{\exp\left(\frac{1}{1-\eta} (a'-a) \right)},$$

The borrowing constraint is:

$$d' \leq \lambda \left[f(k,l) + (1-\delta) k \right].$$

Consider next the first-order conditions that characterize the optimal decision rules.

The optimal amount of debt satisfies:

$$c^{-\gamma} = \beta (1+r) \int \exp \left(\frac{-\gamma}{1-\eta} (a' - a) \right) c'^{-\gamma} \pi (a'|a) \, da' + \Phi (1+r),$$

where Φ is the multiplier on the borrowing constraint. The optimal choice of capital and labor satisfy:

$$\begin{aligned} c^{-\gamma} &= \left(\beta \int \exp \left(\frac{-\gamma}{1-\eta} (a' - a) \right) c'^{-\gamma} \pi (a'|a) \, da' + \Phi\lambda \right) (f_k(k,l) + (1-\delta)), \\ wc^{-\gamma} &= \left(\beta \int \exp \left(\frac{-\gamma}{1-\eta} (a' - a) \right) c'^{-\gamma} \pi (a'|a) \, da' + \Phi\lambda \right) f_l (k,l), \end{aligned}$$

These can be rearranged to yield:

$$\begin{aligned} f_k (k,l) &= \tilde{r} + \delta \\ f_l (k,l) &= w (1+\tilde{r}) \end{aligned}$$

where $\tilde{r}$, the entrepreneur's shadow cost of funds, satisfies:

$$1 + \tilde{r} = (1+r) \left[\lambda + (1-\lambda) \beta (1+r) \int \exp \left(\frac{-\gamma}{1-\eta} (a' - a) \right) \frac{V_n (n', a')}{c^{-\gamma}} \pi (a'|a) \, da' \right]^{-1}.$$

Clearly, these decision rules are similar to those in the original setup. The difference now is that the shadow cost of funds depends on the growth rate of the marginal utility of consumption as it reflects the multiplier on the intertemporal borrowing constraint.

[to be completed]

5. Economy with Exit and Entry

We next ask: does allowing entry and exit overturn our results? Do finance frictions distort the entry/exit margin by preventing productive agents from becoming entrpreneurs? Clearly, if permanent productivity shocks account for most of the unconditional dispersion in productivity, the entry/exit margin can only be distorted if some of the very productive entrpreneurs are relatively poor. But this will not be the case in the ergodic steady-state unless some of these entrepreneurs are young, so that they haven't yet had a chance to accumulate assets and grow out of their borrowing constraint. Hence, in addition to allowing for a choice of entering/exiting entrepreneurship, we also assume that some agents die and are replaced each period by (relatively poor) newborn agents. We assume thus a constant hazard of death each period, $1 - p$. We show below that death is necessary in order to allow the model to account for the fact that some very large plants exit any given period in the data.

A. Environment

As earlier, we assume a continuum of agents of measure 1, indexed by i. Each period the agent decides whether to be an entrepreneur or worker. Switching occupations entails no cost and so these decisions are reversible each period.

A workers supplies 1 unit of labor inelastically at a wage rate W. As earlier, entrepreneurs have access to a technology that produces output using inputs of capital and labor:

$$Y_{it} = A_{it} \left(L_{it}^{\alpha} K_{it}^{1-\alpha} \right)^{\eta},$$

where A_{it} is the agent's productivity as an entrepreneur. We assume

$$\log(A_{it}) = a_{it} = Z_i + \tilde{a}_{it},$$

where, as earlier, Z_i is a permanent productivity component, drawn from a Pareto distribution, and $\tilde{a}_{it}$ is an AR(1) process, as described earlier.

Both types of agents can save using a one period risk-free security. In addition, entrepreneurs can borrow within a period in order to finance labor and capital expenditure, but their ability to borrow is limited by the collateral constraint:

$$WL + K \leq \lambda B.$$

As earlier, we can compute the profits an agent can earn as an entrepreneur:

$$\pi\left(B,A\right) = \max_{K,L} AF\left(K,L\right) - \left(1+r\right)WL - \left(r+\delta\right)K$$

$$s.t.\ WL + K \leq \lambda B$$

We can write the agent's value recursively as:

$$V(B,a) = \max_{B'\geq 0} \frac{C^{1-\gamma}}{1-\gamma} + \beta p \int V\left(B',a'\right)\pi\left(a'|a\right)da'$$

$$\text{where } C = \left(1+r\right)B + \max[\pi(B,A),W] - B',$$

and recall that p is the constant survival probability.

Each period a measure $(1-p)$ of agents are born. At birth agents draw a permanent productivity component, Z_i, from a Pareto distribution characterized by (μ, H) and a variable component $\tilde{a}_{it} = 0$. Moreover, they receive an endowment $B_0\left(Z_i\right)$, deposited in an account with the financial intermediary. We assume that the endowment is potentially a function of the agent's productivity as an entrepreneur. [One could interpret this dependence, as Evans and Jovanovic (1989) do, as reflecting the savings decisions of high-ability people who expected to become entrepreneurs one day. Another interpretation would be seed funding by a venture capitalist who finances the higher-ability would-be entrepreneur]. The newly born agent then chooses its occupation and faces the same problem as an old agent.

We assume, as earlier, that this is a small open economy so that agents can borrow at a risk-free rate r. Let $\mu\left(B,A\right)$ denote the ergodic measure of agents over asset holdings and productivity. Let $I\left(B,A\right) = W > \pi\left(B,A\right)$ denote the choice of becoming a worker. Let $L\left(B,A\right)$ denote the amount of labor demanded by an entrepreneur of type (B,A). The equilibrium wage rate satisfies:

$$\int I\left(B,A\right)d\mu\left(B,A\right) = \int L\left(B,A\right)\left(1 - I\left(B,A\right)\right)d\mu\left(B,A\right)$$

B. Parametrization

In Table 7 we present the moments in the data that we would like our model to account for. These are now computed for the entire sample of plants, including those that are in sample for only a few years. The sample of plants now considerably increases, from 32,000 earlier, to 161,000 for Korea. Since we have shown earlier that the moments for Colombia are similar to those in Korea, we only report the moments for Korea in the Table and discuss below how the Colombian numbers compare.

We report the same set of moments that characterize the distribution of growth rates, persistence, size distribution of plants as earlier. A comparison of the first columns of Table 7 and Table 1 reveals that these moments are very similar for the larger unbalanced panel of plants we consider now.

In addition, we would like our model to account for the age-distribution and exit hazards of plants in the data. Notice in Panel D. of Table 7 that most plants are young (ages 1-5): 51%, with the rest of the sample roughly split between ages 6-10 and 10+. Also notice that there is considerable amount of turnover in the data: the unconditional exit hazard is 1/3, mostly reflecting exit by very small plants. Larger plants, however, exit too. One way to see this is to compute the share of output accounted for by exiting plants. This is equal to 7% in the data, thus suggesting that some very large plants exit as well.

Economy with no startup funds

Consider first the economy in which newly born agents enter with no endowment: $B_0(Z_i) = 0$. We calibrate this economy using a similar procedure as described earlier: now the set of parameters also includes p, the survival probability, and we target the additional set of moments regarding the exit hazards in Panel D. of Table 7. Our model does reasonably well at matching the moments in the data, especially considering that we have only introduced one single parameter and require the model to match the exit hazards and age distribution in the data. As in the data, most plants are young (62% in the model, 51% in the data), exit hazards are large (22% of plants exit in the model, 33% in the data), and exiting plants account for a substantial share of output (7% in the model and in the data).

Panel A of Table 8 reports some of the key predictions of the model. A lot more establishments are now constrained than in the economy without exit and entry (Table 4). The medium external finance premium is 11% and is much more volatile. For example, the 90th percentile is equal to 24% and the 99th percentile is equal to 30%. This dispersion in the internal cost of funds manifests itself in much greater TFP losses. These are 10.6% for our economy, that, recall, is calibrated to the 1.2 debt-to-GDP ratio in Korea. The TFP losses are thus almost 3 times greater than in the economy without exit and entry (recall, equal to 3.6%). Interestingly, most of these losses reflect misallocation of factors among existing plants, not distortions along the entry-exit margin. To see this, we decompose the TFP losses into those arising due to an inefficient allocation of agents into entrepreneurship. These latter losses are much smaller, 0.2%, reflecting that most marginal entrepreneurs are small and account for a small share in aggregate output.

Not only does the model predict much greater TFP losses for Korea, it also predicts that differences in the external-finance ratio generate much greater cross-country TFP differences. The TFP losses in an economy calibrated to the US 2.3 debt-to-GDP ratio are equal to 4.4, while those in an economy calibrated to the Colombian 0.3 debt-to-GDP ratio are equal to 13.1, thus a difference of 9%, much greater than the 4% earlier.

The reason TFP losses are much greater here is that newly born agents that have high ability, Z_i, enter entrepreneurship almost immediately in this version of the model, despite the fact that initially they have little assets and are thus severely constrained. Profits from operating a plant are much greater for highly talented entrepreneurs, in equilibrium, than the relatively low wage they can earn as a workers. Since such entrepreneurs are, initially, very poor, they cannot afford the efficient amount of capital and labor and this reflects in relatively high TFP losses.

We note, however, that this version of the model is at odds with the dynamics of plants in the data. To see this, Figure 5 shows the relationship betwen growth rates and age in the model (dashed line) and compares it to the data (dots). Notice that the youngest establishments grow a lot quicker than in the data (for example, the average growth rate for a 2-year old plant is 25% in the model and only 10% in the data). Panel C of Table 8 shows that young (ages 1-5) plants grow 20% faster in the model than older (ages 10+) plants do. In contrast, they grow only 5% faster in the data. Similarly, plants aged 6-10 grow 6% faster in the model and only 2% faster in the data. Establishments grow much faster in the model because of borrowing constraints: as establishments age, they accumulate internal funds and grow because of the ability to higher capital and labor.

Another way to see that young establishments are too constrained in the model is to compare the returns to capital (recall these are a function of shadow cost of funds) for establishment of different age groups. We compute returns to capital as the average product of capital and find that the model predicts that these returns are much greater for the youngest plants (ages 1-5 and 6-10) than they are in the data. The average product of capital is 30% greater in the model for plants aged 1-5 than for plants that are 10 years old or older; the corresponding statistic is equal to 4% in the data. Similarly, plants aged 6-10 have a much greater average product of capital in the model (17% higher) than in the data (6% higher). Once again, this second measure suggests young entrepreneurs are much too constrained in our model than in the data.[16]

Since the rate of growth and the average product of capital are, in our theory, strongly tied to the extent to which establishments are constrained, we conclude that this version of the model generates TFP losses for the wrong reason, by implying that young establishments are much more constrained than what they are in the data.

Economy with startup funds

The counterfactual predictions above can be easily addressed by assuming that newly entering agents receive an endowment that depends on their ability Z_i. Let

$$B_0(Z_i) = \phi(WL(Z_i) + K(Z_i))$$

where $L(Z_i)$ and $K(Z_i)$ are the efficient amount of labor an entrepreneur would hire absent financing frictions. Here, if $\phi = 1$, entering establishments cannot achieve the efficient scale without borrowing externally. We assume $\phi \in (0, 1)$ and calibrated its size, together with the rest of the parameters, by requiring that the model matches the statistics in Panel C of Table 7 on the relationship between plant growth and age, in addition to the other moments we have targeted above. It turns out that a value $\phi = 0.45$ best fits this feature of the data. Figure 5 shows that now the model fitst the growth-age relationship very well: as in the data, newly entering plants grow about 10% faster. Panel C of Table 8 shows the model also fits well the relationship between the returns to capital and age, though it implies that the youngest plants are somewhat more constrained (a 10% higher average product of capital than 10+ plants) than in the data (4% higher average product of capital).

[16]We have computed similar statistics for establishments in Colombia and found similar numbers. Plants aged 1-5 grow only 11% faster than those older than 10 years, while plants aged 6-10 grow only 2% faster. As for the average product of capital, in Colombia it, in fact, increases with age.

Since entering establishments are now less constrained, the model now produces much smaller dispersion in returns to factors and therefore smaller TFP losses: 5.1% for Korea, 1.5% for US and 6.7% for Colombia. As in the economy without exit and entry, finance frictions account can generate a fairly small TFP gap, about 5%, between US and Colombia, thus about 1/8th of what this gap is in the data.

6. Conclusions

We study a model of establishment dynamics with finance constraints. The model, when parameterized to account for the salient features of the plant-level data, predicts that even extreme financing frictions produce modest (4-5%) TFP losses from misallocation.

References

[1] Angeletos, Marios, 2008, "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," forthcoming, Review of Economic Dynamics

[2] Atkeson, Andrew and Patrick Kehoe, 2007, "Modeling the transition to a new economy: lessons from two technological revolutions," American Economic Review, 97(1): 64-88.

[3] Banerjee, Abhijit V. and Esther Duflo, 2005. "Growth Theory through the Lens of Development Economics,"in: Philippe Aghion & Steven Durlauf (ed.), Handbook of Economic Growth, edition 1, volume 1, chapter 7, pages 473-552.

[4] Bartelsman, Eric J. and Mark Doms, 2000. "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569-594.

[5] Bartelsman, Eric J., John Haltiwanger and Stefano Scarpetta, 2008. "Cross Country Differences in Productivity: The Role of Allocative Efficiency," mimeo.

[6] Basu, Susanto and John Fernald, 1995, "Are apparent productive spillovers a figment of specification error?" *Journal of Monetary Economics*, 36: 165-188.

[7] Basu, Susanto and John Fernald, 1997, "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105(2): 249-283.

[8] Beck, Thorsten, A. Demirguc-Kunt, and Ross Levine, 2000, "A New Database on the Structure and Development of the Financial Sector." *World Bank Economic Review*, 14: 597-605.

[9] Buera, Francisco J. and Yongseok Shin, 2008. "Financial Frictions and the Persistence of History: A Quantitative Explanation," mimeo.

[10] Buera, Francisco J., Joseph P. Kaboski and Yongseok Shin, 2009. "Finance and Development: A Tale of Two Sectors," mimeo.

[11] Caballero, Ricardo J., Eduardo M.R.A. Engel and John C. Haltiwanger, 1995. "Plant-Level Adjustment and Aggregate Investment Dynamcis," *Brookings Papers on Economic Activity*, 26(2), 1-54.

[12] Castro, Clementi, MacDonald, 2009, "Legal Institutions, Sectoral Heterogeneity and Economic Develioment," Review of Economic Studies, 76: 529-561.

[13] Cooley, Thomas and Vincenzo Quadrini, 2001. "Financial Markets and Firm Dynamics," *American Economic Review*, 91(5): 1286-1310.

[14] Gertler, Mark, Simon Gilchrist and Fabio Natalucci, 2007, "External Constraints on Monetary Policy and the Financial Accelerator," *Journal of Money, Credit and Banking.*

[15] Gilchrist, Simon and Jae W. Sim, 2007. "Investment During the Korean Financial Crisis: A Structural Econometric Analysis," *NBER Working Papers*, no. 13315.

[16] Greenwood, Jeremy, Juan Sanchez, and Cheng Wang, 2010, "Quantifying the Impact of Financial Development on Economic Development," mimeo.

[17] Guner, Nezih, Gustavo Ventura and Yi Xu, 2008. "Macroeconomic Implications of Size Dependent Policies," *Review of Economic Dynamics*, 11(4), 721-744.

[18] Hennessy, Christopher A. and Toni M. Whited, 2007. "How Costly is External Financing? Evidence from a Structural Estimation," *Journal of Finance*, 62(4), 1705-1745.

[19] Hopenhayn, Hugo, 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127-1150.

[20] Hopenhayn, Hugo and Richard Rogerson, 1993. "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 101(5), 915-938.

[21] Hsieh, Chang-Tai and Peter Klenow, 2009. "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, forthcoming.

[22] Lagos, Ricardo, 2006. “A Model of TFP,” *Review of Economic Studies*, 73(4), 983-1007.

[23] Moskowitz, Tobias and Annette Vissing-Jorgensen, 2002. “The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?” *American Economic Review*, 92: 745-778.

[24] Restuccia, Diego and Richard Rogerson, 2008. “Policy Distortions and Aggregate Productivity with Heterogeneous Plants,” *Review of Economic Dynamics*, 11(4), 707-720.

[25] Tybout, James R., 2000. “Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?,” *Journal of Economic Literature*, 38(1), 11-44.

Table 1: Establishment-level facts

	Korea	Colombia	Korea (10 + plants)
A. Distribution of growth rates			
$\sigma(\Delta y_{it})$	0.54	0.49	0.53
$kurt(\Delta y_{it})$	12.9	20.8	13.00
$iqr(\Delta y_{it})$	0.49	0.36	0.47
$90\text{-}10\%\ (\Delta y_{it})$	1.13	0.84	1.09
$95\text{-}5\%\ (\Delta y_{it})$	1.63	1.23	1.58
$99\text{-}1\%\ (\Delta y_{it})$	2.96	2.96	2.88
B. Persistence			
$\rho(y_{it}, y_{it-1})$	0.93	0.96	0.92
$\rho(y_{it}, y_{it-2})$	0.91	0.94	0.90
$\rho(y_{it}, y_{it-3})$	0.89	0.93	0.88
$\rho(y_{it}, y_{it-4})$	0.88	0.91	0.87
$\rho(y_{it}, y_{it-5})$	0.86	0.90	0.85
C. Size distribution			
fraction of Y by largest 1%	0.57	0.30	1.37
fraction of Y by largest 5%	0.77	0.61	0.49
fraction of Y by largest 10%	0.84	0.75	0.71
fraction of Y by largest 20%	0.91	0.88	0.80
			0.88
D. Finance			
Debt-to-GDP	1.2	0.3	1.2
# plants	31543	4787	26833

Note: y is the log of value added (revenue net of spending on intermediate inputs)

Table 2: Parameter values

		Benchmark	No iid shocks	No iid and permanent
Assigned parameters				
	γ	1	1	1
	β	0.92	0.92	0.92
	r	0.04	0.04	0.04
	α	0.67	0.67	0.67
	θ	1	1	1
	η	0.85	0.85	0.85
	δ	0.06	0.06	0.06
Calibrated parameters				
	λ	2.30	2.58	3.01
		(0.01)	(0.01)	(0.02)
	ρ	0.72	0.74	0.94
		(0.01)	(0.01)	(0.00)
	$\sigma_{1,\varepsilon}$	0.069	0.092	0.078
		(0.001)	(0.002)	(0.000)
	$\sigma_{2,\varepsilon}$	0.317	0.314	0.336
		(0.001)	(0.002)	(0.001)
	κ	0.065	0.070	0.065
		(0.001)	(0.001)	(0.000)
	σ_z	0.060	-	-
		(0.001)		
	μ	3.89	3.64	-
		(0.02)	(0.02)	
	H	4.66	4.91	-
		(0.02)	(0.02)	

Note: standard errors reported in parantheses

Table 3: Moments in Model and Data

	Data (Korea)	Benchmark	No iid shock	No iid, no permanent
A. Distribution of growth rates				
$\sigma(\Delta y_{it})$	0.54	0.54	0.51	0.51
$kurt(\Delta y_{it})$	12.9	12.9	12.9	18.2
$iqr(\Delta y_{it})$	0.49	0.49	0.47	0.43
$90\text{-}10\%\ (\Delta y_{it})$	1.13	1.15	1.05	0.96
$95\text{-}5\%\ (\Delta y_{it})$	1.63	1.63	1.5	1.37
$99\text{-}1\%\ (\Delta y_{it})$	2.96	2.96	2.96	2.96
B. Persistence				
$\rho(y_{it}, y_{it-1})$	0.93	0.93	0.95	0.95
$\rho(y_{it}, y_{it-2})$	0.91	0.91	0.91	0.91
$\rho(y_{it}, y_{it-3})$	0.89	0.89	0.89	0.87
$\rho(y_{it}, y_{it-4})$	0.88	0.88	0.87	0.83
$\rho(y_{it}, y_{it-5})$	0.86	0.86	0.85	0.78
C. Size distribution				
fraction of Y by largest 1%	0.57	0.58	0.59	0.29
fraction of Y by largest 5%	0.77	0.81	0.83	0.53
fraction of Y by largest 10%	0.84	0.88	0.90	0.66
fraction of Y by largest 20%	0.91	0.93	0.95	0.79
D. Finance				
Debt-to-GDP	1.2	1.2	1.2	1.2
Value of objective RMSE, %		1.9	4.4	23.5

Note: y is the log of value added (revenue net of spending on intermediate inputs)

Table 4: Model predictions

	Benchmark (Korea) $\lambda = 2.3$	US $\lambda = 50$	Colombia $\lambda = 1.2$	No Ext. Finance $\lambda = 1$
A. Size of financial frictions				
Debt-to-GDP	1.2	2.3	0.3	0
Fraction constrained	0.55	0.013	0.815	0.872
Median premium if constrained	0.022	0.018	0.035	0.042
IQR premium if constrained	0.026	0.036	0.036	0.040
90% premium if constrained	0.057	0.086	0.079	0.089
99% premium if constraint	0.147	0.210	0.170	0.183
B. Micro-moments				
$\sigma(\Delta y_{it})$	0.54	0.77	0.39	0.36
$kurt(\Delta y_{it})$	12.9	6.4	27.6	33.1
$\rho(y_{it}, y_{it-1})$	0.93	0.89	0.97	0.97
C.TFP losses from misallocation, %				
Actual losses	**3.6**	**1.3**	**5.2**	**5.3**
Worst-case losses	7.2	7.2	7.2	7.2

Table 5 : Counterfactual experiments

	Data	I. Benchmark	II. No z shock	III. No z, no Z. Match size distribution.	IV. Low ρ Match size distribution	V. Low ρ, same s.d. shocks as III.
A. Distribution of growth rates						
$\sigma(\Delta y_{it})$	0.54	0.54	0.51	1.05	2.17	1.03
$iqr(\Delta y_{it})$	0.49	0.49	0.47	1.22	2.73	1.16
B. Persistence						
$\rho(y_{it}, y_{it-1})$	0.93	0.93	0.95	0.93	0.80	0.77
$\rho(y_{it}, y_{it-3})$	0.89	0.89	0.89	0.80	0.52	0.47
$\rho(y_{it}, y_{it-5})$	0.86	0.86	0.85	0.69	0.34	0.30
C. Size distribution						
fraction of Y by top 1%	0.57	0.58	0.59	0.52	0.44	0.13
fraction of Y by top 10%	0.84	0.88	0.90	0.88	0.88	0.48
D. TFP losses, %						
Korea (1.2)		3.6	4.3	10.5	18.3	6.6
Colombia (0.3)		5.2	5.2	18.1	29.5	10.9
Worst-case		7.2	8.5	54.3	69.9	20.2
Parameters						
ρ		0.72	0.74	0.92	0.80	0.80
$\sigma_{1,\varepsilon}$		0.069	0.092	0.21	0.40	0.21
$\sigma_{2,\varepsilon}$		0.317	0.314	-	-	-
κ		0.065	0.070	-	-	-
σ_z		0.060	-	-	-	-
μ		3.89	3.64	-	-	-
H		4.66	4.91	-	-	-

Table 6 : Robustness checks

	Data	I. β = 0.85 (more impatience)	II. θ = 0.25 (K & L less substit.)	III. η = 0.95 (greater span of control)	IV. Intertemporal debt
A. Distribution of growth rates					
$\sigma(\Delta y_{it})$	0.54	0.49	0.50		
$iqr(\Delta y_{it})$	0.49	0.33	0.54		
B. Persistence					
$\rho(y_{it}, y_{it-1})$	0.93	0.94	0.94		
$\rho(y_{it}, y_{it-3})$	0.89	0.89	0.89		
$\rho(y_{it}, y_{it-5})$	0.86	0.86	0.85		
C. Size distribution					
fraction of Y by top 1%	0.57	0.63	0.48		
fraction of Y by top 10%	0.84	0.93	0.89		
D. TFP losses, %					
Korea (1.2)		7.0			
US (2.3)		2.4			
Colombia (0.3)		8.7			
Worst-case		11.2			
Parameters					
ρ		0.72	0.85	0.80	0.80
$\sigma_{1,\varepsilon}$		0.134	0.011	0.40	0.40
$\sigma_{2,\varepsilon}$		0.299	0.217	-	-
κ		0.069	0.085	-	-
σ_z		0.061	0.046		
μ		4.19	4.3	-	-
H		4.88	2.71	-	-

Table 7: Moments in Economy with Exit/Entry

	Data (Korea)	I. No startup funds	II. With startup funds
A. Distribution of growth rates			
$\sigma(\Delta y_{it})$	0.56	0.57	0.57
$kurt(\Delta y_{it})$	11.4	8.8	6.2
$iqr(\Delta y_{it})$	0.51	0.44	0.45
B. Persistence			
$\rho(y_{it}, y_{it-1})$	0.92	0.93	0.93
$\rho(y_{it}, y_{it-2})$	0.90	0.87	0.87
$\rho(y_{it}, y_{it-3})$	0.88	0.82	0.83
$\rho(y_{it}, y_{it-4})$	0.87	0.78	0.80
$\rho(y_{it}, y_{it-5})$	0.86	0.74	0.78
C. Size distribution			
fraction of Y by largest 1%	0.53	0.34	0.20
fraction of Y by largest 5%	0.72	0.66	0.52
fraction of Y by largest 10%	0.79	0.80	0.70
fraction of Y by largest 20%	0.87	0.89	0.85
D. Age and exit hazards			
fraction age = 1 - 5	0.51	0.62	0.6
fraction age = 6 - 10	0.26	0.16	0.18
fraction age > 10	0.23	0.21	0.23
exit hazard	0.33	0.25	0.22
output share exiting plants	0.07	0.07	0.07
E. Finance			
Debt-to-GDP	1.2	1.2	1.2

Table 8: Predictions of Economy with Exit/Entry

	Data (Korea)	I. No startup funds	II. With startup funds
A. Size of financial frictions			
Fraction constrained		0.83	0.65
Median premium if constrained		0.11	0.08
IQR premium if constrained		0.09	0.04
90% premium if constrained		0.24	0.16
99% premium if constraint		0.30	0.21
B. TFP losses, %			
Total losses		10.6	5.1
Due to misallocation across establishm.		10.4	5.1
US (D/Y = 2.3)		4.4	1.5
Colombia (D/Y = 0.3)		13.1	6.7
C. Characteristics young establishments			
mean Δy if age = 1 - 5 vs. age>10	0.05	0.20	0.06
mean Δy if age = 6 - 10 vs. age>10	0.02	0.06	0.01
Δ Y/K if age = 1 - 5 vs. age> 10	0.04	0.30	0.10
Δ Y/K if age = 6 - 10 vs. age> 10	0.06	0.17	0.05
D. Parameter values			
ρ		0.65	0.65
$\sigma_{I,\varepsilon}$		0.22	0.185
μ		2.4	2.2
H		15	17
p		0.95	0.94
ϕ		-	0.45

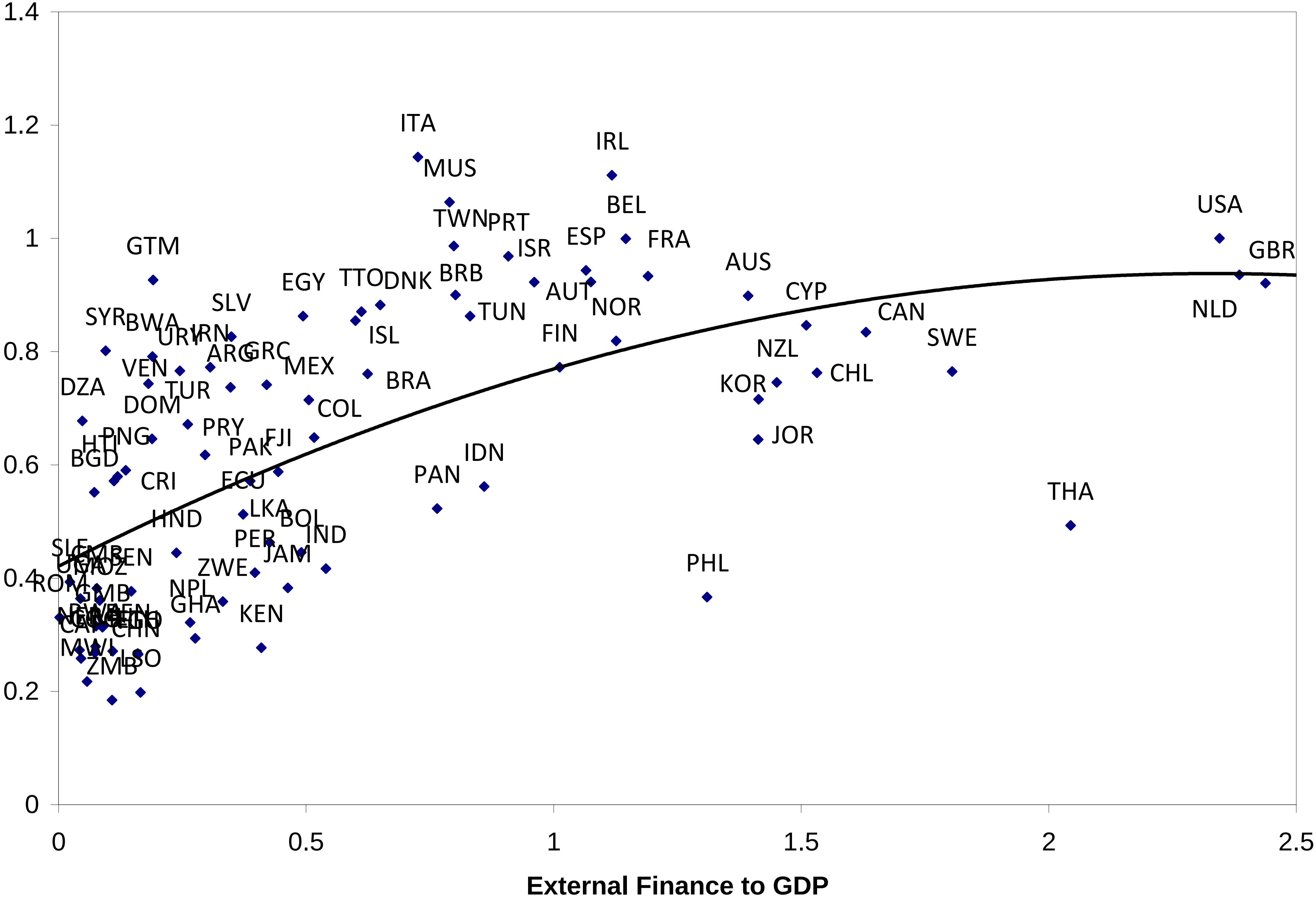

Figure 1: TFP vs. External Finance

Figure 2: Decision rules

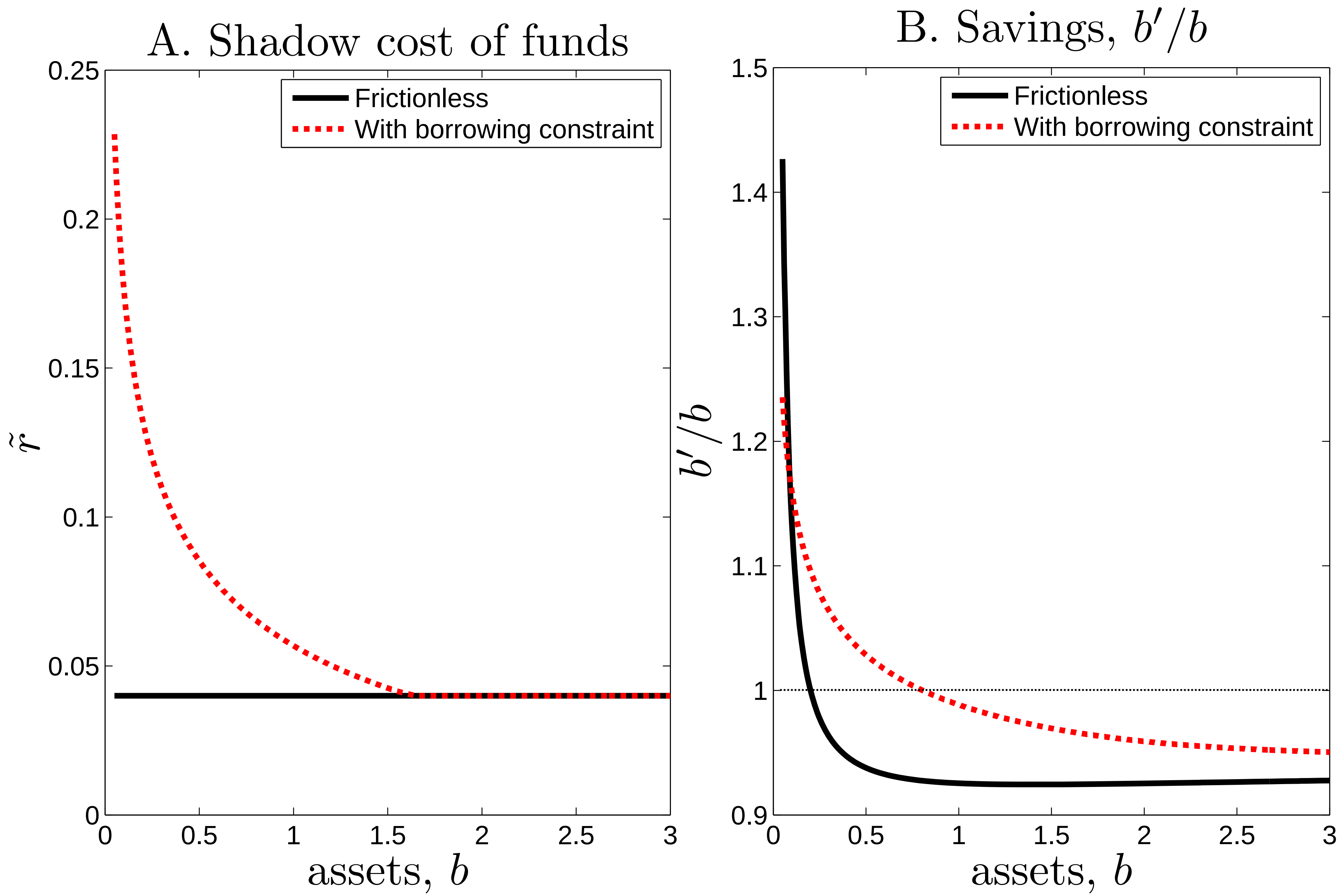

Figure 3: Impulse response to a productivity shock

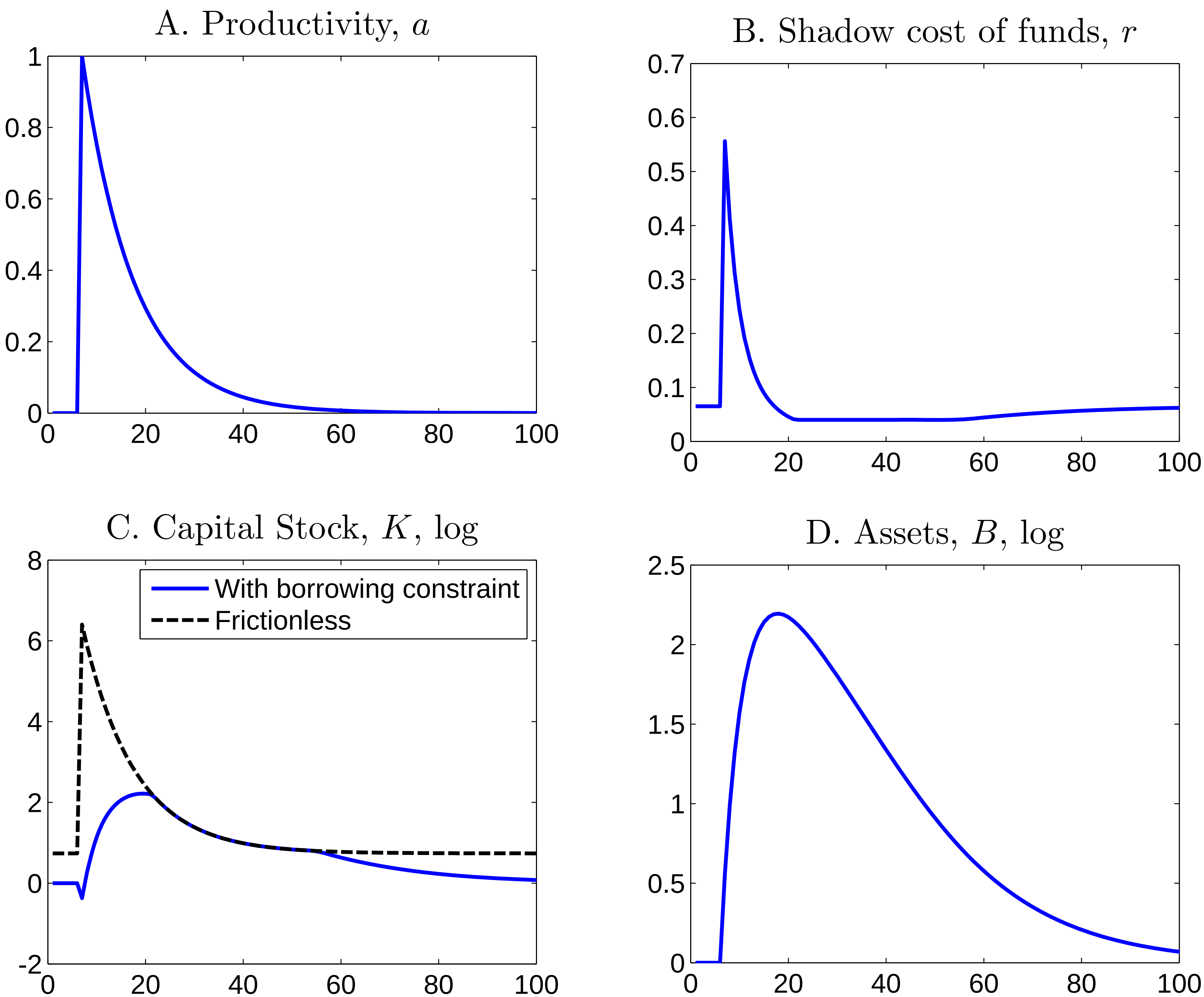

Figure 4: Productivity vs. shadow cost of funds

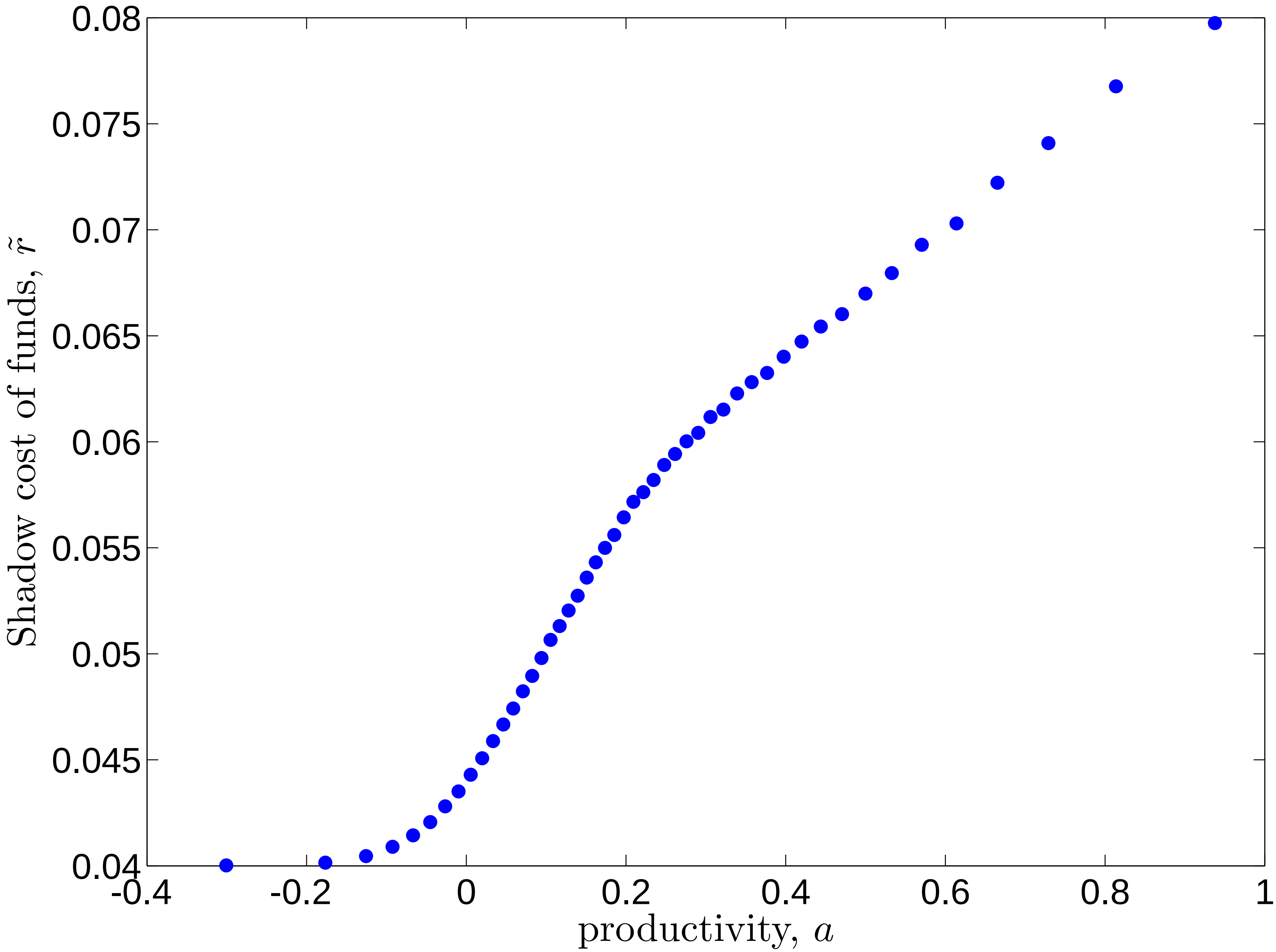

Figure 5: Growth rates vs. age