

# Equilibrium Price Dispersion and Rigidity: A New Monetarist Approach\*

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## Abstract

Why do some sellers set prices in nominal terms that do not respond to changes in the aggregate price level? In many models, prices are sticky by assumption. Here it is a result. We use search theory, with two consequences: prices are set in dollars since money is the medium of exchange; and equilibrium implies a nondegenerate price distribution. When money increases, some sellers keep prices constant, earning less per unit but making it up on volume, so profit is unaffected. The model is consistent with the micro data. But, in contrast with other sticky-price models, money is neutral.

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# 1 Introduction

Arguably the most difficult question in macroeconomics is this: Why do some individual sellers set prices in nominal terms that do not respond to changes in the aggregate price level, ostensibly flying in the face of fundamental microeconomic principles? Of course, some sellers must respond, otherwise the aggregate does not change, but many prices are sticky at least in the short run. In many popular macro models, including those used by most policy makers, this is an *assumption*. We derive it as a *result*. In our model sellers naturally post prices in dollars, since money is the medium of exchange. Dollar prices of many individual sellers may not respond in equilibrium when there is an increase in the aggregate price level generated by an increase in money, even though we allow sellers to change their prices whenever they like and for free (there is no Calvo fairy and no Mankiw cost). In a sense, this provides microfoundations for *the* critical assumption at the core of Keynesian economics. But the policy implications are different. We prove in our model that, when sticky prices emerge as an equilibrium outcome, money is neutral: the central bank cannot engineer a boom or get us out of a slump simply by issuing currency.

Our theory is consistent with what the experts in numerous studies say are the salient features of the data.<sup>1</sup> We calibrate the model to some standard observations, including the average frequency of nominal price changes, and then compare it's predictions with the data in terms of the price-change size (as opposed to frequency) distribution. It turns out the model and data line up quite well. In particular, there is no problem capturing several facts that are deemed important but difficult to match with many existing models. One example is the prevalence of small price changes, defined as less than 5% in absolute value, in combination with fairly large average price changes, which is especially a problem for most menu cost models. Another example is the prevalence of some negative price changes, even during inflation. The model is consistent with some more stylized facts, including the observation that when two sellers adjust their prices at the same time they do not

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<sup>1</sup>A sample of empirical work on price stickiness includes Cecchetti (1986), Carlson (1986), Blinder (1991), Bils and Klenow (2005), Campbell and Eden (2007), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2009), Eichenbaum, Jamovich and Rebelo (2009) and Gagnon (2009). See Klenow and Malin (2010) for a recent survey.

necessarily adjust to the same level, another problem for menu costs. It can match other common observations, such as the fact that prices change more frequently, and not just by larger amounts, when inflation is high, which is a problem for standard Calvo models.

This seems relevant for several reasons. First, despite the successes of the New Classical and Real Business Cycle paradigms, it is hard to deny that at least some nominal prices seem sticky in the sense defined above – they do not respond to changes in the aggregate price level. Moreover, this observation is one of main reasons why many Keynesians are Keynesian. Consider Ball and Mankiw (1994), who we consider representative, when they say: “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky.” They go on to claim that “based on microeconomic evidence, we believe that sluggish price adjustment is the best explanation for monetary nonneutrality.” Furthermore, “As a matter of logic, nominal stickiness requires a cost of nominal adjustment.” Some people that one might not think of as Keynesian present similar positions. For instance, Golosov and Lucas (2003) assert that “menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.”

We interpret the above claims as containing three points related, respectively, to empirics, theory, and policy. The first claim is that price stickiness is “really there” in the data. The quotations assert this, and it is substantiated in the numerous empirical studies mentioned above. We concede this point. The second claim is that this observation implies “as a matter of logic” that economists need to have menu costs, or some related devices, in our models. We prove this wrong: we describe equilibria that match not only the broad observation of price stickiness, but also some of the more detailed empirical findings, with recourse to no such device. The third claim, to which at least Ball and Mankiw seem to subscribe, is that their observations also imply money is not neutral, and this rationalizes Keynesian policy advice. We also prove this wrong: our theory is consistent with the relevant observations, but rather than yielding Keynesian policy implications, money is neutral. Sticky prices simply do not constitute evidence that money is not neutral or that particular policy recommendations are

warranted.<sup>2</sup>

To explain our approach, we begin by pointing out that the issues at hand concern monetary phenomena: Why are prices quoted in dollars in the first place? Why do they not all adjust to changes in the money supply? What does this imply about monetary policy? So we want a monetary model. We work with a version of the New Monetarist framework laid out in Nosal and Rocheteau (2010) or Williamson and Wright (2010*a,b*), based on the model in Lagos and Wright (2005), where money is used as a medium of exchange. For those with a different taste for microfoundations, everything goes through with, say, cash-in-advance or money-in-the-utility-function specifications. But an advantage of the approach adopted here is that agents trade with each other, and not merely against budget equations, so we can meaningfully discuss whether they trade using barter, credit, or money. Following most of the papers in the relevant literature, in at least some circumstances, specialization and matching frictions limit barter, while commitment and information frictions limit credit. Hence, money is essential for at least some exchange, and naturally prices are posted in dollars, because dollars are the objects being traded.

Now consider the following observation. In many popular models, including those in the New Keynesian tradition, following Woodford (2003) or Clarida, Gali and Gertler (1999), there is price dispersion because of nominal rigidities if and only if there is inflation. To understand this point, suppose two sellers set the same  $p_t$  and date  $t$ . Suppose the first one to have a visit from the Calvo fairy has this happen at  $s > t$ . Then the unlucky one who did not receive a chance to change his price is stuck at  $p_t$ , while the lucky one increases his to  $p_s > p_t$ , and hence we get price dispersion, if and only if the inflation rate between  $t$  and  $s$  is strictly positive. But the data suggest that there is dispersion in prices even during episodes of low or zero inflation (this was first impressed on us by Campbell and Eden 2007). Therefore, it seems that it would be good to have a theory that delivers dispersion even with low or no inflation. Although there are options available, for a variety of reasons we use the

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<sup>2</sup>To be clear, our position is not that money is neutral in the real world, and we know of compelling examples (like the one in Lucas 1972) where it is not neutral in theory. The point is to construct a coherent economic environment with two properties: (i) it is consistent with the sticky-price facts; and (ii) it nevertheless delivers neutrality. Money is not superneutral in the model – inflation matters, even if the money stock does not, as in true in many models – but this hardly rationalizes Keynesian policy prescriptions.

approach in Burdett and Judd (1983).

To explain the idea of Burdett-Judd pricing, it helps to go back to the dawn of search theory in economics, say McCall (1970) or Mortensen (1970). A common critique (see e.g. Rothschild 1973) was that in those models agents draw prices (or, in labor applications, wages) from some exogenous distribution  $F$ . From where does  $F$  arise? Diamond (1971) tried to come the rescue by endogenizing the price distribution as follows: Suppose a homogeneous group of buyers each want to buy 1 unit of a good that delivers net payoff  $u - p$ , where  $u$  is utility, and  $p$  is the price in units of transferable utility. At stage 1, sellers with unit cost  $c < u$  each post a price  $p$  taking as given the prices of their competitors. At stage 2 buyers sample randomly from the price distribution, one observation, or more generally at most one observation, at each date until they make a purchase and stop. Diamond's model delivers three strong results: there is a unique equilibrium; all sellers post the same  $p$  (i.e.  $F$  is degenerate); and this price is the monopoly price,  $p = u$ . Not much of a rescue.<sup>3</sup>

Diamond's negative result instigated a wave of research, too big to survey here, trying to come up with models where  $F(p)$  is nondegenerate in equilibrium. One of the nicest contributions in this literature is the Burdett-Judd model, based on a simple twist of Diamond: instead of sampling at most one  $p_t$  at any date  $t$ , buyers sometimes get to sample two or more. Assume for the sake of illustration you see 1 price with probability  $\alpha_1$  and 2 prices with probability  $\alpha_2 = 1 - \alpha_1$ . As special cases,  $\alpha_1 = 1$  yields Diamond (all sellers set  $p = u$ ) and  $\alpha_1 = 0$  yields Bertrand (all sellers set  $p = c$ ). But for any  $\alpha_1 \in (0, 1)$ , there cannot be a single-price equilibrium. In fact, it is shown that distribution has a density  $F'$  on a nondegenerate interval  $[p, \bar{p}]$ , and one can actually solve for  $f(p)$  explicitly. In this set up we get price dispersion, and we get it without inflation, obviously, since the original Burdett-Judd model is a nonmonetary model.

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<sup>3</sup>The proof of Diamond's results is simple: For any distribution  $F(p)$  at stage 2 buyers strategies satisfy a reservation property: there is some  $\bar{p}$  such that buyers accept  $p \leq \bar{p}$  and reject  $p > \bar{p}$ . This means that at stage 1 the choice of any individual seller is clearly to set  $p = \bar{p}$ . So, there is a single price, and we claim it is  $p = u$ . Suppose not; then since  $p > u$  is obviously not a good idea for sellers, they must all set  $p < u$ . But then any individual seller has an incentive to shade up to  $p + \varepsilon$  for some  $\varepsilon > 0$ , since as long as buyers discount the future all they would rather pay  $p + \varepsilon$  than wait for  $p$ . Thus we rule out anything as an equilibrium but  $p = u$  for all sellers. It is easy to see this is in fact an equilibrium.

The first thing we do is embed Burdett-Judd into a dynamic general equilibrium model. In principle, there may be more than one way to do this, but we find it convenient to use the periodic trading structure in Lagos-Wright. In this environment, agents alternate over time between trading in a decentralized market that looks like a lot like Burdett-Judd, and a centralized market that looks like Arrow-Debreu. We think this is a good idea, even without introducing money, say by allowing barter (or transferable utility) in the decentralized market, or perfect credit between the decentralized and centralized markets. But since, as we said, at least some of the issues at hand are monetary, we go on to introduce assumptions limiting barter and credit, so that money has a genuine role as a medium of exchange. The Lagos-Wright model was of course designed to analyze monetary exchange, and while the baseline version of that model uses generalized Nash bargaining, rather than posting, we can easily swap out the Nash bargaining module for a Burdett-Judd price-posting module.<sup>4</sup> In this version of the model, sellers post prices, and they post them in dollars because, given the frictions mentioned above, dollars are the objects being traded.

To preview the way the logic works, at every date  $t$ , theory delivers a distribution  $F_t(p)$  of posted nominal prices for homogeneous goods, with support  $\mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$ . But while  $F_t(p)$  is pinned down, no individual seller cares where he is in this distribution: any  $p \in \mathcal{F}_t$  yields the same profit, simply because lower-price sellers earn less per unit, but make it up on the volume. Indeed,  $F_t(p)$  is constructed by equating profit across all  $p \in \mathcal{F}_t$ . Because any other distribution would imply profitable deviations by sellers, one can say that competition forces the equilibrium distribution to  $F_t(p)$ . As the money supply increases from  $M_t$  to  $M_{t+1}$ , the nominal distribution shifts to  $F_{t+1}(p)$  with new support  $\mathcal{F}_{t+1}$ , but the real price distribution stays the same. This requires that some sellers change their prices, but not all of them: if  $p_t \in \mathcal{F}_t$  and  $p_t \notin \mathcal{F}_{t+1}$  a seller posting  $p_t$  must change, but if  $p_t \in \mathcal{F}_{t+1}$  he may not. In the latter case, real profit per unit falls, but sales increase. Hence, sellers can change prices

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<sup>4</sup>The framework is flexible in this regard, and several mechanisms other than Nash bargaining, have been studied. For instance, Rocheteau and Wright (2005) introduce price posting with directed search and (Walrasian) price taking; Aruoba, Berentsen and Waller (2007) consider alternative bargaining solutions; Galeanois and Kircher (2008) and Dutu, Julien and King (2009) use auctions; and Hu, Kennan and Wallace (2009) use pure mechanism design. No one previously tried (Burdett-Judd) posting with random matching in Lagos-Wright, although it is used in the related model of Shi (1997) by Head and Kumar (2005) and Head, Kumar and Lapham (2010).

infrequently, even though we allow them to change whenever they like and at no cost.

Our theory says that sellers can be rationally inattentive to the aggregate price level and monetary policy, within some range, since as long as  $p_t \in \mathcal{F}_t$  their place in this distribution does not matter. This leads to a certain degree of indeterminacy, since theory pins down only the distribution, not who is where in the support.<sup>5</sup> Despite this indeterminacy, the theory still has many testable predictions. The relative price distribution is determinate, with a simple closed-form solution, and although it is invariant to the price level it does depend on inflation in a precise way. Also, when the price level changes, some sellers drop out of the support and they have to adjust, so the frequency of price changes is predicted to increase with inflation. Also, the degree of indeterminacy shrinks fairly quickly with inflation, so that even for moderately high inflation the predictions are fairly tight. As we demonstrate, the model can be used to confront many empirical issues. Thus, we think it is a reasonable theory of endogenous stickiness. But, again, money is neutral: If we issue extra money at  $t$ , the price distribution jumps immediately, and no real expansion is created, contrasting sharply with what happens in models where prices are sticky for ad hoc reasons. We conclude that nominal price stickiness does not logically justify interventionist policies by central banks.<sup>6</sup>

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<sup>5</sup>This is as it should be, and is not much different from standard models of, say, firm entry and exit (e.g. Pissarides), where theory determines the measure of active firms but not which ones enter from a homogeneous set. Of course, if firms are heterogeneous, with respect to cost, say, then theory does predict which ones enter. Our model similarly relies on at least some homogeneity in firm costs, which we think is fine, especially for retail pricing, where competitive at the wholesale level implies costs are the same. Still, if one were convinced that no two firms could possibly have the same cost, our framework provides a monetary general equilibrium of Burdett-Judd, but not price stickiness. There are other ways to think about refining away the indeterminacy, such as imposing a menu cost  $\varepsilon > 0$  and letting  $\varepsilon \rightarrow 0$ , which intuitively should generate a lot of stickiness. This idea seems worth pursuing, but it is not as easy as one might think.

<sup>6</sup>There are several other interesting models where, despite price stickiness, money may be (sometimes approximately) neutral. These include Caplin and Spulber (1987), Eden (1994), Golosov and Lucas (2007) and Midrigan (2010). Our approach differs in a number of respects. First, we start with a general equilibrium model where money is essential as a medium of exchange. Second, money by design is exactly neutral, although not superneutral, in our environment. Third, stickiness arises entirely endogenously and robustly – it does not depend on particular functional forms, timing, the money supply process, etc. Fourth, the distribution of prices is endogenous and derived from standard microfoundations (Burdett-Judd), instead of simply assuming, say, prices are distributed uniformly on some interval.

## 2 The Model

### 2.1 Preferences, technology and markets

Time is discrete and continues forever. In every period, two markets open sequentially. The first market is a decentralized product market in which buyers and sellers come together through a frictional matching process. In this market, barter is not feasible because buyers do not carry goods that are valued by the sellers, and credit is not feasible because buyers are anonymous. Instead, exchange takes place with fiat money, which is supplied by the government. We will refer to this decentralized and anonymous market as the *Burdett-Judd market* (BJ market). The second market is a centralized product and labor market in which buyers and sellers are recognizable. In this market, exchange may take place using either money or credit. We will refer to this market as the *Arrow-Debreu market* (AD market).

The economy is populated by a continuum of households with measure 1. Each household has preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) + v(x_t) - h_t], \quad (2.1)$$

where  $\beta \in (0, 1)$  is the discount factor,  $u$  is a strictly increasing and strictly concave function defined over the consumption of the good traded in the BJ market (the BJ good),  $v$  is a strictly increasing and strictly concave function defined over the consumption of the good traded in the AD market (the AD good), and  $-h$  is the disutility of working  $h$  hours. For the sake of concreteness, we assume  $u(q) = q^{1-\gamma}/(1-\gamma)$  with  $0 < \gamma < 1$ .

The economy is also populated by a continuum of firms with measure  $s > 0$ . Each firm operates a technology that turns  $h$  hours of labor into  $f(h)$  units of the BJ good, as well as a technology that turns  $h$  hours of labor into  $g(h)$  units of the AD good. For concreteness, we assume that  $f(h) = h/c$  and that  $g(h) = h$ , where  $c > 0$  is the cost of producing a unit of the BJ good relative to the cost of producing the AD good. Firms are owned by the households through a balanced mutual fund.

In the BJ market, each firm posts a nominal price  $p$ , taking as given the amount of money,  $mt$ , brought into the market by each household and the cumulative distribution function,

$F_t(p)$ , of nominal prices posted by all the other firms. Following Burdett and Judd (1983), we assume that each household observes the entire price distribution, but can only purchase the good from a random sample of firms. In particular, the household cannot purchase the good from any firm with probability  $\alpha_0 \in [0, 1)$ . The household can purchase the good from exactly one firm with probability  $\alpha_1 \in (0, \alpha_0)$ . And with probability  $\alpha_2 = 1 - \alpha_0 - \alpha_1$ , the household can purchase the good from two firms. As mentioned above, all transactions in the BJ market are carried out with money.

In the AD market, the government prints money and injects it into the economy through a lump-sum transfer to the households,  $T_t$ . Hence,  $T_t = (\mu - 1)M_t$ , where  $M_t$  is the quantity of money at the beginning of the period and  $\mu > 1/\beta$  is the money growth rate. For the sake of simplicity, we assume that  $\mu$  is constant over time. Each household receives the transfer from the government,  $T_t$ , and a nominal dividend payment from the mutual fund that owns the firms,  $D_t$ . Then, each household chooses how much to work,  $h_t$ , how much of the AD good to consume,  $x_t$ , and how much money to carry into the next BJ market,  $m_{t+1}$ , taking as given the wage and the price of the AD good. Similarly, each firm chooses how much labor to hire and how much of the AD good to produce, taking as given the wage and the price of the AD good. As mentioned above, exchange in the AD market may take place with either credit or money.

## 2.2 The problem of the household

First, consider a household who enters the AD market with  $m_t$  units of money. The lifetime utility of this household is given by

$$\begin{aligned} W_t(m_t) = & \max_{h_t, x_t, m_{t+1}} v(x_t) - h_t + \beta U_{t+1}(m_{t+1}), \\ \text{s.t. } & w_t x_t + m_{t+1} \leq w_t h_t + m_t + D_t + T_t, \\ & x_t \geq 0, m_{t+1} \geq 0. \end{aligned} \tag{2.2}$$

The expression above is intuitive. The household chooses how much to work,  $h_t$ , how much to consume,  $x_t$ , and how much money to hold,  $m_{t+1}$ , so as to maximize the sum of its utility in the current AD market,  $v(x_t) - h_t$ , and its lifetime utility at the beginning of the next BJ

market,  $\beta U_{t+1}(m_{t+1})$ . The household's choice of  $(h_t, x_t, m_{t+1})$  must be affordable given its non-labor income,  $m_t + T_t + D_t$ , and the wage,  $w_t$ .

The household's optimal choices  $(h_t^*, x_t^*, m_{t+1}^*)$  satisfy the conditions

$$\begin{aligned} h_t^* &= x_t^* + w_t^{-1} (m_{t+1}^* - m_t - D_t - T_t), \\ v'(x_t^*) &= 1, \quad \beta U_{t+1}'(m_{t+1}^*) = w_t^{-1}. \end{aligned} \tag{2.3}$$

These optimality conditions are easy to interpret. The optimal choice for  $m_{t+1}$  is such that the disutility from earning an additional unit of money in the AD market,  $1/w_t$ , equates the utility from carrying an additional unit of money into the next BJ market,  $\beta U_{t+1}'(m_{t+1})$ . The optimal choice for  $x_t$  is such that the disutility from working an additional hour, 1, equates the utility from consuming an additional unit of the AD good,  $v'(x_t)$ . The optimal choice for  $h_t$  is such that the household's budget constraint holds with equality.

As in Lagos and Wright (2005), the optimal choices for  $m_{t+1}$  and  $x_t$  are independent of  $m_t$ , and the optimal choice for  $h_t$  is a linearly decreasing function of  $m_t$  with slope  $-1/w_t$ . That is, the amount of money with which the household enters the AD market does not affect the household's decision of how much to consume and how much money to carry to the next BJ market. The amount of money with which the household enters the AD market only affects the household's decision of how much to work and it does so linearly. Given the properties of the optimal choices  $(h_t^*, x_t^*, m_{t+1}^*)$ , it is immediate to verify that the household's lifetime utility  $W_t$  is a linear function of  $m_t$  with slope  $1/w_t$ . Notice that the independence of  $m_{t+1}$  from  $m_t$  greatly simplifies the analysis of the equilibrium as it guarantees that the distribution of money holdings across households in the BJ market is degenerate. Also, notice that the linearity of  $W_t$  with respect to  $m_t$  simplifies the analysis of the equilibrium because it guarantees that the distribution of money holdings across households in the AD market does not affect their valuation of the dividends paid out by the firms.

Next, consider a household who enters the BJ market with  $m_t$  units of money and purchases the good at the price  $p$ . The lifetime utility of this household is given by

$$\begin{aligned} V_t(m_t, p) &= \max_{q_t} \frac{q_t^{1-\gamma}}{1-\gamma} + W_t(m_t - pq_t), \\ \text{s.t.} \quad & 0 \leq pq_t \leq m_t. \end{aligned} \tag{2.4}$$

The expression above is intuitive. The household chooses how much of the good to purchase,  $q_t$ , so as to maximize the sum of its utility in the current BJ market,  $q_t^{1-\gamma}/(1-\gamma)$ , and its lifetime utility from entering the next AD market with  $m_t - pq_t$  units of money,  $W_t(m_t - pq_t) = W_t(0) + (m_t - pq_t)/w_t$ . The choice of  $q_t$  is subject to the cash constraint  $pq_t < m_t$  because money is the only medium of exchange in the BJ market.

The household's optimal choice  $q_t^*$  is given by

$$q_t^*(p) = \begin{cases} m_t/p, & \text{if } p \leq \hat{p}_t, \\ (w_t/p)^{1/\gamma}, & \text{if } p > \hat{p}_t, \end{cases} \quad (2.5)$$

where  $\hat{p}_t = w_t(w_t/m_t)^{\frac{\gamma}{1-\gamma}}$ . If the price  $p$  is smaller than  $\hat{p}_t$ , the cash constraint is binding and the household purchases  $m_t/p$  units of the good. Otherwise, the cash constraint does not bind and the household purchases  $(w_t/p)^{1/\gamma}$  units of the good. Figure I illustrates the household's demand for the BJ good. The black curve is the household's demand given that the cash constraint is binding. The grey curve is the household's demand given that the cash constraint is lax. The household's demand,  $q_t^*$ , is the lower envelope of the black and the gray curves.

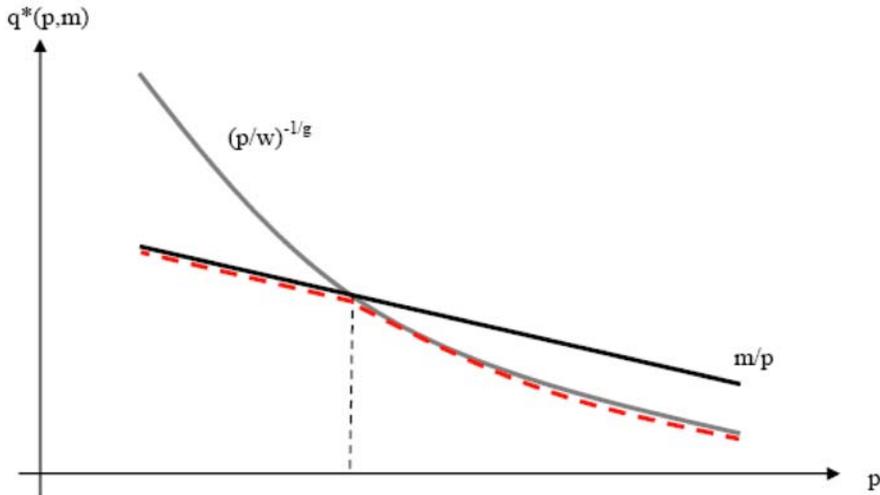


Figure I: Household's demand for the BJ good

Finally, the lifetime utility of a household who enters the BJ market with  $m_t$  units of money is given by

$$\begin{aligned}
U_t(m_t) = & \alpha_0 W_t(m_t) + \alpha_1 \int V_t(p, m_t) dF_t(p) \\
& + \alpha_2 \int V_t(p, m_t) d[1 - (1 - F_t(p))^2].
\end{aligned} \tag{2.6}$$

The expression above is easy to understand. With probability  $\alpha_0$ , the household cannot purchase the BJ good and enters the next AD market with  $m_t$  units of money. With probability  $\alpha_1$ , the household can purchase the good from exactly one firm. The price  $p$  charged by the firm is a random variable with distribution  $F_t(p)$ . With probability  $\alpha_2$ , the household can purchase the good from two firms. The lowest price  $p$  charged by the two firms is a random variable with distribution  $1 - [1 - F_t(p)]^2$ .

After substituting (2.5) into (2.6) and differentiating  $U_t$  with respect to  $m_t$ , we can rewrite the optimality condition for  $m_{t+1}$  as

$$\begin{aligned}
i = & \alpha_1 \int_{\mathbf{1}_{[p \leq \hat{p}_t]}} \left[ \frac{w_t}{p} \left( \frac{m_t}{p} \right)^{-\gamma} - 1 \right] dF_t(p) \\
& + 2\alpha_2 \int_{\mathbf{1}_{[p \leq \hat{p}_t]}} \left[ \frac{w_t}{p} \left( \frac{m_t}{p} \right)^{-\gamma} - 1 \right] [1 - F_t(p)] dF_t(p),
\end{aligned} \tag{2.7}$$

where  $i$  is the net nominal interest rate  $\beta^{-1}(w_{t+1}/w_t) - 1$ . The expression above is intuitive. The left-hand side of (2.7) is the marginal cost of carrying money, i.e. the net nominal interest rate. The right-hand side of (2.7) is the marginal benefit of carrying money, i.e. the value of relaxing the cash constraint in the BJ market. The optimal choice for money holdings,  $m_{t+1}^*$ , equates the marginal cost and the marginal benefit of carrying money.

### 2.3 The problem of the firm

If a firm posts the price  $p$  in the BJ market, its profits are given by

$$\Pi_t(p) = \frac{1}{s} [\alpha_1 + 2\alpha_2(1 - F_t(p)) + \alpha_2 n_t(p)] R_t(p), \tag{2.8}$$

where  $n_t(p)$  and  $R_t(p)$  are defined as

$$\begin{aligned} n_t(p) &= \lim_{\epsilon \rightarrow 0^+} F_t(p) - F_t(p - \epsilon), \\ R_t(p) &= q_t^*(p)(p - cw_t). \end{aligned}$$

The first term on the right-hand side of (2.8) is the number of customers served by the firm. First, there are  $\alpha_1/s$  households who purchase the good from the firm because they are not in contact with any other seller. Second, there are  $2\alpha_2(1 - F_t(p))/s$  households who purchase the good from the firm because the other seller with whom they are in contact charges a price greater than  $p$ . Finally, there are  $2\alpha_2 n_t(p)/s$  households who are in contact with the firm and with another seller that also charges the price  $p$ . Each of these households purchases the good from the firm with probability  $1/2$ .

The second term on the right-hand side of (2.8) is the firm's profit per customer. The firm sells  $q^*(p)$  units to each customer and each unit is sold at the price  $p$  and produced at the cost  $cw_t$ . In Figures II and III, we plot the firm's profits per customer as a function of the price  $p$ . The black curve is the plot of  $(m_t/p)(p - cw_t)$ , which is the profit per customer given that the customer is cash constrained. The gray curve is the plot of  $(w_t/p)^{1/\gamma}(p - cw_t)$ , which is the profit per customer given that the customer is not cash constrained. The firm's actual profit per customer is given by the lower envelope of the black and grey curves. Figure II illustrates a case in which the customer's money holdings are relatively high. In this case, the price that maximizes the firm's profit per customer is  $cw_t/(1 - \gamma)$ . Figure III illustrates the case in which the customer's money holdings are relatively low. In this case, the price that maximizes the firm's profit per customer is  $\hat{p}_t$ . Overall, the price that maximizes the firm's profit per customer is  $p_t^m = \max\{cw_t/(1 - \gamma), \hat{p}_t\}$ . We shall refer to  $p_t^m$  as the *monopoly price*.

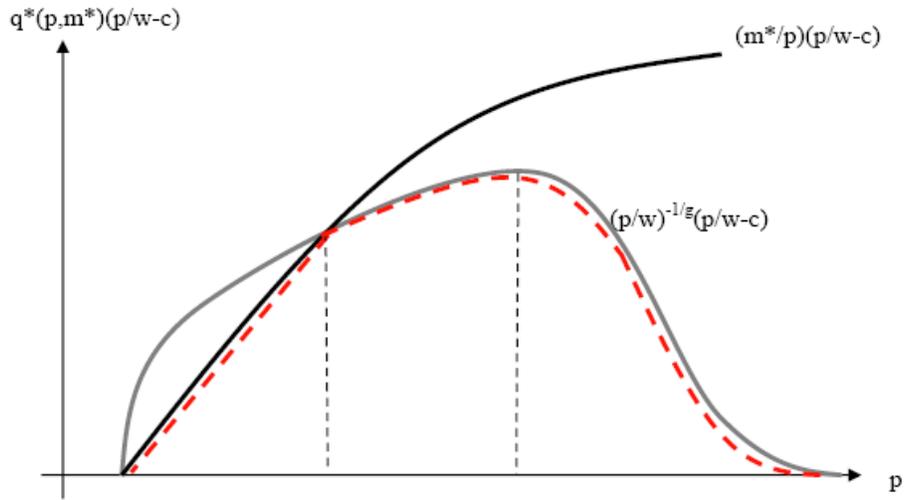


Figure II: Firm's profit per customer in the BJ market

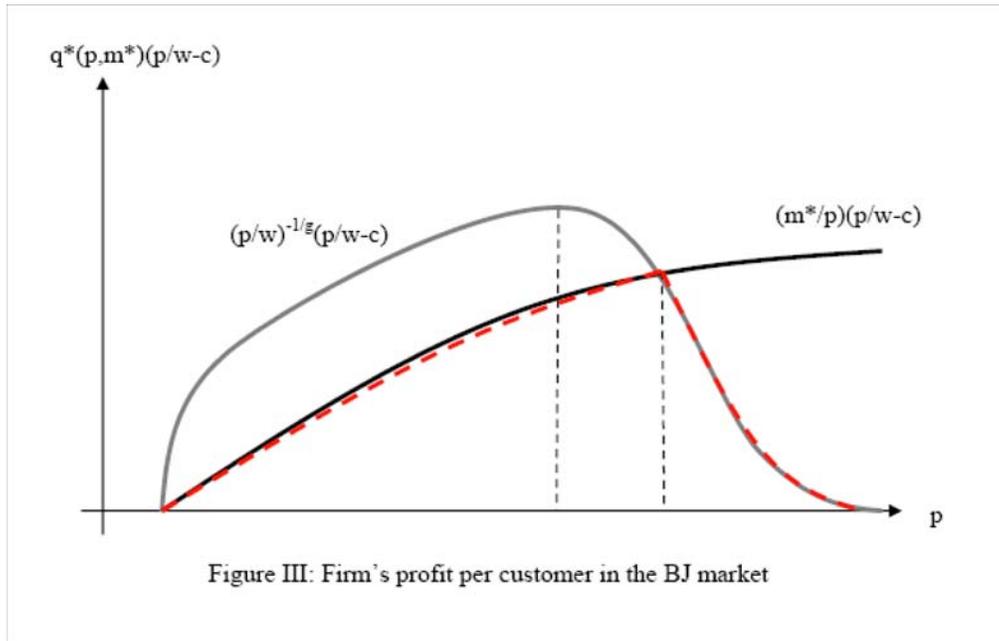


Figure III: Firm's profit per customer in the BJ market

Each firm chooses the price  $p$  so as to maximize the profit function  $\Pi_t(p)$ . Therefore, the price distribution  $F_t$  is consistent with the firms' pricing strategy only if the profit function

$\Pi_t(p)$  is maximized by every price on the support of  $F_t$ , i.e.

$$\begin{aligned}\Pi_t(p_0) &= \Pi_t^* \text{ for all } p \in \text{supp}F, \\ \Pi_t^* &\equiv \max_p \Pi_t(p).\end{aligned}\tag{2.9}$$

The following lemma makes use of condition (2.9) in order to characterize the price distribution  $F_t$ . The proof of the lemma adapts the arguments developed by Burdett and Judd (1983) to characterize the price distribution in a market for an indivisible good where buyers have deep-pockets to a market for a divisible good where buyers are cash-constrained.

**Lemma 1** (Burdett and Judd, 1983): *The unique price distribution that is consistent with the firms' pricing strategy is*

$$F_t(p) = 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{R_t(p_t^m)}{R_t(p)} - 1 \right].\tag{2.10}$$

The price distribution  $F_t$  is continuous and its support is the connected interval  $[\underline{p}_t, \bar{p}_t]$ , where

$$\begin{aligned}R_t(\underline{p}_t) &= \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R_t(p_t^m), \\ \bar{p}_t &= p_t^m.\end{aligned}\tag{2.11}$$

**Proof:** In Appendix A.

Lemma 1 states that the price distribution is continuous. Intuitively, if the price distribution had a mass point at  $p_0$ , a firm that posts  $p_0$  could increase its profits by charging  $p_0 - \epsilon$  instead, as the deviation would leave the firm's profit per customer approximately constant but it would increase the firm's customer base by a discrete amount. Second, lemma 1 states that the support of the price distribution is connected. Intuitively, if the support of the price distribution had a gap between  $p_0$  and  $p_1$ , a firm that posts  $p_0$  could increase its profits by charging  $p_1$  instead, as the deviation would not affect the firm's customer base and it would increase the firm's profits per customer. Third, lemma 1 states that the highest price on the support of the price distribution,  $\bar{p}_t$ , is equal to the monopoly price,  $p_t^m$ . Intuitively, if  $\bar{p}_t$  were smaller than  $p_t^m$ , a firm that posts  $\bar{p}_t$  could increase its profits by charging  $p_t^m$  instead, as the deviation would not affect the firm's customer base and it would increase the firm's profits per customer. If, on the other hand,  $\bar{p}_t$  were greater than  $p_t^m$ , a firm that posts  $\bar{p}_t$

could increase its profits by charging  $p_t^m$ , as the deviation would increase both the firm's customer base and the profit per customer.

Given the above properties of the price distribution and condition (2.9), we can derive the expression (2.10) for  $F_t$  and the expression (2.11) for  $\underline{p}_t$ . Since the price distribution has no mass points, the profit of a firm that charges the price  $p$  is given by

$$\Pi_t(p) = \frac{1}{s} [\alpha_1 + 2\alpha_2(1 - F_t(p))] R_t(p). \quad (2.12)$$

Since the monopoly price is on the support of the price distribution, the maximized profit of the firm is given by

$$\Pi_t^* = \frac{\alpha_1}{s} R_t(p_t^m) \quad (2.13)$$

Finally, since every price on the support of the price distribution must maximize the profit of the firm, it follows that for every  $p \in [\underline{p}_t, \bar{p}_t]$

$$\frac{1}{s} [\alpha_1 + 2\alpha_2(1 - F_t(p))] R_t(p) = \alpha_1 R_t(p_t^m). \quad (2.14)$$

Solving (2.14) with respect to  $F_t$  leads to equation (2.10). In turn, setting  $F_t(\underline{p}_t) = 0$  and solving for  $\underline{p}_t$  leads to equation (2.11).

## 2.4 Equilibrium

We are now in the position to define an equilibrium.

**Definition 2** (Stationary Monetary Equilibrium): *A stationary monetary equilibrium  $\Sigma^*$  is a sequence of quantities  $\{m_t^*, x_t^*, h_t^*, q_t^*\}_{t=0}^\infty$  and prices  $\{w_t^*, F_t^*\}$  that satisfy the following conditions:*

1.  $m_{t+1}^*, x_t^*$  and  $h_t^*$  solve the household's problem in the AD market:

$$i = \int_{\mathbf{1}_{[p \leq \hat{p}_t]}} \left[ \frac{w_t^*}{p} \left( \frac{m_t^*}{p} \right)^{-\gamma} - 1 \right] [\alpha_1 + 2\alpha_2(1 - F_t^*(p))] dF_t^*(p),$$

$$v'(x_t^*) = 1, \quad h_t^*(m) = x_t^* + w_t^{*-1} (m_{t+1}^* - m - D_t - T_t);$$

2.  $q_t^*$  solves the household's problem in the BJ market:

$$q_t^*(p) = \begin{cases} m_t^*/p, & \text{if } p \leq \hat{p}_t, \\ (w_t^*/p)^{\frac{1}{\gamma}}, & \text{if } p > \hat{p}_t, \end{cases}$$

where  $\hat{p}_t = w_t^*(w_t^*/m_t^*)^{\frac{\gamma}{1-\gamma}}$ ;

3.  $F_t^*$  is consistent with the solution to the firm's problem in the BJ market:

$$F_t^*(p) = 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{R_t(p_t^m)}{R_t(p)} - 1 \right] \quad \text{all } p \in [\underline{p}_t, \bar{p}_t],$$

where  $R_t$  is given by (2.8) and  $\underline{p}_t$  and  $\bar{p}_t$  are given by (2.11);

4.  $w_t^*$  is such that the household's money demand equates the government's money supply:

$$m_t^* = M_t;$$

5. nominal variables grow at the rate  $\mu$  and real variables are constant:

$$\begin{aligned} m_{t+1}^* &= \mu m_t^*, & w_{t+1} &= \mu w_t, & F_{t+1}^*(\mu p) &= F_t^*(p), \\ x_{t+1}^* &= x_t^*, & h_{t+1}^* &= h_t^*, & q_{t+1}^*(\mu p) &= q_t^*(p). \end{aligned}$$

To establish the existence of a stationary monetary equilibrium, we proceed in two steps. In the first step, we prove that the prices posted by the firms in the BJ market are decreasing (in the first order stochastic dominance sense) with respect to the amount of money that the households are expected to carry into the BJ market. The intuition for this result is simple. If the households carry more money, the cash constraint is relaxed, and the quantity of the good demanded by a customer at a low-price firm increases relative to the quantity demanded at a high-price firm. For this reason, the profits that a low-price firm makes on each customer increase relative to the profits made by a high-price firm. In order to keep the firms indifferent between posting low and high prices, the distribution of prices must fall so that the number of customers served by a low-price firm declines relative to the number of customers served by a high-price firm.

In the second step, we prove that the amount of money carried by the households in the BJ market is decreasing with respect to the prices posted by the firms in the BJ market.

The intuition for this result is simple. If the prices posted by the firms are higher (in the sense of first order stochastic dominance), the household has a lower probability of meeting a low-price seller and, consequently, a lower probability of being cash constrained. Therefore, if the prices posted by the firms are higher, the value to the household from carrying an additional unit of money in the BJ market falls.

From the above observations, it follows that the amount of money that the households choose to carry into the BJ market is a monotonic function of the amount of money that the firms expect the households to hold. Moreover, we can prove that the amount of money that the households choose to carry into the BJ market is bounded. Hence, from Tarski's fixed point theorem, there exists an  $m_t^*$  such that: (i)  $m_t^*$  is the amount of money that solves the households' problem given that the price distribution is  $F_t^*$ , and (ii)  $F_t^*$  is the price distribution given that the firms expect the households to have an amount of money  $m_t^*$ . That is, from Tarski's fixed point theorem, it follows that there exist  $m_t^*$  and  $F_t^*$  that satisfy the equilibrium conditions (1) and (3). Given  $m_t^*$  and  $F_t^*$ , we can then find the  $h_t^*$ ,  $x_t^*$ ,  $q_t^*$  and  $w_t^*$  that satisfy the remaining equilibrium conditions.

**Theorem 3** (Existence): *For any  $\mu > \beta^{-1}$ , a stationary monetary equilibrium exists.*

**Proof:** In Appendix B.

## 3 Price stickiness and monetary neutrality

### 3.1 Price stickiness

The equilibrium uniquely pins down the aggregate price distribution in the BJ market, but it does not pin down the price of an individual firm. In fact, an individual firm is indifferent between posting any price from the support of the aggregate price distribution, as any one of these prices is profit maximizing. Figure IV illustrates the implications of this property of equilibrium for the dynamics of the aggregate price distribution and for the dynamics of the price of individual firms in the case of an inflationary economy, i.e.  $\mu > 1$ . The black line is the aggregate price distribution in period  $t$ ,  $F_t^*$ , and the red line is the aggregate price distribution in period  $t+1$ ,  $F_{t+1}^*$ . All the firms in the vertically shaded area must change their

price between the two periods as their period- $t$  price does not maximize profits in period  $t+1$ . However, each of the firms in the horizontally shaded area is indifferent between keeping its price constant, posting a new price from the interval  $[\underline{p}_{t+1}, \bar{p}_{t+1}]$ , or randomizing between keeping its price and changing it. The only equilibrium restriction on the price dynamics of individual firms between period  $t$  and  $t+1$  is that the aggregate price distribution in period  $t+1$  has to be  $F_{t+1}^*$ . We formalize this restriction in the following definition.

**Definition 4** (Supportable policies): *The stationary monetary equilibrium  $\Sigma^*$  supports the price policy  $p_{t+1}^*(p)$  if, given that the price distribution in period  $t$  is  $F_t^*(p)$  and all firms follow the policy  $p_{t+1}^*(p)$ , the price distribution in period  $t+1$  is  $F_{t+1}^*(p)$ .*

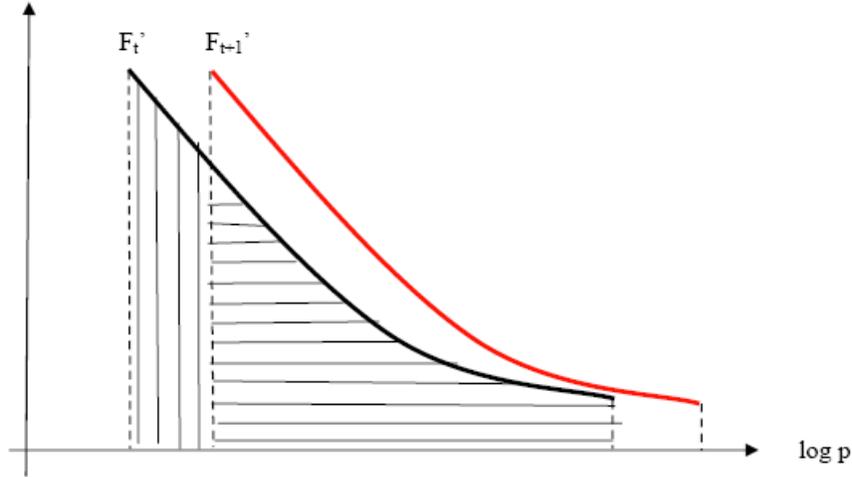


Figure IV: Equilibrium price distribution

In the remainder of the paper, we will restrict attention to the case of an inflationary economy and to pricing policies of the form

$$\begin{aligned}
 p_{t+1}^*(p_t) &= \begin{cases} p_t & \text{w.p. } \rho \\ p' & \text{w.p. } 1 - \rho \end{cases}, & \text{if } p_t \geq \mu \underline{p}_t = \underline{p}_{t+1}, \\
 p_{t+1}^*(p_t) &= p', & \text{if } p_t < \mu \underline{p}_t = \underline{p}_{t+1},
 \end{aligned} \tag{3.1}$$

where  $\rho$  is a parameter between 0 and 1 and  $p'$  is a price randomly drawn from the cumulative

distribution function

$$G_{t+1}^*(p) = \begin{cases} \frac{F_t^*(p/\mu)}{F_t^*(\mu\underline{p}_t) + (1 - \rho) [1 - F_t^*(\mu\underline{p}_t)]}, & \text{if } p < \mu\underline{p}_t, \\ \frac{F_t^*(p/\mu) - \rho F_t^*(p, \mu)}{F_t^*(\mu\underline{p}_t) + (1 - \rho) [1 - F_t^*(\mu\underline{p}_t)]}, & \text{if } p \geq \mu\underline{p}_t. \end{cases} \quad (3.2)$$

According to (3.1), the firm posts a new price in period  $t + 1$  if the price it posted in the period  $t$  is outside of the support of the equilibrium price distribution for period  $t + 1$ . Otherwise, the firm posts the same price in period  $t + 1$  as in period  $t$  with probability  $\rho$  and posts a different price with probability  $1 - \rho$ . Whenever the firm changes price, it draws the new price from the distribution function  $G_{t+1}^*(p)$  defined in (3.2). To see that  $G_{t+1}^*(p)$  is a legitimate cumulative distribution function, notice that  $G_{t+1}^*(p) = 0$  for all  $p \leq \mu\underline{p}_t$ ,  $G_{t+1}^*(p) = 1$  for all  $p \geq \mu\bar{p}_t$ , and  $G_{t+1}^*(p) > 0$  for all  $p$  in the interval between  $\mu\underline{p}_t$  and  $\mu\bar{p}_t$ .

Given that the price distribution in period  $t$  is  $F_t^*$  and that all firms follow the pricing policy  $p_{t+1}^*$ , the price distribution in period  $t + 1$  is given by

$$F_{t+1}(p) = \begin{cases} \left\{ F_t^*(\mu\underline{p}_t) + (1 - \rho) [1 - F_t^*(\mu\underline{p}_t)] \right\} G_{t+1}^*(p), & \text{if } p < \mu\underline{p}_t, \\ \rho F_t^*(p) + \left\{ F_t^*(\mu\underline{p}_t) + (1 - \rho) [1 - F_t^*(\mu\underline{p}_t)] \right\} G_{t+1}^*(p), & \text{if } p \geq \mu\underline{p}_t. \end{cases} \quad (3.3)$$

Using (3.2) to substitute out  $G_{t+1}^*(p)$  in (3.3), we obtain  $F_{t+1}(p) = F_t^*(p/\mu)$  which, in turn, is equal to  $F_{t+1}^*(p)$ . Hence, we have established the following result.

**Proposition 5** (Supported policies): *The pricing policy (3.1) is supported by the stationary monetary equilibrium  $\Sigma^*$  for all  $\rho \in [0, 1]$ .*

The class of pricing policies described by (3.1) is not exhaustive, but it is sufficient to capture a wide range of behaviors in a parsimonious way. For  $\rho = 1$ , the pricing policy (3.1) describes the extreme case in which a firm only changes its price when it is no longer profit maximizing. Clearly, for  $\rho = 1$ , the pricing policy (3.1) attains the smallest fraction of price changes and the highest average price duration that are consistent with equilibrium. For  $\rho = 0$ , the pricing policy (3.1) describes the opposite extreme case in which a firm changes its price in every period. Clearly, for  $\rho = 0$ , the pricing policy (3.1) attains the largest fraction

of price changes and the lowest average price duration that are consistent with equilibrium. For  $\rho$  between 0 and 1, the pricing policy (3.1) describes the intermediate cases in which the firm changes a profit maximizing price with probability  $\rho$ . As the parameter  $\rho$  increases from 0 to 1, the frequency of price changes and the average duration of a price move from one extreme to the other.

For a given  $\rho$ , let us compute the average duration of a price. The cumulative distribution of new prices in period  $t$  is  $G_t^*(p)$ . Let  $N$  denote the largest integer such that  $\mu^N \underline{p}_t \leq \bar{p}_t$ . For  $n = 1, 2, \dots, N$ , a fraction  $G_t^*(\mu^n \underline{p}_t) - G_t^*(\mu^{n-1} \underline{p}_t)$  of new prices lies in the interval  $[\mu^{n-1} \underline{p}_t, \mu^n \underline{p}_t]$ . Also, a fraction  $1 - G_t^*(\mu^N \underline{p}_t)$  of new prices lies in the interval  $[\mu^N \underline{p}_t, \bar{p}_t]$ . Each price in the interval  $[\mu^{n-1} \underline{p}_t, \mu^n \bar{p}_t]$  will be changed in period  $t + i$  (and not before) with probability  $\rho^{i-1}(1 - \rho)$ ,  $i = 1, 2, \dots, n - 1$ , and will be changed in period  $t + n$  with probability  $\rho^{n-1}$ . Therefore, the average duration of prices in the interval  $[\mu^{n-1} \underline{p}_t, \mu^n \bar{p}_t]$  is equal to  $(1 - \rho) + 2\rho(1 - \rho) + \dots + n\rho^{n-1} = (1 - \rho^n)/(1 - \rho)$  periods. Each price in the interval  $[\mu^N \underline{p}_t, \bar{p}_t]$  will be changed in period  $t + i$  with probability  $\rho^{i-1}(1 - \rho)$ ,  $i = 1, 2, \dots, N$ , and will be changed in period  $t + N + 1$  with probability  $\rho^N$ . Therefore, the average duration of prices in the interval  $[\mu^N \underline{p}_t, \bar{p}_t]$  is equal to  $(1 - \rho^{N+1})/(1 - \rho)$  periods. Overall, the average duration of a new price is

$$A(\rho) = \left\{ \sum_{n=1}^N \left[ G_t^*(\mu^n \underline{p}_t) - G_t^*(\mu^{n-1} \underline{p}_t) \right] \frac{1 - \rho^n}{1 - \rho} \right\} + \left[ 1 - G_t^*(\mu^N \underline{p}_t) \right] \frac{1 - \rho^{N+1}}{1 - \rho} \quad (3.4)$$

Notice that, since the ratio  $(1 - \rho^n)/(1 - \rho)$  is increasing in  $\rho$  and  $n$  and the distribution function  $G_t^*$  is increasing in  $\rho$  (in the first order stochastic dominance sense),  $A(\rho)$  is an increasing function of  $\rho$ .

Second, we compute the fraction of prices that change between period  $t$  and  $t + 1$ . The cumulative distribution of prices in period  $t$  is  $F_t^*(p)$ . A fraction  $F_t^*(\mu \underline{p}_t)$  of prices lies in the interval  $[\underline{p}_t, \mu \underline{p}_t]$ , and each of these prices will be changed between period  $t$  and  $t + 1$  with probability 1. A fraction  $1 - F_t^*(\mu \underline{p}_t)$  of prices lies in the interval  $[\mu \underline{p}_t, \bar{p}_t]$ , and each of these prices will be changed between period  $t$  and  $t + 1$  with probability  $1 - \rho$ . Overall, the

fraction of prices that change between  $t$  and  $t + 1$  is given by

$$FR(\rho) = F_t^*(\mu \underline{p}_t) + (1 - \rho) \left[ 1 - F_t^*(\mu \underline{p}_t) \right]. \quad (3.5)$$

Clearly,  $FR(\rho)$  is a decreasing function of  $\rho$ .

Third, we compute the distribution of the magnitude of price changes. The density of firms that post the price  $p$  in period  $t$  and a different price in period  $t + 1$  is given by  $F_t'^*(p)/FR(\rho)$  if  $p$  is smaller than  $\mu \underline{p}_t$ , and by  $(1 - \rho)F_t'^*(p)/FR(\rho)$  if  $p$  is greater than  $\mu \underline{p}_t$ . Among the firms that post the price  $p$  in period  $t$  and a different price in period  $t + 1$ , a fraction  $G_{t+1}^*(p(1 + x))$  increases its price by  $x$  percent or less. Therefore, the cumulative distribution function for the magnitude of price changes is given by

$$H_t(x, \rho) = \frac{1}{FR(\rho)} \int G_{t+1}^*(\rho(1 + x)) \left( 1 - \rho \mathbf{1}[p \geq \mu \underline{p}_t] \right) dF_t'(p). \quad (3.6)$$

From (3.2) and (3.6), it is immediate to verify that the fraction of negative price changes,  $H_t(0, \rho)$ , is strictly positive for all  $\rho < 1$ .

The following theorem contains the main theoretical results of the paper.

**Theorem 6** (Sticky prices): *The stationary monetary equilibrium  $\Sigma^*$ , together with the pricing policy  $p_{t+1}^*$ , is such that the average price duration is  $A(\rho)$  and the frequency of a price change is  $FR(\rho)$ , where  $A(\rho)$  is increasing and  $FR(\rho)$  is decreasing in  $\rho$ .*

(i) *There exists a  $\mu^* > 1$  such that, for all  $\mu \in (1, \mu^*)$  and all  $\rho \in (0, 1]$ , the average price duration  $A(\rho)$  is strictly greater than 1, and the frequency of a price change  $FR(\rho)$  is strictly smaller than 1.*

(ii) *For all  $\mu \in (1, \mu^*)$  and all  $\rho \in [0, 1)$ , the fraction of negative price changes,  $H(0, \rho)$ , is strictly greater than 0.*

**Proof:** In Appendix C.

Part (i) of the theorem shows that, unless the growth rate of money is too high, our model is consistent with the observation that some firms leave their price unchanged for weeks or months in the face of a continuously changing aggregate price level. Our model delivers this result not because there are technological restriction to price adjustment, but

because, due to search frictions, there is an entire interval of prices for which the profits of the firm are maximized. Part (ii) of the theorem shows that our model is consistent with the observation that some firms change their price down in the face of a continuously increasing aggregate price level. Our model delivers this result because of search frictions, not because of idiosyncratic shocks to the firm's cost of production. More broadly, the theorem shows that one should be cautious in inferring the existence of menu costs or Calvo fairies from the observed stickiness of the price of individual sellers. Similarly, the theorem shows that negative price changes in an inflationary economy are not necessarily caused by idiosyncratic productivity shocks.

### **3.2 Monetary neutrality**

In our model, some firms may post the same nominal price for many periods in the face of a continuously increasing aggregate price level. However, in our model, the government cannot increase short-run production or consumption through an unexpected increase in the monetary base. For example, consider what would happen in the government were to unexpectedly double the stock of money at the opening of the AD market. In response to this policy, the amount of money that the households carry into the BJ market would double, but so would the distribution of prices that the firms post in the BJ market. Hence, the quantity of the BJ good traded and produced would remain unchanged. Similarly, in response to this policy, the amount of money that the households spend in the AD market would double, but so would the prices charged by the firms in the AD market. Hence, the quantity of the AD good traded and produced would remain unchanged. Overall, the expansionary monetary policy is completely neutral. Intuitively, money is neutral because, while the price posted by some sellers is rigid, the distribution of prices is perfectly flexible.

## **4 Quantitative evaluation**

In the previous section, we developed a theory of price rigidity that does not rely on the existence of technological frictions to price adjustment (e.g. menu costs or Calvo fairies), but on the existence of search frictions in the product market. Unlike theories of price rigidity

based on technological constraints to price adjustment, our theory implies that money is neutral because the equilibrium price distribution responds fully and instantaneously to increases in the stock of money. In this section, we want to find out whether our theory can account for the empirical facts about prices that have been documented by Klenow and Kryvstov (2008).

## 4.1 Data and calibration

The household's preferences are described by the utility function for the BJ good,  $u(q) = q^{1-\gamma}/(1-\gamma)$ , the utility function for the AD good,  $v(x)$ , and the discount factor,  $\beta$ . The firm's technology is described by the production function for the BJ good,  $f(h) = h/c$ , and the production function for the AD good,  $g(h) = h$ . We restrict attention to the case in which the BJ and AD goods are produced using the same technology, i.e.  $c = 1$ . The firm's pricing behavior is described by the policy function  $p_{t+1}^*(p, \rho)$ . The search frictions in the BJ market are described by the probability distribution  $\{\alpha_i\}_{i=0}^2$ , where  $\alpha_i$  is the probability that a household meets  $i$  firms. We restrict attention to the family of probability distributions  $\{\alpha_i\}_{i=0}^2$  that would obtain if each household could search the BJ market twice and each search would lead to meeting a firm with probability  $\lambda$ . That is, we restrict attention to the family of probability distribution such that  $\alpha_0 = (1-\lambda)^2$ ,  $\alpha_1 = 2(1-\lambda)\lambda$  and  $\alpha_2 = \lambda^2$ . Finally, the monetary policy is described by the growth rate of money,  $\mu$ . Overall, the model is fully characterized by the parameters  $\{\gamma, \beta, \rho, \lambda, \mu\}$  and the utility function  $v(x)$ .

We calibrate the model to the US economy over the period 1988-2004, and we interpret the BJ market as the retail sector and the AD market as the intermediate goods sector. We choose the model period to be a month. We choose the parameter  $\beta$  so that the annual real interest rate in the model,  $\beta^{-12}$ , equals the average real interest rate in the data, 1.035. We choose the parameter  $\mu$  so that the annual inflation rate in the model,  $\mu^{12}$ , equals the average inflation rate in the data, 1.03. We choose the parameters  $\gamma$  and  $\rho$  so as to minimize the distance between the model-generated distribution of price changes in the BJ market,  $H_t(x, \rho)$ , and the empirical distribution of price changes observed in the retail sector (as measured by Klenow and Kryvstov, 2008). Finally, we choose the parameter  $\lambda$  so that

the average mark-up in the BJ market is 30 percent, which is a common estimate of the average mark-up in the retail sector (see Faig and Jerez, 2005). After having calibrated the parameters  $\{\gamma, \beta, \rho, \lambda, \mu\}$ , the predictions of the model regarding the behavior of prices in the BJ market are uniquely pinned down. Since we are interested in comparing these predictions of the model with the empirical behavior of prices in the retail sector, we do not need to calibrate the utility function for the AD good,  $v(x)$ .

There is a simple intuition behind our calibration strategy for  $\gamma$  and  $\rho$ . The parameter  $\gamma$  determines the elasticity of the firm's profit per customer,  $R_t(p)$ , with respect to the firm's price,  $p$ . Hence, the parameter  $\gamma$  affects the equilibrium price distribution,  $F_t^*(p)$ , and the equilibrium price change distribution,  $H_t(x, \rho)$ . Similarly, the parameter  $\rho$  determines the probability that a firm does not adjust its price when the firm is indifferent between adjusting and not adjusting. Hence, the parameter  $\rho$  affects the distribution of prices among firms that do not change their price and, consequently, the distribution of prices among firms that do change their price,  $G_t^*(p)$ , and the price change distribution,  $H_t(x, \rho)$ . Our calibration strategy for  $\lambda$  is also easy to understand. In fact, the equilibrium price distribution,  $F_t^*(p)$ , is decreasing (in the first-order stochastic dominance sense) with respect to the meeting probability  $\lambda$ .

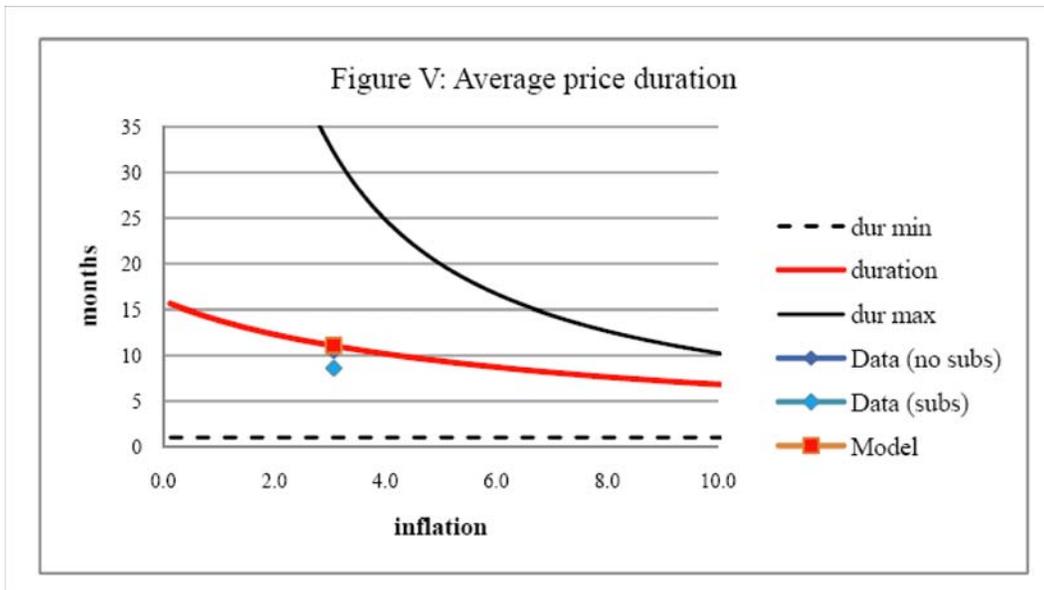
## 4.2 The price of individual sellers

Using the calibrated model, we derive the predictions of the model regarding the behavior of prices and compare these predictions with the empirical findings reported in Klenow and Kryvstov (2008). The bottom line is that our search theory of price rigidity can account quite well for the empirical behavior of prices.

According to the data analyzed by Klenow and Kryvstov (2008), the average duration of a price in the retail sector is between 6.8 and 10.4 months, depending on whether temporary sales and product substitutions are interpreted as price changes or not. In particular, if temporary sales and product substitutions are both interpreted as price changes, the average duration of a price is 6.8 months. If product substitutions are considered as price changes but temporary sales are not, the average duration of a price is 8.6 months. If neither product

substitutions nor temporary sales are considered price changes, the average duration of a price increases to 10.4 months. Following Klenow and Kryvstof (2008), we will take the second case as our benchmark.

The average duration of a price predicted by the model is close to its empirical counterpart. In particular, given an average inflation rate of 3 percent and a calibrated value of  $\rho$  of 0.93, the model predicts that the average duration of a price is 11.6 months. Notice that, for higher values of  $\rho$ , the average price duration predicted by the model would be higher, up to a maximum of xx months. Conversely, for lower values of  $\rho$ , the average price duration implied by the model would be lower, down to a minimum of 1 month. The model would generate the same average price duration as in the data for  $\rho = 0.91$ , a value very close to the one that is obtained with our calibration. Also, notice that the average price duration is decreasing with respect to inflation, as higher inflation increases the fraction of prices that exit the support of the equilibrium price distribution in each period. These properties of the average price duration are illustrated in Figure V.



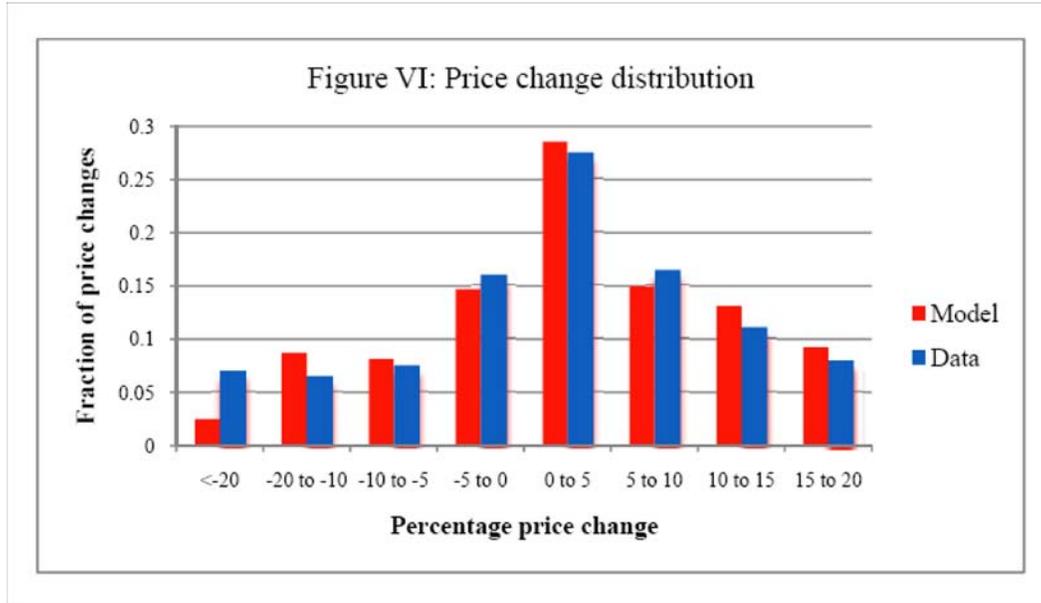
The blue histogram in figure VI is the empirical price change distribution estimated by Klenow and Ksrystof (2008). Three features of the price change distribution are worth

stressing. First, on average, price changes are large. More specifically, the average of the absolute value of price changes is 11 percent. Second, many price changes are small. More specifically, 44 percent of all price changes are smaller than 5 percent in absolute value. Third, many price changes are negative. More specifically, 35 percent of all price changes are negative. Klenow and Krystof (2008), Golosov and Lucas (2007), and Midrigan (2006) interpret the existence of many small and negative price changes as evidence of large and frequent shocks to individual seller's idiosyncratic productivity.

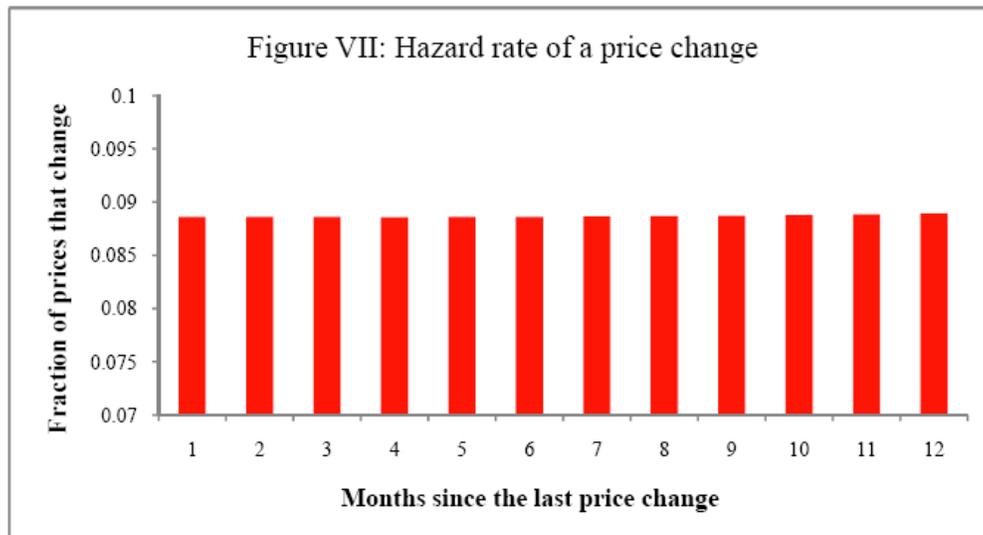
The red histogram in figure VI is the price change distribution predicted by the model. One can immediately see that the model-generated price change distribution is very close to its empirical counterpart and shares the same features. More specifically, for the model-generated price change distribution, the average of the absolute value of price changes is 9 percent, the fraction of price changes between -5 and +5 percent is 43 percent, and the fraction of negative price changes is 35 percent. Interestingly, our model generates a good fit of the empirical price change distribution without seller-specific productivity shocks. According to our model, price changes are large because search frictions create a lot of dispersion in the equilibrium price distribution. For example, the price posted by a firm at the 90th percentile of the equilibrium price distribution is xx times larger than the price posted by a firm at the 10th percentile. Hence, the firms whose price exits the support of the distribution make, on average, a large price adjustment. According to our model, many price changes are small because there are many firms that change their price before it exits the support of the distribution. For the same reason, many price changes are negative.

Klenow and Krystof (2008) estimate the relationship between the probability that a firm adjusts its price for a given item (i.e. the price-change hazard) and the time since the previous price adjustment (i.e. the age of the price). Moreover, they estimate the relationship between the absolute value of the size of a price adjustment (i.e. the price-change size) and the time since the previous price adjustment. After controlling for unobserved heterogeneity across different items, they find that the price-change hazard remains approximately constant during the first 11 months and increases significantly during the 12th month. Similarly, after controlling for unobserved heterogeneity, they find that the price-change size is approximately

independent of the age of the price.



The red histogram in figure VII is the price-change hazard predicted by our model. Notice that, just like in the data, the price-change hazard is approximately constant for the first 11 months of the life of a price. However, unlike in the data, the price-change hazard does not increase on the 12th month of the life of a price. These findings are easy to explain. In our model, the equilibrium price distribution has a wide support. Therefore, during the 12 months after a price change, only few firms need to readjust their price because it is no longer profit maximizing. Instead, during the 12 months after a price change, the majority of firms change their price with probability  $\rho$ , a probability that is independent of the age of a price. The model does not predict a spike in the price-change hazard after 12 months because, unlike in the real world, firms in our model have no seasonal incentives to adjust their price. The red line in figure 2 is the price-change size predicted by the model. For the same reasons that we mentioned above, the price-change size predicted by the model is approximately independent of the age of a price.



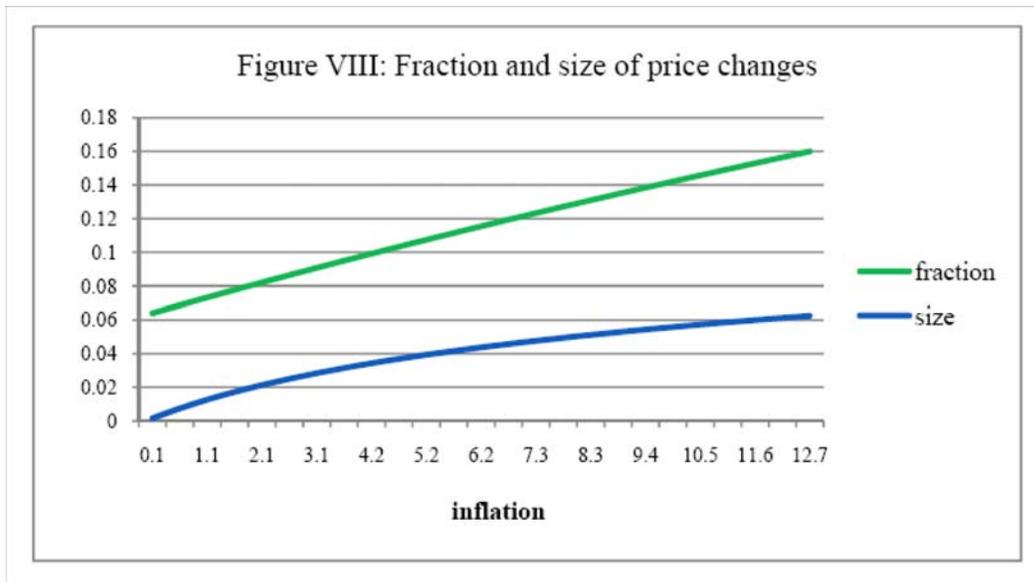
### 4.3 The effect of inflation

Using time-variation in the US inflation rate over the period 1988-2005, Klenow and Kryvstov (2008) measure the effect of inflation on the frequency of price adjustments (i.e. the extensive margin of price adjustments) and on the magnitude of price adjustments (i.e. the intensive margin of price adjustments). They accomplish this task by estimating the coefficient on inflation in the regression of the frequency of price adjustments and in the regression of the magnitude of price adjustments. Their main finding is that inflation has a positive effect on both the frequency and the magnitude of price adjustments. More specifically, they find that a 1 percentage point increase in inflation increases the frequency of price adjustments by 2.38 percentage points and the magnitude of price adjustments by 3.55 percentage points.

Figure VIII illustrates the predictions of our model regarding the effect of inflation on the frequency and magnitude of price adjustments. Notice that, according to our model, an increase in inflation increases both the frequency and the magnitude of price adjustments. These findings are easy to explain. First, an increase in inflation leads to a decline in the real balances carried by the households in the BJ market and, in turn, the decline in real balances leads to a compression of the support of the equilibrium price distribution. Second, given the

length of the support of the equilibrium price distribution, an increase in inflation reduces the time it takes for a price to exit the support. For both reasons, an increase in inflation increases the fraction of prices that are adjusted in every month. For similar reasons, an increase in inflation leads to an increase in the average price adjustment.

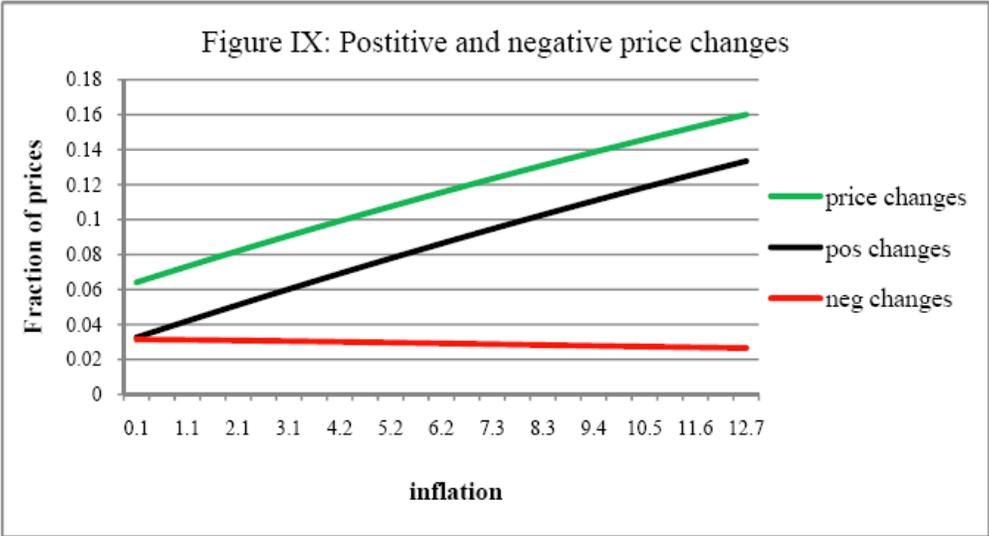
Notice that the effect of inflation on the extensive and intensive margins of price adjustment is the same in the model as in the data. However, note that the magnitude of the effect of inflation on these two margins is different than in the data. In particular, an increase in annual inflation from 3 to 4 percent increases the frequency of price adjustment by approximately 9 percentage points, and it increases the magnitude of price adjustments by approximately 5 percentage points. This discrepancy between the predictions of the model and the results of the regression analysis should not be too surprising nor too much of a concern. In the real world, fluctuations in the inflation rate are likely to be correlated with other shocks (e.g. productivity shocks) that are not controlled for in the regression. In our model, fluctuations in the inflation rate are the only shock.



Finally, Klenow and Kryvstov (2008) measure the effect of inflation on the fraction of prices that increase (i.e. positive price adjustments) and on the fraction of prices that

decrease (i.e. negative price adjustments). Again, they accomplish this task by estimating the coefficient on inflation in the regressions of the fraction of prices that increase and on the fraction of prices that decrease. Their main finding is that inflation has a positive effect on the fraction of price increases and a negative effect on the fraction of prices that decrease. More specifically, they find that a 1 percentage point increase in inflation increases the fraction of positive price changes by 5.48 percentage points and it decreases the fraction of negative price changes by 3.10 percentage points.

Figure IX illustrates the predictions of our model regarding the effect of inflation on positive and negative price changes. Like in the data, our model predicts that an increase in inflation increases the fraction of positive price adjustments, and it reduces the fraction of negative price adjustments. However, the magnitude of the effect of inflation on these variables is different than the one obtain form the regression analysis of the data. In particular, according to our model, an increase in annual inflation from 3 to 4 percent increases the fraction of positive price changes by approximately 10 percentage points, and it decreases the fraction of negative price changes by approximately 2.5 percentage points. For the reasons discussed above, we do not find this discrepancy between the predictions of the model and the outcomes of the regression analysis too surprising.



## 4.4 Summary of results

From the previous paragraphs, it is clear that our theory of price rigidity can account quite well for the empirical behavior of prices in the US retail market. First, the model predicts an average price duration (11.6 months) that is close to the one observed in the data (8.6 months). Second, the model generates a price change distribution that has the same shape and the same features of the empirical price change distribution (i.e. the average magnitude of price changes is large, there are many small price changes, and there are many negative price changes). Third, as it is observed in the data, in the model the probability and magnitude of price adjustments are approximately independent of the age of a price. Fourth, the model correctly predicts that inflation increases both the frequency and the magnitude of price changes. Finally, the model correctly predicts that inflation increases the fraction of positive price changes and reduces the fraction of negative price changes.

In contrast to our model, existing theories of price rigidity cannot account for all these features of the empirical behavior of prices. On the one hand, menu costs theories of price rigidity (e.g. Golosov and Lucas, 2007) cannot simultaneously account for the average duration of prices (which suggests that menu costs are large) and for the large fraction of price changes that are small (which suggests that menu costs are small). On the other hand, time-dependent theories of price rigidity (e.g. Calvo 1983 and Taylor 1980) cannot account for the effect of inflation on the frequency of price adjustment, because this frequency is a technological parameter. To the best of our knowledge, the only theory that matches the empirical behavior of prices as well as ours is the one by Midrigan (2006), which combines elements of state-dependent and time-dependent models. However, Midrigan's theory has very different implications than ours. In Midrigan model money is not neutral. Hence, the central bank can engineer an expansion by injecting money into the economy. In our model, money is neutral and monetary policy cannot be used to generate short-run outbursts of production and consumption.

## 5 Conclusions

We have constructed a theory of nominal price stickiness that is consistent with the empirical evidence. Our theory does not impose ad hoc restrictions on firms' pricing decisions – they are free to change their prices when they like at no cost. Instead, it relies on the existence of standard search frictions in goods markets, which give rise to equilibrium price dispersion. Sticky prices are an obvious corollary of price dispersion. This would be true of relative prices in an economy with perfect credit, in which some individual sellers may keep the same price, measured in terms of a numeraire commodity, in the face of shocks to supply and demand conditions (i.e., technology and preferences). Since many of the claims and arguments in macroeconomics concern monetary matters, we develop the idea in the context of a monetary economy, and show it is also true that individual sellers may keep the same price, measured in terms of dollars, in the face of shocks not only to supply and demand but also to changes in the money supply.

Contrary to what one sees in other models that claim to be consistent with the facts, based on Calvo or Mankiw pricing, in our framework the monetary authority cannot engineer a boom or get us out of a slump simply by issuing currency. We prove that money is neutral in the model, although not superneutral (inflation can have real effects on prices and allocations). Of course, a model is only a model. We did not prove that a central bank could not engineer a real expansion merely by printing money in the real world. And, we did not in this paper get into the question of, even if they could, does this mean they should? All we did was to show that the observation of sticky nominal prices does not logically imply that we need menu costs, or related devices, in our models to match the facts, and that this observation also does not rationalize particular policy prescriptions as some people seem to believe.

## References

- [1] Baharad, E., and B. Eden. 2004. "Price Rigidity and Price Dispersion: Evidence From Micro Data." *Review of Economic Dynamics* 7: 613–41.
- [2] Ball, L., and G. Mankiw. 1994. "A Sticky-Price Manifesto." NBER Working Paper 4677.
- [3] Berentsen, A., G. Menzio, and R. Wright. 2010. "Inflation and Unemployment in the Long Run." *American Economic Review*.
- [4] Bilts, M., and P. Klenow. 2005. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112(5): 947-85.
- [5] Burdett, K., and K. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica* 51: 955-70.
- [6] Burdett, K., and D. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39(2): 257-73.
- [7] Campbell, J., and B. Eden. 2007. "Rigid Prices: Evidence from U.S. Scanner Data." Working Paper 2005-08, Federal Reserve Bank of Chicago, Chicago.
- [8] Calvo, G. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12: 383–398.
- [9] Caplin, A., and D. Spulber. 1987. "Menu Costs and the Neutrality of Money." *Quarterly Journal of Economics*, 102: 703–725.
- [10] Clarida, R., Gali, J., and Gertler, M. 1999. "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature* 37, 1661-1707.
- [11] Dotsey, M., R. King, and A. Wolman. 1999. "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics*, 114: 655–690.
- [12] Eden, B. 1994. "The Adjustment of Prices to Monetary Shocks when Trade is Uncertain and Sequential." *Journal of Political Economy* 102(3): 493–509.

- [13] Eichenbaum, M., N. Jaimovich, and S. Rebelo. 2009. "References Prices, Costs, and Nominal Rigidities," manuscript Northwestern University.
- [14] Faig, M., and B. Jerez. 2005. "A Theory of Commerce." *Journal of Economic Theory*, 122: 60-99.
- [15] Gagnon, E. 2007. "Price Setting During Low and High Inflation: Evidence from Mexico." International Finance Discussion Paper 2007-896, The Federal Reserve Board.
- [16] Golosov, M., and R.E. Lucas, Jr. 2003. Menu Costs and Phillips Curves", NBER Working Paper 10187.
- [17] Golosov, M., and R.E. Lucas, Jr. 2007. "Menu Costs and Phillips Curves", *Journal of Political Economy*, 115(2): 171-99.
- [18] Head, A., and A. Kumar. 2005. "Price Dispersion, Inflation, and Welfare." *International Economic Review* 46(2): 533-72.
- [19] Head, A., A. Kumar, and B. Lapham. 2008. "Market Power, Price Adjustment, and Inflation." *International Economic Review*.
- [20] Jean, K., S. Rabinovich, and R. Wright. 2010. "On the Multiplicity of Monetary Equilibria: Green-Zhou Meets Lagos-Wright." *Journal of Economic Theory*.
- [21] Klenow, P., and O. Kryvtsov. 2008. "State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics* 123(3): 863-904.
- [22] Klenow, P., and B. Malin. 2010. "Microeconomic Evidence on Price-Setting," *Handbook of Monetary Economics*, forthcoming.
- [23] Lagos, R., and R. Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113(2): 463-88.
- [24] Mankiw, G. 1985. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model," *Quarterly Journal of Economics* 100, 529-38.

- [25] McCall, J. 1970. "Economics of Information and Job Search," *Quarterly Journal of Economics* 84, 113-26.
- [26] Mortensen, D. 1970. "A Theory of Wage and Employment Dynamics," in *Microeconomic Foundations of Employment and Inflation Theory*, E.S. Phelps, ed. New York: W.W. Norton.
- [27] Mortensen, D. 2003. *Wage Dispersion: Why Are Similar Workers Paid Differently?* MIT Press.
- [28] Midrigan, V. 2006. "Menu Costs, Multi-Product Firms, and Aggregate Fluctuations." Manuscript, New York University.
- [29] Nakamura, E., and J. Steinsson. "Five Facts About Prices: A Reevaluation of Menu Cost Models." *Quarterly Journal of Economics* 123: 1415-1464.
- [30] Nosal, E., and Rocheteau, G. 2010. *Money, Payments, and Liquidity*, MIT Press, forthcoming.
- [31] Postel-Vinay, F., and J. Robin. 2002a. "The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counter-Offeres," *International Economic Review* 43, 989-1016.
- [32] Postel-Vinay, F. and J. Robin. 2002b. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica* 70, 1295-1350.
- [33] Rothschild, M. 1973. "Models of Market Organization with Imperfect Information: A Survey," *Journal of Political Economy* 81, 1283-308.
- [34] Shi, S. 1997. "A Divisible Model of Fiat Money," *Econometrica* 65, 75-102.
- [35] Taylor, J. 1980. "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88: 1-23.
- [36] Woodford, M. 2003. *Interest and Prices*, Princeton University Press, Princeton NJ.

# Appendix

## A Proof of Lemma 1

We take five steps to prove the lemma.

**Claim 1:** The firm's maximized profits,  $\Pi_t^*$ , are strictly positive.

*Proof:* For any  $\lambda > 1$ , the profits of a firm that posts the price  $\lambda cw_t$  are given by

$$\begin{aligned}\Pi_t(\lambda cw_t) &= \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(\lambda cw_t)) + \alpha_2 n_t(\lambda cw_t)] q_t^*(\lambda cw_t) (\lambda - 1) cw_t \\ &> \frac{\alpha_1}{s} q_t^*(\lambda cw_t) (\lambda - 1) cw_t.\end{aligned}$$

Since  $q_t^*(\lambda cw_t) > 0$  and  $\lambda > 1$ , the previous inequality implies  $\Pi_t(\lambda cw_t) > 0$ . In turn, since  $\Pi_t^* \geq \Pi_t(\lambda cw_t)$ , we have  $\Pi_t^* > 0$ .

**Claim 2:** The price distribution  $F_t$  is continuous.

*Proof:* On the way to a contradiction, suppose that there exists a price  $p_0$  such that  $n_t(p_0) > 0$ . The profits of a firm that posts  $p_0$  are given by

$$\Pi_t(p_0) = \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(p_0)) + \alpha_2 n_t(p_0)] R_t(p_0).$$

Now, consider any  $p_1 < p_0$  such that (i)  $R_t(p_1)$  is strictly positive and (ii)  $\Delta \equiv R_t(p_0) - R_t(p_1)$  is strictly smaller than  $\alpha_2 n_t(p_0) R_t(p_0) / (\alpha_1 + 2\alpha_2)$ . A price  $p_1$  that satisfies conditions (i) and (ii) exists because the function  $R_t(p)$  is continuous in  $p$  and, since  $\Pi_t(p_0) = \Pi_t^* > 0$ ,  $R_t(p_0)$  is strictly positive. The profits of a firm that posts  $p_1$  are given by

$$\begin{aligned}\Pi_t(p_1) &= \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(p_1)) + \alpha_2 n_t(p_1)] R_t(p_1) \\ &\geq \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(p_0)) + 2\alpha_2 n_t(p_0)] [R_t(p_0) - \Delta] \\ &\geq \Pi_t(p_0) + \alpha_2 n_t(p_0) [R_t(p_0) - \Delta] - (\alpha_1 + 2\alpha_2) \Delta\end{aligned}\tag{A.1}$$

Since  $R_t(p_0) - \Delta > 0$  and  $\Delta < \alpha_2 n_t(p_0) R_t(p_0) / (\alpha_1 + 2\alpha_2)$ , it follows from (A.1) that  $\Pi_t(p_1) > \Pi_t(p_0)$ . However, from the fact that  $p_0$  belongs to the support of the price distribution  $F_t$ , it follows that  $\Pi_t(p_0) = \Pi_t^* \geq \Pi_t(p_1)$ . Hence, we have reached a contradiction.

**Claim 3:** The monopoly price,  $p_t^m$ , is the highest price on the support of the distribution  $F_t$ .

*Proof:* On the way to a contradiction, suppose that the highest price on the support of the distribution is  $\bar{p}_t \neq p_t^m$ . The profits of a firm that posts the price  $\bar{p}_t$  are given by

$$\Pi_t(\bar{p}_t) = \frac{\alpha_1}{s} R_t(\bar{p}_t). \quad (\text{A.2})$$

The profits of a firm that posts the price  $p_t^m$  are given by

$$\begin{aligned} \Pi_t(p_t^m) &= \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(p_t^m))] R_t(p_t^m) \\ &\geq \frac{\alpha_1}{s} R_t(p_t^m) \\ &> \frac{\alpha_1}{s} R_t(\bar{p}_t), \end{aligned} \quad (\text{A.3})$$

where the third line makes use of the fact that  $p_t^m$  is the unique maximizer of  $R_t$ . From (A.2) and (A.3), it follows that  $\Pi_t(p_t^m) > \Pi_t(\bar{p}_t)$ . However, from the fact that  $\bar{p}_t$  belongs to the support of the price distribution  $F_t$ , it follows that  $\Pi_t(\bar{p}_t) = \Pi_t^* \geq \Pi_t(p_t^m)$ . Hence, we have reached a contradiction.

**Claim 4:** The support of the price distribution  $F_t$  is connected.

*Proof:* On the way to a contradiction, suppose that  $p_0$  and  $p_1$  are two prices on the support of the distribution such that  $p_0 < p_1$  and  $F_t(p_0) = F_t(p_1)$ . Since  $p_0$  and  $p_1$  belong to the support of the price distribution, they are greater than  $cw_t$  and smaller than  $p_t^m$ . Hence,  $cw_t < p_0 < p_1 \leq p_1$ . The profits of a firm that posts the price  $p_0$  are given by

$$\Pi_t(p_0) = \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(p_0))] R_t(p_0).$$

The profits of a firm that posts the price  $p_1$  are given by

$$\Pi_t(p_1) = \frac{1}{s} [\alpha_1 + 2\alpha_2 (1 - F_t(p_1))] R_t(p_1).$$

First, note that  $\alpha_1 + 2\alpha_2 (1 - F_t(p_1))$  is equal to  $\alpha_1 + 2\alpha_2 (1 - F_t(p_0))$  because  $F_t(p_1) = F_t(p_0)$ . Second, note that  $R_t(p_1) > R_t(p_0)$  because  $cw_t < p_0 < p_1 \leq p_1$  and the function  $R_t(p)$  is strictly increasing for all  $p \in [cw_t, p_t^m]$ . From this observations, it follows that  $\Pi_t(p_1) > \Pi_t(p_0)$ . However, from the fact that  $p_0$  and  $p_1$  belong to the support of the price distribution  $F_t$ , it follows that  $\Pi_t(p_0) = \Pi_t(p_1) = \Pi_t^*$ . Hence, we have reached a contradiction.

**Claim 5:** The price distribution  $F_t$  is given by

$$F_t(p) = 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{R_t(p_t^m)}{R_t(p)} - 1 \right]. \quad (\text{A.4})$$

*Proof:* Since the price distribution has no mass points, the profit of a firm that charges the price  $p$  is given by

$$\Pi_t(p) = \frac{1}{s} [\alpha_1 + 2\alpha_2(1 - F_t(p))] R_t(p).$$

Since the monopoly price is on the support of the price distribution, the maximized profit of the firm is given by

$$\Pi_t^* = \frac{\alpha_1}{s} R_t(p_t^m).$$

Finally, since every price on the support of the price distribution must maximize the profit of the firm, it follows that for every  $p \in [\underline{p}_t, \bar{p}_t]$

$$\frac{1}{s} [\alpha_1 + 2\alpha_2(1 - F_t(p))] R_t(p) = \alpha_1 R_t(p_t^m). \quad (\text{A.5})$$

Solving (A.5) with respect to  $F_t$  leads to equation (A.4).

## B Proof of Theorem 3

We find it convenient to express all the nominal variables in real terms (i.e. units of labor). Specifically, we let  $n_t$  denote the real balances  $m_t/w_t$ , we let  $z_t$  denote the real price  $p/w_t$ , and let  $H_t$  denote the distribution of real prices  $z$ . Then, a stationary monetary equilibrium can be defined as a tuple  $\{n^*, x^*, h^*, q^*, H^*\}$  such that:

(i)  $n^*$  and  $x^*$  and  $h^*$  solve the household's problem in the AD market:

$$i = \int_{\underline{z}(n^*)}^{\hat{z}(n^*)} \left[ \frac{1}{z} \left( \frac{n^*}{z} \right)^{-\gamma} - 1 \right] [\alpha_1 + 2\alpha_2(1 - H^*(z|n^*))] dH^*(z|n^*),$$

$$v'(x^*) = 1, \quad h^*(n) = x^* + (\mu^{-1}n^* - n - D - T);$$

(ii)  $q^*$  solves the household's problem in the BJ market:

$$q^*(z, n) = \begin{cases} n/z, & \text{if } z \leq \hat{z}(n), \\ (1/z)^{\frac{1}{\gamma}}, & \text{if } z > \hat{z}(n); \end{cases}$$

(iii)  $H^*$  is consistent with the solution to the firm's problem in the BJ market:

$$H^*(z|n^*) = 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{R(z^m, n^*)}{R(z, n^*)} - 1 \right] \quad \text{all } z \in [\underline{z}(n^*), \bar{z}(n^*)],$$

where  $R(z, n^*) = q^*(z, n^*)(z - c)$ .

To establish the existence of a stationary monetary equilibrium, it is sufficient to prove that there exists a  $\hat{n}^*$  such that the households' optimal choice of real balances is  $\hat{n}^*$  when the distribution of prices posted by the firms is  $H^*(z|\hat{n}^*)$ . That is, it is sufficient to prove that there exists a  $\hat{n}^*$  such that

$$i = \int_{\underline{z}(n^*)}^{\hat{z}(n)} \left[ \frac{1}{z} \left( \frac{n}{z} \right)^{-\gamma} - 1 \right] [\alpha_1 + 2\alpha_2 (1 - H^*(z|n^*))] dH^*(z|n^*),$$

for  $n = \hat{n}^*$ . We carry out this task in three steps.

**Claim 1:** Let  $\underline{n}^*$  and  $\bar{n}^*$  be defined as

$$\hat{z}(\underline{n}^*) = \frac{c}{1 - \gamma}, \quad \hat{z}(\bar{n}^*) = c \left[ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \frac{1}{n^*} \left( \frac{c}{1 - \gamma} \right)^{-\frac{1}{\gamma}} \left( \frac{\gamma c}{1 - \gamma} \right) \right]^{-1}.$$

For all  $n_0^*$  and  $n_1^*$  such that  $0 < n_0^* < n_1^* \leq \bar{n}^*$ , the price distribution  $H(z|n_0^*)$  first order stochastically dominates the price distribution  $H(z|n_1^*)$ . For all  $n_0^*$  and  $n_1^*$  such that  $\bar{n}^* \leq n_0^* < n_1^*$ , the price distribution  $H(z|n_0^*)$  is equal to the price distribution  $H(z|n_1^*)$ .

*Proof:* For all  $n^* \in (0, \underline{n}^*]$ , the price distribution is given by

$$H(z|n^*) = 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{\bar{z}(n^*)^{-1} n^* (\bar{z}(n^*) - c)}{z^{-1} n^* (z - c)} - 1 \right]$$

and its support is given by  $[\underline{z}(n^*), \bar{z}(n^*)]$ , where

$$\bar{z}(n^*) = n^{*-\frac{\gamma}{1-\gamma}},$$

$$\underline{z}(n^*) = c \left[ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \left( \frac{\bar{z}(n^*) - c}{\bar{z}(n^*)} \right) \right]^{-1}.$$

Consider any  $n_0^*$  and  $n_1^*$  such that  $0 < n_0^* < n_1^* \leq \bar{n}^*$ . Clearly,  $\bar{z}(n_0^*) > \bar{z}(n_1^*)$  and  $\underline{z}(n_0^*) > \underline{z}(n_1^*)$ . For  $z \geq \bar{z}(n_0^*)$ ,  $H(z|n_0^*) = H(z|n_1^*) = 1$ . For  $z \in (\underline{z}(n_1^*), \bar{z}(n_0^*))$ ,  $H(z|n_0^*) < H(z|n_1^*)$  because  $\bar{z}(n_0^*) > \bar{z}(n_1^*)$ . For  $z \leq \underline{z}(n_1^*)$ ,  $H(z|n_0^*) = H(z|n_1^*) = 0$ . Hence,  $H(z|n_0^*)$  first order stochastically dominates  $H(z|n_1^*)$ .

For all  $n^* \in [\underline{n}^*, \bar{n}^*]$ , the price distribution is given by

$$H(z|n^*) = \begin{cases} 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{\bar{z}(n^*)^{-\frac{1}{\gamma}} (\bar{z}(n^*) - c)}{z^{-\frac{1}{\gamma}} (z - c)} - 1 \right], & \text{if } z \in [\hat{z}(n^*), \bar{z}(n^*)], \\ 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{\bar{z}(n^*)^{-\frac{1}{\gamma}} (\bar{z}(n^*) - c)}{z^{-1} n^* (z - c)} - 1 \right], & \text{if } z \in [\underline{z}(n^*), \hat{z}(n^*)] \end{cases}$$

and its support is given by  $[\underline{z}(n^*), \bar{z}(n^*)]$ , where

$$\bar{z}(n^*) = \frac{c}{1 - \gamma},$$

$$\underline{z}(n^*) = c \left[ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \frac{\bar{z}(n^*)^{-\frac{1}{\gamma}} (\bar{z}(n^*) - c)}{n^*} \right]^{-1}.$$

Consider any  $n_0^*$  and  $n_1^*$  such that  $\underline{n}^* \leq n_0^* < n_1^* \leq \bar{n}^*$ . Clearly,  $\bar{z}(n_0^*) = \bar{z}(n_1^*)$ ,  $\hat{z}(n_0^*) > \hat{z}(n_1^*)$  and  $\underline{z}(n_0^*) > \underline{z}(n_1^*)$ . For  $z \geq \hat{z}(n_0^*)$ ,  $H(z|n_0^*) = H(z|n_1^*)$ . For  $z \in [\hat{z}(n_1^*), \hat{z}(n_0^*)]$ ,  $H(z|n_0^*) < H(z|n_1^*)$  because  $z^{-1} n_0^* < z^{-\frac{1}{\gamma}}$ . For  $z \in [\underline{z}(n_0^*), \hat{z}(n_1^*)]$ ,  $H(z|n_0^*) < H(z|n_1^*)$  because  $n_0^* < n_1^*$ . For  $z \leq \underline{z}(n_0^*)$ ,  $H(z|n_0^*) \leq H(z|n_1^*)$ . Hence,  $H(z|n_0^*)$  first order stochastically dominates  $H(z|n_1^*)$ .

For all  $n^* \geq \bar{n}^*$ , the price distribution is given by

$$H(z|n^*) = 1 - \frac{\alpha_1}{2\alpha_2} \left[ \frac{\bar{z}(n^*)^{-\frac{1}{\gamma}} (\bar{z}(n^*) - c)}{z^{-\frac{1}{\gamma}} (z - c)} - 1 \right]$$

and its support is given by  $[\underline{z}(n^*), \bar{z}(n^*)]$ , where

$$\bar{z}(n^*) = \frac{c}{1 - \gamma},$$

$$\underline{z}(n^*) = c + \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \left( \frac{\hat{z}(n^*)}{\bar{z}(n^*)} \right)^{-\frac{1}{\gamma}} (\bar{z}(n^*) - c).$$

In this case, the price distribution does not depend on  $n^*$ . Hence,  $H(z|n_0^*) = H(z|n_1^*)$  for all  $n_0^*$  and  $n_1^*$  such that  $\bar{n}^* \leq n_0^* < n_1^*$ .

**Claim 2:** Given the price distribution  $H(z|n^*)$ , the unique solution for  $n$  to the equilibrium condition (i) is  $\psi(n^*)$ . The solution  $\psi(n^*)$  has the following properties:

- (i) For all  $n_0^*, n_1^*$  such that  $0 < n_0^* < n_1^* \leq \bar{n}^*$ ,  $\psi(n_0^*) \leq \psi(n_1^*)$ ;
- (ii) For all  $n_0^*, n_1^*$  such that  $\bar{n}^* \leq n_0^* < n_1^*$ ,  $\psi(n_0^*) = \psi(n_1^*)$ ;

(iii) For all  $n^* > 0$ ,  $\psi(n^*) \in [\underline{\psi}, \overline{\psi})$ , where  $\underline{\psi} > 0$  and  $\overline{\psi} = \overline{n^*}$ .

*Proof:* Given the price distribution  $H(z|n^*)$ , the equilibrium condition (i) is

$$\begin{aligned} i = & \alpha_1 \int_{\underline{z}(n^*)}^{\hat{z}(n^*)} \left[ \left( \frac{z}{n} \right)^\gamma \frac{1}{z} - 1 \right] dH(z|n^*) \\ & + \alpha_2 \int_{\underline{z}(n^*)}^{\hat{z}(n^*)} \left[ \left( \frac{z}{n} \right)^\gamma \frac{1}{z} - 1 \right] d[1 - (1 - H(z|n^*))^2]. \end{aligned}$$

Let  $\phi(n, n^*)$  denote the right-hand side of the equation above. First, notice that  $\lim_{n \rightarrow 0} \phi(n, n^*) = \infty$ . Second, notice that  $\phi(n, n^*)$  is strictly decreasing in  $n$  for all  $n \in (0, \hat{z}^{-1}(\underline{z}(n^*)))$ . Third, notice that  $\phi(n, n^*) = 0$  for all  $n \geq \hat{z}^{-1}(\underline{z}(n^*)) = \underline{z}(n^*)^{-\frac{1-\gamma}{\gamma}}$ . From these observations, it follows that there exists a unique solution,  $\psi(n^*)$ , to the equation  $\phi(n, n^*) = i$ . Moreover,  $\psi(n^*) > 0$  and  $\psi(n^*) < \hat{z}^{-1}(\underline{z}(n^*))$ .

Consider any  $n_0^*$  and  $n_1^*$  such that  $0 < n_0^* < n_1^* \leq \overline{n^*}$ . In this case, claim 1 implies that  $H(z|n_0^*)$  first order stochastically dominates  $H(z|n_1^*)$  and, consequently, that  $1 - (1 - H(z|n_0^*))^2$  first order stochastically dominates  $1 - (1 - H(z|n_1^*))^2$ . From these observations and from the fact that  $(z/n)^\gamma / z - 1$  is decreasing in  $z$ , it follows that  $\phi(n, n_0^*) \leq \phi(n, n_1^*)$ . Therefore,  $\psi(n_0^*) \leq \psi(n_1^*)$ . Moreover, it is straightforward to verify that  $\psi(n_0^*) \geq \underline{\psi} > 0$ .

Now, consider any  $n_0^*$  and  $n_1^*$  such that  $\overline{n^*} \leq n_0^* < n_1^*$ . In this case, claim 1 implies that  $H(z|n_0^*)$  is equal to  $H(z|n_1^*)$  and that  $1 - (1 - H(z|n_0^*))^2$  is equal to  $1 - (1 - H(z|n_1^*))^2$ . From these observations, it follows that  $\phi(n, n_0^*) = \phi(n, n_1^*)$  and, hence,  $\psi(n_0^*) = \psi(n_1^*)$ . Moreover, it is straightforward to verify that  $\psi(n_1^*) < \overline{n^*}$ .

**Claim 3:** There exists a  $\hat{n}^* \in [\underline{\psi}, \overline{\psi})$  such that  $\psi(\hat{n}^*) = \hat{n}^*$ .

*Proof:* Claim 2 implies that: (a)  $\psi(n^*)$  is an increasing function of  $n^*$ , (b) for all  $n^*$  in the interval  $[\underline{\psi}, \overline{\psi}]$ ,  $\psi(n^*)$  belongs to the interval  $[\underline{\psi}, \overline{\psi})$ . Hence, from Tarski's fixed point theorem, it follows that there exists a  $\hat{n}^* \in [\underline{\psi}, \overline{\psi})$  such that  $\psi(\hat{n}^*) = \hat{n}^*$ . Moreover, claim 2 implies  $\psi(n^*) \geq \underline{\psi}$  for all  $n^* < \underline{\psi}$ , and  $\psi(n^*) < \overline{\psi}$  for all  $n^* \geq \overline{\psi}$ . Hence, there is no  $\hat{n}^* \notin [\underline{\psi}, \overline{\psi})$  such that  $\psi(\hat{n}^*) = \hat{n}^*$ .

## C Proof of Theorem 6

We take two steps to prove Theorem 6.

**Claim 1:** In any stationary monetary equilibrium  $\Sigma^*$

$$\frac{\bar{p}_t - \underline{p}_t}{\bar{p}_t} \geq \Delta > 0, \quad (\text{A.6})$$

where

$$\Delta = \frac{2\alpha_2\gamma}{2\alpha_2 + \alpha_1(1 - \gamma)}.$$

*Proof:* Let  $\Sigma^*$  be an arbitrary stationary monetary equilibrium. From condition (1) in the definition of equilibrium, it follows that either households are cash constrained in all transactions in the BJ market, or they are cash constrained in a positive fraction of transactions and not constrained in the others.

First, suppose  $\Sigma^*$  is such that households are cash constrained in all transactions. In this case, we have

$$\bar{p}_t = w_t^* \left( \frac{w_t^*}{m_t^*} \right)^{\frac{\gamma}{1-\gamma}} \geq \frac{cw_t^*}{1-\gamma}, \quad (\text{A.7})$$

$$\underline{p}_t = \frac{cw_t^*}{\left[ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \frac{\bar{p}_t - cw_t^*}{\bar{p}_t} \right]}. \quad (\text{A.8})$$

From (A.7) and (A.8), it follows that

$$\begin{aligned} \frac{\bar{p}_t - \underline{p}_t}{\bar{p}_t} &= \frac{2\alpha_2(\bar{p}_t - cw_t^*)}{2\alpha_2\bar{p}_t + \alpha_1cw_t^*} \\ &\geq \frac{2\alpha_2\gamma}{2\alpha_2 + \alpha_1(1 - \gamma)} = \Delta, \end{aligned} \quad (\text{A.9})$$

where the second line makes use of the fact that  $\bar{p}_t \geq (1 - \gamma)^{-1}cw_t^*$ .

Next, suppose  $\Sigma^*$  is such that households are cash constrained in some but not all transactions.

In this case, we have

$$\bar{p}_t = \frac{cw_t^*}{1-\gamma} \geq w_t^* \left( \frac{w_t^*}{m_t^*} \right)^{\frac{\gamma}{1-\gamma}}, \quad (\text{A.10})$$

$$\underline{p}_t = \frac{cw_t^*}{\left[ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \left( \frac{\bar{p}_t}{w_t^*} \right)^{-\frac{1}{\gamma}} \left( \frac{\bar{p}_t - cw_t^*}{m_t^*} \right) \right]}. \quad (\text{A.11})$$

From (A.10) and (A.11), it follows that

$$\begin{aligned} \frac{\bar{p}_t - \underline{p}_t}{\bar{p}_t} &= \frac{(\alpha_1 + 2\alpha_2)\gamma m_t^* - \alpha_1\gamma \left(\frac{c}{1-\gamma}\right)^{\frac{\gamma-1}{\gamma}} w_t^*}{(\alpha_1 + 2\alpha_2)m_t^* - \alpha_1\gamma \left(\frac{c}{1-\gamma}\right)^{\frac{\gamma-1}{\gamma}} w_t^*} \\ &\geq \frac{2\alpha_2\gamma}{2\alpha_2 + \alpha_1(1-\gamma)} = \Delta, \end{aligned} \quad (\text{A.12})$$

where the second line makes use of the fact that  $m_t^* \geq [(1-\gamma)/c]^{\frac{1-\gamma}{\gamma}} w_t^*$ . Combining (A.9) and (A.12), we obtain (A.6).

**Claim 2:** Let  $\mu \in (1, \mu^*)$ , where  $\mu^* = (1 - \Delta)^{-1}$ . Then, for any stationary monetary equilibrium  $\Sigma^*$  together with the pricing policy  $p_{t+1}^*(p, \rho)$ , we have

$$\begin{aligned} FR(\rho) &< 1 \text{ and } A(\rho) > 0, & \text{all } \rho \in (0, 1], \\ H_t(0, \rho) &> 0, & \text{all } \rho \in [0, 1). \end{aligned}$$

*Proof:* From claim 1 and  $\mu \in (1, \mu^*)$ , it follows that

$$\mu \underline{p}_t < \frac{1}{1 - \Delta} \underline{p}_t \leq \bar{p}_t. \quad (\text{A.13})$$

For any  $\rho \in (0, 1]$ , the fraction of prices that adjust is

$$\begin{aligned} FR(\rho) &= F_t^*(\mu \underline{p}_t) + (1 - \rho) \left[ 1 - F_t^*(\mu \underline{p}_t) \right] \\ &< F_t^*((1 - \Delta)^{-1} \underline{p}_t) + (1 - \rho) \left[ 1 - F_t^*((1 - \Delta)^{-1} \underline{p}_t) \right] \\ &< 1, \end{aligned}$$

where the second line makes use of (A.13), and the third line makes use of the fact that  $F_t^*((1 - \Delta)^{-1} \underline{p}_t) < F_t^*(\bar{p}_t) = 1$ . Since the fraction of prices that adjust is less than 1, the average duration of a price must be greater than 1 period. That is,  $A(\rho) > 1$ . Finally, it is straightforward to verify that for all  $\rho \in [0, 1)$ , the fraction of negative price changes,  $H_t(0, \rho)$ , is strictly positive.