

Arbitrage-Free Bond Pricing with Dynamic Macroeconomic Models¹

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Abstract

We examine the relationship between monetary-policy-induced changes in short interest rates and yields on long-maturity default-free bonds. The volatility of the long end of the term structure and its relationship with monetary policy are puzzling from the perspective of simple structural macroeconomic models. We explore whether richer models of risk premiums, specifically stochastic volatility models combined with Epstein-Zin recursive utility, can account for these patterns. We study the properties of the yield curve when inflation is an exogenous process and compare this to the yield curve when inflation is endogenous and determined through an interest-rate/Taylor rule. We find that the Epstein-Zin model with moderate risk aversion, persistent volatility of real endowment growth, and exogenous inflation, does a good job of matching the shape of the historical average yield curve. However, it exhibits less volatility in long rates than found in the data. We add to this environment a Taylor rule that raises the short interest rate aggressively in response to current inflation, and re-solve for yields using the endogenous equilibrium process for inflation. We find that risk premiums increase substantially as does the volatility of yields. When we lower the degree of risk aversion in this endogenous-inflation economy, however, we find that the model still fits the shape of the average yield curve and long rates are still substantially less volatile than in the data.

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1 Introduction

The response of long-term interest rates to monetary-policy-induced changes in short term interest rates is a feature of the economy that often puzzles policy makers. For example, in remarks made on May 27, 1994, Alan Greenspan expresses concern that long rates moved too much in response to an increase in short rates:

In early February, we thought long-term rates would move a little higher as we tightened. The sharp jump in [long] rates that occurred appeared to reflect the dramatic rise in market expectations of economic growth and associated concerns about possible inflation pressures.⁶

Then in his February 16, 2005, testimony, he expresses a completely different concern about long rates:

Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. Historically, even distant forward rates have tended to rise in association with monetary policy tightening. ... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.⁷

Greenspan's comments are a reflection of the fact that we do not yet have a satisfactory understanding of how the yield curve is related to features of the macroeconomy such as investors' preferences, fundamental sources of risk, and endogenous monetary policy.

Figure 1 plots the nominal yield curve for a variety of maturities from 1 quarter (which we refer to as the short rate), up to 10 years (40 quarters) for US treasuries starting in the first

⁶Testimony of Chairman Alan Greenspan before the U.S. Senate Committee on Banking, Housing, and Urban Affairs, May 27, 1994. Federal Reserve Bulletin, July 1994.

⁷Testimony of Chairman Alan Greenspan before the U.S. Senate Committee on Banking, Housing, and Urban Affairs, February 16, 2005.

<http://www.federalreserve.gov/boarddocs/hh/2005/february/testimony.htm>

quarter of 1970 and ending in the last quarter of 2005.⁸ Figure 2 plots the average yields curve for the entire sample and for two subsamples. Finally, Figure 3 plots the standard deviation of yields against their maturities. The basic patterns of yields is clear from these figures: (1) On average the yield curve is upward sloping, and (2) There is substantial volatility in yields at all maturities. Greenspan's comments, therefore, must be framed by the fact that long yields are almost as volatile as short rates. The issue, however, is the relationship of this volatility at the long end to the volatility at the short end.

A significant component of long rates is the risk premium. There is now a great deal of evidence that documents that these risk premiums are time-varying and state-dependent. Therefore, movements in long rates could be attributed to changes in expectations of future nominal short rates, movements in risk premiums, or a combination of the two. Moreover, if monetary policy is implemented using an short-interest-rate rule, *e.g.*, a Taylor rule, then the equilibrium endogenous inflation rate will depend on the same risk factors that drive risk premiums in long rates. Monetary policy itself could be a source for fluctuations in the yield curve in equilibrium.

We explore these possibilities by experimenting with a model of time-varying risk premiums generated by the recursive utility model of Epstein and Zin (1989) combined with stochastic volatility of endowment growth. We show how this model can be easily solved using now standard affine term-structure methods. These models have the convenient property that yields are maturity-dependent linear functions of state variables. We examine some general properties of multi-period default-free bonds in these models assuming first that inflation is an exogenous process, then by allowing inflation to be endogenous and controlled by an interest-rate rule.

⁸Yields up to 1991 are from McCulloch and Kwon (1993) then Datastream from 1991 to 2005.

2 The Duffie-Kan Affine Term Structure Model

The Duffie and Kan (1996) class of affine term-structure models, translated into discrete time by Backus et al. (2001), is based on a k -dimensional vector of state variables z that follows a “square-root” model

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + \Sigma(z_t)^{1/2}\varepsilon_{t+1},$$

where $\{\varepsilon_t\} \sim \text{NID}(0, I)$, $\Sigma(z)$ is a diagonal matrix with typical element given by $\sigma_i(z) = a_i + b'_i z$, and b_i has nonnegative elements, and Φ is stable with positive diagonal elements. The process for z requires that the volatility functions $\sigma_i(z)$ be positive, which places further restrictions on the parameters. The pricing kernel takes the form

$$-\log m_{t+1} = \delta + \gamma' z_t + \lambda' \Sigma(z_t)^{1/2} \varepsilon_{t+1},$$

where the $k \times 1$ vector γ is referred to as the “factor loadings” for the pricing kernel, the $k \times 1$ vector λ is referred to as the “price of risk” vector since it controls the size of the conditional correlation of the pricing kernel and the underlying sources of risk, and the $k \times k$ matrix $\Sigma(z_t)$ is the stochastic variance-covariance matrix of the unforecastable shock.

Multi-period default-free discount bond prices are built up using arbitrage-free restriction

$$b_t^{(n)} = E_t[m_{t+1} b_{t+1}^{(n-1)}], \tag{1}$$

where $b_t^{(n)}$ is the price at date- t of a default-free pure-discount bond that pays 1 at date $t + n$, and $b_t^{(0)} = 1$. It is straightforward to show that bond prices of all maturities are log-linear functions of the state:

$$-\log b_t^{(n)} = A^{(n)} + B^{(n)} z_t,$$

where $A^{(n)}$ is a scalar, and $B^{(n)}$ is a $1 \times k$ row vector.

The intercept and slope parameters, which we often refer to as “yield-factor loadings,” of these bond prices can be found recursively according to

$$\begin{aligned} A^{(n+1)} &= A^{(n)} + \delta + B^{(n)}(I - \Phi)\theta - \frac{1}{2} \sum_{j=1}^k (\lambda_j + B_j^{(n)})^2 a_j \\ B^{(n+1)} &= (\gamma' + B^{(n)}\Phi) - \frac{1}{2} \sum_{j=1}^k (\lambda_j + B_j^{(n)})^2 b'_j, \end{aligned} \quad (2)$$

where $B_j^{(n)}$ is the j -th element of the vector $B^{(n)}$. Note that since $b^{(0)} = 1$, we can start these recursions using $A^{(0)} = 0$ and $B_j^{(0)} = 0$, $j = 1, 2, \dots, k$.

We will often work with continuously compounded yields, $y_t^{(n)}$, defined by $b_t^{(n)} = \exp(-ny_t^{(n)})$, which implies $y_t^{(n)} = -(\log b_t^{(n)})/n$.

This is an equilibrium model of bond pricing since it satisfies the no-arbitrage/equilibrium conditions in equation (1), however it is not yet a *structural* model, since the mapping of the parameters of this pricing model to deeper structural parameters of preferences and opportunities has not yet been specified. However, as we will see below, the structural models we consider will all lie within this general class, hence, can be easily solved using these pricing equations.

3 A 2-Factor Model with Epstein-Zin Preferences

We begin our analysis of structural models of the yield curve by solving for equilibrium real yields in a representative agent exchange economy. Following Backus and Zin (2006) we consider a representative agent who chooses consumption to maximize the recursive utility function given in Epstein and Zin (1989)

$$W_t = [(1 - \beta)c_t^\rho + \beta\mu_t(W_{t+1})^\rho]^{1/\rho},$$

where $0 < \beta < 1$ is a discount factor that characterizes impatience, $\rho < 1$ determines the elasticity of intertemporal substitution for deterministic consumption paths, and $\alpha < 1$ determines the degree of relative risk aversion for static gambles. We consider a pure-exchange economy in which the representative agent consumes the stochastic endowment, so that $\log(c_{t+1}/c_t) = x_{t+1}$ where x_{t+1} is the log of the ratio of endowments in $t + 1$ relative to t . Therefore, in equilibrium the marginal rate of intertemporal substitution given by

$$\log m_{t+1} = \log \beta + (\rho - 1)x_{t+1} + (\alpha - \rho) [\log W_{t+1} - \log \mu_t(W_{t+1})], \quad (3)$$

where

$$\log \mu_t(z_{t+1}) = E_t[z_{t+1}] + \frac{\alpha}{2} \text{Var}_t[z_{t+1}],$$

when z_t is conditionally normally distributed. Assume that the endowment-growth process evolves stochastically over time according to

$$x_{t+1} = (1 - \phi_x)\theta_x + \phi_x x_t + v_t^{1/2} \varepsilon_{t+1}^x,$$

and

$$v_{t+1} = (1 - \phi_v)\theta_v + \phi_v v_t + \sigma_v \varepsilon_{t+1}^v$$

is the process for the conditional volatility of endowment growth. We assume that ε^x and ε^v are independent of each other and are both distributed as standard normals independently over time.

The state vector in this model conforms with the setup of the Duffie-Kan model above.

Define the state vector $z_t \equiv [x_t \ v_t]'$, and

$$\begin{aligned}\theta &= [\theta_x \ \theta_v]' \\ \Phi &= \text{diag}\{\phi_x, \phi_v\} \\ \Sigma(z_t) &= \text{diag}\{a_1 + b'_1 z_t, a_2 + b'_2 z_t\} \\ a_1 &= 0, b_1 = [0 \ 1]', a_2 = \sigma_v^2, b_2 = [0 \ 0]'. \end{aligned}$$

It will be simpler to work with the log utility function scaled by consumption. Define $w_t \equiv \log(W_t/c_t)$ which, given the definition of utility, can be written as

$$w_t = \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho \log \mu_t)].$$

We can approximate the right-hand side of this equation around the point $\log(\mu_t) = \bar{m}$ to obtain

$$\begin{aligned}w_t &\approx \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho \bar{m})] + \left[\frac{\beta \exp(\rho \bar{m})}{1 - \beta + \beta \exp(\rho \bar{m})} \right] (\log(\mu_t) - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa \log \mu_t(w_{t+1} + x_{t+1}). \end{aligned}$$

Note that $\kappa < 1$. For the special case $\rho = 0$, *i.e.*, a log time aggregator, this equation holds exactly with $\bar{\kappa} = 1 - \beta$ and $\kappa = \beta$ (see Hansen et al. (2005)). Similarly, if $\bar{m} = 0$, then $\bar{\kappa} = 0$ and $\kappa = \beta$.

Given the state variables and the log-linear structure of the model, we guess a solution for the value function

$$w_t = \bar{\omega} + \omega_x x_t + \omega_v v_t,$$

which implies

$$\begin{aligned}
\log \mu_t &= \log \mu_t (\bar{\omega} + (\omega_x + 1)x_{t+1} + \omega_v v_{t+1}) \\
&= \bar{\omega} + (\omega_x + 1)(1 - \phi_x)\theta_x + \omega_v(1 - \phi_v)\theta_v + (\omega_x + 1)\omega_x x_t + \omega_v \phi_v v_t \\
&\quad + \frac{\alpha}{2}(\omega_x + 1)^2 v_t + \frac{\alpha}{2}\omega_v^2 \sigma_v^2.
\end{aligned}$$

We can use this expression to solve for the value-function parameters and verify its log-linear solution

$$\begin{aligned}
\omega_x &= \kappa(\omega_x + 1)\phi_x \\
\Rightarrow \omega_x &= \left(\frac{\kappa}{1 - \kappa\phi_x} \right) \phi_x \\
\omega_v &= \kappa[\omega_v \phi_v + \frac{\alpha}{2}(\omega_x + 1)^2] \\
\Rightarrow \omega_v &= \left(\frac{\kappa}{1 - \kappa\phi_v} \right) \left[\frac{\alpha}{2} \left(\frac{1}{1 - \kappa\phi_x} \right)^2 \right] \\
\bar{\omega} &= \frac{\bar{\kappa}}{1 - \kappa} + \frac{1}{1 - \kappa} \left[(\omega_x + 1)(1 - \phi_x)\theta_x + \omega_v(1 - \phi_v)\theta_v + \frac{\alpha}{2}\omega_v^2 \sigma_v^2 \right].
\end{aligned}$$

This solution allows us to simplify the new term (in square brackets) in the Epstein-Zin marginal rate of substitution,

$$\begin{aligned}
w_{t+1} + x_{t+1} - \log \mu_t(w_{t+1} + x_{t+1}) &= (\omega_x + 1)[x_{t+1} - E_t x_{t+1}] + \omega_v[v_{t+1} - E_t v_{t+1}] \\
&\quad - \frac{\alpha}{2}(\omega_x + 1)^2 \text{Var}_t[x_{t+1}] - \frac{\alpha}{2}\omega_v^2 \text{Var}_t[v_{t+1}] \\
&= (\omega_x + 1)v_t^{1/2} \varepsilon_{t+1}^x + \omega_v \sigma_v \varepsilon_{t+1}^v - \frac{\alpha}{2}(\omega_x + 1)^2 v_t - \frac{\alpha}{2}\omega_v^2 \sigma_v^2.
\end{aligned}$$

Therefore, the real pricing kernel in this model is of the Duffie-Kan class with 2-factors and

with parameters

$$\begin{aligned}
\delta &= -\log(\beta) + (1 - \rho)(1 - \phi_x)\theta_x + \frac{\alpha}{2}(\alpha - \rho)\omega_v^2\sigma_v^2 \\
\gamma &= [\gamma_x \quad \gamma_v]' \\
&= \left[(1 - \rho)\phi_x \quad \frac{\alpha}{2}(\alpha - \rho) \left(\frac{1}{1 - \kappa\phi_x} \right)^2 \right]' \\
\lambda &= [\lambda_x \quad \lambda_v]' \\
&= \left[(1 - \alpha) - (\alpha - \rho) \left(\frac{\kappa\phi_x}{1 - \kappa\phi_x} \right) \quad - \left(\frac{\alpha}{2} \right) \left(\frac{\kappa(\alpha - \rho)}{1 - \kappa\phi_v} \right) \left(\frac{1}{1 - \kappa\phi_x} \right)^2 \right]'.
\end{aligned}$$

We can now use the recursive formulas in equation (2) to solve for *real* discount bond prices.

Note how the factor loadings and prices of risk depend on the deeper structural parameters, and the greatly reduced dimensionality of the parameter space relative to the general affine model. Also note that for the time-additive expected utility special case, $\alpha = \rho$, the volatility factor does not enter conditional mean of the pricing kernel, $\gamma_v = 0$, and the price of risk for the volatility factor is zero, $\lambda_v = 0$.

Finally, we can see from these expressions for bond prices that the two key preference parameters, ρ and α , provide freedom in controlling both the factor loadings and the prices or risk.

4 Nominal Bond Pricing

The *nominal* pricing kernel is given by

$$\log(m_{t+1}^{\$}) = \log(m_{t+1}) - p_{t+1}, \quad (4)$$

where p_{t+1} is the log of the money-price of goods at $t + 1$ relative to t , *i.e.*, inflation.

4.1 Exogenous Inflation

If we expand the state space to include an exogenous inflation process, p_t , the state vector becomes $z_t = [x_t \ v_t \ p_t]'$. Further, we assume that the stochastic process for inflation is given by

$$p_{t+1} = (1 - \phi_p)\theta_p + \phi_p p_t + \sigma_p \varepsilon_{t+1}^p,$$

where ε_{t+1}^p is also normally distributed independently of the other two shocks. In this case, the parameters for the affine nominal pricing kernel are given by

$$\begin{aligned}\delta^\$ &= \delta + (1 - \phi_p)\theta_p \\ \gamma^\$ &= [\gamma_x \ \gamma_v \ \phi_p]' \\ \lambda^\$ &= [\lambda_x \ \lambda_v \ 1]'. \end{aligned}$$

Note that in this exogenous inflation model, the price of inflation risk is always exactly 1, and does not change with the values of deeper parameters.

4.2 Endogenous Monetary Policy

We begin by assuming that monetary policy follows an very simple nominal interest-rate rule (we will abuse conventional terminology and often refer to this rule as a Taylor rule):

$$\hat{i}_t = \bar{\tau} + \tau_p p_t + s_t, \tag{5}$$

where the monetary policy shock satisfies

$$s_t = \phi_s s_{t-1} + \sigma_s \varepsilon_t^s,$$

and where ε^s is a standard normal shock, independent across time and independent of the other two real shocks. Note that there are a variety of ways to specify Taylor rules (see Ang

et al. (2004)). We begin with this simple rule that targets the short rate as a function of current inflation to derive our basic results. We discuss straightforward extensions of this rule below.

Since this nominal interest rate rule must also be consistent with equilibrium in the bond market, *i.e.*, it must be consistent with the nominal pricing kernel in equation (4) as well as equation (5), we can use these two equations to find the equilibrium process for inflation. Guess a log-linear solution for p_t

$$p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t. \quad (6)$$

Substitute this guess into both the Taylor rule and the nominal pricing kernel and choose parameters $\bar{\pi}$, π_x , π_v and π_s to equate the pricing-kernel-determined short rate with the Taylor-rule-determined short rate. This implies the parameters for the equilibrium inflation process are given by

$$\begin{aligned} \pi_x &= \frac{\gamma_x}{\tau_p - \phi_x} \\ \pi_v &= \frac{\gamma_v - \frac{1}{2}(\lambda_x + \pi_x)^2}{\tau_p - \phi_v} \\ \pi_s &= -\frac{1}{\tau_p - \phi_x} \\ \bar{\pi} &= \frac{1}{\tau_p - 1} \left[\delta - \bar{\tau} + \pi_x(1 - \phi_x)\theta_x + \pi_v(1 - \phi_v)\theta_v - \frac{1}{2}(\lambda_v + \pi_v)^2\sigma_v^2 - \frac{1}{2}\pi_s\sigma_s^2 \right]. \quad (7) \end{aligned}$$

(See Cochrane (2006) for a more detailed account of this rational expectations solution method.)

It is clear from these expressions that the equilibrium inflation process will depend on the preference parameters of the household generally, and attitudes towards risk specifically.

Extending the analysis to include more general Taylor rules is straightforward. For example,

had we written the policy rule as a function of the endowment growth as well as inflation,

$$i_t = \bar{r} + \tau_x x_t + \tau_p p_t + \tau_s s_t,$$

the endogenous inflation process would have the same values of τ_p and τ_s , but the inflation response to changes in endowment growth would become

$$\pi_x = \frac{\gamma_x - \tau_x}{\tau_p - \phi_x}.$$

In a similar fashion, we can extend the analysis to any Taylor rule that is linear in state variables, including lagged short rates, other contemporaneous yields at any maturity, as well as forward-looking rules (as in Clarida et al. (2000)), since in the affine framework, these are all simply linear functions of the current state variables. (See Ang et al. (2004) for concrete examples.)

4.3 A Monetary-Policy Consistent Pricing Kernel

Substitute the equilibrium inflation process from equations (6) and (7) into the nominal pricing kernel to obtain a 3-factor model that is consistent with the nominal-interest rate rule. The state space is given by

$$\begin{aligned} z_t &\equiv [x_t \ v_t \ s_t]' \\ \Phi &= \text{diag}\{\phi_x, \phi_v, \phi_s\} \\ \theta &= [(1 - \phi_x)\theta_x \quad (1 - \phi_v)\theta_v \quad 0]' \\ \Sigma(z_t) &= \text{diag}\{a_1 + b'_1 z_t, a_2 + b'_2 z_t, a_3 + b'_3 z_t\} \\ a_1 &= 0, \quad b_1 = [0 \ 1 \ 0]' \\ a_2 &= \sigma_v^2, \quad b_2 = [0 \ 0 \ 0]' \\ a_3 &= \sigma_s^2, \quad b_3 = [0 \ 0 \ 0]', \end{aligned}$$

and the parameters of the pricing kernel are given by

$$\begin{aligned}\delta^{\$} &= \delta + \bar{\pi} + \phi_x(1 - \phi_x)\theta_x + \pi_v(1 - \phi_v)\theta_v \\ \gamma^{\$} &= [\gamma_x + \phi_x\pi_x \quad \gamma_v + \phi_v\pi_v \quad \phi_x\pi_s]' \\ \lambda^{\$} &= [\lambda_x + \pi_x \quad \lambda_v + \pi_v \quad \pi_s]'\end{aligned}$$

Note that the Taylor-rule parameters, through their determination of the equilibrium inflation process, affect both the factor loadings on the real factors as well as their prices or risk. Potentially, therefore, this endogenous monetary policy can predict significantly different risk premiums in the term structure than the exogenous-inflation model. We explore this possibility through numerical examples.

5 Quantitative Exercises

We calibrate our model to post-war US data as follows:

1. Endowment Growth. $\phi_x = 0.3549$, $\theta_x = 0.0052$, $\sigma_x = 0.0048(1 - \phi_x^2)^{1/2}$;
2. Inflation. $\phi_p = 0.8471$, $\theta_p = 0.0093$, $\sigma_p = 0.0063(1 - \phi_p^2)^{1/2}$;
3. Volatility. $\phi_v = 0.9000$, $\theta_v = 0.0006$, $\sigma_v = 0.0085(1 - \phi_v^2)^{1/2}$;
4. Policy Shock. $\phi_s = 0$, $\sigma_s = 0.0060$.

The endowment growth process is calibrated to quarterly per capita consumption of durables and services and inflation is calibrated to the nondurables and services deflator, as in Piazzesi and Schneider (2006). The volatility process is taken from Bansal and Yaron (2004). The monetary policy shock does not bear any close resemblance to anything measured in data and given its simple form, it does not contribute significantly to the risk premiums in the yield curve.

Figure 4 depicts the average yield curve for the Epstein-Zin stochastic volatility model with both exogenous inflation and endogenous, Taylor-rule driven, inflation. The parameters \bar{r} and β are not separately identified in these models, so we implicitly choose values so that the average short rate matches the data. We fix the other parameters of the model such that $\rho = 0$, $\alpha = -2.91$, $\tau_p = 1.5$. That is, deterministic substitution is logarithmic, risk aversion is moderate, and the feedback of inflation to the short rate through the Taylor rule is governed by a fairly standard parameter value (see Clarida et al. (2000)). The value of α is chosen to create a sufficiently large average risk premium to roughly match the average long yield.

Figure 4 demonstrates that with these parameter values, the exogenous inflation model does a fairly good job of matching the shape of the historical average yield curve. Note that the red *s in the graph represent properties of the data and the solid blue line is a property of the calibrated the model. However, this model exhibits more volatility in short rates and less volatility in long rates than found in the data (the lower line in Figure 5). This is a fairly standard finding for term structure models with stationary dynamics (see Backus and Zin (1994)).

When we add to this environment a Taylor rule that raises the short interest rate aggressively in response to current inflation, $\tau_p = 1.5$, and re-solve for yields using the endogenous equilibrium process for inflation, we see in Figure 4 that risk premiums increase substantially. In addition, Figure 5 demonstrates that the volatility of yields increases as all maturities as a result of this endogenous monetary policy.

The reason for these increases is evident when we look at the yield-factor loadings, $B^{(n)}$, across the exogenous inflation model and the Taylor rule model. Figure 6 plots the loadings on each of the state variables for the two models. The first panel shows that the factor loading on endowment growth increases slightly and second panel shows that the factor loading on the volatility factor increases substantially. This accounts for both the increased premiums and the increased volatility. (The third panel plots the loadings on the exogenous

inflation factor and the monetary policy shock which are not directly comparable.)

Finally, when we can see that if we lower the degree of risk aversion in the endogenous-inflation economy from $\alpha = -2.91$ to $\alpha = -2.77$, we find that the model once again fits the shape of the average yield curve (Figure 7). Moreover, it exhibits the volatility of yields is still high at the short end relative to the data and low at the long end (Figure 8). It appears that when contemplating more serious econometric exercises with this model, it may be difficult to distinguish the monetary policy parameter, τ_p , from the degree of risk aversion, α . (See Cochrane (2006) for a deeper discussion of similar identification issues.)

6 Related Research

This model we develop is similar to a version of Bansal and Yaron (2004) that includes stochastic volatility, however our simple autoregressive state-variable processes do not capture their richer ARMA specifications. This paper adds to a large and growing literature combining structural macroeconomic models that include Taylor rules with arbitrage-free term structures models. Ang and Piazzesi (2003), following work by Piazzesi (2005), have shown that a factor model of the term structure that imposes no-arbitrage conditions can provide a better empirical model of the term structure than a model based on unobserved factors or latent variables alone. Estrella and Mishkin (1997), Evans and Marshall (1998), Evans and Marshall (2001), Hördahl et al. (2003), Bekaert et al. (2005), and Ravenna and Seppala (2006) also provide evidence of the benefits of building arbitrage-free term-structure models with macroeconomic fundamentals. Rudebusch and Wu (2004) and Ang et al. (2004) investigate the empirical consequences of imposing an optimal Taylor Rule on the performance of arbitrage-free term-structure models.

7 Conclusions

This paper demonstrates that an endogenous monetary policy that involves an interest-rate feedback rule, can contribute to the riskiness of multi-period bonds by creating an endogenous inflation process that exhibits significant covariance risk with the pricing kernel. We explore this through a recursive utility model with stochastic volatility which generates sizable average risk premiums. Our results provoke a number of additional questions: The Taylor rule that we work with is arbitrary. How would the predictions of the model change with alternative specification of the rule? In particular, how would adding monetary non-neutralities along the lines of a New Keynesian Phillips curve (as in Clarida et al. (2000) and Gallmeyer et al. (2005)) alter the monetary-policy consistent pricing kernel? What Taylor rule would implement an optimal monetary policy in this context? (Note that since preferences have changed relative to the models in the literature, this is a nontrivial theoretical question.)

In addition, the simple calibration exercise in this paper is not a very good substitute for a more serious econometric exercise. Further research will explore the tradeoffs between shock specifications, preference parameters and monetary policy rules for empirical yield curve models that closer match historical evidence.

Finally, it would be instructive to compare and contrast the recursive utility model with stochastic volatility to other preference specifications that are capable of generating realistic risk premiums. The leading candidate on this dimension is the external habits models of Campbell and Cochrane (1999). We are currently pursuing an extension of the external habits model in Gallmeyer et al. (2005) to include an endogenous, Taylor-rule driven inflation process.

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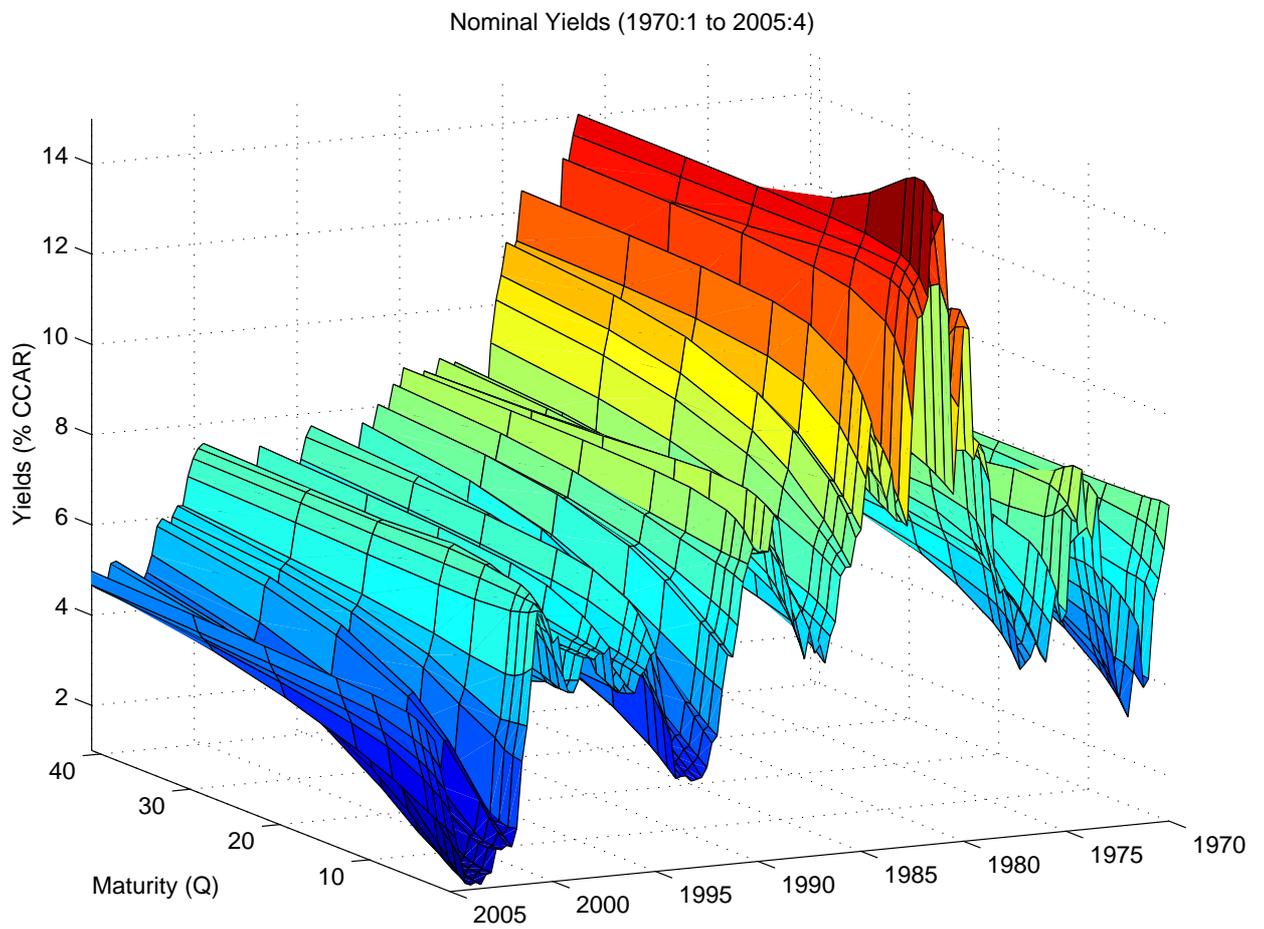


Figure 1: Time series properties of the yield curve, 1970:1 to 2005:4.

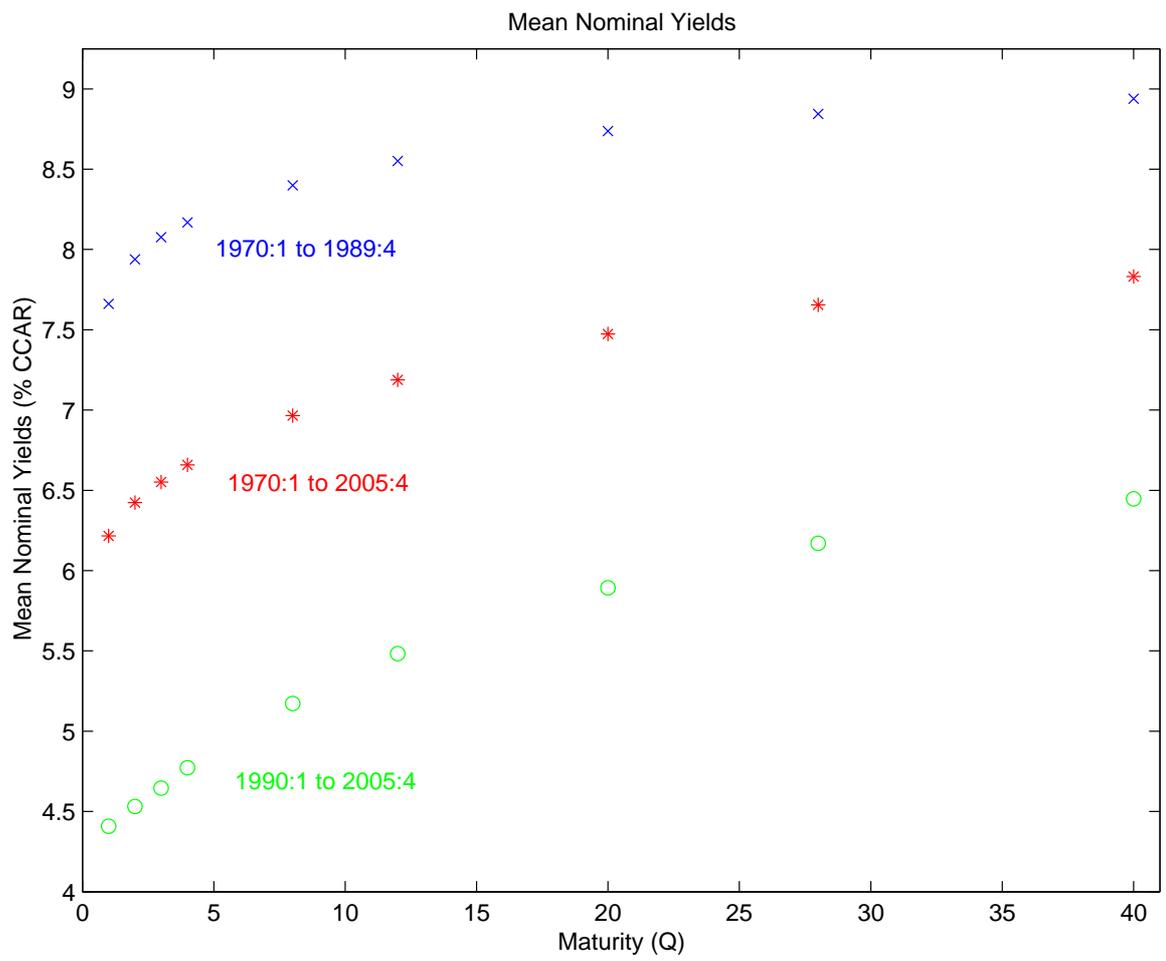


Figure 2: Average yield curve behavior.

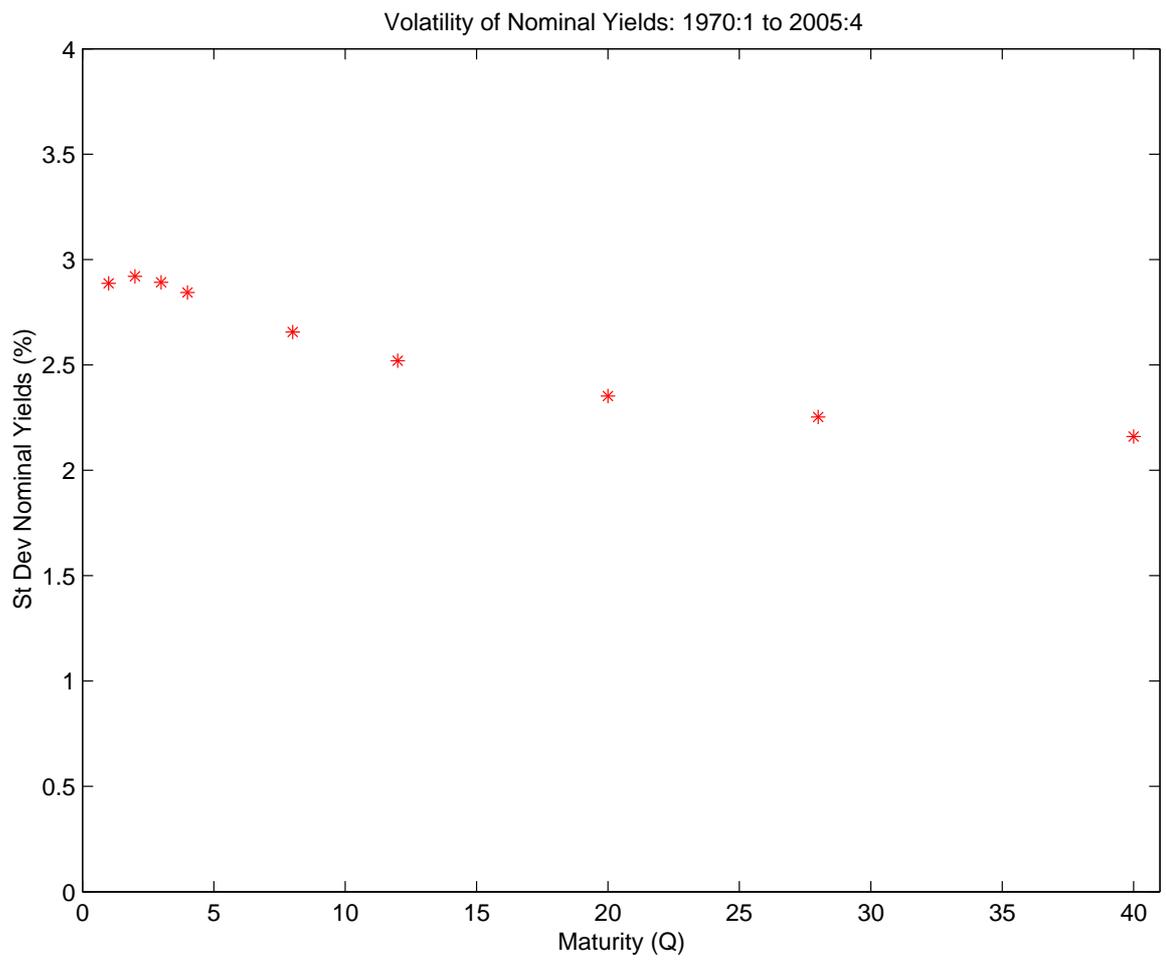


Figure 3: **Volatility of yields of various maturities.**

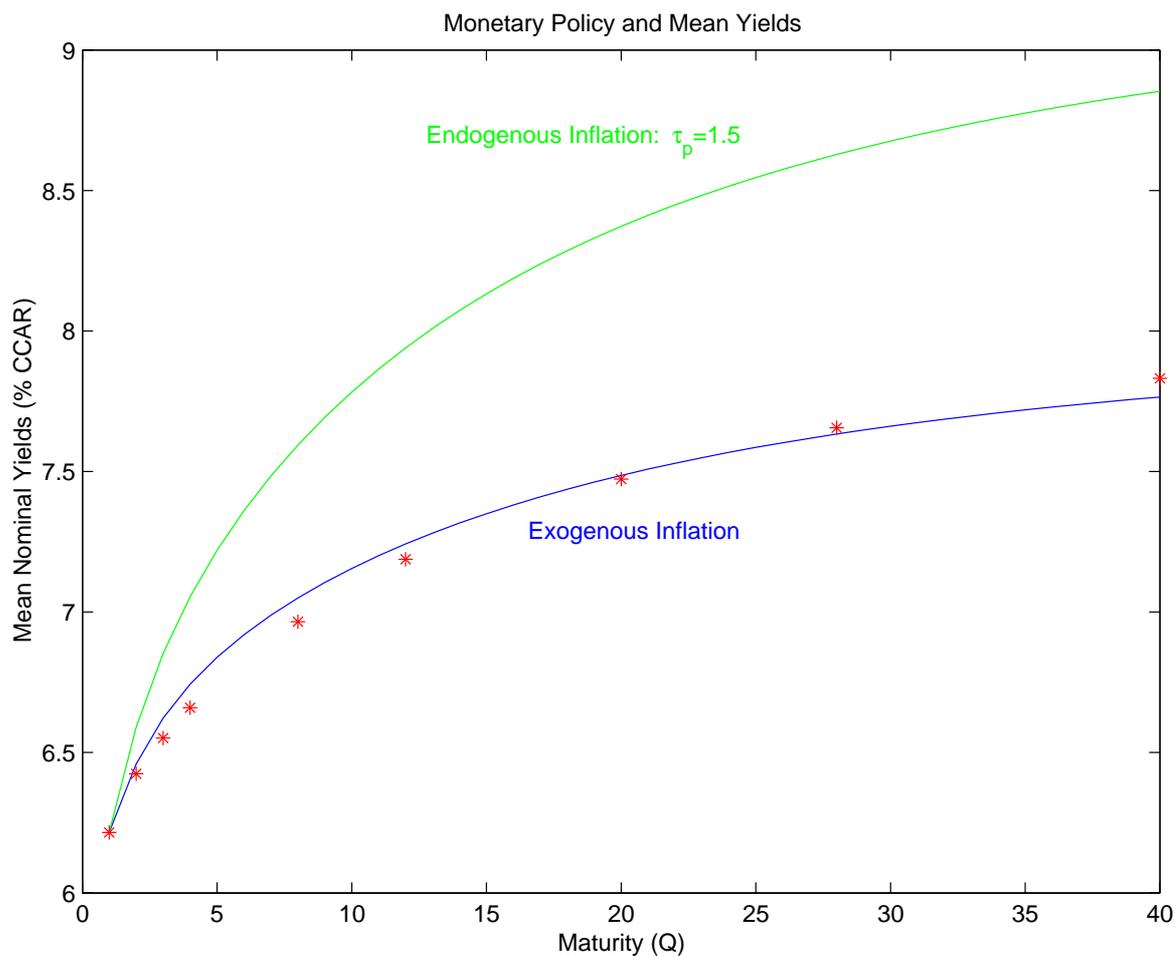


Figure 4: Average yield curve for the Epstein-Zin model with stochastic volatility. Preference parameters: $\rho = 0$, $\alpha = -2.91$. Monetary policy parameter: $\tau_p = 1.5$.

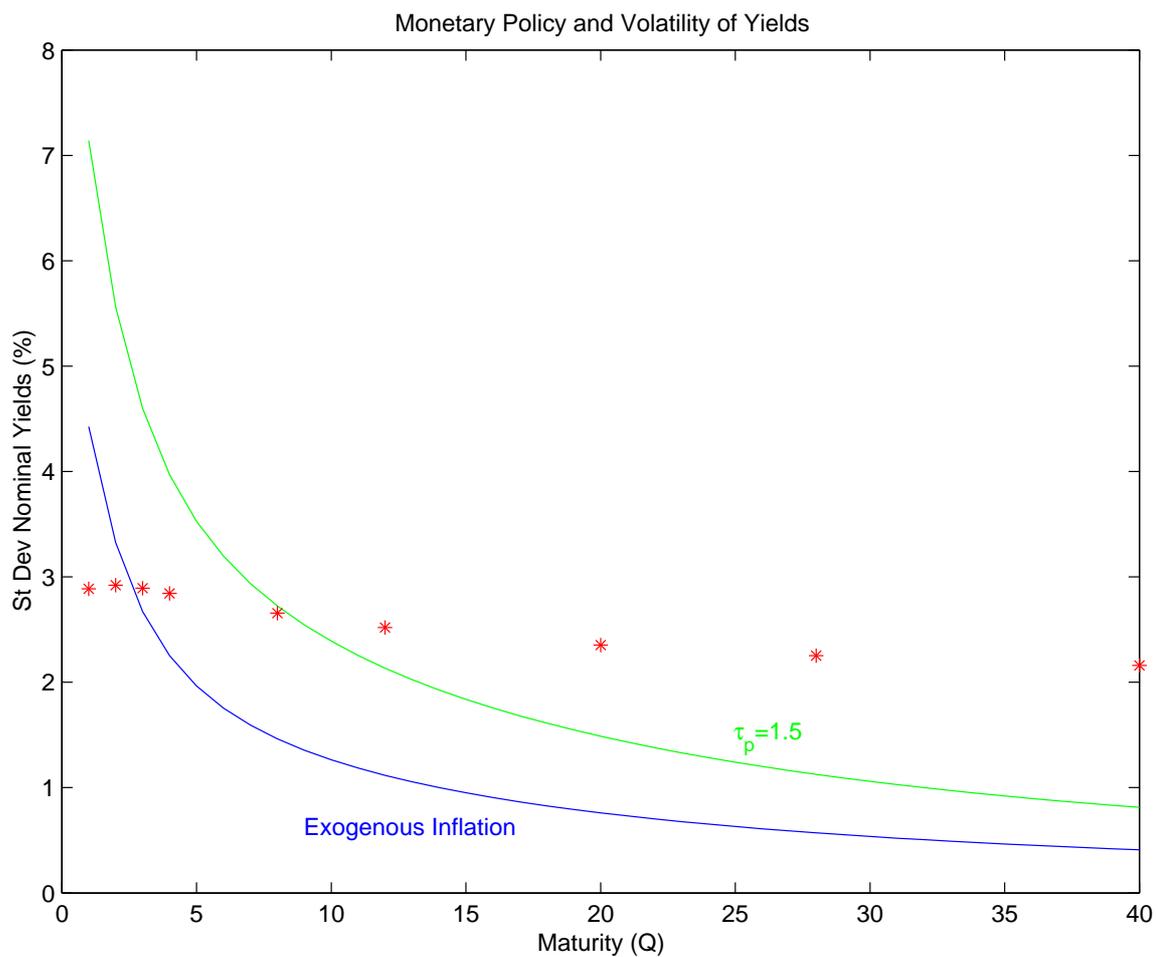


Figure 5: **Volatility of the yield curve for the Epstein-Zin model with stochastic volatility.** Preference parameters: $\rho = 0$, $\alpha = -2.91$. Monetary policy parameter: $\tau_p = 1.5$.

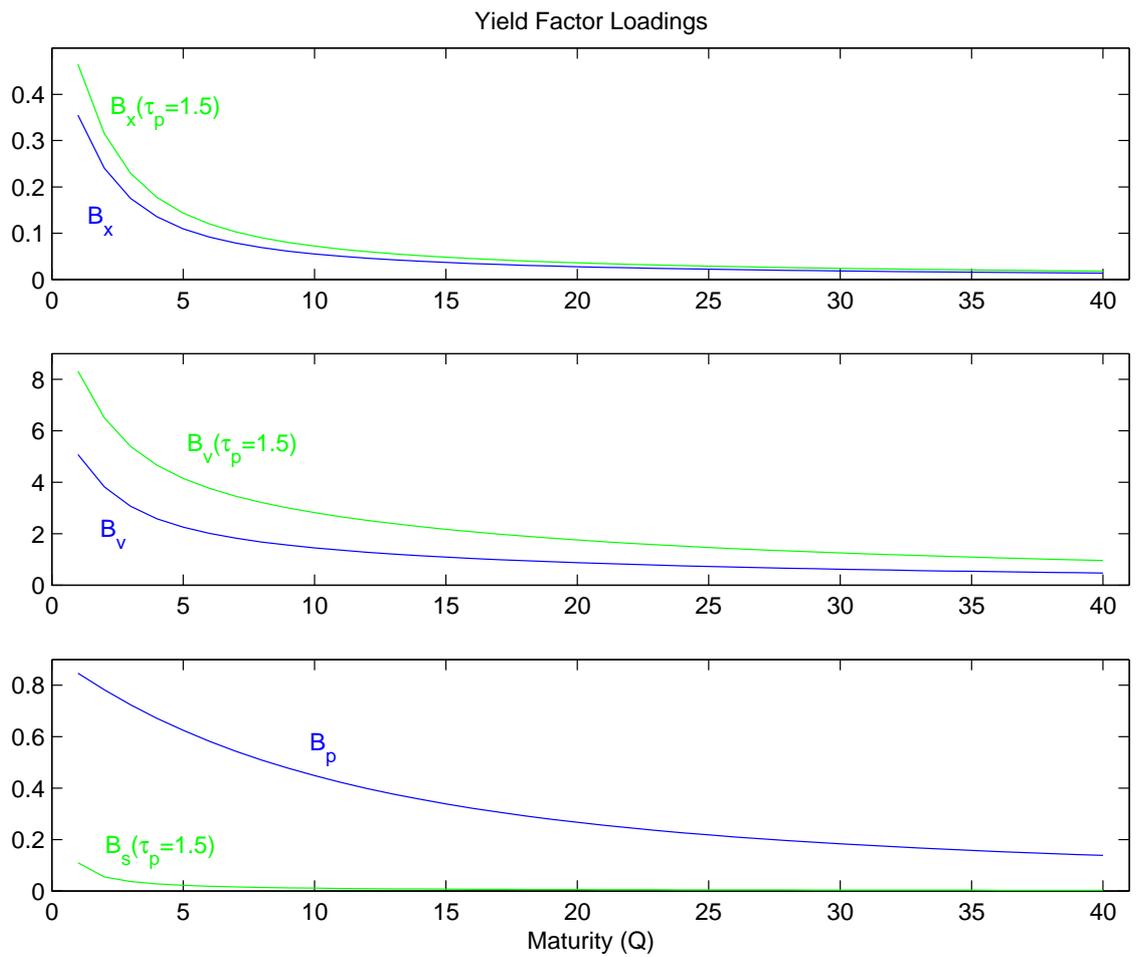


Figure 6: **Yield-Factor Loadings for the Epstein-Zin model with stochastic volatility.** Preference parameters: $\rho = 0$, $\alpha = -2.91$. Monetary policy parameter: $\tau_p = 1.5$.

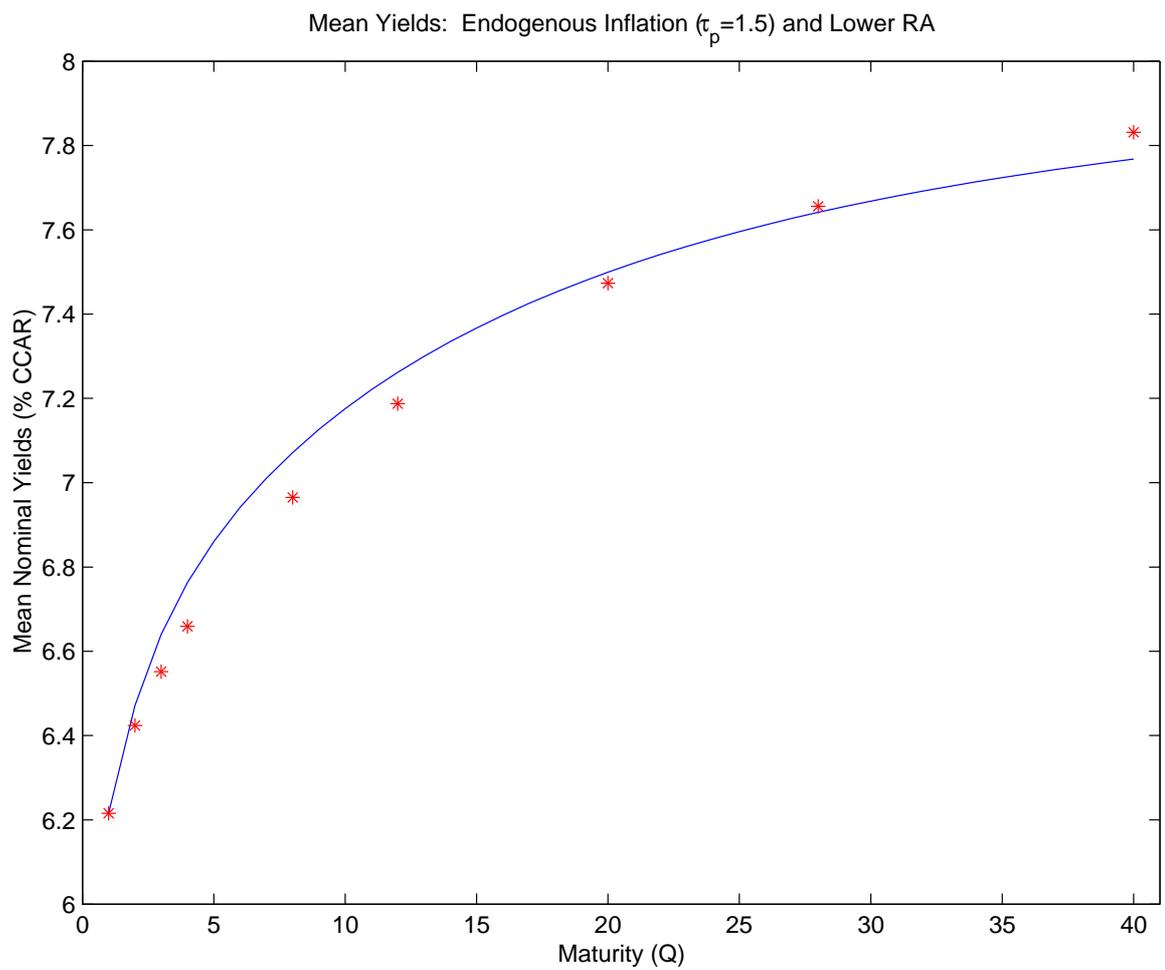


Figure 7: Average yield curve for the Epstein-Zin model with stochastic volatility. Preference parameters: $\rho = 0$, $\alpha = -2.77$. Monetary policy parameter: $\tau_p = 1.5$.

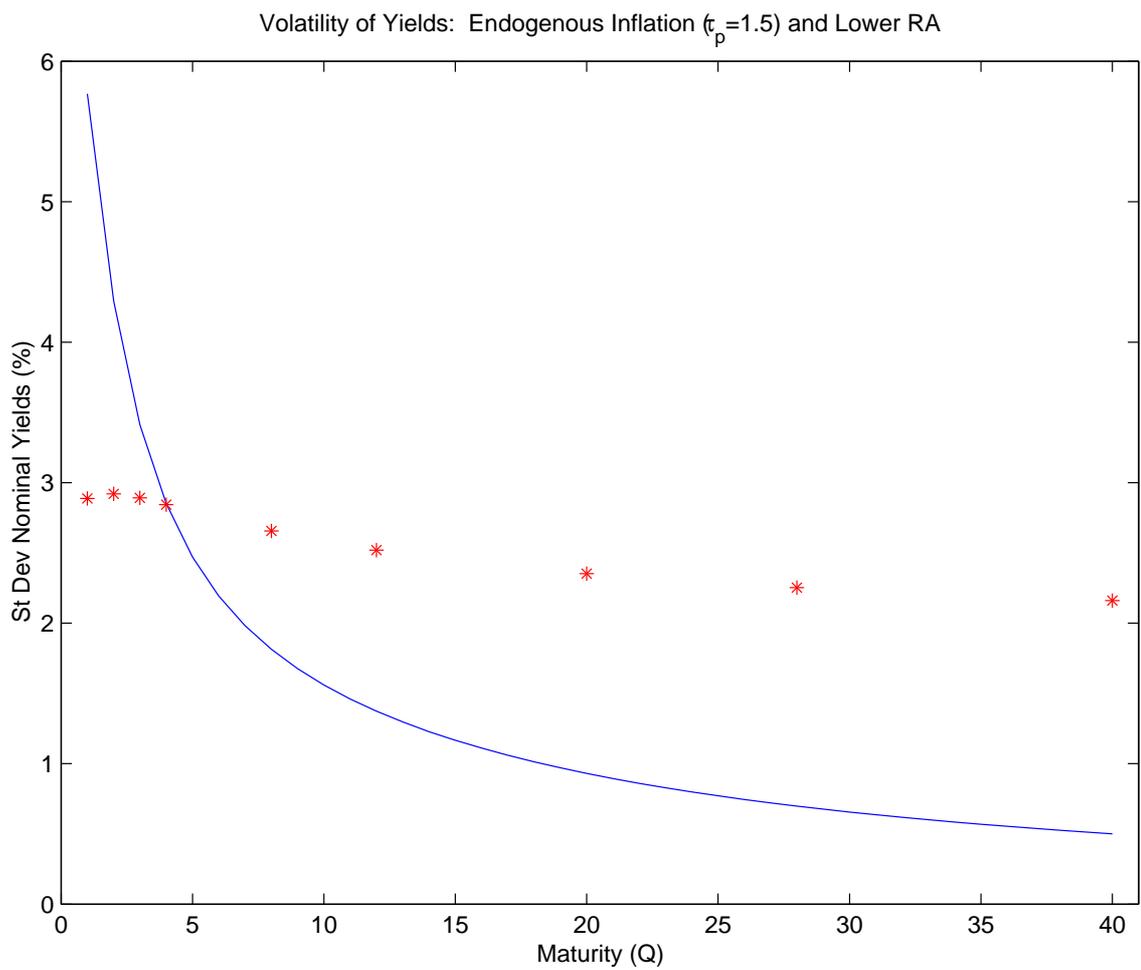


Figure 8: **Volatility of the yield curve for the Epstein-Zin model with stochastic volatility.** Preference parameters: $\rho = 0$, $\alpha = -2.77$. Monetary policy parameter: $\tau_p = 1.5$.