

Comments on “Arbitrage-Free Bond Pricing with Dynamic
Macroeconomic Models” by Gallmeyer, Hollifield, Palomino and Zin
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The questions addressed in this paper are important ones: How does monetary policy affect long-term interest rates? How can we explain the volatility of the long end of the yield curve and its relationship with monetary policy? A successful model is one that answers these questions and is useful to policy makers. To be useful to policy makers, the model should yield quantitative answers to questions like “What will happen to long-term rates if the central bank raises the federal funds rate by 25 basis points? Since money is not introduced in a way that is essential¹, this model is not designed to answer questions about the mechanism by which monetary policy affects the yield curve. This is not necessarily a deficiency since models where money is essential have yet to prove useful in policy discussions and empirical results.² The strength of this model is that it provides quantitative answers to the questions. Hence, it is reasonable to set a standard of success where the model-generated time series must, in some sense, “look like” actual time series.

As in any paper where hard modeling decisions are made, there are both strengths and weaknesses in the choices made by the authors. To describe these choices concisely, I’ll provide a brief overview of the model.

1 Overview of the model

There is a single exogenous, stochastic and perishable endowment good. The endowment grows at the rate x_t with stochastic volatility v_t ,

$$\begin{aligned}x_{t+1} &= (1 - \phi_x)\theta_x + \phi_x x_t + v_t^{\frac{1}{2}} \epsilon_{t+1}^x \\v_{t+1} &= (1 - \phi_v)\theta_v + \phi_v v_t + \sigma_v \epsilon_{t+1}^v.\end{aligned}$$

Hence, the endowment process is characterized by a parameter vector $(\phi_x, \phi_v, \theta_x, \theta_v, \sigma_v)$.

There is a representative agent with Epstein-Zin preferences. The advantages of this preference structure, with its property of separating relative risk aversion from the elasticity of intertemporal substitution, are well known. The preference parameter vector is (β, ρ, α) . The pricing kernel is the intertemporal marginal rate of substitution in consumption, denoted $\log(m_{t+1})$. The price of default-free discount bonds can be determined recursively through an arbitrage-free restriction of the form

$$b_t^{(n)} = E_t m_{t+1} b_{t+1}^{(n-1)}.$$

With this structure for the endowment process and the preferences, the model can be mapped into the Duffie-Kan affine term-structure model with two factors. In particular, the authors

¹Wallace [2001] says that money is “essential” in an economy if it permits allocations that would otherwise not be achieved.

²See Kocherlakota [2002].

guess a form of the value function and then verify this guess is a solution. The value function then implies the form of the real pricing kernel. The process by which the authors relate the deeper preference and endowment parameters to the discrete time affine term structure model is interesting and elegant.

To introduce money and inflation into the model, the authors add a stochastic process to the real pricing kernel

$$\log(m_{t+1}^{\$}) = \log(m_{t+1}) - p_{t+1},$$

where p_{t+1} is inflation. Hence, money, prices and inflation in the model are merely noise, creating a wedge between the real and nominal pricing kernels. Two specifications of the inflation process are studied: exogenous and endogenous.

1.0.1 Exogenous inflation

Inflation is conjectured to take the form

$$p_{t+1} = (1 - \phi_p)\theta_p + \phi_p p_t + \sigma_p \epsilon_{t+1}^p.$$

This specification does not link inflation to the endowment process or to any money growth process. The state space is expanded to x_t, v_t, p_t . They calibrate the model to data and set the parameters values at

$$\begin{aligned}\phi_p &= 0.8471 \\ \theta_p &= 0.0093 \\ \sigma_p &= 0.0063(1 - \phi_p^2)^{\frac{1}{2}}.\end{aligned}$$

The preference parameters are set at $\rho = 0$ and $\alpha = -2.91$, where ρ is the elasticity of intertemporal substitution and α is relative risk aversion.³ and there is little discussion of this choice. With this set of parameter values, the model is used to derive a yield curve. The result is a model-generated yield curve that matches the shape of historical yield curve, but exhibits less volatility in long rate. Since an explicit goal of the paper is to explain volatility at the long end of the yield curve, this answer is not satisfactory. Three comments arise: The first is why are these values for the preference parameters chosen? How does varying the preference parameters change the results? Should we just aimlessly search the parameter space for a better fit? This leads to my second comment, which is that the model, as specified, can be estimated using formal econometric techniques. The endowment process and inflation processes, along with the real and nominal pricing kernels, form a system of equations. Using data on nominal interest rates, inflation and consumption growth, the parameters of the model, and in particular the preference parameters, can be estimated. The preference parameter estimates could then be used to generate a real pricing kernel. What are the estimated preference parameters and does economic intuition suggest that they are sensible? How does the real pricing kernel behave? It seems a missed opportunity.

³These are the values chosen in the version of the paper presented at the conference.

My final comment in this section is that the model, as posed, severely restricts the price of inflation risk by fixing the price of inflation risk at unity. A very simple cash-in-advance model with fixed velocity has the property that inflation is a function of both money growth and output growth, so there is a state-varying inflation risk premium. Even with a zero mean, the inflation risk premium may be an additional source of variability, and may help to remedy the lack of volatility at the long end of the yield curve in the model.

1.0.2 Endogenous Inflation

To make inflation endogenous, the authors assume that monetary policy follows a nominal interest rate rule (Taylor rule) of the form

$$i_t = \bar{\tau} + \tau_p p_t + s_t.$$

This rule raises short-term rates aggressively in response to inflation. There are many other specifications that could be considered and it would be helpful to have a discussion on why this rule is chosen over other specifications. Is this the type of rule the authors believe monetary authorities are using? Is this the rule that gives the best results in the sense that the model-generated yield curve matches the data? Is it chosen for tractability?

This process must be consistent with the other equations in the model, which requires the derivation of an inflation process consistent with the interest rate rule and other equations. To link the rule to the nominal pricing kernel and bond market equilibrium, they use a guess and verify method to derive a consistent inflation process of the form

$$p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t$$

The state space is now $z_t = (x_t, v_t, s_t)$ and there are additional restrictions on the means and conditional variances. Once again they calibrate the model choosing parameter values taken from the data. The calibrated model fits the average yield curve and has a volatility pattern closer to data, especially at long end.

This is a major goal of the paper and, in that sense, is successful, but the question arises as to whether the endogenous inflation process in any way resembles the inflation process in the data. My conjecture is that it does not, and in some important ways, and the differences need to be made explicit. How do the endogenous and exogenous inflation processes compare? If the calibrated model fits the average yield curve and closely matches the volatility, but is based on an inflation process that differs significantly from the actual inflation process, how useful is this to policy makers? How useful is a model that fits the yield curve and its volatility but is greatly at odds with the actual inflation process? Common sense would suggest it is of limited usefulness.

2 Conclusion

The authors are to be commended for devising a model with such a rich potential for explaining yield curves and their volatility. Linking the Epstein-Zin preferences to a discrete

time affine term structure model is no easy task, although they seem to do so effortlessly. The model is devised to be estimated with standard econometric methods using data on bond prices, consumption growth, and inflation. Such an exercise would provide useful insights into the real pricing kernel and the parameters of the Epstein-Zin preferences. The exogenous inflation specification is too restrictive and should permit a variable inflation risk premium. Finally, alternative interest rate rules should be examined with the explicit goal of generating an inflation process using the model that matches the actual inflation process, according to some explicit criterion. While much of this may sound negative, I want to emphasize that this model has the potential to be very useful to policy makers and the steps needed to make it so are straightforward ones to take.

3 References

- Kocherlakota, N. [2002]. “Money: What’s the Question and Why Should We Care about the Answer?” *American Economic Review*, **92**, 2, pp.58–61.
- Wallace, N. [2001]. “Lawrence R. Klein Lecture 2000: Whither Monetary Economics?” *International Economic Review*, **42**,4, pp.847–869.