Long Run Risks and Financial Markets

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Abstract

Recent work shows that concerns about (i) long run expected growth and (ii) uncertainty about future economic prospects, drive asset prices. These two channels of economic risks can account for the risk premia and asset price fluctuations. Hence, the long run risks model potentially provides a coherent and systematic framework for analyzing financial markets.

1 Introduction

Many key features of asset markets are puzzling from the perspective of theoretical models. Among others, these include the equity premium (see Mehra and Prescott, 1985) and asset price volatility (see Shiller, 1981) puzzles, and the large cross-sectional differences in premia across firm characteristic sorted equity portfolios, such as value and growth. In bond and foreign exchange markets the violations of the expectations hypothesis (see Fama and Bliss, 1987; Fama, 1984) and the ensuing return predictability is quantitatively difficult to explain. What risks and concerns can provide an unified explanation for these asset market facts? One potential explanation of all these anomalies is the long run risks model developed in Bansal and Yaron (2004) (BY). In this model the main economic channels that drive financial markets are the fluctuations in the long run growth prospects of the economy and the level of economic uncertainty (consumption or output volatility). Recent work indicates that many of the asset prices anomalies are a natural outcome of these channels. In this article I explain the key mechanisms in the BY model that enable it to account for the asset market puzzles.

In Bansal and Yaron (2004) model, the first economic channel relates to expected growth — consumption and dividend growth rates contain a small long run component in the mean. That is, current shocks to expected growth alter expectations about future economic growth not only for short horizons but also for the very long run. The second channel pertains to varying economic uncertainty — the conditional volatility of consumption is time-varying. Fluctuations in consumption volatility lead to time-variation in risk premia. Agents fear adverse movements in the long run growth and volatility components as they lower the equilibrium consumption, wealth and asset prices. This makes holding equity quite risky, leading to high risk compensation in equity markets.

Epstein and Zin (1989) preferences play an important role in the model. These preferences allow for separation between risk aversion and intertemporal elasticity of substitution (IES) of investors. The magnitude of the risk aversion relative to the reciprocal of the IES determines whether agents prefer early or late resolution of uncertainty regarding the consumption path. In the BY model agents prefer early resolution of uncertainty, that is, risk aversion is larger than the reciprocal of the IES. This ensures that the compensation for long run expected growth risk is positive and quantitatively important. The resulting model is one in which there are three distinct sources of risks that determine the risk premia– short run risk, long run risk, and consumption volatility risk. In the traditional power utility model only the first risk source carries a distinct risk price and the other two risks have zero risk compensation. Separate risk compensation for shocks to consumption volatility and expected consumption growth is a novel feature of the BY model relative to earlier asset pricing models.

To derive model implications for asset prices the preference parameters are calibrated. The calibrated magnitude of the risk aversion and the IES is an empirical issue. Hansen and Singleton (1982), Attanasio and Weber (1989), and Attanasio and Vissing-Jorgensen (2003) estimate the IES to be well in excess of one. Hall (1988) and Campbell (1999), on the other hand, estimate its value to be well below one. Bansal and Yaron (2004) show that even if the population value of the IES is larger than one, the estimation methods used by Hall would measure the IES to be close to zero. That is, in the presence of time-varying consumption volatility, there is a severe downward bias in the point estimates of IES. Bansal, Khatchatrian and Yaron (2005) and Bansal and Shaliastovich (2007) provide further evidence on the magnitude of IES using the data from financial markets.

Different techniques are employed to provide empirical and theoretical support for the existence of long run components in consumption and dividends. While Bansal and Yaron (2004) choose parameters to match the annual moments of consumption and dividend growth rates, Bansal, Gallant and Tauchen (2005) and Bansal, Kiku and Yaron (2006) formally test the model using the efficient and generalized method of moments, respectively. Multivariate analysis of the long-run properties of the consumption and cash flows is presented in Hansen, Heaton and Li (2005), while Colacito and Croce (2006) find statistical support for the long run components in consumption data for the U.S. and other developed economies. Lochstoer and Kaltenbrunner (2006) provide a production-based motivation for the long run risks in consumption. They show that in a standard production economy, where consumption is endogenous, the consumption growth process contains a long run predictable component similar to that in the BY model. There is considerable support for the volatility channel as well. Bansal, Khatchatrian and Yaron (2005) show that consumption volatility is time varying and that its current level predicts future asset valuations (price-dividend ratio) with a significantly negative projection coefficient; this implies that asset markets dislike economic uncertainty. Exploiting the BY uncertainty channel, Lettau, Ludvigson and Wachter (2006) provide interesting market premium implications of the ultra-low frequency decline in consumption volatility.

Using the calibrated model, Bansal and Yaron (2004) show that they can explain the low risk free rate, the equity premium puzzle, high asset price volatility and many of the return and dividend growth predictability dimensions that have been characterized in earlier work. The time-varying volatility in consumption is important to capture some of the economic outcomes that relate to time-varying risk premia.

The arguments presented in their work also has immediate implications for the cross-sectional differences in mean returns across assets. Bansal, Dittmar, and Lundblad (2002 and 2005) show that the systematic risks across firms should be related to the systematic long run risks in firms cash flows that investors receive. Firms whose expected cash-flow (profits) growth rates move with the economy are more exposed to long run risks and hence should carry a higher risk compensation. These authors develop methods to measure the long run risks in cash flows and show that cash flow betas that incorporate long run risks can account very well for for the cross-sectional differences in risk premia of assets. They show that high book-to-market portfolio has larger long run risks beta relative to low book-to-market portfolio of growth firms. Hence, the high mean return of value firms relative to growth firms is not puzzling.

Several recent papers use the long run risk model to address a rich array of asset market questions; among others, these include Kiku (2006), Colacito and Croce (2006), Lochstoer and Kaltenbrunner (2006), Chen, Collin-Dufresne and Goldstein (2006), Chen (2006), Eraker (2006), Piazzesi and Schneider (2005), and Bansal and Shaliastovich (2007). Kiku (2006) shows that the long run risks model can account for the violations of the CAPM and C-CAPM in explaining the cross-sectional differences in mean returns. Further, the model can capture the entire transition density of value or growth returns, which underscores the importance of long run risks in accounting for equity markets behavior. Eraker (2006) and Piazzesi and Schneider (2005) consider the implications of the model for the risk premia on US Treasury bonds and show how to account for some of the average premium puzzles in the term term structure literature. Colacito and Croce (2006) extend the long run risks model to a two country setup and explain the issues about international risk sharing and exchange rate volatility. Bansal and Shaliastovich (2007) show that the long run risks model can simultaneously account for equity markets, yield and foreign exchange behavior, and the explain the nature of predictability and violations of the expectations hypothesis in foreign exchange and Treasury markets. Chen, Collin-Dufresne and Goldstein (2006) and Chen (2006) analyze the ability of the long run risks model to explain the credit spread and leverage puzzles of the corporate sector.

Hansen, Heaton and Li (2005) consider a long run risks model with an unit IES. Using different methods to measure long run risks exposures of book-to-market sorted portfolios, they find that the alternative long run risk measures do line-up in the cross-section with the average returns as in Bansal, Dittmar and Lundblad (2005). However, they also note that the measurement of long run risks can be sensitive to the econometric methods used, given the modest sample size available to an econometrician. Hansen and Sargent (2006) highlight the interesting implications of robust decision making for risks in financial markets when the representative agent entertains the long run risks model as a baseline description of the economy.

The above results indicate that the long run risks model can go a long way towards providing an explanation for many of the asset markets issues.

The remainder of the article has three sections. Section 2 reviews the long run risks setup of Bansal and Yaron (2004) and presents theoretical solutions to the asset prices and discount factor. Section 3 discusses empirical output of the model, and in particular, its implications for the equity, Treasury and currency markets. Section 4 presents concluding comments.

2 Long Run Risks Model

2.1 Preferences and the Environment

Consider a representative agent with the following Epstein and Zin (1989) recursive preferences,

$$U_t = \{(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta}\}^{\frac{\theta}{1-\gamma}},$$

the rate of time preference is determined by δ, with $0 < \delta < 1$. The parameter θ is determined by the risk-aversion and the intertemporal elasticity of substitution (IES) — specifically, $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$, where $\gamma \geq 0$ is the risk-aversion (sensitivity) parameter, and $\psi \geq 0$ is the IES. The sign of θ is determined by the magnitudes of the risk-aversion and the elasticity of substitution. In particular if $\psi > 1$ and $\gamma > 1$ then θ will be negative. Note that when $\theta = 1$, that is $\gamma = \frac{1}{\psi}$, one obtains the the standard case of expected utility.

As is pointed out in Epstein and Zin (1989), when risk aversion equals the reciprocal of IES (expected utility), the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk-version exceeds (is less than) the reciprocal of IES the agent prefers early (late) resolution of uncertainty of consumption path. Hence, these preferences allow for a departure on the agent's preference for the timing of the resolution of uncertainty. In the long run risk model agents prefer early resolution of uncertainty of the consumption path.

The period budget constraint for the agent with wealth W_t and consumption C_t at date t, is

$$W_{t+1} = (W_t - C_t)R_{a,t+1}. \tag{1}$$

The return $R_{a,t+1} = \frac{W_{t+1}}{(W_t - C_t)}$ is the return on aggregate portfolio held by the agent. As in Lucas (1978) we normalize the supply of all equity claims to be one and the risk-free asset to be in zero net supply. In equilibrium, aggregate dividends in the economy (which also include any claims to labor income) equals aggregate consumption of the representative agent. Given a process for aggregate consumption, the return on the aggregate portfolio corresponds to the return on an asset that delivers aggregate consumption as its dividends each time period.

The logarithm of the Intertemporal Marginal Rate of Substitution (IMRS), m_{t+1}, for these preferences (see Epstein and Zin, 1989) is,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} \tag{2}$$

and the asset pricing restriction on any continuous return $r_{i,t+1}$ is

$$E_t \Big[\exp(\theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} + r_{i,t+1}) \Big] = 1, \tag{3}$$

where g_{t+1} equals $\log(C_{t+1}/C_t)$ — the log growth rate of aggregate consumption. The return, $r_{a,t+1}$, is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends each time period.

The return to the aggregate consumption claim, $r_{a,t+1}$, is not observed in the data, while the return on the dividend claim corresponds to the observed return on the market portfolio $r_{m,t+1}$. The levels of market dividends and consumption are not equal; aggregate consumption is much larger than aggregate dividends. The difference is financed by labor income. In the BY model, aggregate consumption and aggregate dividends are treated as two separate processes and the difference between them defines the agent's labor income process.

The key ideas of the model are developed and the intuition is provided via approximate analytical solutions. However, for the key qualitative results the model is solved numerically. To derive the approximate solutions for the model we use the standard approximation utilized in Campbell and Shiller (1988),

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{4}$$

where lowercase letters refer to variables in logs, in particular, $r_{a,t+1} = \log(R_{a,t+1})$ is the continuous return on the consumption claim, and the price to consumption ratio is $z_t = \log(P_t/C_t)$. Analogously, $r_{m,t+1}$ and $z_{m,t}$ correspond to the continuous return on the dividend claim and its log price to dividend ratio. The approximating constants κ_0 and κ_1 are specific to the asset under consideration and depend only on the average level of the asset's valuation ratio z_t. [1] It is important to keep in mind that the average value of z for any asset is endogenous to the model and depends on all its parameters and the dynamics of the asset's

[1] Note that the main approximating constant $\kappa_1 = exp(\bar{z})/(1 + exp(\bar{z}))$. See Campbell and Shiller (1988) for the analogous expression for κ_0.

dividends.

From equation (2) it follows that the innovation in IMRS, m_{t+1}, is driven by the innovations in g_{t+1} and $r_{a,t+1}$. Covariation with the innovation in m_{t+1} determines the risk premium for any asset. We characterize the nature of risk sources and their compensation in the next section.

2.2 Long Run Growth and Economic Uncertainty Risks

The agents' IMRS depends on the endogenous consumption return, $r_{a,t+1}$. The risk compensation on all assets depends on this return which itself is determined by the process for consumption growth. The dividend process is needed for determining the return on the market portfolio. To capture long-run risks, consumption and dividend growth rates, g_{t+1} and $g_{d,t+1}$, are modeled to contain a small persistent predictable component x_t, while fluctuating economic uncertainty is introduced through the time-varying volatility of the cash flows:

$$\begin{aligned} x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\ g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \\ \sigma^2_{t+1} &= \sigma^2 + \nu_1(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \\ & e_{t+1}, u_{t+1}, \eta_{t+1}, w_{t+1} \sim N.i.i.d.(0,1), \end{aligned} \tag{5}$$

with the shocks, e_{t+1}, u_{t+1}, η_{t+1}, and w_{t+1} assumed to be mutually independent. The parameter ρ determines the persistence of the expected growth rate process. First, note that when $\varphi_e = 0$, the processes g_t and $g_{d,t+1}$ have zero auto-correlation. Second, if $e_{t+1} = \eta_{t+1}$, the process for consumption is ARMA(1,1) used in Campbell (1999), Cecchetti, Lam and Mark (1993), and Bansal and Lundblad (2002). If, in addition, $\varphi_e = \rho$, then consumption growth corresponds to an AR(1) process used in Mehra and Prescott (1985). The variable σ_{t+1} represents the time-varying volatility of consumption and captures the intuition that there are fluctuations in the level of uncertainty in the economy. The unconditional volatility of consumption is σ^2. To maintain parsimony, it is assumed that the shocks are uncorrelated, and that there is only one source of time-varying economic uncertainty that affects consumption and dividends.

Two parameters, $\phi > 1$ and $\varphi_d > 1$, calibrate the overall volatility of dividends and its correlation with consumption. The parameter ϕ can be interpreted, as in Abel (1999), as the leverage ratio on expected consumption growth. Alternately, this says that corporate profits, relative to consumption, are far more sensitive to changing economic growth conditions. Note that consumption and dividends are not cointegrated in the above specification — Bansal, Gallant and Tauchen (2005) develop a specification that does allow for cointegration between consumption and dividends.

The better understand the role of long run risks consider the scaled long run variance (or variance ratio) of consumption for horizon J,

$$\sigma^2_{c,J} = \frac{Var[\sum_{j=1}^{J} g_{t+j}]}{J \;\; Var[g_t]}$$

The magnitude of this consumption growth volatility, is the same for all J if consumption is uncorrelated across time. This scaled variance increases with horizon when the expected growth is persistent. Hence, agents face a larger aggregate consumption volatility at longer horizons. As the persistence in x and/or its variance increases the magnitude of long run volatility will rise. In equilibrium, this increase in magnitude of aggregate consumption volatility will require a sizeable compensation if the agents prefer early resolution of uncertainty about the consumption path.

Using multivariate statistical analysis Hansen, Heaton and Li (2005) and Bansal, Kiku and Yaron (2006), provide evidence on the existence of the long run component in observed consumption and dividends. Using simulation methods Bansal and Yaron (2005) document the presence of the long run component in U.S. consumption data while Colacito and Croce (2006) estimate this component in consumption for many developed economies. Note that there can be considerable persistence in the time-varying consumption volatility as well– hence the long run variance of the conditional volatility of consumption can be very large as well.

To see the importance of the small low frequency movements for asset prices, consider the quantity,

$$E_t\Big[\sum_{j=1}^{\infty} \kappa_1^j g_{t+j}\Big].$$

With κ_1 less than one this expectation equals $\frac{\kappa_1 x_t}{1-\kappa_1\rho}$. Even if the variance of x is tiny, but ρ

fairly high, then shocks to x_t can alter growth rate expectations for the long run leading to volatile asset prices. Hence asset markets can legitimately be very concerned about the long run movements in output and cash flows.

2.2.1 Equilibrium and Asset Prices

The consumption and dividend growth rates processes are exogenous in this endowment economy. Further, the IMRS depends on an endogenous return $r_{a,t+1}$. To characterize the IMRS and the behavior of asset returns a solution for the log price-consumption ratio z_t and the log price-dividend ratio $z_{m,t}$ is needed. The relevant state variables for z_t and $z_{m,t}$ are the expected growth rate of consumption x_t and the conditional consumption volatility σ_t^2.

Exploiting the Euler equation (3), the approximate solution for the log price-consumption z_t has the form $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$. The solution for A_1 is,

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}. \tag{6}$$

This coefficient is positive if the IES, ψ, is greater than one. In this case the intertemporal substitution effect dominates the wealth effect. In response to higher expected growth, agents buy more assets, and consequently the wealth to consumption ratio rises. In the standard power utility model with risk aversion larger than one, the IES is less than one, and therefore A_1 is negative — a rise in expected growth potentially lowers asset valuations. That is, the wealth effect dominates the substitution effect.[2]

Corporate payouts (i.e., dividends), with $\phi > 1$, are more sensitive to long run risks and changes in expected growth rate lead to a larger reaction in the price of the dividend claim than in the price of the consumption claim. Hence the expression $A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1,m}\rho}$ for the dividend asset is larger in absolute value relative to the consumption asset.

The solution coefficient A_2 for measuring the sensitivity of price-consumption ratio to

[2] An alternative interpretation with the power utility model is that higher expected growth rates increase the risk-free rate to an extent that discounting dominates the effects of higher expected growth rates. This leads to a fall in asset prices.

volatility fluctuations is

$$A_2 = \frac{0.5[(\theta - \frac{\theta}{\psi})^2 + (\theta A_1 \kappa_1 \varphi_e)^2]}{\theta(1 - \kappa_1 \nu_1)} . \tag{7}$$

An analogous coefficient for the market price-dividend ratio, $A_{2,m}$, is provided in Bansal and Yaron (2004).

The expression for A_2 provides two valuable insights. First, if the IES and risk aversion are larger than one, then θ, and consequently A_2 are negative. In this case a rise in consumption volatility lowers asset valuations and increases the risk premia on all assets. Second, an increase in the permanence of volatility shocks, that is ν_1, magnifies the effects of volatility shocks on valuation ratios as changes in economic uncertainty are perceived by investors as being long lasting.

2.2.2 Pricing of Short Run, Long Run, and Volatility Risks

Substituting the solutions for the price-consumption ratio z_t into the expression for equilibrium return for $r_{a,t+1}$ in equation (4), one can now characterize the solution for the IMRS that can be used to value all assets. The log of the IMRS m_{t+1} can always be stated as the sum of its conditional mean and its one step ahead innovation. The conditional mean is affine in expected mean and conditional variance of consumption growth, and can be expressed as,

$$E_t(m_{t+1}) = m_0 - \frac{1}{\psi} x_t + \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[1 + \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right] \sigma_t^2. \tag{8}$$

m_0 is a constant determined by the preference and consumption dynamics parameters.

The innovation in the IMRS is very important for thinking about risk compensation (risk premia) in various markets. Specifically, it is equal to

$$m_{t+1} - E_t(m_{t+1}) = -\lambda_{m,\eta} \sigma_t \eta_{t+1} - \lambda_{m,e} \sigma_t e_{t+1} - \lambda_{m,w} \sigma_w w_{t+1}, \tag{9}$$

where $\lambda_{m,\eta}$, $\lambda_{m,e}$, and $\lambda_{m,w}$ are the market prices for the short run, long run, and volatility risks. The market prices of systematic risks, including the compensation for stochastic volatility risk in consumption, can be expressed in terms of the underlying preferences and parameters that govern the evolution of consumption growth as,

$$\begin{aligned} \lambda_{m,\eta} &= \gamma \\ \lambda_{m,e} &= (\gamma - \frac{1}{\psi})\Big[\frac{\kappa_1 \varphi_e}{1-\kappa_1 \rho}\Big] \\ \lambda_{m,w} &= (\gamma - \frac{1}{\psi})(1-\gamma)\Big[\frac{\kappa_1 (1 + (\frac{\kappa_1 \varphi_e}{1-\kappa_1 \rho})^2)}{2\ (1-\kappa_1 \nu_1)}\Big]. \end{aligned} \tag{10}$$

The risk compensation for the η_{t+1} shocks is very standard and $\lambda_{m,\eta}$ equals the risk aversion parameter γ. In the special case of power utility, $\gamma = \frac{1}{\psi}$ the risk compensation parameters $\lambda_{m,e}$ and $\lambda_{m,w}$ are zero. Long-run risks and volatility are priced only when the agent is not indifferent to the timing of uncertainty resolution the consumption path, that is, when risk aversion is different from the reciprocal of the IES. For the price of long run risk to be positive and that of volatility risk negative, it is necessary that that the the agent prefer early resolution of uncertainty, that is γ be larger than $\frac{1}{\psi}$. The market prices of long run and volatility risks are sensitive to the magnitude of permanence parameter ρ as well. The risk compensation for long run risks and volatility risks rises as the permanence parameter ρ rises.

The equity premium in the presence of time-varying economic uncertainty is

$$E_t(r_{m,t+1} - r_{f,t}) = \beta_{m,\eta}\lambda_{m,\eta}\sigma_t^2 + \beta_{m,e}\lambda_{m,e}\sigma_t^2 + \beta_{m,w}\lambda_{m,w}\sigma_w^2 - 0.5Var_t(r_{m,t+1}). \tag{11}$$

The first beta corresponds to the exposure to short run risks, and the second to long run risks. The third beta (that is, $\beta_{m,w}$) captures the return's exposure to volatility risks. It is important to note that all the betas in this general equilibrium framework are endogenous. They are completely pinned down by the dynamics of the asset's dividends and the preferences parameters of the agent. The quantitative magnitudes of betas and their effects on the risk premium for the consumption claim is discussed below.

The risk premium on the market portfolio is time-varying as σ_t fluctuates. The ratio of the conditional risk premium to the conditional volatility of the market portfolio fluctuates with σ_t, and therefore the Sharpe ratio is time-varying. The maximal Sharpe ratio in this model economy, which approximately equals the conditional volatility of the log IMRS, also varies with σ_t. During periods of high economic uncertainty (i.e., consumption volatility) all risk premia rise.

The first-order effects on the level of the risk-free rate, as discussed in Bansal and Yaron (2005), are the rate of time preference and the average consumption growth rate, divided by the IES. Increasing the IES keeps the level low. The variance of the risk-free rate is determined by the volatility of expected consumption growth rate and the IES. Increasing the IES lowers the volatility of the risk-free rate. In addition, incorporating economic uncertainty leads to an interesting channel for interpreting fluctuations in the real risk free rate. In addition, this has serious implications for the measurement of the IES in the data which heavily rely on the link between the risk free rate and expected consumption growth. In the presence of varying volatility the estimates of IES based on the projections considered in Hall (1988) and Campbell (1999) are seriously biased downwards.

Hansen, Heaton and Li (2005) also consider a long run risks model. In their model the IES pinned at one. This specific case affords considerable simplicity is solving the model, as the wealth to consumption ratio in this case, is constant. To solve the model at values of IES that differ from one, the authors provide approximations around the case where the IES is one. In a related context Bansal, Kiku and Yaron (2006) provide approximate solution that rely on equation (4); they show how to derive the return $r_{a,t}$ along with the endogenous approximating constants κ_1 and κ_0 for any configuration of preferences parameters.

3 Data and Model Implications

3.1 Data and the Growth Rate Dynamics

Bansal and Yaron (2004) calibrate the model described in (5) at the monthly frequency. From this monthly model they derive time-aggregated annual growth rates of consumption and dividends to match key aspects of annual aggregate consumption and dividends data. For consumption, BEA data on real per capita annual consumption growth of non-durables and services for the period 1929 to 1998 are utilized. Dividends and the value-weighted market return data are taken from CRSP. All nominal quantities are deflated using the CPI.

The annual real per-capita consumption growth mean is 1.8 percent and its standard deviation is about 2.9 percent. Table I, adapted from Bansal and Yaron (2004), shows that in the data, consumption growth has a large first-order autocorrelation coefficient and a small second-order one. The standard errors in the data for these autocorrelations are sizeable. An alternative way to view the long horizon properties of the consumption and dividend growth rates is to use variance ratios which are themselves determined by the autocorrelations (see Cochrane, 1988). In the data the variance ratios first rise significantly and at about 7 years start to decline. The standard errors on these variance ratios, not surprisingly, are quite substantial.

In terms of the specific parameters for the consumption dynamics, BY calibrate ρ at 0.979, which determines the persistence in the long run component in growth rates. Their choice of φ_e and σ ensures that the model matches the unconditional variance and the autocorrelation function of annual consumption growth. The standard deviation of the one-step ahead innovation in consumption, that is σ, equals 0.0078. This parameter configuration implies that the predictable variation in monthly consumption growth is very small as the implied R^2 is only 4.4 percent. The exposure of the corporate sector to long run risks is governed by ϕ, and its magnitude is similar to that in Abel (1999). The standard deviation of the monthly innovation in dividends, $\varphi_d \sigma$, is 0.0351. The parameters of the volatility process are chosen to capture the persistence is consumption volatility. Based on the evidence of slow decay in volatility shocks, they calibrate ν_1, the parameter governing the persistence of conditional volatility at 0.987. The shocks to the volatility process have very small volatility, σ_w is calibrated at 0.23×10^{-5}. At the calibrated parameters the modeled consumption and dividend growth rates very closely match the key consumption and dividends data features reported in Table I. Bansal, Gallant and Tauchen (2005) provide simulation based estimation evidence that supports this configuration as well.

Table II presents the targeted asset market data for 1929 to 1998. The equity risk premium is 6.33% per annum and the real risk free rate is 0.9%. The annual market return volatility is 19.42%, and that of the real risk free is quite small, of about 1% per annum. The volatility of the price-dividend ratio is quite high, and it is a very persistent series. In addition to these data dimensions, Bansal and Yaron also evaluate the ability of the model to capture predictability of returns and the new evidence (see Bansal, Khatchatrian and Yaron, 2005) that price-dividend ratios are negatively correlated with consumption volatility at long leads and lags.

It is often argued that consumption and dividend growth, in the data, is close to being *i.i.d.* Bansal and Yaron (2004) show that their model of consumption and dividends is also consistent with the observed data on consumption and dividends growth rates. However, while the financial market data are hard to interpret from the perspective of the *i.i.d.* growth rate dynamics, Bansal and Yaron show that it is interpretable from the perspective of the growth rate dynamics that incorporate long run risks. Given these difficulties in discrimination across models Hansen and Sargent (2006) utilize features of the long run growth rate dynamics developed for motivating economic models that incorporate robust control.

3.2 Preference Parameters

The preference parameters take account of economic considerations. The time preference parameter $\delta < 1$, and the risk aversion parameter γ is either 7.5 or 10. Mehra and Prescott (1985) argue that a reasonable upper bound for risk aversion is around 10. The IES is set at 1.5 — an IES value that is not less than one is important for the quantitative results.

There is considerable debate about the magnitude of the IES. Hansen and Singleton (1982) and Attanasio and Weber (1989) estimate the IES to be well in excess of 1. More recently, Guvenen (2001) and Attanasio and Vissing-Jorgensen (2003) also estimate the IES over one — they show that their estimates are close to that used in Bansal and Yaron. However, Hall (1988) and Campbell (1999) estimate the IES to be well below one. Bansal and Yaron (2004) argue that the low IES estimates of Hall and Campbell are based on a model without time varying volatility. They show that ignoring the effects of time-varying consumption volatility leads to a serious downward bias in the estimates of the IES. If the population value of the IES in the Bansal and Yaron model is 1.5, then the estimated value of the IES using Hall estimation methods, will be less than 0.3. With fluctuating consumption volatility the projection of consumption growth on the level of the risk free rate does not equal the IES, leading to the downward bias. This suggests that Hall and Campbell's estimates may not a robust guide for calibrating the IES.

In addition to the above arguments, the empirical evidence in Bansal, Khatchatrian and Yaron (2005) shows that a rise in consumption volatility sharply lowers asset prices at long leads and lags, and that higher asset valuations today predict higher corporate earnings growth. Figure 1 to Figure 4, use data from USA, UK, Germany, and Japan to evaluate the volatility channel. The asset valuation measure is the price to earnings ratio, and the consumption volatility measure is constructed by averaging up 8 lags of the absolute value of consumption residuals. It is evident from the graphs that a rise in consumption volatility lowers asset valuations for all counties under consideration—this highlights the volatility channel and motivates the specification of IES larger than one. In a two-country extension of the model, Bansal and Shaliastovich (2007) show that dollar prices of foreign currency and forward premia co-move negatively with consumption volatility differential, while the ex-ante currency returns have positive correlations with it. This provides further empirical support for a magnitude of the intertemporal elasticity of substitution. In terms of growth rate predictability, Ang and Bekaert (2005) and Bansal, Khatchatrian and Yaron (2005) report a positive relation between asset valuations and expected earnings growth. These data features, as discussed in the theory sections above, again require an IES larger than one.

3.3 Asset Pricing Implications

To underscore the importance of two key aspects of the model, preferences and long run risks, first consider the genesis of the risk premium on $r_{a,t+1}$ — the return on the asset that delivers aggregate consumption as its dividends. The determination of risk premia for other dividend claims follows the same logic.

Table III shows the market price of risk and the breakdown of the risk premium from various risk sources. Column I considers the case of power utility as the IES equals the reciprocal of the risk aversion parameter. As discussed earlier, the prices of long run risks and volatility risk are zero. In the power utility case the main risk is the short run risk and the risk premium on the consumption asset equals $\gamma\sigma^2$, which is 0.7% per annum.

Column 2 of Table III considers the case of Epstein and Zin preferences with an IES less than one (set at 0.5). The price of long run growth rate risks is positive, and negative for volatility risks. As $\gamma = 10$, it continues to be larger than the reciprocal of the IES. However, the consumption asset's beta for the long run risks (beta w.r.t to the innovations in x_{t+1}) is negative. This, as discussed earlier, is because A_1 is negative (see equation (6)), implying that a rise in expected growth lowers the wealth to consumption ratio. Consequently, long run risks in this case contribute a negative risk premium of -1.96% per annum. The market price of volatility risk is negative and small, however, the asset's beta for this risk source is large and positive, reflecting the fact that asset prices rise when economic uncertainty rises (see equation (7)). In all, when IES is less than one the risk premium on the consumption asset is negative, which is highly counterintuitive and highlights the implausibility of this parameter configuration.

Column 3 of Table III shows that when IES is larger than one, the price of long run growth risk rises. More importantly, the asset's beta with respect to the long run growth risk is positive, and that for volatility risk is negative. Both these risk sources contribute towards a positive risk premium. The risk premium from long-run growth is 0.76% and that for the short-run consumption shock is 0.73%. The overall risk premia for this consumption asset is 1.52%. This evidence shows that an IES larger than one is required for the long run and volatility risks to carry to a positive risk premium.

It is clear from Table III that the price of risk is highest for the long run risks (see column 2 and 3) and smallest for the volatility risks. A comparison of column 2 and 3 also shows that raising the IES increases the prices of long run and volatility risks in absolute value. The magnitudes reported in Table III are with $\rho = 0.979$ — lowering this persistence parameter also lowers the prices of long-run and volatility risks (in absolute value). Increasing the risk aversion parameter increases the prices of all consumption risks as shown in equation (10). Hansen and Jagannathan (1991) document the importance of the maximal Sharpe Ratio, determined by the volatility of the IMRS, in assessing asset pricing models. Incorporating long run risks increases the maximal Sharpe ratio in the model which easily satisfy the non-parametric bounds of Hansen and Jagannathan (1991).

The risk premium on the market portfolio (i.e., the dividend asset) is also affected by the presence of long run risks. To underscore their importance, assume that consumption and dividend growth rates are *i.i.d.* This shuts-off the long-run risks channel. The market risk premium in this case is

$$E_t(r_{m,t+1} - r_{f,t}) = \gamma cov(g_{t+1}, g_{d,t+1}) - 0.5Var(g_{d,t+1}), \tag{12}$$

and market return volatility equals the dividend growth rate volatility. If shocks to consumption and dividends are uncorrelated then the geometric risk premium is negative and equals $-0.5Var(g_{d,t+1})$. If the correlation between monthly consumption and dividend growth is 0.25, then the equity premium is 0.08 percent per annum — similar to the evidence documented in Mehra and Prescott (1985) and Weil (1989).

Bansal and Yaron (2004) show that the full model that incorporates long run growth rate risks and fluctuating economic uncertainty provides a very close match to the asset market data reported in Table II. That is, this model can account for the *low risk free rate*, *high equity premium*, *high asset price volatility* and *low risk free rate volatility*. This model also quantitatively matches additional data features, such as (i) predictability of returns at short and long horizons using dividend yield as a predictive variable, (ii) time-varying and persistent market return volatility, (iii) negative correlation between market return and volatility shocks, i.e., the volatility feedback effect, (iv) negative relation between consumption volatility and asset prices at long leads and lags, documented in Bansal, Khatchatrian and Yaron (2005). In addition to these puzzles we discuss other application of the model in the following sections.

3.4 Cross-Sectional Implications

Table IV, shows that there are sizable differences in mean real returns across portfolios sorted on the standard book-to-market, size, and momentum (see Jegadeesh and Titman, 1993) for quarterly data from 1967 to 2001. For size and book-to-market sorts, firms are sorted into different deciles once a year and the subsequent return on these portfolios is used for empirical work. For momentum assets CRSP-covered NYSE and AMEX firms are sorted on the basis of their cumulative return over months $t-12$ through $t-1$. The loser portfolio (m1) includes firms with the worst performance over the last year and the winner portfolio (m10) includes firms with the best performance. The data show that subsequent returns on these portfolios have a large spread (i.e., m10 return - m1 return), of about 4.62% per quarter — this is the *momentum spread puzzle*. Similarly, the highest book-to-market firms (b10) earn average real quarterly returns of 3.27% whereas the lowest book-to-market (b1) firms average 1.54% per quarter. The value spread (return on b10 - return on b1) is about 2% per quarter — this is the *value spread puzzle*. What explains these big differences in mean returns across portfolios?

Bansal, Dittmar, and Lundblad (2002 and 2005) connect systematic risks to cash flow risks. They show that the asset's risk measure (that is its beta) is determined by its cash flow properties. In particular, their paper shows that cross-sectional differences in the asset's beta mostly reflects differences is systematic risks in cash flows. Hence systematic risks in cash flows ought to explain differences in mean returns across assets. They develop two ways to measure the long run risks in cash flows. First they model dividend and consumption growth rates as a VAR and measure the discounted impulse response of the dividend growth rates to consumption innovations. This is one measure of risks in cash flows. Their second measure is based on stochastic cointegration — which is estimated by regressing the log level of dividends for each portfolio on a time trend and the log level of consumption. That is consider the projection,

$$d_t = \tau(0) + \tau(1)t + \tau(2)c_t + \zeta_t.$$

The projection coefficient, $\tau(2)$, measures the long run consumption risk in the asset's dividends. The coefficient $\tau(2)$ will be different for different assets.

Bansal, Dittmar, and Lundblad (2002 and 2005) show that the exposure of dividend growth rates to the long run component in consumption has considerable cross-sectional explanatory power. That is, dividend's exposure to long run consumption risks is an important explanatory variable in accounting for differences in mean returns across portfolios. Portfolios with high mean returns also have higher dividend exposure to consumption risks. The cointegration based measure of risk τ_2 also provides very valuable information about mean returns on assets. The cross-sectional R^2 from regressing the mean returns on the dividend based risk measures is well over 65%. In contrast, other approaches find it quite hard to explain the differences in mean returns for the 30 asset menu used in Bansal, Dittmar, and Lundblad. The standard consumption beta's (i.e., C-CAPM), and the market based CAPM asset betas have close to zero explanatory power. The R^2 for the C-CAPM is 2.7%, and that for the market CAPM it is 6.5%, with an implausible negative slope coefficient. The Fama and French three factor empirical specification also generates point estimates with negative, and difficult to interpret, prices of risk for the market and size factors — the cross-sectional R^2 is about 36%. Compared to all these models the cash flow risks model of Bansal, Dittmar, and Lundblad is able to capture a significant portion of the differences in risk premia across assets. Hansen, Heaton and Li (2005) inquire about the robustness of the stochastic cointegration based risk measures considered in Bansal, Dittmar and Lundblad (2002). They argue that the dividend based consumption betas, particularly, the cointegration based risk measures are imprecisely estimated in the time-series. However, in all of the different ways that they consider the mean returns and these risk measures do line up with the mean return of assets. That is, in the cross-section of assets (as opposed to the time-series) the price of risk associated with the long run risk measures is reliably significant.

Bansal, Dittmar and Kiku (2005) derive new results that link this cointegration parameter to consumption beta's by investment horizon and evaluate the ability of their model to explain differences in mean returns for different horizons. They provide new evidence regrading the robustness of the stochastic cointegration based measures of permanent risks in equity markets. Parker and Julliard (2005) evaluate if long run risks in aggregate consumption can account for cross-section of expected returns. Malloy, Moskowitz and Vissing-Jørgensen (2005) evaluate if long run risks in the consumption of stock-holders relative to aggregate consumption has greater ability to explain the cross-section of equity returns, relative to aggregate consumption measures.

3.5 Term Structure and Currency Markets

Colacito and Croce (2006) consider a two country version of the BY model. They show that this model can account for the low cross country consumption growth correlation but high correlation in marginal utilities across countries (high risk sharing despite low measured consumption correlation). This data feature of international data is highlighted in Brandt, Cochrane and Santa-Clara (2005). The key idea that Colacito and Croche pursue is that the long run risks component is very similar across countries, but in the short run consumption growth can be very different. That is countries share very similar long run prospects, but in the short run they can look very different–this dimension, they show, is sufficient to induce high correlation in marginal utilities across countries. It also accounts for high real exchange volatility.

Bansal and Yaron (2004) derive implication for the real term structure of interest rates for the long run risks model. More recent papers by Eraker (2006), Piazzesi and Schneider (2005) also consider the quantitative implications for the nominal term structure using the long run risk model. Bansal and Shaliastovich (2007) show that the BY model can simultaneously account for the upward sloping terms structure, the violations of the expectations hypothesis in the bond markets, the violations in the foreign currency markets, and the equity returns simultaneously. This evidence is an indication that the overall agenda of long run risk can go a long way in providing a baseline model for interpreting financial markets and potentially in the design of policy.

4 Conclusion

The work of Bansal and Lundblad (2002), Bansal and Yaron (2004), Bansal, Dittmar and Lundblad (2005) shows that the long run risks model can help interpret several features of financial markets. These papers argue that investors care about the long run growth prospects and the uncertainty (time-varying consumption volatility) surrounding the growth rate. Risks associated with changing long run growth prospects and varying economic uncertainty drive the level of equity returns, asset price volatility, differences in risk premia across assets, and predicability in financial markets.

Recent papers indicate that the channel in this model can account for nominal yields curve features, such as the violation the expectations hypothesis and the the average upward sloping nominal yield curve. Evidence in Colacito and Croce (2006) and Bansal and Shaliastovich (2007) shows that the model can also account for key aspects of foreign exchange markets. This growing evidence suggests that the model can potentially be used for analyzing implications of policy.

All of this evidence and the economics underlying it support the view that the long run risks and uncertainty channel contain very valuable insights about the workings of financial markets.

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Table I

Time-Series Properties of Data

Variable	Estimate	S.E.
$\sigma(g)$	2.93	(0.69)
$AC(1)$	0.49	(0.14)
$AC(2)$	0.15	(0.22)
$AC(5)$	-0.08	(0.10)
$AC(10)$	0.05	(0.09)
$VR(2)$	1.61	(0.34)
$VR(5)$	2.01	(1.23)
$VR(10)$	1.57	(2.07)
$\sigma(g_d)$	11.49	(1.98)
$AC(1)$	0.21	(0.13)
$corr(g, g_d)$	0.55	(0.34)

Table I, taken from Bansal and Yaron (2004), displays the time-series properties of aggregate consumption and dividend growth rates: g and g_d, respectively. The statistics are based on annual observations from 1929 to 1998. Consumption is real per capita consumption of non-durables and services; dividends are the sum of real dividends across all CRSP firms. $AC(j)$ is the j^{th} autocorrelation, $VR(j)$ is the j^{th} variance ratio, σ is the volatility, and $corr$ denotes the correlation. Standard errors are Newey and West (1987) corrected using 10 lags.

Table II

Asset Market Data

Variable	Estimate	S.E.
	Returns	
$E(r_m - r_f)$	6.33	(2.15)
$E(r_f)$	0.86	(0.42)
$\sigma(r_m)$	19.42	(3.07)
$\sigma(r_f)$	0.97	(0.28)
	Price-Dividend Ratio	
$E(exp(p-d))$	26.56	(2.53)
$\sigma(p-d)$	0.29	(0.04)
$AC1(p-d)$	0.81	(0.09)
$AC2(p-d)$	0.64	(0.15)

Table II, adapted from Bansal and Yaron (2004), presents descriptive statistics of asset market data. The moments are calculated using annual observations from 1929 through 1998. $E(r_m - r_f)$ and $E(r_f)$ are respectively the annualized equity premium and mean risk free-rate. $\sigma(r_m)$, $\sigma(r_f)$, and $\sigma(p-d)$ are the annualized volatilities of the market return, risk-free rate, and the log price-dividend, respectively. $AC1$ and $AC2$ denote the first and second autocorrelations. Standard errors are Newey and West (1987) corrected using 10 lags.

Table III

Risk Components and Risk Compensation

	$\psi = 0.1$	$\psi = 0.5$	$\psi = 1.5$
mpr_η	93.60	93.60	93.60
mpr_e	0.00	137.23	160.05
mpr_w	0.00	-27.05	-31.56
β_η	1.00	1.00	1.00
β_e	-16.49	-1.83	0.61
β_w	11026.45	1225.16	-408.39
prm_η	0.73	0.73	0.73
prm_e	0.00	-1.96	0.76
prm_w	0.00	-0.08	0.03

Table III presents model-implied components of the risk premium on the consumption asset for different values of the intertemporal elasticity of substitution parameter, ψ. All entries are based on $\gamma = 10$. The parameters that govern the dynamics of the consumption process in equation (5) are identical to Bansal and Yaron (2004): $\rho = 0.979$, $\sigma = 0.0078$, $\varphi_e = 0.044$, $\nu_1 = 0.987$, $\sigma_w = 0.23 \times 10^{-5}$ and κ_1 of 0.997. The first three rows report the annualized percentage prices of risk for innovations in consumption, the expected growth risk, and the consumption volatility risk — mpr_η, mpr_e, and mpr_w, respectively. These prices of risks correspond to annualized percentage values for $\lambda_{m,\eta}\sigma$, $\lambda_{m,e}\sigma$, $\lambda_{m,w}\sigma_w$ in equation (9). The exposures of the consumption asset to the three systematic risks, β_η, β_e and β_w, are presented in the middle part of the table. Total risk compensation in annual percentage terms for each risk is reported as prm_*, and equals the product of the price of risk, the standard deviation of the shock, and the beta for the specific risk.

Table IV

Portfolio returns

	Mean	Std. Dev		Mean	Std. Dev		Mean	Std. Dev
S1	0.0230	0.1370	**B1**	0.0154	0.1058	**M1**	-0.0104	0.1541
S2	0.0231	0.1265	**B2**	0.0199	0.0956	**M2**	0.0070	0.1192
S3	0.0233	0.1200	**B3**	0.0211	0.0921	**M3**	0.0122	0.1089
S4	0.0233	0.1174	**B4**	0.0218	0.0915	**M4**	0.0197	0.0943
S5	0.0242	0.1112	**B5**	0.0200	0.0798	**M5**	0.0135	0.0869
S6	0.0207	0.1050	**B6**	0.0234	0.0813	**M6**	0.0160	0.0876
S7	0.0224	0.1041	**B7**	0.0237	0.0839	**M7**	0.0200	0.0886
S8	0.0219	0.1001	**B8**	0.0259	0.0837	**M8**	0.0237	0.0825
S9	0.0207	0.0913	**B9**	0.0273	0.0892	**M9**	0.0283	0.0931
S10	0.0181	0.0827	**B10**	0.0327	0.1034	**M10**	0.0358	0.1139

Table IV, reported in Bansal, Dittmar, and Lundblad (2005), presents descriptive statistics for the returns on the 30 characteristic sorted decile portfolios. Value-weighted returns are presented for portfolios formed on momentum (M), market capitalization (S), and book-to-market ratio (B). M1 represents the lowest momentum (loser) decile, S1 the lowest size (small firms) decile, and B1 the lowest book-to-market decile. Data are converted to real using the PCE deflator. The data are sampled at the quarterly frequency, and cover the 1st quarter 1967 through 4th quarter 2001.

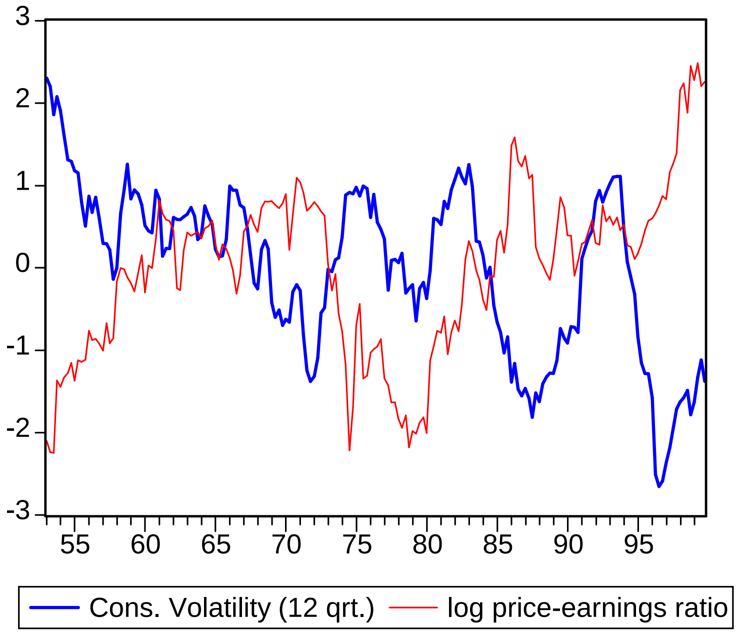

Figure 1. P/E Ratio and Consumption Volatility: USA

Figure 1, taken from Bansal, Khatchatrian, and Yaron (2005), plots consumption volatility along with the logarithm of the price-earnings ratio for the US. Both series are standardized.

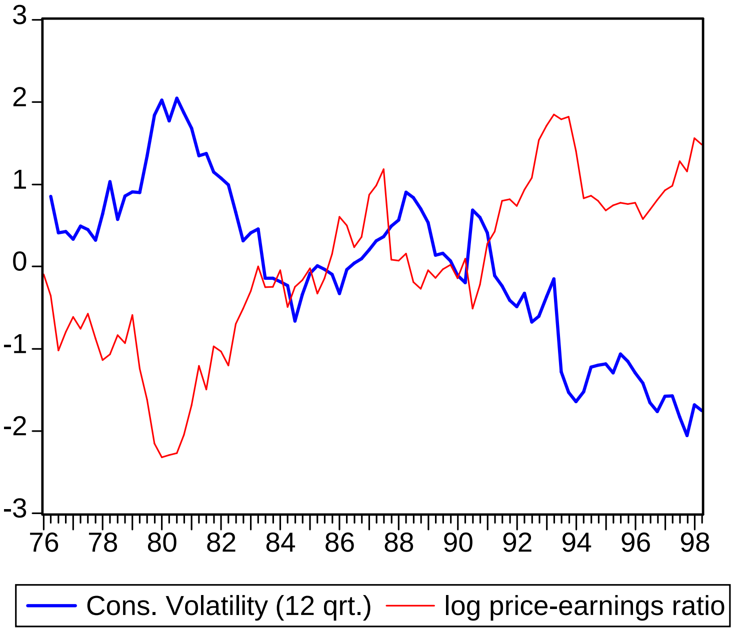

Figure 2. P/E Ratio and Consumption Volatility: UK

Figure 2 plots consumption volatility along with the logarithm of the price-earnings ratio for the UK. Both series are standardized.

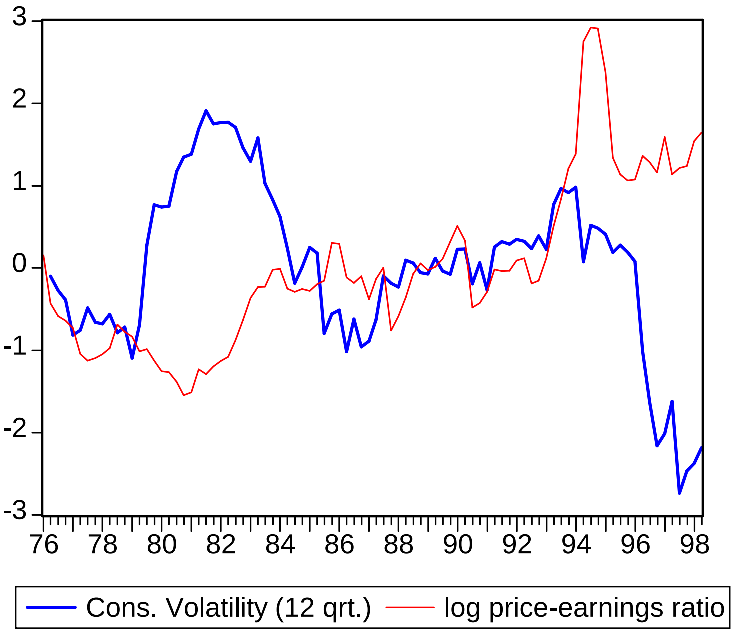

Figure 3. P/E Ratio and Consumption Volatility: Germany

Figure 3 plots consumption volatility along with the logarithm of the price-earnings ratio for Germany. Both series are standardized.

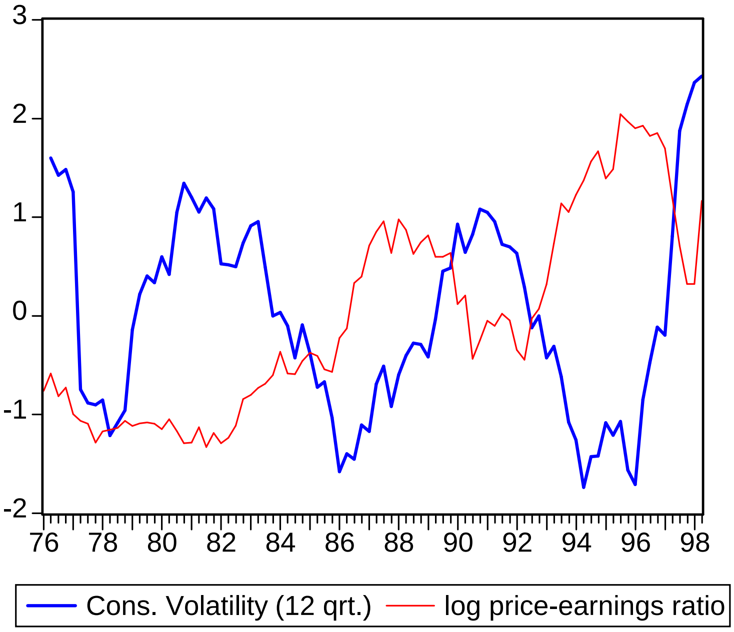

Figure 4. P/E Ratio and Consumption Volatility: Japan

Figure 4 plots consumption volatility along with the logarithm of the price-earnings ratio for Japan. Both series are standardized.