

Repos, Fire Sales, and Bankruptcy Policy

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Abstract

The events from the 2007-2009 financial crisis have raised concerns that the failure of large financial institutions can lead to destabilizing fire sales of assets. The risk of fire sales is related to exemptions from bankruptcy's automatic stay provision enjoyed by a number of financial contracts, such as repo. An automatic stay prohibits collection actions by creditors against a bankrupt debtor or his property. It prevents a creditor from liquidating collateral of a defaulting debtor since collateral is a lien on the debtor's property. In this paper, we construct a model of repo transactions, and consider the effects that a change in the bankruptcy rule regarding the automatic stay has on the activity in repo and real investment markets. We find that exempting repos from the automatic stay is beneficial for creditors who hold the borrowers' collateral. Although the exemption may increase the size of the repo market by enhancing the liquidity of collateral, it can also lead to subsequent damaging fire sales that are associated with reductions in real investment activity. Hence, policy makers face a trade-off between the benefits of investment activity and the benefits of liquid markets for collateral.

Keywords: Repos, fire sales, bankruptcy, JEL classification: G01, G14, G18, G28

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1 Introduction

An institution that suffers large losses may be forced to sell assets at distressed or *fire-sale* prices.¹ If other institutions value their assets at these temporarily low market values, then they too may be forced to sell assets and suffer losses. As a result, the initial sale can set off a cascade of fire sales that inflicts losses on many institutions. A number of commentators have identified fire sales as depleting the balance sheets of financial institutions and aggravating the fragility of the financial system in the recent financial crisis.² Via defaults and fire sales, one troubled institution can damage another and, as a result, reduce the financial system's capacity to efficiently allocate resources.³ The risk of fire sales in the repurchase agreement (repo) market has been a particularly serious concern due to the volume of securities financed in these markets⁴ and to their importance as a source of funding for securities dealers, who play a key role in modern financial systems.⁵ Indeed, recent research⁶ has identified the freezing of the repo market as a key contributor to the recent financial crisis.⁷

This paper develops a model of a repo market and examines the implications of different policy rules on the activity in this market as well as in the market where the collateral of defaulted borrowers can be resold.

A repo is a borrowing arrangement where the first leg of the transaction has one party (the borrower) selling a security to another party (the lender) for cash, and the second leg, which occurs at some predetermined future date, has the borrower repurchasing the security from the lender for cash at a predetermined price. The security that the lender holds in between the two legs is typically referred to as collateral. An important feature of a repo is that the lender (or creditor) holds collateral of the borrower. Under the US Bankruptcy Code, most contracts are subject to an automatic stay when a debtor files for bankruptcy. This stay prevents creditors from initiating collection actions against a bankrupt debtor or his property. Since the collateral is considered to be a lien on the debtor's property, a stay, in practice, delays the ability of a creditor to realize value through a sale of the collateral.⁸

Over the decades since the current framework was established, an ever-increasing set of qualified financial contracts (QFCs), including repos, has been exempted from the stay.⁹ Under current bankruptcy rules, the repo lender can

¹For recent empirical evidence on fire sales, see [14] and [17]

²See [1], [7], [8].

³See [24] and [21].

⁴See [13], [22].

⁵See [3].

⁶See [13], [15], and [5] for a discussion of the risk of fire sales in the tri-party repo market.

⁷See [10] and [23]

⁸Generally speaking, the purpose of an automatic stay is to prevent the destruction of value that can occur when creditors make a *mad dash* to seize the assets of the bankrupt firm. To the extent that the assets used as collateral are financial assets rather than real assets, the destructiveness of this *grab race* is less of a consideration, and so in this paper we focus instead on the effects of a rush to sell these assets in a less-than-perfectly liquid market.

⁹For an account of the changes in the application of bankruptcy law to repos, see [11]. Also notice that in the case of banks taken over by the FDIC or systemically important financial

liquidate the collateral if the borrower defaults before the end of the contract. In effect, the repo contract is exempt from the standard bankruptcy procedure of automatic stay. Such exemption has raised concerns that the default of a large financial institution could trigger destabilizing fire sales of assets. During the recent crisis, similar fears arose on the failure, or near failure of Bear Stearns and Lehman brothers in 2008.¹⁰

Through fire sales, exemption from the stay may then cause allocative inefficiencies: if large amounts of assets are liquidated quickly at distressed prices, the balance sheet of other institutions which must value their assets at such prices, is adversely affected. This in turn may impact their ability to take advantage of productive investment. On the other hand, exemption from bankruptcy stay has been recognized as enhancing the liquidity of the repo market¹¹, which is desirable given that such market provides funding and investment opportunities to many financial institutions.

Therefore, whether exemption from bankruptcy stay for a wide class of financial contracts is desirable or not is an open question. This paper provides an answer to this question.

In our model, the possibility of borrower default motivates lenders to request collateral from buyers as a form of insurance.¹² However, the insurance function of the collateral is imperfect. If there is an automatic stay in place, then the inability to immediately liquidate collateral of defaulted borrowers imposes a cost on holders of collateral. The cost can be associated with the inability to convert relatively illiquid collateral into a liquid asset, or with the risk that the collateral could lose value. If, instead, bankruptcy rules allow the lender to liquidate the borrower's collateral, then the collateral can be sold in a secondary market. Depending on the liquidity of that market, sales of collateral can have important effects on other market participants. We focus on the effect that lenders' sales of collateral have on real investments when investors are using assets similar to the collateral to secure resources for the investment.

Embedded in our model is an externality that implies that lenders do not take into account the effect they have on investors when making their initial repo decisions and their liquidation decisions in the event of borrower's default. Absent this externality, lenders would internalize all the effects of their sales of assets on the economy and make efficient investment decisions in the repo market. The externality is modeled by a trading friction. The trading friction is an important ingredient: in practice repos and other financial instruments that make use of collateral are traded in over-the-counter markets.

Moreover, the trading friction allows us to analyze two distinct mechanisms through which fire sales arise and affect welfare: the first one, and most well

institutions under Dodd-Frank, there may be a stay for a limited time even for QFCs.

¹⁰It is worth noting that these fears were, to a large extent not realized, in part because Bear Stearns was purchased by JP Morgan Chase, and the US broker dealer unit of Lehman did not declare bankruptcy.

¹¹See [11]

¹²See also [18]

known, works through a decrease in the price of assets¹³; the second one works through the frequency of trades. We focus on the latter. Suppose that some agents have access to productive investment funded by selling assets over-the-counter. When a large number of lenders access such market to sell the collateral from defaulted repos, the matching process in the over-the-counter market is affected and the frequency of trades that can fund productive investment decreases.

Thus the externality creates a trade off for policy makers considering bankruptcy rules as policy instruments. On the one hand, exempting repo transactions from the automatic stay is desirable because the ability to liquidate the borrower's collateral increases its value to the lender. This in turn increases the amount that the lender is willing to repo to the borrower in the first place: in this sense, an exemption from the automatic stay makes the repo market more liquid. On the other hand, exemption from automatic stay can result in a reduction of investment activity in case of borrower default, and is thus not desirable. Therefore, the optimal bankruptcy rule depends on the relative size of these two effects; interestingly, however, this is not true when the market where the collateral is sold is very thick. In this case there is no trade off between lenders' need for liquidity and investors' productive opportunities: the market is thick enough to satisfy both and the exemption from bankruptcy stay unambiguously yields a Pareto improvement.

Our paper focuses on the trade-off between the liquidity of the repo market and the potential effects that fire sales related to the exemption from the stay have on the rest of the economy. This trade-off is discussed in the legal literature; see [19]. Duffie and Skeel [9] outline a number of costs and benefits associated with safe harbors from the automatic stay in bankruptcy, including a variety of ways in which the stay can decrease the value of the collateral contract, and on the other side, the potential for costs in a fire sale. They also describe how money market mutual funds holding repos may be forced by regulation to sell collateral in the case of the bankruptcy of a counterparty.

Bolton and Oehmke [6] argue against privileged positions for derivatives in bankruptcy, because it inefficiently undermines the position of other creditors. The paper that is most closely related to our is Acharya, Anshuman, and Viswanathan [2] who also examine the costs of bankruptcy-induced fire sales and argue that the automatic stay provisions for repos may be ex-post optimal when repo borrowers are highly levered. McAndrews and Roberds [16] focus on a different aspect of the bankruptcy rule: they provide an analysis of the benefits of the exemption from the stay associated with close-out netting.

The paper is organized as follows. The basic model, without default, is presented in the next section. The basic model is generalized to allow for defaults

¹³This specific mechanism is less straightforward in the current version of the because *i*) the bargaining power is all on the side of the sellers of assets and *ii*) the strategic interaction in the (over-the-counter) market for collateral assets is static. The combination of these two assumption shuts down this channel: if the larger number of assets available on the market raised the outside option of their buyers (in our model the traders), then this channel would be active again.

in section 3. Section 4 examines the nature of a government policy intervention and carefully analyzes the trade-offs that the government faces. Section 5 offers some concluding comments.

2 The Basic Model

The economy has 3 dates, $t = 1, 2, 3$, and 2 goods, a and c . Good a is durable: it can be costlessly stored from one period to the next. Good c is perishable.

There are 4 types of agents: lenders, L , borrowers, B , investors, I , and traders, T . The measure of each type of agent is n^i , where $i \in \{L, B, I, T\}$.

Lenders and borrowers are born at the beginning of date 1. Borrowers live at dates 1 and 2, and lenders live at dates 1, 2 and 3. Investors and traders are born at the beginning of date 3 and live only at date 3.

Borrowers like to consume good a at date 2. They possess a technology that instantaneously converts good c into good a one-for-one in date 1 or date 2: this technology is costless for borrowers to operate. They can also produce good c , but only at date 2, at a cost of 1 unit of effort per unit of good. The preferences of a borrower, U^B , are given by

$$U^B = a_2^B - c_2^B.$$

where subscripts indicate time of consumption or production.

Lenders want to consume goods a and c at dates 2 and 3; they like good c more than good a . Lenders can produce good c only at date 1, where one unit of costly effort produces one unit of good c . The preferences of a lender, U^L , are given by

$$U^L = -c_1^L + u(c_2^L) + \gamma(a_2^L + a_3^L) + c_3^L,$$

where u is increasing and strictly concave, and $0 < \gamma < 1$.

Traders are endowed with \bar{c} units of good c at date 3. They like to consume goods a and c at date 3, and their preferences, U^T , are given by

$$U^T = a_3^T + c_3^T.$$

Investors are endowed with \bar{a} units of good a at date 3. They like to consume goods a and c , and their preferences, U^I , are

$$U^I = a_3^I + c_3^I.$$

Investors have a costless technology that instantaneously converts good c into good a . Let c^I denote the input into investors' production function at $t = 3$. Unlike the borrower's technology, which is one-to-one, the investor's technology, f , is increasing, strictly concave. The last assumption implies that f is a productive technology in the sense that if the investor could exchange his endowment of good a for \bar{a} units of good c , then marginal return is strictly greater than one for all levels of input $c \in (0, \bar{a}]$.

Agents trade in pairs; that is, they are bilaterally matched. Unless in a match, they are always spatially separated. Agents are matched at the beginning of date 1 and at the beginning of date 3. The date 1 and date 3 matching processes are independent of one another. Since investors and traders are not alive at date 1, only lenders and borrowers enter the matching process at that time.

Some bilateral matches can generate surplus for the agents in the match. For example, borrowers and lenders can benefit from trading good c at date 1 for good c at date 2. In particular, a matched lender can produce good c at date 1 and give it to the borrower, (who converts it into good a). In return, the borrower can produce good c for the lender at date 2. Let this trading arrangement be compactly represented by the “contract” (c_1, c_2) where $c_1 = c_1^L$ and $c_2 = c_2^L = c_2^B$. Note that since the good c that is produced at date 1 (c_1) is converted to good a at $t = 1$ (a_1) one-for-one, then $a_1 = c_1$; and since good a is durable, $a_2 = a_1$. Implicitly embedded in contract (c_1, c_2) is a promise: the borrower promises to produce good c for the lender at date 2. We will assume that agents can commit to any (feasible) promise they make when matched.

Traders and investors can benefit from exchanging good a for good c at date 3. In particular, a matched investor can exchange some of his endowment of good a for some of the trader’s endowment of good c . The trading arrangement between investors and traders can be represented by the contract (a_3, c_3) , i.e., the investor gives up $a_3 = \bar{a} - a_3^I$ units of his endowment of good a and receives c_3 units of the trader’s endowment of good c , so that $c^I = c_3$.

The date-1 contract, (c_1, c_2) , between a matched lender and borrower is determined by bargaining. The lender’s payoff (and surplus) associated with contract (c_1, c_2) is $u(c_2) - c_1$. Since technology and durability of good a implies that $c_1 = a_1 = a_2$, the borrower’s surplus is $c_1 - c_2$. Total match surplus generated by contract (c_1, c_2) is

$$S^{BL} = u(c_2) - c_2.$$

A borrower accepts contract (c_1, c_2) only if $c_1 \geq c_2$ and a lender accepts only if $u(c_2) \geq c_1$. For simplicity, we assume that the lender has all of the bargaining power and makes a take-it-or-leave-it offer to the borrower. This bargaining protocol implies that the lender will choose $c_2 = c_1$ and, hence, receives the entire match surplus.¹⁴ The lender offers contract (c^*, c^*) to the borrower, where $u'(c^*) = 1$, since this offer maximizes match surplus. The borrower will accept this offer.

Let m^{ij} represent the probability that agent i is matched with agent j at date 1, and let m represent the measure of productive date 1 matches. We will assume that the matching technology is Leontief¹⁵ and takes the form $m = \min \{n^L, n^B\}$, $m^{LB} = m/n^L$, $m^{BL} = m/n^B$, and $m^{BB} = m^{LL} = 0$. For this matching technology one can interpret agents as directing their search to a

¹⁴Dividing surplus between the bargainers will not significantly affect our results).

¹⁵All the results can be generalized to an arbitrary matching function with standard properties.

productive partner, where the “short side” of the market determines the number of matches.

Lenders have no incentive to enter the date 3 matching process, independent of being matched or not at date 1, since they have nothing to offer in a date 3 match that could generate a match surplus. Therefore, the expected payoff to a lender—measured before agents are matched at date 1—is $m^{LB} [u(c^*) - c^*]$. Since the lender has all of the bargaining power in a date 1 match with a borrower, the expected payoff to a borrower is zero.

In an investor-trader match, the investor’s payoff associated with contract (a_3, c_3) is $f(c_3) + \bar{a} - a_3$. The surplus that the investor receives is $f(c_3) + (\bar{a} - a_3) - \bar{a} = f(c_3) - a_3$. The trader’s payoff associated with contract (a_3, c_3) is $a_3 + \bar{c} - c_3$, and the surplus he receives is $a_3 + \bar{c} - c_3 - \bar{c} = a_3 - c_3$. Hence the total match surplus is

$$S^{IT} = f(c_3) - c_3.$$

The investor will accept contract (a_3, c_3) only if $f(c_3) > a_3$, and the trader will accept the offer (a_3, c_3) only if $a_3 \geq c_3$. We assume that the investor has all of the bargaining power. The investor will offer contract (a_3, c_3) to the trader, where $a_3 = c_3 = \min\{\bar{a}, \bar{c}\}$, which implies that the match surplus is $f(\min\{\bar{a}, \bar{c}\}) - \min\{\bar{a}, \bar{c}\}$. For convenience, define $\bar{i} \equiv \min\{\bar{a}, \bar{c}\}$, i.e., \bar{i} represents the amount of good c that an investor receives from a trader, and the amount of good a that he gives the trader.

Assumption 1 *Assume and $f'(\bar{i}) > 1$ ¹⁶.*

Let M^{ij} represent the probability that agent i is matched with agent j at date 3, and M represent the measure of date-3 productive matches. For a Leontief matching function, $M = \min\{n^I, n^T\}$, $M^{IT} = M/n^I$, $M^{TI} = M/n^T$, and $M^{II} = M^{TT} = 0$. Since the investor has all of the bargaining power, his payoff is $M^{IT}(f(\bar{i}) - \bar{i}) + \bar{a}$, and the expected payoff to the trader is \bar{c} .

Let p_a represent the value to an investor of having an additional unit of good a , measured in terms of good c , at the beginning of date 3 before matching takes place. Then, when $\bar{i} = \bar{a}$,

$$p_a = M^{IT} f'(\bar{a}) + (1 - M^{IT}),$$

i.e., the investor is indifferent between receiving p_a units of good c for sure, and receiving an additional unit of good a .¹⁷

¹⁶For most of the paper we will focus on the case where $\bar{i} = \bar{a}$; the case where $\bar{i} = \min(\bar{a}, \bar{c}) = \bar{c}$ is less interesting and will be analyzed when relevant.

¹⁷When $\bar{i} = \bar{c}$, then the price of good a is 1 since if the investor is given an additional unit good a he will simply consume it. The *average* price of good a , however, is

$$M^{IT} \left[\frac{\bar{c}}{\bar{a}} \frac{f(\bar{c})}{\bar{c}} + \frac{\bar{a} - \bar{c}}{\bar{c}} \right] + (1 - M^{IT}) > 1.$$

We would argue that in this case, the average price is the relevant statistic when thinking about gains from trade.

Consider the problem of a planner whose objective is to maximize total social surplus, S , subject to the search friction, where

$$\begin{aligned} S &= S^{BL} + S^{IT} \\ &= m(u(c_2) - c_2) + M(f(c_3) - c_3), \end{aligned}$$

since $c_1 = a_2$. Assuming that the planner must respect agent participation constraints, total social surplus will be maximized at $c_2 = c^*$ and $c_3 = \bar{v}$, the take-it-or-leave-it offers made by the lender and investor, respectively.¹⁸ The planner can implement this surplus as long as $u(c_2) \geq a_2 \geq c_2$ and $f(c_3) \geq a_3 \geq c_3$, i.e. agent participation constraints are satisfied. Although the planner can redistribute surplus from the lender to the borrower (by increasing a_2 from c^*) and from the investor to the trader (by increasing a_3 from \bar{a}), he cannot increase total surplus compared to the equilibrium outcome.

The equilibrium in the basic model is Pareto efficient. The basic model lacks frictions that are needed to generate contracts that resemble repo contracts or something that looks like a “fire sale.” In addition, since agents do not default on the their contracts, the basic model can say nothing about bankruptcy or bankruptcy policy. In the next section we introduce a borrower default friction and examine how this affects optimal contracts, and the relationship between bankruptcy policy and fire sales.

3 A Model with Borrower Default

We extend the basic model by introducing the possibility of exogenous default by borrowers. Default is modeled by having the possibility that borrowers die between dates 1 and 2. With probability δ a random fraction Δ of borrowers die, and with probability $1 - \delta$ no one dies.¹⁹ We will refer to the former outcome as the default state, and the latter as the no-default state. From an ex ante date 1 perspective, the probability that a borrower dies is $\delta\Delta$. We use two parameters to describe default so that we can model a rare event, a ‘small’ δ , such as a major financial meltdown, a ‘big’ Δ .

The Section 2 contract between the lender and borrower can be interpreted as an unsecured (by collateral) loan since it is only the borrower’s promise that supports the date 2 payment. In practice, it is not at all unusual for unsecured creditors to receive nothing in the event that the borrower defaults. We model this outcome by assuming that when a matched borrower—holding c_1 units of good a and promising to produce c_2 units of good c at date 2—dies in between dates 1 and 2, the good a he is holding “disintegrates,” and, as a result, the

¹⁸The planner also takes as given the matching technologies and the bargaining protocol of the agents.

¹⁹We can assume that with probability $1 - \delta$, a finite number, i.e., a set of measure zero, of borrowers die. This way there can be defaults even in “good” times, but these defaults are essentially unimportant for the economy. This would correspond to situations (in the real world) where there are “fails” or defaults and these have no significant implications for asset prices or economic activity.

lender receives nothing. Although there is little the lender can do about a borrower's broken promise to supply good c at date 2, since he is spatially separated from other agents, the lender can secure his claim against the borrower by contractually preventing the borrower from holding good a between dates 1 and 2. Such a contract is Pareto optimal: any contract allocating some good a to the borrower between $t = 1$ and $t = 2$ is strictly dominated by a contract that transfers those resources to the lender. The latter contract prevents the societal waste of good a due to the borrower defaulting while holding it. Specifically, the contract can specify that the lender produces good c at date 1 and gives it to a borrower; the borrower then converts good c into good a , and gives good a back to the lender to hold as collateral. At date 2, the collateral—good a —is transferred back to the borrower if he produces good c for the lender; if the borrower does not produce at date 2—because he has died—the collateral becomes the property of the lender. This sort of contract partially insures the lender against a borrower default: If the borrower dies, then the lender has the collateral which is valuable to him at both dates 2 and 3.

Whether one interprets the above contract as a collateralized loan or a repo contract depends on *when* the lender is able to use the collateral. In practice, bankruptcy law specifies when collateral can be used by the lender. Under the US Bankruptcy Code, virtually all collateral is subject to an automatic stay when a debtor files for bankruptcy. This means that a secured (by collateral) creditor is unable to access and use the collateral for a certain period of time after a debtor defaults. However, some financial assets, such as derivatives and repo contracts, are exempt from the automatic stay, which implies that a secured creditor can immediately access and use the collateral as he sees fit. In terms of the model, if the bankruptcy policy dictates an automatic stay in the event of a debtor default, then the contract described above is a *collateralized loan*. In this situation, in the event of a debtor default, the earliest that collateral can be used by the lender is at date 3 after the matching process has been completed. If, instead, the bankruptcy policy exempts the collateral from an automatic stay, then the above contract is *repo*, and the lender can use the collateral as he sees fit starting at date 2, when it becomes known that the debtor has (died and) defaulted on his contractual payment of c_2 . In the next section, we analyze the implications of a bankruptcy policy that exempts the collateralized contracts from an automatic stay. In the subsequent section, we examine the implications of a bankruptcy policy that imposes an automatic stay on collateral.

3.1 Repo contracts

A repo contract is represented compactly by $(\tilde{c}_1, \tilde{c}_2)$, where the initial loan size is \tilde{c}_1 , the amount of collateral is $a_1 = \tilde{c}_1$, and the loan repayment is \tilde{c}_2 . If the borrower does not default, then the lender receives \tilde{c}_2 units of good c from the borrower, and the lender transfers the collateral, \tilde{c}_1 units of good a to the borrower at date 2. If the borrower defaults, then, at date 2 the lender owns the collateral, a , which can be used by him starting at date 2.

Suppose a matched borrower dies. The lender can consume the collateral at

either dates 2 or 3, and his payoff is $\gamma\tilde{c}_1$. Alternatively, the lender can enter the date 3 matching process with his collateral. If he is matched with a trader, then there are gains from trade because the lender's relative valuation of good a to good c is γ and the trader's is 1. Hence, the lender's payoff can exceed $\gamma\tilde{c}_1$ if he is matched with a trader.²⁰

Denote the terms of trade in a lender-trader match by the contract $(\tilde{a}_3, \tilde{c}_3)$, where the lender gives \tilde{a}_3 units of good a to the trader in exchange for \tilde{c}_3 units of good c . The payoff to the lender associated with contract $(\tilde{a}_3, \tilde{c}_3)$ is $\gamma(\tilde{a}_2 - \tilde{a}_3) + \tilde{c}_3$, where \tilde{a}_2 represents the amount of good a that the lender brings into the match, and the payoff to the trader is $\tilde{a}_3 + \bar{c} - \tilde{c}_3$. Total *surplus* in a lender-trader match is $S^{LT} = (1 - \gamma)\tilde{a}_3$.

Assume the lender has all of the bargaining power. The trader will accept the lender's offer only if $\tilde{a}_3 \geq \tilde{c}_3$, and the take-it-or-leave-it assumption implies that $\tilde{a}_3 = \tilde{c}_3$. The payoff to a matched lender holding collateral equal to \tilde{a}_2 is

$$\begin{aligned} \gamma(\tilde{a}_2 - \tilde{a}_3) + \tilde{c}_3 &= \begin{cases} \tilde{a}_2 & \text{if } \tilde{a}_2 \leq \bar{c} \\ \gamma(\tilde{a}_2 - \bar{c}) + \bar{c} & \text{if } \tilde{a}_2 > \bar{c} \end{cases} \\ &= \min\{\tilde{a}_2, \bar{c} + \gamma(\tilde{a}_2 - \bar{c})\} \end{aligned}$$

(a concave function of \tilde{a}_2) and the payoff to the trader is \bar{c} . Since the lender's expected payoff associated with entering the date 3 matching process is strictly greater than $\gamma\tilde{a}_2 = \gamma\tilde{c}_1$, he will always enter the date 3 matching process holding collateral \tilde{a}_2 when his borrower defaults.

The repo contract $(\tilde{c}_1, \tilde{c}_2)$ that the lender offers the borrower in a date 1 match is clearly affected by the possibility that his borrower defaults. Since the lender has all of the bargaining power in the date 1 match, $\tilde{c}_1 = \tilde{c}_2 = \tilde{a}_1 = \tilde{a}_2$. Denote the probability that the lender is matched with a trader in the event that his borrower dies by M_d^{LT} , and let M_d denote the measure of matches between traders and either lenders or investors at date 3 in the default state. For the Leontief matching technology lenders and investors direct their search to traders, and, therefore, $M_d = \min\{n^I + \Delta m, n^T\}$ and

$$M_d^{LT} = \frac{M_d}{\Delta m + n^I}.$$

We can characterize the optimal date 1 repo contract, $(\tilde{c}_1, \tilde{c}_2)$, by considering the following maximization problem, which is to choose the amount of good c_1 to produce.

$$\max_{c_1} -c_1 + (1 - \delta\Delta) u(c_1) + \tag{1}$$

$$\delta\Delta [M_d^{LT} \min\{c_1, \bar{c} + \gamma(c_1 - \bar{c})\} + (1 - M_d^{LT}) \gamma c_1], \tag{2}$$

If the borrower does not die the lender consumes c_1 units of good c at date 2. If the borrower dies, the lender is able to enter the date-3 matching process since

²⁰If the lender is matched with an investor, there are no gains from trade—since both agents have good a —and his payoff will be $\gamma\tilde{c}_1$.

there is an exemption on the automatic stay. He consumes c_1 units of good a if he is not matched. If he is matched, then the amount he consumes depends on whether his collateral is less than or greater than the trader's endowment. The term in brackets arises because if a lender's collateral, c_1 units of good a is less than the trader's endowment of \bar{c} , then he will be able to exchange all of his collateral for good c . On the other hand if his collateral is greater than the trader's endowment, the lender will only be able to exchange part of his collateral for good c . Note that there is a discrete decrease in the *marginal* benefit associated with having an additional unit of good c at $c = \bar{c}$; the expected marginal benefit falls from $\delta\Delta [M_d^{LT} + (1 - M_d^{LT})\gamma]$ to $\delta\Delta\gamma$.

Thus the choice \tilde{c}_1 is characterized by the first order conditions for this problem as follows:

(i) If $(1 - \delta\Delta)u'(\bar{c}) + \delta\Delta\gamma > 1$, then $\tilde{c}_1 > \bar{c}$, where:

$$(1 - \delta\Delta)u'(\tilde{c}_1) + \delta\Delta\gamma = 1; \quad (3)$$

(ii) If $(1 - \delta\Delta)u'(\bar{c}) + \delta\Delta(M_d^{LT} + (1 - M_d^{LT})\gamma) < 1$, then $\tilde{c}_1 < \bar{c}$, where:

$$(1 - \delta\Delta)u'(\tilde{c}_1) + \delta\Delta(M_d^{LT} + (1 - M_d^{LT})\gamma) = 1; \quad (4)$$

(iii) otherwise, $\tilde{c}_1 = \bar{c}$.

Suppose that the default state is realized and, as a result, Δm borrowers die in between dates 1 and 2. Then, at date 3, traders, investors and lenders will enter the matching process. Denote the probability that an investor is matched with a trader in the default state as M_d^{IT} , where $M_d^{IT} = M_d^{LT}$. The terms of trade between an investor and a trader are denoted (\hat{a}_3, \hat{c}_3) and are not a function of the matching probability M_d^{IT} . Hence, the investor exchanges $\bar{\tau} = \min\{\bar{a}, \bar{c}\}$ units of good a for $\bar{\tau}$ units of good c with the trader. From the investor's date 3 perspective, when $\bar{\tau} = \bar{a}$, the price of good a , measured before agents are matched at date 3, p_a^δ , is

$$p_a^\delta = M_d^{IT}f'(\bar{a}) + (1 - M_d^{IT}).$$

It is important to emphasize that $p_a^\delta \leq p_a$, since $M^{IT} \geq M_d^{IT21}$. When $M^{IT} > M_d^{IT}$, $p_a > p_a^\delta$, and the lower price in the default state will be referred to as a "fire sale" of asset a . The value of asset a decreases to investors because lenders' enter the date-3 matching process to sell their collateral and this reduces the probability that the investors are matched with traders. (In the no-default state, an event that occurs with probability $1 - \delta$, the price of asset a is p_a). There are

²¹As above, M^{IT} represents the probability that an investor is matched with a trader in the no-default state. In the no-default state, lenders do not enter the date-3 matching process. When $\bar{\tau} = \bar{c}$, the appropriate measure of gains from trade is the average price of good a , and

$$M_d^{IT} \left[\frac{\bar{c} f(\bar{c})}{\bar{a} \bar{c}} + \frac{\bar{a} - \bar{c}}{\bar{c}} \right] + (1 - M_d^{IT}) \leq M^{IT} \left[\frac{\bar{c} f(\bar{c})}{\bar{a} \bar{c}} + \frac{\bar{a} - \bar{c}}{\bar{c}} \right] + (1 - M^{IT}).$$

real effects associated with the fire sale since the total amount of real investment falls, compared to the situation where borrowers do not default. In fact, in the no default state the repo contract achieves the planner's allocation, since surplus is maximized within each match. In the default state, however, the repo contract may not achieve the planner's allocation since the repo contract allows lenders to enter the $t = 3$ matching process. Once lenders are in a match with a trader, they do maximize the surplus from trading, but such surplus may not maximize aggregate welfare if the surplus from a trader-investor match exceeds the surplus from a trader-lender match. By entering the $t = 3$ matching process, lenders generate a congestion externality: the number of matches between investors and traders decreases, and so does aggregate welfare. If, instead, the surplus from a trader-lender match is larger than the surplus from a trader-investor match then it is desirable to allow lenders in the $t = 3$ matching process.

4 Government policy

Consider now a version of the economy described in the previous section where there is a benevolent government who maximize aggregate welfare according to a utilitarian welfare function. The goal of this section is to characterize a government policy that implements the efficient allocation. In the basic no-default model, the government cannot increase total social surplus, compared to the equilibrium allocation. When borrowers can default, however, a government may be able to increase total social surplus, compared to the equilibrium allocation, by affecting the flow of lenders that enter the date-3 matching process. In particular, the government policy instrument is the specification of automatic stay provisions on collateral. We discuss the role of alternative policies in section 5. Let θ represent the fraction of lenders that are allowed to use the collateral of their defaulting borrower as they see fit starting at the beginning of date 2. An exemption from an automatic stay on collateral for *all* lenders implies that $\theta = 1$, and an automatic stay on all collateral, where lenders are only able to access their collateral in date 3 after the matching process is completed, implies that $\theta = 0$. When $\theta = 1$, the $(\tilde{c}_1, \tilde{c}_2)$ is a repo contract; when $\theta = 0$, it is a collateralized loan contract. Note that $\theta \in (0, 1)$ can be interpreted as a partial exemption from an automatic stay in the sense that some lenders, $\theta \Delta m$ of them, are exempt from an automatic stay and others, $(1 - \theta) \Delta m$ of them, are not. It is important to emphasize that if a lender's collateral is subject to an automatic stay, then he (and his collateral) cannot participate in the date-3 matching process.

Government policy, through its effect on θ , can affect the payoffs and behavior of the various agents in the economy. The expected payoff to a borrower, W_B , is

$$W_B = m(1 - \delta\Delta)(\tilde{a}_1 - \tilde{c}_2),$$

where m is the probability that a borrower is matched with a lender at date 1. The behavior of the borrower can be affected by government policy since policy can affect \tilde{c}_1 , which, in turn, affects \tilde{a}_1 and \tilde{c}_2 . The payoff to the borrower

is unaffected by government policy since the lender has all of the bargaining power, $\tilde{c}_2 = \tilde{a}_1$, which implies that $W_B = 0$.

The payoff to the lender, W_L , is given by

$$W_L = m\{-\tilde{a}_1 + (1 - \delta\Delta)u(\tilde{c}_2) + \delta[\Delta\theta(M_d^{LT}(\tilde{c}_3 + \gamma\tilde{a}_2 - \gamma\tilde{a}_3) + (1 - M_d^{LT})\gamma\tilde{a}_2)] + \Delta(1 - \theta)\gamma\tilde{a}_2\}.$$

Government policy can affect the payoff of the lender directly—since θ appears in W_L —and indirectly through \tilde{c}_1 and \tilde{c}_2 —and, as a result, through \tilde{a}_1 , \tilde{a}_2 , \tilde{c}_3 , and \tilde{a}_3 .

The expected payoff to a trader, W_T , is

$$\begin{aligned} W_T &= (1 - \delta) [M^{TI}(\hat{a}_3 - \hat{c}_3 + \bar{c}) + (1 - M^{TI})\bar{c}] + \\ &\quad \delta [M_d^{TI}(\hat{a}_3 - \hat{c}_3 + \bar{c}) + M_d^{TL}(\tilde{a}_3 - \tilde{c}_3 + \bar{c}) + (1 - M_d^{TI} - M_d^{TL})\bar{c}] \\ &= [(1 - \delta)M^{TI} + \delta M_d^{TI}](\hat{a}_3 - \hat{c}_3) + \delta M_d^{TL}(\tilde{a}_3 - \tilde{c}_3) + \bar{c}, \end{aligned}$$

where \hat{a}_3, \hat{c}_3 denote optimal offers made by the investor to the trader and are characterized below.

$$M_d^{TL} = \frac{M_d}{n^T} \frac{\Delta\theta m}{\Delta\theta m + n^I}$$

and

$$M_d^{TI} = \frac{M_d}{n^T} \frac{n^I}{\Delta\theta m + n^I}.$$

Since investors and lenders have all of the bargaining power in their matches with traders, $\hat{a}_3 = \hat{c}_3$ and $\tilde{a}_3 = \tilde{c}_3$, which implies that $W_T = \bar{c}$. Hence, the payoff to the trader is unaffected by government policy θ . In fact, $\hat{c}_3 = \min\{\bar{c}, \bar{a}\} = \bar{a}$, which is the trade allocation in a trader-investor match in a world without default. Note, however, that \tilde{c}_3 and \tilde{a}_3 can be affected by government policy.

Finally, the payoff to the investor, W_I , is

$$\begin{aligned} W_I &= (1 - \delta) [M^{IT}(f(\hat{c}_3) - \hat{a}_3 + \bar{a}) + (1 - M^{IT})\bar{a}] + \\ &\quad \delta [M_d^{IT}(f(\hat{c}_3) - \hat{a}_3 + \bar{a}) + (1 - M_d^{IT})\bar{a}] \\ &= [(1 - \delta)M^{IT} + \delta M_d^{IT}](f(\hat{c}_3) - \hat{a}_3) + \bar{a}. \end{aligned}$$

Although the behavior of the investor is unaffected by government policy—since $\hat{a}_3 = \hat{c}_3 = \min\{\bar{a}, \bar{c}\} = \bar{a}$ —his payoff is affected since the matching probability M_d^{IT} is a function of θ .

In order to evaluate various government policies, we must understand how the behavior of a lender—which is simply his choice of \tilde{c}_1 —is influenced by changes in θ . The lender's problem is a straightforward generalization of the problem (1) in section 3, to take account of government policy, θ .

$$\begin{aligned} \max_{c_1} & -c_1 + (1 - \delta\Delta)u(c_1) + \\ & \delta\Delta \{ \theta [M_d^{LT} \min\{c_1, \bar{c} + \gamma(c_1 - \bar{c})\} + (1 - M_d^{LT})\gamma c_1] + (1 - \theta)\gamma c_1 \} \quad (G1) \end{aligned}$$

The first-order condition characterizing \tilde{c}_1 is as follows,

$$(i) \text{ If } (1 - \delta\Delta) u'(\bar{c}) + \delta\Delta\gamma > 1, \text{ then } \tilde{c}_1 > \bar{c}, \quad (5)$$

where $(1 - \delta\Delta) u'(\tilde{c}_1) + \delta\Delta\gamma = 1$;

$$(ii) \text{ If } (1 - \delta\Delta) u'(\bar{c}) + \delta\Delta(\gamma + (1 - \gamma)\theta M_d^{LT}) < 1, \text{ then } \tilde{c}_1 < \bar{c}, \quad (6)$$

where $(1 - \delta\Delta) u'(\tilde{c}_1) + \delta\Delta(\gamma + (1 - \gamma)\theta M_d^{LT}) = 1$;

(iii) Otherwise, $\tilde{c}_1 = \bar{c}$.

Proposition 1 demonstrates how loan size, \tilde{c}_1 , for the contract $(\tilde{c}_1, \tilde{c}_2)$ is affected by a change in the government policy variable θ .

Proposition 1 \tilde{c}_1 is weakly increasing in θ .

Proof. If $\tilde{c}_1 < \bar{c}$, then from (6), we have

$$\frac{\partial \tilde{c}_1}{\partial \theta} = - \frac{\delta\Delta(1 - \gamma) \partial(\theta M_d^{LT}) / \partial \theta}{(1 - \delta\Delta) u''(\tilde{c}_1)}. \quad (7)$$

Since

$$\theta M_d^{LT} = \begin{cases} \theta & \text{if } \Delta\theta m + n^I < n^T \\ \frac{\theta n^T}{\Delta\theta m + n^I} & \text{if } \Delta\theta m + n^I > n^T \end{cases},$$

and

$$\frac{\partial(\theta M_d^{LT})}{\partial \theta} = \begin{cases} 1 & \text{if } \Delta\theta m + n^I < n^T \\ \frac{n^I n^T}{(\Delta\theta m + n^I)^2} & \text{if } \Delta\theta m + n^I > n^T \end{cases}, \quad (8)$$

we get that $\partial \tilde{c}_1 / \partial \theta > 0$. If $\tilde{c}_1 \geq \bar{c}$, then from (5), $\partial \tilde{c}_1 / \partial \theta = 0$. ■

The intuition behind this proposition is straightforward. Having access to the date-3 matching process is valuable for the lender. Suppose that $\tilde{c}_1 < \bar{c}$. One can interpret an increase in θ as providing the lender with better insurance against borrower default in the sense that an increase in θ increases the probability that lender will be able to exchange good a —which he values “a little”—for good c —which he values “a lot”—if the borrower defaults. Since the cost associated with borrower default declines as θ increases, the lender has an incentive to increase his date 1 loan, \tilde{c}_1 , and the collateral $\tilde{a}_1 = \tilde{c}_1$ that he holds. Suppose now that $\tilde{c}_1 \geq \bar{c}$. In this situation, the lender has no incentive to increase his date-1 loan size \tilde{c}_1 when θ increases since, independent the lender being matched or not at date 3, the value of an additional unit collateral, conditional on the borrower defaulting, is unchanged and equal to $\gamma < 1$.

The government seeks to maximize total social surplus, S , which is given by

$$S = n^B W_B + n^L W_L + n^I (W_I - \bar{a}) + n^T (W_T - \bar{c}).$$

The assumed bargaining conventions imply that the expression for total social surplus can be simplified to

$$S = n^L W_L + n^I (W_I - \bar{a}),$$

which means we only need to focus on the behavior of and payoffs to lenders and investors.

We now characterize how government policy affects total social surplus. Since $\hat{c}_3 = \bar{i} \equiv \min\{\bar{a}, \bar{c}\}$, the surplus to investors is

$$W_I - \bar{a} = [(1 - \delta) M^{IT} + \delta M_d^{IT}] (f(\bar{i}) - \bar{i}), \quad (9)$$

and government policy θ affects the investor's surplus only through the matching probability, M_d^{IT} .

Proposition 2 *The investor's payoff is weakly decreasing in θ .*

Proof. Note that

$$\frac{\partial M_d^{IT}}{\partial \theta} = \begin{cases} 0 & \text{if } n^T > \Delta\theta m + n^I \\ -\frac{\Delta mn^T}{(\Delta\theta m + n^I)^2} & \text{if } n^T \leq \Delta\theta m + n^I \end{cases}$$

and, since $M^{IT} = \min\{n^I, n^T\}/n^I$, $\partial M^{IT}/\partial \theta = 0$. Therefore, $\partial W_I/\partial \theta = \delta (\partial M_d^{IT}/\partial \theta) (f(\bar{a}) - \bar{a})$ or

$$\frac{\partial W_I}{\partial \theta} = \begin{cases} 0 & \text{if } n^T > \Delta\theta m + n^I \\ -\delta \frac{\Delta mn^T}{(\Delta\theta m + n^I)^2} (f(\bar{a}) - \bar{a}) & \text{if } n^T \leq \Delta\theta m + n^I \end{cases}. \quad (10)$$

■

This proposition accords with intuition. If the measure of traders is relatively large—in the sense that $n^T > \Delta\theta m + n^I$ —then increasing access to the date-3 matching process for lenders has no effect on the investors' surpluses since investors are matched with probability one at date 3. If, however, the number of traders is not relatively large—in the sense that $n^T \leq \Delta\theta m + n^I$ —then increasing access to the date-3 matching process to lenders will reduce the probability that investors are matched with traders and, hence, reduces the payoffs to lenders.

Turning to lenders, since, $\tilde{c}_1 = \tilde{c}_2 = \tilde{a}_2 = \tilde{a}_1$ and $\tilde{a}_3 = \min\{\tilde{c}_1, \bar{c}\}$, the surplus function for a lender can be simplified to

$$W_L = m^{LB} \{-\tilde{c}_1 + (1 - \delta\Delta) u(\tilde{c}_1) + \delta\Delta\gamma\tilde{c}_1 + \delta\Delta\theta M_d^{LT}\tilde{a}_3(1 - \gamma)\}. \quad (11)$$

To assess how its policy affects total social surplus, the government must understand how W_L is affected by a change in θ .

Proposition 3 *The lender's payoff is strictly increasing in θ .*

Proof. The derivative of (11) with respect to θ is:

$$\begin{aligned} \frac{\partial W_L}{\partial \theta} &= m^{LB} \left\{ \frac{\partial \tilde{c}_1}{\partial \theta} [-1 + \delta\Delta\gamma + (1 - \delta\Delta) u'(\tilde{c}_1)] \right\} + \\ &+ m^{LB} \delta\Delta(1 - \gamma) \frac{\partial \tilde{a}_3}{\partial \theta} \theta M_d^{LT} + m^{LB} \delta\Delta(1 - \gamma) \frac{\partial (\theta M_d^{LT})}{\partial \theta} \tilde{a}_3 \end{aligned} \quad (12)$$

The first line of (13) is equal to zero. When $\tilde{c}_1 < \bar{c}$, this is implied by (6), recognizing that $\partial\tilde{c}_1/\partial\theta = \partial\tilde{a}_3/\partial\theta$. When $\tilde{c}_1 \geq \bar{c}$, (5) implies that $\partial\tilde{c}_1/\partial\theta = \partial\tilde{a}_3/\partial\theta = 0$. Therefore

$$\begin{aligned} \frac{\partial W_L}{\partial\theta} &= m^{LB}\delta\Delta(1-\gamma)\left\{\frac{\partial(\theta M_d^{LT})}{\partial\theta}\tilde{a}_3\right\} \\ &= m^{LB}\delta\Delta(1-\gamma)\left\{\begin{array}{ll} \tilde{a}_3 & \text{if } \Delta\theta m + n^I < n^T \\ \frac{n^I n^T}{(\Delta\theta m + n^I)^2}\tilde{a}_3 & \text{if } \Delta\theta m + n^I > n^T \end{array}\right\} \\ &> 0 \end{aligned}$$

■

The intuition behind proposition 2 is straightforward. Holding \tilde{c}_1 constant, an increase in θ increases the chance that the lender will be able to participate in the date-3 matching process. This unambiguously increases the surplus of the lender because, in the event of a borrower default, the value of either part or all of the lender's collateral a increases from γa to a . As well, if $\tilde{c}_1 < \bar{c}$, then, holding the date-3 matching probability constant, an increase in θ optimally increases \tilde{c}_1 and, by construction, the lender's collateral holdings. Since an increase in \tilde{c}_1 is an optimal response to an increase in θ , the lender's surplus must also increase.

Propositions 2 and 3 identify the trade-off that the government faces when choosing its policy. An increase in θ (weakly) lowers the probability that an investor will be matched with a trader and, hence, (weakly) lowers the level of (productive) investment. But an increase in θ strictly increases the probability that a lender will be matched with a trader, in the event of a borrower default, and this enhances the "liquidity" of a lender's collateral. (Collateral becomes more "liquid" in the sense that it can be converted into the consumption good with a higher probability.) To assess a government policy that changes the value of θ , one simply has to compare the "investment effect" with the "liquidity effect." Generally speaking, the net effect can go either way as the magnitudes of the two effects depend upon model parameters.

An increase in θ has also an indirect effect on the liquidity of the repo market at $t = 1$: proposition 1 shows that c_1 increases in θ , which affects the payoff to lender. When choosing the optimal policy parameter θ , however, the marginal effect of c_1 on lender's payoff disappears because of an envelope argument: the lender chooses c_1 at $t = 1$ by exactly offsetting its marginal benefits by its marginal costs. Therefore, the indirect effect that θ has on lenders' welfare through c_1 is zero because when θ is chosen lenders' best response is to choose c_1 to solve problem (2). Nonetheless, the liquidity of the repo market at $t = 1$ affects the speed at which lenders' welfare increases with θ : a higher θ induces a larger marginal benefit to the lender when meeting a trader on the date $t = 3$ market.

Consider first the situation where $n^T > \Delta m + n^I$. One can interpret this situation as one where the date-3 market is "very liquid"—having the capacity always to match both investors and lenders with probability one. In this

situation, the optimal government policy is clear.

Proposition 4 *When $n^T > \Delta m + n^I$, then the optimal government policy provides an exemption from a bankruptcy stay for all lenders.*

Proof. From (10) and (13), when $n^T > \Delta m + n^I$

$$\frac{\partial S}{\partial \theta} = n^L \frac{\partial W_L}{\partial \theta} + n^I \frac{\partial W_I}{\partial \theta} = m\delta\Delta(1-\gamma)\tilde{a}_3 > 0,$$

for all θ . Hence, the government should choose θ “as high as possible,” i.e., $\theta = 1$. ■

Consider now the interesting case where the date-3 market is illiquid from the investor’s perspective in the sense that $n^I > n^T$. For this case, again using (10) and (13), we obtain

$$\frac{\partial S}{\partial \theta} = \frac{1}{(\Delta\theta m + n^I)^2} [(1-\gamma)\tilde{a}_3(\theta) - (f(\bar{i}) - \bar{i})]. \quad (14)$$

From equation (14) it is apparent that whether a marginal increase in θ raises or reduces aggregate surplus depends only on the gap between the payoff to lender and the investor: it is, in fact, independent of the matching probabilities and the effect of θ on the liquidity of the repo market at $t = 1$ (i.e. the amount of the loan between the lender and borrower, c_1).

This is so because the effect of θ on the matching probability is the same for both lenders and investors, since the probability of meeting a trader is the same from their perspective: therefore only the net value of their aggregate payoff determines whether total surplus increases with θ or not. Also, the effect of θ on c_1 does not appear in equation (10) because of an envelope condition argument: the marginal impact of θ on c_1 is weighted by the marginal impact of c_1 on S , which in turn is zero by the optimality conditions to the lender’s decision problem. When the lender makes an offer to the borrower he equalizes marginal benefits and costs of lending c_1 : the optimal bankruptcy policy θ is chosen taking into account the best response of the lender, so the marginal benefits of increasing c_1 (through an increase in θ) are perfectly outweighed by the marginal costs of doing so. Therefore, when $n^I > n^T$, the optimal government policy is determined by comparing the value of $f(\bar{i}) - \bar{i}$ —which is proportional to the investment effect—with that of $(1-\gamma)\tilde{a}_3(\theta)$ —which is proportional to the liquidity effect—for various values of θ . More formally,

Proposition 5 *Suppose $n^I > n^T$. If*

$$(1-\gamma)\tilde{a}_3(0) > (f(\bar{i}) - \bar{i}),$$

then the optimal government policy provides an exemption from a bankruptcy stay of all lenders, i.e., $\theta = 1$. If

$$(1-\gamma)\tilde{a}_3(1) < (f(\bar{i}) - \bar{i}),$$

then the optimal government policy requires a bankruptcy stay for all lenders, i.e., $\theta = 0$.

Proof. Since $\tilde{a}_3 = \min\{\tilde{c}_1, \bar{c}\}$, Proposition 1 implies that $\partial\tilde{a}_3/\partial\theta \geq 0$. If $(1 - \gamma)\tilde{a}_3(0) > (f(\bar{i}) - \bar{i})$, then from (14) $\partial W/\partial\theta > 0$ for all $\theta \in [0, 1]$, and setting $\theta = 1$ is optimal. If $(1 - \gamma)\tilde{a}_3(1) < (f(\bar{i}) - \bar{i})$, then from (14) $\partial W/\partial\theta < 0$ for all $\theta \in [0, 1]$, and setting $\theta = 0$ is optimal. ■

Note the message of the proposition: Even though the date-3 market is illiquid from the perspective of investors—even if lenders are not permitted to participate—it may be optimal for the government to exempt defaulted lenders from a bankruptcy stay. This will happen when, intuitively speaking, the “liquidity value” of allowing lenders to have access to traders is greater than the “investment value” associated with investor-trader matches. It is true that when $\theta = 1$, lenders will displace economy-wide investment when $n^I > n^T$. However, the value of the liquidity, $(1 - \gamma)\tilde{a}_3$, generated by lenders exceeds that of the displaced investment.

The next proposition shows that when $n^I > n^T$ nothing is gained by considering policies with interior $\theta \in (0, 1)$.

Proposition 6 *When $n^I > n^T$, either it is optimal to provide an exemption from the bankruptcy stay to all lenders, $\theta = 1$, or it is optimal to impose a bankruptcy stay on all lenders, $\theta = 0$.*

Proof. Consider the derivative of surplus in formula 14. If there is a strict interior maximum $\hat{\theta}$, then this expression must be positive for values of θ just below $\hat{\theta}$ and negative for values of θ just above $\hat{\theta}$. But since $\tilde{a}_3(\theta)$ is a non decreasing function, this is impossible. ■

When $n^I > n^T$, the optimal government policy is either imposes a bankruptcy stay on all lenders or an exemption from a bankruptcy stay for all lenders. Although a partial exemption, i.e., $\theta \in (0, 1)$, is permitted, it is never optimal, except for the knife edge case where $S'(\theta) = 0$ for all $\theta \in [0, 1]$.²² But even in this knife-edge case, $\theta = 1$ or $\theta = 0$ is an optimal policy. The effect of θ on total surplus equals the total payoff to the lender and the investor evaluated at θ : the payoff to the investor from trading with a trader is independent of θ because it is given by the value of trading \bar{i} (i.e. either \bar{a} or \bar{c}), therefore investors’ loss from an increase in θ is independent of how large θ actually is. On the other hand, the payoff to the lender from trading with a trader is increasing in θ because a larger θ implies a more liquid repo market at $t = 1$ and thus a larger loan from the lender to the borrower (i.e. a larger c_1). This implies that if the lender enters the date $t = 3$ market and is matched with a trader, he can exchange more assets ($\tilde{a}_3 = c_1$) for consumption goods, which increases his payoff. Therefore, if it is optimal to provide exemption from the stay for some lenders (i.e. $\theta^* > 0$), then it must be so for all lenders (i.e. $\theta^* = 1$)²³

²²Among other things, this knife-edge case requires that $\tilde{c}_1 > \bar{c}$.

²³Notice that an increase in θ disrupt the matching process between investors and traders but benefits the matching process between lenders and investors. Since the probability of being matched with a trader is the same for both lenders and investors, then the effect of θ on the matching probabilities is irrelevant for whether total surplus increases or decreases in θ .

The final case in terms of date-3 market liquidity to consider is the intermediate case where $n^I < n^T$ and $\Delta n^L + n^I > n^T$. In other words, there is enough activity in the date 3 market to provide goods to all investors, but not enough to provide goods to all lenders as well. Let θ^* be such that $\theta^* \Delta n^L + n^I = n^T$ —in other words the value of θ which exhausts the supply of traders. Clearly, it would never be optimal for the government to choose a $\theta < \theta^*$. Optimal government policy here somewhat mirrors the case where $n^I > n^T$, except now the lower bound of optimal government policy is θ^* instead of $\theta = 0$. In particular

Proposition 7 *When $n^I < n^T$ and $\Delta n^L + n^I > n^T$ an optimal government policy either provides an exemption from a bankruptcy stay for all lenders, $\theta = 1$, or imposes a bankruptcy stay on fraction $1 - \theta^*$ of lenders.*

Proof. The proof follows those of Propositions 5 and 6. ■

In other words with the Leontief matching technology the essential question is whether there is greater social value from a match by an investor or a match to fulfill liquidity needs of the lenders. If the investments are more valuable, then fire sales should be discouraged to the extent that they crowd out this investment. If the liquidity needs are more valuable, then they should be encouraged through complete exemptions from automatic stays. The larger the liquidity of the repo market at $t = 1$ the more valuable such liquidity needs at $t = 3$ are: the bankruptcy policy affects c_1 and through c_1 it also affects the amount of collateral that lenders want to sell at $t = 3$, \tilde{a}_3 . By inducing an increase in c_1 and thus in \tilde{a}_3 , the exemption from bankruptcy stay unambiguously increases the value of date $t = 3$ liquidity.

This would not necessarily be true if collateral was traded on a competitive market, where the larger amount of assets sales induces a lower price at which those assets are sold (for the same supply). Because of this interaction between the amount of assets sold and the price at which they can be sold, the liquidity needs at $t = 3$ may not necessarily be more valuable to the lender (and a planner) when the exemption from bankruptcy stay is granted to more and more lenders.

5 Alternative policies

The policy described in section 4 restores efficiency in the repo market and the $t = 3$ market and is the policy at the core of financial reforms discussions concerning the exemption from automatic stay for derivatives and repos. Such policy, as characterized in section 4, is clearly an optimal policy.

In this section we analyze alternative policies and discuss how they compare with respect to the exemption from automatic stay.

We focus on the case where $S^{IT} > S^{LT}$: when $n^I \geq n^T$ no lenders should enter the matching process. If the government can tax agents and redistribute resources after trading has happened then the efficient allocation is clearly implementable: the government can tax lenders's purchase of good c in the amount $(1 - \gamma)c_1$ and redistribute it to investors. Such tax system implies that there

exists an equilibrium where lenders do not enter the $t = 3$ matching process since they are indifferent between entering or not.

When $n^I < n^T$ the efficient allocation is such that there is an optimal fraction of lenders, say θ_1 , that should enter the $t = 3$ matching process. If the government can tax agents and redistribute resources after trading has happened then the efficient allocation is implementable by the following tax system: the government taxes lender's entire endowment of good a at $t = 3$, c_1 , so that they cannot actively participate to the $t = 3$ matching process, where only investors participate. Such revenue of good a is redistributed to traders who did not matched with investors; then the government taxes such traders's endowment of good c in the same amount as the tax on lenders, c_1 . The revenue of good c that is thus raised can be redistributed either equally to all lenders or to $\theta_1 m \Delta$ lenders, whereas the remaining $(1 - \theta_1) m \Delta$ lenders are given back their endowment of c_1 units of good a . Alternatively, a tax of $(1 - \gamma) c_1$ units of good a can be levied only on a fraction $(1 - \theta_1)$ of lenders at $t = 3$ if they enter the matching process. Then there exists an equilibrium where only θ_1 lenders enter the matching process and the planner's allocation is implemented.

Notice that no tax system with a unique uniform tax on purchases of good c by lenders could implement the efficient allocation when $n^I < n^T$: such tax would impact all lenders rather than only a fraction $1 - \theta_1$ of them to deter them from entering the $t = 3$ matching process.

Also notice that, being a special case of [20] with no cost of searching, in this model the regressive tax system described in [20] cannot be implemented. If search intensity is costly for each agent then a version of such regressive tax system is equivalent to the government policy we characterize in section 4.

6 Final Remarks

This paper has deals with specific costs and benefits of the exemption from the automatic stay associated with repos in bankruptcy. The benefit we focus on is the improvement in insurance arising from the ability of the lender to dispose of his collateral quickly, and the cost is the disruption of the market for the collateral and investments goods. The desirability of extending the automatic stay to repos depends on the relative importance of these costs and benefits.

The standard argument in favor of automatic stays in the bankruptcy process is the destruction of value associated with an uncoordinated break-up of the bankrupt firm. When the assets being sold are financial instruments rather than real assets, this justification *appears* to be less important. Furthermore, having the option to allow some financial contracts to avoid the automatic stay seems to be desirable as a way of increasing the opportunities for flexibility in a firm's borrowing and thereby reduce borrowing costs. (While other articles, noted in the introduction, have emphasized the costs imposed on less favored lenders, as a first pass, this is a justification for the law to limit the use of this favored treatment itself, not a justification for the prohibition of the favored treatment).

The fire sale cost applies not to the firm itself (in which case initial contracting by the firm with its various counterparties could ultimately resolve the problem) but to other participants in the market for the collateral good. Thus the importance of the cost depends on the effect that the firm’s bankruptcy has on the market—roughly speaking, on the liquidity and depth of that market. In this respect, the conclusions of our model correspond to the comments by Duffie and Skeel (2012), which advocate the exemption from the bankruptcy stay only when the market for the collateral is extremely liquid. (In our model, depth can be associated with the excess of traders willing to take the collateral when lenders attempt to dispose of it). We provide a simple comparison of these costs with the benefits from the improved opportunities of borrowing through increase of the liquidity of the loans provided to the firm initially.

The externality that gives rise to a fire sale in our model is a direct result of trade being mediated by over-the-counter markets. These markets generate a search externality, where sellers of collateral can crowd out investors because both of these parties are searching for the same thing: liquidity. If, alternatively, we model exchange as occurring on purely competitive markets, then the externality disappears. In particular, the optimal bankruptcy policy would exempt the automatic stay because the exemption gives all agents an opportunity to seek liquidity, and competitive markets ensure that liquidity ends up with agents that place the highest value on it. Although this result is interesting, it is not particularly helpful or relevant from a policy perspective, precisely because repo markets are not competitive—they are over-the-counter. Given this, we would argue that the externality that is central to our analysis is both appropriate and realistic. It also turns out that it is extremely tractable to analyze, although other forms of externality, for example through cash-in-the-market pricing (Allen and Gale 2007), will yield similar results.

More specifically, the nature of the inefficiency in this model stems from the non-degenerate type distribution of agents: agents are heterogeneous in their productivity once matched. Since the decision of an agent to enter the matching process alters the distribution of meetings for other agents²⁴, the Hosios condition is no longer sufficient to guarantee efficiency of the equilibrium: the congestion and thick market externalities do not cancel out. The least productive agents, lenders, generate a *congestion* externality on other lenders and, more importantly, on the most productive agents: investors. Thus, in equilibrium less productive matches are generated. Lenders also generate a *thick market*²⁵ externality on the other side of the market: traders. By actively participating to the $t = 3$ matching process lenders make it harder for traders to meet investors. Since investors have all the bargaining power in our benchmark model the thick market externality is less apparent, but if traders can appropriate some of the surplus from trading with investors, then traders would also be adversely affected by lenders’ entry into the $t = 3$ matching process because they have less opportunities to carry out more productive trades. From a pro-

²⁴This is similar to [20].

²⁵Similarly to [20], as discussed on pg.12.

duction efficiency point of view this is not desirable. Despite it is optimal for (at least some) lenders to not search at all, as long as the productivity in a lender-trader match exceeds the marginal cost of searching (which in our model is zero) lenders will prefer to search for traders, thus compromising production efficiency.

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A Appendix on Walrasian $t = 3$ market

This section analyzes equilibria in this model if we replace the date $t = 3$ matching process where traders meet investors and lenders bilaterally, with a competitive market where the asset (good a) is exchanged for good c . With θ

denoting the probability that a lender can access the date $t = 3$ market, lenders maximize expected utility at time $t = 1$ choosing the contract (c_1, a_3^L) that solves:

$$\max_{\{c_1, a_3^L\}} (1 - \delta\Delta) u(c_1) + \delta\Delta\theta \left(\gamma a_3 + \frac{c_1 - a_3}{q} \right) + \delta\Delta (1 - \theta) \gamma c_1 - c_1$$

where the feasibility and participation constraints $c_2 \leq c_1$, $a_3^L \leq a_2 \leq c_1$ have been substituted out. The optimal choice of c_1 satisfies:

$$(1 - \delta\Delta) u'(c_1) + \frac{\delta\Delta\theta}{q} + \delta\Delta (1 - \theta) \gamma = 1 \quad (15)$$

An investor chooses the quantity of goods a and c to consume, denoted a_3^I, c_3^I respectively, and the input of good c to invest in f , denoted c^I , to solve:

$$\begin{aligned} \max_{\{a_3^I, c^I, c_3^I\}} \quad & a_3^I + c_3^I \\ \text{s.t.} \quad & a_3^I \leq \bar{a} \\ & qc^I \leq \bar{a} - a_3^I \\ & c_3^I \leq f(c^I) \end{aligned} \quad (16)$$

where q is the price of good c in terms of good a in the competitive $t = 3$ market. Since f is strictly increasing, the investors' problem boils down to:

$$\max_{\{a_3^I\}} a_3^I + f\left(\frac{\bar{a} - a_3^I}{q}\right)$$

so that investors' choice of a_3^I is such that

$$q = f'\left(\frac{\bar{a} - a_3^I}{q}\right) \quad (17)$$

or $a_3^I = \bar{a}$ if $q > f'(0)$.

A trader chooses the quantity of goods a and c to consume, (a_3^T, c_3^T) , and the quantity of good c to sell on the competitive $t = 3$ market, c_3^s , to solve:

$$\begin{aligned} \max_{\{a_3^T, c_3^T, c_3^s\}} \quad & a_3^T + c_3^T \\ \text{s.t.} \quad & c_3^T + c_3^s \leq \bar{c} \\ & a_3^T \leq qc_3^s \end{aligned}$$

which boils down to:

$$\max_{c_3^s} \bar{c} + (q - 1) c_3^s \quad (18)$$

So that if $q > 1$ then $c_3^s = \bar{c}$; if $q = 1$ then $c_3^s \in [0, \bar{c}]$; if $q < 1$ then $c_3^s = 0$.

Clearly, if an equilibrium with lenders and investors active on the $t = 3$ market exists, then $q \in \left[1, \min\left(\frac{1}{\gamma}, f'(0)\right)\right]$. Since our focus is on the optimal bankruptcy policy, we consider equilibria where both investors and lenders are active. In equilibria where only lenders or investors are active there is no role for policy because there is no trade off between investors' productivity and lender's marginal utility of good c .

Also, let us consider equilibria where traders and lenders, when indifferent, choose $c_3^s = \bar{c}$ and $a_3^L = 0$ respectively, and assume that $\min\left(\frac{1}{\gamma}, f'(0)\right) = \frac{1}{\gamma}$ and f' is convex. This implies that investors choice of c^I is always interior, so (17) holds, and that investors' substitution effect is not too strong with respect to the income effect²⁶ upon an increase in θ inducing an increase in q .

The market clearing condition at $t = 3$ is then:

$$n^I c^I + m\Delta\theta \frac{(c_1 - a_3^L)}{q} = n^T \left(\bar{c} - \frac{a_3^T}{q}\right) \quad (19)$$

where c^I is investors' demand of good c ; $c_3^L = \frac{c_1}{q}$ is lenders' demand of good c ; $m\Delta\theta$ is the measure of lenders who are matched with borrowers at $t = 1$ ($m \leq n^L$), whose borrower defaults at $t = 2$ (Δ) and who are allowed to the date $t = 3$ market (θ). Aggregate supply of good c by traders when they are active in the $t = 3$ market is simply $n^T \bar{c} - \frac{a_3^T}{q}$.

We are now interested in the Pareto ranking of different equilibria with respect to θ : the probability that lenders can access the date $t = 3$ market can in fact be interpreted as a parameter governing the policy on the exemption from automatic stay for repos. Let us define aggregate welfare as the sum of agents' welfare weighted by the relative measure of each agent: $W = n^T W^T + n^I W^I + mW^L$

$$\begin{aligned} W &= n^T W^T + n^I W^I + mW^L \\ &= n^T [\bar{c} + (q-1)c_3^s] + n^I \left[a_3^I + f\left(\frac{\bar{a} - a_3^I}{q}\right) \right] + \\ &\quad m \left[(1 - \delta\Delta) u(c_1) + \delta\Delta \left[\theta \left(\gamma a_3 + \frac{c_1 - a_3}{q} \right) + (1 - \theta) \gamma c_1 \right] - c_1 \right] \end{aligned} \quad (20)$$

Given our specification of preferences, aggregate welfare is always increasing

²⁶More precisely, f' is convex implies that investors' demand of good a is decreasing in θ when q increases ($-\frac{\partial a_3^I}{\partial \theta} < 0$). This is shown below.

in θ for any $\theta \in [0, 1]$ and using (15), we have:

$$\begin{aligned}
\frac{\partial W}{\partial \theta} &= n^T \frac{\partial a_3^T}{\partial \theta} \left[1 - \frac{1}{q} \right] + n^I \frac{\partial a_3^I}{\partial \theta} \left[1 - \frac{1}{q} f' \left(\frac{\bar{a} - a_3^I}{q} \right) \right] + m \delta \Delta \theta \frac{\partial a_3^L}{\partial \theta} \left(\gamma - \frac{1}{q} \right) + \\
&\quad + m \delta \Delta \left[\gamma a_3^L + \frac{c_1 - a_3^L}{q} - \delta \Delta \gamma c_1 \right] + \\
&\quad + \frac{\partial q}{\partial \theta} \left[n^T \frac{a_3^T}{q^2} - n^I f' \left(\frac{\bar{a} - a_3^I}{q} \right) \frac{\bar{a} - a_3^I}{q^2} - m \delta \Delta \theta \frac{(c_1 - a_3^L)}{q^2} \right] \\
&= n^T \frac{\partial a_3^T}{\partial \theta} \left[1 - \frac{1}{q} \right] + m \delta \Delta \theta \frac{\partial a_3^L}{\partial \theta} \left(\gamma - \frac{1}{q} \right) + \\
&\quad + m \delta \Delta \left[\frac{c_1}{q} - \delta \Delta \gamma c_1 \right] + \\
&\quad + \frac{\partial q}{\partial \theta} \left[n^T \frac{a_3^T}{q^2} - n^I \frac{\bar{a} - a_3^I}{q} - m \delta \Delta \theta \frac{c_1}{q^2} \right] \tag{22}
\end{aligned}$$

Since we are focusing on equilibria where traders and lenders are at a corner solution then $\frac{\partial a_3^T}{\partial \theta} = \frac{\partial(q\bar{c})}{\partial \theta} = \bar{c} \frac{\partial q}{\partial \theta}$ and $\frac{\partial a_3^L}{\partial \theta} = 0$. Thus we simply have:

$$\begin{aligned}
\frac{\partial W}{\partial \theta} &= m \delta \Delta c_1 \left[\frac{1}{q} - \delta \Delta \gamma \right] + \\
&\quad + \frac{\partial q}{\partial \theta} \left\{ n^T \left[\bar{c} - \frac{c_3^T}{q} \right] - n^I c^I - m \delta \Delta \theta \frac{c_3^L}{q} \right\}
\end{aligned}$$

In equilibria where $q = 1$ we can use (19) to conclude $\frac{\partial W}{\partial \theta} > 0$.

In equilibria where $q > 1$, if $\frac{\partial q}{\partial \theta} \geq 0$ the term in curly brackets is always positive, so that $\frac{\partial W}{\partial \theta} > 0$. Although not crucial for $\frac{\partial W}{\partial \theta} > 0$, notice that $ac_3^T = 0$ when traders are at a corner solution.

We now argue that if $f'(\cdot)$ is convex then $\frac{\partial q}{\partial \theta} > 0$: differentiating (19) with respect to θ we have:

$$-n^I \frac{\partial a_3^I}{\partial \theta} + m \Delta c_1 + m \Delta \theta \frac{\partial c_1}{\partial \theta} - \frac{1}{q} \frac{\partial q}{\partial \theta} [n^I (\bar{a} - a_3^I) + m \Delta \theta c_1] = 0 \tag{23}$$

where $a_3^L = 0$, $a_3^T = \frac{\bar{c}}{q}$ has been used. From (17) we have that:

$$\frac{\partial a_3^I}{\partial \theta} = - \frac{\frac{1}{q^2} \left[\frac{f'' \left(\frac{\bar{a} - a_3^I}{q} \right)}{q} (\bar{a} - a_3^I) + f' \left(\frac{\bar{a} - a_3^I}{q} \right) \right]}{\left[u_a^{I''} (a_3^I) + \frac{1}{q^2} f'' \left(\frac{\bar{a} - a_3^I}{q} \right) \right]} \frac{\partial q}{\partial \theta}$$

and from (15) we have that:

$$\frac{\partial c_1}{\partial \theta} = \frac{1}{\phi(\theta)} \left\{ \frac{\delta \Delta}{q} u_c^{L'} \left(\frac{c_1 - a_3^L}{q} \right) - \delta \Delta u_a^{L'} (c_1) + \left[\frac{(c_1 - a_3^L) u_c^{L''} \left(\frac{c_1 - a_3^L}{q} \right)}{q} - u_c^{L'} \left(\frac{c_1 - a_3^L}{q} \right) \right] \frac{\delta \Delta \theta}{q^2} \frac{\partial q}{\partial \theta} \right\}.$$

Then (23) becomes:

$$\begin{aligned}
& -n^I \frac{\frac{1}{q^2} \left[\frac{f''\left(\frac{\bar{a}-a_3^I}{q}\right)}{q} (\bar{a} - a_3^I) + f'\left(\frac{\bar{a}-a_3^I}{q}\right) \right]}{\frac{1}{q^2} f''\left(\frac{\bar{a}-a_3^I}{q}\right)} \frac{\partial q}{\partial \theta} + m\Delta c_1 + \\
& \quad + \frac{m\Delta\theta}{\phi(\theta)} \left\{ \frac{\delta\Delta}{q} - \delta\Delta\gamma - \frac{\delta\Delta\theta}{q^2} \frac{\partial q}{\partial \theta} \right\} = \frac{\partial q}{\partial \theta} \frac{1}{q} [n^I (\bar{a} - a_3^I) + m\Delta\theta c_1]
\end{aligned}$$

or simply:

$$\frac{\partial q}{\partial \theta} = q \frac{m\Delta c_1 + \frac{m\Delta\theta}{\phi(\theta)} \left(\frac{\delta\Delta}{q} - \delta\Delta\gamma \right)}{n^I (\bar{a} - a_3^I) + m\Delta\theta c_1 + \frac{m\Delta\theta}{\phi(\theta)} \frac{\delta\Delta\theta}{q} + n^I \left[\frac{(\bar{a}-a_3^I)}{q} + \frac{f'\left(\frac{\bar{a}-a_3^I}{q}\right)}{f''\left(\frac{\bar{a}-a_3^I}{q}\right)} \right]}$$

Therefore if $f'\left(\frac{\bar{a}-a_3^I}{q}\right) \geq f''\left(\frac{\bar{a}-a_3^I}{q}\right) \frac{(\bar{a}-a_3^I)}{q}$ then $\frac{\partial q}{\partial \theta} > 0$. Otherwise it depends: if investors's consumption of good a increases with θ , and such increase is sufficiently large, then $\frac{\partial q}{\partial \theta} < 0$ may also be the case²⁷.

²⁷This happens when the substitution effect is large enough for investors with respect to the income effect of a change in θ .