Changes in Monetary Policy and the Term Structure of Interest Rates: Theory and Evidence

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Abstract

This paper examines how shifts in monetary policy affect the term structure of interest rates. A dynamic asset pricing model is developed for the term structure in an economy where monetary policy is characterized by a responsive policy rule that reflects actual central bank behavior. The rule calls for adjustments in money supply depending on the gap between the current inflation rate and the target rate set by the central bank. It is shown that a multi-factor Cox-Ingersoll-Ross (CIR) model can be obtained as a closed form solution for the term structure of interest rates, with the coefficients in the CIR model being functions of the policy parameters. The model elucidates the underlying mechanism by which changes in monetary policy affects the term structure, and is able to explain the behavior of nominal interest rates across different monetary policy regimes in the United States. The model also provides a framework for formally testing the impact of monetary policy on the term structure. Evidence is found that the shift in the policy regime in late 1979 and the early 1980s indeed led to a structural break in the yield curve in the United States.

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1 Introduction

Studies on monetary policy have suggested that there is a significant difference in the way monetary policy was conducted in the pre- and post-1979 periods in the United States (see Clarida et al. (1998) and McCallum and Nelson (1999) among others). These studies also provide empirical evidence that the Federal Reserve was "accommodative" in the first period and has adopted a proactive stance toward controlling inflation since late 1979 and the early 1980s. While many economists have tried to examine the implications of different monetary policies for macroeconomic performance (see papers in Taylor (1999) eds.), the objective of the present paper is to analyze the impact of such a shift in the monetary policy regime on the term structure of interest rates.

Understanding the properties of the term structure in different monetary policy regimes is important. According to the standard view of the transmission mechanism of monetary policy, the central bank manoeuvres its policy instrument – mostly a very short term interest rate such as the federal funds rate – to affect the interest rates of longer term, and therefore the spending by interest rate sensitive sectors of the economy, and ultimately unemployment and inflation. Hence the central bank's ability to achieve its policy objectives depends to a large extent on the relationship between the short-term interest rate and long-term rates. Nevertheless, this relationship is not likely to remain constant when there is a structural change in monetary policy. Indeed, some empirical evidence has been provided that long-term interest rates seem to have become more sensitive to the movements in monetary policy instrument, such as the federal fund rate, in recent years (see Mehra (1996), Cohen and Wenninger (1994)).

To study the relationship between monetary policy and the term structure of interest rates, a tractable dynamic asset pricing model is developed for an economy where monetary policy is characterized by a responsive policy rule that reflects actual central bank behavior. The rule calls for adjustments in money supply depending on the gap between the current inflation rate and the target rate set by the central bank. Different policy regimes are identified with different degrees of responsiveness of the policy instrument to the inflation pressure. It is a version of policy rule that emerges in both positive and normative analyses of central bank behavior that have appeared in recent literature.[1] Closed form solution of the term structure is obtained.

[1] See Clarida, Gali and Gertler (1999) for a review of recent literature.

It is shown that nominal interest rates are given by affine functions of the exogenous state variables, with the coefficients of the functions depending on the policy parameters. The model elucidates the underlying mechanisms by which monetary policy affects the term structure and offers an explanation for changes in the behavior of nominal interest rates across different monetary policy regimes. Specifically, it implies that when monetary policy becomes more responsive to the inflation gap, long-term interest rates tend to be more sensitive to movements in the short-term rate. Moreover, by stabilizing inflation, a more proactive policy rule will also tend to raise the average level of the yield curve and reduce interest rate volatility. Consistent evidence is found in U.S. interest rate data.

The impact of a shift in the monetary policy rule on the term structure is also formally tested in a parametric model. The closed form solution gives rise to a multi-factor affine model of the term structure. Even though the policy parameter can't be directly estimated in the model, one can identify the qualitative impact on the reduced form coefficients of a change in value of the policy parameter. Therefore, econometric tests for structural changes and parameter instability can be applied to the affine model of the term structure. In the framework of Andrews (1993), the test is first conducted for the case of an exogenously fixed break-point, which is chosen to be October 1979 when monetary policy is supposedly shifted to a new regime. It is confirmed that there was indeed a structural break in the yield curve from the Wald, Lagrange Multiplier and Likelihood Ratio test statistics. Estimates of the key reduced form coefficients in the term structure model are largely consistent with a story of a permanent shift in policy stance toward controlling inflation since 1979. The test is also carried out for the case of an endogenously determined break-point. The data indicate that the structural break in the yield curve indeed occurred in coincidence with the shift in monetary policy in late 1979/early 1980s.

In some ways the results should not be surprising. While the main objective of monetary policy is the control of inflation, inflation in turn directly affects the term structure of nominal interest rates. A nominal risk-less bond can be viewed as a "derivative" asset whose payoff is contingent upon the future inflation rate. Therefore not just expected inflation affects nominal interest rates through the standard Fisher relation, inflation volatility will also play a very important role in the determination of nominal bond prices and hence nominal interest rates. When a monetary policy shift affects the level and volatility of inflation, it is not surprising that we also observe structural changes in nominal interest rates. Of course, monetary policy may also affect nominal interest rates through its impact on the real interest rate. But it is the "inflation channel" that is our focus in this paper. Moreover, while many previous discussions of the relationship between inflation and nominal interest rates have mainly revolved around the expected inflation via the Fisher relation, in the present paper we emphasize the role of inflation volatility.

The study of the relationship between monetary policy and the term structure of interest rates has a long tradition and has produced a large literature. Many of the earlier studies have been devoted to the identification and estimation of the effects of exogenous impulses to monetary policy on long term interest rates. See Akhtar (1995) for a survey on empirical works in this literature and Evans and Marshall (1997) for a recent structural approach. Another strand of research has focused on the the predictive power of the term structure for real activity and inflation and tries to explore the implications for monetary policy (see Fama (1990), Estrella and Mishkin (1995) among others). Meanwhile, in some closely related studies, several researchers have also noticed the changing behavior of the term structure and the connection with monetary policy. Mankiw and Miron (1986) examined the expectation theory of the term structure using data at the short end of the maturity spectrum over different monetary policy regimes. Hamilton (1988) estimated an econometric regime-switching model for the term structure and found substantive evidence on structural change in the interest rates during the monetary experiment of October 1979. More recently, Hsu and Kugler (1997) argued that the varying predictive power of the term spread for the future short-term rate can be attributed to the changes in the policy reaction function adopted by the U.S. monetary authority since the 1980s. Watson (1999) examined the different variability in long term interest rates during 1965-1978 and 1985-1998. Fleming and Remolona (1999) analyzed high-frequency responses of U.S. Treasury yields across the maturity spectrum to macroeconomic announcements. They found the maturity pattern of announcement effects has changed substantially since the late 1970s/early 1980s.

However, most of the former studies on structural change in the interest rate term structure have been done in the framework of the expectation hypothesis without imposing no-arbitrage conditions on the term structure (Fleming and Remolona (1999) is an exception). Moreover none of these studies explicitly models the responsive behavior of the monetary policy and hence suffers from the lack of a formal theory of how policy shifts lead to structural break in the interest rates. In the present paper, we try to fill up the gap with a general equilibrium model of the term structure of interest rates and monetary policy. Our approach draws from two strands of literatures. One is a recent research effort to incorporate a responsive monetary policy rule in a dynamic general equilibrium model of monetary economy (see Rotemberg and Woodford (1999), McCallum and Nelson (1999) among others). The focus of these studies is usually on the evaluation of alternative monetary policy rules for macroeconomic stability. The impact of different monetary policies on financial market is to a large extent ignored. The other strand of literature from which the present paper also draws heavily is the continuous time model of asset price in a general equilibrium setting. The prime general equilibrium model of the term structure of real interest rates is the Cox-Ingersoll-Ross (CIR) model (1985). Other general equilibrium models of the term structure include Vasicek (1977), Longstaff and Schwartz (1992) among others. Sun (1992) considers a discreet time CIR model in a monetary economy by assuming an exogenous inflation process that is correlated with consumption growth. It also discusses the connection between the discreet time model and the continuous time model. A more recent research effort is that of Bakshi and Chen (1996), in which a closed form solution of the term structure of interest rates is obtained for an exogenous money supply process and inflation is determined endogenously together with nominal interest rates. However, treating money as an exogenous variable, the model doesn't allow us to consider the influences of alternative monetary policy regimes on the term structure. The present paper extends the approach of Bakshi and Chen (1996) by considering an endogenous money supply process under a responsive monetary policy rule.

The rest of the paper is organized as follows. Section 2 presents some stylized facts of the term structure of interest rates in the U.S. using a data set of the yields on government pure discount bonds. Section 3 develops the general equilibrium model of the term structure. With the analytical solution, the effects of a shift in the monetary policy rule on the nominal interest rates are examined. I show that the empirical regularities in the behavior of nominal interest rates in the U.S. can be explained by a change of monetary policy as that in late 1979 and the early 1980s. Section 4 formally tests the impact of a shift in monetary policy in a parametric model by exploring the econometric restrictions imposed on the analytical solution of the term structure of interest rates. Section 5 concludes with a discussion of the caveats and extensions of our approach.

2 Descriptive Statistics of the Term Structure

The term structure data used in the present paper are monthly data of the yields on U.S. government pure discount bonds during the period of 1960 - 1995, constructed from the market prices of government coupon bonds.[2] The yields from 1960:01 - 1991:02 are in fact taken from Kwon and McCulloch (1993), and the yields from 1991:03 - 1995:12 are computed by Robert Bliss (1997) using the same McCulloch/Kwon procedure. The data include yields on eighteen nominal bonds of different maturities ranging from 1 month to 10 years. Bond yields are measured as continuously compounded annualized returns on these risk-free zero-coupon bonds.

To see the changing behavior of the nominal interest rates, we break the sample into two periods: 1960:01 - 1979:09 and 1979:10 - 1995:12.[3] We use the 1-month rate to approximate the instantaneous short-term interest rate. In Table 8, we report the results of regressing the long-term rates on the short rate. We can see from the table that the regression coefficient on the the short term rate is much higher in the post-October 1979 period. The difference is not only statistically significant, but also quantitatively important. This result seems to confirm the similar empirical findings from other studies that long-term rates have become more responsive to movements in the short-term rate since late 1979/early 1980s. Such a change in the relationship between the short-term rate and long-term rates can be more clearly seen in Figure 1 where the regression coefficients are plotted against time to maturity for different periods. While in general the responsiveness of long rates to changes in the short-term rate decline as maturity increases, the "response curve" shifts upward significantly after 1979. Moreover, this empirical regularity seems to be invariant with respect to choices of the break point. It can be seen from Figure 1 that in all three sub-sample periods following October 1979, the responsiveness of long-term rates to movements in the short rate are significantly higher compared with the pre-October 1979 period.

In fact, the structural change in the yield curve is not only reflected in

[2]The data set is made available to us by Charles Evans at the Federal Reserve Bank of Chicago. It is the same data set used in Evans and Marshall (1997).

[3]One obvious reason to choose October 1979 as the break point is that Paul Volcker took over as Fed chairman in October 1979 and started an aggressive effort to reign in inflation in the U.S. More discussion of the choice of break point will be offered in section 4.

the correlations between the short-term rate and long-term rates. Table 9 summarizes the average level of the yield curve and volatilities of the interest rates in the two periods. We can see that in the post-1979 period, the yield curve has a higher level on average than in the earlier period. This result holds regardless of whether one looks at the whole post-October 1979 period or the 1983:01-1995:12 period or the more recent 1985:01-1995:12 period (see Figure 2 for plots of the average yield curve in these different periods). As to the volatility of the interest rates, the period between 1979:10 and 1995:12 seems to be characterized by much more volatile nominal interest rates than the earlier period (1960:01-1979:09). However, once we remove the period of 1979:10-1982:12 during which the Fed was in a transition between policy rules and experimenting with different policy instruments and the operating procedures, the standard deviations of the interest rates become much lower. And if we look at the more recent period (1985-1995), the interest rates are in fact less volatile than those in the pre-October 1979 period. Figure 3 includes the plots of standard deviations of the interest rates in these different periods.

To summarize, the term structure of interest rates in the post-1979 period seems to be characterized by higher correlations between the short-term rate and long-term rates than in the previous period. Moreover, the average level of the yield curve is also higher between 1979 and 1995, and volatilities in the interest rates are slightly lower once the transition to the new regime (lasted from 1979 to roughly 1984) is corrected. A model of the term structure should be able to account for these structural changes.

3 A Dynamic Monetary Economy with Policy Rules

Two of the important components that determine nominal interest rates are the real interest rate and inflation. In the present paper, we focus on the impact of the second factor on the term structure of interest rates. We consider a simple representative agent economy with exogenous endowment and flexible prices. Money is made completely neutral in the economy so that monetary policy affects nominal interest rates only through its effect on inflation. The caveats of these simplifying assumptions will be discussed later. Nevertheless, these simplifications lead us to a closed form solution of the term structure of nominal interest rates, with which the impact of a shift in monetary policy can easily be analyzed. Empirical evidence will be presented in the next section.

3.1 The Consumer's Problem

It is assumed that a representative consumer maximizes the following utility function subject to his intertemporal budget constraint:

$$Max\ E_0 \sum_{t=0,\triangle t,2\triangle t,\cdots}^{\infty} e^{-\rho t} u\left(c(t)\right) \triangle t \tag{1}$$

where $u(c) = log(c)$.[4] $\triangle t$ is the length of the time interval during which the consumer makes decisions about consumption, money and asset portfolio holdings. In the following discussion, we will consider the limiting case where $\triangle t \to 0$. $c(t)$ is the consumption flow between $[t, t+\triangle t]$. Money will also be demanded by the agent because consumption transactions are costly and increasing real balance holdings per unit of consumption decrease these transaction costs, which is represented by $f(\frac{m(t)}{c(t)})$, a function of the ratio between the real money balance $m(t)$ and consumption $c(t)$. It is assumed that $f(\cdot)$ is a continuously differentiable decreasing function which reaches its minimum level at some constant k. This implies that there is a satiation level of real cash balance per unit of consumption.[5] The consumer has an exogenous flow of endowment $y(t)$ during $[t, t+\triangle t]$. In equilibrium we have $c(t) = y(t) - f(\frac{m(t)}{c(t)})$. The budget constraint for the consumer can be written as:

$$\begin{aligned}
&P(t)c(t)\triangle t + P(t)f\left(\frac{m(t)}{c(t)}\right)\triangle t + M(t) + e^{-R(t)\triangle t}B(t) + e^{-r(t)\triangle t}P(t)b(t) + \\
&\sum_{i=1}^{N} Q_i(t)S_i(t) \leq P(t)y(t)\triangle t + e^{R^m(t-\triangle t)\triangle t}M(t-\triangle t) + B(t-\triangle t) + P(t)b(t-\triangle t) + \\
&\sum_{i=1}^{N} Q_i(t)S_i(t-\triangle t) + \triangle G(t)
\end{aligned} \tag{2}$$

In the budget constraint, $P(t)$ is the price level at time t, $R(t)$ is the one-period nominal interest rate, $r(t)$ is the one-period real rate. $M(t)$ is nominal cash balances the consumer chooses to hold at time t and carries

[4]The results are not affected if the logarithm utility function is replaced with a more general CRRA utility function.

[5]Wolman (1997) estimated a "transactions technology" based money demand function and found evidence of the presence of a satiation level of cash balances per unit of consumption.

over to time $t + \triangle t$ and $m(t)$ is the real cash balances $\frac{M(t)}{P(t)}$. $B(t)$ is the number of the one-period nominal bond the consumer chooses at time t and holds to time $t + \triangle t$. Similarly, $b(t)$ is the one-period real bond. Other assets in the economy are N nominal long-term, zero-coupon, risk-less bonds represented by $S_i(t)$, for $i = 1, \cdots, N$. The prices of these long-term bonds are $Q_i(t)$. It is assumed that there are a net zero supply of all these assets. $\triangle G(t)$ are government transfers during $[t, t + \triangle t]$.

One of simplifying assumptions made in this paper is that the monetary authority pays interest on the money balance. In particular, when the consumer chooses to hold $M(t - \triangle t)$ at time $t - \triangle t$ and carries it over to time t, he will get extra cash from the monetary authority at a rate $R^m(t - \triangle t)$, which is marginally below $R(t - \triangle)$, the rate on the one-period nominal bond. Hence we abstract from the negative dependence of money demand on positive nominal interest rates. This assumption, borrowed from King and Wolman (1999), results in a simple money demand function with constant velocity when $R^M(t)$ approaches $R(t)$ from below, which in turn leads to closed form solutions for the term structure of nominal interest rates as well as inflation. Empirically, transaction balances have become increasingly interest-rate bearing, which provides some justifications for this simplifying assumption. Moreover, as argued by King and Wolman (1999), the constant velocity money demand function can be thought of as the limiting case that applies when money is interest-bearing, when there is a satiation level of cash balances per unit of consumption, and the interest rate on money is close to the market rate. As we will see below, under this assumption money is completely neutral in the economy, we can therefore focus on the effects of different monetary policies on inflation and explore the implications for the term structure of nominal interest rates.

Finally, besides paying interest $R^M(t - \triangle t)$ on $M(t - \triangle t)$ at time t, new money balances are transfered to the consumer in a lump sum fashion. Hence $G(t)$ satisfies:

$$\triangle G(t) = M(t) - e^{R^M(t-\triangle t)\triangle t} M(t - \triangle t) \tag{3}$$

Given (1) and (2), we can write the first order conditions for the consumer's problem as follows:

$$e^{-\rho t} u'(c_t) = \lambda_t \left[1 - \frac{m_t}{c_t^2} f'\left(\frac{m_t}{c_t}\right) \right] \cdot P_t \tag{4}$$

$$e^{-R_t^m \triangle t} \left(1 + c_t^{-1} f'\left(\frac{m_t}{c_t}\right) \triangle t \right) = E_t \left(\frac{\lambda_{t+\triangle t}}{\lambda_t} \right) \tag{5}$$

$$e^{-R_t \triangle t} = E_t \left(\frac{\lambda_{t+\triangle t}}{\lambda_t} \right) \tag{6}$$

$$Q_{i,t} = E_t \left(\frac{\lambda_{t+\triangle t}}{\lambda_t} Q_{i,t+\triangle t} \right) \tag{7}$$

$$e^{-r_t \triangle t} = E_t \left(\frac{P_{t+\triangle t} \lambda_{t+\triangle t}}{P_t \lambda_t} \right) \tag{8}$$

From (5) and (6), we get an equation relating real cash balance to consumption and nominal interest rates:

$$e^{-R_t^m \triangle t} c_t^{-1} f^{'}(m_t/c_t) \triangle t = e^{-R_t \triangle t} - e^{-R_t^m \triangle t}$$

Note that it is assumed that $f^{'}(\cdot) \leq 0$ and $R_t^m \leq R_t$. The above equation can be simplified by letting $R_t^m \to R_t$ from below, i.e. we let the real cost of holding money goes to zero, then it follows that $f^{'}(\cdot)$ must be zero, which implies that:

$$M_t = k P_t c_t \tag{9}$$

where k is the constant such that $f^{'}(k) = 0$, which represents the satiation level of real cash balances per unit of consumption. We can think of the above money demand function with constant velocity as an approximation when the interest rate on the money balance is close to the market rate. For tractability, we will hence only consider the case where $R^m = R_-$ in the following discussions. Also note that when $R^m = R_-$, the transaction cost $f\left(\frac{m}{c}\right)$ is a constant because $\frac{m}{c}$ is constant. Without loss of generality, we can also simply assume $c(t) = y(t)$ in equilibrium.

3.2 Monetary Policy

Most approaches to asset pricing in a monetary economy consider an exogenous money supply process (e.g. Lucas (1982), Bakshi and Chen (1996)), and therefore leave little room for the discussion of monetary policy. On the other hand, in the literature of monetary economics, great effort has been devoted to the study of monetary policy rules under which the central bank adjusts its policy instrument in response to developments in the economy (see McCallum and Nelson (1999), Leeper (1994) among others). In this literature, policy makers seek to implement a particular equilibrium relationship between the policy instrument and some other endogenous variables by adopting an appropriate money supply process. Hence money supply is endogenous in the sense that the growth rate of money must respond to current and past exogenous monetary policy shocks, as well as other private economy shocks, in a way that is consistent with the policy rule.[6] In contrast, the growth rate of money is usually assumed to follow some exogenous process driven exclusively by monetary policy shocks in most monetary asset pricing models.

In the present paper, I will assume that the central bank pursues a responsive monetary policy using money stock $M(t)$ as its instrument. Researchers have often used the short-term interest rate (Taylor (1993)) or money stock (McCallum (1988)) as the policy instrument in the literature. In either case, the policy rule calls for monetary tightening when in the presence of inflation pressure.[7] In the same spirit, I postulate the following policy rule for the economy under consideration:

$$\frac{\triangle M^s(t)}{M^s(t)} = \frac{\triangle P(t)}{P(t)} - \alpha\left(\frac{\triangle P(t)}{P(t)} - \pi^* \triangle t\right) + \frac{\triangle y(t)}{y(t)} + \frac{\triangle \xi(t)}{\xi(t)} \tag{10}$$

In the above equation, $\triangle X(t) \equiv X(t + \triangle t) - X(t)$, and $\alpha > \alpha^* > 0$ (α^* is some positive constant to be specified in the following), π^* is a target inflation rate set by the central bank. The central bank will reduce the money growth rate if inflation exceeds this target level. Under this specification, the growth rate of money in the economy consists of two components. One is a systematic component which can be represented by a function of endogenous variables (the reaction function). Recent vector autoregression (VAR)-based literature on monetary policy have provided empirical evidence that most of the observed movements in the policy instrument can be explained by macroeconomic conditions (e.g. Bernanke et al. (1997), Christiano et al. (1998b) among others). The other component is an exogenous policy shock $\frac{\triangle\xi(t)}{\xi(t)}$ whose property will be specified below. Possible sources of the policy shock include measurement errors in inflation and some discretionary actions by the central bank.[8]

[6] See Christiano et al. (1998a) for a discussion of endogenous variable policy rules in general equilibrium context.

[7] See McCallum (1997) for a discussion of the issues in the design of monetary policy rules, including the choice of policy instrument.

[8] We can also think that $\xi(t)$ nests an exogenous money demand shock. For example, if money demand is given by $M^d(t) = \psi(t)P(t)y(t)$ instead of (9), where $\psi(t)$ is an exogenous money demand shock, then for a first order approximation, $\frac{\triangle M^d}{M^d} = \frac{\triangle P}{P} + \frac{\triangle y}{y} + \frac{\triangle \psi}{\psi}$. Since

Note that in this endowment economy, since money can not affect output, we assume that the central bank fully accommodates any fluctuations in output by fixing the coefficient on the growth rate of output at 1 in the policy rule. Hence in the light of the money demand function (9), the central bank tries to keep inflation constant when output fluctuates (recall that $c(t) = y(t)$ in equilibrium).

3.3 Exogenous State Variables

We assume that monetary policy shocks and productivity shocks are the only sources of uncertainty in the economy. The exogenous endowment $y(t)$ and policy shocks $\xi(t)$ are assumed to be driven by two independent state variables, $X(t)$ and $Z(t)$ respectively, in the following way when $\triangle t \to 0$:

$$\frac{dy(t)}{y(t)} = (hX(t) - \rho)dt + \epsilon\sqrt{X(t)}dW_1(t) \tag{11}$$

$$\frac{d\xi(t)}{\xi(t)} = (Z(t) - \bar{Z})dt + \omega\sqrt{Z(t)}dW_2(t) \tag{12}$$

where $X(t)$ and $Z(t)$ are characterized by the following stochastic differential equations respectively:

$$dX(t) = k_X(\bar{X} - X(t))dt + \sigma_X\sqrt{X(t)}dW_1(t) \tag{13}$$

$$dZ(t) = k_Z(\bar{Z} - Z(t))dt + \sigma_Z\sqrt{Z(t)}dW_2(t) \tag{14}$$

In the above equations, $W_1(t)$ and $W_2(t)$ are two independent standard Brownian motions, and all the coefficients in the stochastic differential equations are assumed to satisfy regularity conditions so that a unique solution exists for each of the stochastic differential equations. In particular, $k_i > 0$, $\sigma_i > 0$, $k_X\bar{X} > \frac{1}{2}\sigma_X^2$ and $k_Z\bar{Z} > \frac{1}{2}\sigma_Z^2$. These assumptions ensure that $X(t)$ and $Z(t)$ are both strictly positive, stationary, mean-reverting processes with $\bar{X}$ and $\bar{Z}$ being their respective steady state means.

3.4 Equilibrium

An equilibrium is a collection of $c(t)$, $M(t)$, $B(t)$, $b(t)$, $S_i(t)$, $G(t)$ and $P(t)$, $R(t)$, $r(t)$, $Q_i(t)$ with the following properties:

in equilibrium $M^d = M^s$, we have $\frac{\triangle M^s}{M^s} - \frac{\triangle\psi}{\psi} = \frac{\triangle P}{P} + \frac{\triangle y}{y}$. Redefine money demand as in (9), and define money supply as in (10) with the error term $\frac{\triangle\xi}{\xi}$ being the sum of an exogenous policy shock and $-\frac{\triangle\psi}{\psi}$.

(1). $\{c(t), M(t), B(t), b(t), S_i(t)\}$ solves the consumer's problem given $\{P(t), R(t), r(t), Q_i(t), G(t)\}$;

(2). Markets clearing: $c(t) = y(t) - f\left(\frac{m(t)}{c(t)}\right)$,[9] $M(t) = M^s(t)$, $B(t) = 0$, $b(t) = 0$ and $S_i(t) = 0$;

(3). Government budget constraint (3) is satified.

(4). Monetary authority implements the policy rule (10), i.e. as $\triangle t \to 0$,

$$\frac{dM(t)}{M(t)} = \frac{dP(t)}{P(t)} - \alpha\left(\frac{dP(t)}{P(t)} - \pi^* dt\right) + \frac{dy(t)}{y(t)} + \frac{d\xi(t)}{\xi(t)} \tag{15}$$

3.5 Inflation

Consider the limiting case where $R^m(t) \to R(t)$ and $\triangle t \to 0$. Then we have the following result for inflation using (9) and (15) (see Appendix A for details):

Proposition 1 *At the equilibrium of the economy, inflation is given by:*

$$\begin{aligned} \frac{dP(t)}{P(t)} &= \mu_P(t)dt + \sigma_P(t)dW_2(t) \qquad (16) \\ \mu_P(t) &= \pi^* + \frac{1}{\alpha}(Z(t) - \bar{Z}) \\ \sigma_P(t) &= \frac{\omega}{\alpha}\sqrt{Z(t)} \end{aligned}$$

To see the implications of the responsive monetary policy rule (15), let's note that $\mu_P(t)$ has the interpretation of expected inflation at time t, and $\sigma_P^2(t)$ measures inflation volatility at time t. We can see from the above equations that even if monetary authority holds the inflation target constant, differences in policy responsiveness to inflation have important implications for inflation volatility as well as expected inflation. In particular, the more responsive the monetary policy rule is with respective to inflation, as represented by a higher value of α, the lower the inflation volatility, and the smaller the gap between the expected inflation and the target level π^*. In other words, a responsive monetary policy helps stabilize inflation around the target level in this economy. The more aggressive the monetary authority acts against inflation, the more stable the inflation.

Moreover, the covariance between $\frac{dP(t)}{P(t)}$ and $\frac{d\xi(t)}{\xi(t)}$ is equal to $\frac{\omega}{\alpha}Z(t)$, which is greater than zero as long as α and ω are greater than 0. Hence inflation

[9] In deriving the following results, I have assumed $f(k) = 0$.

is positively correlated with exogenous monetary policy expansions. Also note that in the steady state distribution, the mean of $\mu_P(t)$ is π^* since the mean of $Z(t)$ equals $\bar{Z}$. We can therefore also interpret π^* as the "long-run" inflation level.

3.6 The Term Structure of Nominal Interest Rates

Now let's turn to determination of nominal interest rates in the economy. We will first solve for the short-term interest rate, and then derive the term structure of the nominal interest rates.

3.6.1 The Short-Term Nominal Interest Rate

Note that when $f'(\cdot) = 0$, the first order condition (4) implies that $\lambda_t = e^{-\rho t}u'(c_t)/P_t$. It then follows from (6) that $e^{-R_t \triangle t} = e^{-\rho \triangle t} E_t \left(\frac{u'(c_{t+\triangle t})P_t}{u'(c_t)P_{t+\triangle t}} \right)$. Using the Taylor expansion and the above result for inflation (recall again $c(t) = y(t)$ in equilibrium), we have (see Appendix B for details):

Proposition 2 *As $\triangle t \to 0$, the instantaneous short term nominal interest rate is:*[10]

$$R(t) = \left(\pi^* - \frac{\omega^2 \bar{Z}}{\alpha^2} \right) + r(t) + (1 - \frac{\omega^2}{\alpha})(\mu_P(t) - \pi^*) \qquad (17)$$

where $r(t)$ is the real short term interest rate, $\mu_P(t)$ is the expected inflation, and are given respectively by:

$$r(t) = (h - \epsilon^2)X(t) \qquad (18)$$

$$\mu_P(t) - \pi^* = \frac{1}{\alpha}(Z(t) - \bar{Z}) \qquad (19)$$

First note that the standard Fisher relation, which states that the nominal interest rate equals the sum of the real interest rate and expected inflation, is a special case of the above result. If inflation volatility is zero, which is true when $\omega = 0$ (note that the conditional variance of inflation is given by $\frac{\omega^2 Z(t)}{\alpha^2}$), then equation (17) is reduced to $R(t) = r(t) + \mu_P(t)$, which is exactly the Fisher relation. The above result is more general because it takes into account the impact of inflation risk on nominal bond prices.

[10] In order to ensure that $R(t)$ is always positive, we assume that $\alpha > max(\omega^2, \frac{\bar{Z}}{\pi^*})$.

It shows that not just the expected inflation, but also inflation volatility, affects the nominal interest rate in a very important way. In particular, a higher inflation volatility (due to either higher ω or lower α) will actually lead to a lower $R(t)$ holding other things constant, and vice versa. The economic intuitions will be discussed shortly (we will see that this is true for long-term nominal interest rates as well).

The equations also clarify the impact of monetary policy on the short-term nominal interest rate. First, a temporary exogenous monetary policy shock (say an increase in $Z(t)$) affects $R(t)$ only through its effect on expected inflation $\mu_P(t)$. A higher than usual $Z(t)$ drives up $\mu_P(t)$, and hence $R(t)$. In this frictionless economy, monetary expansion immediately translates into higher inflation and has no effect on the real interest rate.

Secondly, if there is a permanent shift of the monetary policy rule, say an increase in the value of α, it will reduce the level of expected inflation $\mu_P(t)$ and hence decrease $R(t)$ through the Fisher relation. However, on the other hand, an increase in α also reduces inflation volatility and hence tends to increase the level of the nominal interest rate through the negative relationship between $R(t)$ and the volatility of inflation. Therefore, in the short run, the immediate effect of such a policy change is ambiguous, and the direction of interest rate movement depends on the relative magnitudes of these two effects. But in the long run, the average level of expected inflation $\mu_P(t)$ converges to the pre-specified target rate π^* (the unconditional mean of $\mu_P(t)$), the average level of $R(t)$ would therefore rise as inflation volatility is reduced by a higher value of α while holding π^* constant.

Moreover, as the policy rule shifts, for example α increases, volatility of the interest rate also changes. In fact, since $X(t)$ and $Z(t)$ are independent, substituting (18) and (19) into equation (17), we have that:

$$Var(R(t)) = (h - \epsilon^2)^2 Var(X(t)) + \left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)^2 Var(Z(t))$$

If we assume that the variance of $X(t)$ and $Z(t)$ remain constant over time, then $Var(R(t))$ will either increase or decrease depending on the values of α. When α is very small, a marginal increase in α will lead to an increase in the volatility of $R(t)$. But if α is large enough (specifically if $\alpha > 2\omega$), an increase in the value of α will reduce the volatility of the interest rate $R(t)$. This result is very intuitive. While a more responsive monetary policy helps stabilize expected inflation around the target level (see equation (15)) and hence reduces the interest rate volatility through this channel, a larger value of α also requires more aggressive movements in the policy instrument when inflation changes and hence tends to increase the interest rate variability. If the policy is not effective enough and inflation is still very volatile, then the "destabilizing" effect on $R(t)$ of a marginal increase in α would dominate the "stabilizing" effect, and we have higher interest rate volatility in equilibrium. But under a very proactive policy, inflation is effectively stabilized around π^* and hence there is little pressure for large movements in the policy instrument even if α is very large. In equilibrium we have lower interest rate volatility as a higher value of α further stabilizes inflation.

This result on interest rate volatility also sheds some light on the issue of interest rate "smoothing" that arises in many discussions of optimal monetary policy. It is sometimes said that central banks should add a lagged interest rate term in the policy reaction function due to the concern that a responsive (with respect to inflation, employment) policy could result in a very volatile short term interest rate. However our model indicates that a proactive policy does not necessarily increase interest rate volatility in equilibrium because of its stabilizing effect on inflation.

3.6.2 Nominal Long-Term Interest Rates

To obtain the solution for the term structure, upon substituting (18) and (19) into (17), we have:

$$
\begin{aligned}
R(t) &= \theta_0 + \theta_1 X(t) + \theta_2 Z(t) \\
\theta_0 &= \pi^* - \frac{\bar{Z}}{\alpha} \\
\theta_1 &= h - \epsilon^2 \\
\theta_2 &= \frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}
\end{aligned}
\tag{20}
$$

Given $X(t)$ and $Z(t)$ that are defined in (13) and (14) respectively, the above result suggests a two-factor affine model of the term structure of interest rates.[11] Hence the term structure of nominal interest rates has the following closed form solution:

[11] It can be show that the market prices of risk for the two factors are proportional to $\sqrt{X_t}$ and $\sqrt{Z_t}$ respectively from the first-order conditions of the consumer's problem. See Appendix C for details

Proposition 3 *At time t, the yield $R(t,\tau)$ on a nominal zero-coupon bond maturing at $t+\tau$ (for $\tau > 0$) is given by:*

$$R(t,\tau) = \theta_0 - \frac{\log H_{1,X}(\tau)}{\tau} - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,X}(\tau)}{\tau}\theta_1 X(t) + \frac{H_{2,Z}(\tau)}{\tau}\theta_2 Z(t) \quad (21)$$

Where $H_{1,i}(\tau)$ and $H_{2,i}(\tau)$ for i = X, Z are given respectively by:

$$\begin{aligned}
H_{1,X}(\tau) &= \left[\frac{2B_X e^{(B_X+A_X)\tau/2}}{(B_X+A_X)(e^{B_X\tau}-1)+2B_X}\right]^{2k_X\bar{X}/\sigma_X^2} \\
H_{2,X}(\tau) &= \frac{2(e^{B_X\tau}-1)}{(B_X+A_X)(e^{B_X\tau}-1)+2B_X} \\
A_X &= k_X + \epsilon\sigma_X \\
B_X &= (A_X^2 + 2(h-\epsilon^2)\sigma_X^2)^{1/2}
\end{aligned}$$

and

$$\begin{aligned}
H_{1,Z}(\tau) &= \left[\frac{2B_Z e^{(B_Z+A_Z)\tau/2}}{(B_Z+A_Z)(e^{B_Z\tau}-1)+2B_Z}\right]^{2k_Z\bar{Z}/\sigma_Z^2} \\
H_{2,Z}(\tau) &= \frac{2(e^{B_Z\tau}-1)}{(B_Z+A_Z)(e^{B_Z\tau}-1)+2B_Z} \\
A_Z &= k_Z + \frac{\omega\sigma_Z}{\alpha} \\
B_Z &= \left[\left(k_Z + \frac{\omega\sigma_Z}{\alpha}\right)^2 + 2\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)\sigma_Z^2\right]^{1/2}
\end{aligned}$$

With the above results, it is now very easy to examine changes in the term structure as the monetary policy shifts. The key policy parameter here is α. Since $H_{i,X}$ (i=1, 2) does not depend on the policy parameter (neither α nor π^*), a shift in monetary policy affects $R(t,\tau)$ through θ_0 and $H_{i,Z}$ (i=1, 2). Hence for the purpose of exposition, let's for the moment ignore the terms associated with $X(t)$ and simply write $R(t,\tau)$ as:

$$R(t,\tau) = \theta_0 - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,Z}(\tau)}{\tau}\theta_2 Z(t) \quad (22)$$

Or

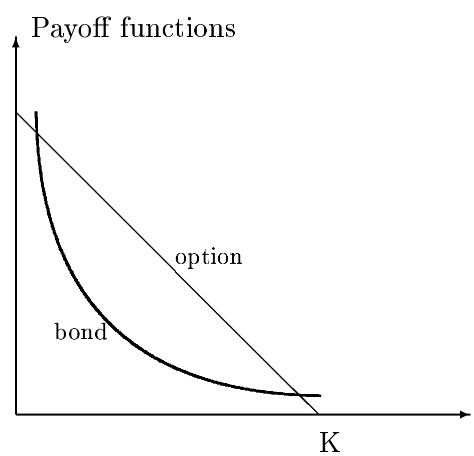
$$\begin{aligned}
R(t,\tau) = {} & \left(\pi^* - \frac{H_{2,Z}(\tau)}{\tau}\frac{\omega^2\bar{Z}}{\alpha^2}\right) - \left(1 - \frac{H_{2,Z}(\tau)}{\tau}\right)\frac{\bar{Z}}{\alpha} - \frac{\log H_{1,Z}(\tau)}{\tau} + \\
& \frac{H_{2,Z}(\tau)}{\tau}\left(1 - \frac{\omega^2}{\alpha}\right)(\mu_P(t) - \pi^*) \quad (23)
\end{aligned}$$

We can then easily see from the above equation that there is a similar negative effect of inflation volatility on the level of long-term nominal interest rates $R(t,\tau)$ as that on the short rate $R(t)$ (see equation (17)). Hence an increase in α tends to raise the entire yield curve because of reduced inflation volatility under a more responsive monetary policy, holding the expected inflation constant.

To understand this seemingly counter-intuitive result – namely that a higher inflation volatility leads to higher prices for nominal bonds and hence lower nominal interest rates and vice versa – let's recall that the time t price of a nominal risk-less bond maturing at $t+\tau$ is given by:

$$\Lambda_{t,\tau} = E_t\left(\Phi_{t,t+\tau} \cdot f(\Pi_{t+\tau})\right)$$

where Φ is some pricing kernel, the payoff function $f(\cdot)$ is given by $f(x) = \frac{1}{x}$, $\Pi_{t+\tau} = \frac{P_{t+\tau}}{P_t}$ and P is the general price level. Hence a nominal bond can be viewed as a "derivative asset" whose payoff is contingent upon the future inflation rate. In fact it mostly resembles a European put option maturing at $t+\tau$ with some strike price K if we compare their respective payoff functions:



An analogy can therefore be drawn between a put option and a nominal risk-less bond. But it is well known that higher volatility of the underlying stock price increases the value of the option, because the owner of the put option benefits from price decreases but has limited downside risk in the event of stock price increases. It is then not surprising that a higher inflation volatility increases the prices of nominal bonds and hence leads to lower nominal interest rates and vice versa, holding other things constant. This analogy shows that inflation volatility plays a critical role in the determination of nominal interest rates, in contrast to many previous studies on the relationship between inflation and nominal interest rates which have mainly focused on the role of expected inflation.

Moreover, note that in (22) as α increases, θ_0 increases. Since θ_0 affects $R(t)$ and $R(t, \tau)$ for any $\tau > 0$ the same way, therefore when the central bank raises the short-term interest rate $R(t)$ by permanently moving toward a more proactive policy (increasing α), long-term rates could rise with almost the same magnitude as the short rate if the effect of α on $H_{1,Z}$ and $H_{2,Z}$ is small. And depending on how α affects $H_{1,Z}$ and $H_{2,Z}$, it is possible for long-term rate $R(t, \tau)$ to increase even more than the short rate $R(t)$ does.

Changes in α not only shift the level of the yield curve, they also affect how long rates respond to movements in the short rate. From equation (20) and (21) we can easily see that if a monetary shock $Z(t)$ moves the short rate $R(t)$ by 1%, the impact on a long rate $R(t, \tau)$ will be given by the factor loading $\frac{H_{2,Z}(\tau)}{\tau}$. Or in other words, as the central bank takes actions to change the short-term rate, the factor loading $\frac{H_{2,Z}(\tau)}{\tau}$ determines the magnitude of the response of the long-term rate $R(t, \tau)$ to movements in the short rate $R(t)$. How changes in α affect the sensitivities of long rates depends on how α affects $\frac{H_{2,Z}(\tau)}{\tau}$. Rewrite the definition of $H_{2,Z}$ here for convenience:

$$H_{2,Z}(\tau) = \frac{2(e^{B_Z \tau} - 1)}{(B_Z + A_Z)(e^{B_Z \tau} - 1) + 2B_Z}$$

Where:

$$\begin{aligned} A_Z &= k_Z + \frac{\omega \sigma_Z}{\alpha} \\ B_Z &= \left[\left(k_Z + \frac{\omega \sigma_Z}{\alpha} \right)^2 + 2 \left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2} \right) \sigma_Z^2 \right]^{1/2} \end{aligned}$$

Even though $H_{2,Z}$ is a complicated nonlinear function of α, large increases in α will usually lead to increases in $H_{2,Z}$, because both A_Z and B_Z are decreasing functions of α for $\alpha > 2\omega^2$, and for large τ (time to maturity) $H_{2,Z}$ can be approximated by $\frac{2}{B_Z + A_Z}$. This suggests that long-term rates tend to be more sensitive to movements in the short-term rate under a more proactive monetary policy rule.

The intuition of this result can be obtained from the derivation of the term structure. In particular, from (31), (35) and (38) in Appendix C, we can see that the short-term interest rate becomes more persistent under a risk neutral probability measure when a larger α reduces the volatility of inflation. But in the risk neutral world, a long term rate can be loosely thought of as the average of the expected future short term rate. Higher persistence in the short rate process therefore leads to larger response of long rates to the movements of the current short rate.

Furthermore, equation (22) implies that the standard deviation of $R(t, \tau)$ is given by:

$$std(R(t,\tau)) = \frac{H_{2,Z}(\tau)}{\tau}\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right) \times std(Z(t)) \tag{24}$$

Since $H_{2,Z}$ tends to increase as we shift toward a more responsive policy rule, a change in α has an ambiguous effect on the volatility of the interest rates. On the one hand, a higher α stabilizes inflation and hence tends to reduce interest rate volatilities. On the other hand, such an increase in α leads to higher factor loadings and hence higher standard deviations of the interest rates. However, following large changes of α, the interest rate volatilities are more likely to decrease a little bit as $std(R(t, \tau))$ will be dominated by $\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)$ for large τ.

Also note that a change of π^*, the target rate of inflation, only affects the term structure through its impact on θ_0. Hence a change in π^* has uniform effect on the interest rates across the maturity spectrum. A higher target rate of inflation leads to higher yield curve as the expected inflation increases but leaves volatilities and correlations among the interest rates unchanged.

To summarize, our model implies that a permanent shift in the monetary policy rule (either a change in π^* or α) is likely to cause structural changes in the term structure of nominal interest rates. In particular, when monetary authority becomes more aggressive to stabilize inflation around a pre-specified target level, not only will the average levels and volatilities of nominal interest rates change, long-term rates will also tend to be more responsive to movements in the short-term interest rate.

In what follows, we will first use a numerical example to show that a shift of policy stance toward controlling inflation like the one in late 1979/early 1980s could in principle generate the empirical regularities in the term structure discussed in section 2. We then go further to formally test the presence of a structural break in the yield curve and show that monetary policy is indeed an important underlying source of the parameter instability in the term structure model.

Figure 4 to 6 demonstrate the effects on the yield curve as we move from a less responsive to a highly responsive policy rule using some ad hoc parameter values. We set $\alpha = 0.2$ and $\alpha = 2$ respectively to represent two policy regimes, with the lower value of α representing a "passive" policy and the larger α representing an "active" policy. For the underlying state variable Z(t), we choose $k_Z = 0.085$, $\bar{Z} = .001$, $\sigma_Z = 0.08$. We also set ω at 0.3. The time to maturity τ is from 1 month to 30 years. These values are chosen so that they are roughly consistent with the econometric estimates in the next section.

Figure 4 is the plot of the factor loading $\frac{H_{2,Z}(\tau)}{\tau}$ against τ. As explained above, this term measures the responsiveness of long rates to movements in the short rate. We can see that when α increases from 0.2 to 2 (representing a shift from a less proactive monetary policy regime to a highly proactive one), the sensitivity of long rates to movements in the short rate increases significantly. The graph also suggests that such an impact is larger at longer maturities. The striking similarity between this figure and Figure 1, which is obtained from the monthly observations on the government bond yields, suggests that the shift in the monetary policy rule has contributed significantly to the structural change in nominal interest rates in the U.S.

More evidence can be obtained from Figure 5, which includes the plot of $\theta_0 - \frac{\log H_{1,Z}}{\tau} + \frac{H_{2,Z}}{\tau}\theta_2\bar{Z}$ for $\alpha = 0.2$ and $\alpha = 2.0$ respectively. It shows how a large increase in α affects the the average position of the yield curve in the steady state distribution. In particular, our model suggests that as α increases from 0.2 to 2, in equilibrium we will tend to have a higher yield curve on average (note that the unconditional mean of Z(t) is $\bar{Z}$). Again, in figure 2 we can see a similar shift of the yield curve in the U.S. data as monetary policy became more responsive in the post-1979 period.

This graph also shows that if an increase of the short rate R(t) by the central bank is due to a permanent shift of α to a higher value, then it is possible that the long rates also move in the same direction with the same or an even bigger magnitude. Hence this result also sheds some light on the puzzling behavior of long term interest rates in early 1994, where a moderate monetary policy tightening led to increases in all the long-term interest rates with similar magnitudes to that in the Federal funds rate[12].

[12]In the spring of 1994, a half percentage point increase in the federal funds rate driven by the Fed led to a half percentage to one percentage point increase in the long rates.

Finally, Figure 6 plots the standard deviation of the nominal interest rates as α increases from 0.2 to 2.0, which confirms that a more responsive monetary policy does not necessary lead to more volatile interest rates. Instead, as inflation stabilizes around the target level, in equilibrium we can have decreased interest rate volatility. Again, we also see similar changes in interest rate volatility as monetary policy rule shifts in Figure 3.

In the following section, we provide formal econometric evidence on the structural break in the yield curve and its relation to monetary policy. In particular, a parameter instability test is conducted against the term structure model introduced in this section. We will show that there is indeed a regime switch in the term structure of nominal interest rates; moreover, such a structural break in the yield curve is largely related to the shift of monetary policy in late 1979 and the early 1980s in the U.S.

4 A Formal Test of a Structural Break in the Yield Curve

The term structure of nominal interest rates is given in equation (21), where $X(t)$ and $Z(t)$ are defined in (13) and (14) respectively. Nevertheless, it is known that the model (21) is under-identified (see Dai and Singleton, 1997). To be able to estimate the model and construct test statistics, let's first define $\lambda_Z = \frac{\omega\sigma_Z}{\alpha}$ and $\lambda_X = \epsilon\sigma_X$, and further normalize σ_X and σ_Z to be 1. With these normalizations, a long-term normal interest rate with maturity τ is then given by:

$$R(t,\tau) = \theta_0 - \frac{\log H_{1,X}(\tau)}{\tau} - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,X}(\tau)}{\tau}\theta_1 X(t) + \frac{H_{2,u}(\tau)}{\tau}\theta_2 Z(t) \quad (25)$$

Where for $s = X$ and Z:

$$\begin{aligned} H_{1,s}(\tau) &= \left[\frac{2\delta_s e^{(\delta_s+k_s+\lambda_s)\tau/2}}{(\delta_s+k_s+\lambda_s)(e^{\delta_s\tau}-1)+2\delta_s}\right]^{2k_s\bar{s}} \\ H_{2,s}(\tau) &= \frac{2(e^{\delta_s\tau}-1)}{(\delta_s+k_s+\lambda_s)(e^{\delta_s\tau}-1)+2\delta_s} \end{aligned}$$

and:

$$\begin{aligned} \delta_X &= ((k_X+\lambda_X)^2+2\theta_1)^{1/2} \\ \delta_Z &= ((k_Z+\lambda_Z)^2+2\theta_2)^{1/2} \end{aligned}$$

Though the policy parameter α can not be directly estimated, from equation (20) we know that it affects θ_0 and θ_2. Moreover, α also affects λ_Z from the above normalization. Let $\beta = (\theta_0, \theta_2, \lambda_Z)'$, and collecting all other parameters of the model in a vector γ, we can therefore form the following testable hypothesis:

$$
\begin{aligned}
H_0: \quad \beta_t &= \beta_0 \text{ for all t} \geq 1 \text{ for some } \beta_0 \in R^3 \\
H_1: \quad \beta_t &= \beta_1 \text{ for t} = 1, \ldots, T_b \\
\beta_t &= \beta_2 \text{ for t} = T_b + 1, \ldots, \text{T}
\end{aligned}
$$

where T_b is the break point which could be known or unknown. In the latter case, it is assumed that we know T_b is in a given interval, say $[T_1, T_2]$.

4.1 The Testing Procedure

Andrews (1993) proposed a general testing procedure for parameter instability and structural change with a known or unknown change point in nonlinear parametric models. The tests apply to a wide class of parametric models that are suitable for estimation by the generalized method of moments procedure.

In particular, let the observed sample be $\{W_t : 1 \leq t \leq T\}$ and assume that the population orthogonality conditions are $Em(W_t, \beta_0, \gamma_0) = 0$ for a specified function $m(\cdot, \cdot, \cdot)$. Andrews proposes that one first obtains a full-sample GMM (FSGMM) estimator $(\tilde{\beta}, \tilde{\gamma})$ with the moment condition $\tilde{m}_T(\beta, \gamma)$ by restricting $\beta_1 = \beta_2$:

$$\tilde{m}_T(\beta, \gamma) = \frac{1}{T} \sum_{1}^{T} m(W_t, \beta, \gamma) \tag{26}$$

One then obtains an unrestricted partial-sample GMM (PSGMM) estimator $(\hat{\beta}_1, \hat{\beta}_2, \hat{\gamma})$ using the moment condition $\hat{m}_T(\beta_1, \beta_2, \gamma, T_b)$, where $\hat{m}_T$ is given by:

$$\hat{m}_T(\beta_1, \beta_2, \gamma, T_b) = \frac{1}{T} \sum_{t=1}^{T_b} \begin{pmatrix} m(W_t, \beta_1, \gamma) \\ 0 \end{pmatrix} + \frac{1}{T} \sum_{t=T_b+1}^{T} \begin{pmatrix} 0 \\ m(W_t, \beta_2, \gamma) \end{pmatrix} \tag{27}$$

For a given break point T_b, Wald $(W_T(T_b))$, the Lagrange Multiplier $(LM_T(T_b))$ and the Likelihood Ratio $(LR_T(T_b))$ test statistics can be constructed based on the full-sample and partial-sample estimators under general regularity conditions (see Appendix D for details). All the test statistics have asymptotic χ^2 distributions under the null hypothesis.

When the break point T_b is only known to be in the interval $[T_1, T_2]$, the test statistics are of the form:

$$\sup_{T_b \in [T_1, T_2]} W_T(T_b), \qquad \sup_{T_b \in [T_1, T_2]} LM_T(T_b), \qquad \sup_{T_b \in [T_1, T_2]} LR_T(T_b) \tag{28}$$

Andrews shows that the above test statistics are asymptotically equivalent and the asymptotic null distributions are given by the supremum of the square of a standardized tied-down Bessel process of order $p \geq 1$ (where in our case $p = 3$) under suitable conditions. Asymptotic critical values are obtained through Monte Carlo simulations and are reported in Andrews (1993).

Former GMM estimations of the CIR term structure model include Gibbons and Ramaswamy (1993) among others. Many studies have also used other methods to estimate the term structure model, such as the efficient method of moment (EMM) and the maximum likelihood method. To our knowledge, the present paper is the first to apply the PSGMM procedure to test for parameter instability and structural break in the CIR model.

4.2 The Moment Conditions

To apply the above testing procedure, we have to first construct a set of moment conditions. In section 2, we have showed that the average levels, the volatilities and the co-movements of the interest rates seem to have experienced a structural change when monetary policy shifted in late 1979. In section 4, we constructed a model to give a theoretical explanation of the observed differences in the interest rates. Hence, it is natural to use the mean, variance and covariance of the interest rates as the moment conditions in our estimation and testing.

From (25) and the definition of $X(t)$ and $Z(t)$ as in (13) and (14), we can easily obtain the mean $ER(t,\tau)$ and variance $Var\,(R(t,\tau))$ of the nominal interest rates of different maturities (τ). For the eighteen interest rates we have (τ from 1 month to 10 years), these means and and variances give us 36 moment conditions. Moreover, the covariances between the short term interest rate (τ_0 = 1-month) and long term interest rate (τ > 1-month) $Cov\,(R(t,\tau_0), R(t,\tau))$ give us 17 more moment conditions.

Table 1: Test statistics of the 2-factor model

Wald	LM	LR
2322.16	27.28	18765.90

Table 2: Testing the over-identification restrictions of the 2-factor model

FSGMM	D.F.	PSGMM	D.F.
76.79	44	104.13	94

4.3 The Results

We first fix T_b at September 1979 in the light of the above discussions. The test statistics are summarized in Table 1. Parameter estimates are presented in Table 10.

Since the three test statistics have the same asymptotic χ^2 distribution with 3 degrees of freedom, the three tests therefore all reject the null hypothesis at the 1% significance level. Notice that though the three test statistics have the same asymptotic distribution, their finite sample values are quite different. As another piece of evidence on the structural change, it is also interesting to notice that while the over-identification restriction is rejected for the full-sample (restricting $\beta_1 = \beta_2$) GMM estimation, the over-identification restriction can't be rejected for the partial-sample (unrestricted) GMM specification. This suggests that the data favor unrestricted estimates of β_1 and β_2 over the restricted ones. Table 2 gives the values of the test statistics for over-identification restrictions and their degrees of freedom (D.F.) for the full-sample GMM (FSGMM) and the partial-sample GMM (PSGMM) estimations respectively.

Of course, one may suspect that the rejection of the parameter stability or the rejection of the over-identification restrictions in the full-sample estimation are due to a mis-specification of the model. One source of mis-specifications is omitted factors. If the instantaneous short-term interest rate is driven by three factors instead of two, we could have parameter instability if we fit the data with a two-factor model. Recall that to get (25), we have used (20) for the instantaneous short term interest rate: $R(t) = \theta_0 + \theta_1 X(t) + \theta_2 Z(t)$. But we can also add a third independent

Table 3: Test statistics of the 3-factor model

Wald	LM	LR
4015.53	21.45	9627.93

Table 4: Testing the over-identification restrictions of the 3-factor model

FSGMM	D.F.	PSGMM	D.F.
82.16	40	103.86	90

factor to the model and apply the same estimation procedure and test the same null hypothesis of parameter stability $\beta_t = \beta_0$ against the alternative of structural change. The test statistics are presented in Table 3, and the parameter estimates are summarized in Table 10.

Again, all three tests overwhelmingly reject the null hypothesis of no structural change. Moreover, the over-identification restrictions are rejected for the full-sample estimation while such restrictions can't be rejected for the partial-sample estimation. See Table 4 for the test statistics on over-identification restrictions. Hence omitted factors are unlikely to be the source of parameter stability.

It therefore seems to be safe to conclude from the above results that there is indeed a structural change in the term structure of interest rates around October 1979 as the U.S. monetary policy shifts. However, to conclude that the policy shift is the source of the regime switch in the term structure, we have to further examine the point estimates of the parameters from the partial-sample GMM to see if they are consistent with our model's predication.

Table 5 and Table 6 present the estimates of $\hat{\theta}_0$, $\hat{\theta}_2$ and $\hat{\lambda}_Z$ in the pre- and post-1979 periods from the two-factor model and three-factor model respectively. Numbers in parentheses are Newey-West estimates of standard errors.

Recall that $\hat{\theta}_0$, $\hat{\theta}_2$ and $\hat{\lambda}_Z$ are related to the policy parameter α in the following way:

$$\hat{\theta}_0 = \pi^* - \frac{\bar{Z}}{\alpha}$$

Table 5: Point estimates of the two-factor model

Pre-1979			Post-1979		
$\hat{\theta}_0$	$\hat{\theta}_2$	$\hat{\lambda}_Z$	$\hat{\theta}_0$	$\hat{\theta}_2$	$\hat{\lambda}_Z$
.0030	.0150	-.1036	.0069	.0256	-.1791
(.0044)	(.0023)	(.0182)	(.0062)	(.0040)	(.0241)

Table 6: Point estimates of the three-factor model

Pre-1979			Post-1979		
$\hat{\theta}_0$	$\hat{\theta}_2$	$\hat{\lambda}_Z$	$\hat{\theta}_0$	$\hat{\theta}_2$	$\hat{\lambda}_Z$
.0098	.0034	.2891	.0204	.0050	.0117
(.0288)	(.0005)	(.0098)	(.0287)	(.0006)	(.0061)

$$\hat{\theta}_2 = \left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)\sigma_Z^2$$
$$\hat{\lambda}_Z = \frac{\omega\sigma_Z}{\alpha}$$

We can see that in both 2-factor model and 3-factor model $\hat{\theta}_0$ becomes much smaller in the post-1979 period than in the pre-1979 period, which is consistent with the hypothesis of an increased α. The estimates also suggest that $\hat{\theta}_2$ in both models increase in the latter period. As explained above, the effect of changes in monetary policy on θ_2 depends on the relative value of α and ω. If ω^2 is larger than $\alpha/2$, an increase in α will lead to an increase in θ_2.

More evidence can be obtained from the estimates of $\hat{\lambda}_Z$. In the 3-factor model, $\hat{\lambda}_Z$ decreases from 0.2891 to 0.0117 after 1979, which implies that ratio of the post-1979 α and the pre-1979 α is around 24.7. If we compute the ratio of $\hat{\theta}_2 + \hat{\lambda}_Z^2$, which is equal to σ_Z^2/α, we get an estimate of the ratio of the post-1979 α and the pre-1979 α around 17.0.

Recall that the monetary policy rule is given by:

$$\frac{dM(t)}{M(t)} = \frac{dP(t)}{P(t)} - \alpha\left(\frac{dP(t)}{P(t)} - \pi^* dt\right) + \frac{dy(t)}{y(t)} + \frac{d\xi(t)}{\xi(t)}$$

In the above policy reaction function, α measures the responsiveness of the monetary policy to inflation pressure. If α is less than 1, the monetary policy is thought of as accommodative in a sense that money growth is not reduced when the rate inflation exceeds the target. In contrast, an α with a value of greater than 1 would imply lower money growth when the rate of inflation exceeds the target level. The maintained hypothesis is that since late 1979 and early 1980's, the monetary policy has become more pro-active towards controlling inflation with the α increases from less than 1 to a value much greater than 1. And if this is the case, we should have the ratio of α from those two periods in the magnitude of 10 or 20. Therefore our estimates are fully consistent with the maintained hypothesis.

Nevertheless, the estimates of $\hat{\lambda}_Z$ from the 2-factor model seem to suggest that α actually decreases in the post-1979 period as the absolute value of the estimate of $\hat{\lambda}_Z$ increases from 0.1036 to 0.1791. But recall that $\hat{\lambda}_Z$ is a function of ω and σ_Z too ($\hat{\lambda}_Z = \frac{\omega\sigma_Z}{\alpha}$). By definition, ω measures the volatility of monetary shocks, which will hardly remains constant across those two period. If ω also increased in the post-1979 period, we could have a moderate increase in the estimate of $\hat{\lambda}_Z$ even if α increased. Note that the increase in $\hat{\lambda}_Z$ in the 2-factor model is only 0.07. So the third factor in the 3-factor model could be interpreted as the volatility of monetary shocks. Once we factor that into the model of the term structure, we have a constant ω in the 3-factor model and we have an estimate of $|\hat{\lambda}_Z|$ which decreases as α increases.

Finally, we can obtain further evidence about the relationship between the monetary policy and the term structure by endogenously determining the break point. Indeed, when we assume that the break point is known, the choice of T_b seems to be quite arbitrary. It is hard to argue that the structural break occurred exactly in October 1979 instead of some other date, even though many take the former date as a turning point in the U.S. monetary policy. Moreover, the limitation of our tests is that they can only be used to test for a one-time change, while the monetary policy may have changed several times in the past and the changes are gradual. Hence it is more realistic to relax the assumption of a known break point and test whether there is a possible structural change during a specific period of time. This approach treats the break point as an endogenous variable. It therefore also produces a natural estimate of the break point, which can give us further insight into the source of the structural change. For example, we know as a matter of fact that the U.S. monetary policy experienced dramatic change in late 1979 and the early 80s. If our estimate of the break point for the term structure lies inside that period of time, it would provide further evidence that the shift in monetary policy is the source of structural change

Table 7: Estimates of the endogenous break point

Two-factor Model		Three-factor Model	
$\sup_{T_b} LM_T(T_b)$	T_b^*	$\sup_{T_b} LM_T(T_b)$	T_b^*
31.4041	November 1980	25.2326	November 1980

in the term structure.

Therefore, instead of fixing T_b at September 1979, we will assume that $T_b \in [T_1, T_2]$, where T_1 corresponds to January 1970 and T_2 corresponds to December 1985. This period is long enough to include several important episodes, such as the collapse of the fixed exchange regime in 1971, the Oil Crises in the 1970s,[13] and the shift of the U.S. monetary policy during 1979-82, etc. We test whether there is a structural break in the term structure of interest rates in the U.S. during this 15-year period, and, if there is indeed a structural break, when it occurred.

When T_b is unknown and the time interval $[T_1, T_2]$ is very large, the most convenient test is the Lagrange multiplier test because it only requires one full-sample estimation, while the Wald and likelihood ratio tests require one partial-sample GMM estimation for each point of time in the interval and are very time consuming. Given that these three tests are asymptotically equivalent, we will only construct the Lagrange multiplier test. Moreover, from the above results when T_b is fixed, it seems that the LM test is the most conservative one in finite samples even though the three test statistics have the same asymptotic distribution. Table 7 gives the test statistics and the estimate of the break point T_b^* for both two-factor and three-factor models.

The test statistics again reject the null hypothesis of no structural change at 1% level and suggest a break point which is consistent with the timing of the monetary policy shift. In summary, these results provide empirical support to the theoretical model about the relationship between the monetary policy and the term structure in the last chapter. They show that the shift of monetary policy is largely responsible for the structural break

[13] Some researchers have suggested that supply shocks during the 1973-74 Oil Crisis are responsible for a structural change in the term structure, not the shift of monetary policy in late 1979 and the early 1980s. See Lu (1999).

in the yield curve. Moreover they also show that we can extract information about the monetary policy, which can not be directly observed, from the term structure of nominal interest rates due to the close relationship between they two.

5 Concluding Remarks

In the present paper, we examine the impact of monetary policy on the term structure of nominal interest rates. By developing a tractable general equilibrium model, we argue that the permanent shift to a more proactive policy stance towards controlling inflation in late 1979/early 1980s is an important underlying source of the structural changes in the yield curve that we observe in the data. The model also offers a testable parametric model of the term structure. Results from formal econometric tests further confirm our conjecture. This study contributes to the literature in several ways. Many studies have looked at the implications of alternative monetary policy rules for macroeconomic performance. Nevertheless, the impact of monetary policy on asset prices has been to a large extent ignored. This paper is an effort to investigate the interactions between monetary policy and the financial market. Moreover, the term structure is the main channel of monetary transmission. A better understanding of the impact of monetary policy on the term structure is crucial for evaluating policy effectiveness.

Our study also contributes to the literature on term structure models. Recognizing the important role of monetary policy in determining the short-term interest rate, several recent studies have explicitly incorporated the monetary policy behavior in a model of the term structure, For example, Rudebush (1995) estimates a daily model of Federal Reserve interest rate targeting behavior and explores the implications for the yield curve in the framework of expectation hypothesis for the term structure. In continuous time models, Piazzesi (1999) develops a factor model of the term structure which incorporates macroeconomic jump effects due to monetary policy actions in response to inflation pressure. Due to their considerations of high frequency data (daily, weekly), these studies have adopted a partial equilibrium approach which treats inflation as exogenous. The feedback effect from monetary policy on inflation, which is supposedly the main concern of monetary policy, is ignored. In the present study, inflation and the nominal short-term interest rate are determined jointly in a general equilibrium model and we show that the inflation process is not invariant to policy actions.

However, one major caveat of our approach is that monetary policy affects nominal interest rates only through its impact on inflation. Other channels (such as the real interest rate) through which monetary policy may affect the term structure of nominal interest rates are shut down in the model for tractability. To fully understand the relationship between monetary policy and the term structure, it is necessary to incorporate the real effect of monetary policy into the model. Another weakness of our model is the independence of inflation and the real interest rate, which seems to contradict the empirical evidence (Sun, 1992).

Our model can also be extended to study the implications of different monetary policies for other nominal asset prices such as exchange rates. Studies have suggested that asymmetries in the conduct of monetary policy in different countries may be the source of the forward premium anomaly (Bansal, 1997). These extensions are left for future research.

A Proof of Proposition 1

First note that when $f^{'}(m_t/c_t) = 0$, the transaction cost is a constant. So without loss of generality, let's assume that $y(t) = c(t)$.

Consider the case where $\triangle t \rightarrow 0$. Assuming that $\frac{dP(t)}{P(t)} = \mu_P(t)dt + \sigma_1(t)dW_1(t) + \sigma_2(t)dW_2(t)$ and applying Ito's Lemma to (9):

$$\frac{dM(t)}{M(t)} = (\mu_P(t) + \mu_y(t) + \sigma_y(t)\sigma_1(t))dt + (\sigma_1(t) + \sigma_y(t))dW_1(t) + \sigma_2(t)dW_2(t) \quad (29)$$

where $\mu_y(t) = hX(t) - \rho$, $\sigma_y(t) = \epsilon\sqrt{X(t)}$.

From (15) we have the policy rule:

$$\frac{dM(t)}{M(t)} = \frac{dP(t)}{P(t)} - \alpha\left(\frac{dP(t)}{P(t)} - \pi^* dt\right) + \frac{dy(t)}{y(t)} + \frac{d\xi(t)}{\xi(t)} \quad (30)$$

where $\frac{d\xi(t)}{\xi(t)}$ is given in (12).

Using the above two equations, we have the following relations because of the unique representation of the diffusion process M(t):

$$\sigma_1(t) + \sigma_y(t) = (1-\alpha)\sigma_1(t) + \sigma_y(t)$$
$$\sigma_2(t) = (1-\alpha)\sigma_2(t) + \sigma_\xi(t)$$
$$\mu_P(t) - \alpha(\mu_P(t) - \pi^*) + \mu_y + \mu_\xi(t) = \mu_P(t) + \mu_y(t) + \sigma_y(t)\sigma_1(t)$$

where from (12) we have $\mu_\xi(t) = Z(t) - \bar{Z}$, $\sigma_\xi(t) = \omega\sqrt{Z(t)}$.
Solving the above three equations gives us Proposition 1.

B Proof of Proposition 2

Using the first order conditions (4) and (6) from the consumer's problem, when $f^{'}(m_t/c_t) = 0$, we have:

$$e^{-R(t)\triangle t} \quad = \quad e^{-\rho\triangle t} E_t\left[\frac{P(t)c(t)}{P(t+\triangle t)c(t+\triangle t)}\right]$$

Using the Taylor expansion for the left-hand side,

$$e^{-R(t)\triangle t} = 1 - R(t)\triangle t + \circ(\triangle t)^{3/2}$$

Similarly for the right-hand side:

$$e^{-\rho\triangle t} E_t\left[\frac{P(t)c(t)}{P(t+\triangle t)c(t+\triangle t)}\right] = (1 - \rho\triangle t + \circ(\triangle t)^{3/2}) \times$$
$$E_t\left[\left(1 - \frac{\triangle P_t}{P_t} + \left(\frac{\triangle P_t}{P_t}\right)^2 + \circ(\triangle t)^{3/2}\right)\left(1 - \frac{\triangle c_t}{c_t} + \left(\frac{\triangle c_t}{c_t}\right)^2 + \circ(\triangle t)^{3/2}\right)\right]$$

using the facts that $c_t = y_t$ and:

$$\frac{\triangle y_t}{y_t} = \mu_{y,t}\triangle t + \sigma_{y,t} W_{1,t+\triangle t}\sqrt{\triangle t}$$
$$\frac{\triangle P_t}{P_t} = \mu_{P,t}\triangle t + \sigma_{1,t} W_{1,t+\triangle t}\sqrt{\triangle t} + \sigma_{2,t} W_{2,t+\triangle t}\sqrt{\triangle t}$$

where $W_{1,t}$ and $W_{2,t}$ are two independent standard normal variables. We hence have as $\triangle t \to 0$:

$$R(t) = \rho + \mu_y(t) + \mu_P(t) - \sigma_y^2(t) - (\sigma_1^2(t) + \sigma_2^2(t)) - \sigma_1(t)\sigma_y(t)$$

Substituting in relevant terms in the above equation leads us to Proposition 2.

C Proof of Proposition 3

From the first order conditions we have that the state price deflator is given by $\pi(t) = \frac{e^{-\rho t}}{P(t)c(t)}$. Hence,

$$\begin{aligned}
\frac{d\pi(t)}{\pi(t)} &= -R(t)dt - (\epsilon, \frac{\omega}{\alpha}) \begin{pmatrix} \sqrt{X(t)} & 0 \\ 0 & \sqrt{Z(t)} \end{pmatrix} \begin{pmatrix} dW_1(t) \\ dW_2(t) \end{pmatrix} \\
&= -R(t)dt - (\lambda_r, \lambda_u) \begin{pmatrix} \sqrt{r(t)} & 0 \\ 0 & \sqrt{u(t)} \end{pmatrix} \begin{pmatrix} dW_1(t) \\ dW_2(t) \end{pmatrix}
\end{aligned}$$

Where $\lambda_r = \frac{\epsilon}{\sqrt{\theta_1}}$, $\lambda_u = \frac{\omega}{\alpha\sqrt{\theta_2}}$. See (20) for the definitions of θ_1 and θ_2.

Let $r(t) = \theta_1 X(t)$, $u(t) = \theta_2 Z(t)$, the instantaneous short-term interest rate in (20) can be written as:

$$R(t) = \theta_0 + r(t) + u(t) \tag{31}$$

$r(t)$ and $u(t)$ are given respectively by:

$$dr(t) = k_X(\bar{r} - r(t)) + \sigma_r \sqrt{r(t)} dW_1(t) \tag{32}$$

$$du(t) = k_Z(\bar{u} - u(t)) + \sigma_u \sqrt{u(t)} dW_2(t) \tag{33}$$

where $\bar{r} = \theta_1 \bar{X}$, $\sigma_r = \sqrt{\theta_1}\sigma_X$, $\bar{u} = \theta_2 \bar{Z}$ and $\sigma_u = \sqrt{\theta_2}\sigma_Z$.
Hence under the Equivalent Martingale Measure (EMM), we can rewrite (32) and (33) as:

$$dr(t) = \tilde{k}_r(\tilde{r} - r(t))dt + \sigma_r \sqrt{r(t)} d\tilde{W}_1(t) \tag{34}$$

$$du(t) = \tilde{k}_u(\tilde{u} - u(t))dt + \sigma_u \sqrt{u(t)} d\tilde{W}_2(t) \tag{35}$$

where $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ are two independent standard Brownian motions under EMM, and the coefficients are given in the following equations:

$$\tilde{k}_r = k_X + \lambda_r \sigma_r \tag{36}$$

$$\tilde{r} = \frac{k_X \bar{r}}{k_X + \lambda_r \sigma_r} \tag{37}$$

$$\tilde{k}_u = k_Z + \lambda_u \sigma_u \tag{38}$$

$$\tilde{u} = \frac{k_Z \bar{u}}{k_Z + \lambda_u \sigma_u} \tag{39}$$

Using the well-known results of the multi-factor Cox-Ingersoll-Ross Term Structure Model (e.g. Duffie 1996), Proposition 3 follows.

D The Wald, Lagrange Multiplier and Likelihood Ratio Test Statistics for Structural Break

The following results can be found in Andrews (1993). First let $\tilde{S}$ be a consistent estimator of S, where S is defined as (See (25) in section 4.1 for the definition of $\tilde{m}_T(\cdot,\cdot)$):

$$S = \lim_{T\to\infty} Var\left(\sqrt{T}\tilde{m}_T(\beta_0,\gamma_0)\right)$$

Define $\pi = T_b/T$. Let $\hat{S}(T_b) = Diag(\pi\hat{S}_1(T_b),(1-\pi)\hat{S}_2(T_b))$ be a consistent estimator of $S(\pi)$, where $S(\pi)$ is given by:

$$S(\pi) = \begin{bmatrix} \pi S & 0 \\ 0 & (1-\pi)S \end{bmatrix}$$

Then $(\tilde{\beta},\tilde{\gamma}) = \arg\inf(\tilde{m}_T'\hat{S}^{-1}\tilde{m}_T)$ and $(\hat{\beta}_1,\hat{\beta}_2,\hat{\gamma}) = \arg\inf(\hat{m}_T'\hat{S}(T_b)^{-1}\hat{m}_T)$. The definition of $\hat{m}_T$ is given by equation (26) in section 4.1.

Further define $\tilde{M}$ and $\hat{M}_r(T_b)$ for $r=1,2$ respectively as:

$$\begin{aligned}
\tilde{M} &= \frac{1}{T}\sum_{1}^{T}\frac{\partial m(W_t,\tilde{\beta},\tilde{\gamma})}{\partial\beta'} \\
\hat{M}_1(T_b) &= \frac{1}{T_b}\sum_{1}^{T_b}\frac{\partial m(W_t,\hat{\beta}_1,\hat{\gamma})}{\partial\beta_1'} \\
\hat{M}_2(T_b) &= \frac{1}{T-T_b}\sum_{T_b+1}^{T}\frac{\partial m(W_t,\hat{\beta}_2,\hat{\gamma})}{\partial\beta_2'}
\end{aligned}$$

With the above definitions, the Wald statistic $W_T(T_b)$ is then given by:

$$\begin{aligned}
W_T(T_b) &= T\left(\hat{\beta}_1(T_b)-\hat{\beta}_2(T_b)\right)' \\
&\times\left(\frac{T}{T_b}\hat{V}_1(T_b)+\frac{T}{T-T_b}\hat{V}_2(T_b)\right)^{-1}\left(\hat{\beta}_1(T_b)-\hat{\beta}_2(T_b)\right) \quad (40)
\end{aligned}$$

where $\hat{V}_r(T_b) = \left(\hat{M}_r(T_b)'\hat{S}_r(T_b)^{-1}\hat{M}_r(T_b)\right)^{-1}$ for $r=1,2$.

The Lagrange Multiplier test statistic is given by;

$$LM_T(T_b) = \frac{T}{\pi(1-\pi)}\tilde{m}_{1,T}(\tilde{\beta},\tilde{\gamma})'\tilde{S}^{-1}\tilde{M}(\tilde{M}'\tilde{S}^{-1}\tilde{M})^{-1}\tilde{M}'\tilde{S}^{-1}\tilde{m}_{1,T}(\tilde{\beta},\tilde{\gamma}) \quad (41)$$

where $\tilde{m}_{1,T}(\tilde{\beta},\tilde{\gamma}) = \frac{1}{T}\sum_{1}^{T_b} m(W_t,\tilde{\beta},\tilde{\gamma})$.

Finally, the Likelihood Ratio statistic is just given by the difference between the PSGMM objective function evaluated at the full sample GMM estimator $(\tilde{\beta},\tilde{\gamma})$ and the partial sample GMM estimator $(\hat{\beta}_1,\hat{\beta}_2,\hat{\gamma})$:

$$\begin{aligned} LR_T(T_b) \quad = \quad & T\hat{m}_T(\tilde{\beta},\tilde{\beta},\tilde{\gamma},T_b)'\hat{S}(T_b)^{-1}\hat{m}_T(\tilde{\beta},\tilde{\beta},\tilde{\gamma},T_b) - \\ & T\hat{m}_T(\hat{\beta}_1,\hat{\beta}_2,\hat{\gamma},T_b)'\hat{S}(T_b)^{-1}\hat{m}_T(\hat{\beta}_1,\hat{\beta}_2,\hat{\gamma},T_b) \end{aligned} \tag{42}$$

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Table 8: OLS regression of long rates on the short rate

$R(t,\tau)$	Constant	std. error	$R(t)$	std. error	$D(t) \cdot R(t)$	std. error	R-square
3-month	0.2626	0.0024	1.0014	8.783E05	0.0176	3.712E05	0.9978
6-month	0.4922	0.0068	1.0074	0.0003	0.0135	0.0001	0.9956
9-month	0.6828	0.0105	0.9924	0.0004	0.0208	0.0002	0.9938
1-year	0.9299	0.0137	0.9541	0.0005	0.0416	0.0002	0.9921
1.25-year	1.1824	0.0166	0.9113	0.0006	0.0675	0.0003	0.9908
1.5-year	1.4006	0.0188	0.8748	0.0007	0.0885	0.0003	0.9898
1.75-year	1.5742	0.0207	0.8471	0.0008	0.1017	0.0003	0.9888
2-year	1.7231	0.0222	0.8239	0.0008	0.1107	0.0004	0.9880
2.5-year	1.9983	0.0247	0.7803	0.0009	0.1288	0.0004	0.9865
3-year	2.2487	0.0274	0.7396	0.0010	0.1489	0.0005	0.9853
4-year	2.6275	0.0312	0.6786	0.0010	0.1787	0.0005	0.9837
5-year	2.9030	0.0347	0.6353	0.0011	0.1987	0.0005	0.9823
6-year	3.1318	0.0385	0.5996	0.0012	0.2159	0.0006	0.9810
7-year	3.3064	0.0416	0.5728	0.0013	0.2275	0.0006	0.9800
8-year	3.4408	0.0440	0.5523	0.0015	0.2350	0.0006	0.9794
9-year	3.5515	0.0461	0.5354	0.0014	0.2409	0.0006	0.9790
10-year	3.6456	0.0480	0.5207	0.0015	0.2465	0.0007	0.9787

$R(t)$ is the 1-month rate, $D(t) = 1$ if $t > 1979:09$ and 0 otherwise. The standard errors are computed using the Newy-West procedure.

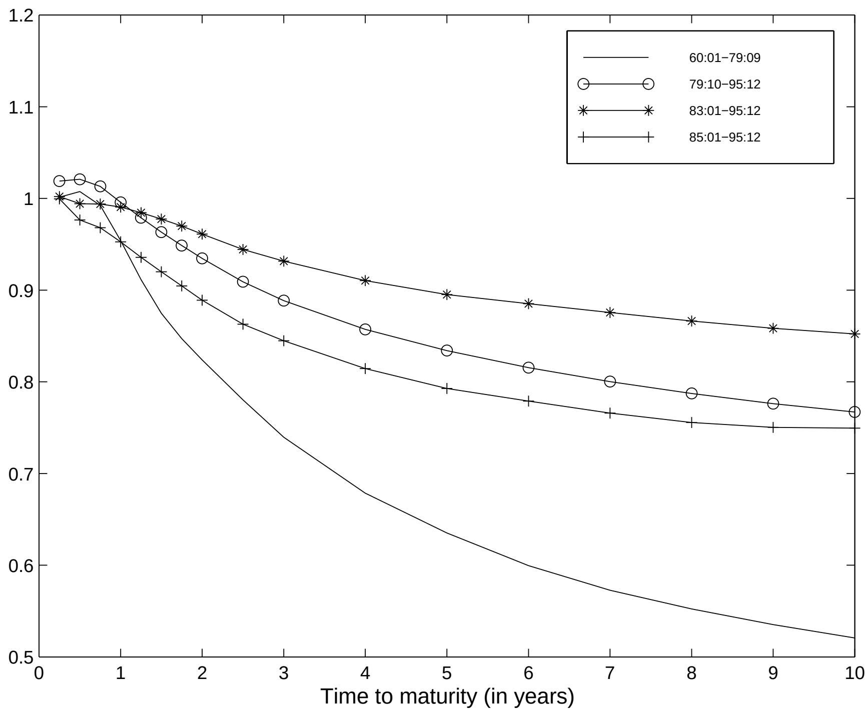
Table 9: Mean and std. dev. of the interest rates in different periods

	1960:01 - 1979:09		1979:10 - 1995:12		1983:01 - 1995:12		1985:01 - 1995:12	
$R(t,\tau)$	Mean	Std	Mean	Std	Mean	Std	Mean	Std
3-month	5.1536	1.9162	7.5314	3.1769	6.3622	2.0394	5.8327	1.7253
6-month	5.4137	1.9379	7.7728	3.2128	6.5736	2.0722	6.0191	1.7109
9-month	5.5244	1.9413	7.9164	3.1912	6.7333	2.0972	6.1677	1.7167
1-year	5.5744	1.9071	8.0508	3.1298	6.9021	2.1204	6.3215	1.7138
1.25-year	5.6098	1.8600	8.1937	3.0726	7.0760	2.1262	6.4832	1.6931
1.5-year	5.6433	1.8200	8.3091	3.0203	7.2209	2.1216	6.6199	1.6662
1.75-year	5.6768	1.7907	8.3840	2.9749	7.3229	2.1175	6.7149	1.6426
2-year	5.7083	1.7676	8.4368	2.9320	7.3996	2.1098	6.7862	1.6176
2.5-year	5.7615	1.7273	8.5417	2.8510	7.5448	2.0854	6.9269	1.5705
3-year	5.8030	1.6927	8.6578	2.7796	7.6957	2.0579	7.0788	1.5326
4-year	5.8681	1.6437	8.8326	2.6692	7.9229	2.0061	7.3070	1.4539
5-year	5.9197	1.6158	8.9570	2.5914	8.0862	1.9798	7.4711	1.4156
6-year	5.9611	1.6011	9.0705	2.5235	8.2335	1.9593	7.6206	1.3933
7-year	5.9938	1.5942	9.1497	2.4644	8.3387	1.9322	7.7279	1.3579
8-year	6.0193	1.5925	9.2024	2.4092	8.4134	1.8986	7.8088	1.3207
9-year	6.0387	1.5943	9.2444	2.3554	8.4753	1.8612	7.8825	1.2919
10-year	6.0525	1.5983	9.2839	2.3048	8.5329	1.8230	7.9550	1.2718

Table 10: Point estimates of the multi-factor CIR models

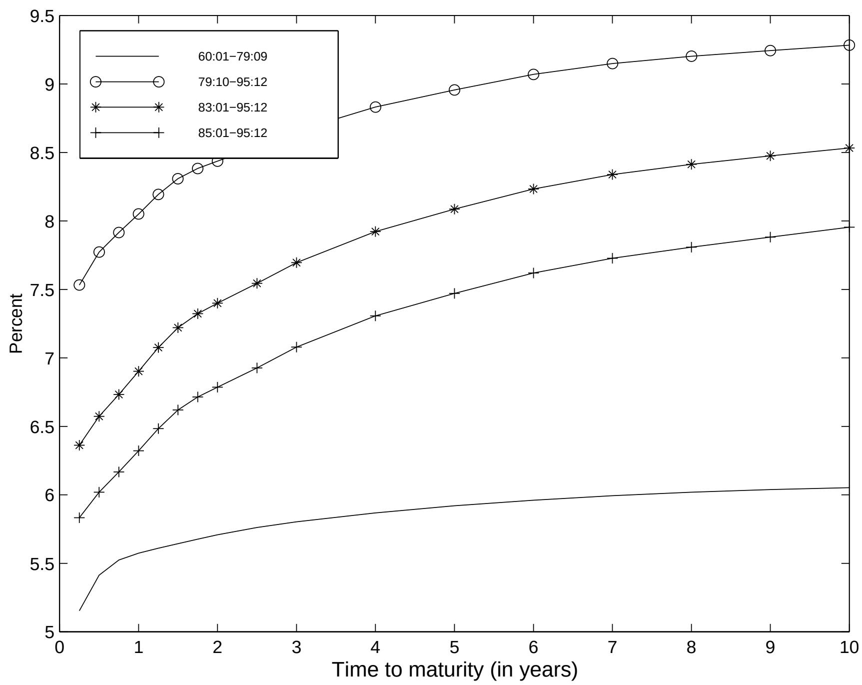
Coefficients	2-factor CIR	Std Error	3-factor CIR	Std Error
$\theta_{0,1}$	.0030	.0044	.0098	.0288
$\theta_{0,2}$	.0069	.0062	.0204	.0287
$\theta_{Z,1}$	.0151	.0024	.0034	.0005
$\theta_{Z,2}$	.0256	.0040	.0050	.0006
$\lambda_{Z,1}$	-.01036	.0182	.2891	.0098
$\lambda_{Z,2}$	-.1791	.0242	.0117	.0061
θ_X	.0047	.0019	.0053	.0256
θ_Y			.0205	.0050
λ_X	-4.2988	.7515	-3.9385	52.56
λ_Y			-5.9829	.1857
k_Z	.4568	.0180	.0855	.0059
k_X	11.7567	.7936	4.3022	52.50
k_Y			13.1867	.2110
$\bar{Z}$	1.7545	.4860	6.7367	1.3375
$\bar{X}$	3.6241	.7601	.4236	6.243
$\bar{Y}$			.5724	.1431

Figure 1: OLS regression coefficients of long rates on the short rate



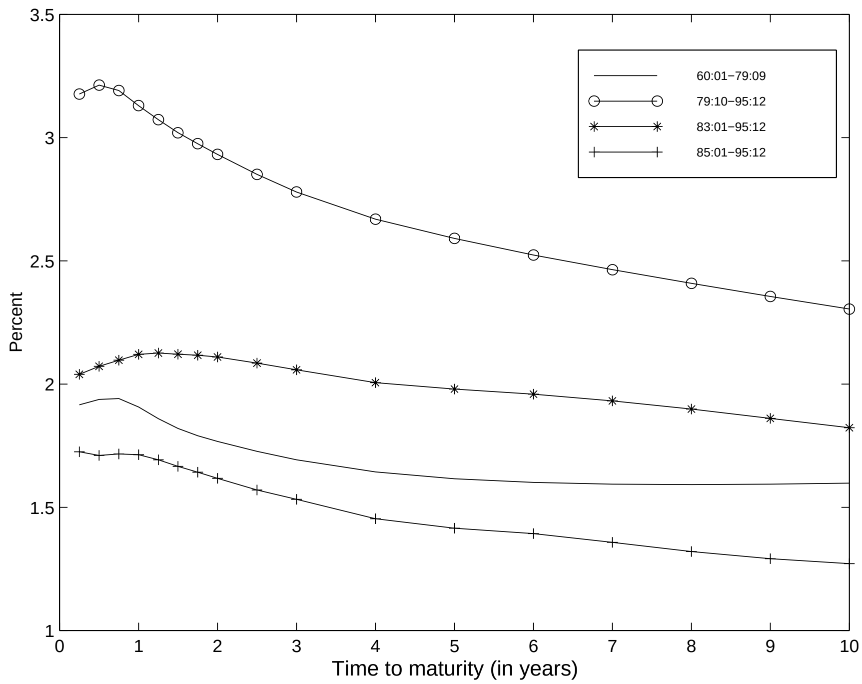
The data are monthly observations of the nominal yields on government pure discount bonds from 01:1960 – 12:1995. There are a total of 17 bonds of different maturities ranging from 3 months to 10 years. The short term interest rate is approximated by the 1-month rate.

Figure 2: Average level of the yield curve



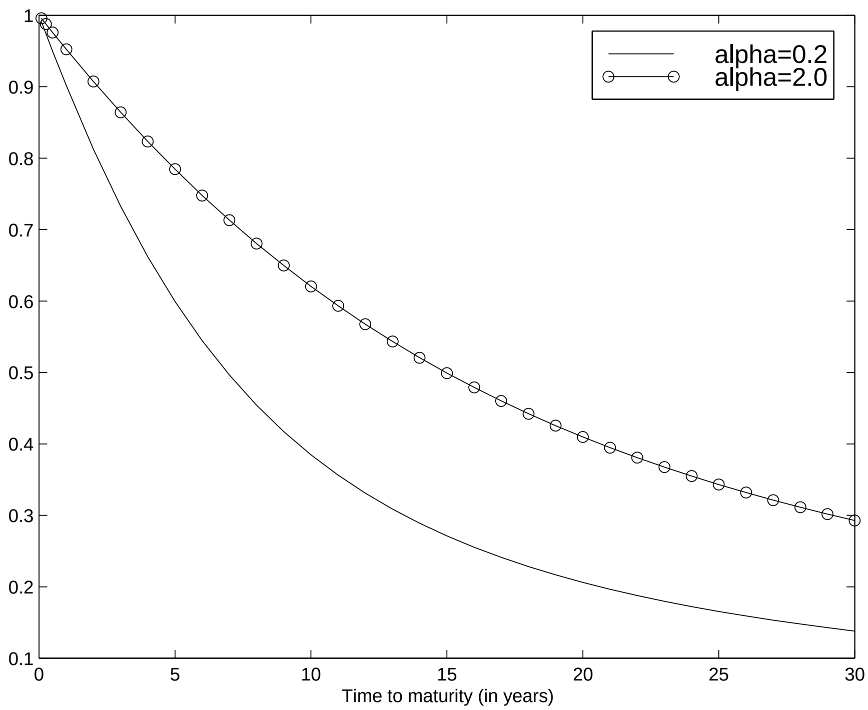
The curves are plots of average interest rates against time to maturity over different periods. The data are monthly observations of the nominal yields on government pure discount bonds from 01:1960 – 12:1995. There are a total of 17 bonds of different maturities ranging from 3 months to 10 years.

Figure 3: Interest rate volatility



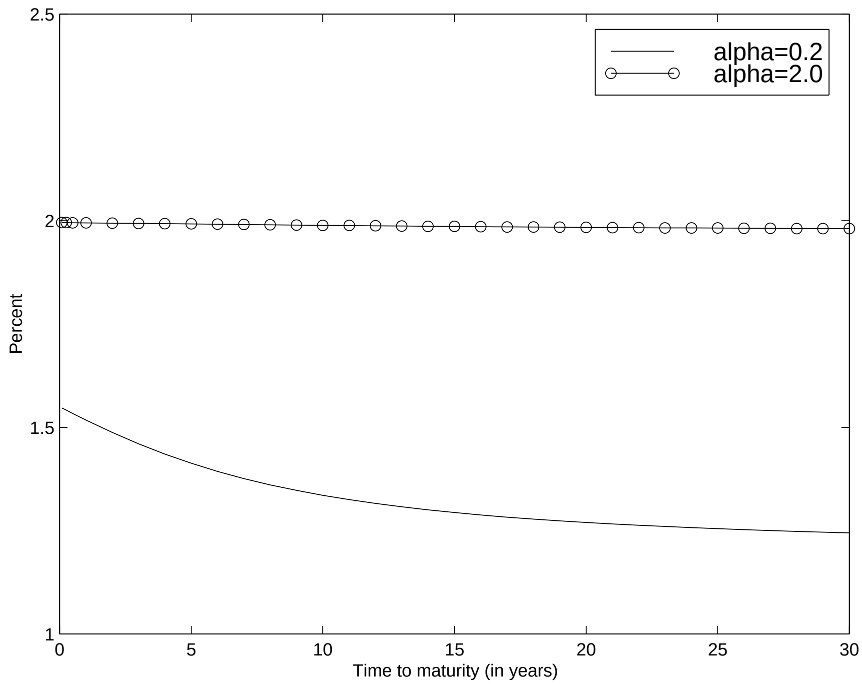
The curves are plots of interest rate volatilities against time to maturity over different periods. The data are monthly observations of the nominal yields on government pure discount bonds from 01:1960 – 12:1995. There are a total of 17 bonds of different maturities ranging from 3 months to 10 years. The volatilities are measured by the standard deviations of the nominal interest rates.

Figure 4: The impact of a policy change on the responsiveness of long rates



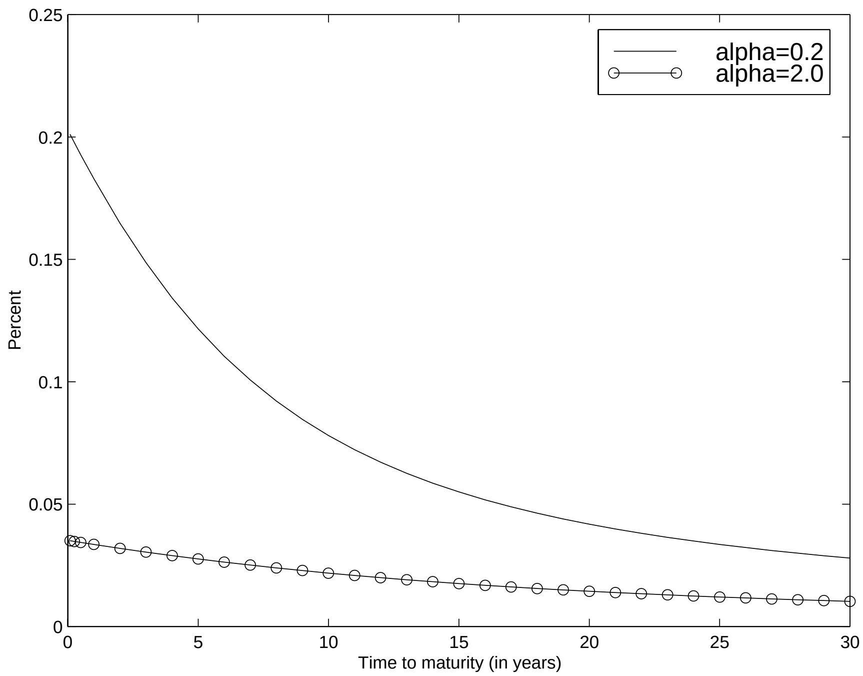
The curves are the plots of $\frac{H_{2,Z}(\tau)}{\tau}$ in equation (20) for $\alpha = 0.2$ and $\alpha = 2.0$ respectively. They measure the responsiveness of long rates to movements in the short term rate.

Figure 5: The impact of a policy change on the average level of the yield curve



The curves are the plots of $\theta_0 - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,Z}(\tau)}{\tau}\theta_2 Z(t)$ in equation (20) for $\alpha = 0.2$ and $\alpha = 2.0$ respectively, replacing $Z(t)$ with its unconditional mean $\bar{Z}$. They measure the impact of changes in α on the average level of the yield curve in the steady state distribution, holding everything else constant.

Figure 6: The impact of a policy change on the volatilities of long rates



The curves are the plots of $\frac{H_{2,Z}(\tau)}{\tau}\theta_2 \times std(Z(t))$ in equation (17) for $\alpha = 0.2$ and $\alpha = 2.0$ respectively. They measure the impact of changes in α on the standard deviations of the nominal interest rates of different maturities.