General Equilibrium of a Monetary Model with State-Dependent Pricing*

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Abstract

There is a long standing debate on whether nominal shocks have real effects on the economy. According to one theory, frictions in the price adjustment process can lead to the non-neutrality of money. Macroeconomic models of optimal price setting that nest these price adjustment frictions, however, have proven to be difficult to construct and apply to the data. This paper provides a rational expectations equilibrium model of optimal price setting that is solved numerically. The solution requires the specification and estimation of a price forecast rule. The structural parameters of the model, focusing on the parameters of the price adjustment cost process, are estimated through an indirect inference procedure using aggregate data from the U.S. economy. According to the estimated results, large and variable adjustment costs are required for the model to match up against U.S. data.

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1 Introduction

There is a long-standing debate on whether nominal shocks have real effects on the economy. According to one theory, frictions in the price adjustment process can lead to the non-neutrality of money. If firms fail to adjust their prices in response to an increase in the nominal money supply, the result will be an increase in output due the increase in nominal demand. One primary mechanism used to explain the infrequency of price adjustment is the presence of a fixed cost of changing prices. When faced with this type of cost, firms will optimally weigh the present and future benefits of resetting the price against the fixed cost of the price change. Numerous theoretical studies have examined the implications of price adjustment costs, but empirical evidence has been relatively scarce. One of the main reasons for this lack of empirical evidence is that the models have been constructed under restrictive assumptions that make it difficult to apply to data.

Very little is known empirically about the magnitude and structure of price adjustment costs faced by firms. In one of the only studies that directly measures adjustment costs, Levy, Bergen, Dutta and Venable (1997) study the costs of changing prices in supermarkets. The amount of time required at each step of the price-change process is recorded, and then wage data is used to compute the actual cost of a price change. They find that adjustment costs comprise 0.7 percent of annual revenues, which is a nontrivial amount according to the theoretical literature.[1]

In several other recent studies, the parameters of the adjustment cost process are estimated indirectly. Willis (2000) estimates the fixed price-adjustment costs for the newsstand prices of magazines. The results indicate that these costs are economically significant, between 2 to 4 percent of annual revenues. In addition, the estimates indicate that the costs

[1] According to calculations from Blanchard and Kiyotaki (1987), menu costs as small as 0.08 percent of revenues may be sufficient to prevent price adjustment in response to a 5 percent change in aggregate demand.

vary over time with a high degree of persistence. In another study, Slade (1998) estimates the existence, type, and magnitude of adjustment costs using weekly retail price data on saltine crackers. The results of her estimation indicate that the fixed cost is the important determinant of price behavior, rather than a variable cost, and that the magnitude of the cost is somewhat higher than the estimate of Levy et al.[2] Aguirregabiria (1999) estimates fixed costs of price adjustment jointly with fixed costs of ordering in the supermarket industry. His results are similar in magnitude to those of Slade.

The distinguishing feature between the studies listed above and this paper is the incorporation of a rational expectations equilibrium (REE) into the model. In models of state dependent pricing, firm demand is usually a function of the firm's relative price. Therefore, in order for firms to make an optimal pricing decision, they must have a forecast of inflation. In Willis (2000) the inflation process is assumed to be exogenous. Thus, there is no requirement that the firms' pricing decisions aggregate to match the exogenous inflation process. In Slade, the model is assumed to have zero inflation over the two year period of study. Instead of the partial-equilibrium style framework in these papers, I will assume that agents have a specific forecast rule used to form inflation expectations. In the presence of persistence in the adjustment costs process, a perfect forecast rule would require that agents have complete information, including the prices and adjustment costs of all other firms. Inclusion of all of those state variables in a computational solution, however, would be extremely difficult. Therefore, I will follow an approach similar to Krusell and Smith (1998) in which firms are assumed to be boundedly rational, meaning that firms will make their inflation forecasts based upon a restricted set of information. The aim is to find a simple rule which provides a satisfactory inflation forecast for firms.

The contribution of this paper is to provide a framework in which the underlying struc-

[2] The difference between the estimates of Levy et al. and Slade point toward differences in the respective definitions of menu costs. By directly measuring the adjustment cost, Levy et al. use a strict definition of the value of labor required to enact a price change. Through structural estimation, Slade captures the cost of any rigidity in the price-setting process, of which labor costs are a subset.

tural parameters of a model of monopolistic competition for an economy can be estimated to satisfy the conditions for REE and match simulated data from the model to U.S. aggregate data. The key component of the model will be the forecast rule used by firms to predict inflation. In a rational expectations equilibrium, the forecast rule must be self-validating. Therefore, any estimation procedure must require that the forecast rule be a "good" predictor of the resulting inflation series.

A similar REE model is solved by Dotsey, King and Wolman (1999).[3] In their model, firms face price adjustment costs that are independently and identically distributed (i.i.d.) over time and across agents. This assumption is necessary to obtain a solution that can be log-linearized and analyzed around the steady state. They then analyze their model through calibration exercises.

The flexibility of this model is one of the main distinguishing features from the study by Dotsey, King and Wolman. Their assumption of i.i.d. adjustment costs dictates that all firms choosing to adjust prices will select the same new price, regardless of the length of time since the previous change. In a model with persistence in the adjustment cost, the expectations of future adjustment costs will be conditional on the current cost. A firm that currently faces a high adjustment cost will expect a high cost in the future, and therefore will make a larger change in price than a firm with a lower cost in order to reduce the frequency in which the cost is paid. As indicated by the results in Willis (2000), the persistence of adjustment costs may be high for some firms in the economy.

A second distinguishing feature of this model is that it lends itself easily to estimation. The structural parameters governing the adjustment cost process are estimated to match simulated aggregate data from the model against macroeconomic data from the U.S. economy. The forecast rule via which firms form their inflation expectations will be calculated from the data. An indirect inference estimation procedure then provides a method to es-

[3]For other examples of more restrictive general equilibrium models, see Caplin and Leahy (1991) and Benabou (1992)

timate parameters of the unobserved adjustment cost process so that the conditions for a REE are satisfied and selected aggregated moments from the simulated data match the actual data.

The results indicate that the adjustment cost process has a moderate degree of persistence and that firms on average pay adjustments costs of around 5 percent of revenues, a value that is much higher than the partial-equilibrium estimates in the literature. In addition, the estimated model does not produce impulse response functions of monetary shocks that resemble those from U.S. data.

2 State-Dependent Pricing Model

I analyze the problem of a monopolistically competitive firm with a given initial price in an economy consisting of n agents. Demand for the firm's output is determined by the level of real balances in the economy and the firm's price relative to the aggregate price index. Firm profits are determined by firm revenues minus the cost of production.

Each period, the firm decides whether or not to adjust its price by computing the discounted expected benefit of changing the price compared to the cost of adjustment. At the time of the decision, the firm knows its relative price in the previous period, the real money supply in the previous period, its fixed cost of price adjustment in the current period, and the current level of nominal money growth, which is exogenously set by the monetary authority. The firm does not know the inflation rate for the current period, since the change in the price level is a consequence of the joint action of the firms in their simultaneous price setting decisions. The firms, therefore, will need to use a forecast of current inflation as they make their optimal decisions.

The benefit of price adjustment today is that the forward-looking firm can choose the optimal price based upon current expected demand and conditional expectation of demand in the future. The cost of price adjustment appears in the form of an idiosyncratic fixed cost.

The structure of the adjustment cost process will affect both the intensive and extensive margins of the firm's decision due to the dynamic aspect of the model. If the firm decides not to change its price, then its relative price will be deflated by changes in the aggregate price index. For example, an increase in the aggregate price index would cause the firm's relative price to fall, leading to an increase in demand for the firm's output. Assuming that the firm will meet excess demand, firm profits will fall as a result of producing in excess of the optimal monopolistic output level and selling at a lower relative price.

2.1 Environment

This specification follows the standard model used in the menu cost literature with the addition of heterogeneous costs of price adjustment.[4] Formally, I assume that there are n producers who each produce a single differentiated good. The contemporaneous real profit for firm i in period t is

$$\Pi_{i,t} = \left(\frac{P_{i,t}}{P_t}\right) Y_{i,t} - \frac{d}{\gamma} Y_{i,t}^{\gamma} \tag{1}$$

where

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \frac{M_t}{P_t} \tag{2}$$

and

$$P_t = \left(\frac{1}{n}\sum_{j=1}^{n} P_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{3}$$

Here $Y_{i,t}$ represents firm output, $P_{i,t}$ is the firm's price, M_t is the nominal money supply, P_t is the CES aggregate price index, θ is the elasticity of substitution across goods, and γ and d are parameters of the cost function.

In the dynamic setting, the firm will make its pricing decision based upon its price at the end of the previous period, $P_{i,-1}$, the real money supply at the end of the previous

[4] See Blanchard and Kiyotaki (1987) or Ball and Romer (1990).

period, $\frac{M_{-1}}{P_{-1}}$, the current growth rate of nominal money, $\%\Delta M$, the current idiosyncratic adjustment cost that must be paid in the event of a price adjustment, ψ_i, the current aggregate inflation rate, $(\pi = \Delta \ln P)$, and expectations of future realizations of these state variables, which are dependent upon the current state. Since firms do not know the current inflation rate, they will make a distributional forecast based upon the current information set, Ω. The nature of the forecast rule will be elaborated upon below.

If there is a positive trend in nominal money growth, then the nominal variables will not be stationary. To frame the problem in a stationary environment, two of the firm's state variables will be rewritten as the relative price at the end of the previous period $\left(p_{-1} = \frac{P_{i,-1}}{P_{-1}}\right)$ and the real money supply in the previous period $\left(m_{-1} = \frac{M_{-1}}{P_{-1}}\right)$. These two relative variables along with the nominal money growth rate and the inflation forecast, $\pi(\Omega)$, summarize the relevant information for the firm's optimization problem. The current expected level of real money balances can be computed using the previous level of real money, the current growth rate of money, and the forecast of inflation $\left(\hat{m} = m_{-1}\frac{1+\ln \Delta M}{1+\pi(\Omega)}\right)$. The actual inflation rate used to depreciate the relative prices, $\bar{\pi}$, will be that determined by the firms' simultaneous pricing decisions. Price changes are assumed to go into effect immediately.

The optimization problem is formulated using a dynamic programming approach. Based upon the current realization of the states, $S = \{p_{-1}, m_{-1}, \ln \Delta M, \pi(\Omega), \psi_i\}$, the firm will compare the value of changing its price and paying the adjustment cost, V^C, against the value of keeping its price fixed, V^{NC}. The value function is expressed as

$$V(S) = \max\left(V^C, V^{NC}\right) \tag{4}$$

where

$$\begin{aligned} V^C(S) &= \max_p \left\{\Pi(p, \hat{m}) - \psi_i + \beta E_{S'|S}\left[V(S')\right]\right\} \\ S' &= \{p, \hat{m}, \ln \Delta M', \pi(\Omega'), \psi_i'\} \end{aligned} \tag{5}$$

and

$$
\begin{aligned}
V^{NC}(S) &= \Pi\left(\frac{p_{-1}}{1+\pi(\Omega)}, \hat{m}\right) + \beta E_{S'|S}\left[V(S')\right] \\
S' &= \left\{\frac{p_{-1}}{1+\pi(\Omega)}, \hat{m}, \ln \Delta M', \pi(\Omega'), \psi_i'\right\}
\end{aligned} \tag{6}
$$

and

$$
\hat{m} = m_{-1}\left(\frac{1+\ln \Delta M}{1+\pi(\Omega)}\right)
$$

.

Expectations are taken over the exogenous variables using conditional distributions. I assume that nominal money growth follows a stationary autoregressive process independent of firm i's decision and denote the conditional distribution of this process as $\Phi_1(\ln \Delta M' | \ln \Delta M)$. The adjustment cost process is modeled as an autoregressive log-normal process subject to idiosyncratic shocks:

$$
\log\left(\psi_{i,t}\right) = \mu + \rho \log\left(\psi_{i,t-1}\right) + \varepsilon_{i,t} \tag{7}
$$

The structural parameters of interest in this study are those describing the adjustment cost process: the mean, persistence, and standard deviation of the innovations to the process, $\{\mu, \rho, \sigma_\varepsilon\}$. In this model, firms will make their pricing decisions based upon not only the current realization of the adjustment cost, but also upon expectations of future realizations, where the conditional distribution for the adjustment cost process is $\Phi_2(\psi_i'|\psi_i)$. The parameters of this process will affect both the discrete choice decision as well as the optimal price decision conditional upon the firm deciding to change its price. The adjustment cost process also may affect the rational expectations equilibrium through the effect on pricing decisions in the economy and potentially through the inflation forecast.

2.2 Rational expectations equilibrium

Solving for the equilibrium of the model entails selecting an appropriate inflation forecast rule based upon the given information set. This forecast rule will be expressed as a law of motion for inflation based upon current state variables and potentially other lagged variables of the economy. Using this forecast rule, the firm will solve the optimization problem in (4) by determining a policy function for the updating of prices: $p = f\left(p_{-1}, m_{-1}, \ln \Delta M, \pi\left(\Omega\right), \psi_i\right)$.

The recursive equilibrium of the model consists of the functions V and f along with the inflation forecast rule, $\pi\left(\Omega\right)$, such that (i) V and f solve the firm's optimization problem and (ii) $\pi\left(\Omega\right)$ matches the actual inflation resulting from firms' pricing decisions in a simulated economy.

2.3 Computational framework

Due to the discrete nature of the adjustment decision combined with potential serial correlation in the adjustment cost process, the derivation of an analytic solution to the firm's problem is not feasible. I solve the model numerically using value function iteration, which yields policy functions dependent on the state variables.[5] The implications of the solution are investigated via simulation. The numerical results are then later used in the estimation.

All components of the state space take values in a discrete set. The bounds of the relative price state space are set wide enough to include all optimal price decisions, and the space is divided into a grid with 1% increments. The autoregressive process for nominal money growth is transformed into a discrete-valued Markov chain following Tauchen (1986). There are three points in the state space for this variable, and the values used to parameterize this process will be described in the data section below. The adjustment cost process is also specified as a first-order Markov transition matrix using the same method. The adjustment cost state space consists of five discrete points bounded within two standard deviations of the mean.

As for the inflation forecast, I construct a transition process similar to the one used by Krusell and Smith (1998). Ideally, one should model the inflation forecast rule as a function

[5] Similar solution methods are used in Rust (1987) and Cooper, Haltiwanger and Power (1999) in the investment literature.

of the relative prices and price-adjustment costs of all agents in addition to the past values of nominal money growth and the real money supply. Computationally, however, inclusion of so many state variables quickly leads to the curse of dimensionality. This is caused by the fact that value function iteration requires the solution of policy functions for every combination of states in the state space. In order to avoid this complexity, I will assume that agents only have a limited amount of information to use for forecasting, but they will act rationally based upon the information given to them.

To further simplify the computational solution, the inflation forecast will be expressed as a forecast of the real money supply. Since agents know the current nominal money growth rate and the previous period's real money supply, a forecast of the current real money supply is a sufficient statistic for the current inflation rate.

From this description, a natural starting point for a forecast of current real money supply is a simple linear projection using the current nominal money growth rate and lagged real money supply.

$$m_t = \alpha_0 + \alpha_1 \ln \Delta M_t + \alpha_2 m_{t-1} \tag{8}$$

The forecast rule is summarized by the coefficients of the linear equation: $\Gamma = \{\alpha_1, \alpha_2\}$. The first coefficient, α_0, is determined by the normalization of the real money supply to be mean 1.[6] Based upon this equation, a transition matrix can be calculated as described for the exogenous variables in the model. This transition matrix will then allow firms to calculate expectations of the real money supply, and therefore inflation. Referring back to the state space, S, firms now have transition matrices or transition functions for all of the variables in the state space. In the simulation, the actual updating of the real money

[6] This normalization is used to ease comparison with the real money supply in the U.S. data. A subsequent normalization is undertaken in the model so that the real money supply has a mean equal to the symmetric equilibrium of the economy in the absence of price-adjustment costs:

$$m = \left(\frac{\theta - 1}{d\theta}\right)^{\frac{1}{\gamma - 1}}$$

supply from one period to the next will be based upon the exogenous nominal money growth rate and the inflation rate resulting from the pricing decision of firms. In equilibrium, the forecast and the simulated outcome will be identical.

The solution of the rational expectations equilibrium can be derived in several different ways. First, the structural parameters of the model could be selected along with an initial guess of the forecast parameters, Γ_0. Then, an iterative procedure can be initiated to find a fixed point of Γ. After one solution and simulation of the model, the forecast regression can be run on the simulated data to produce Γ_1, which can then replace Γ_0 before resolving the model. This process continues until a fixed point is reached. It is possible that there could be multiple fixed points for any given set of structural parameters. A second method would be to preselect the forecast rule based upon known beliefs of the agents. Then, a set of structural parameters can be estimated such that the forecast rule matches up with the simulated data.

It is helpful to think of the two solutions in terms of linking three key components: 1) data; 2) beliefs of agents; 3) simulated data using optimal policy rules from the model incorporating the beliefs of the agents into the decision problem. The first solution method solves for the beliefs of agents such that the beliefs match the resulting simulated data, where the structural parameters other than the forecast parameters are preselected. The second method uses data to gather information on the beliefs of agents, and then the structural parameters are selected to match the given beliefs against the resulting simulated data. The first method is useful in illustrating the model solution method and for comparison purposes when the structural parameters have been calibrated from data. The second method described is more applicable for the current problem due to the fact that many of the structural parameters, such as those governing the adjustment cost process, are unknown, whereas data are available for parameterization of the forecast rule.

3 Empirical evidence

To calibrate the exogenous nominal money growth process and to provide an forecast of the real money supply, I use data from the U.S. economy. The time frequency is annual, and data from 1959 through 1991 is used. This date range is selected because the velocity of M2 has been stationary over this period.[7] This implies that the real money supply, once adjusted for real output growth, has also been stationary over this period. The model described above has no output growth and the real money supply is stationary. Annual, rather than quarterly, data are used in an effort to capture the general overall relationships between money, output and prices.

The nominal money growth process is assumed to follow an exogenous autoregressive process. The parameters of this process are estimated via an AR(1) regression. The autoregressive coefficient is 0.423 (0.164) and the variance of the residuals is 0.00059.[8] One drawback of this characterization of the monetary process is that the fit of this regression is very low: the R^2 is 0.19. Additional lags are not statistically significant. This autoregressive process is then transformed into a discrete-valued Markov process.[9]

The assumed forecast equation used by the firms, expressed in (8), is estimated from the data to provide the beliefs of agents in the model. The estimated coefficients are listed in Table 1.

These coefficients are assumed to capture the beliefs of firms regarding the forecast of inflation in the model, expressed here via a forecast rule for the current real money supply. As with the money growth process, this forecast rule is also transformed into a Markov process to provide firms with transition matrices to be used in the calculation of future expectations.

[7] See Hallman, Porter and Small (1991).

[8] The standard error of the autoregressive coefficient is in parentheses.

[9] The state space for nominal money growth consists of three discrete points: $\{0.46\%, 4.47\%, 8.48\%\}$. This simple representation is used to keep the overall size of the state space managable.

Regarding the other parameters of the model, I assume that the annual discount rate for the firm, β, is 0.95. The scalar on the cost function, d, affects the steady state level of output, but not the growth levels that will be the focus of the estimation. As such, d is not identified given our data. The value of d is set at 0.5.

4 Estimation

In order to determine if the model is capable of explaining the relationship between money, output, and prices, the structural parameters will be estimated using an indirect inference procedure proposed by Gourieroux, Monfort and Renault (1993). This procedure consists of estimating auxiliary parameters from U.S. data and from simulated data from the rational expectations equilibrium of the model. In my estimation, the auxiliary parameters are coefficients of the firm's forecast-rule regression and coefficients of a regression of inflation on lagged inflation, money growth, and output growth. The criterion function is a weighted average of the sum of squared errors from each equation, and the coefficients are estimated by OLS. The structural parameters are then estimated by matching the two sets of OLS estimates (from the data and the simulation of the model) according to a minimum distance function. The benefits of this procedure are that it provides a convenient, indirect formulation of moments relating to unobserved variables, such as price adjustment costs in this example, and the resulting estimates have well-behaved asymptotic properties when the criterion function and the auxiliary parameters are well chosen. The parameters to be estimated are those governing the adjustment cost process $\{\rho, \mu, \sigma_\varepsilon\}$, the elasticity of substitution across goods, θ, and the curvature parameter for the firm's cost function, γ. This parameter vector is denoted as $\phi \equiv \{\gamma, \theta, \rho, \mu, \sigma_\varepsilon\}$.

4.1 Auxiliary parameters

The selection of auxiliary parameters, hence referred to as moments, is crucial in order to obtain meaningful estimates of the structural parameters. If the moments from the simulated data are not sensitive to changes in the structural parameters, then little information can be gained via estimation. It is through the selection of "good" moments that identification is achieved. One statistic that is a useful indicator concerning the selection of moments is the standard error of the estimated parameter since it is a function of the derivative of the moment with respect to the structural parameter.[10]

A total of five moments are selected to estimate the five structural parameters. The first two moments are the regression coefficients used in the solution for the rational expectations equilibrium: $\{\alpha_1, \alpha_2\}$ from (8). Since these coefficients must match those obtained from the data used to specify the beliefs of agents, they are natural moments to include in the estimation. The second set of moments come from a regression of inflation on lagged inflation, lagged nominal money growth, and lagged output growth.[11]

$$\pi_t = \delta_0 + \delta_1 \ln \Delta M_{t-1} + \delta_2 \Delta Y_{t-1} + \delta_{3\pi_t} \pi_{t-1}$$

This regression is chosen to capture the dynamics of inflation as it relates to changes in the main aggregate variables of the economy. The forecast of inflation used in solving the rational expectations equilibrium does not imply a specific dynamic relationship between inflation and lagged aggregate variables. Including these coefficients in the estimation introduces this important dynamic aspect, and the estimation results will reveal if a model of state-dependent pricing is capable of reproducing the inflation dynamics from the data.

4.2 Indirect inference

From this specification, the auxiliary parameters are $\lambda = \{\alpha_1, \alpha_2, \delta_1, \delta_2, \delta_3\}$. Following Gourieroux et al., the criterion is specified as $Q_T(y_T, x_T, \lambda)$, where $y_T = (y_1, ..., y_T)$

[10] For a more detailed description of this estimation method, see the discussion of indirect inference in Willis (2000).

[11] In the simulated data, aggregate output is calculated by summing up the demands for each firm's production.

$$Y_t = \sum_{i=1}^{n} \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} m_t$$

represents the endogenous series for inflation, real money balances, and output and x_T $= (x_1, ..., x_T)$ represents the exogenous nominal money growth data. The criterion function in this case is the sum of the negative sum of squared errors from the two regression equations.

Denote $\hat{\lambda}_T$ as the solution to the maximization of the criterion function

$$\hat{\lambda}_T = \arg\max_{\lambda} Q_T\left(y_T, x_T, \lambda\right)$$

For a given set of structural parameters, ϕ, I construct a simulated dataset of 200 firms over 200 years based upon independent draws of the innovations to the exogenous processes and on initial values z_0^s.

$$\left[y_t^s\left(\phi\right), x_t, t = 0, ..., T\right]$$

For each simulation dataset, I maximize the same criterion function, replacing the observed data with the simulated data.

$$\hat{\lambda}_T^s\left(\phi\right) = \arg\max_{\alpha} Q_T\left(y_T^s\left(\phi\right), x_T, \lambda\right) \tag{9}$$

The indirect estimator of ϕ is defined as the solution to the following minimization problem

$$\hat{\phi} = \arg\min_{\phi} \left[\hat{\lambda}_T - \hat{\lambda}_T^s\left(\phi\right)\right]' \hat{\Omega}_T \left[\hat{\lambda}_T - \hat{\lambda}_T^s\left(\phi\right)\right]$$

where $\hat{\Omega}_T$ is a positive definite weighting matrix.

The weighting matrix is calculated via the bootstrap method. After an initial set of estimates are obtained using the identity matrix as an initial weight matrix, the model is simulated 200 times using different draws for the innovations to the exogenous processes. After each simulation, the auxiliary parameters are calculated and stored. The weighting matrix is then calculated as the inverse of the covariance matrix of the 200 sets of auxiliary parameters. A second estimation is then computed using this weighting matrix

4.3 Results

The results of the two regressions using data from the U.S. economy are located in the first column of Table 2. For the forecast regression, the real money supply process is very persistent, with a coefficient of 0.986 on the lagged value of real money. Increases in nominal money growth have a conditionally positive relationship with the real money supply. Regarding the inflation dynamics, inflation is also very persistent in the data. The autoregressive coefficient is 0.945. Nominal money growth has an insignificant negative conditional correlation with inflation, while output growth has a marginally significant positive relationship with inflation.

The structural parameters for the firm's optimization problem are estimated using the indirect inference procedure described above, and the results are displayed in Table 3. The estimate of the cost parameter, γ, is 1.25. This indicates that firms face diminishing returns to production. The CES parameter estimate, θ, translates to a markup of 94 percent. This estimate is much higher than recent findings of markups on the order of 20 to 30 percent.[12]

The estimates of the adjustment cost process are expressed in log-normal terms. The median adjustment cost faced by firms is 7.3 percent of revenues. This is not, however, the average cost paid by firms. As illustrated in Table 4, firms facing lower costs adjust prices more frequently. The first column lists the five costs in the discrete state space. The costs are expressed as a percentage of steady state revenue. The second column displays the distribution of the adjustment costs across firms in a single simulation dataset. The third column lists the fraction of firms that adjust when faced with the respective adjustment cost. This fraction is 0.37 for firms facing the lowest adjustment cost. As the adjustment cost rises, the fraction of firms adjusting falls. The highest cost paid is 43.07 percent of revenue, but only 0.3 percent of the firms facing that adjustment cost choose to adjust.

The mean adjustment cost paid is 4.9 percent of revenue. In comparison to results from

[12] See Domowitz, Hubbard and Petersen (1988)

a partial equilibrium analysis of the magazine industry in Willis (2000), this estimate is higher than average cost estimate for that industry (4.04 percent of revenues). Results from the supermarket literature find costs in the range of 0.7 to 1.9 percent of revenues. The persistence parameter of the adjustment cost process is estimated as 0.49, which is lower than the estimate of 0.68 for the magazine industry.

5 Impulse response

In order to assess the implications of the structural estimates, I examine the impulse response functions of the aggregate variables in response to a monetary shock. To capture the dynamic effects of a monetary policy shock, I use a modified version of the identification strategy proposed by Bernanke and Blinder (1992). A VAR composed of nominal money growth, detrended output growth, and inflation is estimated from U.S. aggregate data under the restriction that nominal money growth is an exogenous autoregressive process.[13] The frequency and time range of the data are the same as that used in the estimation procedure above. The impulse response function is then generated by decomposing the errors to identify the effect of the monetary policy shock on the variables of the system.

Figure 1 displays the impulse response to a 1% innovation to nominal money growth based upon U.S. data. The response of output growth matches the results from Bernanke and Mihov (1998), which uses the federal funds as the indicator of monetary policy, in that output growth initially falls in response to the shock, then growth peaks after approximately 1 year before falling back to the steady state growth rate. The inflation impulse response is also similar to that reported by Bernanke and Mihov. Inflation decreases over the first year, then increases to a level above the steady state before exponentially falling back to the steady state level.

[13]Bernanke and Blinder argue that the federal funds rate is a better predictor of major macroeconomic variables than M2, but in their analysis, M2 does a reasonable job of predicting movements in the real economy. Since the effect of the nominal money supply on the monopolistic competition model is very transparent, I choose to use M2 as the monetary policy indicator rather than the federal funds rate.

An impulse response function from the model is generated via simulation. Based upon a sample of 200 firms simulated for 200 periods, aggregate data series are computed. A VAR is then calculated using the same identifying restrictions as for the U.S. data. Figure 2 displays the impulse response functions. Since firms observe the current nominal money growth rate before selecting prices, they have the opportunity to react immediately. This is illustrated by the initial increase in inflation to 4.9%. The higher inflation does not completely offset the money growth, so the real money supply increases, translating to an increase in output through higher aggregate demand. In the second period, inflation is higher than money growth. Thus, real money balances are falling in the period, resulting in negative output growth. These responses are very similar to the response functions generated by Dotsey, King and Wolman.

After the 5th period, money growth has returned to its steady state level, but inflation remains higher than steady state until almost the 10th period. Output growth gradually increases to the steady level after reaching its lowest level in the 3rd period.

The two impulse responses do not appear to have much in common. The surge in output growth occurs in the first period in the model, but in the second period in the data. Inflation only responds with a long lag in the data, contrary to the immediate response in the model. If the informational assumptions were altered in the model to delay adjustment, this would only lead to a larger initial output surge as the real money supply would initially be higher.

6 Alternative forecast specifications

Thus far, the inflation forecasts of firms have been based on a simple forecast regression of the real money supply on current nominal money growth and lagged real money. This forecast rule may be too limited in the sense of not providing an accurate enough forecast. In order to evaluate whether firms would benefit from additional information, the forecast rule can be expanded to be a function of additional variables that might increase the forecast accuracy. The obvious benefit is that firms will have better expectations of inflation as they make their pricing decision. The cost of augmenting the forecast function is that this adds more variables to the state spaces, increasing the dimensionality of the computational solution.

As an initial specification test, the current inflation forecast rule (Model A) will be compared against a forecast rule that also incorporates the lagged value of nominal money growth into the information set (Model B). In order to compare these forecast rules, the structural parameters are set at the values produced in the estimation. The solution of the fixed point of the forecast rule is solved for each rule, and the forecasts are evaluated based upon calculating the sum of squared inflation forecast errors.

The resulting forecast rules are displayed in Table 6. The forecast rule solving for the REE in Model B predicts a real money process with less persistence, indicated by the lower coefficient on m_{t-1}. Increases in current nominal money growth rate have a positive correlation with real money balances, as in Model A. An increase in the lagged value of nominal money growth has a negative correlation. This is likely due to the expectation that there will be a large number of firms who did not initially adjust in response to the higher nominal money growth. One period after the increase, some of these non-adjusting firms will decide to change prices either because they have received a lower adjustment cost or because their relative price has now fallen too low. Controlling for the relationship with current nominal money growth, this surge in adjustment (i.e. increase in aggregate price level) will be associated with a decline in the real money supply.

Comparing the two forecast rules, the sum of squared forecast errors associated with Model B is slightly lower than Model A. This is not surprising since the firms now have more information on which to base their forecast. Figure 3 plots the expected inflation and realized inflation for 100 simulated periods for Model A in the top panel and Model B in the lower panel. The realizations of the exogenous processes for nominal money growth are identical for both models. While both graphs look similar, there is a noticeable improvement in the forecast of inflation during periods when the inflation rate is increasing. In particular, during the upswings of inflation around periods 30 and 63, the inflation forecast is much more accurate in Model B. Future research will further explore the benefit of expanded forecast rules, potentially including additional lags of money growth and real money as well as moments from the distribution of prices, which may help predict a surge in adjustment caused by many firms having low relative prices.

7 Conclusion

The objective of this paper is to determine if the standard model of monopolistic competition is capable of explaining the dynamic relationship between money, output, and prices observed in U.S. aggregate data. To answer this question, a rational expectations equilibrium (REE) for the model must be found. This is accomplished by establishing a simple forecast rule for inflation under the assumption of bounded rationality for the agents, similar to the assumptions used in Krusell and Smith (1998). Given this forecast rule, the beliefs of agents are obtained from U.S. data, and then the structural parameters of the model are estimated by an indirect inference procedure. This estimation procedure requires that the REE condition of the model is satisfied along with matching inflation dynamics from the data.

The estimated price adjustment cost process is moderately persistent. The mean adjustment cost paid by firms is over 5 percent of revenues, a value much higher than partial-equilibrium estimates for various industries. The estimated price markup for firms, over 90 percent, is also much higher than recent estimates in the literature. In an additional comparison of the model and the data, impulse response functions reveal different dynamics for inflation and output in response to a monetary shock.

Future research will examine the benefits of an expanded forecast rule. The inclusion of additional information to assist firms in forming inflation expectations may result in an improved REE solution. Early results indicate that the addition of lagged nominal money growth to the forecast rule leads to a reduction in the inflation forecast error.

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Table 1: Coefficient Estimates of Forecast Rule for Real Money

	Coefficients on		
constant	$\ln \Delta M_t$	m_{t-1}	R^2
-0.008	0.477	0.986	0.71
(0.126)	(0.157)	(0.122)	

Table 2: Coefficient Estimates of Auxiliary Parameters

	Data (1)	Model (2)
Forecast regression		
$\Delta \ln M_t$	0.477	0.502
	(0.157)	
m_{t-1}	0.986	0.991
	(0.122)	
R^2		
Inflation dynamics		
$\Delta \ln M_{t-1}$	-0.086	-0.037
	(0.1)	
$\Delta \ln Y_{t-1}$	0.223	0.239
	(0.12)	
π_{t-1}	0.945	0.692
	(0.104)	
R^2		

NOTE: Standard errors are in parentheses.

Table 3: Estimates of Structural Parameters

γ	1.249
	(0.042)
θ	2.069
	(0.027)
σ_ε	1.185
	(0.044)
μ	-2.620
	(0.089)
ρ	0.534
	(0.010)

Table 4: Distribution of Adjustment Costs

adjustment cost (% of "steady state" revenue)	realizations	fraction adjusting
1.23	4977	0.37
2.99	9015	0.30
7.28	11911	0.17
17.70	9097	0.05
43.07	5000	0.003
mean adjust cost paid = 4.90 %		

Table 5: Alternative Forecast Rules for Real Money

	Forecast coefficients			sum of squared
	$\ln \Delta M_t$	m_{t-1}	$\ln \Delta M_{t-1}$	forecast errors
Model A	0.477	0.986	–	0.037
Model B	0.221	0.877	-0.151	0.035

Figure 1: Impulse Response Functions based upon U.S. Data

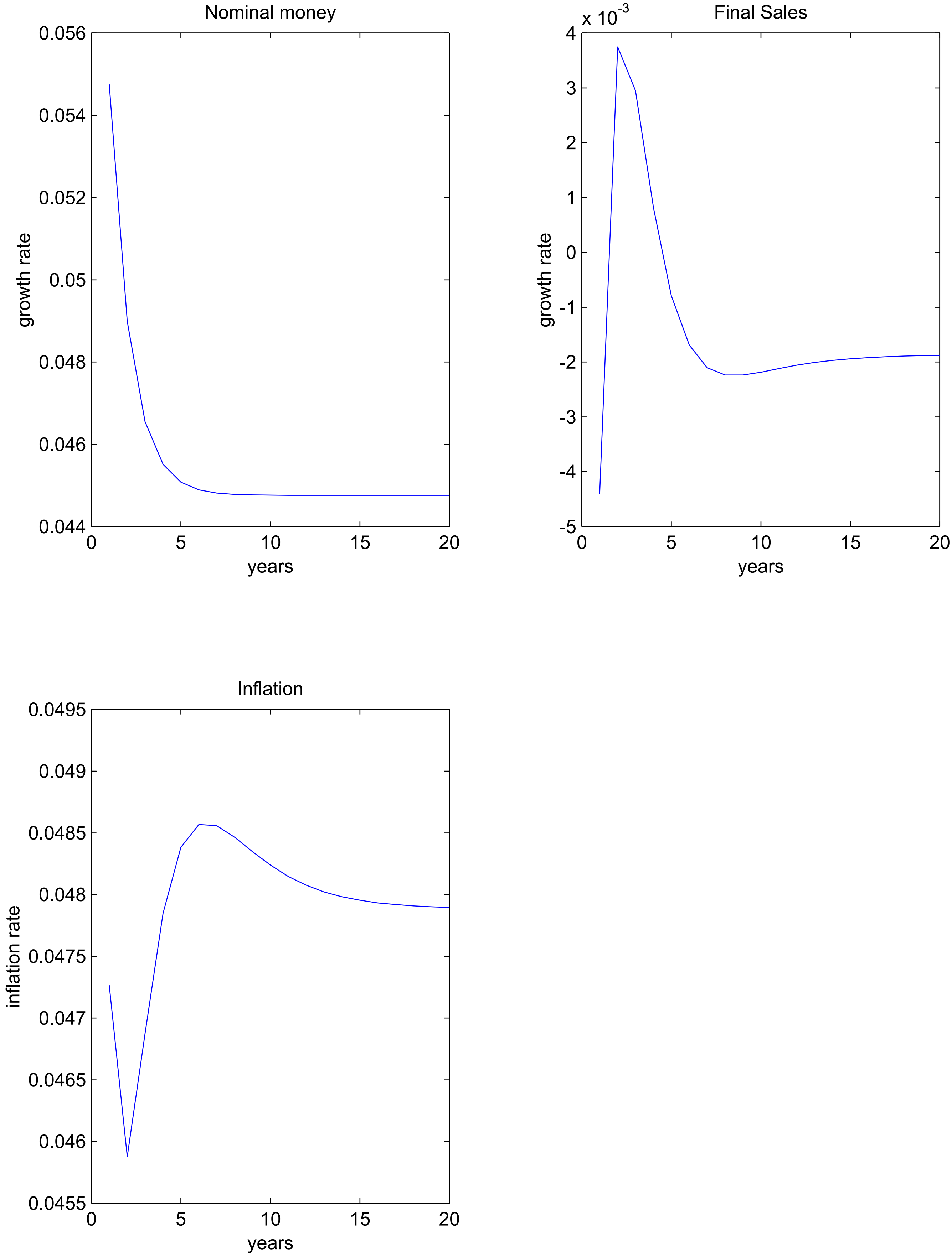

Figure 2: Impulse Response Functions Based Upon Simulated Data

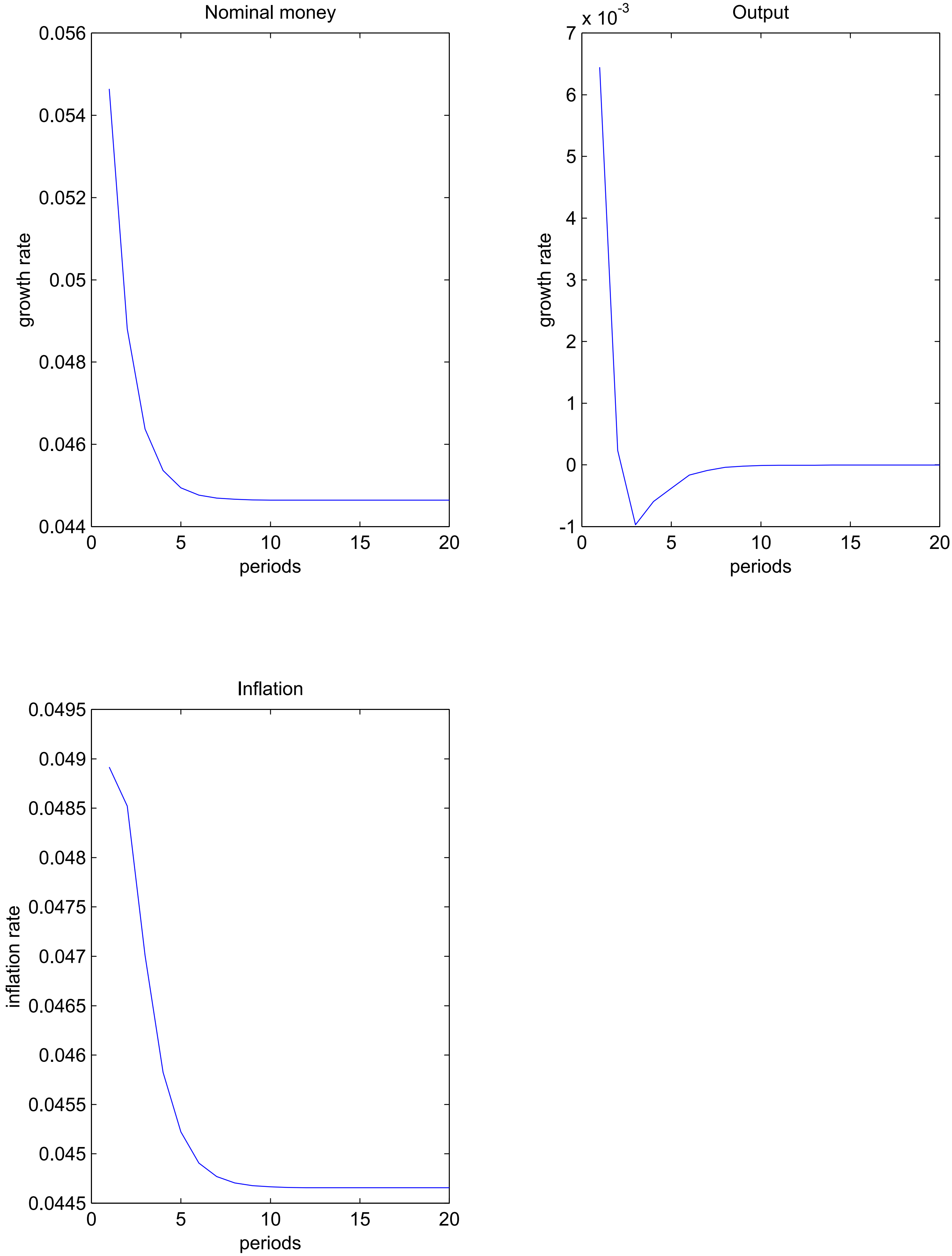

Figure 3: Comparison of Inflation Forecasts