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# Adverse Selection and the Market for Health Insurance in the U.S.

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## Abstract

Several studies have examined the market for employer-provided group health insurance in the United States. The theoretical side of the literature has struggled with the existence of equilibrium due to the adverse selection problem inherent in the sale of health insurance. The empirical side of the literature has had trouble estimating the price elasticity of demand for health insurance, in part, because many of the empirical papers are not based upon any of the theoretical work. The purpose of this paper is to present a screening model of the market for health insurance that will attempt to address both problems. The model will discuss the existence of equilibrium and lends itself more easily to empirical applications than previous models. Unlike Rothschild and Stiglitz (1976) or Wilson (1977), I can show that both a unique separating equilibrium and multiple pooling equilibria exist in my model under the assumption that a worker's health type is private information.

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## I. Introduction

The market for employment-based group health insurance in the United States has several interesting features. One of the most interesting features is the presence of adverse selection inherent in the sale of insurance. Because a consumer's health "type" is unknown to the insurance company, it is difficult for insurance companies to price policies appropriately. Incorrect pricing could potentially lead to a market failure. This implies that adverse selection creates problems when attempting to prove than an equilibrium exists in this market. Another important feature of this market is the tax subsidy for health insurance.

How does this tax subsidy work? Any money spent on a group health insurance plan by employers can be deducted as a business expense. In addition, this money is not taxed as income to the employee. Thus, employees can choose between buying health insurance with pre-tax dollars or buying other goods with after-tax dollars. Obviously, this tax subsidy has a profound effect on the demand for health insurance. Evidence that health insurance leads to the over-consumption of medical care implies there is concern that subsidizing the purchase of health insurance is one factor leading to rising nominal health care costs.

The classic papers in the theoretical literature on the market for health insurance include Phelps (1973), Goldstein and Pauly (1976), Rothschild and Stiglitz (1976), and Wilson (1977). Phelps (1973) carefully described the consumer optimization problem associated with the purchase of health insurance. Unfortunately he only presented a partial equilibrium model, so there was no mention of the production of insurance policies or the existence of equilibrium. Phelps (1973) also allowed consumers to buy insurance policies directly, so there was no discussion of the impact of the provision of insurance through the employer.

Goldstein and Pauly (1976) applied the model of local public good provision presented in Tiebout (1956) to the employer provision of group health insurance. In their model, employees vote on the insurance policy they are provided and the policy chosen is that preferred by the median voter / employee. One implication of their model is that, in equilibrium, the employees of each firm will be homogeneous with respect to their insurance preferences - a separating equilibrium. Although Goldstein and Pauly (1976) discussed some of the necessary assumptions for the existence of equilibrium, they did not formally state and prove an existence theorem. They also pointed out several cases where equilibrium may not exist. Like Phelps (1973), Goldstein and Pauly (1976) ignored the production of insurance in their model.

Rothschild and Stiglitz (1976) and Wilson (1977) presented "screening" models that have consumers purchasing policies from different insurance companies. Like Goldstein and Pauly (1976), both Rothschild and Stiglitz (1976) and Wilson (1977) discussed the existence of equilibrium in their models. They describe both pooling and separating equilibrium. Unfortunately, they also had problems proving that either of these equilibria exists. In addition, they excluded employer provision of health insurance from their models.

The model I present in this paper will extend this analysis by combining the careful description of insurance policies and consumer behavior presented in Phelps (1973) with the discussion of the production of policies and the existence of equilibrium presented in Rothschild and Stiglitz (1976), Goldstein and Pauly (1976), and Wilson

(1977). In my model, consumers purchase policies through their employers. This allows me to examine the behavior of the three major players in the health insurance market (consumers, employers, and the insurance company) in the same model. Another advantage of this model is that it suggests a specific empirical framework for the calculation of the price elasticity of demand for health insurance.

An accurate calculation of the price elasticity of demand for health insurance is required to estimate the effects of the elimination (or reduction) of the tax subsidy. The empirical literature in this area has come up with a wide range of elasticity estimates.[1] This is true in part because there is not a close connection between much of this empirical work and the theoretical models I discussed above. I hope that the model that I present in this paper will lend itself more easily to empirical applications than the previous models.

The remainder of this paper is organized as follows: In section II, I will present the model. In section III, I will discuss an empirical application of the model. The appendices will describe some of the details of the existence proof and lemmas.

## II. The Health Insurance Model

### 1. Overview of the Model

This model is best seen as a game with two stages. In Stage One there are an infinite number of workers who inelastically supply one unit of labor to one of

[1] The papers I am referring to include Goldstein and Pauly (1976), Long and Scott (1982), Talyor and Wilensky (1983), Holmer (1984), and Phelps (1986b). Their price elasticity estimates range from the - .16 found in Holmer (1984) to the - 1.81 found in Phelps (1986b).

q = {1, …, Q} firms.[2] Every worker is of a specific health "type." Although the set of health types is common knowledge, each worker's health type is not directly observed either by their employer or the insurance company. Each worker is one of a finite number of ages (which is common knowledge). I will vary the amount of information an employee's age conveys about their health type and examine how the set of equilibria changes as this information changes. For example, in the zero information / zero correlation case I will assume that a worker's age is not correlated with their health type. This implies that age conveys no information about health type. The perfect information / perfect correlation case will assume that a worker's age is perfectly correlated with their health type. Therefore, knowing a worker's age implies full knowledge of their health type. I will show that both a unique separating equilibrium and multiple pooling equilibria exist in the zero information case. In the perfect information case there is no adverse selection problem, because an employee's health type is no longer private information. This implies that the insurance company can perfectly price discriminate between health types and a unique equilibrium exists.

Firms produce the numeraire with a constant returns to scale technology. I will also assume that this is a perfectly competitive economy so the price of one unit of labor is given. The numeraire firms then compete for labor by offering different portfolios of insurance policies. Workers base their decision on where to work by evaluating the utility they would receive from the particular set of polices offered by each numeraire firm.

[2] Here Q is a strictly positive integer greater than 1.

In Stage Two I will assume that different states of the world create different levels of illness. Each numeraire firm has a Borel subset of the space of risk-averse employees. These employees are all endowed with the same strictly positive level of pre-tax income from supplying labor in Stage One. Because employees don't know with certainty which state of the world will occur, they use some of their endowment to purchase a health insurance policy from the subset of insurance policies offered by their employer to protect their income from random losses due to illness. There is one risk-neutral insurance company that uses premium payments from the employees to produce health insurance policies using a constant returns to scale technology. Finally, since this is a perfectly competitive economy, employees and the health insurance company take the price of insurance policies as given.

In order for an equilibrium to exist in this model, the insurance company must be able to charge a premium for each policy that allows them to at least break even. This requires that the insurance company be able to acquire some information about each employee's health type. How does the insurance company acquire this private information in the zero information case? Their information comes from the policy chosen by each employee in Stage Two out of the set of contracts offered in Stage One. There are certain sets of contracts that provide the insurance company with a signal about each employee's health type.

Consider a Pooling Equilibrium, where each health type buys the same policy. Each numeraire firm will also hire the same proportion of workers from each health type, so the distribution of health types in every firm is identical to the distribution of health types in the population. The insurance company can use actuarial data to estimate the probability of each state of the world for the population and apply these population estimates to each firm. This allows the insurance company to charge the correct premium on average.

The optimal behavior for employees in this two-stage game is found by using backward induction, so I will describe Stage Two, then Stage One. Finally, I will discuss the existence of equilibrium in the model.

## 2. Stage Two – The Insurance Market

### A. States of the World

Suppose that there are two states of the world. In state one no health care is required and in state two $h \in \mathbb{R}_{++}$ units of medical care is required at a price of $p \in \mathbb{R}_{++}$ units of numeraire.

### B. Employees

Denote the measure space of employees in the economy by $(\mathcal{I} = [0, 1], \beta(\mathcal{I}), \lambda)$. Here $\beta(\mathcal{I})$ represents the Borel $\sigma$-field of subsets of $\mathcal{I}$, and $\lambda$ is the Lebesgue measure. Denote a typical Borel subset of $\mathcal{I}$ by $I \in \beta(\mathcal{I})$. In Stage Two I will focus on the employees in numeraire producing firm q. Firm q hires a Borel subset, $I_q$, of the continuum of workers in the economy.

### Health Types

Employees can be divided into a finite set of different health types. I will denote the set of health types by i = {1, ..., T} with T a strictly positive integer greater than 1. A health type is defined by its probability of state two occurring. For example, employee i faces the probability $\pi_i$ of state two occurring and m < n implies $\pi_m < \pi_n$. I will also assume $0 < \pi_1 < \pi_T < 1$. This implies that type 1 faces the lowest expected medical expenses and type T faces the highest expected medical expenses.

**Age**

Each employee is one of a finite number of ages. Denote employee i 's age by $a_i \in \mathbb{R}_{++}$ and the set of ages by $A = \{a_1, \ldots, a_T\}$. In the zero information case, an employee's age is not correlated with their health type. In the perfect information case, as employee's age is perfectly correlated with their health type. In this case, I will assume that health type i has age $a_i$ and $a_m < a_n$ for $m < n$.

**Insurance Policies**

In order to describe what an insurance policy (the differentiated good) looks like in this context, some notation is needed:

$C_j \in [0, 1]$ is the coinsurance rate provided by policy j.

$D_j \in [0, p * h]$ is the deductible provided by policy j.[3]

$B_j$ is the benefit payment provided by policy j.

$R_j \in \mathbb{R}_{+}$ is the premium for policy j.

An insurance policy's level of coverage is defined by its coinsurance rate and its deductible.[4]

Define the benefit payment of policy j as the amount of numeraire the insurance company would pay in medical expenses if state two occurred:

[3] The existence of equilibrium in this model is unaffected by my choice of insurance policy characteristics. Therefore I could add HMO-style insurance characteristics, such as the degree of choice among providers, without losing my existence result.

[4] The deductible is a fixed amount a consumer must pay toward medical bills each year before their insurance company begins to pitch in. The coinsurance rate refers to the percentage, C, of the medical bill paid by an insured consumer after they have exceeded their deductible. The insurance company pays the other (1 - C) percent of the bill above the deductible.

$$B_j = (1 - C_j) * (p * h - \min (p * h, D_j)).$$[5]

It should be clear that $B_j \in [0, p * h] \subset \mathbb{R}_+$.

It will be more convenient to work with the benefit payment of policy j, $B_j$, to describe the level of coverage provided by policy j. However, there is not a one-to-one relationship between a (coinsurance rate, deductible) pairing, (C, D), and a benefit payment B. It is easy to derive two different (coinsurance rate, deductible) pairings that produce the same benefit payment. I will later assume that an employee's utility depends only on their income, so that they will be indifferent between different (coinsurance rate, deductible) pairings that produce the same benefit payment. This is because these pairings will produce the same level of residual income for a given premium.

**Endowments**

I will assume that all health types have an identical marginal product, denoted by $MP \in \mathbb{R}_{++}$, in terms of their production of numeraire. Because workers are paid their marginal product in Stage One, each health type enters Stage Two with an identical endowment of pre-tax income, denoted by $W^p = MP \in \mathbb{R}_{++}$. Assume $0 < p * h < W^p$.

**Residual Income**

Employee i is endowed with a pre-tax income of $W^p$ from her work in Stage One. Denote employee i 's marginal tax rate by $t \in (0, 1)$ and her after-tax income by $W^a$. Due to the tax subsidy on employer-provided health insurance, employee i pays her insurance

[5] The term $(p * h - \min (p * h, D_j))$ represents any medical expenses above the deductible. For example, if $\min (p * h, D_j) = p * h$, then the medical expenses fall below the deductible and the employee must pay for these medical expenses out of pocket. If $\min (p * h, D_j) = D_j$, then $(p * h - \min (p * h, D_j)) = (p * h – D_j)$. This represents the portion of the medical expenses (above the deductible) that the insurance company applies the coinsurance rate to.

premium $R_j$ with pre-tax dollars. She then pays for her remaining medical bills with after-tax dollars. This implies that employee i 's after-tax income can be written as:

$$W^a = (1 - t) * (W^p - R_j) = [\ (1 - t) * (W^p) + (t * R_j)\ ] - R_j.$$

To simplify this notation, define $W = [\ (1 - t) * (W^p) + (t * R_j)\ ]$. This implies that employee i 's after-tax income can be defined as:

$$W^a = W - R_j.$$

Define employee i 's residual income in each state with insurance policy $(B_j, R_j)$ as follows:

| **State** | **Residual Income** |
|---|---|
| 1 | $I_{i1} = W - R_j$ |
| 2 | $I_{i2} = W - R_j - p * h + B_j$ |

Notice that employee i will be fully insured whenever $B_j = p * h$. When this is true, in either state of the world employee i will have residual income equal to $W - R_j$.

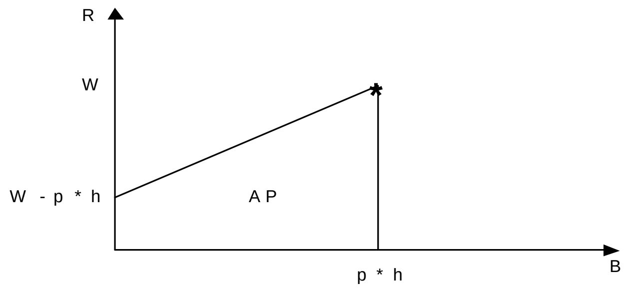
**The Allowable Set of Insurance Policies**

I will restrict the set of policies considered in this model to those that generate non-negative levels of consumption. Define the allowable set of policies as follows:

$$AP = \{\ (B_j, R_j)\ 0\ [0, p * h] \times \mathbb{R}_+ \ \mid\ W - R_j \geq 0 \ \text{ and } \ W - R_j - p * h + B_j \geq 0\ \}.$$[6]

Here is a graph of AP:

[6] It should be clear that AP is a compact set.




Define $AP_q$ to be the **compact** subset of AP offered by numeraire producing firm q. This means that the employees of firm q can choose between all of the benefit payment, premium pairs contained in $AP_q$ in Stage Two. Which pair will each employee choose? That depends on their preferences, which are described next.

**Preferences**

Under fairly general conditions, Herstein and Milnor (1953) have shown that i 's preferences over distributions of different states of the world can be represented by an expected utility function. I will write i 's expected utility function as:

$$EU_i = (1 - \pi_i) * U(I_{i1}) + \pi_i * U(I_{i2}).$$

I will make several assumptions about utility in this model:

**U1.** The sub-utility function U(() is twice continuously differentiable, strictly increasing, and strictly concave function that maps from $\mathbb{R}_{++}$ to $\mathbb{R}$.

**U2.** Each employee has the same level of risk aversion. This implies that the marginal utility of income is positive, but decreases as income increases.[7] Therefore, i 's preferences are monotone with respect to income.

**U3.** The marginal utility of income is bounded from below: $U_{I_{is}} > z > 0$ for some positive real constant z for s = {1, 2}.

[7] In other words, $\partial U / \partial I_{is} = U_{I_{is}} > 0$ and $\partial^2 U / \partial I_{is}^2 < 0$ for s = {1, 2}.

**U4.** The sub-utility function $U(I_{is}) = -\infty$ for $I_{is} \leq 0$ for $s = \{1, 2\}$.[8]

I can rewrite employee i 's expected utility function by plugging in the definition of the budget constraint:

$$EU_i = (1 - \pi_i) * U(W - R_j) + \pi_i * U(W - p * h - R_j + B_j).$$

**Lemma 1:** $EU_i$ is a concave, twice continuously differentiable function.

**Proof:** See Appendix II.

**Commodity Space**

I will show that an equilibrium exists in stage two by citing an existence theorem presented in Marton (2000). In order to see that Stage Two of the Health Insurance Model presented in this paper is just a special case of the more general model presented in Marton (2000), I will describe the commodity space in Stage Two using the notation of Marton (2000).

In this paper one can think of an insurance policy as a differentiated commodity. This is because an insurance policy has multiple characteristics and it can only be consumed in integer amounts. The other commodity in the Health Insurance Model is numeraire. Numeraire can be thought of as a non-differentiated commodity. It is not necessarily consumed in integer amounts.

Since an insurance policies coinsurance rate and deductible can be described by its benefit payment, I will define the insurance policy characteristics space as:

$$K^d = [0, p * h].$$

$K^d$ is a compact metric space. The overall commodity space is then defined to be:

[8] This prohibits employee from using all of their endowment of wealth to purchase an insurance policy and having no residual income.

$$K = K^d \cup \{\text{numeraire}\} = [0, p * h] \cup \{\text{numeraire}\}.$$

It should be clear that K is also a compact metric space. Let $\beta(K)$ be the Borel $\sigma$-field of subsets of K and let V denote a typical Borel subset of K.

I will now informally describe the consumer's choice problem. The consumer is endowed with some level of numeraire from their work in Stage One. They then choose a benefit payment, $B_j$, and a premium, $R_j$, from the set of benefit payment, premium pairs offered by their employer. They then consume their insurance policy and the numeraire they have left over after paying their taxes and for their uninsured medical expenses (if any).

**Commodity Bundles**

In this model an individual commodity bundle can be thought of as a non-negative bounded Borel measure on K such that $m(K^d) = 1$. M(K) is defined to be the set of bounded, signed measures on K. I will endow M(K) with the weak star topology.

Define the individual commodity bundle, $m_j$, associated with policy $(B_j, R_j)$ 0 $AP_q$ as follows:

$$m_j \mid K^d{}_{B_j} (V) = 1 \quad \text{if} \quad B_j \; 0 \; V \quad \text{and} \quad 0 \text{ elsewhere.}$$ [9]

The consumption set that each employee i of firm q faces, denoted $\Omega_q \subset M(K)$, is the set of all individual commodity bundles associated with each policy in $AP_q$. $\Omega_q$ is endowed with the relative topology from M(K).

[9] This is just the Dirac measure at $B_j$.

## Optimization Problem

As mentioned, employee i begins Stage Two with an endowment of $W^p$ units of numeraire from her work in Stage One to use towards the purchase of an insurance policy. Therefore, one can think of her endowment as a measure $\omega$ defined as:

$$\omega(\text{numeraire}) = W^p = MP\ 0\ |_{++} \quad \text{and} \quad \omega \mid K^d = 0.$$

Given her endowment, employee i chooses the consumption bundle $m_i$ 0 $\Omega_q$ that maximizes her expected utility.

This is equivalent to saying that employee i chooses the insurance policy $(B_j, R_j)$ 0 $AP_q$ that maximizes her expected utility:

$$EU_i(B_j, R_j) = (1 - \pi_i) * U(W - R_j) + \pi_i * U(W - p * h - R_j + B_j). \qquad (*)$$

Notice that the expected utility function can be expressed in terms of residual income or in terms of an insurance policy $(B_j, R_j)$ and money, as in the equation $(*)$ above. This will be important in Appendix I where I show that consumers satisfy the assumptions of the existence proof presented in Marton (2000).

## Optimal Behavior for Employees

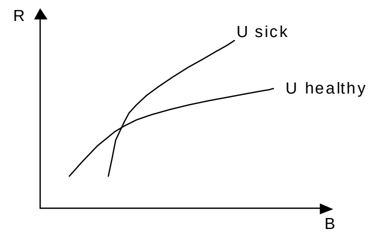
The slope of employee i 's indifference curve through policy $(R_j, B_j)$ is defined as:

$$MRS_{ij} = \frac{-\,\partial EU_i / \partial B_j}{\partial EU_i / \partial R_j} = \frac{\pi_i * U'(W - p * h - R_j + B_j)}{(1 - \pi_i) * U'(W - R_j) + \pi_i * U'(W - p * h - R_j + B_j)}$$

**Lemma 2 (Single Crossing Condition):** $\pi_m > \pi_n$ implies that $MRS_{mj} > MRS_{nj} > 0$.

**Proof:** See Appendix II.

This says that when the benefit payment is measured on the horizontal axis, the indifference curve of a less healthy type is always steeper than the indifference curve of a more healthy type. Here is an illustration of the single crossing condition for two health types:




The single crossing condition implies that a sick person must pay more (in terms of utility) for a given increase in coverage than a healthy person.

Define the highest level of expected utility that employee i can achieve from choosing a policy in $AP_q$ as:

$$EU_i^*(AP_q) = \max \{ EU_i(B_j, R_j) \mid (B_j, R_j)\ 0\ AP_q \}.$$

Next define the preferred set of employee i given $AP_q$ as the set of policies in $AP_q$ that provide this maximum level of expected utility:

$$K_i^*(AP_q) = \{ (B_j, R_j)\ 0\ AP_q \mid EU_i(B_j, R_j) = EU_i^*(B_j, R_j) \}.$$

**Lemma 3:** Let $m < n$. If $(B_m, R_m)\ 0\ K_m^*$ and $(B_n, R_n)\ 0\ K_n^*$, then $B_m \leq B_n$ and $R_m \leq R_n$.

**Proof:** See Appendix II.

This lemma says that if two different health types are faced with the same set of insurance choices, the relatively sicker health type (n) will always prefer a policy with at least as much coverage, if not more, than the relatively healthier type (m).

**Lemma 4:** Let $m < i < n$. If $K_m^* \cap K_n^* \neq \emptyset$, then $K_m^* \cap K_n^*$ consists of exactly one policy and $K_i^* = K_m^* \cap K_n^*$.

**Proof:** See Appendix II.

Lemma 4 says that only one policy can be in the most preferred set of two different health types and if such a policy exists, it must be the only policy in the preferred set of the intermediate health types.

**What premium, $R_j$, would the insurance company offer with benefit payment $B_j$ to employee i if it knew i 's health type?**

The insurance company will offer a premium that at least covers the expected benefit payment of the policy:

$$E(B_j)_i = \pi_i * B_j.$$

In addition, the insurance company incurs non-medical / administrative costs in the process of honoring the policy.  These administrative costs depend on the level of coverage provided by policy j and will be described in more detail later.  The premium offered with policy j must also reflect these administrative costs.  Define the increasing, real-valued function $P(B_j) \in \mathbb{R}_+$ as the “loading fee” of policy j.  The loading fee is a percentage of the expected benefit payment the insurance company charges on top of the expected benefit payment to cover these administrative costs.  You can think of the loading fee as the actual price of the plan.[10]  The loading fee is defined as an increasing function of a policy's benefit payment because as a policy's coverage level increases the administrative costs associated with the policy (such as the paperwork involved in processing claims) increases.

[10] I will prove later that $P(*)$ is continuous in equilibrium.

This implies that if the insurance company knew employee i 's health type, in equilibrium, they would offer the following premium with benefit payment, $B_j$:[11]

$$R_{ij} = (1 + P(B_j)) * E(B_j).$$

Again, I will assume that this is a perfectly competitive economy so that employees and the insurance company take the price of the policy, $P(B_j)$, as given.

**B. The Insurance Company**

Assume that there exists one risk neutral insurance company. The insurance company uses the non-differentiated good, money/numeraire, as the input into the production of insurance policies. Recall my assumption that this is a perfectly competitive economy, so the insurance company takes prices as given.

Insurance policy j is completely described by its benefit payment, $B_j$, and its premium, $R_j$. I will define the increasing, real valued function $G(B_j)$ to be the non-medical (administrative) costs associated with providing policy j. G is an increasing function of a policy's benefit payment because as a policy's coverage level increases the administrative costs associated with the policy (such as the paperwork involved in processing claims) increases.

To make things simple, I will assume that administrative costs can be described as a fixed percentage, g 0 [0, 1], of each dollar of a policy's benefit payment. This implies:

$$G(B_j) = g * B_j.$$

For example, if g = .10, then a policy's administrative cost equals 10 cents for every dollar of coverage it provides.

[11] Phelps (1973) modeled a premium as proportional to the amount of medical care consumed.

**Net-put Measures and the Production Set**

I defined the compact metric space of overall commodity characteristics as:

$$K = K^{d} \cup \{\text{numeraire}\} = [0, p * h] \cup \{\text{numeraire}\}.$$

As above, let $\beta(K)$ be the Borel $\sigma$ - field of subsets of K and let V be a typical Borel subset of K.

I will define $Y^{\sim}$ to be the following subset of M(K):

$$Y^{\sim} = \{\, y^{\sim}\ 0\ M(K) \ \mid\ y^{\sim} \mid K^{d}{}_{q} \geq 0 \quad \text{and} \quad y^{\sim}(\text{numeraire}) = - \mu_{K^{d}{}_{q}} G\ dy^{\sim} \,\}.$$[12]

Given this definition, I will define the insurance company's production set, Y, as follows:

$$Y = \{\, y\ 0\ M(K) \ \mid\ \exists\, y^{\sim}\ 0\ Y^{\sim} \text{ such that } y \leq y^{\sim} \,\}.$$

The interpretation here is that y(numeraire) is the total amount of the input (numeraire) required for the production of the outputs (insurance policies) described by net-put measure y. $\mu_{K^{d}{}_{q}} G\ dy$ is the total administrative costs required for the production of the policies described by y. It should also be noted that Y is endowed with the relative topology from M(K).

**Expected Profits**

Suppose that the insurance company sells policy $(B_j, R_j)$ to employee i of firm q. The expected profits from this sale are:

$$E(\text{Profits})_{i,\,j,\,q} = R_j - \pi_i * B_j - G(B_j).$$

Aggregating over all employees in firm q gives the expected profits from selling policy j to the employees in firm q:

[12] Here $y^{\sim} \mid K^{d}{}_{q}$ is the measure $y^{\sim}$ restricted to the domain $(K^{d}{}_{q}, \beta(K^{d}{}_{q}))$.

$$E(\text{Profits})_{.,\,j,\,q} = \mu_{I_q} [\, R_j - \pi_i * B_j - G(B_j) \,]\, d\lambda.$$

Aggregating over all plans gives the insurance company's total expected profits from selling policies to the employees of firm q:

$$E(\text{Profits})_{.,\,.,\,q} = \mu_{AP_q}\, \mu_{I_q} [\, R_j - \pi_i * B_j - G(B_j) \,]\, d\lambda\; dy.$$

**Optimization Problem**

Given risk neutrality, the insurance company's optimization problem is to choose the production vector y such that expected profits are maximized.

**First Order Conditions**

There are an infinite number of first order equations, one for each plan. A typical FOC is given below:

$$\mu_{I_q} [\, R_j - \pi_i * B_j - G(B_j) \,]\, d\lambda = 0.$$

This equation implies:

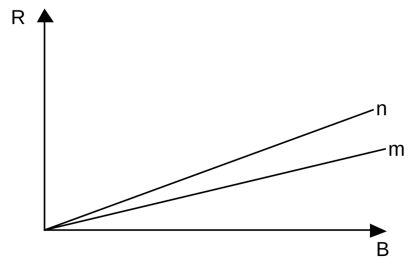
$$\mu_{I_q} R_j\, d\lambda = \mu_{I_q} [\, \pi_i * B_j + G(B_j) \,]\, d\lambda.$$

Therefore, for each policy ($B_j$, $R_j$), the insurance company chooses a production level such that the total premiums collected for policy j equals the total expected costs of policy j.

Consider the sale of a policy to employee m. The set of policies that break even when sold to employee m can be described by the following equation:

$$R = \pi_m * B + G(B) = \pi_m * B + g * B = (\pi_m + g) * B.$$

This implies that the insurance company will break even by charging a loading fee P(B) = G(B). The set of polices that break even when sold to employee m is represented by the line labeled m in the graph below. Also represented in the graph is the set of policies that break even when sold to employee n, for n > m.




**Lemma 5:** If $m < n$, then for any policy $(B_j, R_j)$, $E(Profits)_{n, j, .} \leq E(Profits)_{m, j, .}$ if and only if $B_j \geq 0$.

**Proof:** See Appendix II.

Lemma 5 says that the insurance company will make at least as much money by selling policy j to health type m as opposed selling policy j to health type n, when health type m is relatively more healthy than health type n. This is true as long as the benefit payment of policy j is non-negative.

**C. Stage Two Equilibrium**

Given an endowment of pre-tax income ($W^p$) for each employee in firm q from their work in Stage One and one consumption set, $\Omega_q$, from which they all must choose a consumption bundle from, a **Stage Two Equilibrium** consists of:

i) A consumption bundle (measure) $m_i^*$ 0 $\Omega_q$ for each employee in firm q that describes the insurance policy $(B_j^*, R_j^*)$ that maximizes their expected utility:

$$EU_i(B_j^*, R_j^*) = (1 - \pi_i) * U(W - R_j^*) + \pi_i * U(W - p * h - R_j^* + B_j^*).$$

ii) A production measure $y^*$ 0 Y for the health insurance company that maximizes its expected profits from selling policies to the employees of firm q:

$$E(Profits)_{., ., q} = \mu_{AP_q} \mu_{I_q} [\, R_j - \pi_i * B_j - G(B_j) \,]\, d\lambda \, dy^*.$$

iii) a price vector p* that describes the loading fee of each plan $P(B_j)$.

### 3. Stage One – The Labor Market

The labor market in Stage One will be constructed as a screening model.[13] I will vary the amount of knowledge the numeraire firms (and the insurance company in Stage Two) have about a worker's health type. Despite this lack of information, the numeraire producing firms move first. They offer a wage rate and a subset of the insurance policy characteristics space to potential workers. Workers then choose which numeraire firms to work for based upon these offers.

**Notation**

As mentioned, the measure space of all workers is $(\mathcal{I} = [0,1], \beta(\mathcal{I}), \lambda)$. Denote a typical worker by $i \in \mathcal{I}$. Each worker is endowed with one unit of labor and inelastically supplies that labor to one numeraire firm. I will assume this is a competitive labor market. Therefore the price of labor for each worker is defined to be their common marginal product.

**Labor Demand**

There are q = {1, …, Q} numeraire producing firms, each with the same constant returns to scale production function $F(*)$: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$. Numeraire firm q hires a Borel subset of workers, $I_q$, of $\mathcal{I}$ that depends upon the portfolio of insurance options $AP_q$ it offers. The marginal product for each worker i is the constant value MP.[14]

[13] The label "screening model" is used because the uniformed party (the numeraire producing firm) moves first. For more on the difference between screening models and signaling models, see Stiglitz and Weiss (1984).

[14] As mentioned above, for each health type i, $MP = W^p$.

Recall that $\lambda$ is the Lebesgue measure on the measure space of employees. Now define the total labor input used by numeraire firm q, $L_q$, as:

$$L_q = \mu_{I_q\,(AP_q)}\ MP\ d\lambda.$$

Firm q produces $F(L_q)$ units of numeraire. The total cost of production for firm q, $TC_q$, is:

$$TC_q = \mu_{I_q\,(AP_q)}\ MP\ d\lambda.$$

Now we can define the profit function for numeraire firm q:

$$\text{Profits}_q\ (AP_q)\ =\ F(L_q)\ -\ TC_q.$$

In order to simplify things, I will assume that the production function for each firm can be described as:

$$F(L_q) = L_q.$$

The intuition here is that firm q uses only labor to produce numeraire with no fixed costs and a constant returns to scale technology. If firm q hires a worker and each worker's marginal product is five, then this worker will produce five units of numeraire. Firm q can sell these five units for five dollars, which it must pay to the worker for their labor.

Numeraire firm q chooses the subset of allowable polices, $AP_q$, which maximizes its profits. It should be pointed out that the numeraire firms bear none of the costs associated with the provision of health insurance to their employees. Despite this fact, employees still prefer to acquire health insurance through their employer because of the tax subsidy described earlier. The discussion above implies that each numeraire firm will make zero profits in equilibrium no matter which insurance portfolio it offers or its level of production of numeraire.

**Labor Supply**

Workers must choose the numeraire firm to which they sell their endowment of labor. They do so by evaluating the utility they would receive in Stage Two with the $AP_q$ offered by each numeraire firm. Each worker chooses to work for the numeraire firm with the $AP_q$ that maximizes their utility in Stage Two.

Define the indirect utility function of worker i as follows:

$$IU_i(AP_q) = \max EU_i(B_j^*, R_j^*) = (1 - \pi_i) * U(W - R_j^*) + \pi_i * U(W - p * h - R_j^* + B_j^*).$$

In other words, the worker's utility in Stage Two depends on their choice of insurance policy $(B_j^*, R_j^*)$, which in turn depends on the portfolio of choices $(AP_q)$ offered by their employer in Stage One.

**Stage One Equilibrium**

A **Stage One Equilibrium** consists of:

i) A set of insurance contracts $(AP_1, \ldots, AP_Q)$ such that each numeraire firm q offers the set of contracts $AP_q$ that maximizes their profits in Stage One:

$$\text{Profits}_q (AP_q) = F(L_q) - TC_q = 0.$$

ii) An allocation of disjoint subsets of $\mathcal{I}$ to the numeraire firms $(I_1, \ldots, I_Q)$ such that each worker i is working for the firm q with the $AP_q$ that maximizes their indirect utility in Stage One:

$$IU_i(AP_q) = \max EU_i(B_j^*, R_j^*) = (1 - \pi_i) * U(W - R_j^*) + \pi_i * U(W - p * h - R_j^* + B_j^*).$$

iii) A price system where each worker is paid their marginal product: $W^p = MP \; \forall i$.

**4. Equilibrium**

I will now define what an equilibrium looks like in this model. The key to the existence of equilibrium in this model is whether or not a Stage One Equilibrium can be supported in Stage Two. In order for a Stage One Equilibrium to be supported in Stage Two, the set of contracts offered by the numeraire firms in Stage One must be able to convey enough information about employee's health types through their choices in Stage Two so that the insurance company can break even.

**Definition:** An Equilibrium in this model consists of a Stage One Equilibrium and a Stage Two Equilibrium for each numeraire firm q = {1, …, Q}.

**Lemma 6:** If $\bigcap_q K^{d2}_q$ represents an equilibrium set of insurance policies, then

$E(\text{Profits})_{\cdot,\cdot,q} = 0 \ \forall q.$

Lemma 6 says that in equilibrium the insurance company will make zero profits from the sale of insurance policies to the employees of firm q, for q = {1, …, Q}.

**Proof:** See Appendix II.

The types of equilibria (pooling or separating) that we see in this model depend upon the relationship between a worker's age and their health type. As mentioned, I will assume that a worker's age (a signal) is common knowledge and break up the relationship between age and health type into different cases.

**A. Case 1, Zero Information / Zero Correlation**

Here a worker's age provides no information to the insurance company about their health type. In this case there exists a unique Separating Equilibrium, where each health type purchases a distinct policy, and multiple Pooling Equilibria, where each health type purchases the same policy.

**The Separating Equilibrium**

I will first describe the (unique) set of policies that are offered in the separating equilibrium. The set of policies is derived by first assigning employees of health type T their most preferred policy, among those which earn non-negative profits for employees of type T. Type T - 1 employees are next assigned their most preferred policy from among those which earn non-negative profits for type T - 1 employees **AND** which are not preferred by type T employees. This process continues until health type 1 is reached.

Define the set of policies that make non-negative expected profits when sold to health type T as:

$$S_T = \{ (B_j, R_j) \ 0 \ AP_q \mid E(\text{Profits})_{T,\, j,\, .} \geq 0 \}.$$ [15]

Define policy $(B_T, R_T)$ 0 $S_T$ as the policy in $S_T$ that maximizes type T 's expected utility. In other words, $(B_T, R_T)$ 0 $K_T^*(S_T)$.

For i < T, define the set of policies that make non-negative expected profits when sold to health type i employees **AND** which are not preferred by type i + 1 employees as:

$$S_i = \{ (B_j, R_j) \ 0 \ AP_q \mid E(\text{Profits})_{i,\, j,\, .} \geq 0 \text{ and } EU_{i+1}(B_j, R_j) \leq EU_{i+1}(B_{j+1}, R_{j+1}) \}.$$ [16]

Define policy $(B_i, R_i)$ 0 $S_i$ as the policy in $S_i$ that maximizes type i 's expected utility. In other words, $EU_i(B_i, R_i) \geq EU_i(B_j, R_j)$ for every $(B_j, R_j)$ 0 $S_i$.

I will denote the set of policies described above as follows:

$$S = \{ (B_1, R_1), \ldots, (B_i, R_i), \ldots, (B_T, R_T) \}.$$

**Lemma 7:** The following properties hold with respect to the set of policies S described above:

(a) $(B_{i+1}, R_{i+1}) > (B_i, R_i) > 0$.

(b) $E(\text{Profits})_{i,\, i,\, .} = 0$.

(c) $EU_{i+1}(B_{i+1}, R_{i+1}) = EU_{i+1}(B_i, R_i)$.

---

[15] $S_T$ is a compact set.
[16] $S_i$ is also a compact set for every i.

(d) $(B_i, R_i)$ is unique for each i.

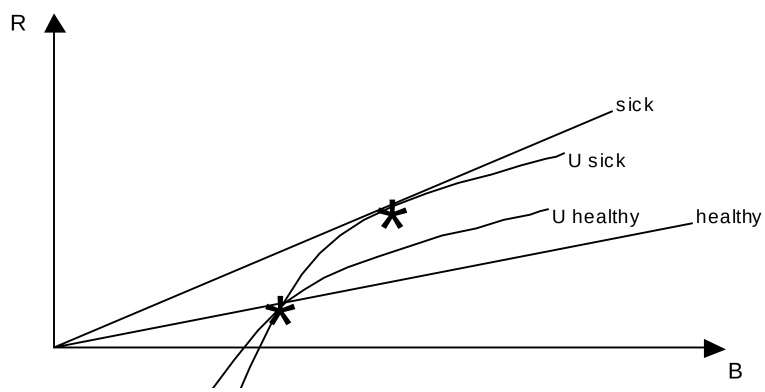
**Proof:** See Appendix II.

Condition (a) says that the optimal policy for health type i + 1 has a higher benefit payment and premium than health type i. Condition (b) says that the insurance company will make zero profits from selling type i his or her optimal policy. Condition (c) says that health type i + 1 is indifferent between their optimal policy and the optimal policy of type i. Finally, condition (d) says that each health type's optimal policy is unique.

**Theorem 1:** A unique Separating Equilibrium exists in the zero information case. The set S of insurance policies described above constitute this unique Separating Equilibrium. In other words, if this set of policies is offered by employers in Stage One, each health type i will maximize their utility by choosing to purchase policy i, the numeraire firms will break even, and the insurance company will break even.

**Proof:** See Appendix III.

Here is an illustration of the unique separating equilibrium for two health types:




It should be mentioned that as long as the union of all policies offered by the numeraire firms in Stage One equals S, it does not matter which firms offer each plan. For example, a Separating Equilibrium can exist where each firm offers a different plan (if the number of firms (Q) equals the number of health types (T)). In this case, each firm would specialize in hiring one specific health type. A Separating Equilibrium can also exist where each of the Q firms offers the entire set S. In this case, each firm could specialize in hiring one specific health type or they could each hire a representative sample of the population. Obviously there are other possibilities.

**The Pooling Equilibria**

Define the intermediate health type, $\pi^*$, as follows:

$$\pi^* = (\pi_1 + \pi_2 + \ldots + \pi_T) / T.$$

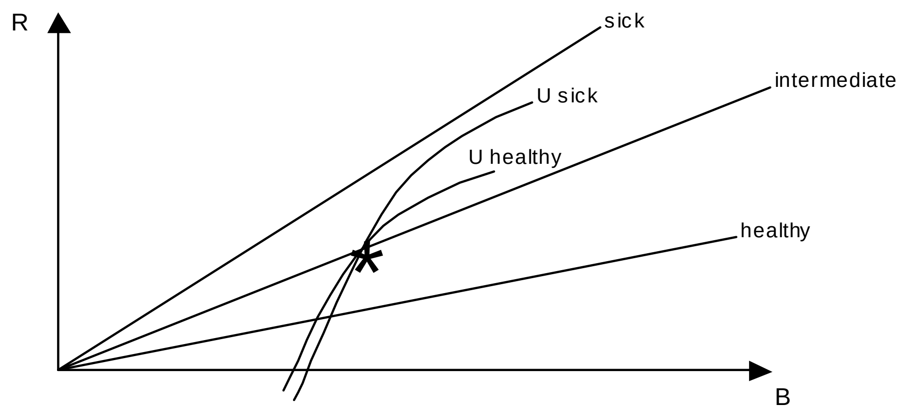
Define the set of policies that make zero expected profits when sold to the intermediate health type as follows:

$$P_* = \{ (B_j, R_j)\ 0\ Aq_+ \mid R_j = \pi^* * B_j + g * B_j \}.$$

**Theorem 2:** Multiple Pooling Equilibria exist in the zero information case. Any policy in the set $P_*$ described above constitutes a Pooling Equilibrium. In other words, if any one policy in this set is offered by all employers in Stage One and each employer hires a representative sample of workers, each health type i will maximize their utility by choosing to purchase this policy, the numeraire firms will break even, and the insurance company will break even.

**Proof:** See Appendix III.

Here is a graph of one Pooling Equilibrium for two health types:




**How do these results differ from what is typically found in the literature?**

In most screening models, a pooling equilibrium does not exist. The non-existence of a pooling equilibrium is one of the major results both of Rothschild and Stiglitz (1976) and Wilson (1977). Because this is such a famous result, I think that it is important to explain why my model provides a different prediction. The primary difference between my model and their models is that in my model the numeraire firms choose which policies to offer consumers. In both Rothschild and Stiglitz (1976) and Wilson (1977), the insurance companies sell policies directly to consumers. In their model a pooling equilibrium can always be destroyed by an insurance company offering a new policy that will only be preferred by the lower health types in the pool (the relatively healthy types) and that will make strictly positive profits. In the pooling equilibrium the insurance company was making zero profits, so they have an incentive to deviate from the pooling equilibrium and offer this new policy, thus breaking up the pool. In my model this will not happen because the numeraire firms cannot increase their profits by changing the set of policies being offered. Even though the insurance company has something to gain by altering the set of policies being offered, the numeraire firms do not. Therefore, in my model the incentives of the numeraire firms and the insurance company are not aligned.

Another common result in the screening literature is that in certain situations a separating equilibrium may also fail to exist. Again, the fact that the numeraire firms choose the set of policies being offered assures the existence of a separating equilibrium in my model.

**B. Case 2, Perfect Information / Perfect Correlation**

As mentioned, there is no adverse selection problem in this case. Assume that each numeraire firm then offers the full set of allowable insurance policies, AP.

**Lemma 8:** In the perfect correlation case, each health type will choose the same benefit payment, B*. This benefit payment is the same as that chosen by health type T in he separating equilibrium of the zero information case. Therefore, $B^* = B_T$.

**Proof:** See Appendix II.

For each health type i, the premium associated with this benefit payment is:

$$R_i^* = (\pi_i + g) * B^*.$$

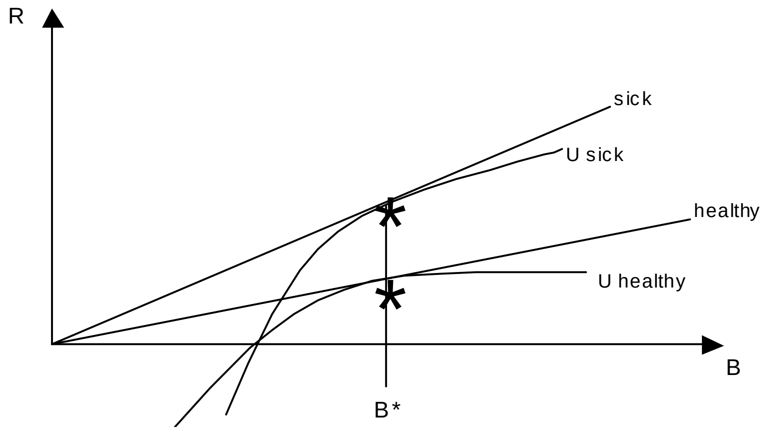
This defines a unique set of policies:

$$F = \{ (B^*, R_1^*), \ldots, (B^*, R_T^*) \}.$$

**Theorem:** A unique equilibrium exists in the perfect correlation case. The set F of insurance policies described above constitutes this unique equilibrium.

**Proof:** See Appendix III.

Here is the graph for two health types:




## III. An Empirical Application - Estimating the Price Elasticity of Demand

The theory presented above suggests the following structural model of the market for employer-provided group health insurance:

**Supply Side:**

$Q_S^{C,D} = \beta_0 + \beta_1$ * price of policy C,D + $\beta_2$ * total administrative costs associated with C,D + u

**Demand Side:**

$Q_D^{C,D} = \alpha_0 + \alpha_1$ * price of policy C,D + $\alpha_2$ * health-type + $\alpha_3$ * age + $\alpha_4$ * pre-tax income + $\alpha_5$ * price of medical care + $\alpha_6$ * quantity of medical care + $\alpha_7$ * level of risk aversion + v

**Equilibrium condition:**

$Q^{C,D} = Q_D^{C,D} = Q_S^{C,D}$

I plan on estimating this structural model using data from the 1987 National Medical Expenditure Survey, a large cross-sectional survey compiled by the Agency for Healthcare Research and Quality (AHRQ). I can then use the structural model to estimate the price elasticity of demand for employer-provided group health insurance.

The difference between this approach and some of the studies mentioned in the introduction is that many of these early studies wrote down a reduced form insurance demand equation without formally deriving the structural model from economic theory. In doing so, these studies completely ignored the supply side of the market and market clearing conditions.

Although I wrote the supply and demand relationships in the structural model as linear equations, there is no reason for this to be the case. Therefore, I will estimate the structural model using a non-parametric approach. The benefit of such an approach is that you do not have to make assumptions about the specific functional form of the equations you are estimating.

## IV. Conclusion

The purpose of this paper is to present a model of the market for health insurance that focuses on two main issues. The first is the existence of equilibrium, which is complicated by the familiar adverse selection problem. Given the assumptions of the model, I can show that a unique Separating Equilibrium and multiple Pooling Equilibrium and exist in the Zero Information case. In the Perfect Information case, I can show that a unique equilibrium exists. The second issue is applying the model to estimating price elasticities. This model suggests a structural model that can be used to estimate the price elasticity of demand.

## Appendix I

I will show that consumers and the insurance company in Stage Two satisfy the assumptions necessary for the existence of equilibrium in a more general model presented in Marton (2000). In Marton (2000), I proved the following existence theorem:

**Theorem 1 - Marton (2000):** Suppose consumers satisfy assumptions P1 through P5 placed on preferences and assumptions C1 through C4 placed on consumer characteristics. If, in addition, the firm satisfies assumptions F1 through F10, then an equilibrium ($p^*$, $\tau^*$, $y^*$, $\varepsilon$) exists for an economy $\varepsilon$.

To apply this existence theorem to Stage Two of the Health Insurance Model, I must show that consumers and the insurance company satisfy the assumptions listed in the theorem. I will first show that consumers in the Health Insurance Model meet all of the assumptions listed in the theorem. Next I will show that the insurance company satisfies the assumptions for the firm listed in the theorem.

**Step 1: Show that consumers in the Health Insurance Model satisfy the assumptions of the existence theorem**

**P1.** This follows from equation ($*$) in Section II and the continuity of U($*$). Q.E.D.

**P2.** This is true by the definition of the budget constraint in Section II. Q.E.D.

**P3.** Here consumption bundle m offers no money. This would imply an income less than or equal to zero by the definition of the budget constraint in Section II. Therefore, assumption U4 in the Health Insurance Model implies that the expected utility associated with m is $-\infty$. Because m' provides a non-negative level of income, m' is strictly preferred to m due to the monotonicity of preferences with respect to income in the Health Insurance Model. Q.E.D.

**P4.** Assumption P4 holds in the health insurance model if for any consumption bundle there is some amount of money that could be given to employee i that would make her better off. Because preferences are monotone with respect to income or money and the marginal utility of income is bounded below, assumption P4 is satisfied in the Health Insurance Model. Q.E.D.

**P5.** This says that if you consider two bundles m and m' such that both give the same level of money and similar (but not identical) levels of health insurance, there is some amount of money that can be added to bundle m which would make is strictly preferred to m'. The interesting case here is where bundle m provides less insurance than bundle m'.

The definition of the budget constraint in Section II implies that you can compensate a bundle with a lower level of insurance (m) with enough money such that it would provide a higher level of **income** than any bundle with a higher level of insurance (m'). This is because there is a trade-off between insurance and money in creating income. Because expected utility is continuous and monotone with respect to income, you can compensate a bundle with a lower level of insurance (m) with enough money such that it would provide a higher level **of expected utility** than any bundle with a higher level of insurance (m'). Q.E.D.

**C1.** This holds because each employee has the same endowment, so E is a point. Q.E.D.

**C2.** This holds because each employee has the same sub-utility function. Q.E.D.

**C3.** Assumption C3 holds as long as assumptions P5 and C2 hold. Q.E.D.

**C4.** Because each employee is endowed with a strictly positive amount of income from Stage One ($W^p > 0$), this holds in the Health Insurance Model. Q.E.D.

**Step 2: Show that the insurance company in the Health Insurance Model satisfies the assumptions of the existence theorem**

I will check each of the assumptions F1 through F10 individually below.

**F1.** I must show that y 0 Y implies $\alpha$y 0 Y for any scalar $\alpha \geq 0$.

**<u>Case 1</u>** Assume $\alpha = 0$. This implies that $\alpha y = 0$. I have shown by verifying assumption F5 below that 0 0 Y, so y 0 Y implies $\alpha$y 0 Y for $\alpha = 0$.

Before proving Case 2, I must show that $\tilde{y}_0$ 0 $\tilde{Y}$ implies $\alpha\tilde{y}_0$ 0 $\tilde{Y}$ for $\alpha > 0$. It should be clear that $\alpha\tilde{y}_0 \mid K^d \geq 0$. It is also clear that $\alpha\tilde{y}_0$(numeraire) = $- \mu_{K^d}$ G $d\alpha\tilde{y}_0$. This implies that $\tilde{y}_0$ 0 $\tilde{Y}$ implies $\alpha\tilde{y}_0$ 0 $\tilde{Y}$ for $\alpha > 0$.

**<u>Case 2</u>** Assume $\alpha > 0$. In this case, $\alpha$y is obviously a bounded and signed measure on K, so $\alpha$y 0 M(K). Because y 0 Y, I know $\exists$ $\tilde{y}_0$ 0 $\tilde{Y}$ such that $y \leq \tilde{y}_0$. This implies that

$\exists$ $\tilde{y}_1 = \alpha\tilde{y}_0$ 0 $\tilde{Y}$ such that $\alpha y \leq \tilde{y}_1 = \alpha\tilde{y}_0$. Therefore, $\alpha$y 0 Y.

Therefore, I have shown that y 0 Y implies $\alpha$y 0 Y for any scalar $\alpha \geq 0$. Q.E.D.

**F1.** This is true because Y is endowed with the weak star topology. See Theorem 6.1 in Parthasarathy (1967). Q.E.D.

**F2.** This will hold in the Health Insurance Model since, by definition, the only measure shared by both Y and $\Omega_q$ is the zero measure. Q.E.D.

**F3.** Before proving the main result, I must first prove that the set M(K) is closed under addition and the set $\tilde{Y}$ is closed under addition.

**<u>M(K) is closed under addition</u>**: Consider two measures a 0 M(K) and b 0 M(K). Define the sum of these two measures to be c = a + b. It is easy to see that c is a bounded and signed measure. This implies c 0 M(K).

**$\tilde{Y}$ is closed under addition**: Consider two measures $\tilde{y}_0$ 0 $\tilde{Y}$ and $\tilde{y}_1$ 0 $\tilde{Y}$. Define the sum of these two measures as $\tilde{y}_2 = \tilde{y}_0 + \tilde{y}_1$. Because M(K) is additive, $\tilde{y}_2$ 0 M(K). It is also easy to see that $\tilde{y}_2 \mid K^d \geq 0$ and $\tilde{y}_2$(numeraire) = - $\mu_{K^d}$ G d $\tilde{y}_2$. This implies $\tilde{y}_2$ 0 $\tilde{Y}$.

To prove Y is convex, I must show that for any two plans $y_0$, $y_1$ 0 Y, the plan $y_2 = \alpha y_0 + (1 - \alpha)y_1$ 0 Y for all $\alpha$ 0 [0, 1]. Because Y is a CRS technology, I know $\alpha y_0$ 0 Y and $(1 - \alpha)y_1$ 0 Y for all $\alpha$ 0 [0, 1]. This implies:

$$\exists\ \tilde{y}_0 \text{ 0 } \tilde{Y} \text{ such that } \alpha y_0 \leq \tilde{y}_0 \quad \text{and} \quad \exists\ \tilde{y}_1 \text{ 0 } \tilde{Y} \text{ such that } (1 - \alpha)y_1 \leq \tilde{y}_1.$$

Because $\tilde{Y}$ is closed under addition, $\tilde{y}_2 = \tilde{y}_0 + \tilde{y}_1$ 0 $\tilde{Y}$. Because M(K) is closed under addition, I have:

$$y_2 = \alpha y_0 + (1 - \alpha)y_1 \text{ 0 M(K) for all } \alpha \text{ 0 } [0, 1].$$

Therefore $\exists\ \tilde{y}_2 = \tilde{y}_0 + \tilde{y}_1$ 0 $\tilde{Y}$ such that $y_2 = \alpha y_0 + (1 - \alpha)y_1 \leq \tilde{y}_2$ for all $\alpha$ 0 [0, 1]. This implies $y_2$ 0 Y for all $\alpha$ 0 [0, 1], so Y is convex. Q.E.D.

**F5.** Consider - $\Omega_q$. This set consists of all bounded measures m such that:

$$m \mid K^d \;=\; m \mid K^d{}_B (V) = -1 \quad \text{if} \quad B \text{ 0 } V \quad \text{and} \quad 0 \quad \text{elsewhere.}$$

Any measure in this subset is by definition a member of Y. Therefore, - $\Omega_q \subset$ Y. Q.E.D.

**F6.** I will define -$\tilde{Y}$ to be the following subset of M(K):

$$-\tilde{Y} = \{\ \tilde{y} \text{ 0 M(K)} \ \mid\ \tilde{y} \mid K^d \leq 0 \ \text{ and } \ \tilde{y}(\text{numeaire}) = \mu_{K^d} \text{ G dy }\}.$$

Given this definition, I will define -Y as follows:

$$-Y = \{\ y \text{ 0 M(K)} \ \mid\ \exists\ \tilde{y} \text{ 0 } -\tilde{Y} \text{ such that } y \geq \tilde{y}\ \}.$$

The only measure shared by Y and this set is the zero measure. Q.E.D.

**F7.** Here W is the aggregate initial endowment. This implies that $\Omega$ - W consists of all consumption measures besides the aggregate initial endowment measure. The only measure shared by this set and Y is the zero measure, $Y \cap (\Omega - W) = \{0\}$. Therefore, the fact that the space $Y \cap (\Omega - W)$ is bounded follows trivially. Q.E.D.

**F8.** This follows from Lemma 6.3 of Parthasarathy (1967). Q.E.D.

**F9.** In the Health Insurance Model, the only input to production is money in the form of premium payments. Q.E.D.

**F10.** Again, this holds in the Health Insurance Model since there is only one input, money. Q.E.D.

**Appendix II: Proofs of the Lemmas**

**Lemma 1:** This follows from the definition of the sub-utility function U($*$). Q.E.D.

**Lemma 2:** This is just a restatement of Lemma 3 of Wilson (1977). Q.E.D.

**Lemma 3:** This is just a restatement of part of Lemma 4 of Wilson (1977). Q.E.D.

**Lemma 4:** This is a restatement of the second part of Lemma 4 of Wilson (1977). Q.E.D.

**Lemma 5:** I will show that if $B_j \geq 0$ and m < n, then for any policy $(B_{j,}\ R_j)$, $E(\text{Profits})_{n,\ j,\ .} \leq E(\text{Profits})_{m,\ j,\ .}$. Assume $B_j > 0$. If m < n, then $\pi_n > \pi_m$. This implies the following inequality:

$$[\ R_j - \pi_n * B_j - G(B_j)\ ] < [\ R_j - \pi_m * B_j - G(B_j)\ ].$$

Therefore, $E(\text{Profits})_{n,\ j,\ .} < E(\text{Profits})_{m,\ j,\ .}$. Now suppose that $B_j = 0$. If this is true, then $E(\text{Profits})_{n,\ j,\ .} = E(\text{Profits})_{m,\ j,\ .} = R_j$.

Proving the other case, if m < n and for any policy $(B_{j,}\ R_j)$, if $E(\text{Profits})_{n,\ j,\ .} \leq E(\text{Profits})_{m,\ j,\ .}$, then $B_j \geq 0$, is trivial. Q.E.D.

**Lemma 6:**

**Lemma 7:** This proof will proceed by induction. Suppose for some $i < T$, it has been shown that (a) through (d) hold $\forall j$ such that $i < j < T$. I will show that these conditions hold for all i.

**step 1, proof of (b):** This is shown to be true in Lemma 9 of Wilson (1977).

**step 2, prove $B_i < B_{i+1}$ and $R_i < R_{i+1}$:** This is shown to be true in Lemma 9 of Wilson (1977).

**step 3, proof of (c):** Start by noticing that $E(Profits)_{i,\, i,\, .} = 0$ implies that:

$$(B_i, R_i) = (\varepsilon, (\pi_i + g) * \varepsilon) \text{ for some } \varepsilon\ 0\ |.$$

By induction hypothesis (a) and the result $B_i < B_{i+1}$ and $R_i < R_{i+1}$, it follows that $\varepsilon < B_T$.

By Lemma (I don't have), $dEU_i(\varepsilon, (\pi_i + g) * \varepsilon) / d\varepsilon > 0$.

By the definition of $S_i$, $EU_i(\varepsilon, (\pi_i + g) * \varepsilon) > EU_i(B, R)$ for every (B, R) 0 $S_i$.

This implies $EU_{i+1}(\varepsilon, (\pi_i + g) * \varepsilon) = EU_{i+1}(B_{i+1}, R_{i+1})$. This proves that:

$$EU_{i+1}(B_i, R_i) = EU_{i+1}(B_{i+1}, R_{i+1}).$$

**step 4, proof of (d):** This is shown to be true in Lemma 9 of Wilson (1977).

**step 5, prove $B_i > 0$ and $R_i > 0$:** Note that by condition (a), I know

$$(B_{i+1}, R_{i+1}) = (\varepsilon, (\pi_{i+1} + g) * \varepsilon) \quad \text{for some } 0 < \varepsilon < B_T.$$

Lemma (I don't have), implies that $EU_{i+1}(0) < EU_{i+1}(B_{i+1}, R_{i+1})$.

Therefore, the previous argument implies $B_i > 0$ and $R_i > 0$.

A similar proof can be used to establish (a) through (d) for i = (T - 1). Q.E.D.

**Lemma 8:**

**Appendix III: Proofs of Theorems**

**Theorem 1:** Suppose that the union of the set of policies offered by each firm q,

$\bigcap_q AP_q$, equals the set S described above. These firms have no incentive to deviate from offering these policies because they will make zero profits regardless of the set of policies they offer. According to Lemma 6, each worker i will maximize their utility given S in Stage Two by choosing to work for the firm that offers policy $(B_i, R_i)$ in Stage One. Each worker will then receive their most preferred policy given S in Stage Two and be paid their marginal product. This implies that workers have no incentive to switch firms because they cannot increase their utility by doing so. Therefore all of the conditions for a Stage One Equilibrium are satisfied.

Given $AP_q \subseteq S$ for every numeraire firm q and the fact that each worker i chooses in Stage One to work for the numeraire firm q that offers policy $(B_i, R_i)$, I can apply the existence theorem from Marton (2000) to each firm q in order to show that an equilibrium exists in Stage Two. I show in Appendix I that the health insurance model satisfies the assumptions of this existence theorem. Q.E.D.

**Theorem 2:** Numeraire firms make zero profits regardless of the set of insurance policies they offer. Assume that they each offer policy P 0 $P_*$ in Stage One. No numeraire firm has any incentive to deviate and offer a different set of policies.

If workers are allocated to numeraire firms in the way described above, each worker will receive a wage equal to their marginal product and the ability to choose the policy in Stage Two that maximizes their utility, given that there is only one choice. Therefore, no worker has any incentive to move to a different employer in Stage One because they cannot achieve a higher level of utility by doing so. Therefore all of the conditions for a Stage One Equilibrium are satisfied.

As mentioned, in Stage Two employees will choose policy P, this maximizes their utility given that there is only one choice. Because each numeraire firm has the same distribution of health types as the population, the insurance company can apply these population estimates to each firm and break even, on average, by producing and selling policy P. This implies that I can apply the existence theorem from Marton (2000) to each firm q to show that an equilibrium exists in Stage Two. I show in Appendix I that the health insurance model satisfies the assumptions of this existence theorem. Q.E.D.

Notice that this proof applies to every P 0 $P_*$.

**Theorem 3:** Because I have assumed that each numeraire firm q offers the full set of insurance policies, I can focus on one numeraire firm without loss of generality. This firm has no incentive to deviate from offering AP because they will make zero profits regardless of the set of policies they offer. Each worker i will choose to work for this firm in Stage One because they can then pick the policy, ($B^*$, $R_i^*$), that maximizes their expected utility in Stage Two. This implies that no worker has an incentive to switch to a different employer. Therefore all of the conditions for a Stage One equilibrium are satisfied.

In Stage Two, each worker chooses the policy that maximizes their utility given their health type and the insurance company will break even by offering this set of policies. Therefore, the fact that a Stage Two equilibrium exists can be proved by applying the existence theorem from Marton (2000) to each health type, as if there are identical numeraire firms for each health type. Because each numeraire firm has a constant returns to scale technology, there is no real difference between one firm hiring all of the workers and each health type working for a different firm. Q.E.D.

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**<u>Uniqueness Theorem</u>**: The equilibrium price (premium) vector of insurance policies in Stage Two, p*, is unique.

**<u>Proof of Uniqueness Theorem</u>**

Suppose first that the vector of premiums is $p_0 \leq p^*$. For any of the plans with the premium set below the equilibrium premium, the insurance company cannot cover the administrative costs involved in producing the plans. This implies zero production of those insurance policies and a "bidding up" of their premiums.

Now suppose the premium vector is some $p_1 \geq p^*$. For any of the plans with the premium set above the equilibrium premium, the insurance company can never reach the profit maximizing level of production because no one would want to buy these policies. This implies a “bidding down” of their premiums.

**Extensions**

One extension I am interested is to examining is the existence of equilibria in the labor market when a workers’ marginal product is only partially correlated with their health type. I would like to see how strong the correlation must be before a separating equilibrium exists.