The Competition from a Distant Firm may be Good for Firms in Duopoly Markets

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Abstract

In a linear-city framework, we show that the competition from a distant firm may reduce the competition between the firms in a duopoly market and increase the prices and the profits of the duopolists.

JEL Classifications: L1, D4, C4

1. Introduction

Classic models of monopoly suggest that a monopolist has the ability to exercise his market power by charging the monopoly price to earn a monopoly profit. A durable good monopolist, however, may not have such ability, as pointed out by Coase (1972) initially, and then formalized and proved by Stokey (1981), Bulow (1982), Gul, Sonnenschein and Wilson (1986), Kuhn (1986), and Bond and Samuelson (1984), using different frameworks. The intuition of the Coase Conjecture is as follows. Suppose that a monopolist has a durable good for sale and it charges the monopoly price in the first period. Consumers who have valuations higher than the price will buy it and the other consumers whose valuations are below the monopoly price will remain in the market. When the monopoly price is above the marginal cost of the good, the monopolist will have every incentive to cut his price in the second period to serve the remaining consumers and make additional profits. This process continues until the price charged by the monopolist equals the marginal cost. Anticipating the monopolist's price-cutting behavior, consumers will choose to postpone purchasing the good. When the consumers' cost of waiting is zero, the monopolist will lose his market power completely and will only be able to sell his good at marginal cost.

The result of the Coase Conjecture can be attributed to the inability of the monopolist to commit to maintaining sufficiently high prices in the future. Therefore, anything that helps the monopolist to sustain sufficiently high prices in the future would break the logic of the Coase Conjecture and result in a higher profit for the firm. Using an infinitely repeated game framework, Ausubel and Deneckere (1987) and Gul (1987) demonstrate that the existence of another firm in the market or even the threat of entry by another firm may prevent the incumbent from cutting prices in the future because of the potential punishment from the other firm or the potential entry of the other firm, and cause the incumbent to earn monopoly profits. As Ausuble and Deneckere (1986) put it, "if you cannot punish yourself, find someone else to punish you."

In this paper, the logic of Ausubel and Deneckere (1987) and Gul (1987) is applied to a duopoly market in which two firms located side by side have an identical good for

sale. Without the competition from the outside of the duopoly market, the duopolists would engage in a Bertrand type competition. Consequently their prices and profits at the equilibrium would be low. The existence of a firm located a distance away from the market creates an extra competition for the duopolists which makes the duopolists' reduction in price more costly, and therefore reduces the level of competition between the duopolists and increases their prices and profits.

The result of this paper is different from the results of Ausubel and Deneckere (1987) and Gul (1987) in two respects. First, their results indicate that "two may be better than one" and are derived with respect to durable goods. Our result suggests that "three may be better than two" and is derived in terms of a general good in a horizontally differentiated market. Second, in their models, tacit collusion is the device for the firms to reap the static monopoly profits in duopoly market and a framework of infinitely repeated game is used to accommodate such collusion, as well as the Coase Conjecture. In contrast, firms in our model engage in no collusion and the framework of our model is a one-shot game. The duopolists may earn greater profits with the existence of a distant firm because the competition from the distant firm may increase the cost of a price reduction and therefore reduce the competition between the duopolists. In another related collusion analysis, Werden and Baumann (1986) show that four firms may be few but three firms are not to form a cartel so that four may be better than three. Their result is similar to the result of this paper in that both suggest that more firms in a market may lead to higher profits. However, they focus on the number of firms needed for the formation of a cartel and it is the collusion that may increase the profits of the firms. We focus on the competition among the firms and it is the competition from a distant firm that may increase the profits of the firms in a duopoly market.

The paper is organized as follows. In the next section, we establish a model that features both the internal competition between the duopolists and the external competition between the duopolists and a firm that is located a distance away from the duopoly market. In section 3, the equilibrium prices and profits of the duopolists are derived corresponding to both situations when a distant firm exists and when it does not. Section 4 compares the equilibria under the two situations. Section 5 is the conclusion.

2. The Model

Consider a linear city with unit length.[1] There are three firms producing a homogeneous good with the same constant marginal cost normalized to zero. The cost of land in this city varies significantly which is higher in the middle and lower at the border so that all firms are located at the ends of the city, firm 1 and firm 2 are located at the left end and firm 0 is located at the right end. Consumers are uniformly distributed along the city and the unit transportation cost is t. The consumption value of the good is $\bar{v}$ and the prices of the good charged by the firms are p_1, p_2 and p_0, respectively.[2] For convenience, the market of firms 1 and 2 is called the duopoly market or market L and the market of firm 0 is called market R.

To derive the demand for market L and market R, we need to determine the factors that affect a consumer's decision in choosing between the markets. The transportation cost and the prices of the good charged by the firms are two such factors. In addition, market L is a duopoly market and market R is a single-firm market. The advantage to visit a duopoly market is that such a market provides a greater variety of goods for consumers to browse. It may also have a better market environment because of the non-price competition between the firms. We use u to denote the extra utility a consumer would obtain to visit the duopoly market, but we set it equal to zero since our result is independent of the value of u as long as it is small.

The disadvantage to visit a duopoly market is that the firms are located side-by-side and a consumer may easily make a mistake to buy the good from the higher-price firm as a result of the consumer's occasional losing memory about the price difference, occasional losing ability to compare prices or occasional running into a friend that causes him to ignore the price difference in the market. Since such a random reference is not exactly known at the time that consumers makes decisions on choosing the markets, we use a random variable to describe it. The probability distribution of the random variable represents the expectation of the consumers' random preferences. The random variable is

[1] See Hotelling (1929) or Tirole (1988) for a description of the classical linear city model.

defined as $\Theta = wq$, where w is a Bernoulli random variable with the outcomes of (-1, 1) and the probability distribution of (0.5, 0.5); q $(0 \le q \le 1)$ is a continuous random variable with a density function $r(q)$. w and q are independent. The sign of Θ indicates which firm is randomly preferred. If it turns out to be 1, then firm 1 is randomly preferred. If it turns out to be –1 then firm 2 is preferred. q measures the degree of a consumer's random preference. A higher value of q represents a stronger random preference. It is assumed that all Θ s for all consumers are independent and identically distributed. Note that the consumer preference for firm characterized by Θ is symmetric and all consumers have the same expectation of the random preference.

Let $D_L(p, p_0)$ be the demand function of market L, where p is the expected price in market L and p_0 is the price of firm 0 in market R. Given p_0, the demand in market L depends on the expected price in market L which in turn depends on the prices of firms 1 and 2, the probability distribution of random preference $r(q)$, and the probability at which a random preference is corrected, as we show below. Suppose that $p_1 \ge p_2$. For a typical consumer who visits the duopoly market, if $\Theta < 0$, then the consumer prefers firm 2 and in this case he will definitely buy the good from firm 2 since the price of firm 2 is also lower; if $\Theta > 0$, then the consumer prefers firm 1. In this case, the random preference and the lower-price preference of the consumer work in the opposite directions and the consumer's selection of a firm will depend on the degrees of these two preferences. We assume that a random preference dominates a lower-price preference if and only if $q > X$, where $X = \dfrac{p_1 - p_2}{p_1}$ is the relative price. Thus, the probability that a random preference dominates a lower-price preference is $1 - R(X)$, where $R(q)$ $(0 \le q \le 1)$ is the cumulative distribution function of q . Obviously, the greater is the relative-price-difference, the stronger the consumer's preference for the lower-price firm, and therefore the lower the probability that the consumer's random preference suppresses his lower-price preference. On the other hand, a random preference is fundamentally an error.

[2] The value of the good, the prices and the profits of the firms can be normalized with respect to the transportation cost, t, as we will see in sections 3 and 4.

Therefore, it may be corrected before the consumer actually buys the good. We assume that the probability of a random preference being corrected is $1-\frac{D_L(p_1,p_0)}{D_L(p,p_0)}$. Under these assumptions, a consumer who enters the duopoly market will buy from firm 1 when $p_1 \geq p_2$ if and only if all of the following three conditions are satisfied: he randomly prefers firm 1; his random preference for firm 1 suppresses his lower-price preference for firm 2; and he does not correct his random preference. The probability for all these three events to take place is $w = 0.5(1-R(X))\frac{D_L(p_1,p_0)}{D_L(p,p_0)}$. Note that w is the probability that a consumer mistakenly purchases the good from the higher-price firm. Consequently, the expected price in the market is $p = wp_1 + (1-w)p_2$ and the corresponding market demand is $D_L(p,p_0)$.

When $p_1 = p_2$, the expected price of the good in market L, p, equals the prices of the firms so that the demand of market L is $D_L(p_1,p_0)$. Since the probability for each individual consumer to randomly prefer a firm is $\frac{1}{2}$, the demand for each firm is $\frac{D_L(p_1,p_0)}{2}$. When $p_1 > p_2$, the expected price of market L equals $p = wp_1 + (1-w)p_2$ and the market demand is $D_L(p,p_0)$. Because the probability for an individual consumer to buy from firm 1 is w, the demand for firm 1 is simply $wD_L(p,p_0) = \frac{D_L(p_1,p_0)}{2}(1-R(X))$. The demand for firm 2 is the residual market of firm 1 that equals $D_L(p,p_0) - \frac{D_L(p_1,p_0)}{2}(1-R(X))$.

By the symmetries of the firms and consumers' random preferences, the demand for firm 1 and 2 when $p_1 < p_2$ can be derived analogously. The summarized demand function for each firm in market L, given the price of its competitors, is listed below.

$$q_1 = \begin{cases} \dfrac{D_L(p_1, p_0)}{2}(1 - R(X)) & p_1 > p_2 \\ D_L(p, p_0) - \dfrac{D_L(p_2, p_0)}{2}(1 - R(X)) & p_1 \le p_2, \end{cases} \tag{1}$$

and

$$q_2 = \begin{cases} \dfrac{D_L(p_2, p_0)}{2}(1 - R(X)) & p_2 > p_1 \\ D_L(p, p_0) - \dfrac{D_L(p_1, p_0)}{2}(1 - R(X)) & p_2 \le p_1, \end{cases} \tag{2}$$

where, $p = wp_{\max} + (1-w)p_{\min}$ is the expected price of market L, $p_{\max} = \max\{p_1, p_2\}$, $p_{\min} = \min\{p_1, p_2\}$, $w = \dfrac{0.5D_L(p_{\max}, p_0)(1 - R(X))}{D_L(p, p_0)}$ is the probability of buying from the higher-price firm, and $X = \dfrac{p_{\max} - p_{\min}}{p_{\max}}$ is the relative price. Obviously, the demand functions are continuous when both $R(X)$ and $D_L(p, p_0)$ are continuous.

Given the price of firm 0 in market R, $D_L(p, p_0)$ is uniquely determined as in the standard linear city model. To derive it, let U_R and U_L be the net utilities of buying the good from markets R and L respectively, and x be the distance from the location of a consumer to the location of the duopolists in market L. Given p_0, the location of the consumer who is indifferent between buying from market R and not buying, x_0, can be solved from $U_R = \bar{v} - p_0 - t(1 - x_0) = 0$ which is $x_0 = 1 - \dfrac{\bar{v} - p_0}{t}$. Let $\tilde{p}(p_0)$ be the expected price in market L such that the consumer located at x_0 is indifferent between buying from market L and not buying. From the net utility equation $U_L = \bar{v} - \tilde{p}(p_0) - tx_0 = 0$, it is easy to find that $\tilde{p}(p_0) = 2\bar{v} - t - p_0$.

$\tilde{p}(p_0)$ is the critical price for the firms in the duopoly market to have an external competition with firm 0 in market R when the price of firm 0 is p_0. When the expected price in market L, p, is above $\tilde{p}(p_0)$, there will be no competition between the firms in

markets L and R. In this case the demand in market L is $D_L(p, p_0) = \frac{\bar{v} - p}{t}$ and the demand in market R is $D_R(p, p_0) = \frac{\bar{v} - p_0}{t}$. When p is below $\tilde{p}(p_0)$, firms in market L will be in direct competition with firm 0 in market R. The demand in market L and market R in this case can be solved from $U_L = U_R$, where U_L and U_R are the net utility of buying from markets R and L, respectively, and we find that $D_L(p, p_0) = x = \frac{t + p_0 - p}{2t}$ and $D_R(p, p_0) = 1 - x = \frac{t + p - p_0}{2t}$.

In summary, the kinked demand in the duopolistic market L given p_0 is given by

$$D_L(p, p_0) = \begin{cases} \frac{\bar{v} - p}{t} & \text{if } p \geq \tilde{p}(p_0) \\ \frac{t + p_0 - p}{2t} & \text{if } p < \tilde{p}(p_0) \end{cases}. \tag{3}$$

Where, $\tilde{p}(p_0) = 2\bar{v} - t - p_0$. The corresponding demand in market R is given by

$$D_R(p, p_0) = \begin{cases} \frac{\bar{v} - p_0}{t} & \text{if } p \geq \tilde{p}(p_0) \\ \frac{t + p - p_0}{2t} & \text{if } p < \tilde{p}(p_0) \end{cases}. \tag{4}$$

3. The equilibrium prices and profits with and without firm 0 in the market

It has been shown in Lu (1999) that a unique non-trivial symmetric equilibrium, $(p_1, p_2) = (\hat{p}, \hat{p})$, exists in a duopoly market with heterogeneous consumers with random preferences, as long as the market demand curve is concave and the cumulative distribution function of q, $R(X)$, is regular ($0 \leq R'(0) < 1$). The equilibrium price, $\hat{p}$, satisfies

$$e_{\hat{p}^-} \leq 1 - R'(0) \leq e_{\hat{p}^+}, \tag{5}$$

where, $e_{\hat{p}^-}$ and $e_{\hat{p}^+}$ are the price elasticities of demand when price tends to $\hat{p}$ from below and above. We assume that $R(X)$ is regular ($0 \le R'(0) < 1$) and by (3), the demand for market L is obviously concave. Thus, there exists a unique non-trivial equilibrium in the duopolistic market L and it satisfies $e_{\hat{p}^-} \le 1 - R'(0) \le e_{\hat{p}^+}$. From now on, we use R' to denote $R'(0)$ for simplicity.

Using the result stated above, we first calculate the equilibrium prices and profits of the firms in the three-firm market. The calculation is carried out in three steps. First, the equilibrium price in market L is derived as a function of the price of firm 0 (Lemma 1) which we call "the reaction function of market L". Second, based on the reaction function of market L, we find firm 0's three local profit-maximizing prices (Lemma 2). In the third step, the three local maximum profits corresponding to the three local profit-maximizing prices are compared and the price corresponding to the highest profit is the equilibrium price for firm 0 (Proposition 1).

Lemma 1. (a) If $p_0 > (\bar{v} - t) + \frac{\bar{v}}{2 - R'}$, then the equilibrium price in market L is $\hat{p}(p_0) = \frac{1 - R'}{2 - R'}\bar{v}$ which is above $\tilde{p}(p_0)$ and the profit of firm 0 is $p_0(\hat{p}(p_0), p_0) = p_0 \frac{\bar{v} - p_0}{t}$.

(b) If $(\bar{v} - t) + \frac{\bar{v}}{3 - 2R'} \le p_0 \le (\bar{v} - t) + \frac{\bar{v}}{2 - R'}$, then the equilibrium price in market L is $\hat{p}(p_0) = \tilde{p}(p_0)$ and the profit of firm 0 is $p_0(\hat{p}(p_0), p_0) = p_0 \frac{\bar{v} - p_0}{t}$.

(c) If $p_0 < (\bar{v} - t) + \frac{\bar{v}}{3 - 2R'}$, then the equilibrium price in market L is $\hat{p}(p_0) = \frac{1 - R'}{2 - R'}(t + p_0)$ which is below $\tilde{p}(p_0)$ and the profit of firm 0 is $p_0(\hat{p}(p_0), p_0) = p_0 \frac{(3 - 2R')t - p_0}{2t}$ (All proofs are relegated to the appendix).

As shown in section 2, given the price of firm 0, the demand curve in market L is kinked at $p = \tilde{p}(p_0)$, where $\tilde{p}(p_0) = 2\bar{v} - t - p_0$. From the formula it is easy to see that $\tilde{p}(p_0)$ is inversely related to the price of firm 0, p_0. Note that the price at the kink, $\tilde{p}(p_0)$, is the critical price for a competition between markets L and R to occur. If firm 0 charges a higher price, $\tilde{p}(p_0)$ will be lower, therefore it is less likely for firm 0 to have a competition with the duopolists in market L. Lemma 1 states that as long as the price of firm 0 is above $(\bar{v} - t) + \frac{\bar{v}}{2 - R'}$, the corresponding equilibrium price in market L exceeds the price at the kink and markets L and R are independent. The profit of firm 0 in this case is $p_0(\hat{p}(p_0), p_0) = p_0 \frac{\bar{v} - p_0}{t}$. When firm 0's price is between $(\bar{v} - t) + \frac{\bar{v}}{3 - 2R'}$ and $(\bar{v} - t) + \frac{\bar{v}}{2 - R'}$, the equilibrium price in market L is equal to the price at the kink, $\tilde{p}(p_0)$. At this price, the duopolists attract those and only those consumers who do not buy from firm 0. Therefore, the whole city is covered by the firms while no direct competition occurs between the firms in markets L and R. The profit for firm 0 is $p_0(\hat{p}(p_0), p_0) = p_0 \frac{\bar{v} - p_0}{t}$. Only when firm 0's price is below $(\bar{v} - t) + \frac{\bar{v}}{3 - 2R'}$, direct competition between the firms in markets L and R occurs. In this case, the profit of firm 0 is $p_0(\hat{p}(p_0), p_0) = p_0 \frac{(3 - 2R')t - p_0}{2t}$.

Note that firm 0 acts as a leader and the duopolists in market L act as followers in this pricing game. The leadership position that firm 0 enjoys reflects the market power that firm 0 possesses which originates from its advantageous location: there is no firm competing with it at its location while the duopolists are located side by side competing with each other.

Knowing the reaction function of the duopolists in market L, firm 0 will choose a price that maximizes its profit. Lemma 2 provides firm 0's three local profit-maximizing prices and the corresponding local maximum profits. The results are obtained by

maximizing the three profit functions of firm 0 derived in Lemma 1. For convenience, we denote $V_1 = \frac{2(2-R')t}{4-R'}$, $V_2 = \frac{2(3-2R')t}{5-2R'}$ and $V_3 = \frac{(5-2R')(3-2R')t}{8-4R'}$.

Lemma 2: (a) If $p_0 \geq (\bar{v}-t) + \frac{\bar{v}}{2-R'}$, then firm 0's profit maximizing price is

$$p_0 = \begin{cases} \frac{\bar{v}}{2} & \text{if } 0 \leq \bar{v} < V_1 \\ (\bar{v}-t) + \frac{\bar{v}}{2-R'} & \text{if } \bar{v} \geq V_1 \end{cases}.$$

The corresponding maximum profit of firm 0 is

$$p_0(\hat{p}(p_0), p_0) = \begin{cases} \frac{\bar{v}^2}{4t} & \text{if } 0 \leq \bar{v} < V_1 \\ ((\bar{v}-t) + \frac{\bar{v}}{2-R'})(1 - \frac{\bar{v}}{(2-R')t}) & \text{if } \bar{v} \geq V_1 \end{cases}.$$

(b) If $(\bar{v}-t) + \frac{\bar{v}}{3-2R'} \leq p_0 \leq (\bar{v}-t) + \frac{\bar{v}}{2-R'}$, then firm 0's profit maximizing price is

$$p_0 = \begin{cases} (\bar{v}-t) + \frac{\bar{v}}{2-R'} & \text{if } 0 \leq \bar{v} \leq V_1 \\ \frac{\bar{v}}{2} & \text{if } V_1 < \bar{v} \leq V_2 \\ (\bar{v}-t) + \frac{\bar{v}}{3-2R'} & \text{if } \bar{v} > V_2 \end{cases}.$$

The corresponding maximum profit of firm 0 is

$$p_0(\hat{p}(p_0), p_0) = \begin{cases} ((\bar{v}-t) + \frac{\bar{v}}{2-R'})(1 - \frac{\bar{v}}{(2-R')t}) & \text{if } 0 \leq \bar{v} \leq V_1 \\ \frac{\bar{v}^2}{4t} & \text{if } V_1 < \bar{v} \leq V_2 \\ ((\bar{v}-t) + \frac{\bar{v}}{3-2R'})(1 - \frac{\bar{v}}{(3-2R')t}) & \text{if } \bar{v} \geq V_1 \end{cases}.$$

(c) If $p_0 \leq (\bar{v}-t) + \frac{\bar{v}}{3-2R'}$, then firm 0's profit maximizing price is

$$p_0 = \begin{cases} (\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'} & \text{if } 0 \le \bar{v} \le V_3 \\ \dfrac{(3 - 2R')t}{2} & \text{if } \bar{v} \ge V_3 \end{cases}.$$

The corresponding maximum profit is

$$p_0(\hat{p}(p_0), p_0) = \begin{cases} ((\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'})(1 - \dfrac{\bar{v}}{(3 - 2R')t}) & \text{if } 0 \le \bar{v} \le V_3 \\ \dfrac{((3 - 2R')t)^2}{8t(2 - R')} & \text{if } \bar{v} \ge V_3 \end{cases}.$$

The results of Lemma 2 are summarized in Table 1.

Table 1. Firm 0's Local Profit-Maximizing Prices and Corresponding Profits

$\bar{v}$ / p_0	$0 \le \bar{v} \le V_1$	$V_1 \le \bar{v} \le V_2$	$V_2 \le \bar{v} \le V_3$	$\bar{v} \ge V_3$
Scenario 1: $p_0 \ge (\bar{v} - t) + \dfrac{\bar{v}}{2 - R'}$ ($\hat{p}(p_0) \ge \tilde{p}(p_0)$)	$p_0 = \dfrac{\bar{v}}{2}$ $p_0 = \dfrac{\bar{v}^2}{4t}$	$p_0 = (\bar{v} - t) + \dfrac{\bar{v}}{2 - R'}$ $p_0 = ((\bar{v} - t) + \dfrac{\bar{v}}{2 - R'})(1 - \dfrac{\bar{v}}{(2 - R')t})$		
Scenario 2: $p_0 \le (\bar{v} - t) + \dfrac{\bar{v}}{2 - R'}$ $p_0 \ge (\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'}$ ($\hat{p}(p_0) = \tilde{p}(p_0)$)	$p_0 = (\bar{v} - t) + \dfrac{\bar{v}}{2 - R'}$ $p_0 = ((\bar{v} - t) + \dfrac{\bar{v}}{2 - R'})(1 - \dfrac{\bar{v}}{(2 - R')t})$	$p_0 = \dfrac{\bar{v}}{2}$ $p_0 = \dfrac{\bar{v}^2}{4t}$	$p_0 = (\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'}$ $p_0 = ((\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'})(1 - \dfrac{\bar{v}}{(3 - 2R')t})$	
Scenario 3: $p_0 \le (\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'}$ ($\hat{p}(p_0) \le \tilde{p}(p_0)$)	$p_0 = (\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'}$ $p_0 = ((\bar{v} - t) + \dfrac{\bar{v}}{3 - 2R'})(1 - \dfrac{\bar{v}}{(3 - 2R')t})$			$p_0 = \dfrac{(3 - 2R')t}{2}$ $p_0 = \dfrac{(3 - 2R')^2 t}{8(2 - R')}$

Note: $\tilde{p}(p_0) = 2\bar{v} - t - p_0$ is the price at the kink of the demand curve in market L given p_0.

The rows in table 1 represent various scenarios for firm 0 to choose from. Corresponding to a high p_0 (scenario 1), medium p_0 (scenario 2), and low p_0 (scenario 3), the equilibrium price in market L will be above, exactly at, or below the price at the kink of the demand of market L, and markets L and R will be independent, accommodating, or directly competing with each other, respectively. The columns of the table represent different values of the good. If the value of the good $\bar{v}$ is in $[0, V_1]$, for instance, the profit-maximizing price for firm 0 would be $p_0 = \frac{\bar{v}}{2}$ if he were to choose a price above $(\bar{v} - t) + \frac{\bar{v}}{2 - R'}$ and the corresponding profit would be $p_0 = \frac{\bar{v}^2}{4t}$.

In order to find firm 0's global profit-maximizing price of the good, the local maximum profits of firm 0 in three different scenarios are compared. Lemma 3 are some mathematical results that are useful for the comparison.

Lemma 3.

(a) $\frac{\bar{v}^2}{4t} > ((\bar{v} - t) + \frac{\bar{v}}{3 - 2R'})(1 - \frac{\bar{v}}{(3 - 2R')t})$.

(b) $\frac{\bar{v}^2}{4t} > ((\bar{v} - t) + \frac{\bar{v}}{2 - R'})(1 - \frac{\bar{v}}{(2 - R')t})$.

(c) $((\bar{v} - t) + \frac{\bar{v}}{3 - 2R'})(1 - \frac{\bar{v}}{(3 - 2R')t}) < ((\bar{v} - t) + \frac{\bar{v}}{2 - R'})(1 - \frac{\bar{v}}{(2 - R')t})$ if $\bar{v} \le V_1$ and

$((\bar{v} - t) + \frac{\bar{v}}{3 - 2R'})(1 - \frac{\bar{v}}{(3 - 2R')t}) > ((\bar{v} - t) + \frac{\bar{v}}{2 - R'})(1 - \frac{\bar{v}}{(2 - R')t})$ if $\bar{v} \ge V_2$.

(d) $\frac{((3 - 2R')t)^2}{8t(2 - R')} \ge ((\bar{v} - t) + \frac{\bar{v}}{3 - 2R'})(1 - \frac{\bar{v}}{(3 - 2R')t})$.

With the results of Lemma 3, the global profit-maximizing prices for firm 0 corresponding to different values of the good are easily derived. Specifically, if $0 \le \bar{v} \le V_2$, firm 0 chooses $p_0 = \frac{\bar{v}}{2}$ to maximize its profit and its maximum profit is $p_0 = \frac{\bar{v}^2}{4t}$. If

$V_2 \le \bar{v} \le V_3$, firm 0 chooses $p_0 = (\bar{v} - t) + \frac{\bar{v}}{3-2R'}$ to maximize its profit and its maximum profit is $p_0 = ((\bar{v} - t) + \frac{\bar{v}}{3-2R'})(1 - \frac{\bar{v}}{(3-2R')t})$. If $\bar{v} \ge V_3$, firm 0 chooses $p_0 = \frac{(3-2R')t}{2}$ to maximize its profit and its maximum profit is $p_0 = \frac{(3-2R')^2 t}{8(2-R')}$. Because the duopolists in market L are maximizing their own profits by choosing the prices specified by the reaction function of market L in Lemma 1, once we find the profit-maximizing price for firm 0, we find the corresponding equilibrium prices of the duopolists, and consequently the equilibrium of this three-firm market. Proposition 2 provides the equilibrium prices and profits of the firms that vary with respect to the value of the good.

Proposition 1. In the three-firm market with $t > 0$,

$$(\hat{p}_0, \hat{p}_i (i = 1,2)) = \begin{cases} (\frac{\bar{v}}{2}, \frac{(1-R')\bar{v}}{2-R'}) & \text{if } 0 \le \bar{v} \le V_1 \\ (\frac{\bar{v}}{2}, \frac{3\bar{v}}{2} - t) & \text{if } V_1 \le \bar{v} \le V_2 \\ ((\bar{v} - t) + \frac{\bar{v}}{3-2R'}, \frac{2(1-R')\bar{v}}{3-2R'}) & \text{if } V_2 \le \bar{v} \le V_3 \\ (\frac{(3-2R')t}{2}, \frac{(1-R')(5-2R')t}{2(2-R')}) & \text{if } \bar{v} > V_3 \end{cases}$$

is the unique market equilibrium. The corresponding profits are

$$(p_0, p_i (i = 1,2)) = \begin{cases} (\frac{\bar{v}^2}{4t}, \frac{(1-R')\bar{v}^2}{2t(2-R')^2}) & \text{if } 0 \le \bar{v} \le V_1 \\ (\frac{\bar{v}^2}{4t}, \frac{1}{2}(\frac{3\bar{v}}{2} - t)(1 - \frac{\bar{v}}{2t})) & \text{if } V_1 \le \bar{v} \le V_2 \\ ((\bar{v} - t) + \frac{\bar{v}}{3-2R'})(1 - \frac{\bar{v}}{(3-2R')t}), \frac{(1-R')\bar{v}^2}{(3-2R')^2 t}) & \text{if } V_2 \le \bar{v} \le V_3 \\ (\frac{(3-2R')^2 t}{8(2-R')}, \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}) & \text{if } \bar{v} > V_3 \end{cases}$$

When the value of the good is low ($0 \le \bar{v} \le V_1$), knowing the reaction function of the duopolists in market L, firm 0 chooses not to compete with the duopolists in market L

by selecting a high price which results in two virtually independent markets L and R. Some of the consumers choose not to buy at all because of the low value of the good compared to the high price and transportation cost. When the valuation of the good is medium ($V_1 \le \bar{v} \le V_3$), firm 0 chooses such a price that the duopolists in market L will respond with prices that attract exactly all the consumes who do not buy from firm 0. Only when the value of the good is high ($\bar{v} \ge V_3$), firm 0 chooses a low price to attract a large number of consumers and a direct competition with the duopolists in market L occurs.

Next, we calculate the equilibrium prices and profits of the firms in the duopoly market assuming that firm 0 were to leave the market and firms 1 and 2 were unable to relocate because of the high cost of relocation. Let $\tilde{p}$ be the critical expected price in market L such that the consumer located at $x = 1$ is indifferent between purchasing and not purchasing the good. Then, $\tilde{p} = \bar{v} - t$ which is derived from $U_L = \bar{v} - p - t = 0$. If the expected price in market L, *p*, is higher than $\tilde{p}$, then the demand will be $\frac{\bar{v} - p}{t}$; otherwise it equals one, i.e.,

$$D_L(p) = \begin{cases} \dfrac{\bar{v} - p}{t} & \text{if } p > \tilde{p} \\ 1 & \text{if } p \le \tilde{p} \end{cases} \tag{6}$$

Based on the derived demand function we obtain the following result.

Proposition 2. Without firm 0 in the market, if $\bar{v} \le V_a = (2 - R')t$, then the equilibrium prices are $\hat{p}_1 = \hat{p}_2 = \frac{1 - R'}{2 - R'}\bar{v} > \tilde{p}$ and the equilibrium profits are $p_1 = p_2 = \frac{(1 - R')\bar{v}^2}{2t(2 - R')^2}$; if $\bar{v} > V_a$, then the equilibrium prices are $\hat{p}_1 = \hat{p}_2 = \tilde{p} = \bar{v} - t$ and the equilibrium profits are $p_1 = p_2 = \frac{\bar{v} - t}{2}$.

When firm 0 is not in the market, the duopolists, firm 1 and firm 2, are the only players in the game of price competition. The firms, together with the consumers, will jointly determine the price of the good and the profits of the firms. Note that the demand

curve is kinked at $p = \tilde{p}$ in this case. With consumers who may accidentally buy the good from the higher-price firm, when the consumption value of the good is relatively low the competition between the firms is relatively weak. Consequently, the equilibrium price of the good will be greater than $\tilde{p}$. If the consumption value of the good is relatively high, then the duopolists will compete forcefully with each other and the resulting equilibrium price will be exactly at $\tilde{p}$, which is the lowest price that the duopolists will charge.[3]

4. The effect of the existence of firm 0 on the duopolists' prices and profits, and social welfare

According to the existing oligopoly theory, when the number of firms in a market increases, the level of competition increases and as a result the prices of the good and the profits of the firms decrease in the absence of collusion among the firms. We show that this is not necessarily true. The competition from a spatially differentiated good may increase both the prices and profits of the existing firms. We show this by comparing the prices and profits of the duopolists with and without the existence of firm 0 in the market. The welfare effect of the existence of firm 0 is also presented in this section.

Note that $V_a > V_3$, where $V_a = (2 - R')t$ and $V_3 = \dfrac{(5-2R')(3-2R')t}{2(4-2R')}$. Table 2 lists the prices and profits of the duopolists with and without firm 0 in the market, which are derived in Propositions 1 and 2, where, $\hat{p}_1 = \hat{p}_2 = \hat{p}$ and $p_1 = p_2 = p$.

[3] The price will not go any lower because further reduction in price will not generate higher profit than the profit at $\tilde{p}$.

Table 2. The Duopolists' Prices and Profits with and without Firm 0 in Market

Situations \ $\bar{v}$	$0 \le \bar{v} \le V_1$	$V_1 \le \bar{v} \le V_2$	$V_2 \le \bar{v} \le V_3$	$V_3 \le \bar{v} \le V_a$	$\bar{v} \ge V_a$
Without Firm 0	$\hat{p} = \frac{(1-R')\bar{v}}{2-R'}$ $p = \frac{(1-R')\bar{v}^2}{2t(2-R')^2}$				$\hat{p} = \bar{v} - t$ $p = \frac{\bar{v}-t}{2}$
With Firm 0	$\hat{p} = \frac{(1-R')\bar{v}}{2-R'}$ $p = \frac{(1-R')\bar{v}^2}{2t(2-R')^2}$	$\hat{p} = \frac{3\bar{v}}{2} - t$ $p = \frac{1}{2}(\frac{3\bar{v}}{2} - t)(1 - \frac{\bar{v}}{2t})$	$\hat{p} = \frac{2(1-R')\bar{v}}{3-2R'}$ $p = \frac{(1-R')\bar{v}^2}{(3-2R')^2 t}$	$\hat{p} = \frac{(1-R')(5-2R')t}{2(2-R')}$ $p = \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$	

Proposition 3.

(a) If $0 \le \bar{v} \le V_1$, the existence of firm 0 has no effect on the prices and the profits of the duopolists in market L.

(b) If $V_1 \le \bar{v} \le V_3$, the existence of firm 0 increases both the prices and the profits of the duopolists in market L.

(c) If $V_3 \le \bar{v} \le V_a$, the existence of firm 0 increases the prices of the duopolists. There are four scenarios in terms of the profits of the duopolists. If $0.79 \le R' < 1$, the existence of firm 0 increases the profits. If $0.29 \le R' \le 0.79$ and $V_3 \le \bar{v} \le \frac{(5-2R')t}{2\sqrt{2}}$, the existence of firm 0 increases the profits. If $0.29 \le R' \le 0.79$ but $\frac{(5-2R')t}{2\sqrt{2}} \le \bar{v} \le V_a$, the existence of firm 0 decreases the profits. Finally, if $0 \le R' < 0.29$, the existence of firm 0 decreases the profits.

(d) If $V_a \le \bar{v} \le V_b$, where $V_b = \frac{(9-9R'+2R'^2)t}{2(2-R')} \ge V_a$, the existence of firm 0 increases the prices but decreases the profits of the duopolists in market L.

(e) If $\bar{v} \geq V_b$, the existence of firm 0 decreases both the prices and the profits of the duopolists in market L.

We compare the effects of the existence of firm 0 on the prices and the profits of the firms by the value of the good. When the value of the good is low, i.e., $0 \leq \bar{v} \leq V_1$, firm 0, if it exists, will choose to charge a high price to maximize its profit. Consequently, its demand will be low and the markets L and R will be independent.[4] Therefore, the existence of firm 0 does not have any effects on the prices and the profits of the duopolists in market L.

When $V_1 \leq \bar{v} \leq V_3$, it is not profitable for firm 0, if it exists, to charge a high price to avoid interaction with the duopolists, or to charge a low price to initiate a direct competition with the duopolists in markets L. The profit-maximizing strategy for firm 0 is to charge such a price to induce the duopolists to serve exactly its residual market. Any reduction in price by a duopolist from that, which would be profitable if firm 0 did not exist, would initiate a direct competition with firm 0 and therefore would be unprofitable. As a result, the competition between the duopolists is reduced by the existence of firm 0 and both the prices and the profits of the duopolists are increased.

When $V_3 \leq \bar{v} \leq V_a$ and firm 0 is not in the market, the competition between the duopolists are strong and the prices of the duopolists are low. The competition from firm 0, which will always occur when the value of the good is above V_3, creates additional impedance preventing the duopolists from reducing their prices. Consequently, the prices of the duopolists are higher when firm 0 is in the market. The profits of the duopolists, however, may either be increased or decreased by the existence of firm 0, depending on the amount of reduction in demand resulted from the existence of firm 0. When $0.79 < R' < 1$, consumers are very responsive to the lower price of the duopolists in market L so the duopolists are more willing to lower their prices. Faced with such consumers and duopolists, firm 0 charges a relatively higher price to reduce the competition with the duopolists. As a result, the duopolists' demand are higher, so are

their profits compared to their profits when firm 0 is not in the market. When $0 \le R' \le 0.29$, consumers' responsiveness to the lower price of the duopolists in market L is weak and the result of the previous case is just reversed, i.e., the existence of firm 0 reduces the demand and the profits of the duopolists in market L. When $0.29 \le R' \le 0.79$, consumers' responsiveness to lower price in market L is not decisive. Whether or not the existence of firm 0 increases the profits of the duopolists depends on the value of the good. If the value is relatively low, i.e., $V_3 \le \bar{v} \le \frac{(5-2R')t}{2\sqrt{2}}$, firm 0 charges a relatively higher price. The demand and the profits of the duopolists are greater. If $\frac{(5-2R')t}{2\sqrt{2}} \le \bar{v} \le V_a$, then, firm 0 charges a relatively lower price. The demand and the profits of the duopolists in market L are lower.

When $V_a \le \bar{v} \le V_b$, the prices of the duopolists are still higher when firm 0 is in the market. But, the profits of the duopolists are lowered by the existence of firm 0 despite the fact that the prices are higher. The reason is that the duopolists serve the whole city when firm 0 is not in the market and the existence of firm 0 dramatically reduces the demand for the duopolists.

When $\bar{v} \ge V_b$, the duopolists serve the whole city if firm 0 is not in the market. With such a high value of the good, the prices of the duopolists are lowered by the existence of firm 0 because of the extra competition from firm 0. The profits of the duopolists are also lowered by the existence of firm 0 because both the demand and the prices are lowered.

Proposition 3 shows that the existence of a distant firm may have no effect on the prices and profits of the duopolists in market L, increase both the prices and profits of the duopolists, increase the prices but reduce the profits, or reduce both the prices and profits of the duopolists. Which result would occur depends mainly on the value of the good. Proposition 3 indicates that it may be good for duopolists engaging in stiff competition to have a distant firm to compete with them since the competition from a distant firm may

[4] Due to the low value of the good, competition between the markets may never materialize regardless of what firm 0 does.

provide extra impedance for the duopolists to reduce their prices, therefore increase the duopolists' prices and profits.

To provide some concrete ideas about the intervals for the value of the good specified in Proposition 3, we calculate the values of V_1, V_3, V_a and V_b for $R'=0$, 0.5 and 1, respectively, and list them in the following table.

Table 3. The Examples of the V_1, V_3, V_a and V_b Values

	V_1	V_3	V_a	V_b
$R'=0$	t	$1.875t$	$2t$	$2.25t$
$R'=0.5$	$0.86t$	$1.33t$	$1.5t$	$1.67t$
$R'=1$	$0.67t$	$0.75t$	T	t

Consider the case $R'=0$ as an example. $R'=0$ means that consumers are totally unresponsive to the lower price in market L. In this case if the value of the good is less than t, the transportation cost of crossing the city, then the prices and the profits of the duopolists in market L are independent of the existence of firm 0. If $\bar{v}$ is between t and $1.875t$, then, both the prices and the profits of the duopolists are increased as a result of the existence of firm 0. If $\bar{v}$ is between $1.875t$ and $2.25t$, the prices of the duopolists are increased but the profits are lowered by the existence of firm 0. Only when the value of the good is greater than $2.25t$, the competition from firm 0 causes lower prices and lower profits for the duopolists.

Notice that in table 3 when R' is greater, the intervals $[0, V_1]$ and $[V_1, V_3]$ are smaller but $[V_b, \infty)$ is larger. Proposition 4 shows that this is always true when $0 \le R' < 1$.

Proposition 4: The intervals of the value of the good $[0, V_1]$ and $[V_1, V_3]$ are negatively related to R'; the interval $[V_b, \infty)$ is positively related to R'.

Note that R' measures the consumers' responsiveness to the lower price in market L. Higher R' implies greater responsiveness of consumers to the lower price in market L. Proposition 4 states that when consumers are more responsive to the lower price in market L, the prices and profits of the duopolists in market L are less likely to be independent of the existence of firm 0 since $[0, V_1]$ is smaller; the prices and profits of the duopolists are less likely to be increased simultaneously by the existence of firm 0 since $[V_1, V_3]$ is smaller, but they are more likely to be decreased simultaneously by the existence of firm 0 since $[V_b, \infty)$ is greater, and vice versa (under the assumption that the valuation of good is a random draw from a uniform distribution).

Next we explore the effect of the existence of firm 0 on social welfare. To begin with, we derive the demand functions of the markets with and without the existence of firm 0, respectively, using the formula of prices and profits listed in tables 1 and 2. The results are summarized in Table 4.

Table 4. The Demand in the Markets with and without the Existence of Firm 0

Situations \ $\bar{v}$	$0 \le \bar{v} \le V_1$	$V_1 \le \bar{v} \le V_2$	$V_2 \le \bar{v} \le V_3$	$V_3 \le \bar{v} \le V_a$	$\bar{v} \ge V_a$
Without Firm 0	$D_L = \frac{\bar{v}}{(2-R')t}$				$D_L = 1$
With Firm 0	$D_R = \frac{\bar{v}}{2t}$ $D_L = \frac{\bar{v}}{(2-R')t}$	$D_R = \frac{\bar{v}}{2t}$ $D_L = 1 - \frac{\bar{v}}{2t}$	$D_R = 1 - \frac{\bar{v}}{(3-2R')t}$ $D_L = \frac{\bar{v}}{(3-2R')t}$	$D_R = \frac{3-2R'}{4(2-R')}$ $D_L = \frac{5-2R'}{4(2-R')}$	

Based on the demand information listed in Table 4, total social welfares are calculated for the two-firm market when firm 0 is not in the market and the three-firm market, respectively, and compared. The result is given in Proposition 5.

Proposition 5: Independent of the valuation of the good, social welfare is always higher when firm 0 is in the market.

There are two fundamental reasons why the presence of firm 0 increases total social welfare. First, the existence of firm 0 in market R reduces the transportation cost of the consumers. Second, the existence of firm 0 opens the market to the consumers who otherwise would not purchase the good.

As is shown in sections 3 and 4, firm 0 acts as a leader in the three-firm pricing game because of its advantageous location in the city. The next proposition shows that firm 0 benefits from its leadership position.

Proposition 6. (a) If $0 \le R' < 0.5$, then $p_0 < \hat{p}$ for $\bar{v} \in [t, \frac{(3-2R')t}{2}]$ and $p_0 > \hat{p}$ for all other values of $\bar{v}$; if $0.5 < R' < 1$, then $p_0 > \hat{p}$ for all $\bar{v} \in (0, \infty)$.

(b) For all $0 \le R' < 1$ and $\bar{v} \in (0, \infty)$, $p_0 > p_1 = p_2$.

Proposition 6 states that the profit of firm 0 is always greater than the profit of a duopolist in market L, which is a reflection of its advantageous location in the city. The price charged by firm 0, however, may be lower than the prices charged by the duopolists in market L, which happens when consumers are not very responsive to the lower price in market L and the value of the good is between *t*, the transportation cost from market L to market R, and $\frac{(3-2R')t}{2}$.

6. Conclusion

The main purpose of the paper is to show a theoretical possibility that the competition from a distant firm, which creates an additional impedance for duopolists to reduce their prices, may increase both the prices and the profits of the firms engaging in

stiff competition in a duopoly market. Since firms may make more profits when there are three firms instead two in the market, “three may be better than two.”

Note that the goods produced by the firms in this paper are assumed to be identical, but they are indeed horizontally differentiated. The goods in the duopoly market are differentiated by consumers’ random preferences. The good produced by the distant firm is spatially differentiated from the goods produced by the firms in the duopoly market. Therefore, the result of the paper can be restated as that the additional competition from a spatially differentiated good may increase the profits of the firms that produce horizontally differentiated goods in a duopoly market. This result is derived using a stylized linear-city model in this paper, but it may be generally true that the competition from a differentiated good may increase the profits of the firms that produce differentiated goods.

Appendix

Proof of Lemma 1. (a) $p_0 > (\bar{v} - t) + \frac{\bar{v}}{2 - R'}$ if and only if $1 - R' > \frac{2\bar{v} - t - p_0}{t + p_0 - \bar{v}}$. By (3), $\frac{2\bar{v} - t - p_0}{t + p_0 - \bar{v}}$ equals the elasticity of demand at $\tilde{p}(p_0)$ when p approaches $\tilde{p}(p_0)$ from above, $e_{\tilde{p}^+(p_0)}$. Therefore, solving $e_p = 1 - R'$, where $e_p = \frac{p}{\bar{v} - p}$, we get the equilibrium price in market L $\hat{p}(p_0) = \frac{1 - R'}{2 - R'}\bar{v}$. Since $e_{\hat{p}(p_0)} = 1 - R' > \frac{2\bar{v} - t - p_0}{t + p_0 - \bar{v}} = e_{\tilde{p}^+(p_0)}$, we have $\hat{p}(p_0) > \tilde{p}(p_0)$. Because there is no competition between markets L and R, the demand in market R is $\frac{\bar{v} - p_0}{t}$ and the profit of firm 0 is $p_0 \frac{\bar{v} - p_0}{t}$.

(b) $(\bar{v} - t) + \frac{\bar{v}}{3 - 2R'} \le p_0 \le (\bar{v} - t) + \frac{\bar{v}}{2 - R'}$ if and only if $\frac{2\bar{v} - t - p_0}{2(t + p_0 - \bar{v})} \le 1 - R' \le \frac{2\bar{v} - t - p_0}{t + p_0 - \bar{v}}$. By (3), $\frac{2\bar{v} - t - p_0}{2(t + p_0 - \bar{v})} = e_{\tilde{p}^-(p_0)}$, and $\frac{2\bar{v} - t - p_0}{t + p_0 - \bar{v}} = e_{\tilde{p}^+(p_0)}$. Therefore, the equilibrium price in market L $\hat{p}(p_0) = \tilde{p}(p_0)$. Since there is no competition between markets L and R, the profit of firm 0 is still $p_0 \frac{\bar{v} - p_0}{t}$.

(c) $p_0 < (\bar{v} - t) + \frac{\bar{v}}{3 - 2R'}$ if and only if $1 - R' < \frac{2\bar{v} - t - p_0}{2(t + p_0 - \bar{v})}$. By (3), $\frac{2\bar{v} - t - p_0}{2(t + p_0 - \bar{v})} = e_{\tilde{p}^-(p_0)}$. Therefore, solving $e_p = 1 - R'$, where $e_p = \frac{p}{t + p_0 - p}$, we get the equilibrium price in market L, $\hat{p}(p_0) = \frac{1 - R'}{2 - R'}(t + p_0)$. By $e_{\hat{p}(p_0)} = 1 - R' < \frac{2\bar{v} - t - p_0}{2(t + p_0 - \bar{v})} = e_{\tilde{p}^-(p_0)}$, we obtain $\hat{p}(p_0) < \tilde{p}(p_0)$. Because in this case

the demand for firm 0 is $\frac{t+\hat{p}(p_0)-p_0}{2t}$ which equals $\frac{(3-2R')t-p_0}{2t(2-R')}$, the profit of firm 0 is $p_0\frac{(3-2R')t-p_0}{2t(2-R')}$. Q.E.D.

Proof of Lemma 2. (a) By Lemma 1, if $p_0 \geq (\bar{v}-t)+\frac{\bar{v}}{2-R'}$, $p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}$. Maximizing p_0 we get $p_0=\frac{\bar{v}}{2}$. If $\frac{\bar{v}}{2}>(\bar{v}-t)+\frac{\bar{v}}{2-R'}$ or equivalently $\bar{v}<V_1$, then the profit-maximizing price is $p_0=\frac{\bar{v}}{2}$ and the maximum profit is $p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}=\frac{\bar{v}^2}{4t}$. If $\frac{\bar{v}}{2}\leq(\bar{v}-t)+\frac{\bar{v}}{2-R'}$ or equivalently $\bar{v}\geq V_1$, then the profit maximizing price is $p_0=(\bar{v}-t)+\frac{\bar{v}}{2-R'}$ and the corresponding profit is

$$p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}=((\bar{v}-t)+\frac{\bar{v}}{2-R'})(1-\frac{\bar{v}}{(2-R')t}).$$

(b) By Lemma 1, if $(\bar{v}-t)+\frac{\bar{v}}{3-2R'}\leq p_0\leq(\bar{v}-t)+\frac{\bar{v}}{2-R'}$, $p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}$. Maximizing p_0 we get $p_0=\frac{\bar{v}}{2}$. If $\frac{\bar{v}}{2}\geq(\bar{v}-t)+\frac{\bar{v}}{2-R'}$ or equivalently $\bar{v}\leq V_1$, then the profit maximizing price is $p_0=(\bar{v}-t)+\frac{\bar{v}}{2-R'}$ and the corresponding profit is

$$p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}=((\bar{v}-t)+\frac{\bar{v}}{2-R'})(1-\frac{\bar{v}}{(2-R')t}).$$ If

$(\bar{v}-t)+\frac{\bar{v}}{3-2'}\leq\frac{\bar{v}}{2}\leq(\bar{v}-t)+\frac{\bar{v}}{2-R'}$ or equivalently $V_1\leq\bar{v}\leq V_2$, then the profit maximizing price $p_0=\frac{\bar{v}}{2}$ and $p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}=\frac{\bar{v}^2}{4t}$. If $\frac{\bar{v}}{2}\leq(\bar{v}-t)+\frac{\bar{v}}{3-2R'}$ or equivalently $\bar{v}\geq V_2$, then, the profit maximizing price is $p_0=(\bar{v}-t)+\frac{\bar{v}}{3-2R'}$ and the corresponding profit is $p_0(\hat{p}(p_0),p_0)=p_0\frac{\bar{v}-p_0}{t}=((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t})$.

(c) By Lemma 1, if $p_0 \le (\bar{v}-t)+\frac{\bar{v}}{3-2R'}$, $p_0(\hat{p}(p_0),p_0)=p_0\frac{(3-2R')t-p_0}{2t(2-R')}$.

Maximizing p_0 we get $p_0=\frac{(3-2R')t}{2}$. If $\frac{(3-2R')t}{2}\ge(\bar{v}-t)+\frac{\bar{v}}{3-2R'}$ or equivalently $\bar{v}\le V_3$, then the profit maximizing price is $p_0=(\bar{v}-t)+\frac{\bar{v}}{3-2R'}$ and the corresponding profit is $p_0(\hat{p}(p_0),p_0)=p_0\frac{(3-2R')t-p_0}{2t(2-R')}=((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t})$.

If $\frac{(3-2R')t}{2}\le(\bar{v}-t)+\frac{\bar{v}}{3-2R'}$ or equivalently $\bar{v}\ge V_3$, then $p_0=\frac{(3-2R')t}{2}$ and $p_0(\hat{p}(p_0),p_0)=p_0\frac{(3-2R')t-p_0}{2t(2-R')}=\frac{((3-2R')t)^2}{8t(2-R')}$. Q.E.D.

Proof of Lemma 3. (a) $\frac{\bar{v}^2}{4t}>((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t})$ is equivalent to,

$(3-2R')^2\bar{v}^2>4((4-2R')\bar{v}-A)(A-\bar{v})$, where $A=(3-2R')t$. But,

$(3-2R')^2\bar{v}^2-4((4-2R')\bar{v}-A)(A-\bar{v})=((5-2R')\bar{v}-2A)^2>0$. Therefore (a) holds.

(b) $\frac{\bar{v}^2}{4t}>((\bar{v}-t)+\frac{\bar{v}}{2-R'})(1-\frac{\bar{v}}{(2-R')t})$ is equivalent to,

$(2-R')^2\bar{v}^2>4((3-R')\bar{v}-B)(B-\bar{v})$, where $B=(2-R')t$. But,

$(2-R')^2\bar{v}^2-4((3-R')\bar{v}-B)(B-\bar{v})=((4-R')\bar{v}-2B)^2>0$. Therefore (b) holds.

(c) Let $g(x)=((\bar{v}-t)+\bar{v}x)(1-\frac{\bar{v}}{t}x)$, then $g'(x)=\frac{\bar{v}}{t}(2t-\bar{v}-2\bar{v}x)$ so that $g(x)$ reaches its maximum at $x=\frac{2t-\bar{v}}{2\bar{v}}$. If $\bar{v}\le V_1$, then $\frac{1}{3-2R'}<\frac{1}{2-R'}\le\frac{2t-\bar{v}}{2\bar{v}}$, hence,

$g(\frac{1}{3-2R'})<g(\frac{1}{2-R'})$, or

$((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t})<((\bar{v}-t)+\frac{\bar{v}}{2-R'})(1-\frac{\bar{v}}{(2-R')t})$. If $\bar{v}\ge V_2$, then

$\frac{2t-\bar{v}}{2\bar{v}} \le \frac{1}{3-2R'} < \frac{1}{2-R'}$, hence, $g(\frac{1}{3-2R'}) > g(\frac{1}{2-R'})$, or

$((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t}) > ((\bar{v}-t)+\frac{\bar{v}}{2-R'})(1-\frac{\bar{v}}{(2-R')t})$.

(d) $\frac{((3-2R')t)^2}{8t(2-R')} \ge ((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t})$ is equivalent to

$A^2 \ge \frac{8(2-R')}{(3-2R')^2}((4-2R')\bar{v}-A)(A-\bar{v})$, where $A=(3-2R')t$, or

$((5-2R')A-4(2-R')\bar{v})^2 \ge 0$, which always holds. Q.E.D.

Proof of Proposition 1. If $\bar{v} \in [0,V_1]$, by Lemma 3 (a) and (b), $\hat{p}_0 = \frac{\bar{v}}{2}$ ($p_0 = \frac{\bar{v}^2}{4t}$) is the profit-maximizing price for firm 0 and the corresponding equilibrium prices for the duoplolists are $\hat{p}_1 = \hat{p}_2 = \frac{(1-R')\bar{v}}{2-R'}$. The profits of the duopolists are $p_1 = p_2 = \frac{1}{2}\hat{p}_1\frac{\bar{v}-\hat{p}_1}{t} = \frac{(1-R')\bar{v}^2}{2t(2-R')^2}$. If $\bar{v} \in [V_1,V_2]$, by Lemma 3 (a) and (b), $\hat{p}_0 = \frac{\bar{v}}{2}$ ($p_0 = \frac{\bar{v}^2}{4t}$) is the profit-maximizing price for firm 0 and the corresponding equilibrium prices for the duopolists are $\hat{p}_1 = \hat{p}_2 = \tilde{p}(\hat{p}_0) = \frac{3\bar{v}}{2} - t$. The profits of the duopolists are $p_1 = p_2 = \frac{1}{2}\hat{p}_1\frac{\bar{v}-\hat{p}_1}{t} = \frac{1}{2}(\frac{3\bar{v}}{2}-t)(1-\frac{\bar{v}}{2t})$. If $\bar{v} \in [V_2,V_3]$, by Lemma 3 (c), firm 0's profit-maximizing price is $\hat{p}_0 = (\bar{v}-t)+\frac{\bar{v}}{3-2R'}$ ($p_0 = ((\bar{v}-t)+\frac{\bar{v}}{3-2R'})(1-\frac{\bar{v}}{(3-2R')t})$) and the corresponding equilibrium prices in market L are $\hat{p}_1 = \hat{p}_2 = \tilde{p}(\hat{p}_0) = \frac{2(1-R')\bar{v}}{3-2R'}$. The profits of the duopolists are $p_1 = p_2 = \frac{1}{2}\hat{p}_1\frac{\bar{v}-\hat{p}_1}{t} = \frac{(1-R')\bar{v}^2}{(3-2R')^2 t}$. If $\bar{v} \in [V_3,\infty]$, by Lemma 3 (d) and (c), $\hat{p}_0 = \frac{(3-2R')t}{2}$ ($p_0 = \frac{(3-2R')^2 t}{8(2-R')}$) is the profit-maximizing price

for firm 0 and the corresponding equilibrium price in market L is $\hat{p}_1 = \hat{p}_2 = \frac{(1-R')(5-2R')t}{2(2-R')}$ which is solved from $1-R' = e_p = \frac{\hat{p}}{t+\hat{p}_0-\hat{p}}$ because in this case there is a direct competition between the firms in markets L and R. The profits of the duopolists are $p_1 = p_2 = \frac{1}{2}\hat{p}_1 \frac{\hat{p}_0 + t - \hat{p}_1}{2t} = \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$. Q.E.D.

Proof of Proposition 2. By (6), the elasticity of demand is $e_p = \begin{cases} \frac{p}{\bar{v}-p} & \text{if } p > \tilde{p} \\ 0 & \text{if } p \le \tilde{p} \end{cases}$, where $\tilde{p} = \bar{v} - t$. Solving $e_p = \frac{p}{\bar{v}-p} = 1-R'$ we get $\hat{p} = \frac{1-R'}{2-R'}\bar{v}$. When $\bar{v} \le V_a = (2-R')t$, $\tilde{p} \le \hat{p}$. Thus, the non-trivial equilibrium price in market L is $\hat{p} = \frac{1-R'}{2-R'}\bar{v}$. The profits are $p_1 = p_2 = \hat{p}D_L(\hat{p}) = \frac{(1-R')\bar{v}^2}{2t(2-R')^2}$. When $\bar{v} > V_a$, the non-trivial equilibrium price is $\hat{p} = \tilde{p} = \bar{v} - t$ because at $\tilde{p}$, $e_{p^+} = \frac{\bar{v}-t}{t} \ge 1-R' \ge e_{p^-} = 0$. The equilibrium profits are $p_1 = p_2 = \frac{\bar{v}-t}{2}$ since the demand in this case is 1. Q.E.D.

Proof of Proposition 3. First, we compare the prices with and without firm 0 in the market (see table 2 for reference). When $\bar{v} \in [0, V_1]$, the prices are the same. When $\bar{v} \in [V_1, V_2]$, $\bar{v} > V_1 = \frac{2(2-R')t}{4-R'}$ which implies that $\frac{3\bar{v}}{2} - t > \frac{(1-R')\bar{v}}{2-R'}$. When $\bar{v} \in [V_2, V_3]$, it is obvious that $\frac{2(1-R')\bar{v}}{3-2R'} > \frac{(1-R')\bar{v}}{2-R'}$. When $\bar{v} \in [V_3, V_a]$, $\bar{v} < V_a = (2-R')t < \frac{(5-2R')t}{2}$, thus, $\frac{(1-R')(5-2R')t}{2(2-R')} > \frac{(1-R')\bar{v}}{2-R'}$. When $\bar{v} \in [V_a, V_b]$,

$\bar{v} < V_b = \frac{(9-9R'+2R'^2)t}{2(2-R')}$ which implies that $\bar{v} - t < \frac{(1-R')(5-2R')t}{2(2-R')}$. When $\bar{v} \in [V_b, \infty)$, $\bar{v} > V_b$, which implies that $\bar{v} - t > \frac{(1-R')(5-2R')t}{2(2-R')}$.

Second, we compare the profits with and without firm 0 in the market. When $\bar{v} \in [0, V_1]$, the profits are the same. When $\bar{v} \in [V_1, V_3]$, both equilibria lie on the upper portion of the demand curve $D(p) = \frac{\bar{v} - p}{t}$. Because both prices are below the monopoly price and the price is higher when firm 0 is in the market, by monotonicity, the profits are higher when firm 0 is in the market. For $\bar{v} \in [V_3, V_a]$, we need to consider three scenarios: $0.79 < R' < 1$, $0 \le R' < 0.29$ and $0.29 \le R' \le 0.79$. If $0.79 < R' < 1$, then $V_a < \frac{(5-2R')t}{2\sqrt{2}}$, therefore, for all $\bar{v} \in [V_3, V_a]$, $\bar{v} \le V_a < \frac{(5-2R')t}{2\sqrt{2}}$, or $\frac{(1-R')\bar{v}^2}{2t(2-R')^2} < \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$. If $0 \le R' < 0.29$, then $V_3 > \frac{(5-2R')t}{2\sqrt{2}}$, therefore, for all $\bar{v} \in [V_3, V_a]$, $\bar{v} \ge V_3 > \frac{(5-2R')t}{2\sqrt{2}}$, or $\frac{(1-R')\bar{v}^2}{2t(2-R')^2} > \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$. If $0.29 \le R' \le 0.79$, then $V_3 \le \frac{(5-2R')t}{2\sqrt{2}} \le V_a$. In this case, if $\bar{v} \in [V_3, \frac{(5-2R')t}{2\sqrt{2}}]$, then $\frac{(1-R')\bar{v}^2}{2t(2-R')^2} < \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$, if $\bar{v} \in [\frac{(5-2R')t}{2\sqrt{2}}, V_a]$, then $\frac{(1-R')\bar{v}^2}{2t(2-R')^2} > \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$. When $\bar{v} \in [V_a, \infty]$, $\bar{v} \ge V_a > \frac{57-49R'-4R'^2}{8(2-R')^2}$ which is equivalent to $\frac{\bar{v}-t}{2} > \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$. Q.E.D.

Proof of Proposition 4. Since $\frac{dV_1}{dR'} = \frac{d}{dR'}(\frac{2(2-R')t}{4-R'}) = -\frac{4t}{(4-R')^2} < 0$, $[0, V_1]$ is negatively related to R'. Since $\frac{dV_b}{dR'} = \frac{d}{dR'}(\frac{(9-9R'+2R'^2)t}{2(2-R')}) = \frac{t}{2(2-R')^2}(-9+8R'-2R'^2) < 0$, $[V_b, \infty)$ is positively

related to R'. $V_3 - V_1 = \frac{(5-2R')(3-2R')t}{4(2-R')} - \frac{2(2-R')t}{4-R'}$. Let $y = 2-R'$. Then, $V_3 - V_1 = \frac{t}{4}(\frac{4y^3 - y - 2}{y(2+y)})$. Since $\frac{d(V_3 - V_1)}{dy} = \frac{t}{4}(\frac{4y^4 + 16y^3 + 3y^2 + 4y + 4}{y^2(2+y)^2}) > 0$, $\frac{d(V_3 - V_1)}{dR'} = \frac{d(V_3 - V_1)}{dy}\frac{dy}{dR'} < 0$. Therefore, $[V_1, V_3]$ is negatively related to R'. Q.E.D.

Proof of Proposition 5. The coverage of the markets is illustrated in Figure 1. If $\bar{v} \in [0, V_1]$, the existence of firm 0 in the market does not change the behaviors of the duopolists so the coverage of market L remains the same. On the other hand, the existence of firm 0 creates a new market for the consumers who are located near firm 0. Therefore, social welfare is increased by the existence of firm 0. If $\bar{v} \in [V_1, V_2]$, then $\frac{\bar{v}}{(2-R')t} > 1 - \frac{\bar{v}}{2t}$, where $[0, \frac{\bar{v}}{(2-R')t}]$ is the coverage of market L when firm 0 is not in the market and $[0, 1 - \frac{\bar{v}}{2t}]$ is the coverage of market L when firm 0 is in the market. The welfare gains from trade with the consumers who are located to the left of $1 - \frac{\bar{v}}{2t}$ are the same with or without the existence of firm 0 because the transportation costs of the consumers are the same. The welfare gains from trade with the consumers located between $1 - \frac{\bar{v}}{2t}$ and $\frac{\bar{v}}{(2-R')t}$, a total sale of $\frac{\bar{v}}{(2-R')t}$-$(1 - \frac{\bar{v}}{2t})$, when firm 0 is not in the market are lower than the welfare gains from trade with the consumers located to the right of $1 - (\frac{\bar{v}}{(2-R')t} - (1 - \frac{\bar{v}}{2t}))$, a total sale of $\frac{\bar{v}}{(2-R')t}$-$(1 - \frac{\bar{v}}{2t})$, because of the lower transportation costs. In addition, the existence of firm 0 increases trade by $1 - \frac{\bar{v}}{(2-R')t}$. Therefore, the existence of firm 0 increases social welfare. Using the same arguments we can prove the result for $\bar{v} \in [V_2, V_3]$ and $\bar{v} \in [V_3, V_a]$. For $\bar{v} \in [V_a, \infty]$, the result is true simply because the existence of firm 0 reduces consumers' transportation costs. Q.E.D.

Proof of Proposition 6. (a) If $\bar{v} \in [0, V_1]$, $p_0 = \frac{\bar{v}}{2} > \frac{(1-R')\bar{v}}{2-R'} = \hat{p}$ for all $0 < R' < 1$. If $\bar{v} \in [V_1, V_2]$, then $p_0 = \frac{\bar{v}}{2}$ and $\hat{p} = \frac{3\bar{v}}{2} - t$, and $p_0 > \hat{p}$ if and only if $t > \bar{v}$. When $0 < R' < 0.5$, $t \in (V_1, V_2)$, hence, $p_0 > \hat{p}$ for $\bar{v} \in [V_1, t)$ and $p_0 < \hat{p}$ for $\bar{v} \in (t, V_2]$. When $0.5 < R' < 1$, $t > V_2 \geq \bar{v}$, hence, $p_0 > \hat{p}$ for all $\bar{v} \in [V_1, V_2]$. If $\bar{v} \in [V_2, V_3]$, $p_0 = (\bar{v} - t) + \frac{\bar{v}}{3-2R'}$ and $\hat{p} = \frac{2(1-R')\bar{v}}{3-2R'}$, and $p_0 > \hat{p}$ if and only if $\bar{v} > \frac{(3-2R')t}{2}$. When $0 < R' < 0.5$, $\frac{(3-2R')t}{2} \in (V_2, V_3)$, hence, $p_0 < \hat{p}$ for $\bar{v} \in [V_2, \frac{(3-2R')t}{2})$ and $p_0 > \hat{p}$ for $\bar{v} \in (\frac{(3-2R')t}{2}, V_3]$. When $0.5 < R' < 1$, $\frac{(3-2R')t}{2} < V_2 \leq \bar{v}$, hence, $p_0 > \hat{p}$ for all $\bar{v} \in [V_2, V_3]$. If $\bar{v} \in [V_3, \infty)$, then $p_0 = \frac{(3-2R')t}{2} > \frac{(1-R')(5-2R')t}{2(2-R')} = \hat{p}$ for all $0 < R' < 1$. Therefore, if $0 < R' < 0.5$ and $\bar{v} \in [t, \frac{(3-2R')t}{2}]$, then $p_0 < \hat{p}$; otherwise, $p_0 > \hat{p}$.

(b) If $\bar{v} \in [0, V_1]$, then $p_0 = \frac{\bar{v}^2}{4t} > \frac{(1-R')\bar{v}^2}{2t(2-R')^2} = p$. If $\bar{v} \in [V_1, V_2]$, then, $p_0 = \frac{\bar{v}^2}{4t} > \frac{1}{2}(\frac{3\bar{v}}{2} - t)(1 - \frac{\bar{v}}{2t}) = p$. If $\bar{v} \in [V_2, V_3]$, $p_0 = ((\bar{v} - t) + \frac{\bar{v}}{3-2R'})(1 - \frac{\bar{v}}{(3-2R')t})$ and $p = \frac{(1-R')\bar{v}^2}{(3-2R')^2 t}$. $p_0 > p$ if and only if $f(K) = (5-3R')K^2 - (3-2R')(5-2R')K + (3-2R')^2 < 0$, where $K = \frac{\bar{v}}{t}$. Since $V_2 \leq \bar{v} \leq V_3$, $\frac{V_2}{t} \leq K \leq \frac{V_3}{t}$. It is easy to show that $f(\frac{V_2}{t}) = -(3-2R')^2(1 + 4(1-R')^2) < 0$ and $f(\frac{V_3}{t}) = -\frac{(3-2R')^2}{16(2-R')^2} g(R')$, where $g(R') = 11 - 21R' + 16R'^2 - 4R'^3$. Since the solutions to $g'(R') = 0$ are $R' = 1.16, 1.5$ and $g'(R')$ is concave, $g'(R') < 0$ when $0 < R' < 1$, hence, $g(R') > g(1) = 2 > 0$. Therefore

$f(\frac{V_3}{t}) < 0$. But, $f(K)$ is convex when $0 < R' < 1$, therefore $f(K) < 0$ for $0 < R' < 1$. In other words, $p_0 > p$ for all $\bar{v} \in [V_2, V_3]$. If $\bar{v} \in [V_3, \infty)$, $p_0 = \frac{(3-2R')^2 t}{8(2-R')}$ and $p = \frac{(1-R')(5-2R')^2 t}{16(2-R')^2}$. $p_0 > p$ if and only if $g(R') = 11 - 21R' + 16R'^2 - 4R'^3 > 0$ which has been shown to be true. Q.E.D.

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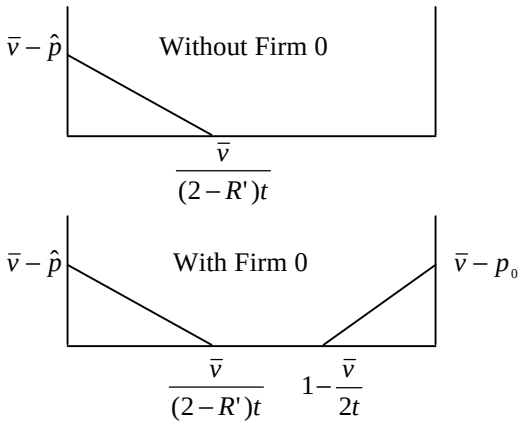
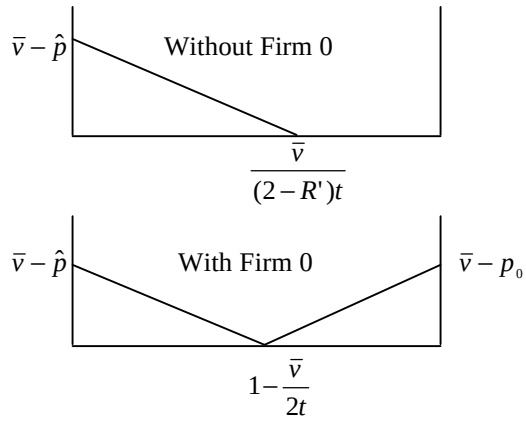
Figure 1. The Coverage of the Markets by the Value of the Good[5]

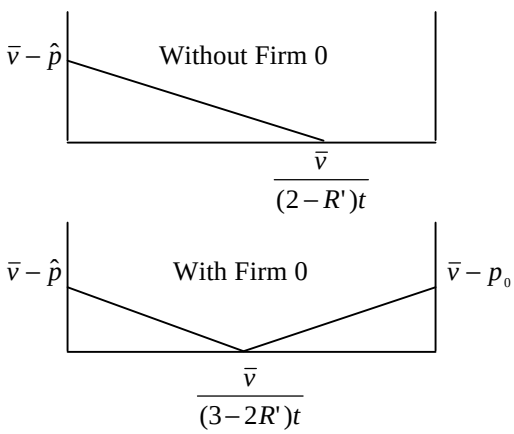
(a) $\mathbf{0 \le \bar{v} \le V_1}$



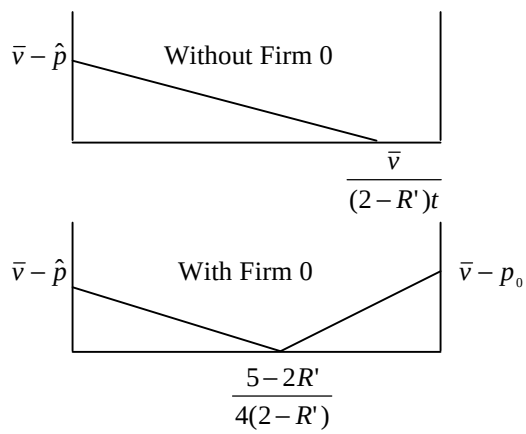
(b) $\mathbf{V_1 \le \bar{v} \le V_2}$



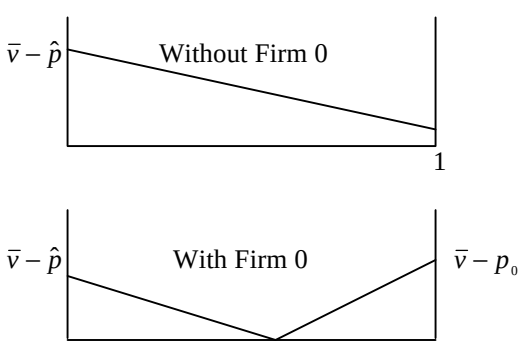
(c) $\mathbf{V_2 \le \bar{v} \le V_3}$



(d) $\mathbf{V_3 \le \bar{v} \le V_a}$



(e) $\mathbf{\bar{v} \ge V_á}$



[5] The vertical axis measures the consumer's net utility of buying the good.

$$\frac{5-2R'}{4(2-R')}$$