A General Theory of Employment, Interest and Money: with MSIAH VECM Markov-Switching Evidence

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Abstract

This paper delivers a theory and estimation of a Vector Error Correction Model with one cointegrating relation for the US federal funds rate from 1960 to 2012. Interest rates are explained here by the M2 money supply growth rate, the unemployment rate and the inflation rate. Regime shifts are identified through Markov Switching analysis with three key regimes resulting: 1) one similar to NBER contractions, 2) one similar to NBER expansions, and 3) one similar to negative real interest rate periods including most of the post 2000 "Unconventional" period. Each regime is characterized not only by its transition probability but also its speed of adjustment in returning to equilibrium after shocks. Results indicate that the stochastic monetary process strongly explains postwar interest rates along with inflation and unemployment, in a way consistent with certain DSGE theory. Empirically, the nominal interest rate relationship loses it typically cashless character.

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1 Introduction

Lucas and Stokey (1983) provide the foundation for why inflation rate targeting has become the dominant international monetary policy. Successful Inflation targeting turns nominal government debt into real debt, eliminates sudden expropriation of lenders' capital through inflation increases, and allows for optimal tax smoothing of both fiscal taxes and the inflation tax. How to do inflation targeting has evolved from a monetary approach to the Taylor (1993) rule and its extensions. This shift towards interest rate targeting occured in the wake of the view that money supply targeting could not succede in inflation targeting because money demand was viewed to be instable after the financial deregulation that began in the early 1980s. Subsequently taking money supply out of the monetary policy models became the standard approach until the recent financial sector collapse and near 1930's style contraction.

It is well known that the Taylor rule in its simplest static form can be derived from the balanced growth path equilibrium of standard intertemporal euler conditions of the nominal interest rate within standard monetary models, albeit with a coefficient of one on the inflation rate, Irving Fisher style. However by endogenizing velocity in a cash-in-advance economy, Davies et al. (2013) show that the coefficient on inflation, for the log-linearized, dynamic Euler equation as solved for the nominal interest rate in a forward looking form, is always greater than one for positive nominal interest rates. This reproduces the above-one inflation rate parameter in an equilibrium condition that in a different appproach is viewed as a key part of the Taylor rule, this being the so-called Taylor principle. Davies et al. show that estimation of a "Taylor rule" in such an economy can only spurriosly be called a reaction function since they estimate only an equilibrium condition in an economy in which the central bank supplies money at a stochastic rate of growth.

This paper pushes this further by introducing money supply growth directly into the nominal interest rate dynamic equilibrium condition. This results by combining the intertemporal Euler equation for the nominal interest rate with the cash-in-advance constraint so as to bring the money supply growth rate directly into the Euler condition. Here the inflation rate is targeted successfully by a long run stationary mean for the money supply growth rate process, as qualified by allowing a theoretic drift in the mean to occur as is found in US postwar data. The paper then turns to US postwar data and estimates this nominal interest rate model with the money supply growth rate as a varable. It results that the money supply growth rate is a key fourth variable in a cointegrating relation among the nominal interest rate, the inflation rate, the unemployment rate and the money supply growth rate. These four variables are found to be all statistically integrated of order 1 [I(1)], suggesting the desirability of a cointegration approach with a vector error correction (VECM) study of the dynamics.

In the VECM, we allow for Markov Switching (MS) in the money supply growth rate process and find three marked regimes: Regime 1 for contractions, Regime 2 for expansions, and Regime 3 with a characteristic of capturing much of the negative real interest rate periods. This third regime mainly includes the post 2001-2004 period of pegged low nominal interest rates and the "unconventional" latter part of this latter decade that again includes a period of pegged low nominal rates. In both expansion and contraction, VECM dynamics show that past period unemployment changes explains the current period interest rate changes, as is consistent with a real interest rate effect; this unemployment effect is much stronger during contractions, a plausible result. Also in contractions, past money supply growth changes explain current interest rate changes, suggestive of an "active" countercyclical monetary policy, while in expansions, past inflation rate changes explain current interest rate changes instead of past money supply changes, suggestive of a more "passive" monetary stance in expansions. In all three regimes, the past nominal interest rate changes also explain the current interest rate changes. But in the "unconventional" third regime, this past nominal interest rate change is the only significant variable, making the dynamics of the third regime similar to a random walk in the nominal interest rate.[1]

Our empirical results that regimes represent different sets of monetary shocks in accordance to whether the economy is in an expansion, recession, or some type of crisis that might involve fear of a rare crisis event or lost decade. In this sense we are in accordance with Pakos et al.'s (2013) three state Markov-switching identification of postwar US regimes in terms of expansions, recessions, and periods of potential lost decades. In our allowing for the money supply process to have mean drift, related work on general parameter drift is found in Fernández-Villaverde et al. (2010). Other recent related work includes Bianchi and Ilut (2013) who identify regimes by chronological Fed chairmen and their so-called "monetary/fiscal mix", now within a DSGE new-keynesian model. This work has roots in Leeper and Zha (2003). As our regimes instead come from the Markov-switching methodology for the money supply growth rate, there is no ready link to chronological Fed chairmen; instead we find a type of institutional consistency across chairmen in the postwar US period, with only contractions and expansions the key feature. However our Regime 3 might be pinned on certain Fed chairman as it is more of a chronological period.

Our results give an interest rate equation that loses its typically cashless character (see Thornton, 2014, Woodford, 2008, Leeper and Roush, 2003). This results are our VECM approach gives due to the problem of unbalanced regressions in Taylor rules, as focused on by Siklos and Wohar (2005), and allows problems of nonlinearities and time-varying parameters that arise with interest rate rule regressions to be associated with the properties of the money growth stochastic process. This results in a longer period of validity for the estimated long run interest rate cointegrating vector than that found for stationary policy rules in Davig and Leeper (2007). Albeit without the reaction function connotations, our VECM can be thought of as consistent with

[1] Friedman's (1968) AEA Presidential address pointed out how Wicksell and he himself agreed that monetary can peg nominal rates but not real rates, and that such pegging induces liquidity effect distortions to the real interest rate: what could be considered an unconventional approach to monetary policy. Hayek's (1931) *Prices and Production* also continually weighed in on this point.

dynamic movement in the inflation targets such as in Ireland (2007), Erceg and Levin (2003), Smets and Wouters (2007), Cogley and Sbordone (2005), Gavin et al. (2005), Roberts (2006), and Salemi (2006).

Section 2 sketches the theoretical economy of Benk et al. (2010) and derives the Euler condition on interest rates as combined with the cash-in-advance constraint so as to bring the money supply growth rate into the interest rate equilibrium condition. Section 3 provides the methodological framework of the empirical analysis and the cointegraton analysis. Section 4 presents the Markov-Switching (MS) analysis with non-stationary variables and presents the results of the estimation of a three-state MS VECM. Section 5 presents robustness tests through a Rolling Trace test. And Section 6 provides interpretation of the cointegrating vector erro term and Section 7 concludes. Appendixes are devoted to: A. data description and model selection procedure of a congruent VAR; B. model selection procedure in Markov-switching VECM framework; C. an estimation (as a robustness check) of a two-state MS VECM; and D. various test statistics used in the estimation.

2 Theoretical Monetary Economy

Benk et al. (2013) presents a cash-in-advance monetary economy with costly credit provided by the finanical intermediation sector so as to endogenize the velocity of money. In addition it uses endogenous growth that Davies et al. (2013) show results in the "target" parameters of the log-linearized Euler condition, which is observationally equivalent to various extended forms of the Taylor rule equation, that are actually the balanced growth path (BGP) endogenous equilibrium values of the variables rather than exogenously specified parameters.

Consider first what happens to the Euler condition and its log-linearization when the model omits the addition of the credit alternative means of exchange that enables the representative consumer to avoid the inflation tax. This standard cash-in-advance economy, still using endogenous growth although that could be omitted as well if exogenous "targets" are more to one's liking, is stated as follows.

Standard monetary real business cycle model shocks are to the goods sector productivity, z_t, and to the money supply growth rate, ζ_t. Shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process. :

$$s_t = \Phi_s s_{t-1} + \varepsilon_{st}, \tag{1}$$

where the shock vector is $s_t = [z_t \ \zeta_t]'$, the autocorrelation matrix is $\Phi_s = diag\left\{\varphi_z, \varphi_\zeta\right\}$ and $\varphi_z, \varphi_\zeta \in [0,1]$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{st} = [\epsilon_{zt} \ \epsilon_{\zeta t}]' \sim N(\mathbf{0}, \mathbf{\Sigma})$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix $\mathbf{\Sigma}$.

2.1 Cash Only Economy

A representative consumer has current period constant elasticity of substitution (CES) utility from consumption of goods, c_t, and leisure, x_t : $\frac{(c_t x_t^{\psi})^{1-\sigma}}{1-\sigma}$, with time discount factor $\beta \in (0,1)$, and with $\psi > 0$ and $\sigma > 0$. Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in goods production (G) and human capital investment (H), whereby $s_{Gt} + s_{Ht} = 1$. The agent allocates a time endowment of one between leisure, x_t, labor in goods production, l_{Gt}, and time spent investing in the stock of human capital, $l_{Ht} : l_{Gt} + l_{Ht} + x_t = 1$. Output of goods can be converted into physical capital, k_t, without cost and is thus divided between consumption goods and investment, denoted by i_t, net of capital depreciation. The capital stock used for production in the next period is therefore given by: $k_{t+1} = (1-\delta_k)k_t + i_t = (1-\delta_k)k_t + y_t - c_t$.The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $l_{Ht}h_t$, with $A_H > 0$ and $\eta \in [0,1]$, such that the human capital flow constraint is $h_{t+1} = (1-\delta_h)h_t + A_H(s_{Ht}k_t)^{1-\eta}(l_{Ht}h_t)^{\eta}$.

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t (l_{Gt} + l_{Ft}) h_t$ and $P_t r_t s_{Gt} k_t$, and a nominal transfer from the government, T_t. With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1-\delta_k)k_t$, and investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds, $B_{t+1} - B_t(R_t)$, where R_t is the gross nominal interest rate, the consumer's budget constraint is:

$$\begin{aligned} & P_t w_t (l_{Gt} + l_{Ft}) h_t + P_t r_t s_{Gt} k_t + T_t \\ \geq \; & P_t c_t + P_t k_{t+1} - P_t(1-\delta_k)k_t + M_{t+1} - M_t \\ & + B_{t+1} - B_t(R_t). \end{aligned} \tag{2}$$

The standard money-only cash-in-advance (CIA) constraint is

$$M_t + T_t \geq P_t c_t. \tag{3}$$

Given k_0, h_0, and the evolution of M_t $(t \geq 0)$ as given by the exogenous monetary policy in equation (4) below, the consumer maximizes the lifetime discounted utility flow subject to the budget and exchange (2)-(3).

The firm maximizes profit given by $y_t - w_t l_{Gt} h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital: $y_t = A_G e^{z_t}(s_{Gt}k_t)^{1-\alpha}(l_{Gt}h_t)^{\alpha}$. The first order conditions for the firm's problem yield the standard expressions for the wage rate and the rental rate of capital: $w_t = \alpha A_G e^{z_t}\left(\frac{s_{Gt}k_t}{l_{Gt}h_t}\right)^{1-\alpha}$, $r_t = (1-\alpha)A_G e^{z_t}\left(\frac{s_{Gt}k_t}{l_{Gt}h_t}\right)^{-\alpha}$. It is assumed that government policy includes sequences of nominal transfers as given by:

$$T_t = \Theta_t M_t = (\bar{\Theta} + e^{u_t} - 1)M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}, \tag{4}$$

where Θ_t is the growth rate of money and $\bar{\Theta}$ is the stationary gross growth rate of money.

The equilibrium intertemporal Euler condition in this model with leisure is standard; given the inflation rate π_{t+1} defined by P_{t+1}/P_t, this condition is

$$\frac{1}{R_t} = \beta E_t \left\{ \frac{c_{t+1}^{-\sigma} x_{t+1}^{\psi(1-\sigma)}}{c_t^{-\sigma} x_t^{\psi(1-\sigma)}} \frac{1}{\pi_{t+1}} \right\}. \tag{5}$$

A log-linearized form of this equation, with over-bars indicating net rates, and $\overline{g}_c$ and $\overline{g}_x$indicating the growth rate of the subscripted variables in net terms, is then

$$\overline{R}_t - \overline{R} = E_t \left(\overline{\pi}_{t+1} - \overline{\pi} \right) + \sigma E_t \left(\overline{g}_{c,t+1} - \overline{g} \right) - \psi \left(1 - \sigma \right) E_t \overline{g}_{x,t+1}. \tag{6}$$

We consider in such a model in which one minus leisure, $1 - x_t$, is productively employed labor, or a type of employment rate relative to the "labor force" of the representative agent. Then changes in leisure, x_t, can well be thought of as changes in the unemployment rate. This is a simple interpretation within neoclassical models of voluntary unemployment that abstracts from the more complex Mortenson-Pissarides approach of frictional unemployment. And it is consistent with defining the unemployment rate as the percent of the labor force that is unemployed and then defining employment as the percent of the labor force that is employed, or one minus the unemploment rate.

The Euler condition looks a bit some form of a Taylor (1993) equation in which the growth in consumption and in the unemployment rate replace the so-called "output gap" or real interest rate components of the model. In addition, the coefficient on the inflation term is Fisher-like, at one, rather than Taylor like at above one.

Money has not been introduced into the Euler equation but now it will be in an alternative equilibrium condition of the model, one of which could also, alternatively, be the focus of interest rate determination.

Therefore take the cash-in-advance constraint over two time periods and combine them so that

$$\frac{M_{t+1}}{M_t} = \frac{P_{t+1} c_{t+1}}{P_t c_t}.$$

this leads, using the money supply growth notation of Θ_t,to an expression for c_{t+1}/c_t : $\Theta_{t+1}/\pi_{t+1} = c_{t+1}/c_t$, than can be substituted into the Euler equation using appropriate expected values. The result with log-linearization is the following

$$\overline{R}_t - \overline{R} = \left(1 - \sigma \right) E_t \left(\overline{\pi}_{t+1} - \overline{\pi} \right) + \sigma E_t \left(\Theta_{t+1} - \bar{\Theta} \right) - \psi \left(1 - \sigma \right) E_t \overline{g}_{x,t+1}.$$

This model brings in the money supply growth rate into the interest rate determination, but the coefficient on the inflation term will either be less than one if the CES utility coefficient $\sigma < 1$, or negative if $\sigma > 1$. The latter value will also cause the unemployment effect on the interest rate to be negative. So this model ends up being inconsistent with the data in at least these two ways, as we show in the empirical analysis below.

2.2 Endogenous Velocity Extension of the CIA Economy

Instead this theoretical failing of the simple CIA economy induces us to follow the endogenous velocity approach of Benk et al. (2010) and the Euler derivation of that model in Davies et al. (2013) in order to derive a theoretical model that offers a consistent theoretical backing of our empirical findings. This extends the above model by bringing in a bank sector that uses the financial intermediation approach, of Clark (1984) and Hancock (1985) and Humphries and Berger (1997), to produce the means to avoid the inflation tax through using credit to purchase goods during the period, and paying off only at the end of the period.

Let the consumption normalized real money demand, notated by m_t/c_t (also known as the inverse consumption velocity of money), use the notation $a_t \equiv \frac{m_t}{c_t}$ (as in Gillman and Kejak, 2005). Then following Benk et al. (2013), the CIA constraint lets money purchases be given by

$$M_t + T_t = a_t c_t,$$

and credit purchases be given by $(1 - a_t)\, c_t$. Now there is time l_{Qt} allocated to the credit sector with a production of credit q_t given by $q_t = A_Q e^{v_t} \left(l_{Qt} h_t\right)^{\gamma} d_t^{1-\gamma}$, where d_t are deposits made by the consumer in the bank each period, $A_Q \in R+$ and $\gamma \in [0, 1)$. From these deposits cash is taken and credit payments are made so that at the end of each period the consumer is constrained by $c_t = d_t$.

Davies et al. (2013) show that the resulting intertemporal capital Euler condition is extended, and has some significant differences from the above Euler equation:

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\sigma} x_{t+1}^{\psi(1-\sigma)}}{c_t^{-\sigma} x_t^{\psi(1-\sigma)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{7}$$

where $\tilde{R}_t$ represents one plus a 'weighted average cost of exchange' as follows:

$$\tilde{R}_t \equiv 1 + a_t(R_t) + \gamma\,(1 - a_t)\,(R_t).$$

Since γ is the coefficient of labor in the production of credit q_t, and it is less than one, the cost of exchange on average is lowered by using credit, even as scarce time is used up in the process of avoiding the inflation tax (which is not socially optimal, but is privately optimal for the consumer). In contrast, with simple CIA, $a_t = 1$ and $\tilde{R}_t = 1 + R_t$. This gives rise to an log-linearized Euler equation that has a coefficient Ω on the inflation term that is always greater than one for any positive interest rate, thereby giving an equilibrium condition that is observationally equivalent to certain classes of the so-called Taylor rules:

$$\begin{aligned} \overline{R}_t - \overline{R} \;&=\; \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega \sigma E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega \psi\,(1-\sigma)\, E_t \overline{g}_{x,t+1} \\ &\quad + (\Omega - 1)\, \overline{R} \frac{a}{1-a} E_t \overline{g}_{a,t+1} - (\Omega - 1)\, E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{8}$$

where $\Omega \equiv 1 + \frac{(1-\gamma)(1-a)}{\left(1+\overline{R}\right)[\gamma + a(1-\gamma)]} \geq 1$, and where a is the BGP solution for $a_t : \frac{m}{c} = 1 - A_Q \left(\frac{\overline{R}\gamma A_Q}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1$. Since $\Omega \geq 1$ (=1 only if $R = 0$), the forward-looking interest rate term enters the equation, along with a velocity growth term $\overline{g}_{a,t+1}$; these extra terms drop out for $a = 1$, at the social optimum of $R = 0$, as the equation reduces back to the form found in the simple CIA economy. One clear advantage of this extension relative to empirical work is that the coefficient on the inflation term Ω is above one as is found also in the Taylor literature, with the Davies et al. (2013) point that estimation of this equilibrium condition is observationally equivalent to estimation of a differently motivated "reaction-function" Taylor equation.

A way to re-write the Euler equation is again to combine it with the CIA constraint. This results in a modified log-linearized equilibrium condition of

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\sigma E_t \left(\Theta_{t+1} - \bar{\Theta}\right) - \Omega\psi\left(1-\sigma\right) E_t \overline{g}_{x,t+1} \\ &\quad + \left[\left(\Omega - 1\right)\overline{R}\left(\frac{a}{1-a}\right) - \Omega\sigma\right] E_t \overline{g}_{a,t+1} - \left(\Omega - 1\right) E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{9}$$

Consider that on the BGP stationary equilibrium, with a balanced growth rate of g, it results that the nominal interest rate is directly related to the money supply growth rate:

$$\begin{aligned} 1 &= \beta E_t \left\{ \frac{c_{t+1}^{-\sigma} x_{t+1}^{\psi(1-\sigma)}}{c_t^{-\sigma} x_t^{\psi(1-\sigma)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\} = \beta\left(1+g\right)^{-\sigma}\left(1+r\right), \\ \left(1+\bar{R}\right) &= \left(1+\bar{\pi}\right)\left(1+r\right), \ \left(1+\bar{\Theta}\right) = \left(1+\bar{\pi}\right)\left(1+g\right), \\ \left(1+\bar{\Theta}\right) &= \left(1+\bar{R}\right)\beta\left(1+g\right)^{1-\sigma}, \end{aligned}$$

and so $\left(1+\bar{\Theta}\right) = \left(1+\bar{R}\right)\beta\left(1+g\right)^{1-\sigma}$, or $\bar{R} \simeq \bar{\Theta} + \rho + (\sigma - 1)\, g$, where $\beta \equiv \frac{1}{1+\rho}$. This BGP expression for R implies that in the "long run" the interest equals the money supply growth rate.

If the forward-looking expected interest rate term is replaced simply by the expected money supply growth rate, and if the velocity term is dropped since in the estimation below this variable is found to drop out of the cointegration analysis, then we could write the log-linearized equilibrium condition as

$$\overline{R}_t - \overline{R} = \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \left[\Omega\sigma - \left(\Omega - 1\right)\right] E_t \left(\Theta_{t+1} - \bar{\Theta}\right) - \Omega\psi\left(1-\sigma\right) E_t \overline{g}_{x,t+1}. \tag{10}$$

If these four variables are cointegrated, that is R, , π, Θ, and the unemployment rate x here (but using u below) then the above equation would provide for restrictions on the expected parameters of the cointegrating vector.

In particular, $\sigma < 1$ would give a negative coefficient for the unemployment rate. The Ω would be the coefficient on the inflation rate term. and the money supply term coefficient of $\Omega\sigma - (\Omega - 1)$ would be negative for example if $\sigma = 0.5$, and $\Omega = 4$. Then $\Omega\sigma - (\Omega - 1) = -1$, so that the money supply growth would have a negative effect, or in this case a coefficient of -1.

An important note theoretically is that a fixed point solution to the model around which a log-linearization can be done requires that if the money supply growth rate follows a unit root, it must do so within some bounded range in order for a *BGP* solution to exist. As we do find such a unit root below in the data, the importance of the idea of bounded-ness in the money supply process comes through, thereby using this crucial concept that has been popularized by the work of Leeper and Zha (2003) and the subsequent related work.

3 Empirical Methodology

Consider a general specification of the stochastic money supply growth rate process Θ_t as we turn to the econometric estimation. Consider allowing for either some persistence in the BGP mean $\bar{\Theta}$ or non-stationarity and that Θ_t responds to the whole set of shocks $[s_t]$ following a process subject to regime-switching:[2]

$$\Theta_t = (1-\tau)\bar{\Theta}(s_t) + \tau\Theta_{t-1}(s_t) + \sigma_{u,t}\xi_t + \sigma_{z,t}z_t + \sigma_{v,t}v_t \ , \tag{11}$$

where $\tau \in (-1, 1]$ is the persistence parameter and $\sigma_{u,t} \equiv \sigma_u(s_t^*), \sigma_{z,t} \equiv \sigma_z(s_t^*), \sigma_{v,t} \equiv \sigma_v(s_t^*)$ are regime-switching standard deviations that deliver the possibility of differentiated effects of shocks on money growth under different regimes (s_t^*). Also the additional shock v_t is added from the bank sector shocks that Benk et al. (2010) also include. However these different shocks are presented for intuition only in that the empirical work does not distinguish the source of the ultimate monetary shock. The intuition suggests how the fiscal authority induces monetary shocks through effects on its tax financing of expenditure from the well-known goods sector productivity shock, monetary authority self-induced shocks such as from fixing nominal interest rates, and especially in the last recession potential banking shocks (see. Benk et al. 2005, 2010, for identification of such banking sectoral productivity shocks; many other bank shocks now have been postulated in the literature.)

According to Equation (11), consider Case I where $\tau \in (-1, 1)$ and $\bar{\Theta} = \Theta^*(s_t)$: the monetary authority (CB) has a long-run stationary money supply growth rate with a regime dependent target $\Theta^*(s_t)$ and is potentially able to reach the target. In this case, steady state solutions associated with the given $\bar{\Theta} = \Theta^*(s_t)$ are expected to be stationary while regime changes will modify the target and the dynamics of the model in line with the traditional approach to regime switching Taylor rules (see Valente, 2003; Francis and Owyang, 2005; Assenmacher-Wesche, 2006; Castelnuovo et al., 2012; Trecroci and Vassalli, 2010). This case is compatible with the assumption of stationarity made by Clarida et al. (2000). In Case II where $\tau = 1$ then $\bar{\Theta} = \Theta_{t-1} = \Theta_t$ and the money supply growth tends to stay where the past history has driven it.

In sum, the two possible assumptions on $\bar{\Theta}$ as follows:

$$\begin{cases} \text{Case I:} \quad \tau \in (-1, 1) \ \Rightarrow \overline{\Theta} = \Theta^*(s_t) \ \ \text{CB has a stable(regime dependent) monetary target} \\ \text{Case II:} \ \ \tau = 1 \ \Rightarrow \overline{\Theta} = \Theta_{t-1} \quad \text{CB does not correct the effects of shocks} \end{cases}$$

In Case II, the stochastic trend of the nominal interest rate can be thought of as a monetary behaviour of the central bank that tends to undergo exogenous shocks such

[2] See for example Liu et al. (2011).

as may be imposed by fiscal authorities as a result of goods productivity shocks or banking sector shocks. A *reaction function* interpretation of this formulation is that of a central bank that permanently (or highly persistently) modifies money growth to take account of various shocks, and consequently modifies nominal rates.[3]

Results find that Θ_t is an I(1) process and this excludes Case I, leaving us in the realm of Case II only. Since both velocity growth and consumption growth are stationary, the specification process results in a reduction of the number of variables in y_t to $y_t = \left[\overline{R}_t, \overline{\pi}_t, u, \Theta_t\right]$.[4] One can also view this inclusion of the non standard variables in light of suggestion by Sims and Zha (2006) who suggest the existence of an omitted variable bias in traditional estimates of the Taylor rule.

In Case II we face an empirical framework different from the traditional one considered by Clarida et al. (2000). The empirical analysis is developed in a system framework considering also the possible presence of regimes, in a Markov-Switching Cointegrated Vector Error Correction Model (MS-VECM). Our approach makes it possible to check which of the two variables (Θ_t, $\overline{R}_t$) changes when $\overline{R}_t$ is outside the equilibrium value in different periods and under different regimes.

Note that for the US data period of 1960-2012, applying simple OLS to the variables of $\overline{R}_t, \overline{\pi}_t$, and u or to the extended model of $\overline{R}_t, \overline{\pi}_t, u$, and Θ_t gives poor results in terms of the dimension of the coefficients. This could reflect a strong endogeneity problem which would then require instrumental variables, but also we know from Stock (1987) that superconsistency of integrated series holds only for very large samples. Moreover, estimating the equation in its different inversions produces different estimates of the equilibrium parameters, as we would expect. As stated by Hall (1986), Stocks' (1987) theorem 3 establishes that the estimates of the cointegrating regression are consistent but subject to a finite sample bias. This bias relates to the overall goodness of fit of the regression. Therefore, even considering the I(1) nature of our variables, the OLS estimation of a single equation, in the framework of the two-stage Engle-Granger procedure, is an inadequate procedure, and in our case a VAR approach is strongly suggested. So we proceed with a VECM methodology.

3.1 A linear VECM representation of the dataset

The data is monthly for the United States from 1960.1 to 2012.12, as given in the Federal Reserve Bank of St. Louis FRED database. This comprises the monthly Federal funds rates for $\bar{R}_t$; the percentage change in the CPI for the inflation rate $\bar{\pi}_t$ [$\bar{\pi}_t = (\Delta_{12} cpi_t)100$) where cpi_t is the log transformed consumer price index ($\ln CPI$)]; the log of the unemployment rate u_t; the growth rate in $M2$ [($\Theta = \Delta_{12} \ln M2$)100] for

[3] Smets and Wouters (2007) propose an example of a monetary policy reaction function that is characterised by the presence of a non-stationary process in the *usually constant* term. Also see Woodford (2008) for a model with non-stationary targets and some comments on this feature of Smets and Wouters's (2003) model.

[4] We included these two variables as exogenous variables in the short run VECM specification, with consumption growth being found to be significant. However the Markov Switching regimes, specified below, remain unchanged and so these results are not reported.

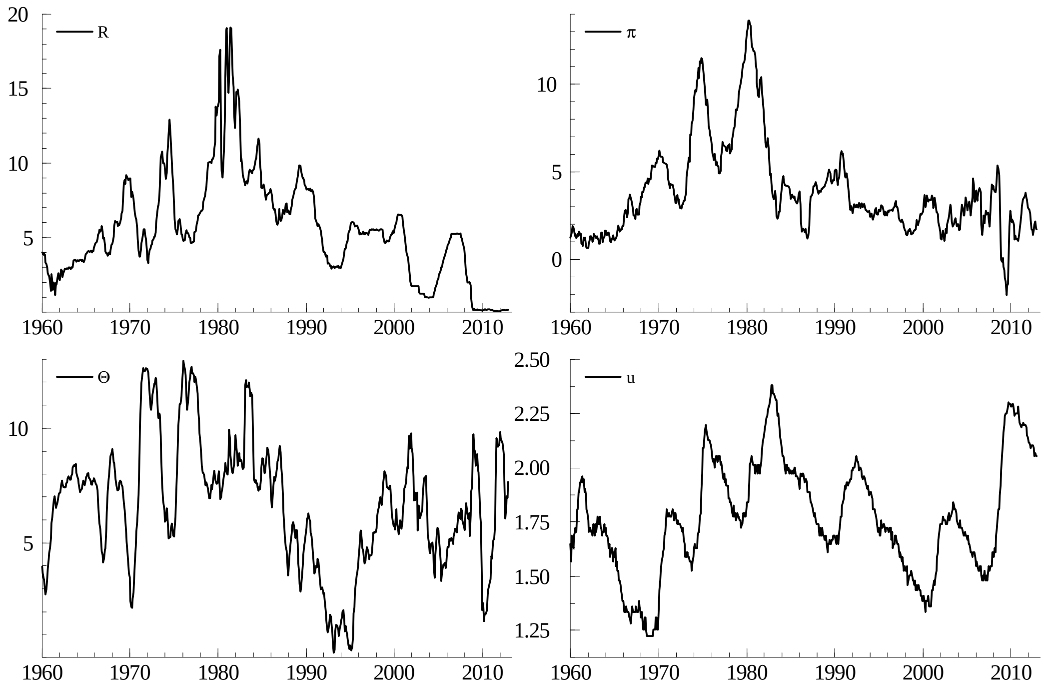


Figure 1: Federal Fund Rate, inflation rate, rate of growth of M2, log of unemployment rate

the monetary aggregate. Figure 1 graphes each of the four variables.

Our series are well characterised as an I(1) process.[5] Therefore, we assume that their true dynamics can be approximated by a VAR(k) system, which can be more conveniently written as a VECM($k-1$) (Vector Error Correction Model):

$$\Delta y_t = \upsilon + \Pi y_{t-1} + \sum_{j=1}^{k-1} \Upsilon_j \Delta y_{t-j} + \Sigma \varepsilon_t \ , \tag{12}$$

where $y_t = \begin{bmatrix} \overline{R}_t & \overline{\pi}_t & \Theta_t & u_t \end{bmatrix}'$, υ is the vector of intercept terms, the matrices Υ_j contain the short-run information, while the long-run information of the data is found in Π. $\Sigma\varepsilon_t$ is a vector of errors.[6] It is important in this framework to lay emphasis on the misspecification problems and the required properties for a satisfactory description of the data. Therefore, after an accurate check to determine the maximum lag length of the system (12), we apply Johansen's (1988) (1991) approach by estimating a VAR(6) and testing for the reduced rank of Π in order to define $\Pi = \alpha\beta'$.

The results of the cointegration test (i.e. the trace and the maximum eigenvalue

[5]We check for the presence of a unit root by means of the ADF test and the DF-GLS test (Elliott *et al.*, 1996), allowing for an intercept as the deterministic component. The unit root null cannot be rejected at the 5% level in all cases. KPSS stationarity tests (Kwiatkowski *et al.*, 1992) confirm this result. Differencing the series induces stationarity. These results are confirmed in the multivariate framework. Results are reported in Appendix D.

[6]With $\varepsilon_t \sim i.i.d.N(0, I)$. The assumption is that the reduced form shocks follow a multivariate normal distribution, $\Sigma\varepsilon_t \sim N(0, \Omega)$, where Ω denotes the variance-covariance matrix of the errors.

test) are reported in Table 1. There is a clear indication that the long-run matrix Π has a reduced rank ($r = 1$). Hence, we conclude that there is exactly one cointegrating relationship between the four variables analyzed.[7] Therefore, it is possible to define $\Pi = \alpha\beta'$, where α and β are (4×1) vectors. Specifically, we obtain the results that are reported in Table 2.

Table 1: Test for cointegrating rank

Rank	0	1	2	3
Trace test [Prob]	79.50[0.000]**	32.35[0.098]	16.12[0.172]	5.20[0.272]
Max test [Prob]	47.15[0.000]**	16.23[0.293]	10.92[0.267]	5.20[0.272]
Trace(T-nm) [Prob]	76.46[0.000]**	31.11[0.129]	15.50[0.203]	5.00[0.294]
Max(T-nm) [Prob]	45.35[0.000]**	15.61[0.339]	10.51[0.301]	5.00[0.293]

Note. The trace test and the max test are the log-likelihood ratio tests (LR), which are based on the four eigenvalues (0.072, 0.025, 0.017 and 0.008). The VAR tested for cointegration is a VAR(6) with an intercept in the cointegrating vector. The row denoted as rank reports the number of cointegrating vectors, and [prob] indicates the p-value computed from critical values by Doornik (1998). The last two rows report small sample correction.

Table 2: Cointegrated coefficients and loading coefficients

	Cointegrated coefficients	*Loading coefficients*
$\bar{R}_t$	1	$\alpha_R = -0.012$ (0.004)
$\bar{\pi}_t$	-2.519 (0.295)	$\alpha_\pi = 0.002$ (0.003)
Θ_t	0.927 (0.282)	$\alpha_\Theta = -0.011$ (0.003)
u_t	10.952 (2.913)	$\alpha_u = -0.001$ (0.0002)
$Const.$	-21.475 (5.212)	

Note. The standard errors are presented in the round parentheses

3.2 Restricted Cointegrating Vector

We test if Θ is a relevant variable for cointegration, and the LR test on $\beta_\Theta = 0$ strongly rejects the hypothesis that it is not relevant: $\chi^2(1) = 7.301[0.0069]^{**}$. In addition, as the coefficient α_π is not significantly different from zero, we also test the restriction[8] $\alpha_\pi = 0$, which is not rejected.[9] Further, testing the hypothesis that $\beta_\Theta = 1$ results in it not being rejected ($\chi^2(2) = 0.44821[0.7992]$). These results are presented in Table 3. Tables 2 and 3 both show that with reference to the entire period all the variables react

[7] This finding is corroborated by looking at the roots of the companion matrix of the chosen VAR(6), which show that there are three common trends.

[8] Table 2a also reports the tests of weak exogeneity on all variables.

[9] This means that π is a weak exogenous variable.

to the equilibrium error with the expected sign. The only exception is the inflation rate $\bar{\pi}_t$, which is a weakly exogenous variable.

The results imply for the restricted cointegrating vector, with the nominal interest rate put on the lefthandside, a coefficient of -1 for the money supply growth rate on the nominal interest rate. In addition the inflation rate has a significant effect with a coefficient of the magnitude seen in many Taylor type estimations. The results show that the estimation is consistent with the theory in Section 2 in which the money supply growth rate plays a key role. It suggests there may be problems of bias due to an omitted variable, in particular Θ, in typical Taylor type estimation.

Table 3:. Multivariate cointegration analysis

	Cointegrated coefficients	Loading coefficients
$\bar{R}$	1	$\alpha_R = -0.011$ (0.0038)
$\bar{\pi}_t$	-2.572 (0.304)	$\alpha_\pi = 0$
Θ_t	1	$\alpha_\Theta = -0.011$ (0.0029)
u_t	12.145 (3.074)	$\alpha_u = -0.001$ (0.0002)
$Const.$	-23.900 (5.395)	
	Test of weak exogeneity	LR test of restrictions:
Restriction:	$\alpha_R = 0$	$\chi^2(1) = 6.2675[0.0123]^*$
Restriction:	$\alpha_\pi = 0$	$\chi^2(1) = 0.4375[0.5083]$
Restriction:	$\alpha_\Theta = 0$	$\chi^2(1) = 9.4585[0.0021]^{**}$
Restriction:	$\alpha_u = 0$	$\chi^2(1) = 11.336[0.0008]^{**}$

Note.The standard errors are presented in the round parentheses, while the p-values are reported in the square brackets

Figures 1 and 2 in combination show that the stationary equilibrium error is closely related to the cycle of inflation. Figure 2 shows the graph of the long-run equilibrium error, $\beta' y_t^*$, which gives a picture of the periods where $\bar{R}_t$ is higher or lower than its equilibrium values in association with the different tenures of Chairmen. Figure 2 shows also that there are two periods where the evolution of the interest rate seems more strictly determined by the forces underlying our long-run equilibrium relationship: the longest period is identifiable with the Greenspan tenure while the second is shorter and coincident with the first part of the Martins tenure. Between 1968 and the end of 1987, we can observe the longest period when the rate of interest is largely out of equilibrium, showing large fluctuations. This period predominantly coincides with the Burns-Miller and Volker tenure. A high disequilibrium also seems to characterize the more recent period under the Bernanke tenure (i.e. between 2007 and 2012). Within the period of higher disequilibrium, we can notice that under the Burns-Miller tenure the interest rate is prevalently below the equilibrium. Meanwhile, in the Volker period the interest

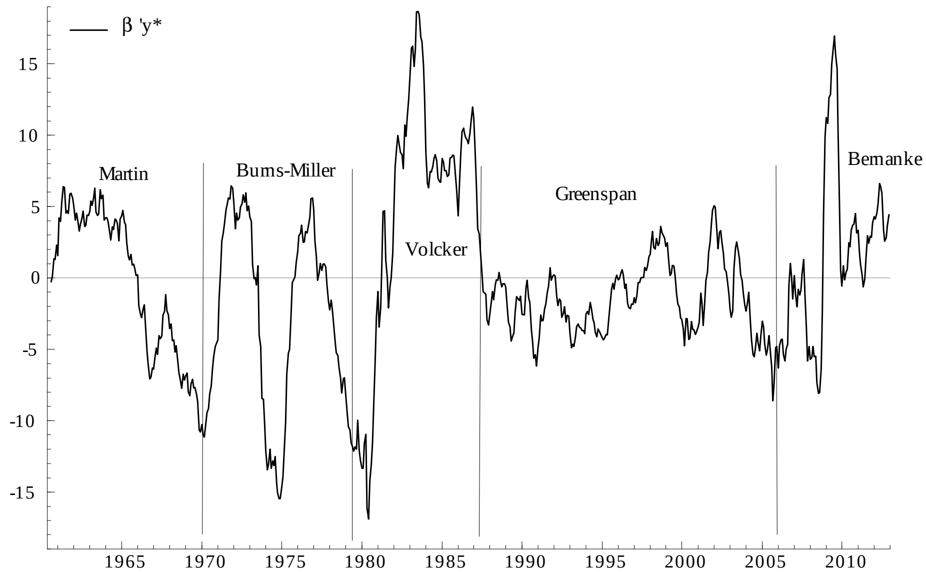


Figure 2: Equilibrium error $\beta' y_t^* = \bar{R}_t - 1.6\bar{\pi}_t + (\Theta_t - \bar{\pi}_t) + 12.2u_t - 23.9$ and US Federal Bank Chairmen's tenures

rate is above the equilibrium. In addition, during the Greenspan administration the rate of interest remains predominantly below the equilibrium.

The estimated equilibrium error is prevalently positive during the Volker period and negative between 1987 and 2007. The latter is in accordance with the opinion that the Federal Reserve maintained the Fed funds rate below the level suggested by the Taylor rule between 1987 and 2007 (See Hayford and Malliaris, 2005). Similarities between the Volker and the Bernanke period include a sharp increase of the equilibrium error (respectively, between 1981 to 1983 and 2008 to 2009) as associated with a sharp increase of u_t and Θ_t and the two negative peaks of the equilibrium interest rate for the whole period.

In the following section we will extend our analysis to include regime shifts in the short-run dynamics, given this estimated long-run equilibrium. We shall see that by introducing nonlinearities the responses of $\bar{R}_t$, $\bar{\pi}_t$, Θ_t and u_t to the equilibrium error are different in some sub-periods under different regimes.

4 Three State Markov-switching VECM Analysis

The analytical framework of this section studies a multivariate linear system of non-stationary time series that is subject to regime shift. Consequently, we follow the works by Krolzig (1997), (1998), who employs a Markov regime switching vector equilibrium correcting model (MS-VECM) to allow for state dependence in the parameters.[10]

[10] The MS-VAR model was proposed by Krolzig (1997). It is a multivariate generalisation of Hamilton (1989) to non-stationary cointegrated VAR systems.

Krolzig's procedure consist of a two-step approach:[11] the first step corresponds to a cointegration analysis in a standard linear model while in the second step the analysis applies the Markov Switching methodology to account for regime shifts in the short-run parameters of the estimated VECM.[12]

The Markov regime-switching model is based on the idea that the parameters of a VAR depend upon a stochastic, unobservable regime variable $s_t \in (1, ..., M)$. Therefore, it is possible to describe the behaviour of a variable (or the behaviour of a combination of variables) with a model that describes the stochastic process that determines the switch from one regime to another by means of an ergodic Markov chain defined by the following transition probabilities:

$$p_{ij} = \Pr(s_{t+j} = j \,| s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1, \quad i, j \in \{1, ..., M\}$$

The cointegrating relations as found are included in the MS(M)-VECM($k - 1$) as exogenous variables, which are assumed to remain constant, where k denotes the number of lags and M the number of regimes.[13] There are many types of MS-VAR models and in this framework the model selection is more complex than in a linear model. We have to decide the maximum lag, which parameters are allowed to vary and how many regimes are to be estimated. The letters following MS stand for the respective parameters varying, specifically: I for the intercept, A for the short-run coefficients, and H for the covariance matrix. A Markov-switching MSIAH VECM that generalizes the system (12) to account for regime shifts in all these components has the following specification:

$$\Delta y_t = \upsilon(s_t) + \alpha(s_t)\beta' y_{t-1} + \sum_{j=1}^{k-1} \Upsilon_j(s_t)\Delta y_{t-j} + \Sigma(s_t)\varepsilon_t, \qquad (t = 1, .., T) \tag{13}$$

where[14] $\Sigma(s_t)\varepsilon_t \sim N(0, \Omega(s_t))$, $\Omega(s_t) = \Sigma(s_t)\Sigma'(s_t)$, $s = 1, .., M$ and the parameters $\upsilon(s_t)$, $\alpha(s_t)$, $\Upsilon_j(s_t)$, and $\Omega(s_t)$ describe the dependence on a finite number of regimes s_t. Hansen and Johansen (1998) have shown that shifts in $\upsilon(s_t)$ are decomposed into shifts in the mean of the equilibrium error $\mu(s_t)$ and shifts in the short-run drifts $\delta(s_t)$ of the system.

[11] For this analysis it can be assumed that the error term is not normally distributed. Johansen (1991, p. 1566) shows that the assumption of Gaussian distribution is not relevant for the results of the asymptotic analysis.

[12] Saikkonen (1992) and Saikkonen and Luukkonen (1997) show that most of the asymptotic results of Johansen (1988 and 1991) for estimated cointegration relations remain valid and can be extended to include the data generated by an infinite non-Gaussian VAR.

[13] In this contest the usual estimation method of parameters is the maximum likelihood and, since the state variable s_t is unobservable, Hamilton (1989) suggests using a maximum likelihood estimation technique via an Expectation Maximization (EM) algorithm. For a detailed description see Krolzig (1998).

[14] Model (13) is indicated as MSIAH(M)-VECM($k - 1$) and could be considered the more general model in terms of changing coefficients.

First we proceed to investigate the presence of nonlinearities allowing regime shifts in the unrestricted intercept (I), in the adjustment coefficients (A), and in the variance-covariance matrix (H), MSIAH-VECM or MSIAH-VARX, where X means that in specification (13) the equilibrium relation obtained in the first step ($\beta' y_{t-1}$) is exogenous. Therefore, the model captures shifts in the mean of the equilibrium error[15] along with shifts in the drift and in the variance-covariance matrix of the innovations. At the same time we relax the assumption of linear adjustment towards the equilibrium, letting the vector of adjustment coefficients $\alpha(s_t)$ and the matrices of the autoregressive part also be regime-dependent. Allowing switches in the loading matrix provides some interesting interpretation for our approach.

As stated previously, we choose the number of regimes and the model in relation to the possible combination of changing parameters, fundamentally between MSIAH, MSAH, MSIH and MSH. To operate a model selection we adopt an intuitive approach, which is inspired to the Krolzig's (1997) suggestion. As a first step, within a given regime (M) and a given MS specification,[16] we choose the best model in terms of maximum lag using the Information Criteria (IC). We then compare the various MS specifications, choosing the model that dominates in term of IC and LR test. The model selection procedure is repeated for different regimes and, finally, the chosen models with different regimes are compared and selected with the usual IC criteria.[17]

The likelihood ratio is higher with 3 regimes than 2 and with the more general MSIAH(3)-VECM(1) specification. This MSIAH allows in addition for shifts in the intercept of the VECM, relative to a MSAH(3)-VECM(1). For the information criteria (AIC), it also results that 3 regimes appear better than two.

Reported in Appendix C in Table 1B and in Table 2B, on the basis of the the information criteria alone it is difficult to choose between the models MSAH(3)-VECM(1) and MSIAH(3)-VECM(1). And so based on likelihood ratios we choose the more general MSIAH(3)-VECM(1). Note that there is no difference in terms of the dating of the regimes between these two specifications, and also no difference with reference to weak exogeneity and volatility. Appendix C also presents the results for the less statistically prefered two-state Markov model which leaves out a key aspect of our results: the Regime 3 that we report in this section.

Therefore here we report the results of the three-state Markov-switching VECM of the MSIAH(3)-VECM(1) form. All the tests support the non-linearity (LR linearity test: 1327.2753, $\chi^2(68) = [0.0000]^{**}$, $\chi^2(74) = [0.0000]^{**}$). Moreover, the Davies (1977) upper bound test does not reject the non-linear model: $DAVIES = [0.0000]^{**}$.

[15] The coefficient $\mu(s_t)$ includes all the target terms of the theoretical model.

[16] This procedure was done for each combination of changing parameters (MSIAH, MSAH, MSIH, MSH).

[17] Results are provided upon request.

Table 4: Estimated coefficients in the non linear VECM(1)

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.758**	0.150	0.021	**0.041**
$\Delta\overline{R}_{t-1}$	**0.319**	0.059	**−0.119**	−0.001
$\Delta\overline{\pi}_{t-1}$	0.033	0.179	−0.194	−0.004
$\Delta\Theta_{t-1}$	**0.741**	**−0.269**	**0.237**	0.001
Δu_{t-1}	**−12.01**	−1.302	−0.471	0.161
$\beta' y_{t-1}$	**−0.029**	−0.008	−0.001	**−0.001**
SE (Reg.1)	1.037	0.397	0.407	0.032
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	0.001	−0.112	**0.221**	**0.026**
$\Delta\overline{R}_{t-1}$	**0.481**	0.106	**−0.219**	**−0.016**
$\Delta\overline{\pi}_{t-1}$	**0.103**	**0.314**	**−0.190**	0.004
$\Delta\Theta_{t-1}$	0.042	−0.021	0.575	0.004
Δu_{t-1}	**−1.663**	0.032	**−0.272**	**−0.217**
$\beta' y_{t-1}$	−0.0002	0.005	**−0.009**	**−0.001**
SE (Reg.2)	0.205	0.252	0.252	0.026
Regime 3	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.094**	**−0.656**	**0.729**	−0.015
$\Delta\overline{R}_{t-1}$	**0.660**	**0.946**	**−0.865**	−0.025
$\Delta\overline{\pi}_{t-1}$	−0.013	**0.345**	**−0.315**	−0.006
$\Delta\Theta_{t-1}$	0.009	0.005	**0.308**	0.006
Δu_{t-1}	0.127	−1.621	−1.140	0.195
$\beta' y_{t-1}$	**−0.003**	**0.026**	**−0.029**	0.001
SE (Reg.3)	0.051	0.458	0.625	0.021

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Table 4 reports the estimated transition matrix and the regime properties. It presents the distinct set of the estimated parameters of the VECM in each regime, endogenously separated by Markov switching methodology. The three distinct regimes provide a picture that mostly differs with respect to the coefficients of adjustment to the equilibrium error, to the variance-covariance matrix of the innovations and to the cyclical phase. Regime 1, in Figure 4, exhibits a higher interest rate volatility, is strongly characterised by the adjustment of the interest rate to the equilibrium error and by the absence of an adjustment of money growth, which is weakly exogenous as the inflation rate. In general, we can observe that the dating of regime 1 probabilities is consistent with the findings of Sims and Zha (2002),[18] Francis and Owyang (2005) and also with models for the dating of recession periods according to NBER (see Figure 4).[19] Regime 1 captures roughly all of the post 1960 recessions except 1991, and adds one extraneous short period around 1985. This includes all of the "inflation scare" periods that were indicated by Goodfriend (1993) and Goodfriend and King (2005).[20]

The second regime is characterised by the moderate volatility of all of the variables and tends to coincide with NBER expansions (see Figure 5). The interest rate and inflation do not adjust to the equilibrium error becoming weak exogenous, while money growth becomes reactive. The dating of this regime seems to associate the Greenspan period with the Martin and Burns-Miller periods, with the exception of the recession periods. In regime 2 there is an important change in the adjustment coefficients since both inflation and the interest rate do not adjust to the equilibrium error (i.e. weak exogeneity) while the unemployment rate and money growth adjust to the equilibrium error. Moreover, the inflation rate is now a strongly exogenous variable.

Regime 3 as shown in Figure 6 prevalently captures the more recent periods, from 2004 to 2012, and it becomes prevailing from 2009 up to the present day. Both money growth and the interest rate adjust to the equilibrium error, the coefficient of adjustment of money growth is higher than in regime 2 while the interest rate adjustment is lower than in regime 1. This is also the only regime where the inflation rate is not weakly exogenous. The unemployment rate becomes a strongly exogenous variable. This regime exhibits very low volatility in the interest rate and higher volatility of money growth and inflation rate. This is a regime where a negative real interest rate coincides with its occurrence in 1971, and after 2003, although it misses the 1980 negative real interest rate by a couple of years.

Roughly identifying regime 1 with NBER recessions and regime 2 with NBER expansions, Table 5 shows that: a) there is an higher probability to pass from a recession to an expansion than vice versa; b) there is an higher probability to persist in expansions than in recessions; c) the probability to pass to regime 3 when the economy is in expansion is lower than during recessions; d) when the economy is in regime 3, there is an higher probability to pass to an expansion than to a recession period.

[18] See State 3 in Figure 1, pag 6.

[19] See Hamilton (1989). Moreover, we observe that the first regime mostly coincides with the dating that we found for the first regime in the two-state Markov-switching VECM. This confirms the robustness of the identification of regime 1.

[20] Goodfriend (1993) indicates the period between 1979.12 and 1980.2 as the first inflation scare, the period between 1981.1 and 1981.10 as the second inflation scare, and the period between 1983 and 1984 as the third inflation scare, and 1987 as the fourth inflation scare.

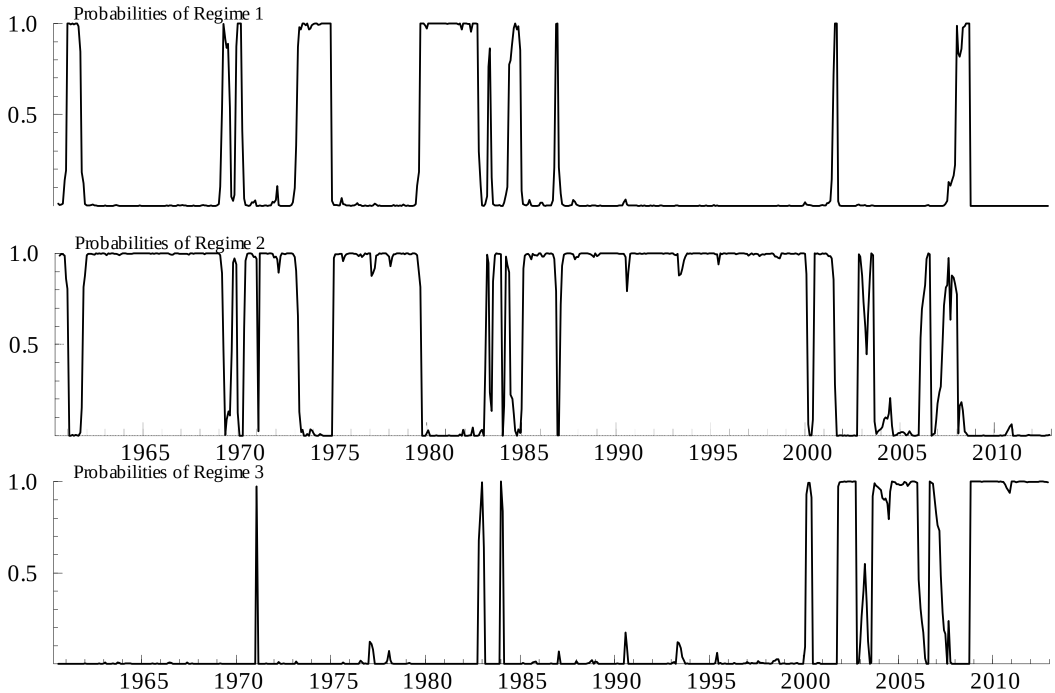


Figure 3: Conditional (smoothed) probabilities of the three regimes obtained from MSIAH(3)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.6\overline{\pi}_t + \Theta_t + 12.2u_t$ restricted as exogenous variable.

Table 5: Transition probabilities and Regime properties

Transition probabilities	p_{1i}	p_{2i}	p_{3i}
Regime 1	0.89	0.03	0.0002
Regime 2	0.08	0.96	0.08
Regime 3	0.03	0.02	0.92
Regime properties	*nObs*	*Prob*	*Duration*
Regime 1	103.6	0.161	9.35
Regime 2	413.4	0.648	22.94
Regime 3	112.0	0.191	11.94

5 Robustness: Rolling trace test

Our results for the US from 1960-2013 show how a crucial property of the nominal rate- a unit root- is not treated in conjunction with the cointegration approach in estimating so-called Taylor rules. Granger and Newbold (1974) and Phillips (1986) pathbreaking work shows that a static regression in levels, when some of the variables in the regression have unit roots, is spurious. Evidence of non-stationarity of the typical Taylor variables for US data has long been reported such as by Bunzel and Enders (2005) and Siklos

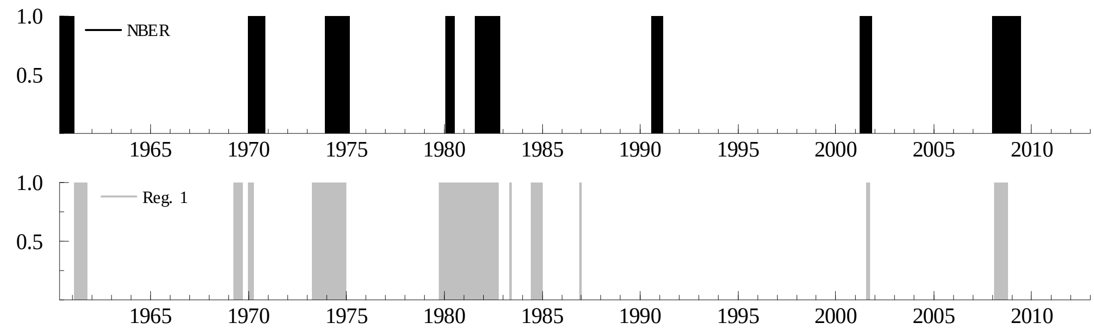


Figure 4: NBER recession dates (shadowed black areas) compared with smoothed probabilities of regime 1 (shadowed grey areas)

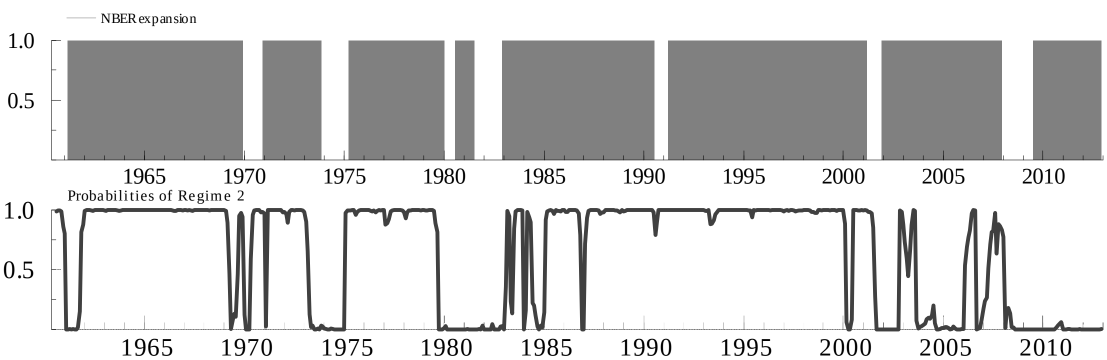


Figure 5: NBER expansions dates (shadowed grey areas) compared with smoothed probabilities of regime 2

and Wohar (2006). In confirmation of this issue, Gerlach-Kristen (2003) and Österholm (2005) find signs of instability, misspecification and inconsistencies in estimated Taylor rules, mainly due to mistreatment of the non-stationarity of the data.

Nor does an interest rate relation with output and inflation, as in a standard Taylor rule, necessarily identify an interest rate reaction function (see also also Orphanides, 2003) or a central bank reaction function (see Minford et al., 2002). Our results imply that the traditional Taylor rule hides the role of money supply in essence by sweeping components of the cointegrating relation into the error term and/or the constant term. In terms of our Case I, with a stationary money growth process, and other vwe expect that the traditional Taylor rule performs well, while in periods where the money supply growth rate is I(1) or near I(1), we expect that a Taylor rule with money growth shows better results. So we perform a rolling cointegration trace test with both a Taylor rule with money and without money, and compare the results together with a rolling integration test on money growth.

The rolling window technique (Rangvid and Sorensen, 2002) is based on keeping constant the size of the sub-sample and then rolling through the full sample both

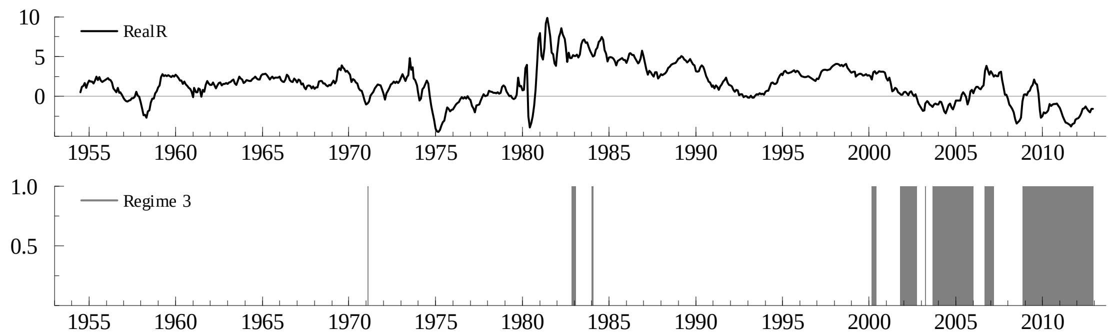


Figure 6: Regime 3 compared with the real interest rate.

the first and last observation in the subsample. The size of the sub-sample is thus a constant fraction of the size of the full. The rolling window focuses on changes in the presence of cointegration during the full sample, and provides a more refined tool to investigate the presence of common stochastic trend along the period. It is a sort of dynamic cointegration analysis. It is a more powerful methodology with respect to recursive techniques since, as shown by Rangvid and Sorensen (2002), the expansion of the sample size in the Johansen (1991) cointegration test provides increasing values of the trace statistics. On the contrary, an increasing values for the rolling trace test could be interpreted as an increasing support for cointegration,

The continuous plot of trace test statistics for a rolling, fixed length, window provides essential information about the time varying pattern of the number of cointegrating vectors and the force towards convergence, expressed by the magnitude of the trace coefficient. The test statistics are calculated for a rolling 150 observations (which corresponds to 12 years and half) time window[21] by adding one observation to the end and removing the first observation and so on. That is, starting with observations 1–150, we calculate the first trace test statistics; then, we calculate the trace tests for observations 2– 151, 3–152, and so on. The sequences of these statistics are scaled by their 5% critical values[22]. A value of the scaled test statistic above one means that the corresponding null hypothesis can be rejected at the 5% level for the specified sub-sample period. Figure1 plots the scaled trace test statistics for the null hypotheses r = 0, against the alternative r = 1. The graph refers, respectively, to the cointegrating relation between R, π, Θ, u (the black continuous line) and between R, π, u (the dashed line). Figure1 shows evidence of a stable cointegrating relation for both up to the end of the 1982, but a different behavior after that date. More precisely, cointegration in the traditional Taylor rule formulation disappears after 1982 and this implies that all the traditional Taylor relations, estimated as static relations, are candidate to be spurious regressions; this is true even if a smoothing term is provided in it. On the contrary,

[21] Several trials with larger windows and various lags in the VAR specification have been made with similar results.

[22] We will compute the critical values for the test using MacKinnon-Haug-Michelis (1999) p-values.

the modified Taylor rule (with money growth) shows the presence of a more clear and stable cointegration. It must be stressed that the only period, dated from 1991 up to the end of 1994, where cointegration appears to weaken, corresponds to the well known break in M2. It is usually recognized that estimating across this period is problematic and relies on essentially *ad hoc* dummies variables.

Therefore we consider the reported results as evidence that the traditional static Taylor equation estimated from the beginning of the 80's is candidate to be a spurious regression while the smoothing version is mispecified since the Engle-Granger (1987) theorem asserts that this dynamic specification is admitted only in presence of cointegration between the involved variables. On the contrary, the nominal interest rate equation with money growth does not suffer from this misspecification as the cointegration exist for all the periods.

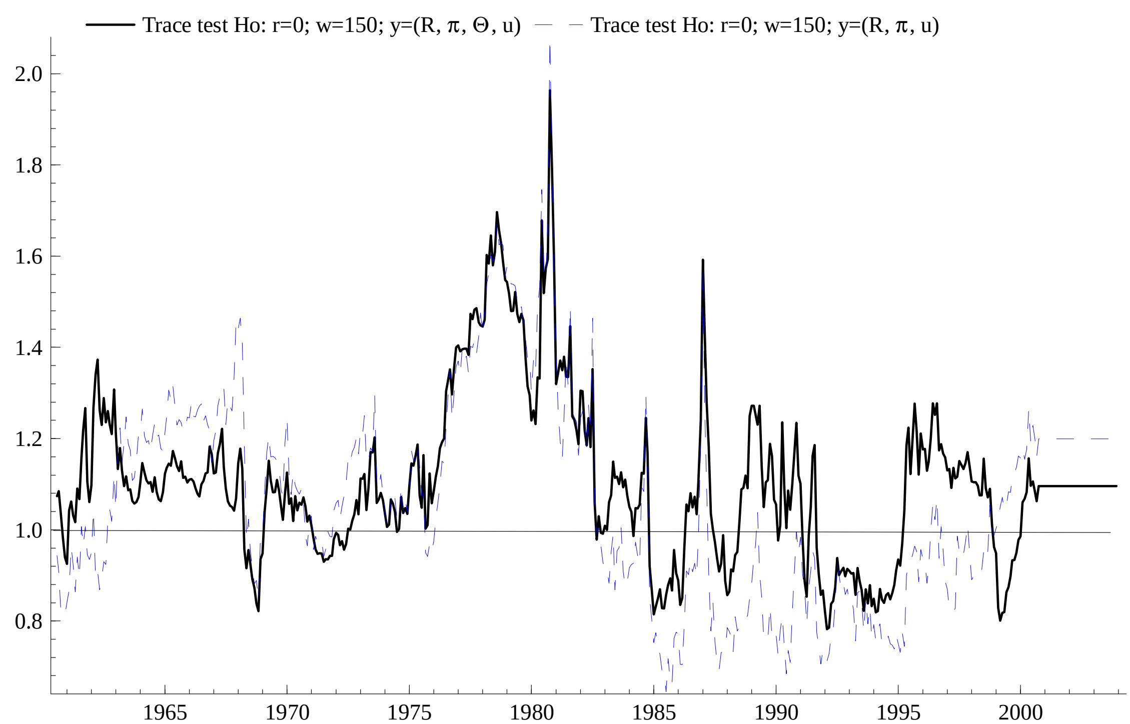


Figure 7: Trace test computed for a window equal to 150 and for a different set of data, respectively, the modified Taylor relation, which involve R, π, Θ, u (the black continuous line) and the traditional Taylor relation which involves R, π, u (the dashed line).

In Figure 8 we report the rolling DF-GLS test on unit root that gives an insight on the dynamics of the non-stationarity for all the four variables. Here again the test was normalized and when the DF-GLS test is above 1 non-stationarity is rejected. Therefore, we can see that all the variables are I(1) along all the period, with the only exception of Θ, in the period 1960 up to 1975. This exception could thereby correspond to a period of validity for the nominal interest estimation as in a traditional Taylor rule. However the overall period provides a cointegrated relation and confirms our hypothesis that the nominal interest rate estimation without the money supply growth rate is mispecified over this postwar US period. This is in line with Minford (2002) who notes that "Taylor (1999) himself emphasised that his rule mimicked the interest rate behaviour one would expect from a k% money supply rule".

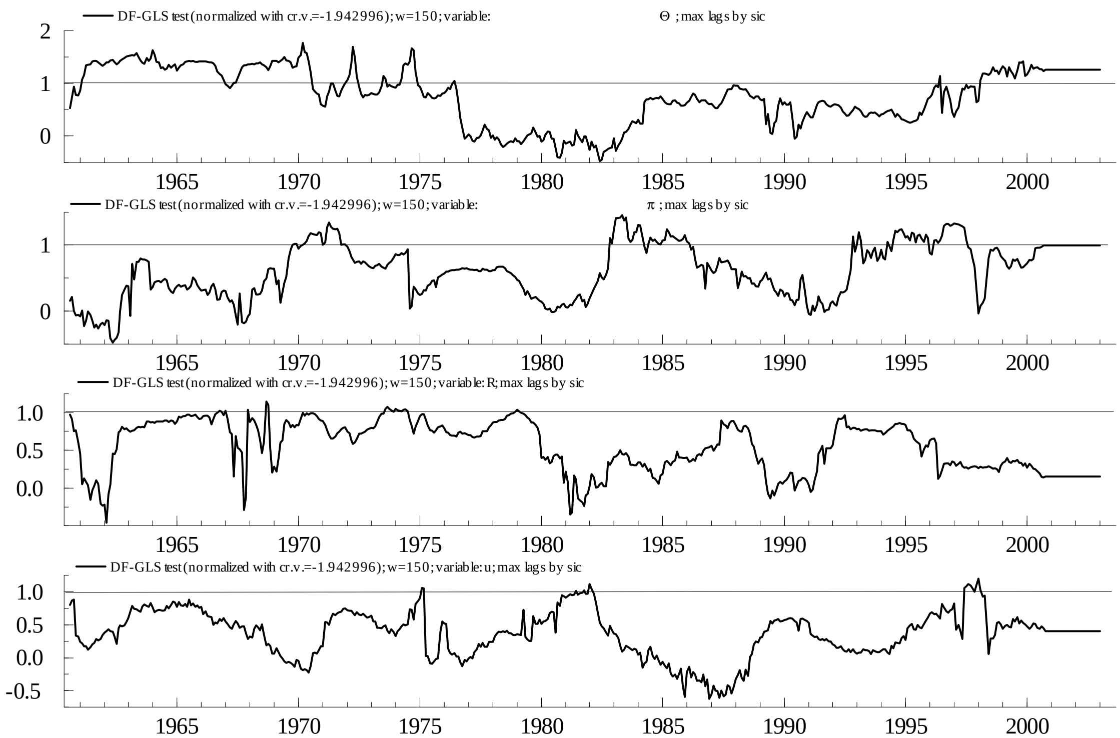


Figure 8: Rolling DF-GLS test computed for Θ, π, R and u for a window equal to 150.

In Figure 9 and 10 we report the rolling trace test for all possible trivariate and pairwise combinations of the four variables. The analysis for the trivariate case shows that there is no clear stable cointegration in all combinations, with the only exception already discussed for the traditional Taylor rule in the first period of the sample. For the pairwise combinations Figure 4 shows that there in no stable pairwise contegration.

6 Interpretation of VECM Error: Liquidity Effect and the Fisher Effect of Money Supply Growth

For our discussion consider Figure 11 which graphs the actual money supply growth rate of M2 minus the inflation rate in the dashed line. It is clear that with real money

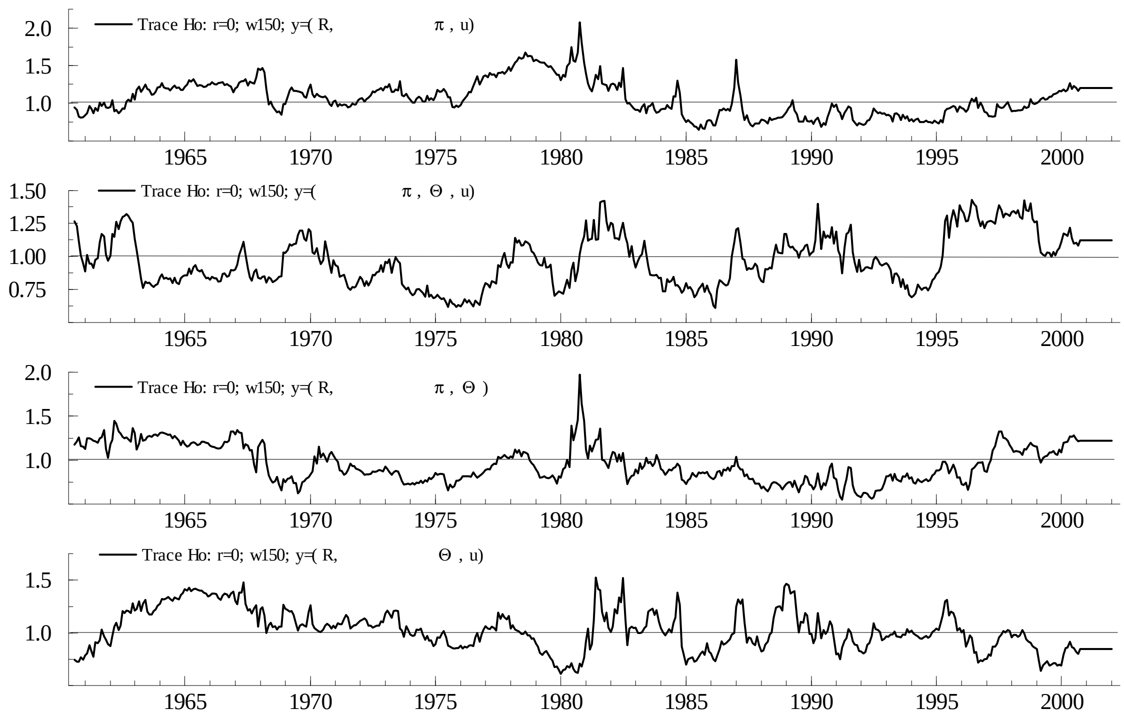


Figure 9: Rolling trace test for all possible trivariate combinations of the four variables Θ, π, R and u.

demand defined as the money stock M divided by the price level P, that the growth rate in the real money demand equals the growth rate in the money supply minus the inflation rate. Therefore Figure 11 is graphing the growth rate in the real money demand assuming clearing in the money market, or more simply the actual money supply growth rate minus the actual inflation rate. Comparing Figure 11 to the errors of the cointegrating vector as graphed in Figure 2, shown in Figure 11 as the solid line, a remarkable correlation results of 0.80.

Since the cointegrating vector includes the money supply growth with a negative one relation to the nominal interest rate, it is already including a classic "liquidity" effect of money on the nominal interest rate: Given that the inflation tax effect on the nominal rate is already accounted for in the positive inflation rate term with a Taylor type magnitude of the coefficient, money supply growth rate increases cause the nominal interest rate to rise relative to the current inflation rate in what appears to be anticipated higher future inflation.

The error is showing that the money supply growth rate and the inflation rate follow each other in the long run, with the residual of this relation being stationary. The un-accounted for, or "unexpected" fluctuations in the cointegrating relation are the error term and this shows a stationary residual that mirrors a close relation of the money supply growth minus the inflation rate. This suggests a residual Fisherian interest rate effect of higher expected inflation from higher money supply growth such

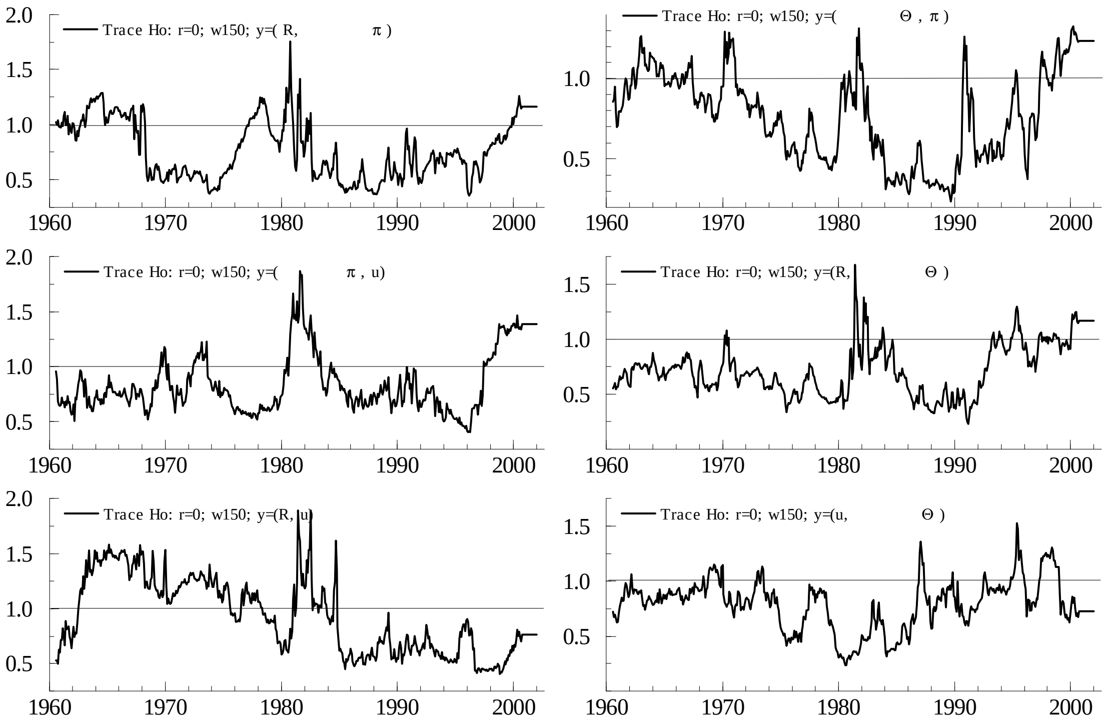


Figure 10: Rolling trace test for all possible pairwise combinations of the four variables Θ, π, R and u.

that causes a higher nominal interest rate than what is already accounted for within the cointegrating vector that includes a strong liquidity effect.

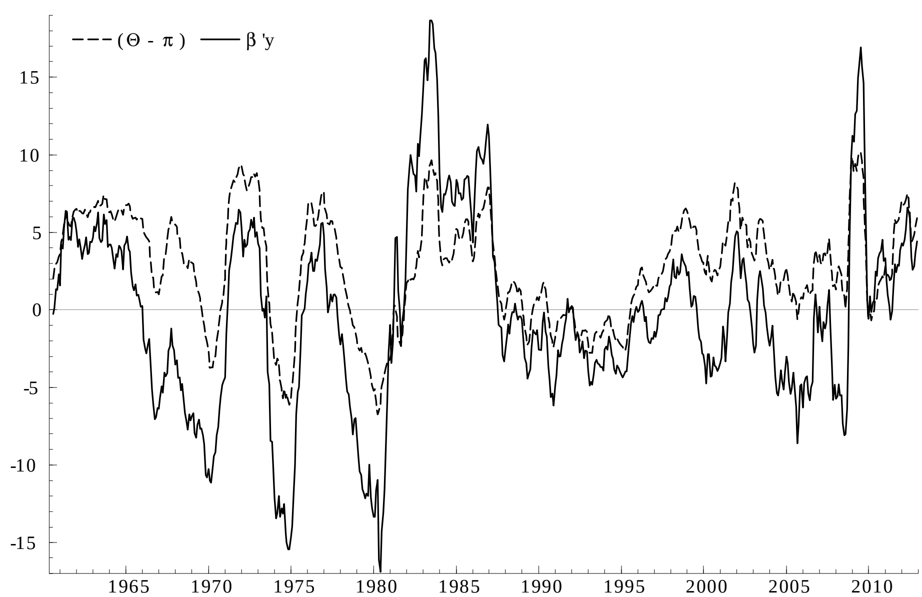
The residual error happens historically and so has ready interpretation within this framework. For example, the most readily interpretable periods are the lead up to the peak inflation of the early 1980s, and the subsequent rapid decline in the inflation rate. Figure 11 shows that during the lead up to 1980 the money supply growth was less than the inflation rate, just as was the error of the cointegrating vector. This implies that given both the expected liquidity effect from money supply growth rate and the expected inflation effect, on the nominal interest rate, the nominal interest rate was lower than expected as the actual inflation rate outpaced the money supply growth rate. This could be from the liquidity effect of the money supply growth being stronger than expected or from the inflation rate being higher than was expected, both of which could well be expected to occur simultaneously.

Similarly during the sudden de-acceleration of the money supply growth rate following 1980, the actual money supply growth rate exceeded the inflation rate so as to cause the nominal interest rate to be higher than was predicted by the cointegrating vector. Using similar logic this was due to a liquidity effect of the money supply growth rate decrease that was less negative than was anticipated or an inflation rate that was lower than was expected according to the cointegrating relation, and again both are likely to have occurred at once.

This cointegrating relation and its error term thereby explains the well-known higher nominal interest than that predicted by the Taylor equation before 1980, and the lower nominal interest than is predicted by the Taylor equation after 1980. So now a completely different explanation from a Taylor reaction function emerges from the equilibrium money supply Euler condition, yet one that is observationally equivalent to the Taylor-type reaction function explanation. This implies that we have supplied strong evidence that estimation of Taylor relations could simply be spurriously associated to reaction functions when these estimations are actually the result of estimating the equilibrium asset price of the nominal interest rate within a money based general equilibrium economy.

This begs the question as to what the money supply growth rate shocks are being driven by. We prefer to call them the financing needs of the government as it attempts to optimally smooth both fiscal and monetary taxes over time through an inflation targeting strategy from which they must depart during wartime (Vietnam) or bank crisis combined with war (what many call the expected upcoming inflation increase that markets anticipate in wake of the huge build up of Fed assets during a time when banks are unable to lend out the reserves more profitably than keeping them at the Fed). In our model, these occasional fluctuations in the inflation rate are part of the stochastic drift of the money supply growth rate around some bounded mean area as fiscal demands dictate. That monetary policy is viewed as an integral part of fiscal policy, rather than the central bank being some independent entity that can do whatever it wants, is a dictum of considering the government budget and tax policy including the inflation tax in a unified fashion.

vs EqError



$_1$1.*pdf*

Figure 11: Growth rates of real balances (M2) and the VECM Equilibrium Error Term

Note that another way to simply state the nature of the error term of the nominal interest rate of the cointegrating relation is that it is the growth rate of velocity. Denoting this growth by "gv" in Figure 12, with it superimposed upon the error term as in Figure 11, the correlation of the two series is somewhat lower at 0.64 than the 0,80 of Figure 11, but still a strong relation. Further the gv variable is the near inverse of the growth of the inverse consumption velocity variable, or the g_a that is found in the equation (9) that describes the equilibrium interest rate condition. The sign of g_a is plausibly negative in equation (9) and this in turn corresponds to a positive sign for the growth rate of the consumption velocity of money, denoted by g_v in our modelling notation, if we were to re-write equation (9) instead with the g_v replacing the g_a variable. This positive sign is what we see in Figure 12 below since it shows the positive correlation of the error term and gv, which is the same as our g_v, and this error term positively affects the nominal interest rate.

Including the growth in velocity (gv) in the dynamic VECM finds that it is insignificant for all three regimes. This leaves the velocity as the lion's share of the unexpected part of the equation. This means that we can interpret the residual of a tax smoothing government with respect to the nominal interest rate estimation to be simply characterized as the growth rate in the consumption velocity of money demand of Figure 12, or perhaps better by the growth in real money balances $(\Theta - \pi)$ of Figure 11.

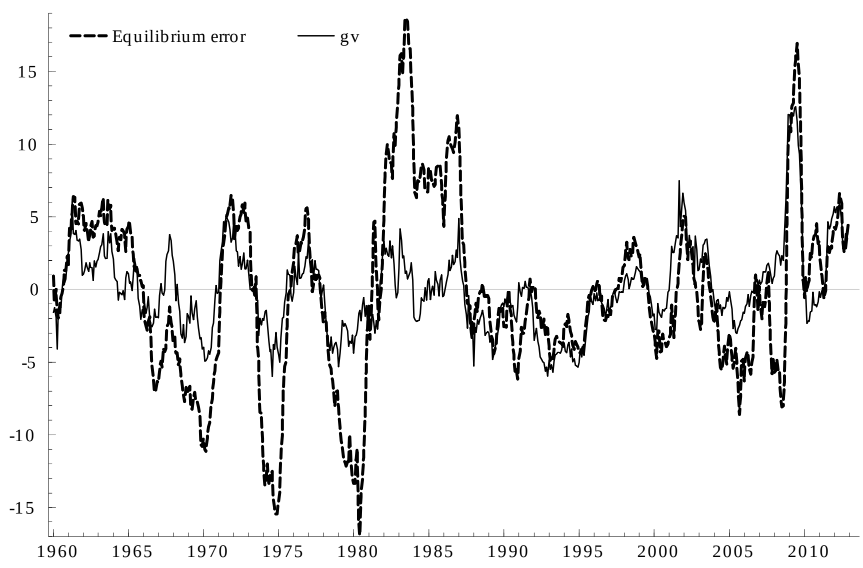


Figure 12: Growth rates the Consumption Velocity of Money (gv) and the Equilibrium VECM Error Term

7 Conclusion

The paper presents evidence of a cointegrated relationship between the nominal interest rate, inflation, the unemployment rate and money growth, for the US 1960-2013 period. This postwar period is longer than that typically seen for Taylor rule estimations. The cointegrating equilibrium relationship is characterised by a stable greater-than-one coefficient for inflation as is observationally equivalent to the Taylor principle coefficient, both a liquidity effect from money supply growth and a Fisherian inflation tax effect associated with money supply growth, as well as the absence of breaks, and the inclusion of regimes. We interpret the results in relation to the Taylor rule-type literature but here our approach is from an equilibrium Euler equation combined with an exchange constraint that brings money into the mix. A crucial role is found for the money growth process, so that the nominal interest rate estimation loses its typically cashless character.

The paper estimates short run dynamic equations with a regime switching error correction mechanism. We find historically meaningful regimes in the short run coefficient of inflation (and other variables) as a result of changes in the adjustment to the equilibrium relationship. We have also find an important role for regimes in volatility. One particular result found is how the unemployment rate reduces the nominal interest rate both in the cointegrating vector and in the dynamics for both Regimes 1 and 2 that are characterized as contractions and expansions respectively. However the dynamics show no significant effect during the Regime 3's unconventional Fed policy period of late, a time when the unemployment rate has been stressed by policymakers. An interpretation of this is that the policymakers may be stressing unemployment in discourse during the Regime 3 period, but in fact unemployment has not affected nominal interest rate dynamics during this Regime 3 period. This does not make the policymakers particularly wrong in that the longer run cointegrating vector includes a strong negative unemployment effect, but such stress on unemployment for the nominal interest rate dynamics could even more strongly be voiced during normal expansions and contractions.

In sum, by adding in the significant money supply growth rate variable, we find that a Fed emphasis on the dual mandate of employment and inflation as key factors in nominal interest rate determination are supported by the results here in general. This means that even if the Fed verbally makes the nominal interest rate its target, the money supply growth rate changes in a stable relation with the nominal interest rate along with the inflation rate and unemployment. Further, the error term of this cointegrating relation shows a stable long run one-to-one relatoin between the money supply growth rate and the inflation rate, even as the cointegrating vector includes a money supply induced liquidity effect. Both of these money supply effects on interest rates are absent from New Keynesian models of employment and interest rates without money. However in this paper we provide a general theory of employment, interest and money.

References

Alvarez, F., Lucas Jr., R. E., Weber, W. E., 2001. Interest Rate and Inflation. *American Economic Review*, 91(2), 219-225.

Ang, A., Boivin, J., Dong, S., Loo-Kng, R., 2011. Monetary Policy Shifts and the Term Structure. *Review of Economic Studies*, 78, 429-457.

Assenmacher-Wesche, K., 2006. Estimating Central Banks' preferences from a time-varying empirical reaction function. *European Economic Review*, 50(8), 1951-1974.

Beck, G. W., Wieland, V., 2008. Central bank misperceptions and the role of money in interest-rate rules. *Journal of Monetary Economics*, 55, S1-S17.

Belke, A., Cui, Y., 2010. US-Euro Area Monetary Policy Interdependence: New Evidence from Taylor Rule-based VECMs. *The World Economy*, 33(5), 778-797.

Benk S., Gillman M., Kejak M., 2005, Credit Shocks in the Financial Deregulatory Era: Not the Usual Suspects. *Review of Economic Dynamics*, 8(3), 668-687.

Benk S., Gillman M., Kejak M., 2010. A banking explanation of the US velocity of money: 1919-2004. *Journal of Economic Dynamics and Control*, 34(4), 765-779.

Bianchi, F., Ilut, C., 2013. *Monetary/Fiscal Policy Mix and Agents' Beliefs.* Economic Research Initiatives at Duke (ERID), Working Paper No. 119, July.

Bunzel, H., Enders, W., 2005. Is the Taylor rule missing? a statistical investigation. Department of Economics, Iowa State University, Working Paper Series.

Castelnuovo, E., Greco, L., Raggi, D., 2012. Policy rules, regime switches, and trend inflation: an empirical investigation for the United States. *Macroeconomic Dynamics*, 1(1), 1-23.

Christensen, A. M., Nielsen, H. B., 2003. *Has US monetary policy followed the Taylor rule? A cointegration analysis 1988-2002.* Danmarks Nationalbank Working Papers 11.

Christensen, A. M., Nielsen, H. B., 2009. Monetary Policy in the Greenspan Era: A Time Series Analysis of Rules vs. Discretion. *Oxford Bulletin of Economics and Statistics*, 71(1), 69-89.

Clarida, R., Gali, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. *The Quarterly Journal of Economics*, 115(1), 147-180.

Clarida, R. H., Sarno, L., Taylor, M. P., Valente, G., 2006. The Role of Asymmetries and Regime Shifts in the Term Structure of Interest Rates. *Journal of Business*, 79, 1193-1224.

Clark, J. A., 1984. Estimation of Economies of Scale in Banking Using a Generalized Functional Form. *Journal of Money, Credit and Banking*, 16(1), 53-68.

Cogley, T., Sbordone, A. M., 2005. *A Search for a Structural Phillips Curve.* Staff Report 203. New York: Federal Reserve Bank of New York.

Davies, R. B., 1977. Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika*, 64, 247-254.

Davies C., Gillman M., Kejak M., 2012. *Deriving the Taylor Principle when the*

Central Bank Supplies Money. Economics Working Paper E2012/20, Cardiff University.

Davig T., Leeper E. M., 2007. Generalizing the Taylor Principle. *American Economic Review*, 97(3), 607-635.

Doornik, J. A., 1998. Approximations to the asymptotic distribution of cointegration tests. *Journal of Economic Surveys,* 12, 573-593.

Eleftheriou, M., 2009. Monetary policy in Germany: A cointegration analysis on the relevance of interest rate rules. *Economic Modelling*, 26, 946-960.

Elliott, G., Rothenberg, T., & Stock, J. H., 1996. Efficient tests for an autoregressive unit root. *Econometrica*, 64, 813-836.

Engle, R. F., Granger, C. W., 1987. Co-integration and error correction: representation, estimation, and testing. *Econometrica*, 55(2), 251-276.

Erceg, C., Levin, J., Andrew, T., 2003. Imperfect credibility and inflation persistence, *Journal of Monetary Economics*, 50(4), 915-944.

Fernández-Villaverde, J., Guerrón-Quintana, P. A., Rubio-Ramírez, J., 2010. *Reading the recent monetary history of the US, 1959-2007.* NBER Working Paper 15929.

Fitzgerald, T. J., Holtemeyer, B., Nicolini, J. P., 2013. *Is There a Stable Phillips Curve After All?.* The Region, Federal Reserve Bank of Minneapolis, December, 4-11.

Francis, N., Owyang, M., T., 2005. Monetary Policy in a Markov-Switching VECM: Implications for the Cost of Disinflation and the Price Puzzle. *Journal of Business and Economic Statistics*, 23(3), 305-13.

Gavin, W. T., Keen, B. D., Pakko, M. R., 2005. The Monetary Instrument Matters. *Federal Reserve Bank of St. Louis Review*, 87, 633-58.

Gerlach-Kristen, P., 2003. *Interest rate reaction functions and the Taylor rule in the Euro area.* Working Paper Series 0258, European Central Bank.

Gerberding, C., Scharnagl, M., Seitz, F., 2007. *Simple interest rate rules with a role for money.* Deutsche Bundesbank, Discussion Paper, Series 1, No. 31/2007.

Gillman, M., Kejak, M., 2005. Inflation and balanced-path growth with alternative payment mechanisms. *Economic Journal*, 115(500), 247-270.

Gillman, M., Kejak, M., Pakos, M., 2014. *Learning about Disaster Risk: Joint Implications for Stocks and Bonds.* Center for Economic Research and Graduate Education, Prague, manuscript.

Gomme, P., 1993. Money and growth revisited: Measuring the costs of inflation in an endogenous growth model. *Journal of Monetary Economics*, 32(1), 51-77.

Goodfriend, M., 1993. Interest rate policy and the inflation scare problem: 1979–1992. *Economic Quarterly, Federal Reserve Bank of Richmond*, 79(1), 1-24.

Goodfriend, M., 1998. Using the term structure of interest rates for monetary policy. *Economic Quarterly, Federal Reserve Bank of Richmond*, 84(3), 13-30.

Goodfriend, M., King, R. G., 2005. The incredible Volcker disinflation. *Journal of Monetary Economics*, 52, 981-1015.

Granger, C. W., Newbold, P., 1974. Spurious regressions in econometrics. *Journal of Econometrics*, 2(2), 111-120.

Hayford, M. D., Malliaris, A. G., 2005. How did the Fed react to the 1990s stock market bubble? Evidence from an extended Taylor rule. *European Journal of Operational Research* 163, 20-29.

Hall, S. G., 1986. An application of the Grander and Engle two-step estimation procedure to united kingdom aggregate wage data. *Oxford Bulletin of Economics and Statistics*, 48(3), 229-239.

Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57, 357-384.

Hancock, D.,1985. The financial firm: production with monetary and nonmonetary goods. *Journal of Political Economy*, 93(5), 859-880.

Hansen P., Johansen S., 1998. *Workbook on Cointegration*. Oxford: Oxford University Press.

Ireland, P. N., 1999. Does the time-consistency problem explain the behavior of inflation in the United States?. *Journal of Monetary Economics*, 44(2), 279-291.

Ireland, P. N., 2003. *Irrational Expectations and Econometric Practice - Discussion of Orphanides and Williams: Inflation Scares and Forecast-Based Monetary Policy*, Federal Reserve Bank of Atlanta, Working Paper 2003-22.

Ireland, P. N., 2007. Changes in the Federal Reserve's inflation target: Causes and consequences. *Journal of Money, Credit and Banking*, 39 (8), 1851-1882.

Johansen, S., 1988. Statistical analysis of cointegrating vectors. *Journal of Economic Dynamics and Control*, 12, 231-254.

Johansen, S., 1991. Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models. *Econometrica*, 59, 1551-1580.

Keating, J., Smith A. L., 2013. *Price Versus Financial Stability: A role for money in Taylor rules?*. Working Paper Series in Theoretical and Applied Economics 201307, University of Kansas, Department of Economics.

Kocherlakota, N., 2013. A Time of Testing. St. Paul Chamber of Commerce, St. Paul, Minnesota, November 12. Available at:
http://www.minneapolisfed.org/news_events/pres/speech_display.cfm?id=5202.

Krolzig, H-M., 1997. *Markov-Switching Vector Autoregressions*, New York: Springer.

Krolzig, H-M., 1998. *Econometric modeling of Markov-switching vector autoregressions using MSVAR for Ox*, Discussion Paper, Institute of Economics and Statistics, Oxford.

Kurz, C., Kurz-Kim, J. R., 2011. Taylor Rule Revisited: from an Econometric Point of View. *Review of Economics and Financ*e, 1, 46-51.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., & Shin, Y., 1992. Testing the null hypothesis of stationary against the alternative of a unit root. *Journal of Econometrics*, 54, 159-178.

Leeper, E. M., Roush, J. E., 2003. Putting "M" back in monetary policy. *Journal of Money, Credit and Banking*, 35(6), 1217-1256.

Leeper, E. M., Zha, T., 2003. Modest policy interventions. *Journal of Monetary*

Economics, 50(8), 1673-1700.

Liu, Z., Waggoner, D. F., Zha, T., 2011. Sources of macroeconomic fluctuations: A regime-switching DSGE approach. *Quantitative Economics,* 2(2), 251-301.

Lucas, Jr., R. E., Stokey, N. L., 1983. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, 12, 55-93.

McCallum, B. T., Nelson E., 2011. Money and inflation: Some critical issues, in Friedman, B. M., and Woodford, M. (Eds.), *Handbook of Monetary Economics*, 3, 97-153. Elsevier.

MacKinnon, J.G., Haug, A.A., Michelis, L., 1999. Numerical distribution functions of likelihood ratio tests for cointegration. *Journal of Applied Econometrics*, 14, 563–577.

Minford, P., Perugini, F., Srinivasan, N., 2002. Are interest rate regressions evidence for a Taylor rule?. *Economics Letters*, 76, 145-150.

Österholm, P., 2005. *The Taylor Rule: A Spurious Regression?*. Bulletin of Economic Research, 57(3), 217-247.

Orphanides, A., 2003. Historical monetary policy analysis and the Taylor rule. *Journal of Monetary Economics*, 50(5), 983-1022.

Orphanides, A., Williams, J. C., 2005. Inflation scares and forecast-based monetary policy. *Review of Economic Dynamics*, 8(2), 498-527.

Phillips, P. C., 1986. Understanding spurious regressions in econometrics. Journal of econometrics, 33(3), 311-340.

Rangvid, J., Sørensen, C., 2002. Convergence in the ERM and declining numbers of common stochastic trends. *Journal of Emerging Market Finance*, 1(2), 183-213.

Roberts, J. M., 2006. Monetary Policy and Inflation Dynamics. *International Journal of Central Banking*, 2, 193–230.

Saikkonen, P., 1992. Estimation and testing of cointegrated systems by an autoregressive approximation. *Econometric Theory*, 8, 1-27.

Saikkonen, P., Luukkonen, R., 1997. Testing cointegration in infinite order vector autoregressive processes. *Journal of Econometrics*, 81, 93-126.

Salemi, M. K., 2006. Econometric Policy Evaluation and Inverse Control. *Journal of Money, Credit, and Banking*, 38, 1737–64.

Sauer, S., Sturm, J. E., 2007. Using Taylor rules to understand European Central Bank monetary policy. *German Economic Review*, 8(3), 375-398.

Shadman-Mehta, F., 2001. A Re-evaluation of the Relation between Wages and Unemployment in the United Kingdom. *Economica*, 68(272), 567-585.

Siklos P. L., Wohar M. E., 2006. Estimating Taylor-type Rules: An Unbalanced Regression?. *Advances in Econometrics*, 20, 239-276.

Sims C. A., Zha, T., 2002. *Macroeconomic switching.* manuscript, Princeton University.

Sims, C. A., Zha, T., 2006. Were there regime switches in US monetary policy?. *The American Economic Review*, 96(1), 54-81.

Smets, F., Wouters, R., 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *The American Economic Review*, 97(3), 586-606.

Stock, J. H., 1987. Asymptotic properties of least squares estimators of cointegrating vectors. *Econometrica*, 55, 1035-1056.

Taylor, J. B., 1993. Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195-214.

Taylor, J.B., 1999. An historical analysis of monetary policy rules. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press for NBER, Chicago.

Thornton, D. L., 2014. Monetary Policy: Why Money Matters (and Interest Rates Don't). *Journal of Macroeconomics*, forthcoming.

Tillmann, P., 2007. Inflation regimes in the US term structure of interest rates, *Economic Modelling*, 24, 203-223.

Tobin, J., 1965. Money and economic growth. *Econometrica* 33 (4), 671–684.

Trecroci, C., Vassalli, M., 2010. Monetary policy regime shifts: new evidence from time-varying interest rate rules. *Economic Inquiry*, 48(4), 933-950.

Valente, G., 2003. Monetary Policy Rules and Regime Shifts. *Applied Financial Economics*, 13, 525–535.

Walsh, C. E., 2003. Speed limit policies: the output gap and optimal monetary policy. *The American Economic Review*, 93(1), 265-278.

Woodford, M., 2008. How important is money in the conduct of monetary policy?. *Journal of Money, Credit and Banking*, 40(8), 1561-1598.

APPENDIX A: Data Analysis

First is a data description by graph in Figure 1A and through and preliminary analysis in Table 1A. The stationarity of the change in the variables entering the cointegrating vector can be seen in both the figure and the table.

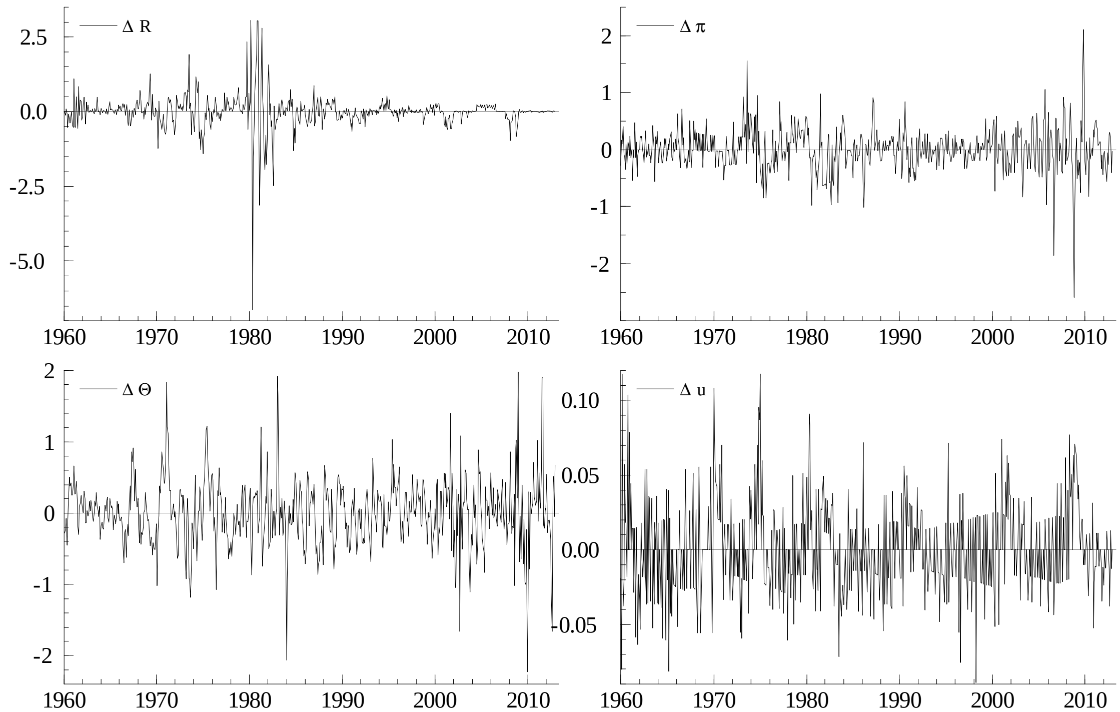


Figure1A: Differenced data. The sample is 1960.1 2012.12.

Table 1A - *Statistics and normality test*

Mean	Max	Min	Std. Dev	Skewness	Kurtosis	Normality test $\chi^2(2)$
5.5659	19.1	0.07	3.5490	0.9387	1.5279	87.080[0.0000] $**$
3.9202	13.6	-2.0	2.7162	1.4034	1.9223	374.73[0.0000] $**$
6.6964	12.9	0.21	2.7405	-0.0069	-0.1194	0.1771[0.9153]
1.7725	2.38	1.22	0.2619	0.07433	-0.5077	8.4066[0.0149]$*$

Next is choosing the congruent VAR specification. Starting with a VAR(7), we first conduct tests of model reduction within a framework of nested models specification. On the basis of the AIC information criteria we choose a VAR(6). On the contrary, the SC and HQ information criteria choose a more parsimonious formulation, like a VAR(2) (see Table2Aa). Therefore, in order to choose between the VAR(6) and VAR(2) parameterization, we adopt also the F-test on a group of coefficients: Table 3Aa shows that the only reduction which is not rejected is that from a VAR(7) to VAR(6); all other reductions are rejected and we observe that a VAR(2) is never accepted if tested against all the other lags.

Moreover, we conduct the LM test on autocorrelation both for a VAR(6) and a VAR(2) specification and we see that there is no autocorrelation of order 1 and 6 in the VAR(6), but there is autocorrelation of order 1 and 2 in the VAR(2). Undertaking the same model selection procedure, but starting from a maximum lag of nine periods, we reach the same conclusion (see Tables 2Ab and 3Ab).

More importantly, we find a confirmation of our choice also if we check for autocorrelation, which is the major concern in the choice of the congruent VAR. Table 4A reports the tests for autocorrelation, respectively, of order 6 and order 1 in a VAR(6) specification: the first row of Panel a reports the test for the system, while the other rows report the autocorrelation tests in each single equation of the system. Table 5A does the same in the VAR(2) specification.

Table 2Aa - *Progress to date [starting with a VAR(7)]*

Model	T	n		log-likelihood	SC	HQ	AIC
VAR(7)	629	116	OLS	551.18073	-0.56414	-1.0654	-1.3837
VAR(6)	**629**	**100**	**OLS**	**540.55244**	-0.69426	-1.1263	$\mathbf{-1.4008}$
VAR(5)	629	84	OLS	522.22564	-0.79991	-1.1629	-1.3934
VAR(4)	629	68	OLS	497.15841	-0.88413	-1.1779	-1.3646
VAR(3)	629	52	OLS	474.41971	-0.97575	-1.2004	-1.3431
VAR(2)	**629**	**36**	**OLS**	**445.22451**	$\mathbf{-1.0468}$	$\mathbf{-1.2024}$	-1.3012
VAR(1)	629	20	OLS	210.36027	-0.46397	-0.55039	-0.60528

Table 2Ab - *Progress to date [starting with a VAR(9)]*

Model	T	n		log-likelihood	SC	HQ	AIC
VAR(9)	627	148	OLS	587.81135	-0.35465	-0.9957	-1.4029
VAR(8)	627	132	OLS	561.61874	-0.43546	-1.0072	-1.3704
VAR(7)	627	116	OLS	547.32004	-0.55421	-1.0566	-1.3758
VAR(6)	627	100	OLS	536.44018	-0.68387	-1.1170	$\mathbf{-1.3922}$
VAR(5)	627	84	OLS	518.28313	-0.79031	-1.1541	-1.3853
VAR(4)	627	68	OLS	493.28874	-0.87495	-1.1695	-1.3566
VAR(3)	627	52	OLS	470.25051	-0.96582	-1.1910	-1.3341
VAR(2)	627	36	OLS	441.16852	$\mathbf{-1.0374}$	$\mathbf{-1.1933}$	-1.2924
VAR(1)	627	20	OLS	207.35097	-0.45595	-0.54258	-0.5976

Table 3Aa - *Tests of model reduction (models are nested for test validity)*

VAR(7)->VAR(6):F(16,1824)=1.2684[0.2088]
VAR(7)->VAR(5):F(32,2203)=1.7405[0.0063]**
VAR(7)->VAR(4):F(48,2301)=2.1867[0.0000]**
VAR(7)->VAR(3):F(64,2339)=2.3515[0.0000]**
VAR(7)->VAR(2):F(80,2357)=2.6273[0.0000]**
VAR(7)->VAR(1):F(96,2367)=7.7594[0.0000]**
VAR(6)->VAR(5):F(16,1836)=2.2106[0.0038]**
VAR(6)->VAR(4):F(32,2217)=2.6424[0.0000]**
VAR(6)->VAR(3):F(48,2317)=2.7084[0.0000]**
VAR(6)->VAR(2):F(64,2355)=2.9623[0.0000]**
VAR(6)->VAR(1):F(80,2373)=9.0458[0.0000]**
VAR(5)->VAR(4):F(16,1848)=3.0546[0.0000]**
VAR(5)->VAR(3):F(32,2232)=2.9360[0.0000]**
VAR(5)->VAR(2):F(48,2332)=3.1889[0.0000]**
VAR(5)->VAR(1):F(64,2370)=10.678[0.0000]**
VAR(4)->VAR(3):F(16,1861)=2.7857[0.0002]**
VAR(4)->VAR(2):F(32,2247)=3.2164[0.0000]**
VAR(4)->VAR(1):F(48,2347)=13.066[0.0000]**
VAR(3)->VAR(2):F(16,1873)=3.6124[0.0000]**
VAR(3)->VAR(1):F(32,2262)=18.075[0.0000]**
VAR(2)->VAR(1):F(16,1885)=32.634[0.0000]**

Table 3Ab - *Tests of model reduction (models are nested for test validity)*

VAR(9) –> VAR(8): F(16,1793)= 3.1086 [0.0000]**
VAR(9) –> VAR(7): F(32,2166)= 2.4130 [0.0000]**
VAR(9) –> VAR(6): F(48,2263)= 2.0490 [0.0000]**
VAR(9) –> VAR(5): F(64,2300)= 2.0949 [0.0000]**
VAR(9) –> VAR(4): F(80,2318)= 2.3015 [0.0000]**
VAR(9) –> VAR(3): F(96,2327)= 2.4076 [0.0000]**
VAR(9) –> VAR(2): F(112,2333)= 2.6047 [0.0000]**
VAR(9) –> VAR(1): F(128,2337)= 6.5150 [0.0000]**
VAR(8) –> VAR(7): F(16,1806)= 1.6980 [0.0407]*
VAR(8) –> VAR(6): F(32,2181)= 1.5007 [0.0357]*
VAR(8) –> VAR(5): F(48,2278)= 1.7344 [0.0014]**
VAR(8) –> VAR(4): F(64,2315)= 2.0718 [0.0000]**
VAR(8) –> VAR(3): F(80,2333)= 2.2369 [0.0000]**
VAR(8) –> VAR(2): F(96,2343)= 2.4865 [0.0000]**
VAR(8) –> VAR(1): F(112,2349)= 6.9054 [0.0000]**
VAR(7) –> VAR(6): F(16,1818)= 1.2984 [0.1889]
VAR(7) –> VAR(5): F(32,2195)= 1.7453 [0.0061]**
VAR(7) –> VAR(4): F(48,2294)= 2.1869 [0.0000]**
VAR(7) –> VAR(3): F(64,2331)= 2.3611 [0.0000]**
VAR(7) –> VAR(2): F(80,2349)= 2.6323 [0.0000]**
VAR(7) –> VAR(1): F(96,2359)= 7.7394 [0.0000]**
VAR(6) –> VAR(5): F(16,1830)= 2.1897 [0.0042]**
VAR(6) –> VAR(4): F(32,2210)= 2.6271 [0.0000]**
VAR(6) –> VAR(3): F(48,2309)= 2.7107 [0.0000]**
VAR(6) –> VAR(2): F(64,2347)= 2.9605 [0.0000]**
VAR(6) –> VAR(1): F(80,2365)= 9.0141 [0.0000]**
VAR(5) –> VAR(4): F(16,1842)= 3.0453 [0.0000]**
VAR(5) –> VAR(3): F(32,2225)= 2.9501 [0.0000]**
VAR(5) –> VAR(2): F(48,2324)= 3.1937 [0.0000]**
VAR(5) –> VAR(1): F(64,2362)= 10.645 [0.0000]**
VAR(4) –> VAR(3): F(16,1855)= 2.8227 [0.0001]**
VAR(4) –> VAR(2): F(32,2240)= 3.2281 [0.0000]**
VAR(4) –> VAR(1): F(48,2340)= 13.025 [0.0000]**
VAR(3) –> VAR(2): F(16,1867)= 3.5980 [0.0000]**
VAR(3) –> VAR(1): F(32,2254)= 17.991 [0.0000]**
VAR(2) –> VAR(1): F(16,1879)= 32.481 [0.0000]**

Table 4A - *Testing for error autocorrelation in VAR(6)*

Panel a: Testing for Vector error autocorrelation from lags 1 to 6
$\chi^2(96)$= 173.64 [0.0000]** and F-form F(96,2288)= 1.7926 [0.0000]**
$\bar{R}_t$: AR 1-6 test: F(6,598) = 1.5800 [0.1504]
$\bar{\pi}_t$: AR 1-6 test: F(6,598) = 1.6038 [0.1436]
Θ_t: AR 1-6 test: F(6,598) = 1.0429 [0.3962]
u_t: AR 1-6 test: F(6,598) = 1.3518 [0.2321]
Panel b: Testing for Vector error autocorrelation from lags 1 to 1
$\chi^2(16)$= 14.074 [0.5932] and F-form F(16,1824)= 0.84205 [0.6378]
$\bar{R}_t$: AR 1-1 test: F(1,603) = 0.3156 [0.5745]
$\bar{\pi}_t$: AR 1-1 test: F(1,603) = 0.0512 [0.8211]
Θ_t: AR 1-1 test: F(1,603) = 1.3351 [0.2484]
u_t: AR 1-1 test: F(1,603) = 0.8565 [0.3551]

Table 5A - *Testing for error autocorrelation in VAR(2)*

Panel a: Testing for Vector error autocorrelation from lags 1 to 2
$\chi^2(32)$=95.089 [0.0000]** and F-form F(32,2247)=3.0202 [0.0000]**
$\bar{R}_t$: AR 1-2 test: F(2,618) = 5.4561 [0.0045]**
$\bar{\pi}_t$: AR 1-2 test: F(2,618) = 0.0111 [0.9889]
Θ_t : AR 1-2 test: F(2,618) = 2.1549 [0.1168]
u_t : AR 1-2 test: F(2,618) = 10.6740 [0.0000]**
Panel b: Testing for Vector error autocorrelation from lags 1 to 1
$\chi^2(32)$=95.089 [0.0000]** and F-form F(32,2247)=3.0202 [0.0000]**
$\bar{R}_t$: AR 1-1 test: F(1,619) = 6.1284 [0.0136]*
$\bar{\pi}_t$: AR 1-1 test: F(1,619) = 0.0216 [0.8832]
Θ_t : AR 1-1 test: F(1,619) = 2.1777 [0.1405]
u_t : AR 1-1 test: F(1,619) = 1.8212 [0.1777]

APPENDIX B: Markov Switching Lags and Regimes

Table 1B reports all the model selection criteria in the MSIAH framework. More specifically, in the case of two regimes it is difficult to choose the maximum lag, since there is not a coherent indication given by the information criteria. The AIC criterion tends to over-parameterize, but the HQ and SC choose a VECM(1). In the case of three regimes a MSIAH(3)-VECM(1) is preferred, and this model also dominates the two-regimes version. This means that, although the estimation of three regimes increases the number of parameters, it dominates the two-regimes model, since the improvement in the likelihood outweighs the cost of estimating a model with a greater number of parameters and this is indicated by all the information criteria for more than one lag (e. g. AIC clearly prefers the three-regimes model even up to the fifth lag). Tables 1aB and 1bB report all other information in terms of probability and duration of regimes, respectively, in the two-state and three-state Markov switching estimation.

Table 1B - *Model selection criteria in the MSIAH-VECM framework*

Model	fitting	Information Criteria			n
MSIAH(M)-VECM(k-1)	Log-likelihood	AIC	HQ	SC	
MSIAH(2)-VECM(5)	1005.7386	−2.5683	−2.0249	−1.1694	198
MSIAH(2)-VECM(4)	972.7061	−2.5650	−2.1094	−1.3922	166
MSIAH(2)-VECM(3)	953.4520	**−2.6056**	−2.2378	−1.6588	134
MSIAH(2)-VECM(2)	921.1298	−2.6045	−2.3246	−1.8839	102
MSIAH(2)-VECM(1)	880.2042	−2.5762	**−2.3840**	**−2.0816**	70
MSIAH(3)-VECM(5)	1133.9300	−2.6516	−1.8282	−0.5320	300
MSIAH(3)-VECM(4)	1088.8023	−2.6607	−1.969	−0.8803	252
MSIAH(3)-VECM(3)	1096.3686	−2.8374	−2.2775	−1.3961	204
MSIAH(3)-VECM(2)	1125.5045	−3.0827	−2.6545	−1.9805	156
MSIAH (3)-VECM(1)	**1090.2598**	**−3.1232**	**−2.8268**	**−2.3602**	**108**

n is the number of parameters and k is the maximun lag in VAR specification

Table 1aB - *Regime properties of MSIAH(M)-VECM(k-1)*

MSIAH(M)-VECM(k-1)	p_{11}	p_{12}	duration 1	duration 2
MSIAH(2)-VECM(5)	0.81	0.95	5.29	19.63
MSIAH(2)-VECM(4)	0.81	0.95	5.15	22.15
MSIAH(2)-VECM(3)	0.89	0.97	8.71	37.12
MSIAH(2)-VECM(2)	0.86	0.97	7.36	33.06
MSIAH(2)-VECM(1)	0.86	0.97	6.98	33.93

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration i" denotes the expected duration (in months) of each regime i.

Table 1bB- *Transition probabilities and regime properties of MSIAH(M)-VECM(k-1)*

MSIAH(M)-VECM(k-1)	p_{11}	p_{12}	p_{13}	duration regime 1	duration regime 2	duration regime 3
MSIAH(3)-VECM(5)	0.6057	0.9499	0.6288	2.54	19.97	2.69
MSIAH(3)-VECM(4)	0.5825	0.9455	0.6205	2.39	18.33	2.64
MSIAH(3)-VECM(3)	0.7072	0.9306	0.6778	3.42	14.42	3.10
MSIAH(3)-VECM(2)	0.9052	0.9364	0.8227	10.55	15.73	5.64
MSIAH(3)-VECM(1)	0.8930	0.9564	0.9163	9.35	22.94	11.94

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration regime i" denotes the expected duration (in months) of each regime i.

Table 2B reports all the model selection criteria in the alternative MSAH framework. Observations done so far for the MSIAH model are valid also in this contest and the preferred model is a MSAH(3)-VECM(1). Tables 2aB and 2bB report all other information in terms of probability and duration of regimes, respectively, in the two-state and three-state Markov switching estimation. Although the conclusions are the same, we report also all correspondent tables for the MSH(M)-VECM(k-1) specification.

It must be stressed that we have also estimated other versions of Markov-Switching VECM, but for space considerations, we report only the three versions which are interesting to the analysis. Here we just want to observe that the dominant aspect in this model selection procedure is a clear improvement when introducing the shift in the variance-covariance matrix.

Table 2B - *Model selection criteria in the MSAH-VECM framework*

Model	fitting	Information Criteria			n
MSIAH(M)-VECM(k-1)	Log-likelihood	AIC	HQ	SC	
MSAH(2)-VECM(5)	1006.9289	−2.5848	−2.0524	−1.2141	194
MSAH(2)-VECM(4)	970.8371	−2.5718	−2.1272	−1.4272	162
MSAH(2)-VECM(3)	963.4160	**−2.6500**	−2.2932	−1.7315	130
MSAH(2)-VECM(2)	924.2287	−2.6271	−2.3581	−1.9347	98
MSAH(2)-VECM(1)	886.8219	−2.6099	**−2.4288**	**−2.1436**	66
MSAH(3)-VECM(5)	1166.1133	−2.7794	−1.9780	−0.7163	292
MSAH(3)-VECM(4)	1159.3932	−2.9106	−2.2410	−1.1867	244
MSAH(3)-VECM(3)	1146.6899	−3.0229	−2.4849	−1.6380	196
MSAH(3)-VECM(2)	1112.0541	−3.0654	−2.6592	−2.0197	148
MSAH (3)-VECM(1)	**1078.4998**	**−3.1113**	**−2.8368**	**−2.4047**	**100**

n is the number of parameters and k is the maximun lag in VAR specification

Table 2aB - *Regime properties of MSAH(M)-VECM(k-1)*

MSAH(M)-VECM(k-1)	p_{11}	p_{12}	duration 1	duration 2
MSAH(2)-VECM(5)	0.9428	0.7931	17.47	4.83
MSAH(2)-VECM(4)	0.9393	0.7303	16.47	3.71
MSAH(2)-VECM(3)	0.9431	0.7988	17.56	4.97
MSAH(2)-VECM(2)	0.9623	0.8176	26.56	5.48
MSAH(2)-VECM(1)	0.9670	0.8585	30.27	7.07

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration i" denotes the expected duration (in months) of each regime i.

Table 2bB - *Transition probabilities and regime properties of MSAH(M)-VECM(k-1)*

				duration		
MSAH(M)-VECM(k-1)	p_{11}	p_{12}	p_{13}	regime 1	regime 2	regime 3
MSAH(3)-VECM(5)	0.7726	0.9332	0.8553	4.40	14.96	6.91
MSAH(3)-VECM(4)	0.8322	0.9485	0.9372	5.96	19.42	15.92
MSAH(3)-VECM(3)	0.8673	0.9186	0.7988	7.54	12.29	4.97
MSAH(3)-VECM(2)	0.8635	0.9301	0.8450	7.32	14.31	6.45
MSAH(3)-VECM(1)	0.8957	0.9582	0.9152	9.58	23.91	11.79

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration regime i" denotes the expected duration (in months) of each regime i.

Table 3B - *Model selection criteria in the MSH-VECM framework*

Model	fitting	Information Criteria			n
MSH(M)-VECM(k-1)	Log-likelihood	AIC	HQ	SC	
MSH(2)-VECM(5)	925.1765	−2.5920	−2.2901	−1.8148	110
MSH(2)-VECM(4)	910.5296	−2.5963	−2.3383	−1.9321	94
MSH(2)-VECM(3)	889.6734	**−2.5808**	−2.3668	−2.0297	78
MSH(2)-VECM(2)	869.0603	−2.5662	−2.3960	−2.1281	62
MSH(2)-VECM(1)	840.1273	−2.5250	**−2.3988**	**−2.2000**	46
MSH(3)-VECM(5)	1059.4854	−2.9745	−2.6342	−2.0984	124
MSH(3)-VECM(4)	1050.5644	−2.9970	−2.7006	−2.2340	108
MSH(3)-VECM(3)	1036.0713	**−3.0018**	−2.7493	−2.3518	92
MSH(3)-VECM(2)	1006.6770	−2.9592	−2.7506	−2.4223	76
MSH (3)-VECM(1)	991.9332	−2.9632	**−2.7985**	**−2.5393**	60

n is the number of parameters and k is the maximun lag in VAR specification

Table 3aB - *Regime properties of MSH(M)-VECM(k-1)*

MSH(M)-VECM(k-1)	p_{11}	p_{12}	duration 1	duration 2
MSH(2)-VECM(5)	0.8265	0.9543	5.76	21.89
MSH(2)-VECM(4)	0.8247	0.9540	5.70	21.76
MSH(2)-VECM(3)	0.8288	0.9550	5.84	22.21
MSH(2)-VECM(2)	0.8261	0.9577	5.75	23.64
MSH(2)-VECM(1)	0.8224	0.9556	5.63	22.51

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration i" denotes the expected duration (in months) of each regime i.

Table 3bB - *Transition probabilities and regime properties of MSH(M)-VECM(k-1)*

				duration		
MSH(M)-VECM(k-1)	p_{11}	p_{12}	p_{13}	regime 1	regime 2	regime 3
MSH(3)-VECM(5)	0.8692	0.9502	0.9091	7.65	20.07	11.00
MSH(3)-VECM(4)	0.8782	0.9456	0.8799	8.21	18.37	8.33
MSH(3)-VECM(3)	0.8762	0.9487	0.8916	8.08	19.50	9.23
MSH(3)-VECM(2)	0.8765	0.9598	0.9439	8.10	24.90	17.82
MSH(3)-VECM(1)	0.8723	0.9510	0.8991	7.83	20.40	9.91

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration regime i" denotes the expected duration (in months) of each regime i.

We may conclude that the specification with one lag is superior even in the two-regime model[23], and the comparison between three and two regimes is favorable to the three-regimes specification, in terms of all the information criteria. This conclusion is also confirmed by all the LR tests we have done. Comparing the information criteria reported in Table 1B and in Table 2B, it is difficult to choose between the models MSAH(3)-VECM(1) and MSIAH(3)-VECM(1). However, there is no difference in terms of the dating of the regimes, and also there is no difference with reference to all other important information related to the concept of weak exogeneity and volatility.[24] We report only the results of the MSIAH(2)-VECM(1) and MSAH(3)-VECM(1) models, since these are more informative with respect to the shift in the constant and the adjustment coefficients.

The dating of regimes for the MSAH(3)-VECM(1) version is presented in Figure 1B, and Table 4B reports the estimated coefficients.

[23] It is important to stress that adding more dynamics does not change the main information regarding the dating of the regimes and the statistical significance of the adjustment coefficients.

[24] Moreover, for the two models the underlying assumptions concerning autocorrelation and normality appear to be satisfied.

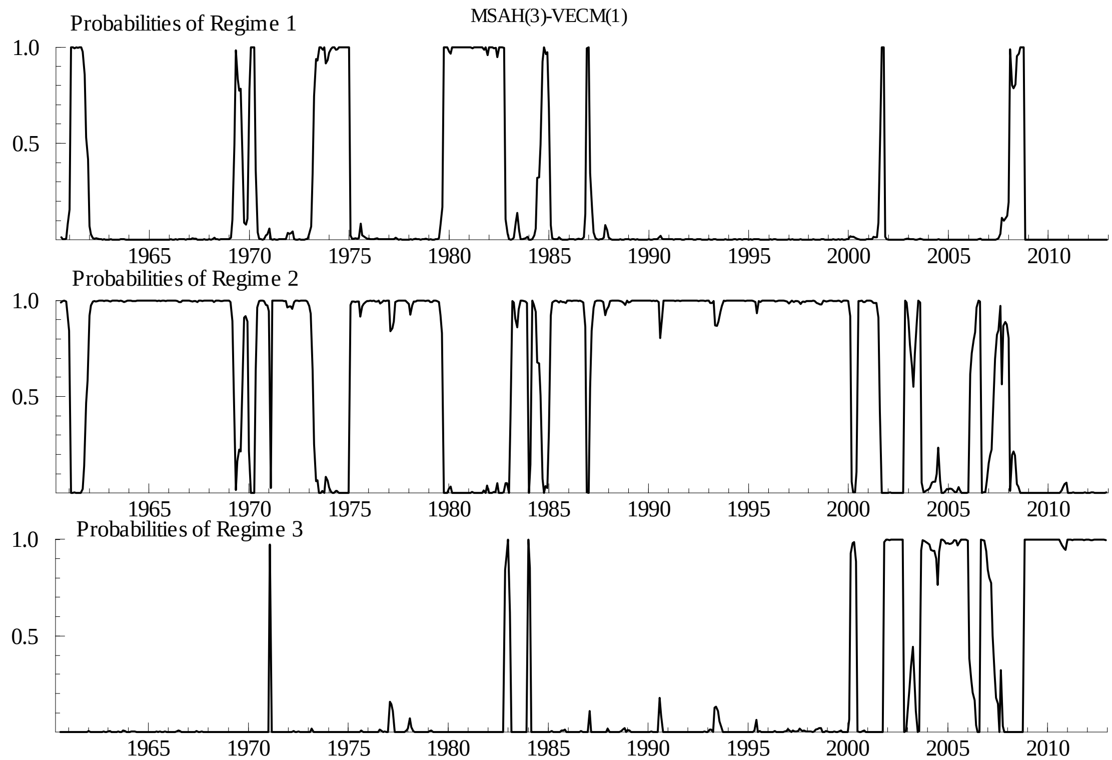


Figure 1B: Conditional (smoothed) probabilities of the three regimes obtained from MSAH(3)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.6\overline{\pi}_t + \Theta_t + 12.2u_t$ restricted as exogenous variable.

Table 4B- *Estimated coefficients in the non-linear VECM(1)*

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.017950**	0.003856	0.003423	**−0.002338**
$\Delta\overline{R}_{t-1}$	**0.321372**	0.053472	**−0.119070**	−0.000321
$\Delta\overline{\pi}_{t-1}$	0.033179	0.196502	−0.207637	−0.006186
$\Delta\Theta_{t-1}$	**0.754147**	**−0.281400**	**−0.239370**	−0.001810
Δu_{t-1}	**−11.827021**	−1.612646	−0.705350	**0.285474**
$\beta' y_{t-1}$	**−0.033263**	−0.004253	−0.001561	**−0.001627**
SE (Reg.3)	1.052867	0.397654	0.413859	0.033853
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.017950**	0.003856	0.003423	**−0.002338**
$\Delta\overline{R}_{t-1}$	**0.472565**	0.117516	**−0.219777**	**−0.014840**
$\Delta\overline{\pi}_{t-1}$	**0.092320**	**0.312353**	**−0.181082**	−0.004065
$\Delta\Theta_{t-1}$	0.035742	−0.016915	0.571675	0.003124
Δu_{t-1}	**−1.634772**	0.011515	−0.237931	**−0.211028**
$\beta' y_{t-1}$	0.000528	0.003243	**−0.008594**	**−0.001213**
SE (Reg.2)	0.208758	0.255221	0.251750	0.026394
Regime 3	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	
$Const.$	**0.017950**	0.003856	0.003423	**−0.002338**
$\Delta\overline{R}_{t-1}$	**0.663402**	**0.924057**	**−0.842949**	−0.025673
$\Delta\overline{\pi}_{t-1}$	−0.014583	**0.351489**	**−0.319073**	−0.005763
$\Delta\Theta_{t-1}$	0.008864	0.011150	**0.303993**	0.005797
Δu_{t-1}	0.111979	−1.540130	−1.204324	0.191255
$\beta' y_{t-1}$	**−0.002725**	**0.022789**	**−0.027218**	0.000571
SE (Reg.1)	0.051414	0.460859	0.624440	0.021234

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Appendix C. A Two-State Markov-switching VECM

In this section we report the results of the two-state Markov-switching VECM [more precisely: MSIAH(2)-VECM(1)]. In this framework, all the tests support the non-linearity hypothesis: LR linearity test=907.1641, $\chi^2(34)$=[0.0000]**, $\chi^2(36)$=[0.0000]**. Moreover, the Davies (1977) upper bound test does not reject the non-linear model: DAVIES=[0.0000]**. Tables 1C reports regime properties, and the matrix $\hat{P}$ is the estimated transition matrix. Table 2C presents the distinct set of the estimated parameters of the VECM in each regime endogenously separated by Markov-Switching methodology.

The two distinct regimes mostly differ with respect to a different adjustment to the equilibrium error and with respect to volatility.

Figure 2C shows that the most remarkable periods prevailing in regime 1 are clearly identified as NBER recession periods also when we consider only two-state Markov-switching.

$$\hat{P} = \begin{pmatrix} \hat{p}_{11} & \hat{p}_{12} \\ \hat{p}_{21} & \hat{p}_{22} \end{pmatrix} = \begin{pmatrix} 0.86 & 0.13 \\ 0.03 & 0.97 \end{pmatrix}$$

Table 1C - *Regime properties*

	nObs	Prob	Duration
Regime 1	108.0	0.1706	6.98
Regime 2	521.0	0.8294	33.93

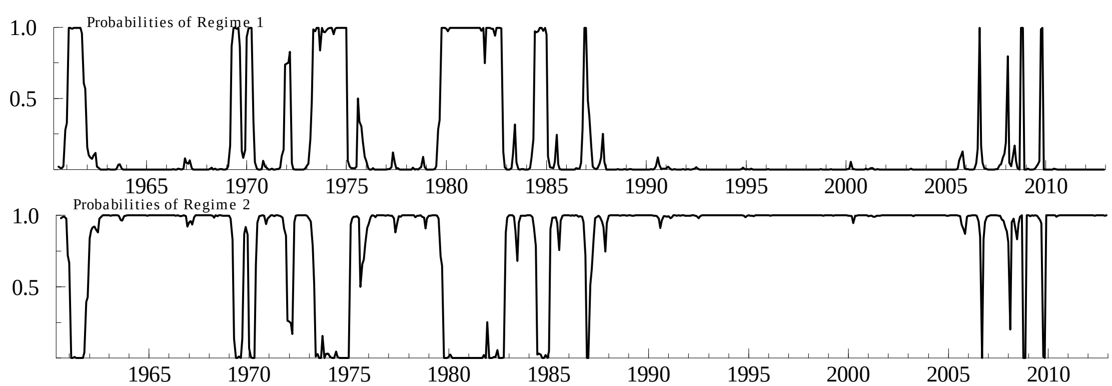


Figure 1C: Conditional (smoothed) probabilities of the three regimes obtained from MSIAH(2)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.6\overline{\pi}_t + \Theta_t + 12.2u_t$ restricted as exogenous variable.

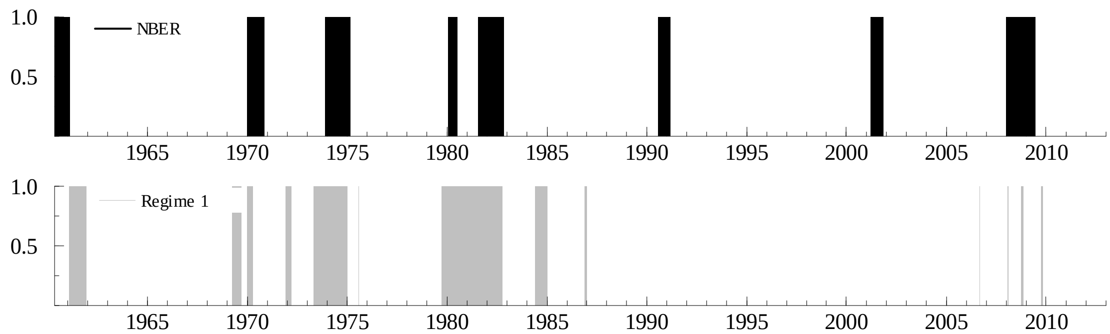


Figure 2C: NBER recession dates (shadowed black areas) compared with smoothed probabilities of Regime 1 (shadowed grey areas).

Table 2C - *Estimated coefficients in the non linear VECM(1)*

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.618940**	−0.116170	0.055603	**0.036655**
$\Delta\overline{R}_{t-1}$	**0.304398**	0.043001	**−0.116891**	−0.000576
$\Delta\overline{\pi}_{t-1}$	0.199760	**0.375792**	−0.101656	−0.006421
$\Delta\Theta_{t-1}$	**0.784600**	**−0.489908**	**0.439680**	−0.005025
Δu_{t-1}	**−11.61071**	−1.537988	−1.277024	**0.214426**
$\beta' y_{t-1}$	**−0.022735**	0.003336	−0.003136	**−0.001237**
SE (Reg.1)	1.020657	0.474899	0.326292	0.031569

Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	0.009180	−0.042348	**0.319242**	**0.013478**
$\Delta\overline{R}_{t-1}$	**0.520426**	**0.182668**	**−0.272291**	**−0.021101**
$\Delta\overline{\pi}_{t-1}$	0.027547	**0.291142**	**−0.312431**	−0.004958
$\Delta\Theta_{t-1}$	0.024176	0.017141	**0.422738**	0.004043
Δu_{t-1}	**−1.393898**	−0.107293	−0.186801	**−0.110677**
$\beta' y_{t-1}$	−0.000292	0.002110	**−0.012857**	**−0.000651**
SE (Reg.2)	0.180859	0.283529	0.389653	0.026396

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Appendix D: ADF, DF-GLS and KPSS Tests

Table 1D shows the results of the ADF tests, DF-GLS tests (Elliott *et al.*, 1996) and KPSS tests (Kwiatkowski *et al.*, 1992), allowing for an intercept, as the deterministic component, in the level of g_c. The column denoted Lags reports the maximum lag, which was selected on the basis of the Akaike information criterion (AIC) and also chosen in order to avoid autocorrelated residuals of each ADF regression. All the results show the presence of a unit root in levels of the variables, since we were unable to reject the unit root null hypothesis at conventional levels of significance, and KPSS stationarity tests confirm this result. Panel b of Table 1 shows that differencing the series induce stationarity in each case without ambiguity. Therefore, we conclude that the examined series are a realization from a stochastic process integrated of order one I(1).

Table 1D - *Unit-root test*

	Panel a: Variables in levels			
Variables	*Lags*	*ADF*	*DF GLS*	*KPSS*
$\overline{R}$	13	-2.426	-1.908	1.259
$\overline{\pi}$	13	-2.186	-1.729	0.876
Θ	12	-2.291	-1.367	1.026
u	3	-2.192	-1.925	1.388
	Panel b: Variables in differences			
Variables	*Lags*	*ADF*	*DF GLS*	*KPSS*
$\Delta\overline{R}$	13	-6.270	-4.765	0.089
$\Delta\overline{\pi}$	13	-7.245	-5.204	0.067
$\Delta\Theta$	12	-8.375	-4.923	0.030
Δu	3	-8.029	-3.737	0.097

Note. Critical values at the 5 and 1 percent significance levels for the ADF test for the unit root null, in the case of a constant in the regression, are -2.87, -3.44, respectively. Critical values at the 10, 5 and 1 percent significance levels for the DF-GLS test (Elliott et al., 1996) for the unit root null, in the case of a constant as the deterministic component of the regression, are -2.62, -2.03 and -1.73, respectively. The column denoted Lags reports the maximum lag, which was selected on the basis of the Akaike Information Criterion (AIC) and to avoid autocorrelated residuals of each ADF regression. Critical values at the 10, 5 and 1 percent significance levels for the KPSS test (Kwiatkowski et al., 1992) for the null of stationarity, in the case of a constant as the deterministic component of the regression, are 0.35, 0.46 and 0.74, respectively.

A highly persistent series, with a root very near to unity, is in practice indistinguishable from a true unit root and it is better approximated by I(1) process than by stationary ones (while acknowledging the alternative of fractional cointegration). Moreover, as shown by Johansen (2006), the cost of treating near unit roots as stationary is that the standard asymptotic distributions provide very poor approximations to the finite sample distributions of the estimated steady-state values.

Figure 1D and Table 2D show clearly, as stated in the paper, that the growth rate of the real consumption (g_c) and the rate of change of velocity of circulation of money (g_v) are stationary variables.

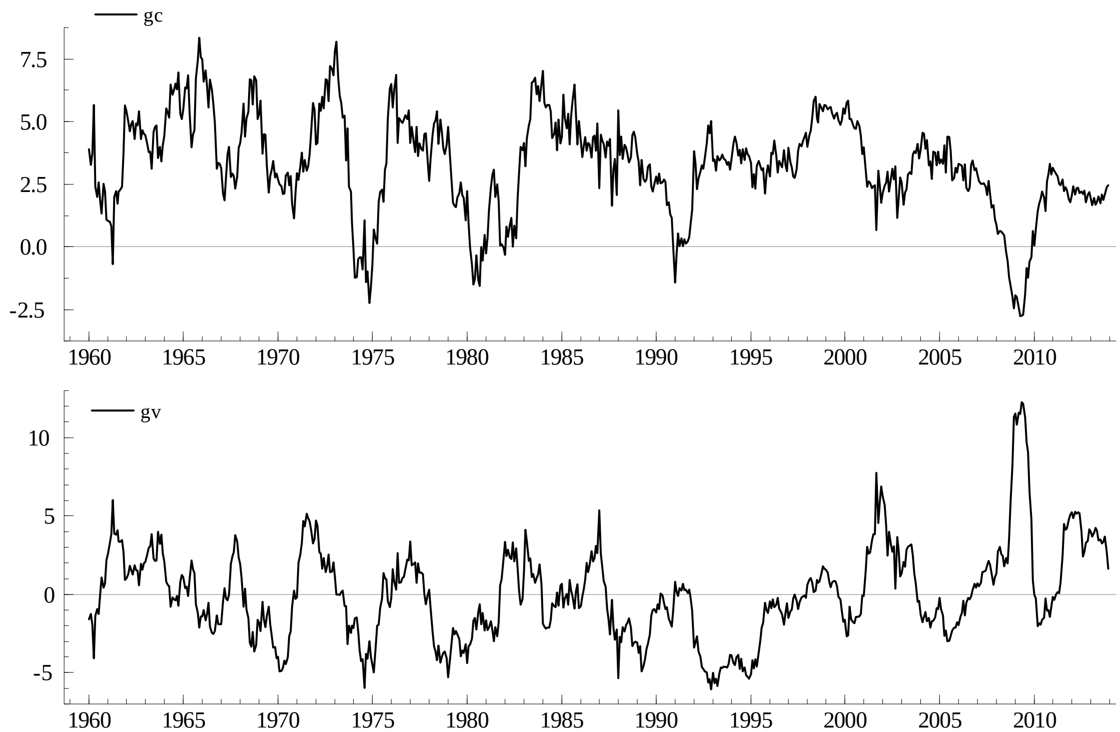


Figure 1D - gc and gv are, respectively, the rate of growth of the real consumption and of the money velocity.

Table 2D - *Unit-root test*

	Panel a: Variables in levels			
$Variables$	$Lags$	ADF	$DFGLS$	$KPSS$
g_c	12	-4.370	-3.371	0.351
g_v	12	-3.436	-2.087	0.445
	Panel b: Variables in differences			
$Variables$	$Lags$	ADF	$DFGLS$	$KPSS$
Δg_c	11	-8.352	-2.190	0.013
Δg_v	11	-10.466	-10.396	0.037

See Table 1D