Liquidity Shortages in a Model with Equilibrium Shirking

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Abstract

This paper develops a model in which idea-rich, cash-poor entrepreneurs develop risky investment projects that are subject to future stochastic funding requirements and moral hazard associated with the potential of entrepreneurs to reduce their work effort and affect the likelihood that the project will succeed. The model suggests that the incentive for the entrepreneur to shirk increases during times of economic distress, and this heightened degree of shirking in the economy exacerbates both the magnitude and duration of the subsequent economic contraction. This result occurs without significant additional cuts in factor employment due to shirking, but is instead the consequence of fewer projects being successfully completed due to the weakened incentives that the economic environment provides to the entrepreneurs to ensure their project's success.

Keywords: Credit-rationing, moral hazard, equilibrium shirking.

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1 Introduction

The recent financial crisis has focused the attention of the profession on the important role that financial frictions can play in affecting the onset, depth, and duration of economic downturns. There has been renewed interest in the financial accelerator models of Williamson (1986,1987) and Bernanke and Gertler (1989) in which a contraction in the net worth of the businesses during recessions may limit their ability of obtain funding and exacerbate the decline in economic activity. These models are based on the early work of Diamond (1984) in which moral hazard associated with the borrowers' private information concerning project outcomes is factored into incentive constraints embedded in financial contracts.

Among the recent papers that have extended this literature are Gertler and Kiyotaki (2010) and Curdia and Woodford (2010), who construct models with exogenous financial frictions that are able to simulate the observed behavior of selected key financial and macroeconomic variables during the recent credit crisis in the United States. They are primarily concerned with examining the appropriate monetary policy responses to economic shocks originating in the financial sector. However, these models do not address the endogenous emergence of liquidity shortages *per se*, i.e., shortages of funds needed to meet sudden, unanticipated expenditures associated with ongoing business activities. Such liquidity shortages are regular features of recessions, and were particular prominent during the recent financial crisis.[1]

Liquidity shortages are characterized by Holmstrom and Tirole (1998), hereafter HT, and Atolia, Einarsson, and Marquis (2010), hereafter AEM, as arising from the limited pledgeable future income associated with funded projects in the presence of moral hazard. Adverse shocks to firms may result in termination of ongoing projects, which in aggregate could reduce overall economic activity if the provision of private liquidity is curtailed. HT examine conditions under which the government may usefully supplement the supply of liquidity to the economy.[2] AEM examine how moral hazard in lending can induce liquidity shortages during severe economic downturns and thereby exacerbate the economic contraction that ensues.

In the model of AEM, incentive constraints in equity contracts are always seen to induce maximum work effort. The consequence is a reduction in lending below the socially optimal level of funding. These incentive constraints were seen to bind only occasionally when adverse aggregate economic shocks were sufficiently large, and

[1] One branch of this literature that deals with the liquidity issues of financial institutions is represented by the bank runs model of Diamond and Dybvig (1983) and models in which financial fragility serves as a commitment mechanism as in Diamond and Rajan (2001). These models rely on adverse selection associated with investor types and are not the subject of this paper which is more concerned with how the interaction of liquidity shortages and moral hazard during economic downturns can heighten the economy's contraction.

[2] Kiyotaki and Moore (2005, 2008) examine insufficient aggregate liquidity arising from liquidity constraints that limit the supply of new equity issues due to the inalienable human capital of entrepreneurs and from limited marketability of some existing equity shares, thus giving rise to a demand for money. The purpose of their paper is to capture some asset-pricing anomalies and demonstrate the potential role of open market operations that are conducted in the equity markets in mitigating the aggregate liquidity shortages.

credit rationing would result. These results do not depend on a financial accelerator associated with fluctuations in a borrower's net worth, as in much of the recent literature. This result occurs solely from the unwillingness of lenders to support as many firms that experience unanticipated idiosyncratic liquidity shocks and ration the credit extended to them.

A shortcoming of the AER model is the rigid structure of the incentive compatibility constraint that rules out equilibrium shirking. In practice, shirking cannot be completely eliminated, nor is it necessarily desirable to do so. This paper examines the consequences of contracts that allow some degree of shirking to occur in equilibrium. The benefit to entrepreneurs from shirking varies across firms and becomes known to the lender after initial funding of the projects. Firms found to be subject to greater moral hazard end up shirking. However, despite shirking an entrepreneur may receive additional funding for his project, if the project's unanticipated liquidity needs are sufficiently small. That is, while shirking dims the success rate of the project, it does not necessarily make the alternative of no additional funding and zero return with certainty the dominant outcome.

Equilibrium shirking qualitatively alters the incidence of moral hazard affecting the rationing of credit to firms. For example, in the AEM model, incentive constraints associated with moral hazard bind only occasionally, i.e., during severe economic downturns, and affect all firms. By contrast, credit rationing due to moral hazard in the case of equilibrium shirking affects only the marginal firms seeking the benefits of shirking and thus operates at the extensive rather than the intensive margin.[3]

In essence, firms are all identical when they approach the lenders with two-period, risky investment projects. The lenders observe the aggregate productivity shock and know the stationary distributions of firms' idiosyncratic liquidity shocks and of the firm's benefit from shirking that are realized in the second period. They then choose the amount of funding to offer each firm by purchasing equity shares in the firms' proposed projects. Firms in the second period of the projects receive their liquidity shocks in the form of unanticipated costs that must be met if the firm is to bring their project to fruition. Shirking occurs that reduces the likelihood of the risky projects succeeding with a positive payoff and also diminishes the expected consumer surplus from successful projects.

The total amount of initial funding for the project that is chosen by investors equates their expected return to their outside investment option, while entrepreneurs select their shares of the project so that they can extract the entire economic surplus from the projects. Adverse aggregate productivity shocks reduce the likelihood of successful completion of projects by exacerbating moral hazard. This moral hazard reduces project profitability and investors' initial funding, thus rendering investment in the projects procyclical. Reduced profitability, however, causes a disproportionately larger fall in surplus relative to project revenues, thus lowering the inside equity

[3]This model also includes reproducible physical capital which was absent in AEM. The presence of capital in the model has two significant effects on the model's results. It allows households to smooth consumption through investment, while introducing greater volatility in output due to the persistence effects of fluctuations in reproducible capital on output. See Gibson (2011) for an analysis of the output and welfare effects in the AEM model with capital added.

needed by the entrepreneur to extract the surplus, thereby rendering outside shares countercyclical.

With persistent productivity shocks, an increase in moral hazard due to adverse shocks not only reduces first-period funding but also the amount of aggregate liquidity set aside for future funding of liquidity needs. These factors combine to exacerbate economic downturns through investment channels. In the presence of equilibrium shirking, both the amplitude and duration of economic contractions are more severe than they would otherwise be in the absence of shirking. This outcome is significant in that contracts could have been written that enforced a higher level of work effort, but were not freely selected. Greater volatility in consumption, output, and labor income is the result.

2 The Model

The principal focus of the model is on the aggregate consequences of moral hazard when shirking may occur for some firms that are subject to idiosyncratic liquidity shocks. To ensure perfect risk-sharing within the representative household setting, the following assumptions are made. There is a continuum of households, each of which consists of an investor and continuums of workers and entrepreneurs. Each entrepreneur owns an investment project that requires labor services and rented capital that is supplied from outside the household, the funding for which requires external finance. The workers within the household and the household capital stock are supplied to entrepreneurs of other households in exchange for labor and capital income. The investor manages the household's assets, which include capital, equity shares in the projects of other households, and a liquid real asset called 'money,' All entrepreneurs' projects are ex ante identical and are traded in a single equity market where they receive identical share prices. A household is able to completely diversify the idiosyncratic risk by taking equal equity positions in all projects offered by other households' entrepreneurs.

In this economy, the household provides neither investment funding nor labor services to its own projects. At the beginning of the period, the members of the household separate, perform their assigned tasks, and then reunite at the end of the period, when they pool their resources and consume together. This structure of the representative household ensures labor and equity markets in which moral hazard issues may arise.

2.1 Project Implementation and Financing

Each entrepreneur has a two-period risky project that requires external finance. An initial investment is made to cover the rental of capital and the wage bill. This investment is received in exchange for equity shares in the project. After one period, an aggregate productivity shock is observed and an idiosyncratic liquidity shock occurs, where the latter is modeled as an additional amount of labor needed to bring the project to completion. At this stage, the investors decide whether to provide

additional funding or abandon the project.

If the project is continued, it may succeed and provide a positive payoff or it may fail and yield a zero return. The probability of success is determined by whether the entrepreneur chooses to shirk. If he chooses not to shirk the project succeeds with probability p_H, if he chooses to shirk, he receives a private benefit of J and the project succeeds with probability $p_L < p_H$, thus inducing a dead-weight loss for the economy. In a departure from Atolia, Einarsson, and Marquis (2010), the benefit from shirking, J, is uncertain at the time the project is initially funded, however, its distribution is known. The actual realization occurs at the beginning of the second period when the liquidity financing decision is made.

Since the initial financing decision is made prior to the realization of J, all projects receive the same funding. Given the heterogeneity in the realized shirking benefit, in equilibrium, it is optimal for entrepreneurs to shirk if the benefit from shirking is above a threshold. However, as the realization of J is known at the beginning of the next period before the liquidity needs are financed and can be observed by the investors, the financing of second-period liquidity needs can be made contingent on this realization.[4] The timing of decisions and resolution of uncertainty with respect to projects is shown Figure 1.

At the beginning of each period, projects of total measure one are initiated by the entrepreneurs from each household. These projects are indexed by $i \in [0, 1]$. If the project is taken to completion and is successful, its output is determined by the amount of labor employed in the initial period and the amount capital rented in the first period and deployed in the second period. Let y^i_{t+1} denote the output of a successfully implemented project that was begun in period t. Then,

$$y^i_{t+1}\left(\theta_{t+1}\right) = \theta_{t+1}(n^i_{1,t})^{\alpha}\left(k^i_{t+1}\right)^{\gamma}, \qquad \alpha, \gamma > 0, \quad \alpha + \gamma \leq 1, \tag{1}$$

where $n^i_{1,t}$ is the outside labor employed by the household in period t, k^i_{t+1} is the capital employed, and θ_{t+1} is the realization of the stochastic aggregate productivity parameter at the beginning of time $t + 1$. Revenue from this project is denoted $\hat{R}^i\left(\theta_{t+1}\right)$ and given by:

$$\hat{R}^i\left(\theta_{t+1}\right) \equiv q^i_{t+1} y^i_{t+1}\left(\theta_{t+1}\right) = q^i_{t+1}\theta_{t+1}(n^i_{1t})^{\alpha}\left(k^i_{t+1}\right)^{\gamma}, \tag{2}$$

where q^i_{t+1} is the price of good i produced by entrepreneur i's project. Note that both the project's output and revenue are stochastic.

At the beginning of period $t + 1$, each project begun in period t experiences the same aggregate productivity shock, and a project-specific liquidity shock that determines the amount of additional funding needed to bring the project to conclusion. The shock is denoted ρ^i_{t+1} with distribution $F(\rho)$ and density $f(\rho)$ which are known at date t when the initial financing decision is made. An entrepreneur receiving this shock must hire an additional $n^i_{2,t+1}$ outside workers, such that

[4] We do not impose any costly state verification, i.e., an entrepreneur's "type" or private benefit that would accrue in the event of shirking is revealed without additional costs to the investors.

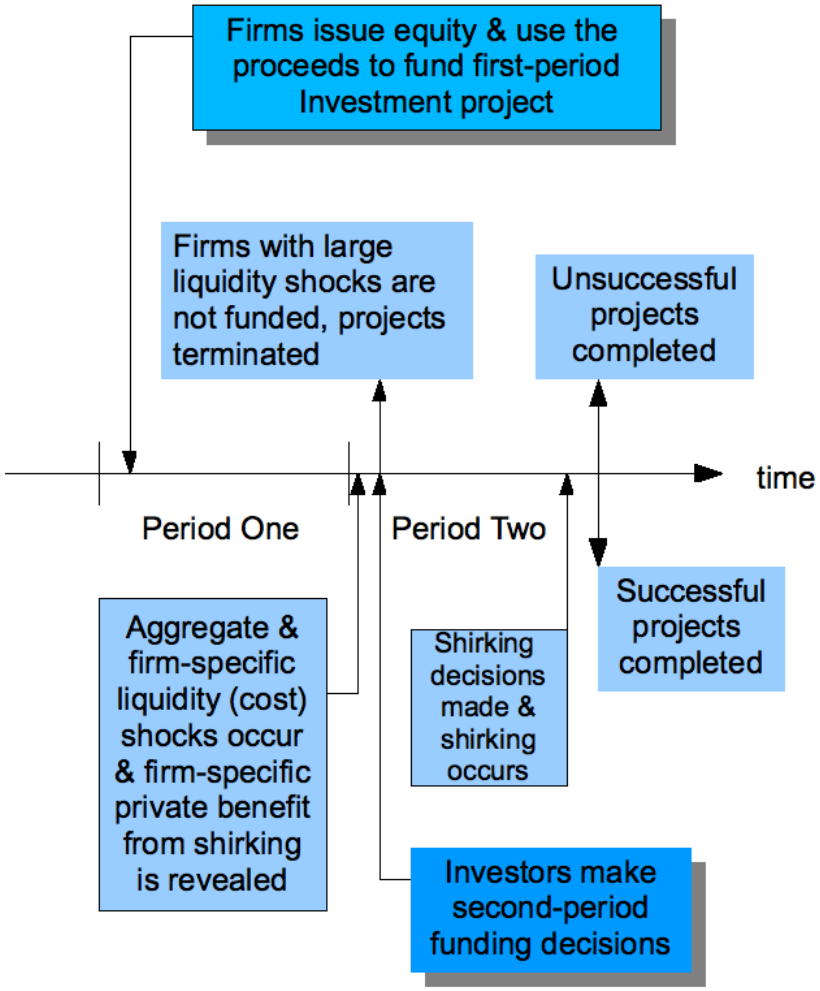

Figure 1: Timing of Project Financing Decisions

$$n_{2,t+1}^{i} = \rho_{t+1}^{i}. \tag{3}$$

This shock is labeled a liquidity shock reflecting the constraint that its financing requires external funding with a liquid asset as described below.

All projects require 100 percent external financing. In the first period, entrepreneurs issue equity shares to investors and use the proceeds to finance the first-period wage bill and the capital rental expense. While investors are aware of the stochastic process generating the aggregate productivity shocks and the distributions of liquidity shocks and the entrepreneurs' private benefits, they do not observe these shocks until the beginning of the second period, at which time they must decide whether to fund the observed liquidity shocks and allow the projects to run to completion or simply refuse the second-period funding and effectively terminate the projects.

With all of the project's costs paid upfront, sales revenues represent net profits to be distributed among shareholders, i.e., investors and entrepreneurs. Denote external shares in project i issued by the entrepreneur to investors by s^i and normalize total shares per project to one. Then, the first-period financing constraint is given by:

$$p_t s_t^i = w_t n_{1,t}^i + r_t k_{t+1}^i, \tag{4}$$

where p_t is the share price associated with a project started in period t, which thus receives $p_t s_t^i$ total initial funding, and w_t and r_t are the wage rate and capital rental in period t.

If the liquidity need is financed, the probability of the successful completion of the project depends on satisfying the incentive-compatibility constraint for the entrepreneur, which depends on his private benefit from shirking, or

$$p_H \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{t+1}\right) \geq p_L \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{t+1}\right) + J_{t+1}^i s_t^i, \tag{5}$$

where the total benefit from shirking, $J_{t+1}^i s_t^i$, with $J_{t+1}^i > 0$, is an increasing function of outside equity shares, s_t^i, reflecting the lower stake that entrepreneurs hold in the project, i.e., as s increases. The benefit J_{t+1}^i is drawn from distribution $H(J)$ with density $h(J)$ which is known at the time of financing the project in period t. Note that there is one incentive compatibility constraint for each aggregate state. Therefore, for each aggregate state, θ_{t+1}, there exists a threshold value

$$J_{t+1}^{i*} \left(\theta_{t+1}\right) = \Delta p \frac{1 - s_t^i}{s_t^i} \hat{R}^i \left(\theta_{t+1}\right) \tag{6}$$

with $\Delta p = p_H - p_L$, such that all projects with $J_{t+1}^i > J_{t+1}^{i*} \left(\theta_{t+1}\right)$ are subject to shirking and for these projects the probability of success falls from p_H to p_L.

Investors realize that there will be a need for liquidity financing in the second period for some fraction of the projects in which they purchase equity shares. This second-period financing requires liquid assets that have been set aside in the previous period for this purpose. For each firm, once ρ_{t+1}^i is observed, the investor decides whether to fund the liquidity need. Conditional on being financed, the expected

benefit for the investor is identical for all continued projects. Therefore, there exists a threshold value of ρ_{t+1} such that all projects with lower liquidity needs than the threshold are financed.

The threshold value depends on the aggregate productivity shock and whether shirking is expected to take place. Denote the threshold cutoff value for projects with a high probability of success (no shirking) by $\rho_{t+1}^{H*}(\theta_{t+1})$ and with a low probability of success (shirking) by $\rho_{t+1}^{L*}(\theta_{t+1})$. The dependence of the threshold value of the liquidity shock that determines whether financing is forthcoming on the probability of success owes to: (i) the fact that the probability of success affects the expected benefit of liquidity financing, (ii) the assumption that the actual realization of the benefit from shirking (which affects probability of success) occurs before the decision to finance the liquidity need is made and that this realization can be observed by the investors, and (iii) the assumption that the liquidity shock and the benefit from shirking are independent. The functional dependence of ρ^*'s on θ_{t+1} arises from the fact that the project revenue (conditional on probability of success) which determines the benefit of liquidity financing, depends on θ_{t+1}.

2.2 The Household sector

This section describes the decisions of the representative household – excluding the entrepreneurs' decisions. The entrepreneur's problem is treated separately for added emphasis.

The representative household maximizes lifetime utility, with period utility, $U(C, L)$, defined over consumption and leisure. The varieties of consumption goods produced by different projects are perfect substitutes, implying that consumption can be defined by a linear aggregator of different varieties, or

$$C_t = \int_0^1 c_t^k dk, \tag{7}$$

where c_t^k is the consumption of variety or good k. While the household consumes jointly, the individual members of the household – the investor, the entrepreneurs, and the workers – specialize in different income-earning activities. The workers provide the labor, n_t, which generates labor income for the household. Entrepreneurs start new projects in each period and retain $(1 - s_t^i)$ shares in the *new* projects. Profits from *maturing* projects are denoted Π_t^l, which provides another source of income for the household.

The investor manages the household's assets, making the optimal consumption-saving decision and the portfolio allocation decision for investing the household's savings among three assets. The investor chooses k_{t+1} units of capital to carry to the next period which it rents at rate r_t. The rent on capital is paid in period t. Second, it buys s_t^j shares of projects externally operated by other households, where $j \in [0, 1]$.[5]

[5]We note that the households in the economy are of measure one, and each household starts projects of measure one. Thus, a double continuum of projects are started every period. However, in the spirit of the representative agent assumption and to simplify notation, we avoid the double

With the number of project shares normalized to 1, the household is entitled to the fraction s_t^j of the gross revenue from sales of the project's output in period $t+1$, provided the project is carried through to completion and is successful. In order that the project be completed, its random liquidity need that is realized at the beginning of period $t+1$ must be financed. This liquidity need arises from the fact that the entrepreneur must pay for unanticipated extra costs of operations in period $t+1$ before production is possible. Setting aside funds for future liquidity financing is the third investment option for the household. These funds are carried forward in the form of M_{t+1} units of liquid assets, which comprise the economy's composite goods that can be costlessly stored intertemporally, but yield zero net return.

The final decision of the household's investor is to determine which of the ongoing projects that he has initially funded are to receive additional funding in the second period to absorb the liquidity need and enable the projects to be carried to completion. This decision is made after observing the current period aggregate shock, θ_t, and the project-specific liquidity shock, ρ_t^k, and the revelation of the private benefit, J_t^k, that the entrepreneur managing the project would receive if he were to chose to shirk and lower the probability that the project will succeed. The superscript k differs from j used earlier to indicate projects that were begun and financed in the previous period, $t-1$. As discussed earlier, this decision would take the form of a cut-off value of the liquidity shock, ρ_t^{H*} or ρ_t^{L*} depending on the realization of the benefit from shirking, J_t^k, and the consequent high or low probability of success of the project.

Let $m_t^k\left(\rho_t^k\right)$ denote the liquidity need per share that the household must choose whether to fund, given the number of shares s_{t-1}^k that were issued in the previous period. Then, in equilibrium, given the total liquidity need for project k, $m_t^k\left(\rho_t^k\right)$ satisfies:

$$m_t^k\left(\rho_t^k\right) s_{t-1}^k = \rho_t^k w_t. \tag{8}$$

Denoting the household's total income by X_t, it can be expressed as:

$$X_t = \begin{array}{c} w_t n_t + r_t k_{t+1} + \int_0^1 \Pi_t^l dl + \\ \int_0^1 s_{t-1}^k \hat{R}_t^k(\theta_t)\left[p_H I_{[\rho_t^k \le \rho_t^{H*}]} I_{[J_t^k \le J_t^*(\theta_t)]} + p_L I_{[\rho_t^k \le \rho_t^{L*}]} I_{[J_t^j > J_t^*(\theta_t)]}\right] dk \end{array}, \tag{9}$$

where I denotes the indicator function that is one when the condition in its subscript is true and zero otherwise.

The consumption-based price index for the aggregate good is

$$Q_t = \min_k q_t^k, \tag{10}$$

which in equilibrium will imply that

$$q_t^k = q_t = Q_t = 1, \tag{11}$$

index notation for the projects. Furthermore a single index notation is sufficient for our purposes as a single continuum of projects allows the household to completely diversify idiosyncratic risk.

for all varieties k that are produced in equilibrium as the composite good is the numeraire.

Thus, the household's budget constraint is

$$C_t + \int_0^1 p_t s_t^j dj + \int_0^1 m_t^k(\rho_t^k) s_{t-1}^k \left[I_{[\rho_t^k \leq \rho_t^{H*}]} I_{[J_t^k \leq J_t^*(\theta_t)]} + I_{[\rho_t^k \leq \rho_t^{L*}]} I_{[J_t^k > J_t^*(\theta_t)]} \right] dk \quad (12)$$
$$+ M_{t+1} + [K_{t+1} - (1-\delta) K_t] \leq M_t + X_t,$$

where the right-side has the total funds available to the household: the liquidity and the income described in (9). The left-hand side is the use of those funds: consumption, the purchase of shares in new projects, funds needed to meet the liquidity needs of existing projects, provision for the liquidity needs for the next period, and the household's gross investment. In addition to the budget constraint, the ability of the household to meet the current liquidity needs is constrained by the liquidity carried over from the previous period, implying the following inequality:

$$\int_0^1 m_t^k(\rho_t^k) s_{t-1}^k \left[I_{[\rho_t^k \leq \rho_t^{H*}]} I_{[J_t^k \leq J_t^*(\theta_t)]} + I_{[\rho_t^k \leq \rho_t^{L*}]} I_{[J_t^k > J_t^*(\theta_t)]} \right] dk \leq M_t. \quad (13)$$

At the beginning of period 0, the household takes as given its initial asset holdings that includes shares in its own projects $(M_0, K_0, s_{-1}^k, s_{-1}^l)$ and solves the following problem:

$$\max_{\substack{\{C_t, L_t, n_t, M_{t+1}, K_{t+1}, \\ s_t^i, s_t^j, \rho_t^{H*}, \rho_t^{L*}\}}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \quad (14)$$

subject to

$$n_t + L_t \leq 1, \quad (15)$$

and $(12-13)$. For the economy as a whole $n_{1,t-1}$ is also given.

The optimization problem can be reformulated as a dynamic program, where the household-specific state variables are $\left(M, K, s_{-1}^l, s_{-1}^k\right)$, with s_{-1}^l representing shares of projects started by the household in the previous period, s_{-1}^k representing shares of projects of other households in which the representative household had invested in the previous period. Shares of projects undertaken by the household in the current period are denoted s^i, and shares of projects begun by other households in which the representative household is investing in the current period are denoted s^j. The aggregate state variables are θ and $n_{1,-1}$. The problem can then be written recursively as:

$$V\left(M, K, s_{-1}^l, s_{-1}^k; \theta, n_{1,-1}\right) = \max_{\substack{\{C, L, n, M', K', \\ s^i, s^j, \rho^{H*}, \rho^{L*}\}}} \left\{ \begin{array}{c} U(C, L) + \\ \beta E_{\theta'} \left[V\left(M', K', s^i, s^j; \theta', n_1\right) \right] \end{array} \right\}, \quad (16)$$

subject to $(12-13)$ and (15).

2.3 Entreprenuer's Problem

Recall that the project under management by the entrepreneur is subject to moral hazard. The probability of success of the project depends on the effort of the entrepreneur. If the entrepreneur exerts effort, the probability of success is p_H, and if he shirks, the probability falls to $p_L < p_H$. Shirking provides an exogenous benefit to the entrepreneur. As investors are aware of this possibility, they limit funding to the point where their expected return equates to the return from alternative investment options.

We assume that the entrepreneur maximizes his expected profits subject to his incentive compatibility constraint and his first-period funding needs. He is the residual claimant to the fraction $(1 - s_t^i)$ of period $t + 1$ gross revenues that are realized if the project succeeds. His share of these revenues corresponds to the entire consumer surplus expected to be realized from the investment. His objective therefore corresponds to maximizing the consumer surplus that is generated by the project. Ex-post heterogeneity in the benefit from shirking across entrepreneurs allows for the possibility of equilibrium shirking. The incentive-compatiblity constraint in equation (4) defines the threshold value (J^*) of the shirking benefit J (see eq. (5)). For $J > J^*$ the entrepreneur shirks.

The entreprenuer's profits, therefore, are $(1 - s_t^i)\,\hat{R}_{t+1}^i$ with probability $p_H F\left(\rho_{t+1}^{H*}\right)$ if he does not shirk and $p_L F\left(\rho_{t+1}^{L*}\right)$ otherwise. Recall, ρ_{t+1}^{H*} $\left(\rho_{t+1}^{L*}\right)$is the maximum liquidity need that is financed by the investor when the probability of success of the project is high (low). Thus, the entrepreneur's objective becomes

$$\max_{\{s_t^i, n_{1,t}^i, k_{t+1}^i\}} \left\{ \begin{array}{c} (1 - s_t^i)\, p_H E_{t,\theta}\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \hat{R}^i\left(\theta_{t+1}\right) F\left(\rho_{t+1}^{H*}\right) | J_{t+1} \leq J^*\left(\theta_{t+1}\right)\right] + \\ (1 - s_t^i)\, p_L E_{t,\theta}\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \hat{R}^i\left(\theta_{t+1}\right) F\left(\rho_{t+1}^{L*}\right) | J_{t+1} > J^*\left(\theta_{t+1}\right)\right] + \\ s_t E_{t,\theta}\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \int_{[J_t^k > J_t^*(\theta_t)]} J dH\left(J\right)\right] \end{array} \right\} \tag{17}$$

where the profits are discounted back to time t using the household's stochastic discount factor and $E_{t,\theta}$ denotes expectation over θ_{t+1} conditional on information at date t and recall that $H\left(.\right)$ is the distribution of J. Equation (17) also makes use of the fact that the liquidity shock and the benefit from shirking are independent. The maximization of (17) is subject to the first-period financing constraint, equation (4).

3 Solving the Model

The solution procedure begins with the household's problem followed by that of the entrepreneur. Of interest is the extent to which the measure of projects subject to moral hazard (due to the incentive compatibility constraint for the entrepreneurs) fluctuates over the business cycle. Moreover, attention will focus on the effect of a tightening of the incentive compatibility constraint during a significant adverse shock and how it negatively impacts access to credit to new projects and the funding of liquidity needs for contuining projects, thereby exacerbating economic downturns.

3.1 Solution to Household's Problem

The first-order conditions for the household's problem yield the familiar Euler equation for the household's labor-leisure choice:

$$w_t U_{C_t} = U_{L_t} \tag{18a}$$

The consumption-savings decision of the household, where savings takes the form of gross investment in capital, is altered only slightly from its familiar form due to the payment in advance that is required by the two-period nature of the projects. In this case, the interest is earned in the current period of the investment and the Euler equation becomes:

$$(1 - r_t)\, U_{C_t} = \beta\,(1-\delta)\, E_{t,\theta}\left[U_{C_{t+1}}\right] \tag{18b}$$

The optimality conditions for the choice of liquidity (M_{t+1}), levels of investment in projects (s_t^j), and the decision to finance the liquidity needs of previous-period projects (ρ_t^{H*} and ρ_t^{L*}) are:

$$U_{C_t} = \beta E_{t,\theta}\left[U_{C_{t+1}}\left\{\frac{p_H \hat{R}_{t+1}\left(\theta_{t+1}\right)}{m_{t+1}\left(\rho_{t+1}^{H*}\right)}\right\}\right], \tag{18c}$$

$$U_{C_t} = \begin{array}{c} \beta E_{t,\theta}\left[U_{C_{t+1}}\left\{\frac{p_H \hat{R}_{t+1}(\theta_{t+1}) F\left(\rho_{t+1}^{H*}\right)}{p_t}\right\}\left\{1 - \frac{\bar{m}_{t+1}^{H}\left(\rho_{t+1}^{H*}\right)}{m_{t+1}\left(\rho_{t+1}^{H*}\right)}\right\} H\left(J^*\left(\theta_{t+1}\right)\right)\right] + \\ \beta E_{t,\theta}\left[U_{C_{t+1}}\left\{\frac{p_L \hat{R}_{t+1}(\theta_{t+1}) F\left(\rho_{t+1}^{L*}\right)}{p_t}\right\}\left\{1 - \frac{\bar{m}_{t+1}^{L}\left(\rho_{t+1}^{L*}\right)}{m_{t+1}\left(\rho_{t+1}^{L*}\right)}\right\}\left[1 - H\left(J^*\left(\theta_{t+1}\right)\right)\right]\right] \end{array}, \tag{18d}$$

$$U_{C_t} + \lambda_t = U_{C_t} \frac{p_H \hat{R}_t\left(\theta_t\right)}{m_t\left(\rho_t^{H*}\right)}, \tag{18e}$$

$$U_{C_t} + \lambda_t = U_{C_t} \frac{p_L \hat{R}_t\left(\theta_t\right)}{m_t\left(\rho_t^{L*}\right)}. \tag{18f}$$

where λ_t is the Lagrange multiplier on the liquidity constraint and

$$\bar{m}_{t+1}^{H}\left(\rho_{t+1}^{H*}\right) = \int_0^{\rho_{t+1}^{H*}} m_{t+1}(\rho_{t+1}) \frac{f(\rho)}{F\left(\rho_{t+1}^{H*}\right)} d\rho, \tag{19a}$$

$$\bar{m}_{t+1}^{L}\left(\rho_{t+1}^{L*}\right) = \int_0^{\rho_{t+1}^{L*}} m_{t+1}(\rho_{t+1}) \frac{f(\rho)}{F\left(\rho_{t+1}^{L*}\right)} d\rho. \tag{19b}$$

are the average liquidity needs, conditional on the need being financed, when the probability of successful completion of the project is high and low respectively.

Equations (18c) and (18d) reflect the optimal consumption-savings decisions where savings takes the form of money and equity shares. The right-hand side of (18c) represents the discounted expected benefit of foregoing a unit of consumption today in exchange for an increase in the stock of money available to meet future liquidity

needs. That is, the term in curly braces is the additional revenues per unit of money carried forward. In equation (18d), the first term in curly braces on the first line of the equation is the expected revenues per share divided by the share price when there is no shirking. The second term in curly braces reflects the additional costs of ownership that the average liquidity needs will require. That is, when the average liquidity need is zero, or $\bar{m} = 0$, then this term is one. These expected returns are valued at next period's marginal utility, weighted by the probability that no shirking will occur, and discounted back. The second row of (18c) has a similar interpretation for the case of shirking.

Equations (18e) and (18f) reflect the marginal decisions on funding the second-period liquidity needs for the cases when shirking does not occur, (18e), and when shirking does occur, (18f). In the discussion, attention is restricted to (18e), the first-order condition for ρ_t^{H*}, which on simplification yields:

$$\rho_t^{H*}(\theta_t) = \frac{1}{1 + \frac{\lambda_t}{U_{C_t}}} \frac{p_H s_{t-1}^k \hat{R}_t^k(\theta_t)}{w_t}. \tag{20}$$

This condition on financing the liquidity need is very intuitive. When liquidity is in abundant supply, λ_t is zero, giving:

$$\rho_t^{H*}(\theta_t) w_t = p_H s_{t-1}^k \hat{R}_t^k(\theta_t), \tag{21}$$

where the left-hand side is the liquidity need of the marginal project with high probability of success and the right-hand side is the expected revenue accruing to the investor, conditional on the liquidity need being financed. The liquidity need of a project will be financed up to this amount because the past investment decision is not relevant for liquidity financing. In addition, since the investor is diversified over a large number of identical projects, he is risk-neutral with respect to the liquidity funding of any single project. When liquidity is limited, λ_t is positive and the amount of liquidity supplied to projects is reduced accordingly as indicated in equation (20).

3.2 Solution to Entrepreneur's Problem

The entrepreneur's objective in (17) is strictly increasing in $\hat{R}_{t+1}^i(\theta_{t+1})$, the project revenues in the case of successful completion. Given that production costs are paid in advance, the objective is strictly increasing in $n_{1,t}^\alpha k_{t+1}^\gamma$ irrespective of the future aggregate shock. Thus, it is worthwhile to simplify the problem by first maximizing $n_{1,t}^\alpha k_{t+1}^\gamma$ subject to the first-period financing constraint to solve for the optimal choice of k_{t+1} as a function of $n_{1,t}$. This gives:

$$k_{t+1} = \left(\frac{\gamma}{\alpha}\frac{w_t}{r_t}\right) n_{1,t}. \tag{22}$$

Using (22), the financing constraint can be written as:

$$p_t s_t = \left(\frac{\alpha + \gamma}{\alpha}\right) w_t n_{1,t}. \tag{4'}$$

Now (4′) can be used to solve for both $n_{1,t}$ and k_{t+1} in terms of s_t as:

$$n_{1,t} = \frac{\alpha}{\alpha+\gamma}\frac{p_t}{w_t}s_t, \tag{23a}$$

$$k_{t+1} = \frac{\gamma}{\alpha+\gamma}\frac{p_t}{r_t}s_t. \tag{23b}$$

In what follows, it is assumed that the liquidity shock is uniformly distributed over $[0, \bar{\rho}]$ so that:

$$F(\rho) = \frac{\rho}{\bar{\rho}}, \qquad 0 \leq \rho \leq \bar{\rho}. \tag{24a}$$

and the benefit from shirking J is uniformly distributed over $\left[0, \bar{J}\right]$ so that:

$$H(J) = \frac{J}{\bar{J}}, \qquad 0 \leq J \leq \bar{J}. \tag{24b}$$

The first-order condition for this problem, on simplification, yields

$$\begin{gathered}
\left[2(\alpha+\gamma+1)(1-s_t)-1\right]\left(\frac{p_L^2}{p_H^2}\right)E_{t,\theta}\left[\beta\frac{U_{c,t+1}}{U_{c,t}}\theta_{t+1}F\left(\rho_{t+1}^{H*}\right)\right]+ \\
\left\{\begin{array}{c}\left[\left[2+3(\alpha+\gamma)\right](1-s_t)-2\right]\left(1-\frac{p_L^2}{p_H^2}\right)- \\ \left[\frac{3}{2}(\alpha+\gamma)(1-s_t)-1\right]\Delta p\frac{p_L}{p_H^2}\end{array}\right\}E_{t,\theta}\left[\beta\frac{U_{c,t+1}}{U_{c,t}}\theta_{t+1}F\left(\rho_{t+1}^{H*}\right)H\left(J^*\left(\theta_{t+1}\right)\right)\right]+ \\
\frac{2+\alpha+\gamma}{2}\frac{s_t\bar{J}}{n_{1,t}^{\alpha}k_{t+1}^{\gamma}}\frac{p_L}{p_H^2}E_{t,\theta}\left[\beta\frac{U_{c,t+1}}{U_{c,t}}F\left(\rho_{t+1}^{H*}\right)\right]=0
\end{gathered} \tag{25}$$

3.3 Imposing the Equilibrium

The only goods that are produced in equilibrium are from the projects that received a sufficiently low liquidity shock, i.e., for the projects with $\rho_t^i \leq \rho_t^{H*}$ or $\rho_t^i \leq \rho_t^{L*}$ depending on whether the entrepreneur shirks. All projects are *ex ante* identical and all goods enter symmetrically into the utility function. Thus, for each project with $\rho_t^i \leq \rho_t^{H*}$ or $\rho_t^i \leq \rho_t^{L*}$. Equilibrium conditions become:

$$s_t^i = s_t \tag{26a}$$

$$y_t^i = y_t = \theta_t(n_{1,t-1})^{\alpha}(k_t)^{\gamma} \tag{26b}$$

$$q_t^i = q_t = Q_t = 1 \tag{26c}$$

$$\hat{R}_t^i = \hat{R}_t = y_t \tag{26d}$$

with labor market equilibrium given by:

$$n_{1,t} + \bar{n}_{2,t}^H\left(\rho_t^{H*}\right)F\left(\rho_t^{H*}\right)H\left(J^*\left(\theta_t\right)\right) + \bar{n}_{2,t}^L\left(\rho_t^{L*}\right)F\left(\rho_t^{L*}\right)\left[1-H\left(J^*\left(\theta_t\right)\right)\right] = n_t \tag{27}$$

where

$$\bar{n}_{2,t}^{H}\left(\rho_t^{i*}\right)=\int_0^{\rho_t^{H*}}\rho\frac{f\left(\rho\right)}{F\left(\rho_t^{H*}\right)}d\rho,\quad\text{and}\quad\bar{n}_{2,t}^{L}\left(\rho_t^{L*}\right)=\int_0^{\rho_t^{L*}}\rho\frac{f\left(\rho\right)}{F\left(\rho_t^{L*}\right)}d\rho,\tag{28}$$

are the average additional labor requirements, conditional on shirking and on the liquidity need being financed. Furthermore, the household's time constraint must be satisfied.

$$n_t+L_t=1.\tag{29}$$

The clearing of the market for the aggregate good requires

$$C_t+M_{t+1}-M_t+K_{t+1}-\left(1-\delta\right)K_t=Y_t,\tag{30}$$

where

$$Y_t=y_t\left(\theta_t\right)\left\{p_HF\left(\rho_t^{H*}\right)H\left(J^*\left(\theta_t\right)\right)+p_LF\left(\rho_t^{L*}\right)\left[1-H\left(J^*\left(\theta_t\right)\right)\right]\right\},\tag{31}$$

is the output of the aggregate good.

The equilibrium demand for liquidity cannot exceed the supply so that

$$s_{t-1}\left[\begin{array}{c}H\left(J^*\left(\theta_t\right)\right)\int_0^{\rho_t^{H*}}m_t(\rho)f\left(\rho\right)d\rho+\\\left[1-H\left(J^*\left(\theta_t\right)\right)\right]\int_0^{\rho_t^{L*}}m_t(\rho)f\left(\rho\right)d\rho\end{array}\right]\leq M_t\tag{32}$$

The equations for $(18a-18f)$, $(23a-23b)$, (25), $(26b-26d)$, (27), and $(29-32)$ contain the following endogenous variables: s_t, p_t, y_t, $n_{1,t}$, ρ_t^{H*}, ρ_t^{L*}, q_t, $\hat{R}_t$, w_t, r_t, L_t, n_t, K_{t+1}, C_t, M_{t+1}, Y_t, and λ_t. The model thus consists of 17 variables and 17 equations which can be solved.

4 Calibrating the Model

The functional forms are first specified, followed by the calibration of the model to the data.

4.1 Functional Forms etc.

The utility function is assumed to be log-linear:

$$U\left(C,L\right)=\ln C+\eta\ln L,\ \eta>0\tag{33}$$

The aggregate productivity shock follows an autoregressive process:

$$\ln\theta_t=\psi_\theta\ln\theta_{t-1}+\varepsilon_t,\tag{34}$$

with serial correlation ψ_θ where the innovation to aggregate productivity, ε_t, is assumed to be normally distributed with mean zero and a standard deviation of σ, but

truncated at some lower bound, $\varepsilon_t \geq \varepsilon_L$. Hence, in the non-stochastic steady state $\theta_{ss} = 1$. The truncation of ε is necessary under a continuous distribution in order to prevent shirking in any aggregate state in the nonshirking version of the model. See Atolia, Einarsson, and Marquis (2010) for further details.

Given the distribution of the liquidity shock in $(24a)$ and the equilibrium liquidity funding condition (8), equations (28) and (19) can be written as:

$$\bar{n}^H_{2,ss}(\rho^{H*}_{ss}) = \frac{\rho^{H*}_{ss}}{2}, \quad \text{and} \quad \bar{n}^L_{2,ss}(\rho^{L*}_{ss}) = \frac{\rho^{L*}_{ss}}{2}, \tag{28$_{ss}$}$$

$$\bar{m}^H(\rho^{H*}_{ss}) = \frac{w_{ss}\rho^{H*}_{ss}}{2s_{ss}} = \frac{m^H(\rho^*_{ss})}{2}, \quad \text{and} \quad \bar{m}^L(\rho^*_{ss}) = \frac{w_{ss}\rho^{L*}_{ss}}{2s_{ss}} = \frac{m^L(\rho^{L*}_{ss})}{2} \tag{19$_{ss}$}$$

Using the functional form of the utility function, the optimality conditions $(18a - 18f)$ can be simplified as follows:

$$\frac{w_{ss}}{C_{ss}} = \frac{\eta}{L_{ss}}, \tag{18a$_{ss}$}$$

$$(1 - r_{ss}) = \beta\,(1 - \delta)\,, \tag{18b$_{ss}$}$$

$$1 = \beta\frac{p_H\hat{R}_{ss}}{m(\rho^{H*}_{ss})} = \beta\frac{s_{ss}p_H\hat{R}_{ss}}{\rho^{H*}_{ss}w_{ss}}, \tag{18c$_{ss}$}$$

$$p_{ss} = \beta\hat{R}_{ss}\left[\begin{array}{c} p_H F(\rho^{H*}_{ss})\left\{1 - \frac{\bar{m}^H(\rho^{H*}_{ss})}{m(\rho^{H*})}\right\} H\left(J^*\left(1\right)\right) + \\ p_L F(\rho^{L*}_{ss})\left\{1 - \frac{\bar{m}^L(\rho^{L*}_{ss})}{m(\rho^{L*})}\right\}\left[1 - H\left(J^*\left(1\right)\right)\right] \end{array}\right], \tag{18d$_{ss}$}$$

$$\lambda_{ss} = \frac{1}{C_{ss}}\left[\frac{p_H\hat{R}_{ss}}{m(\rho^{H*}_{ss})} - 1\right], \tag{18e$_{ss}$}$$

$$\lambda_{ss} = \frac{1}{C_{ss}}\left[\frac{p_L\hat{R}_{ss}}{m(\rho^{L*}_{ss})} - 1\right]. \tag{18f$_{ss}$}$$

The first-order condition for the entrepreneur's problem also simplifies considerably in the steady state to:

$$\begin{gathered} \left[2\left(\alpha + \gamma + 1\right)\left(1 - s_t\right) - 1\right]\left(\frac{p_L^2}{p_H^2}\right) + \\ \left\{\begin{array}{c} \left[\left[2 + 3\left(\alpha + \gamma\right)\right]\left(1 - s_t\right) - 2\right]\left(1 - \frac{p_L^2}{p_H^2}\right) - \\ \left[\frac{3}{2}\left(\alpha + \gamma\right)\left(1 - s_t\right) - 1\right]\Delta p\frac{p_L}{p_H^2} \end{array}\right\} H\left(J^*\left(1\right)\right) + \\ \frac{2 + \alpha + \gamma}{2}\frac{s_t\bar{J}}{n^{\alpha}_{1,t}k^{\gamma}_{t+1}}\frac{p_L}{p_H^2} = 0 \end{gathered} \tag{25$_{ss}$}$$

4.2 Calibration

Using $(18c_{ss} - 18d_{ss})$ and $(4')$, one can solve for

$$n_{1,ss} = \frac{\alpha}{\alpha+\gamma}\frac{\left(\rho_{ss}^*\right)^2}{2\bar{\rho}}\left[H\left(J_{ss}^*\right) + \frac{p_L^2}{p_H^2}\left(1 - H\left(J_{ss}^*\right)\right)\right] = \frac{\alpha}{2\alpha+\gamma}n_{ss}, \tag{35}$$

where the last equality follows from (27) and (28_{ss}).

The model is calibrated so that in the non-stochastic steady state $n_{ss} = .36$, which approximates a 40-hour workweek and is consistent with the survey data discussed in Juster and Stafford (1991). This implies a value of $\eta = 0.8773$ for the parameter on leisure in the utility function in the case of shirking. In the the version without shirking, the implied value is 0.9906. For an annual calibration, β is set to the usual value of .96. In the production function, α and γ are set to 1/3. The coefficient on capital, γ, is broadly in line with US post-War aggregate data. The labor share parameter, α, is lower than typically assumed in standard (RBC) models. This reflects the fact that the amount of labor devoted to new projects (i.e. iniated in the current period) is only one part of the total hours worked in the economy. The remaining part, $(n - n_1)$, arises from the liquidity shock. Although essential to bring a project to completion, it is 'unproductive' in the sense that the quantity of output is unaffected. This also implies that the marginal products of k and n_1 do not match (are in fact higher than) r and w respectively. Treating entrepreneurs' share $(1-s)$ of net revenue, i.e. $(Y - wn - rk)$, as labor income (payoff for exerting effort), the total share of labor in final output amounts to about 60 percent. Finally, the innovation to aggregate productivity, σ, is set at .0075 and .0120 in the shirking and nonshirking cases respectively. This equalizes output volatility across the two model versions. The aggregate shock has a serial correlation of $\psi_\theta = .80$, a value widely assumed in annually calibrated RBC models, broadly equivalent to the quarterly value of 0.95. [See, e.g., Kydland and Prescott (1982).]

To ensure that shirking is costly, p_H is given a relatively high value of .9 and p_L is set to a low value of .4. The liquidity shock distribution parameter $\bar{\rho}$ is set to .7, implying that 90 per cent of the second-period liquidity needs of nonshirking entrepreneurs are financed. For the shirkers, the financing ratio is 40 per cent. In the model without shirking, this financing ratio is approximately 85 per cent. The maximum private benefit from shirking, $\bar{J}$, is set to 0.1393, implying that 20 per cent of entrepreneurs are shirking in the nonstochastic steady state. In the model where shirking is not allowed, the fixed value of J is set at 0.0862, which implies that the IC constraint in (5) binds roughly one fifth of the time in stochastic simulations.

The preference and technology parameters are listed in Table 1, along with the calibrated steady states of the shirking and nonshirking model versions.

5 Results

In this section, the effect of equilibrium shirking on business cycle dynamics is discussed. Its effects on the second moments and correlation properties of key aggregate variables in simulation exercises are reported. It is seen that shirking adds volatility to output and consumption, and reverses the correlation with output of the threshold projects that determine the mass of projects whose second-period liquidity needs are financed, and hence are allowed to go to completion. Impulse response functions both in the presence and in the absence of equilibrium shirking are then examined, including exercises that simulate severe contractions. In the latter exercises, shirking is seen to exacerbate the magnitude and duration of downturns in the economy.

5.1 Business Cycles with Equilibrium Shirking Due to Financial Frictions (Moral Hazard)

Table 2 presents results for the second moments from simulations of the model with and without equilibrium shirking. As shown, when shirking is permitted, entrepreneurs assume a smaller stake in the success of their projects, i.e., $1-s$ falls, and shirking becomes highly countercyclical, with the threshold value J^* nearly perfectly correlated with and as volatile as output. As a consequence, output becomes more volatile due to volatility in the success rate of completed projects. To maintain the same output volatility in the no-shirking case requires a productivity shock that is approximately 40% smaller (.0075 vs. .0120) than in the case of shirking. Much of this volatility is absorbed by consumption and labor income, where the latter results from the added volatility of the wage rate rather than employment.

From the business cycle properties of the model, it appears that inefficiencies arising on the production side from frictions resulting from moral hazard impair the ability of capital (or investment) to smooth consumption making it more closely correlated with output. The countercyclical behavior of s, prevents project size (as measured by ps) to rise commensurately with the rise in productivity. This tones down the demand for capital (and the increase in capital rental), thereby attenuating the investment response.

The countercyclical behavior of s in the model can be understood as follows. In this economy, investors behave competitively and as a result all the surplus from the project goes to the entrepreneur. The entrepreneur acquires this surplus through his equity stake $(1-s)$ in the project. In bad times, for a given project size (in terms of n_1 and k), two factors act to reduce the surplus from the project due to the greater probability of shirking arising from a fall in J^*. First, more projects are now likely to succeed with lower probability p_L conditional on their liquidity need being financed. Second, as $\rho^{L*} \ll \rho^{H*}$, more projects are now subject to stricter credit-rationing in the second period. While the value of the outside option of the investor fluctuates over the business cycle (see, for example, the volatility of r_K), this variation is moderate relative to the decline in the surplus of the project due to financial frictions. As a result, the surplus becomes a smaller fraction of the value of the project and hence, a smaller value of $(1-s)$ allows the entrepreneur to extract the surplus making s

countercyclical.

The cyclical properties of the second-period liquidity financing is also significantly affected by equilibrium shirking. The mass of projects that have their liquidity needs funded is mildly procyclical in the absence of shirking, but becomes countercyclical when shirking is present. ρ^{H*} shows a strongly countercyclical behavior which implies firms not subject to binding moral hazard face a tighter liquidity constraint in the second period in good times. The reason for this seeming anomaly lies in the fact that while the supply of liquidity (M) is strongly procyclical, an increase in J^* causes an even stronger increase in demand for liquidity in good times.[6] This response occurs from the increase in J^* that allows more firms to become eligible for liquidity funding as $\rho^{H*} \gg \rho^{L*}$. Furthermore, as the liquidity need is denominated in labor units, the demand for liquidity also rises in good times due to highly procyclical wages. In the no-shirking case only this second effect operates which reduces the correlation of ρ^{H*} with output relative to that of aggregate liquidity M from 0.98 to 0.49, which is still positive. In the current setup, a further increase in liquidity demand due to the first effect described above, causes a negative correlation of ρ^{H*} with output. However, the correlation property of ρ^{H*} exaggerates the cyclical behavior of the availability of liquidity in the model as it fails to take into account the compositional effects arising from transition of firms from being subject to shirking to being free from shirking and vice versa over the business cycle. The weighted average, $\rho^{A*} = M/w = \rho^{H*}H(J^*) + \rho^{L*}(1 - H(J^*))$, which does not suffer from this limitation is much less countercyclical. The countercyclicality of ρ^{A*} is merely a reflection of the excess volatility of the wages over the business cycle that makes liquidity demand measured in units of goods highly procyclical.

5.2 Financial Frictions and the Response to an Adverse Shock

Figures 2-4 compare the impulse responses of key variables of the two economies to a sequence of adverse shocks. In the economy with no shirking, moral hazard causes the incentive constraint to bind only during severe downturns. Therefore, to contrast the equilibrium shirking case from the no-shirking case in presence of moral hazard, both economies are subjected to a two standard deviation adverse productivity shock, beginning in an initial state of economic distress, with below normal productivity ($\theta = .988$).

These adverse shocks exacerbate the moral hazard problem as J^* immediately falls (see Figure 2) and recovers slowly as the adverse shock dies out. Rising moral hazard causes output to contract much more severely with shirking. As seen from the second moments earlier in Table 2, most of this excess impact on output is absorbed by consumption, thereby increasing consumption volatility and its correlation with

[6]The countercyclicality of ρ^{H*} squares with the intuition of the cyclical nature of corporate finance. Firms that are always free from moral hazard in the real world (with low J^* in the model) do find it easy to have their liquidity need financed in bad times when there is a 'flight to quality.' On the other hand, firms on the margin of being subject to moral hazard do indeed see their liquidity constraint becoming less binding in good times as the threat to "project success" posed by moral hazard diminishes.

output. With the same number of projects started every period, average project output declines causing the value of firm shares (p) to fall precipitously along with output, because s does not change much as we shall see later.

The fall in output in the economy (especially on impact) is brought about by a reduction in the success rate of projects, $p_H F\left(\rho^{H*}\right) H\left(J^*\right) + p_L F\left(\rho^{L*}\right)\left[1 - H\left(J^*\right)\right]$ caused by the rising problem of moral hazard and not by reduction in factor usage. As Figure 3 shows, y, the firm output conditional on being successful, is actually higher in the model with equilibrium shirking. A steep fall in wages helps the firm increase first-period labor, n_1 and initially there is very little change in the capital stock. On the other hand, with rising moral hazard (lower J^*), more firms face a binding moral hazard constraint. As the marginal firms become subject to moral hazard, their individual success rate falls from p_H to p_L causing the overall success rate of projects to decline.

The presence of moral hazard causes firms' access to financing to fall in bad times both with and without shirking. As Figure 4 shows, the effect is, however, stronger with equilibrium shirking because of an added reduction in the success rate of projects. First-period financing (ps) declines more, as the fall in the success rate of projects causes a larger decline in average output, even as investors' stake (s) rises modestly during downturns. Second-period liquidity provisioning (M) also falls more with equilibrium shirking. However, note that, as discussed above, due to the sharp decline in the wage rate, despite a lesser availability of aggregate liquidity, the threshold of workers that firms can hire in the second period to meet unexpected project needs goes up: both ρ^{H*} and ρ^{A*} rise. In essence, access to credit falls only for the marginal firms that, during downturns, become subject to moral hazard. Thus, in the model with equilibrium shirking, credit tightening is on the extensive margin, whereas in the no shirking model, it was on the intensive margin as every firm experienced a reduced access to credit.

Summarizing the results, the effect of equilibrium shirking when the economy is in distress and subjected to a very sharp adverse productivity shock is seen to significantly amplify the magnitude and the duration of the economy's contraction. Most of this loss in output is absorbed with persistent declines in consumption and provisions for future liquidity financing, with little additional effects on capital investment or the rental rate. Employment is also seen to be little affected by shirking; however, the wage rate declines sharply. Taken together with the sharp increase in shirking, corresponding to a decline in J^*, is seen to lower output, not because factor employment has fallen, but rather because the success rate for completed projects declines. [See equations (30)and (31).]

6 Conclusions

Idea-rich, cash-poor entrepreneurs must seek funding to be rewarded for product or process innovations that they can bring to the market. However, the success of their entrepreneurial activities can depend on their willingness to exert the effort required to make their project successful. During times of economic stress, the expected rewards

to entrepreneurship falls as the funding for those investments contracts. The incentive for entrepreneurs to be diligent in ensuring success of their projects is reduced and private benefits to shirking are more alluring.

This paper develops a model in which outside equity financing of projects is required along with short-term liquidity needs that arise unexpectedly that require immediate funding for the project to be completed. These projects are subject to the moral hazard associated with entrepreneurial shirking. In this model, equilibrium shirking is present and countercyclical. The consequences of operative contracts that permit some degree of shirking to occur are shown to permit greater volatility in output and consumption. Simulations suggest that shirking amplifies the magnitude and duration of the contraction that accompanies large negative shocks impacting an already distressed economy. However, factor employment per se is little affected by shirking. The major consequences of lower output and consumption result from the effect that greater shirking has on the likelihood of the successful completion of projects.

In comparison to AEM, this paper makes a more realistic assumption on the nature of moral hazard. It allows the benefit from shirking to vary across firms with the actual benefit to a firm or entrepreneur becoming known to the investor (or becoming public knowledge) after the first-period financing has been made. In such a setting, it naturally follows that incentivizing all firms under all conditions will not be optimal. Allowing shirking in equilibrium is therefore a natural outcome. The paper shows that this turns out to be fruitful. Financial frictions are found to be more effective in amplifying the impact of an exogenous shock on the economy in the presence of equilibrium shirking compared to the model in AEM where incentive constraints associated with moral hazard bind occasionally but does not lead to shirking. In this respect, this paper shows that equilibrium shirking seems to be a more promising approach. However, the AEM model can explain the documented asymmetry in business cycles, with downturns being more severe, as the moral hazard (incentive) constraint only binds during bad economic times.

There are several extensions that this paper has suggested to the authors. How important is entrepreneurial net worth to the incentive for entrepreneurs to shirk and investors to invest in uncertain projects that are subject to this type of moral hazard? In the present set up, credit tightening occurs on the extensive margin alone as wages become highly procyclical. Reducing wage flexibility, for example, by introducing labor market frictions can potentially tighten credit on the intensive margin as well.

References

Atolia, Manoj, Tor Einarsson, and Milton Marquis. 2010. "Understanding Liquidity Shortages During Severe Economic Downturns," Journal of Economic Dynamics and Control 35: 330-343.

Bernanke, Ben and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations," American Economic Review 79:14-31.

Curdia, Vasco and Michael Woodford. 2010. "Conventional and Unconventional Monetary Policy, " Manuscript.

Christiano, Lawrence J and Jonas Fisher. 1994. "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints," Federal Reserve Bank of Chicago Working Paper Series (Macroeconommic Issues), WP-94-6.

Diamond, Douglas. 1984. "Financial Intermediation and Delegated Monitoring," Journal of Economic Studies 51:393-414.

Diamond, Douglas and Philip Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy 91:401-419.

Diamond, Douglas and Raghuram Rajan. 2001. "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," Journal of Political Economy April.

Gertler, Mark and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis, " Manuscript.

Gibson, John. 2011. "The Business Cycle Effects of Occasionally Binding Incentive Constraints in Two-Period Risky Projects with Capital," Manuscript. Florida State Universtiy.

Holmstrom, Bengt and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector," Quarterly Journal of Economcis 112:662-692.

Holmstrom, Bengt and Jean Tirole. 1998. "Private and Public Supply of Liquidity," Journal of Political Economy 106:1-40.

Holmstrom, Bengt and Jean Tirole. 2001. "LAPM: A Liquidity-Based Asset Pricing Model," Journal of Finance 56:1837-1867.

Juster, F. Thomas and Frank P. Stafford. 1991. "The Allocatio of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," Journal of Economic Literaure, 29 (June): 471-522.

Kiyotaki, Nobuhiro and John Moore. 2005. "Liquidity and Asset Pricing," International Economic Review 46:317-349.

Kiyotaki, Nobuhiro and John Moore. 2008. "Liquidity, Business Cycles, and Monetary Policy," Working Paper. Princeton University.

Kydland Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations," Econometrica 50(4) (November): 1345-1370.

Marcet, Albert. 1988. "Solving Non-linear Models by Parameterized Expectations," Manuscript. Carnegie-Mellon University, Graduate School of Industrial Administration.

Tirole, Jean. 2006. The Theory of Corporate Finance, Princeton University Press: Princeton and Oxford.

Williamson, Stephen D. 1986. "Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing," Journal of Monetary Economics 18:159-179.

Williamson, Stephen D. 1987. "Financial Intermediation, Business Failures, and Real Business Cycles," Journal of Political Economy 95:1196-1216.

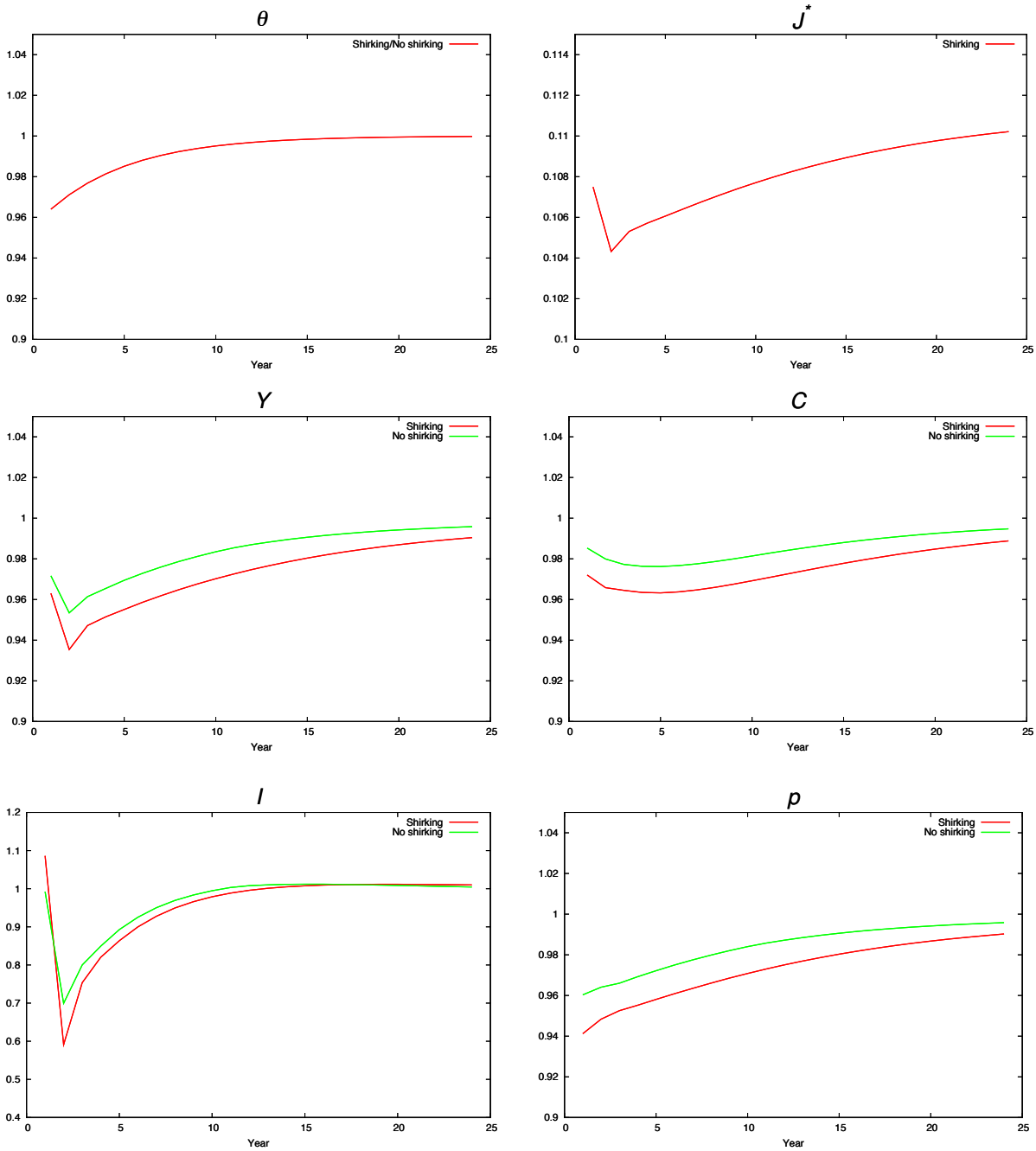

Figure 2. Impulse response functions.

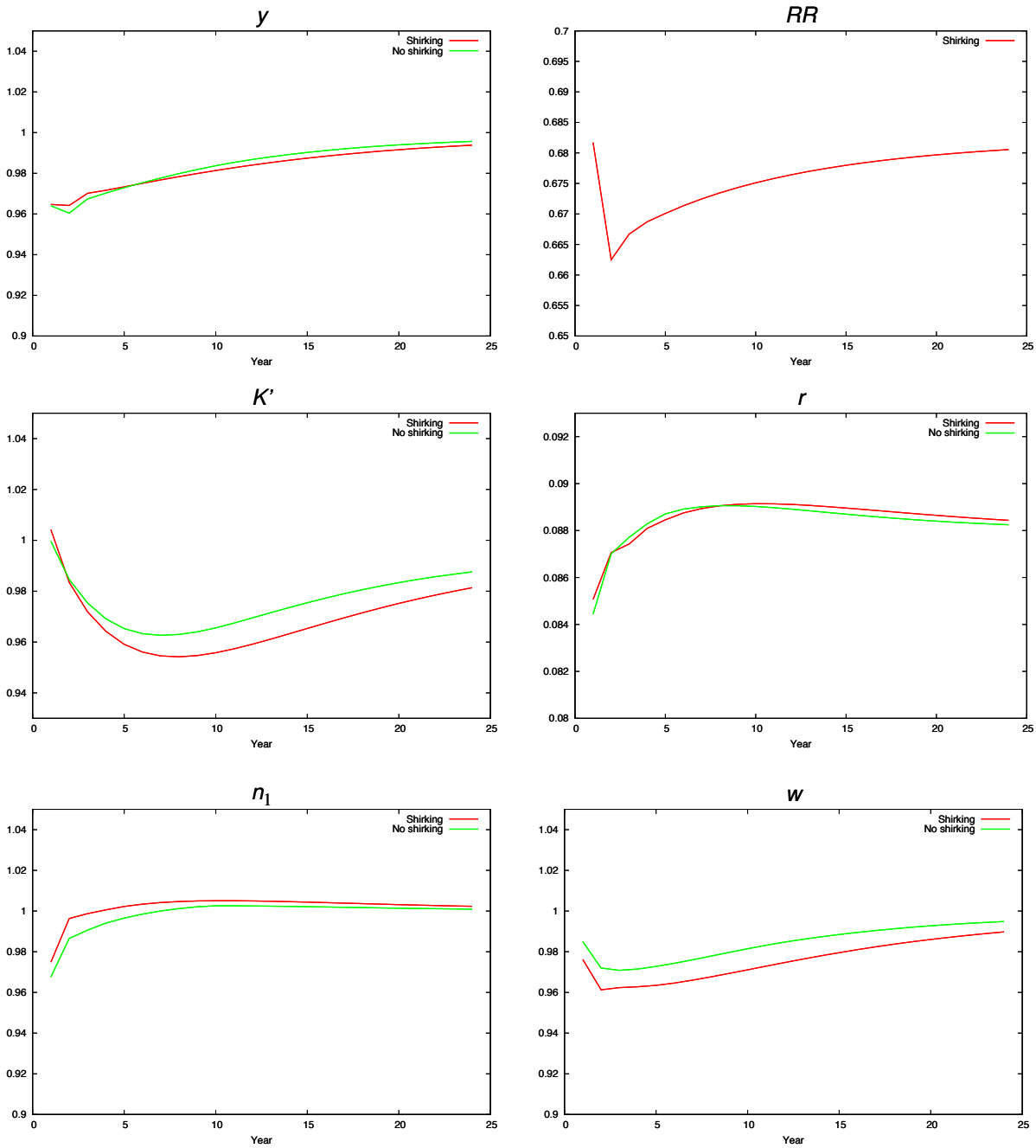

Figure 3. Impulse response functions.

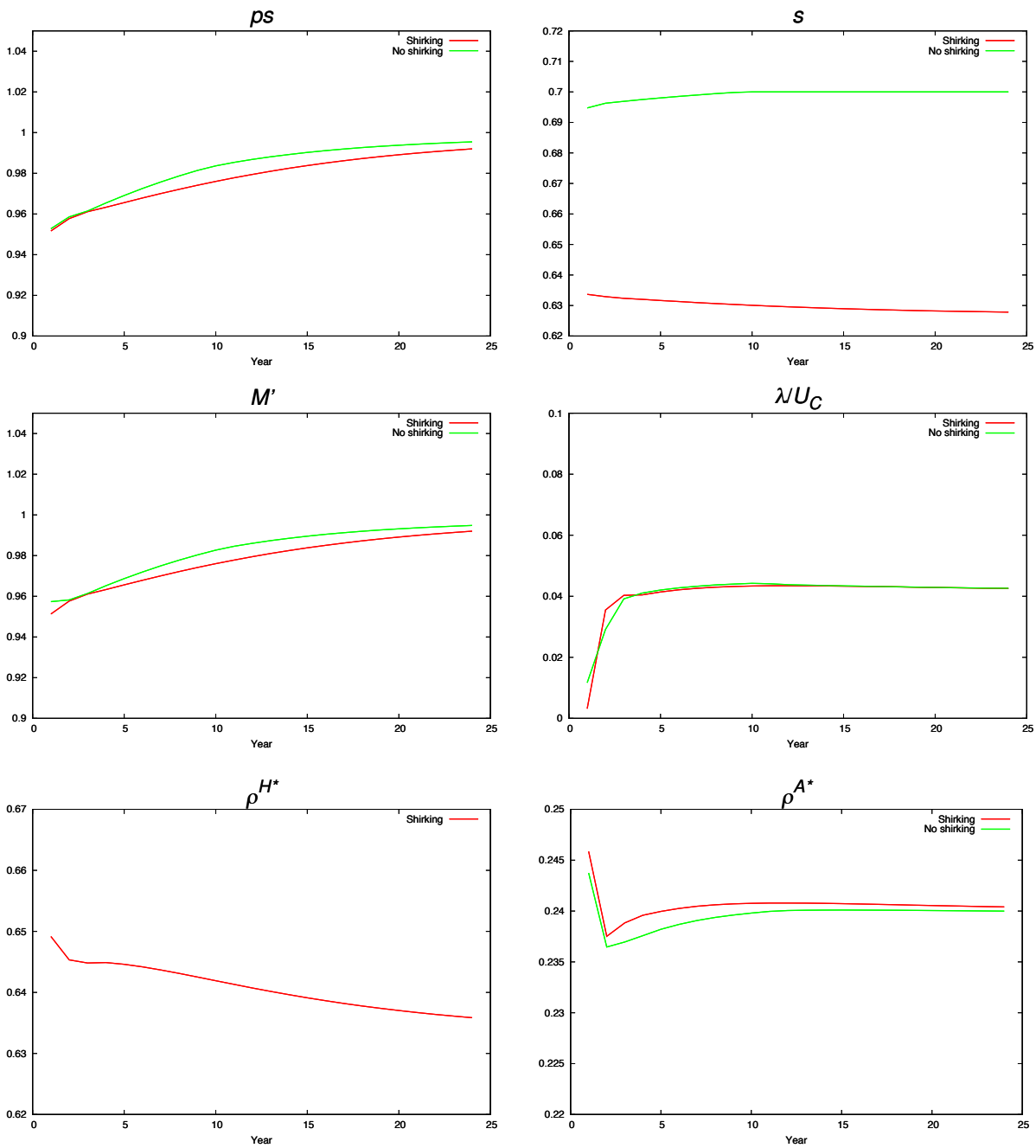

Figure 4. Impulse response functions.

Table 1

Preference Parameters

$\beta = 0.96, \;\; \eta = 0.8773$ (shirking), $\eta = 0.9906$ (no shirking).

Maximum Shirking Threshold

$\bar{J} = 0.1393$

Production Parameters

$\alpha = 1/3$

$\gamma = 1/3$

$\delta = 0.05$

$p_H = 0.9, \;\; p_L = 0.4$

$\bar{\rho} = 0.70$

Calibrated Steady State, Shirking

$n = 0.36,$

$\rho^{H*} = 0.6326, \;\; \rho^{H*}/\bar{\rho} = 0.90$

$\rho^{L*} = 0.2812, \;\; \rho^{L*}/\bar{\rho} = 0.40$

$J^* = 0.1114, \;\; J^*/\bar{J} = 0.8$

$C = 0.2337, \;\; M = 0.0769, \;\; M/C = 0.33$

$\hat{R} = 0.3742, \;\; Y = 0.2556, \;\; K = 0.4369$

$p = 0.1227, \;\; s = 0.6268$

$w = 0.3204, \;\; r = 0.0880$

Calibrated Steady State, No Shirking

$n = 0.36,$

$\rho^* = 0.5797, \;\; \rho^*/\bar{\rho} = 0.83$

$J = 0.0846 \;\; J^* = 0.0885,$

$C = 0.2785, \;\; M = 0.1035, \;\; M/C = 0.37$

$\hat{R} = 0.4132, \;\; Y = 0.3079, \;\; K = 0.5879$

$p = 0.1478, \;\; s = 0.7000$

$w = 0.4311, \;\; r = 0.0880$

Table 2

Liquidity Model w/Capital and Equilibrium Shirking

	No Shirking		**Shirking**	
	$\sigma = 0.0120$		$\sigma = 0.0075$	
Variable	stdev	corr w/Y	stdev	corr w/Y
Y	3.10	1.00	3.11	1.00
C	2.51	0.94	2.68	0.97
I	12.78	0.76	11.81	0.72
K'	3.74	0.82	3.39	0.84
θ	1.96	0.92	1.23	0.85
ρ^{H*}	0.41	0.45	1.35	-0.97
ρ^{A}	0.82	0.45	0.61	-0.19
w	2.69	0.97	2.59	0.98
M'	3.12	0.98	2.50	0.98
s	0.30	0.69	0.53	-0.98
sp	3.14	0.98	2.50	0.98
r	2.21	0.01	1.97	-0.20
p	2.93	0.98	3.03	0.98
J^*	0.00	-	3.29	0.99
J^*s	-	-	2.77	0.99

Notes: $\rho^A = M/w$

The simulations are based on samples, 2000 periods long.