

*The Economics of Counterfeiting**

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Abstract

This paper develops a new tractable strategic theory of counterfeiting as a competition between good and bad guys. There is free entry of bad guys, who choose whether and what note to counterfeit, and what quality to produce. Good guys select a costly verification effort. Along with the quality, this effort fixes the chance of finding counterfeits, and induces a collateral “hot potato” passing game among good guys — seeking to avoid counterfeits passed around. We find a unique equilibrium of the entwined counterfeiting and verifying games. With log-concave verification costs, counterfeiters produce better quality at higher notes, but verifiers try sufficiently harder that the verification rate still rises. We prove that the unobserved counterfeiting rate is hill-shaped in the note, vanishing at extremes. We also deduce comparative statics in legal costs and the technology. We find that the very stochastic nature of counterfeiting limits its social cost.

Our theory applies to fixed-value counterfeits, like checks, money orders, or money. Focusing on counterfeit money, we assemble a unique data set from the U.S. Secret Service. We identify key time series and cross-sectional patterns, and explain them: (1) the ratio of all counterfeit money (*seized* or *passed*) to passed money rises in the note, but less than proportionately; (2) the passed-circulation ratio rises in the note, and is very small at \$1 notes; (3) the vast majority of counterfeit money used to be *seized* before circulation, but now most passes into circulation; and (4) the share of passed money found by Federal Reserve Banks generally falls in the note, as does the ratio of the internal FRB passed rate to the economy-wide average. Our theory explains how to estimate from data both the street price of counterfeit notes and the small costs of verifying counterfeit notes.

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1 Introduction

Counterfeiting is a major economic problem, called “the world’s fastest growing crime wave” Phillips (2005). And specifically, the counterfeiting of stated value financial documents like money, checks, or money orders, is both centuries-old *and* a large and growing economic problem. Domestic losses from check fraud, for instance, may have exceeded \$20 billion in 2003. This scourge has risen greatly from years earlier with growth of internet-circulated Nigerian scams.¹ Counterfeit money is much less common but still quite costly: The counterfeiting rate of the U.S. dollar is about one per 10,000 notes, with the direct cost to the domestic public amounting to \$61 million in fiscal year 2007, which is up 66% from 2003. The indirect counterfeiting costs for money are much greater, forcing a U.S. currency re-design every 7–10 years. As well, many costs are borne by the public in checking the authenticity of their currency.

When we write *counterfeit* money (or checks), we have in mind two manifestations of it. *Seized* money is confiscated before it enters circulation or is passed. *Passed* money is found at a later stage, and leads to losses by the public. We have gathered an original data set mostly from the Secret Service on seized and passed money over time and across denominations. In the USA, all passed counterfeit currency must be handed over to the Secret Service, and so very good data is available (in principle).

We develop a simple and tractable equilibrium theory of counterfeiting that also explains the data on counterfeit money. The key stylized facts of counterfeit money in the USA are best expressed in terms of two measures — the *counterfeit-passed ratio* (seized plus passed over passed) and the *passed rate* (passed over circulation):

- #1. *The counterfeit-passed ratio rises, but less than proportionately with the note.*
- #2. *The passed rate is small for low notes, greatly rises, and levels off or drops.*
- #3. *Since the 1970s, the counterfeit-passed ratio has dramatically fallen about 90%.*
- #4. *The fraction of counterfeit notes found by Federal Reserve Banks falls in the note.*

We build a strategic model of the struggle between “bad guys” who may counterfeit and “good guys” who must transact (a continuum of each). In this large game, we allow a single variable decision margin for each side, and a free entry choice by bad guys.

Good guys expend efforts screening out passed counterfeit money handed them; more effort yields stochastically better scrutiny. In the world of counterfeit goods with no middlemen, only bad guys pass on the fake merchandise. But with counterfeit money or counterfeit goods resold, a much larger collateral game emerges: Good guys unwittingly pass on the counterfeit goods or money they acquire in an anonymous

¹Data here is sketchy. This estimate owes to a widely-cited Nilson Report (www.nilsonreport.com).

random matching exchange economy. This becomes a game of strategic complements (i.e. it is *supermodular*), since the more others verify, the more one should do likewise to protect oneself. We think this simple “hot-potato” game is novel in monetary theory.

Bad guys supply counterfeits. Their choice variables are whether and what value to counterfeit, and if so, what *quality* to produce. A counterfeit with twice the quality costs twice as much to catch with any given probability — the *verification rate*. With this cardinal notion of quality, vigilance efforts equal quality times an increasing and convex verification cost function. This prism through which good guys efforts and bad guys quality translate into a verification rate is at the core of our theory. Better verification in turn depresses the *passing fraction* of counterfeits into circulation by bad guys, and raises the *discovery rate* of passed money by good guys.

Equilibrium in our game can be recursively computed in two stages. Incentives in the counterfeit entry game pin down the quality and verification effort; meanwhile, the equilibrium effort in the passing game fixes the counterfeiting rate. No counterfeiting equilibrium exists at low but strictly positive value notes or goods, since it cannot pay for the expected legal costs even if all counterfeits certainly pass. Strictly above this threshold, we establish a unique Nash equilibrium of our model (Theorem 1).

Near the least value counterfeit, verification effort and counterfeit quality vanish. But counterfeiters have proportionately so much more to gain as the value rises. So counterfeit quality swamps verification effort in this limit, and verification vanishes (Theorem 2); the marginal verification cost vanishes too. In the hot potato game, this cost margin is the product of the counterfeiting rate, the counterfeit value and the discovery rate. Thus, the counterfeiting rate vanishes at low notes, and so too does the passed rate — its product with the discovery rate. So the passed rate vanishes at the least value counterfeits — the first part of stylized fact #2 (Corollary 6-*a*).

The paper revolves around the unfolding clash between verification effort and counterfeit quality either as the stakes amplify, or other features of the counterfeiting game change, like legal, production, or verification costs. Suppose the note value rises. The verification effort then rises (Theorem 3) — for if not, counterfeiting would prove more profitable at higher values. In a key result (Theorem 4), we prove that if the verification cost function is log-concave, then the counterfeit quality rises in the value. We document this conclusion, and then show that this explains stylized fact #1. The reason is that greater counterfeit quality costs more at higher notes, raising expected revenues too (Corollary 2), i.e. the passing fraction times the counterfeit value rises in the counterfeit value. Since the counterfeit-passed ratio is inverse to the passing fraction, it then cannot rise 1-for-1 with the counterfeit value (Corollary 3).

For the next major result (Theorem 5), we determine that verifiers eventually win out in the battle with counterfeiters. While quality rises in the counterfeit value, effort rises so much faster that the resulting verification rate steadily increases. Our only proviso is that the counterfeit cost elasticity does not fall in quality — as is true of most standard cost functions. The measured passing fraction falls in the counterfeit value, and the counterfeit-passed ratio accordingly rises (Corollary 2 and Corollary 3).

While the counterfeiting rate is the fake fraction of the circulation, it is not merely a statistical yardstick: In fact, this risk measure equilibrates the passing game played by verifiers, just as prices clear markets. We also prove that the counterfeiting rate is approximately the ratio of verification costs and unit counterfeiting costs. Not only does the counterfeiting rate vanish at low notes, it also does so at very high notes — since quality explodes in the counterfeit value (Theorem 6). We then bound the counterfeiting rate, and deduce a rough hill-shape in terms of the counterfeit value (Theorem 7). We illustrate this and all findings in a worked example. Scaling it by the discovery chance yields the observed passed rate; this rate shares hill-shape, thus explaining the second half of stylized fact #2 (Corollary 6-*b*).

We next turn to a welfare analysis of the costs of counterfeiting, since we can easily quantify costs to counterfeiters and verifiers. We quantify these social costs, and show that they are bounded below the counterfeit value. We argue that this exception to Tullock’s Theorem owes to the stochastic nature of counterfeiting (Theorem 8).

Our large game is sufficiently tractable that we can easily analyze the thrust and parry of the competition between good and bad guys. We show that if technological progress occurs in counterfeiting, then verifiers try harder in equilibrium, and also counterfeit quality rises. With “neutral progress”, the equilibrium verification rate is unchanged but the counterfeiting rate falls if the progress was “quality-augmenting”, in an intuitive sense that we define (Theorem 9). Turning to our other comparative static, Theorem 10 discovers a perverse effect of greater legal costs, crowding out verifier effort, reducing the verification rate. This underscores that the Treasury or producers of counterfeited goods should be more concerned about how readily checked are the money or goods rather than how steep are the legal penalties.

We show that Theorem 9 helps explain stylized fact #3. Most counterfeit money used to be seized, while now the reverse holds. This owes to a technological transformation in counterfeiting, first with office copiers in the 1980s and then digital means (computers with ink jet printers) in the 1990s (Corollary 4). Theorem 9 also captures the classic cat-and-mouse game between counterfeiter and originator: Easier verification is tantamount to neutral technological regress; therefore, effort and quality equally

fall, verification is unchanged, and the counterfeiting rate falls Corollary 1.

Our model also admits expressions for several economically meaningful variables. For example, the street price of counterfeit notes (19) can be approximated using the counterfeit-passed ratio. This owes to equilibrium behavior by bad guys. The implied prices agree with typical estimates and anecdotal evidence. Meanwhile, equilibrium behavior by good guys in the passing game affords a glimpse into currency verification costs incurred by the public. Marginal verification costs equal the passed rate times the denomination, and so amount to at most 1/4 cent for the \$100 bill! Our microeconomics foundation may be more aptly thought of “nano-economics”. That such small verification costs explain the data testifies to the power of even slight incentives.

The passed rate reflects the incentives of individuals as they notice counterfeit money. The paper ends with a reverse test for the paper, focusing on money that verifiers miss, and is ultimately caught by Federal Reserve Banks (FRB). For instance, the FRB actually finds a majority of \$1 passed notes, and their share of passed money falls in the denomination, except for the \$100 note (stylized fact #4). We then argue that this reflects two features of our theory — that the more valuable notes are both better quality and better verified by the public. Also, the internal FRB counterfeiting rate is likewise a decreasing ratio of the overall passed rate, until the \$100 note. We argue in Corollary 8 and Corollary 9 and that both facts owe to the rising verification rate, and behavior in the hot potato passing game.

RELATIONSHIP TO THE LITERATURE. Despite how common and time-eternal a problem it is, counterfeit money has been very much a blackbox to economists. To be sure, the published literature is very small. There are some purely theoretical papers inspired by the classic money matching model of Kiyotaki and Wright (1989) and the more closely-related Williamson and Wright (1994). Aside from the subject matter, our link to this literature is minimal: Ours is partial equilibrium behavioral model, while these are general equilibrium papers seeking to price the counterfeits. None aspires to explain data, or could explain the current data, as we argue in the paper. Since they assume fixed signals of the authenticity of money, they share neither our main novel strategic core nor our conclusions about the counterfeiting rate, and passed and seized money. In Green and Weber (1996), only government agents can descry the counterfeit notes, whose stock is assumed exogenous, unlike here. Williamson (2002) admits counterfeits of private bank notes that are discovered with fixed chance; counterfeiting does not occur in most of his equilibria. Recognition of counterfeits is also stochastic and exogenous in Nosal and Wallace (2007), who find no counterfeiting in equilibrium with a high enough cost of counterfeit. By contrast, in our model,

counterfeit quality is endogenous, and a high enough note must be counterfeited.

For a key point of comparison, the papers cited above assume that transactors get a free signal of the money quality *after* acquiring it. We instead posit that individuals verify when it can affect choice, namely when handed it. This is important, producing the strategic complements hot potato game. It also agrees with how most individuals behave: At the moment we acquire money, we check it; otherwise, it lives in our wallet.

We lay out the model in §2. Innocent verifiers care about the behavior of each other when they acquire money that is surely passed on, but perhaps not for checks. *For definiteness, we then use the language of counterfeit money.* In §3, we establish equilibrium existence, and then illustrate it in a fully solved example using geometric verification and counterfeit quality cost functions. All theorems in sections 3–5 apply equally to both counterfeits. We then focus exclusively on counterfeit money, and show how our model explains the behavior of seized money in §6, and of passed money in §7. We conclude in §8 with a different data set from the Federal Reserve Banks. Technical proofs are deferred to the appendix, and intuitively explained in the text.

2 The Model

We will construct a dynamic discrete time model in which a continuum of notes of denomination Δ transact once per “period” — where the time period is specific to Δ . Counterfeiting for each Δ plays out as a separate game, and we take the denominations as given. Our data will come from the U.S. dollar denominations \$1, \$5, . . . , \$100. We will focus on the story and language of counterfeit money, since the theory we develop is largely applicable without change to counterfeit goods. We identify where these changes occur. In particular, Δ is the sales value of the good to be counterfeited.

There are two types of maximizing risk neutral agents: a continuum of bad guys (counterfeiters) and good guys (transactors). Everyone therefore acts competitively, believing he is unable to affect the actions of anyone else. Counterfeiters choose whether to enter, and if so, they select the quality of money to produce and distribute, and then are eventually jailed. There is an infinitely elastic supply of counterfeiters with free entry; each earns zero profits, taking account of the legal penalty (“crime does not pay”). Each piece of money changes hands in chance pairwise transactions from bad guy to good guy, or from good guy to good guy. Counterfeiters who transact are indistinguishable from good guys. Good guys choose an effort level to examine notes that they are handed. Some unknowingly acquire counterfeit currency and some do not. We ignore payoff discounting, since any note acquired is soon spent.

2.1 The Hot Potato Game

If an innocent individual attempts to spend “hot” money, and this is noticed, then it becomes worthless — since knowingly passing on counterfeit currency is illegal.² We simply assume that this extra crime of “uttering” is not done.

Faced with this prospect, individuals choose how carefully to check the authenticity of any money *before* they accept it. *Verification* is a stochastic endeavor that transpires note by note — as more valuable notes will command closer scrutiny. We write the verification *rate* (or *intensity*) as the chance $v \in [0, 1]$ that one correctly identifies a given note as counterfeit. We assume real notes are never mistaken for counterfeit.

Verification costs are smooth, increasing and strictly convex in the verification rate v . We write them as $q\chi(v)$, where $q > 0$ will soon be interpreted as quality. We assume that $\chi'(0) = 0$, with $\chi'(v) > 0$ and $\chi''(v) > 0$ when $v > 0$.³

Each period, innocent transactors either go to the bank (unlike counterfeiters) or meet a random verifier for transactions. These events are not choices, and occur with fixed chances β and $1 - \beta$, respectively. Banks have verifying machines or capable staff who can better spot counterfeit money than individuals, but still imperfectly. Write their verification intensity as $\alpha \in (0, 1)$. Indeed, from \$5–10 million of passed money hits the Federal Reserve yearly, missed by banks (see Table 7). All told, any counterfeit money is found in a transaction with the *discovery rate* $\rho(v) = \alpha\beta + (1 - \beta)v > 0$.

Assume that a fraction κ of all Δ notes tendered in transaction is counterfeit, with an average verification rate v . As notes are spent upon acquisition, transactors choose their intensity \hat{v} to minimize losses from counterfeit money and verification efforts:

$$\kappa(1 - \hat{v})\rho(v)\Delta + q\chi(\hat{v}) \tag{1}$$

A verifier incurs a loss in the triple event that (i) he is handed a counterfeit note, (ii) his verifying efforts miss this fact, *and* (iii) the next transaction catches it. These are independent events with respective chances κ , $1 - \hat{v}$, and $\rho(v)$.

This is a doubly *supermodular game*: One’s verification intensity \hat{v} is a *strategic complement* to the average intensity v and the counterfeiting rate κ . The incentive to verify money that one acquires is stronger the more intensely others check their notes, or the more prevalent counterfeit money is. Thus, the verification best response function \hat{v} rises in v and κ . Supermodular games in economics may have multiple

²Title 18, Section 472 of the U.S. Criminal Code

³Weak convexity in this case is remarkably without loss of generality. For one can always secure an (expected) verification chance of v at cost $(\chi(v - \varepsilon) + \chi(v + \varepsilon))/2$ instead by flipping fair coin, and verifying at rates $v - \varepsilon$ or $v + \varepsilon$. In other words, we must have $\chi(v) \leq (\chi(v - \varepsilon) + \chi(v + \varepsilon))/2$.

ranked equilibria, but here there is a unique symmetric Nash equilibrium.

The second order condition for minimizing (1) is met given strictly convex costs χ . Agents must choose a common verification intensity $\hat{v} = v$ in the verification game. There are no asymmetric equilibria. Since $\beta\alpha > 0$, the corner solution is not optimal. When $\kappa > 0$, all verifiers will choose the same effort level, inducing the same positive verification rate v , because $\chi'(0) = 0$. Substituting this into the first order condition yields the equilibrium equation that the marginal costs and benefits of effort coincide.

$$q\chi'(v) = \kappa\rho(v)\Delta \quad (2)$$

The counterfeiting rate κ acts like a market-clearing price, quantifying the risk.

From the supermodular structure, the marginal benefit on the left side of (2) linearly rises both in κ and in v . This yields an economic expression for the counterfeiting rate:

$$\kappa = \frac{q\chi'(v)}{\rho(v)\Delta} = \frac{\text{marginal verification cost}}{\text{discovery rate} \times \text{denomination}} \quad (3)$$

The right side is a quotient of two increasing functions of v : Marginal verification costs rise in v by convexity, and while marginal verification gains rise linearly in v . In the later equilibrium, the counterfeiting rate will equilibrate both factors.

Finally, *we address counterfeit checks. Since they are not resold, the discovery rate $\rho(v)$ might not appear in (1), and thus in the first order conditions (2)–(3).*

2.2 Currency Verification and Counterfeit Quality

Among the many decisions made by counterfeiters, we center our theory on the quality choice. Better quality notes look and feel more authentic, and so pass more readily.⁴ This singular focus is motivated by the concerns of law enforcement and bank officials, and the fact that it affords a parsimonious theory that explains the key facts.

We now introduce a specific cardinal meaning for the quality of counterfeit notes turning on how it impairs verification. Verification rate $v \in [0, 1]$ for a quality $q > 0$ note costs effort $e = q\chi(v)$: Doubling the quality requires twice the effort to produce the same verification intensity. There is another economic motivation for this key functional form. Counterfeiters and verifiers have strategically opposed preferences over the verification rate. Verifiers do not know the quality of any note, even if they can

⁴The European Central bank has adopted the catch phrase “feel-look-tilt” in its ad campaign for the security features of the Euro, where tilt refers to the hologram image.

infer its equilibrium level.⁵ So their choice variable is not v but instead the effort e , and this choices depend on their expected costs (1). Thus, it makes most economic sense that counterfeiters have a convex cost over quality, which linearly increases these costs.

Write the verification rate as the induced function $v = V(e, q)$ of effort and quality. We have the useful inverse relations $V(q\chi(v), q) \equiv v$ and $q\chi(V(e, q)) \equiv e$, and so $V(e, q) = \chi^{-1}(e/q)$. Since v is a probability, we further define $V(e, q) = 1$ if $e > q\chi(1)$. Thus, at any effort level e , verification is perfect for low enough quality.

If we differentiate the identity $q\chi(V(e, q)) \equiv e$, then we discover $q\chi'V_q + \chi \equiv 0$ and $q\chi'V_e \equiv 1$. Further differentiating reveals that $q\chi'V_{ee} + q\chi''V_e^2 \equiv 0$. Summarizing:

Lemma 1 (Verification Function) *Fix the verification effort $e > 0$ and counterfeit note quality $q > 0$ so that $v = V(e, q) < 1$. Verification intensity rises in e and falls in q :*

- (a) *Verification rises in effort, with slope $V_e(e, q) = 1/q\chi'(v) > 0$.*
- (b) *Verification falls in quality, with slope $V_q(e, q) = -\chi(v)/q\chi'(v) < 0$.*
- (c) *Marginal returns to verification effort fall: $q^2V_{ee}(e, q) = -\chi''(v)/(\chi'(v))^3 < 0$.*

Greater counterfeit quality harms verification, $V_q < 0$, this damage intuitively should obey the law of diminishing returns. Differentiating the identity $q\chi'V_q + \chi \equiv 0$:

$$q^2V_{qq} = \frac{\chi}{\chi'} + \left(\frac{\chi}{\chi'}\right)^2 \left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'}\right)$$

Diminishing returns to greater quality necessarily arises when this is always positive — which is only necessary for verification costs χ log-concave in the rate v : $(\log \chi)'' \leq 0$, and thus $(\chi'/\chi)' \leq 0$, or $\chi''/\chi' \leq \chi'/\chi$. This discipline on the cost convexity is natural since quality and verification costs interact multiplicatively. *We maintain this log-concavity assumption throughout the paper.* It is critical, but not too restrictive — for instance, it merely precludes any verification cost that are more convex than the exponential function; geometric costs $\chi(v) = \lambda v^r$ with any exponent $r > 1$ work.⁶

Lemma 2 (Verification with Log-concavity) *The marginal returns to quality fall, or $V_{qq} > 0$, while quality and effort are strategic substitutes in the verification rate:*

$$q^2V_{eq} = \frac{\chi}{(\chi')^2} \left(\frac{\chi''}{\chi'} - \frac{\chi'}{\chi}\right) \leq 0 \tag{4}$$

⁵In principle, equilibrium quality could be random, in which case we interpret q as the mean quality.

⁶Log-concavity is a standard assumption for probability densities (see Burdett (1996) or Bagnoli and Bergstrom (2005)). Our application to cost functions like χ may well be novel.

2.3 Verification and the Passing Fraction

A counterfeiter produces an illegal good, which may be *seized* prior to passing it onto the public. Police may either uncover the counterfeit note “factory” or catch the crook in the act of transporting the money. We summarize the hurdles of passing notes by the *equilibrium passing fraction* $0 < f \leq 1$. This is the chance that any given note passes, or in our continuum model, the share of production that the counterfeiter passes.

This paper turns on the role of individual verification efforts in preventing counterfeit money. Such efforts intuitively facilitate police seizures, by providing clues into ongoing counterfeit operations. So we assume that police seize a fraction $0 < s(v) < 1$ of counterfeit money production. The passing fraction reflects seizure and verification via $f(v) = (1 - s(v))(1 - v)$. Loosely, the notes must pass through two filters — police then the first verifier. Passing is thus choked off with perfect verification ($f(1) = 0$), and some passing occurs when no one verifies ($f(0) = 1 - s(0) > 0$). We assume that the resulting passing fraction continuously falls in verification ($f'(v) < 0$). Since $1 - v > f(v)$, a “good guy” passes a counterfeit note more often than a “bad guy”.

If seizures were a fixed fraction s of production, then a unit elasticity of $f(v) = (1 - s)(1 - v)$ would arise: $\mathcal{E}_{1-v}(f) = 1$. If verifier activity enhances police seizures,⁷ then this elasticity exceeds one. *We assume for simplicity a constant passing elasticity:*

$$\Upsilon \equiv \mathcal{E}_{1-v}(f) = -(1 - v)f'(v)/f(v) \in [1, 2) \quad (5)$$

This implies that $f(v) = f(0)(1 - v)^\Upsilon$. Notice that *the constant elasticity passing fraction is strictly log-concave*, since $(\log f)' = f'/f = -\Upsilon/(1 - v)$ falls in v .

2.4 The Counterfeiter’s Problem

While counterfeiting of money or goods is a dynamic process, we project it to a static optimization. We consider legal, production, and distribution costs.

Firstly, a counterfeiter may be caught: The present value of the punishment loss is $\ell > 0$. Next, a counterfeiter incurs a fixed cost for the human and physical capital, and a small marginal cost of production. Given the increasing returns, the optimization of the counterfeiter might not imply a finite expected quantity.⁸ But the distribution costs

⁷On its web page, the Secret Service also advises anyone receiving suspected counterfeit money: “Do not return it to the passer. Delay the passer if possible. Observe the passer’s description.”

⁸ “If a counterfeiter goes out there and, you know, prints a million dollars, he’s going to get caught right away because when you flood the market with that much fake currency, the Secret Service is going to be all over you very quickly. They will find out where it’s coming from.” — interview with Jason Kersten, author of Kersten (2005) [*All Things Considered*, July 23, 2005].

are surely convex in quantity, since each passing attempt incurs the risk of discovery. While we address endogenous counterfeit quantity in §6.3, we pursue a more focused theory centered on the quality choice. We assume that counterfeiters choose an optimal quality q for producing the *expected quantity* $x > 0$ of counterfeit money. The cost $c(q)$ is increasing and convex, with $c', c'' > 0$ and continuous, and $c'(q)$ unbounded as $q \rightarrow \infty$. The *producer surplus* of quality q $c'(q) - c(q)$ is initially zero, and then rising.

A counterfeiter cares about his quality, and how carefully his notes are scrutinized. Counterfeiters do not attempt to pass their money at a bank, and so face a verification intensity $v = V(e, q)$. The expected revenues for quality q of note Δ are $f(v)x\Delta$, while their costs are $c(q) + \ell$. Given free entry, expected profits vanish:

$$\Pi(q, e, \Delta) \equiv \Delta x f(V(e, q)) - c(q) - \ell = 0 \quad (6)$$

Better quality simultaneously raises the passing fraction and the counterfeiters' costs.

Provided there is any counterfeiting, the optimal quality q is positive and finite because the returns to greater quality are bounded, given $f \leq 1$. Also, zero quality yields perfect verification $v = 1$, and precludes all passing. Since the profit function is smooth and strictly concave in quality by Lemma 2 and $f' < 0 < f''$, first order conditions define a unique counterfeiter's quality optimization:

$$\Pi_q(q, e, \Delta) \equiv \Delta x f'(V(e, q)) V_q(e, q) - c'(q) = 0 \quad (7)$$

3 Counterfeiting Equilibrium

3.1 Existence and Uniqueness

Our model consists of enmeshed hot-potato and counterfeiting games pitted in a “large game” — i.e. a game with a continuum of players (initiated by Schmeidler (1973)). Since quality and verification effort are unobserved, this dynamic Bayesian game can be solved using Nash equilibrium. For as we have seen in §2.1, there is a unique optimal verification rate for any expected quality. This in turn implies a unique effort. Also, we will argue that the optimal quality is unique given effort. In summary, for a fixed denomination Δ , a symmetric *equilibrium* will be a triple (q, e, κ) , such that:

- (a) Counterfeiters' quality $q > 0$ maximizes profits $\Pi(q, e, \Delta)$, and so (7) holds.
- (b) Verifiers' effort $e > 0$ ensures that counterfeiters earn no profits, so (6) holds.

- (c) The counterfeiting rate is $\kappa \in (0, 1)$, so that each verifier's effort $e = q\chi(v)$ solves the optimization (1) for the quality q and the verification rate v .

For any quality and verification effort $q, e > 0$, equilibrium obtains in the hot-potato game of §2.1 for a unique counterfeiting rate $\kappa > 0$ in (3). This recursive structure allows us to solve for the *counterfeiting equilibrium* (q, e) in isolation first. That $\kappa < 1$ is mathematically immaterial in the verifier's optimization (1), but is needed for any economic sense. In Theorem 7, we will derive sufficient conditions for this bound.

The two nonlinear equations (6) and (7) in two unknowns have a unique solution if the note is high enough: For the counterfeiter must pay a fixed legal cost $\ell > 0$ irrespective of the note that he counterfeits, since he is eventually caught. So only high enough notes are counterfeited. For greater notes, verification effort is needed to preclude counterfeiting profits, and positive quality precludes perfect verification.

Theorem 1 (Existence and Uniqueness) *For any $\Delta > \underline{\Delta} \equiv \ell/(xf(0))$, there exists a unique counterfeiting equilibrium (q, e) ; it is differentiable in Δ , and the verification rate, effort, quality are positive. No counterfeiting equilibrium exists for $\Delta \leq \underline{\Delta}$.*

Absent verification, counterfeiting is profitable, and counterfeit money circulates. But then verification has positive marginal benefits, and zero marginal costs $\chi'(0) = 0$. With perfect verification, counterfeiters lose money. So $0 < v < 1$, as assumed in (2).

To see nonexistence: At any $\Delta < \underline{\Delta}$, it is impossible to satisfy zero profits, since $c(q) + \ell \geq \ell = \underline{\Delta}xf(0) > \Delta xf(v)$ whenever $q, v \geq 0$. If $\Delta = \underline{\Delta}$, zero profits requires that quality vanish. But then perfect verification is achievable at arbitrarily small cost, and this forces negative profits. The paper henceforth assumes a denomination $\Delta > \underline{\Delta}$.

3.2 An Illustrative Example of a Counterfeiting Equilibrium

Geometric cost functions verification and quality production result in an example fully solvable in closed form. So assume $\chi(v) = v^B$ and $c(q) = q^A$, where $A, B > 1$. Clearly, both cost functions are convex and χ is log-concave. Let us reformulate the first order condition (7) for quality instead in (q, v) -space, substituting from Lemma 1:

$$qc'(q) = -\Delta xf'(v) \frac{\chi(v)}{\chi'(v)} \quad (8)$$

Simply assume that the police do not diminish the passing chance, so that $\Upsilon = 1$ and thus $f(v) = 1 - v$. The zero profit equation (6) and revised quality FOC (8) are then:

$$\Delta x(1 - v) - q^A - \ell = 0 \quad \text{and} \quad Aq^A - \Delta xv/B = 0$$

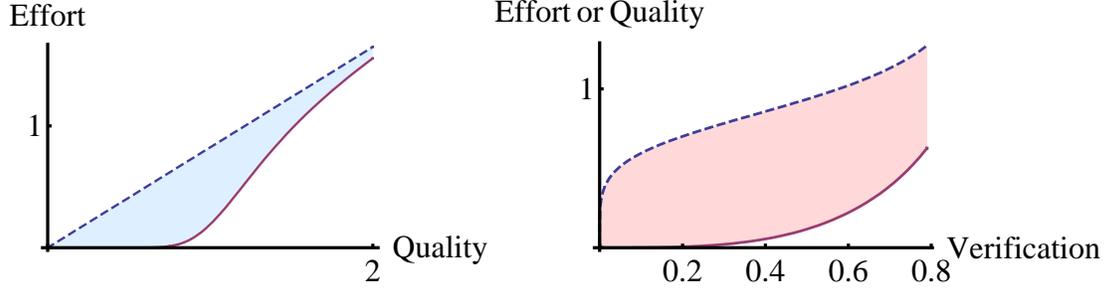


Figure 1: **Effort, Quality, and Verification.** At left, equilibrium verifier effort is graphed as a function of equilibrium counterfeit quality for our example (with $A = 5$, $B = 3$, $x = 2$ and $\ell = 10$), as the note passes $\underline{\Delta} = 5$. At right, quality (dashed) and effort (solid) are graphed as a function of the verification rate. The effort-quality ratio and so the verification rate rise from 0. As effort and quality explode in Δ , their ratio tends to the dashed line with slope $\chi(\bar{v})$, with limit verification $\bar{v} = 0.8$.

By Theorem 1, the least counterfeit denomination is $\underline{\Delta} = \ell/x f(0) = \ell/x$. One can check that this is consistent with the boundary condition $v = q = 0$. Solving these two equations in q and v , we find that the limit verification rate is $\bar{v} = AB/(1 + AB) < 1$:

$$q^A = (1 - \bar{v})(\Delta - \underline{\Delta}) \quad \text{and} \quad v = \bar{v}(1 - \underline{\Delta}/\Delta) \quad (9)$$

So verification rises, but is forever imperfect, as the counterfeiting problem persists. Also, the verification rate rises in the convexity measures A and B . Next, verifier effort e can be deduced by combining both expressions in (9):

$$e = qv^B = (1 - \bar{v})^{1/A} \bar{v}^B \Delta^{-B} (\Delta - \underline{\Delta})^{B+1/A} \quad (10)$$

As seen in Figure 1, quality in (9) initially rises much faster than effort, since $B > 0$. To this point in the model, the bank behavior is irrelevant. But now the discovery rate comes into play. The counterfeiting rate is found by substituting equilibrium quality and verification from (9) into (3) — namely, into $\kappa = Bqv^{B-1}/(\rho(v)\Delta)$. Absent a banking sector, the discovery rate is $\rho(v) = v$, and the resulting counterfeit rate is a hill-shaped function of the note Δ (Figure 2), vanishing as $\Delta \uparrow \infty$ or $\Delta \downarrow \underline{\Delta}$:

$$\kappa = Bx^{1/A} (1 - \bar{v})^{1/A} \Delta^{-1+1/A} \bar{v}^{B-2} (1 - \underline{\Delta}/\Delta)^{B+1/A-2} \quad (11)$$

This example has illustrated the recursive structure of counterfeiting equilibrium — first find quality and effort, and then the counterfeiting rate. We now explore the model for general cost functions, and see that the properties of this example are quite robust.

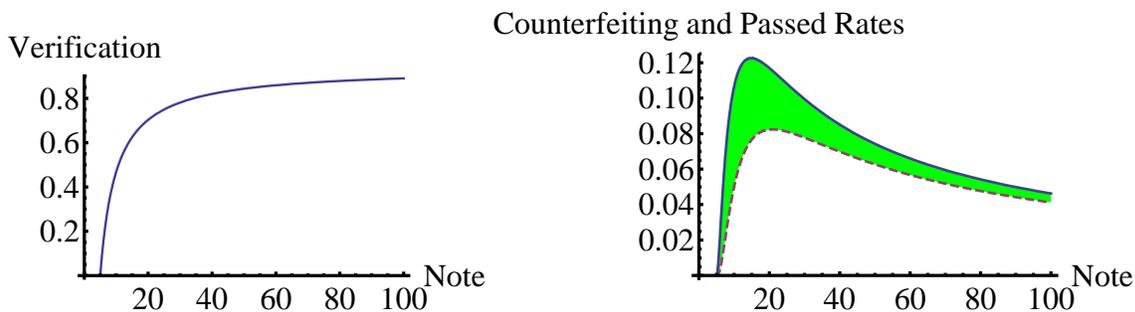


Figure 2: **Verification and the Counterfeiting Rate.** At left is the plot of the rising verification rate in our example. Derived in the counterfeiting game, it yields the counterfeiting rate in the hot-potato verification game. This counterfeiting rate (right solid curve) is rising and then falling in the note. The dashed product of these two curves is the passed counterfeiting rate (20) — the share of counterfeit notes found by innocent verifiers (see §7). It starts at zero, rises steeply, and eventually falls off here.

4 Equilibrium Across Denominations

The denomination measures the stakes in the strategic battle between counterfeiters and verifiers. As in our example in §3.2, effort and quality vanish near the least stakes and monotonely grow without bound in the stakes. We explore how good and bad guys respond differently as the stakes intensify in the denomination. As a result, the verification rate monotonely rises from 0, while the counterfeiting rate rises and falls.

4.1 Rising Verification Effort Meets Rising Counterfeit Quality

The first general feature of the example in §3.2 is that verifier effort, counterfeit quality, and the verification rate all vanish at low notes, as does the slope of effort in quality. As the note passes $\underline{\Delta}$, profits and counterfeit losses both rise a little. Since this is an infinite proportion of counterfeiting profits and a negligible fraction of verifier losses, the counterfeit quality response is infinitely more elastic than the effort response.

Theorem 2 (Lowest Notes) *The counterfeit quality q , the verification effort e and rate v all vanish as $\Delta \downarrow \underline{\Delta}$. Effort vanishes proportionately faster than quality near $\underline{\Delta}$.*

If verification did not vanish, counterfeiting would be strictly profitable at notes just below $\underline{\Delta}$. The second last claim — seen in (10) — formally owes to l’Hopital’s Rule. For since $q, e \rightarrow 0$ as $\Delta \rightarrow 0$, we have $\lim_{\Delta \rightarrow 0} de/dq = \lim_{\Delta \rightarrow 0} e/q = \lim_{\Delta \rightarrow 0} \chi(v) = 0$.

Verifiers pay greater heed to more valuable notes, as their losses from acquiring bad money are greater. For if verification effort did not rise, then criminals would find higher notes more profitable to counterfeit. For a proof, differentiate the zero-profit

identity (6) in Δ to get $\Pi_q \dot{q} + \Pi_e \dot{e} + \Pi_\Delta = 0$. Since $\Pi_q = 0$ in equilibrium by (7), and $\Pi_e = \Delta f' V_e < 0 < f = \Pi_\Delta$, a positive effort slope $\dot{e} > 0$ follows from:

$$\Delta f' V_e \dot{e} + f = 0 \quad (12)$$

Theorem 3 (Effort) *Verification effort e rises in the note Δ .*

Next, a higher note pushes up the marginal gain to quality for counterfeiters, while greater effort pushes it down by (4). The net effect is unclear. But from the log-concave verification cost function χ and passing fraction f , we can deduce that quality rises.

Theorem 4 (Quality) *Counterfeit quality q rises in the note Δ .*

Just as the effort comparative static is driven by incentives in the entry game by bad guys, the quality comparative static turns on incentives in the hot-potato game.

Loosely, log-concavity precludes local “near jumps” of an increasing function, like the verification cost χ , and local “near flats” of a decreasing fraction, like the passing function f .⁹ If the note just rises “a little”, then so does the verification effort $e = q\chi(v)$, by Theorem 3. To sustain zero profits (6), the passing fraction $f(v)$ must fall “a little”. If f is not log-concave, then v could rise “a lot”, and so $\chi(v)$ could rise “a lot” too. Alternatively, if χ is not log-concave, then $\chi(v)$ could rise “a lot” even if v only rises “a little”. Either way, the quality $q = e/\chi(v)$ could fall.

4.2 The Rising Verification Rate

Theorem 3 and Theorem 4 predict an intensifying duel between verification efforts and counterfeit quality as the denomination rises. The verification rate rises when effort $e \equiv q\chi(v)$ rises proportionately more than quality q . While a verifier may study a \$100 note with greater care than a \$5 note, the \$100 passes more readily if its quality is sufficiently higher. Or quality could improve sufficiently faster than effort so that the verification rate falls. For general cost functions, we prove that this occurs.

Our insight into the verification rate comes by relating it to quality. So motivated, we eliminate the note Δ from the zero profit and optimal quality conditions (6)–(7):

$$\frac{qc'(q)}{c(q) + \ell} = \frac{-f'(v)}{f(v)} \frac{\chi(v)}{\chi'(v)} \quad (13)$$

⁹Since log-concavity says $\chi(v+e)\chi(v-e) \leq \chi(v)^2$ for all $e > 0$, the ratio $\chi(v+e)/\chi(v)$ cannot exceed $\chi(v)/\chi(v-e) > 1$, which rules out “steep rises” in χ . Just as well, since $f(v)/f(v-e) < 1$ is an upper bound on $f(v+e)/f(v)$, the decreasing function f cannot have a “near flat”.

Since $f(v) = f(0)(1-v)^{\Upsilon}$ and χ is log-concave, the right side of (13) rises in v . When q is so small that legal costs exceed producer surplus from quality, or $\ell > qc'(q) - c(q)$, the ratio $q/(c(q) + \ell)$ rises in q , and so does the left side of (13). Since quality rises in the note Δ by Theorem 4, so does verification v . For larger qualities q , if qc'/c is nondecreasing, then the left side of (13) rises in q , as $c(q)/(c(q) + \ell)$ does. *A fortiori*,

Theorem 5 (Verification) (a) *The verification rate rises at low enough notes $\Delta > \underline{\Delta}$.*
(b) *If the cost elasticity qc'/c weakly rises in q , the verification rate rises in the note.*
(c) *The verification of any denomination $\Delta > \underline{\Delta}$ is at most $1 - (\underline{\Delta}/\Delta)^{1/\Upsilon}$.*

The appendix proves part (c). The cost elasticity qc'/c is constant for geometric costs, like $c(q) = q^A$ in the example in §3.2, and increasing for exponential costs, such as $c(q) = e^q$. For economic insight into its role, let the denomination Δ rise. Then the marginal benefit of quality rises too. If the cost elasticity fell, then marginal costs might flatten, and quality thereby rise so much that verification drops.

Theorem 5 also asserts that the verification rate is bounded strictly below one at each fixed note Δ . It is silent on whether the verification rate rises to 1. It need not: The verification rate in the example in §3.2 is uniformly bounded below one across all notes, and we have no reason to disbelieve this possibility from our evidence in §6.

Theorem 6 (Highest Notes) *Both effort e and quality q explode as the note $\Delta \uparrow \infty$.*

These explosions occurred in the example in §3.2. In light of Theorem 5 and $e = q\chi(v)$, it suffices that $q \rightarrow \infty$. Re-write the zero profit condition (6) in (q, v) -space:

$$\Delta x f(v) - c(q) - \ell = 0 \tag{14}$$

Absent a quality explosion, verification shoots to 1 too fast for optimality. Namely, $\Delta(1-v)^{\Upsilon}$ is bounded in (14), whereas optimality entails $\Delta(1-v)^{\Upsilon-1}$ bounded in (8).

4.3 A Hill-Shaped Bound for the Unobserved Counterfeiting Rate

With free entry by counterfeiters, the counterfeiting rate is a free variable in our model. While a function of the quantity of counterfeit notes, this unobserved fraction acts as a price — the risk level that clears the verification effort and counterfeit quality market.

We now bound the counterfeiting rate using primitives. Just as in the example in §3.2, we prove that the counterfeiting rate vanishes at the least and highest notes. First, examining equation (3), $\kappa \rightarrow 0$ near the least counterfeit note $\underline{\Delta}$ since quality and the verification rate vanish, while $\Delta \geq \underline{\Delta} > 0$ and the discovery rate $\rho \geq \alpha\beta > 0$.

To see why counterfeiting disappears at high notes, eliminate Δ from (3) using the first order condition (7), and simplify it with Lemma 1 and $f(v) = (1 - s(0))(1 - v)^\Upsilon$:

$$\kappa = \frac{q\chi'(v)}{\rho(v)} \frac{x f'(v) V_q(e, q)}{c'(q)} = (1 - s(0))^\Upsilon (1 - v)^{\Upsilon-1} \frac{x\chi(v)}{\rho(v)c'(q)} \quad (15)$$

Since $\rho(v) \geq \alpha\beta$ and $\chi(v) \leq \chi(1) < \infty$, if $\Upsilon = 1$ (no police), *the counterfeiting rate is a ratio of verification costs and marginal costs of quality*. By Theorem 6, quality explodes in the note, and thus so does its marginal cost $c'(q)$. Then $\kappa \rightarrow 0$ by (15).

We next globally bound the counterfeiting rate. This bound rises if counterfeiting is easier — lower legal costs ℓ , seizure rate $s(0)$, or counterfeit cost parameter c_0 , or a higher production level x . It falls when verification is more effective — a higher banking verification rate $\beta\alpha$, or a lower perfect verification marginal cost $\chi'(1)$.

Theorem 7 (Counterfeiting) (a) *The counterfeiting rate vanishes as $\Delta \downarrow \underline{\Delta}$ or $\Delta \uparrow \infty$.* (b) *Given a geometric bound $c'(q) \geq c_0 q^\eta$ for $c_0 > 0$ and $\eta > 0$, the counterfeiting rate is bounded by:*

$$\kappa < \frac{x(1 - s(0))\chi'(1)}{\alpha\beta(c_0\ell^\eta)^{1/(\eta+1)}} \quad (16)$$

While counterfeiting never disappears, it can spiral out of control if it is cheap. Completing the existence theorem for a counterfeiting equilibrium, *we assume that the upper bound (16) is less than one, and so the counterfeiting rate is less than one.*

4.4 The Social Costs of Counterfeiting and Tullock's Bound

A passed counterfeit note incurs one counterfeiting cost, but many verification costs until discovery. The counterfeiting rate (3) balances costs in the battle between good and bad guys. Multiplying equation (15) by $\frac{q/x}{q/x}$ yields an approximate reformulation:

$$\kappa \approx (1 - \text{police seizure rate}) \cdot \frac{\text{unit verification costs}}{\text{unit production costs}} \quad (17)$$

This sheds a unique light on the development of fiat currency — i.e. non-commodity money whose face value exceeds its intrinsic cost. This required technologies that could produce documents whose counterfeits could be descried at an effort cost far below their unit production cost. If not, counterfeiting would have spun out of control.

This suggests a quick consistency test of the model, that the stochastic struggle for the Δ note should incur expected social costs of at most Δ , as Tullock (1967) predicts:

Parties to a transfer/theft of D dollars should be collectively willing to spend up to D to affect the transfer/theft. We now uncover a novel limitation on this insight:

Theorem 8 (Social Costs) *The average costs of counterfeiting a Δ note are at most $(1-v)\Delta$, and the average total costs of verifying a circulating Δ note are at most $\kappa v\Delta$.*

Owing to the stochastic nature of the discovery process, total counterfeiting expenses (apart from law enforcement) have a ceiling $(1 - v + \kappa v)\Delta$, bounded below Tullock's Δ upper bound. The maximum social costs are *lower* when prevention efforts v are *greater*. The proof also reveals that more effective counterfeiting interdiction curiously lessens criminal production costs of counterfeiting: Counterfeiting costs are farther from the upper bound $(1 - v)\Delta$ the greater is the police seizure rate $s(v)$.

5 Counterfeiting as Technology or Legal Costs Change

To explain why counterfeiting costs have so greatly fallen (§6.2), we now develop a model of technological progress. Let $Q(q, t)$ denote quality “cost units”, so that quality q costs $c(Q(q, t))$ given the technology t . Then $Q_t < 0$ for a cost-lowering technology, while $Q_q > 0$ since higher quality costs more. For the separable case $Q(q, t) = q/t$, the cost of any quality level q falls from $c(q)$ to $c(q/t)$ as technology rises to $t > 1$ from 1. We call this *neutral technological progress*, as it treats all quality levels alike. Technology might better reduce the cost of higher quality levels, so that the ratio of quality cost units falls: $Q(q_2, t)/Q(q_1, t)$ is weakly decreasing in t , for $q_2 > q_1$. Such *quality-augmenting technological progress* soon explains the data in §6.2. Conversely, *quality-reducing* technological progress better raises higher quality cost levels.

Theorem 9 (Technology) (a) *Verification effort rises for any technological progress.*
 (b) *If the cost elasticity qc'/c weakly rises, quality rises with technological progress.*
 (c) *The verification rate falls (rises) for quality-augmenting (quality-reducing) progress.*
 (d) *The verification rate is constant and counterfeiting rate rises for neutral progress.*

Proofs of (a)–(c) are in Appendix A.8. For part (d), consider neutral progress. Quality rises, verification is unchanged, and so the counterfeiting rate in expression (3) rises.

We have no tractable analytic example of globally quality-reducing or augmenting technological progress. In the geometric family, $Q(q, t) = q^{1/t}$ yields quality-augmenting technology improvement in t iff $q > 1$. This reduces to a parameterized cost function $c(q) = q^{A/t}$ in our example in §3.2. The equilibrium quality in (9) rises

in t for $q > 1$, and falls for $q > 1$, while the verification rate in (9) always falls in t . Finally, the counterfeiting rate (11) rises in t at low notes, and then falls for high notes.

Theorem 9 sheds light on the so-called “cat and mouse” nature of the real world competition between counterfeiters and governments. Improved monetary security mimics technological regress by counterfeiters, raising counterfeiting costs. Verifiers relax their vigilance and quality falls by parts (a), (b). By part (c), effort falls so much that the verification rate falls when the security effects “quality-reducing regress” — namely, disproportionately inflating costs of mimicking the low cost security features. An excellent application of this principle occurred in Canada. As color was introduced on each note in the 1970s, the counterfeiting rate almost vanished for a couple years.

The flip side of counterfeiting progress is easier verification. Both changes are captured in the Bureau of Printing and Engraving’s motto for the new currency is “Safer. Smarter. More Secure.” It boasts that the money is “harder to fake and easier to check”. Yet making a currency harder to counterfeit or easier to verify are intuitively similar. We now exploit a joint homogeneity of degree one, and find when they are equivalent: A security feature *uniformly* halving verification costs is the same as a neutrally less efficient technology with larger cost quality units $2q$ in lieu of q .¹⁰

Corollary 1 (Easier Verification) *If the cost function χ falls to $\gamma\chi$, where $\gamma < 1$, then effort and quality fall, the verification rate is unchanged, and the counterfeit rate falls.*

Unlike when a currency is more costly to counterfeit, if it is more *uniformly* readily verified, then effort is not crowded out so much that verification falls.

Consider another government policy intervention. If it raises the legal punishment for counterfeiting, then the least notes can no longer be profitably counterfeited. But among those notes that are still counterfeited, verification effort drops. This crowding out from government intervention is so strong that the verification rate falls.

Theorem 10 (Legal Costs) *Assume a weakly rising cost elasticity qc'/c .*

- (a) *As legal costs rise, the least counterfeit note rises, and effort and verification fall.*
- (b) *Absent police, and so with unit passing elasticity $\Upsilon = 1$, quality falls in legal costs.*
- (c) *If $\Upsilon > 1$, then quality falls for the lowest notes, and rises for the highest notes.*
- (d) *If $\chi'(v)/v$ rises in v , then the counterfeiting rate falls in legal costs if quality does.*

In the example in §3.2 with passing elasticity $\Upsilon = 1$, the least counterfeit note $\underline{\Delta} = \ell/x$ rises in the legal costs ℓ , the counterfeit quality and verification rate (9) fall, and the counterfeit rate (3) falls with sufficiently convex verification cost $B > 2$.

¹⁰Changing the verification cost function to $\hat{\chi} = \frac{1}{2}\chi$ is equivalent changing the counterfeiting cost function to $\hat{c}(q) = c(2q)$ in terms of the new quality units $\hat{q} = q/2$, so that $\hat{c}(\hat{q}) \equiv c(q)$.

6 Evidence from Seized Counterfeiting Money

Our model is testable, and admits expressions for the levels of both seized and passed counterfeit money. We first explore consequences of the counterfeit entry game.

6.1 The Counterfeit-Passed Ratio Across Denominations

Since $f'(v) < 0$, the passing fraction falls in the note, by Theorem 5. If quality were fixed, then to ensure zero profits in (6), the equilibrium passing fraction would scale by one-half as the note doubled from \$5 to \$10 or \$10 to \$20. Its elasticity in Δ would then be -1 . But Theorem 4 proves that endogenous quality optimally rises in the note. This eats into profits as the note rises. So the passing fraction falls less than proportionately in Δ , and its elasticity exceeds -1 . Together, Theorems 4–5 yield:

Corollary 2 (Passing Fraction) *The passing fraction Δ -elasticity is $\mathcal{E}_\Delta(f) \in (-1, 0)$.*

Counterfeit money is eventually either *seized* from the criminals by police or the first verifiers, or *passed* onto the public, and later lost by an unwitting individual.

Call these levels $S[\Delta]$ and $P[\Delta]$. For simplicity, we pursue a steady-state analysis and do comparative statics and dynamics using a comparison of steady-states. Since these approaches give similar ordinal implications, the costly generality is not needed.

The values $S[\Delta]$ and $P[\Delta]$ obey two steady-state conditions. First, the *counterfeit value* of seized plus passed money $C[\Delta] = S[\Delta] + P[\Delta]$ equals the value of counterfeit money leaving circulation. Second, passed money circulating is constant: To wit, the outflow of passed money from circulation equals the inflow of new counterfeit money passing into circulation. We assume that counterfeiters attempt to pass all production, so that seized money represents failed passed money.¹¹ The inflow of passed money then equals the passing fraction times the counterfeit production. Immediately, we get:

$$P[\Delta] = f(v[\Delta]) \cdot (\text{production value}) = f(v[\Delta]) \cdot C[\Delta] \quad (18)$$

The importance of the *counterfeit-passed ratio* $C[\Delta]/P[\Delta]$ is apparent. It inherits the passing fraction properties from Corollary 2, offering easy testable implications.

Corollary 3 (Counterfeit-Passed Ratio) *The counterfeit-passed ratio rises in the note Δ , with elasticity*

$$0 < \mathcal{E}_\Delta(C/P) = -\mathcal{E}_\Delta(f) < 1$$

¹¹This is an overestimate because some money might be seized before any passing attempt, perhaps found in the counterfeiter's possession or after he is followed back to his lair. So to make sense of our data application below, we assume that this overestimate does not vary in the denomination.

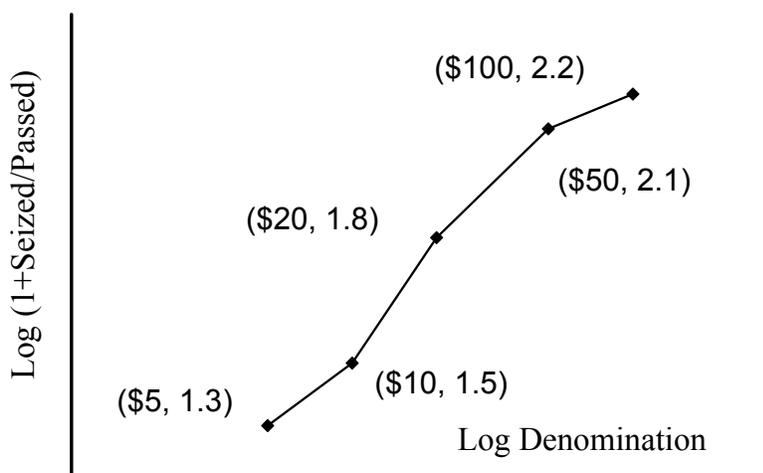


Figure 3: **USA Counterfeit Over Passed, Across Denominations.** These are the counterfeit-passed ratios, averaged over 1995–2007, for non-Colombian counterfeits in the USA. Clearly, they rise in Δ . The sample includes almost ten million passed notes, and about half as many seized notes. Data points are labeled by pairs $(\Delta, C(\Delta)/P(\Delta))$. So for every passed \$5 note, 0.33 have been seized on average. For this log-log graph, slopes are elasticities — positive and below one. We do not have data for this time span for the \$1 note; it averages 0.23 for the years 1998 and 2005–7.

This explains our result in Figure 3 (described in Appendix B) that the counterfeit-passed ratio has risen in the denomination in the USA 1995–2007 (as well as separately for 1995–99 and 2000–04). This trend also holds in Canada over the span 1980–2005 for all six paper denominations.¹² Corollary 3 also *correctly predicts that the slopes in this log-log diagram (i.e. elasticities) are not only positive but also less than 1.*

This analysis sheds light on the criminal marketplace. If producers sell to middlemen, then legal costs are borne by both parties, and average costs overstate the “street price” of counterfeit notes: Our two expressions for the passing fraction (6) and (18) from theory and data yield a simple upper bound on these prices:¹³

$$\text{street price} \leq \text{average costs} = \frac{c(q[\Delta]) + \ell}{\Delta x} = f(v[\Delta]) = \frac{\text{passed}}{\text{seized} + \text{passed}} \quad (19)$$

¹²For Canada, from 1980-2005, the counterfeit-passed ratios are respectively 0.095, 0.145, 0.161, 0.184, 0.202, and 3.054 over the notes \$5, \$10, \$20, \$50, \$100, and \$1000. Production of the \$1000 note was discontinued in 2000 to counter money laundering and organized crime.

¹³We thank Pierre Duguay for this insight; he said the predicted street prices are realistic. In one recent American case, a Mexican counterfeiting ring discovered this year sold counterfeit \$100 notes at 18% of face value to distributors, who then resold the counterfeit notes for 25–40% of face value. The money was transported across the border by women couriers, carrying the money.

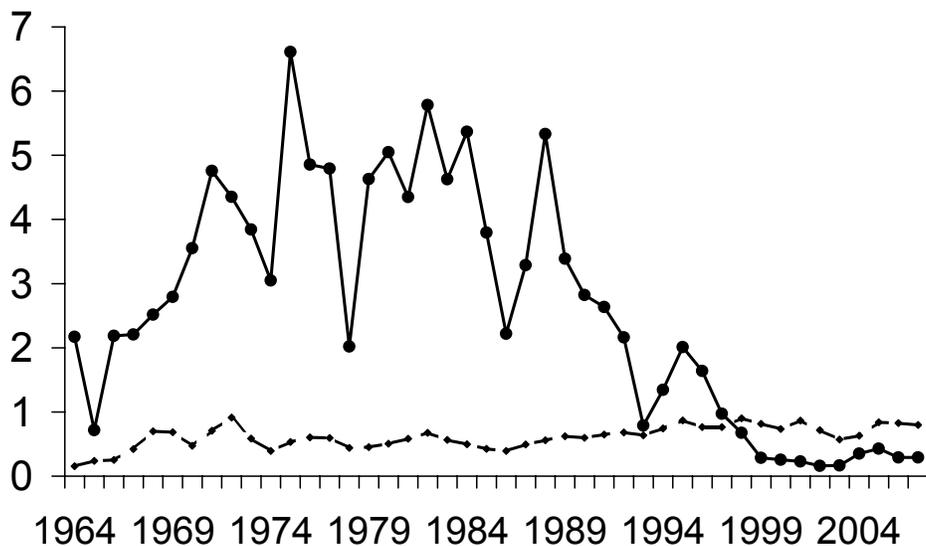


Figure 4: **USA Passed and Seized, 1964–2007.** The units here are per thousand dollars of circulation across all denominations. The dashed line represents seizures, and the solid line passed money. From 1970–85, the vast majority of counterfeit money (about 90%) was seized. The reverse holds (about 20%) for 2000–2007. Two down-spikes in 1986 and 1996 roughly correspond to the years of technological shifts.

The implied US street price ceilings can be computed from Figure 3, to get \$3.37, \$5.95, \$9.30, \$19.20, \$35.70, respectively. Testing this awaits data.

As an aside, if the counterfeit-passed ratio varies across denominations, then so must the verification rate, by Corollary 3. This empirical regularity is incompatible with a constant verification rate. It cannot be stochastic but exogenous, as in any paper that presumes verifiers observe a fixed authenticity signal — like Williamson (2002).

6.2 The Falling Counterfeit-Passed Ratio Over Time

There has been a sea change in the seized and passed time series since 1980. For the longest time, seized vastly exceeded passed counterfeit money, as seen in Figure 4. But starting in 1986, and accelerating in 1995, the counterfeit-passed ratio began to tumble. Tables have turned: By far, most counterfeit money now is passed,¹⁴ and the passing fraction has risen roughly from 10% to 80%. Our theory explains this change.

Appendix B documents two technological revolutions in counterfeiting during this time span: In the 1980’s, photocopiers became a tool of choice by counterfeiters. Next

¹⁴The Annual Reports of the USSS supplied earlier data, and the Secret Service itself gave us more recent data. Seized is a more volatile series, as seen in Figure 4, as it owes to random, maybe large, counterfeiting discoveries, and is also contemporaneous counterfeit money. By contrast, passed money is twice averaged: It has been found by thousands of individuals, and may have long been circulating.

Table 1: **Fraction of Notes Digitally Produced, 1995–2004.** This Secret Service data encompasses all 8,541,972 passed and 5,594,062 seized counterfeit notes in the USA, 1995–2004. Observe (a) the growth of inexpensive digital methods of production, and (b) lower denomination notes are more often digitally produced.

Note	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	avg.
\$5	.250	.306	.807	.851	.962	.972	.986	.980	.974	.981	.901
\$10	.041	.095	.506	.851	.908	.911	.961	.963	.971	.978	.756
\$20	.139	.295	.619	.882	.902	.926	.929	.961	.974	.983	.823
\$50	.276	.335	.546	.768	.777	.854	.911	.828	.822	.857	.755
\$100	.059	.066	.147	.263	.239	.314	.267	.251	.307	.399	.250

came digital counterfeiting technology in the 1990s — scanners and ink jet printers (see Table 1). Also, as Appendix B shows, this technology was smaller scale. We reconcile this technology change with the falling counterfeit-passed ratio.

Corollary 4 (Digital Technology) *The counterfeit-passed ratio is lower with a new quality-augmenting technology, or with a smaller scale technology.*

This follows from equation (18) since verification and thus the passing fraction drops with quality-augmenting technology change by Theorem 9. It also drops with a lower quantity by Theorem 5, since this has the same effect as a smaller note (see (6)).

6.3 The Rising Counterfeit Scale Across Denominations

We turn now to the cross-sectional observation that counterfeit scale and quality both rise in the note. As Table 1 depicts, the digitally-produced fraction falls in the note. In lieu of digital production, Judson and Porter (2003) find that 73.6% of passed \$100 notes were *circulars* — many notes from the same source (i.e. large scale production). This was 19.2% of \$50 notes, and less than 3% of other notes. Circulars are usually produced with printing presses, and are much higher quality. The “Supernote” is the highest quality counterfeit on record. First found in 1990, this deceptive North Korean counterfeit \$100 note was made from bleached \$1 notes, with the intaglio printing process used by the Bureau of Engraving and Printing — missed even by banks.¹⁵

Our model is readily amenable this richness. Suppose that in addition to $(x, c(q), \ell)$, there exists a *large-scale* (printing press) production $(X, Sc(q), \ell)$. Let output scale up more than production costs — with legal costs scaling up even more: $\ell/\ell > X/x > S$.

¹⁵Once a counterfeit hits a Federal Reserve Bank, it is almost impossible to trace it back to the original depositor. As such, counterfeit money that is so high quality as to escape earlier detection ought not affect incentives of individuals in our model, which might understate the quality rise at the highest notes.

For the chance of being found out rises more than proportionately with output (see footnote 8). This inequality ensures that neither technology is globally preferred. Since quantity and denomination are complements in profits, we get part (a) below:

Corollary 5 (Scale) (a) *Counterfeiters use large scale production for the highest notes.*
(b) *Counterfeit quality jumps up when switching to the larger scale.*

We twice apply our theory for part (b): First, legal costs rise moving from $(x, c(q), \ell)$ to $(x, c(q), \ell/S)$, as $\ell/S > \ell$. By Theorem 10, quality rises at least at the highest notes. Next, shifting to $(X/S, c(q), \ell/S)$ yields the same quality as $(X, Sc(q), \ell)$. Since $X/S > x$, this amounts to a higher note, and quality further rises by Theorem 4.

This corollary is silent about how the verification rate changes at the jump. Higher legal costs push down verification at the jump, while a higher currency lifts it up. In other words, verification falls if the legal cost scale up much more than the output does.

7 Evidence from Passed Counterfeit Money

7.1 Passed Counterfeit Rates Across Denominations

We turn to passed counterfeit money, fleshing out implications of the hot potato game. Figure 5 plots at the left the average fraction $p[\Delta]$ of passed \$1 notes for 1990–1996, and of the \$5, \$10, \$20, \$50, \$100 notes for 1990–2004. These ratios per million have averaged 1.96, 19.46, 71.21, 72.03, 49.94, 81.43, respectively. See Appendix B.¹⁶

The total supply of counterfeit and genuine Δ notes has *value* $M[\Delta] > 0$; we treat this as invariant to the supply of counterfeit notes. Recall that the value $P[\Delta]$ of passed money of denomination Δ is the discovery rate $\rho[\Delta]$ times the circulating counterfeit money $\kappa[\Delta]M[\Delta]$. The *passed rate* $p[\Delta] \equiv P[\Delta]/M[\Delta]$ is the fraction of all circulating Δ -notes per period that are discovered. Then we have from equation (3):

$$p[\Delta] = \rho[\Delta]\kappa[\Delta] = \frac{q[\Delta]\chi'(v[\Delta])}{\Delta} = \frac{\text{marginal verification cost}}{\text{denomination}} \quad (20)$$

The implied verification costs in (20) are easily measured by $\Delta p[\Delta]$. These are quite miniscule even for the highest notes. The passed rate is at most 1 per 10,000 *annually*. Suppose the \$100 note transacts at least four times per year. Then the passed rate $p[\Delta]$ is at most 1 in 40,000, and marginal verification costs are at most \$100/40,000, or *one*

¹⁶The common claim that the most counterfeited note domestically on an annualized basis is the \$20 is false over our time span. Accounting for the higher velocity of the \$20, on a per-transaction basis (the relevant measure for decision-making), the \$100 note is unambiguously the most counterfeited note.

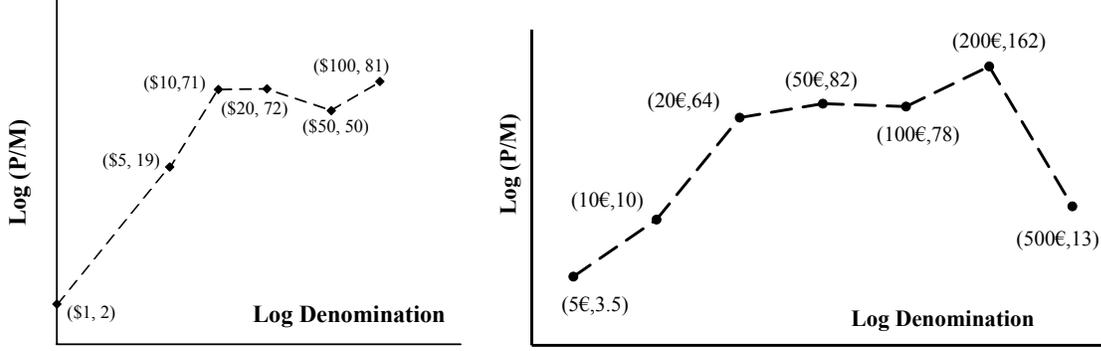


Figure 5: **Passed Over Circulation, Dollar and Euro.** At left are the average ratios of passed domestic counterfeit notes to the (June) circulation of the \$1 note for 1990-96, 1998, 2005-7, and the \$5, \$10, \$20, \$50, \$100 notes for 1990-2007, all scaled by 10^6 . At right is the Euro data. The data points are labeled by the pairs $(\Delta, P(\Delta)/M(\Delta))$.

quarter penny per note. Yet such tiny verification costs drive our theory. Surprisingly, incentives explain behavior even when costs are very small.

Since quality and verification vanish as Δ tends down to $\underline{\Delta} > 0$ by Theorem 2, the marginal verification cost in (20) vanishes as $\Delta \downarrow \underline{\Delta} > 0$. Without appealing to the elasticity or log-concavity assumptions, Theorem 2 and equation (20) at once imply:

Corollary 6 (Passed Money) (a) *The passed rate $p[\Delta]$ vanishes as the note $\Delta \downarrow \underline{\Delta}$.*
 (b) *The passed-rate $p[\Delta]$ drops for very large notes under Theorem 7's assumptions.*

Corollary 6 (a) obtains practically without caveat, and is strongly predictive of the data. For instance, the counterfeiting rate (11) in our example in §3.2 yields a passed rate $p = \rho(v)\kappa$ proportional to $\Delta^{-1+1/A}(1 - \underline{\Delta}/\Delta)^{B+1/A-1}$. This vanishes for Δ near $\underline{\Delta}$, given any $B > 1$. Corollary 6 (b) predicts a falling passed rate at theoretically high enough notes, but this is not apparent in the US dollar data. Yet the Euro offers two higher value notes; the passed rate clearly drops at the 500 Euro note in Figure 5.

The counterfeiting rate $\kappa[\Delta]$ is unobserved, and the passed-rate $p[\Delta] = \rho[\Delta]\kappa[\Delta]$ is its observable manifestation. While the passed rate is an imperfect proxy for the counterfeit rate, the Secret Service and the Federal Reserve may treat them as synonymous. Since the discovery rate $\rho(v[\Delta])$ rises in the note, $p[\Delta]$ is an increasing multiple of $\kappa[\Delta]$. So its peak must occur at a higher note, as seen in Figure 2. Also, *the passed rate will increasingly understate the actual counterfeiting problem at low notes.*

Our theory assumes that notes trade hands once per “period”. Unlike with the counterfeit-passed ratio, the passed rate is a flow over a stock, which skews the *per transaction* meaning. Yet the velocity is intuitively falling in the note.¹⁷ The higher the

¹⁷Lower denomination notes wear out faster, surely due to a higher velocity. Longevity estimates by

note, fewer transaction opportunities a year represents. Interpreting annualized passed data in this light, the relevant “per transaction passed rate” rises from \$50 to \$100 note, and might always rise in the denomination. Yet this falling velocity surely cannot account for the more than twelve-fold drop in the passed rate at the 500 Euro note.

7.2 The Stable Passed Rate Over Time

We see in Figure 4 that while the seized levels have dramatically fallen, passed money rates have proven quite stable through time. Our theory makes sense of this. The conflict between quality and verification effort induces the quality and verification rate variables to co-move. Quality-augmenting technological changes raises counterfeit quality and lessens the verification rate (Theorem 9). Likewise as legal costs change, quality and the verification rate move in opposition for most notes (Theorem 10).

The passed rate is also perfectly buffered to changes in banking verification rates. The counterfeit rate κ explicitly depends in (3) on the banking verification rate α and banking chance β , while the passed rate p in (20) does not. So if banks more effectively verify, then the counterfeit rate falls while the passed rate is constant. While there is less circulating counterfeit money with greater α or β , it is found at a faster rate. On balance, these effects exactly cancel, and the verification rate only indirectly affects the passed counterfeit money through the marginal verification cost.

8 Evidence from Passed Money in the Banking System

We turn to the last piece of evidence for our costly stochastic verification story, this one solely applicable to money. The banking sector offers a reverse test of the model — for unlike how passed money is found, counterfeit money hitting banks *has missed* earlier detection. Ideally, this data would reflect just our behavioral assumptions of verifiers, and not of banks. While not quite possible, the evidence is still compelling.

We have maintained (*bank model #1*) that banks find counterfeit notes at a fixed rate $\alpha \in (0, 1)$. The equilibrium discovery rate $\rho[\Delta] = \beta\alpha + (1 - \beta)v[\Delta]$ thus rises in the note. Since we assume that counterfeiters do not attempt to pass their money in a bank, this simple model of bank behavior is moot for equilibrium predictions of the effort, quality, and verification rate (as seen in our example in §3.2). While the counterfeiting rate expression reflects the discovery rate, the passed rate does not.

the Federal Reserve Bank of NY [www.newyorkfed.org/aboutthefed/fedpoint/fed01.html] are 1.8, 1.3, 1.5, 2, 4.6, and 7.4 months, respectively, for \$1, . . . , \$100. Observe the disproportionate upward jump from \$20 to \$50 and then from \$50 to \$100. FRB (2003) has close longevity estimates.

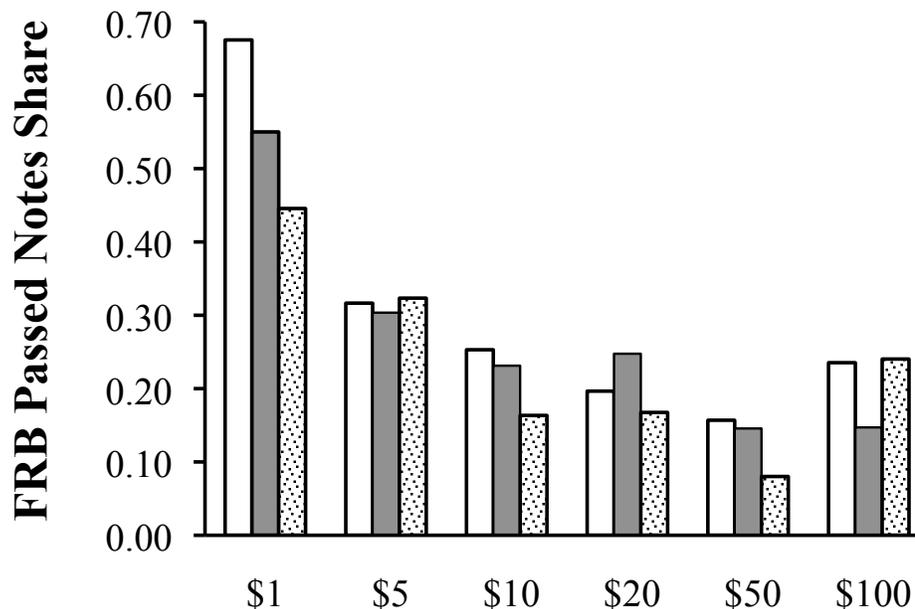


Figure 6: **FRB Share of Passed Notes.** The bars are the fractions of all passed notes across denominations found by Federal Reserve Banks in 1998, 2002, and 2005.

Two other parsimonious models of bank behavior might better apply for all notes. Since we argued that counterfeit money produced by large scale printing presses occurs at high notes and has a distinctly better quality by Corollary 5, we could just posit a lower fixed bank discovery rate $\underline{\alpha} < \alpha$ for these higher notes (*bank model #2*). Alternatively, we could build more closely on our verification model (*bank model #3*): Here, we venture the same verification cost function for banks as verifiers — it costs effort $q\chi(\alpha)$ to check a quality q note with intensity α — but that banks verify all notes with equal diligence — spending the same effort $b = q\chi(\alpha)$ per note.¹⁸ In this case, unlike bank model #2, the bank verification rate always falls in Δ due to the rising counterfeit quality, but again drops discontinuously if quality jumps.

Commercial banks transfer damaged or unneeded notes to the Federal Reserve Banks (FRB). The FRB found 21% of all passed counterfeits in 2002, but a much larger portion of the low denomination notes. *A priori*, this reverse monotonicity might seem surprising since the lowest notes are easiest for verifiers to catch. This anomaly offers more support for our model, and is fleshed out more fully in Figure 6.¹⁹

¹⁸Bank tellers told us that they were neither encouraged nor incentivized to treat different notes differently. They simply go by the feel of the note, and skip its other security features. That banks are surely more effective verifiers is then captured by assuming a large enough parameter b .

¹⁹See Table 6.1 in Treasury (2000), Table 6.3 in Treasury (2003), and Table 5 in Judson and Porter (2003).

To begin with, observe that intuitively, the fraction $\phi[\Delta]$ of notes that banks transfer to the FRB each period should fall in Δ , since longevity rises in the denomination. We first consider banks, for which we lack data, but have less couched predictions.

Corollary 7 (Bank Passed Note Share) *Assume the transfer rate ϕ does not fall too fast in Δ . Then the fraction of passed Δ notes found by banks falls in Δ in bank models #1–#3. The bank share falls less, or rises more the faster transfer rate ϕ drops.*

To see this, observe that a bank finds a passed note when (i) it is fake (chance κ), and (ii) the last verifier prior to the bank missed it (chance $1 - v$), and then (iii) deposited it in the bank (chance β), and then (iv) the bank finds it (chance α). Conditional on (i), events (ii)–(iv) are independent. So the reciprocal bank share of passed notes is:

$$\begin{aligned} \frac{1}{\mu[\Delta]} &= \frac{\text{passed notes found by verifiers, commercial banks, or an FRB}}{\text{passed notes found by commercial banks}} \\ &= \frac{\kappa v + \kappa(1 - v)\beta\alpha + \kappa(1 - v)\beta(1 - \alpha)\phi}{\kappa(1 - v)\beta\alpha} = \frac{v}{(1 - v)\beta\alpha} + 1 + \frac{(1 - \alpha)\phi}{\alpha} \end{aligned}$$

The nonconstant terms are (resp.) increasing and falling due to ϕ . All told, the bank share $\mu[\Delta]$ of passed notes falls in Δ if the transfer chance $\phi[\Delta]$ does not fall too fast.

Corollary 8 (FRB Passed Notes Share) *Assume the transfer rate ϕ does not fall too fast in Δ . Then the fraction of all passed Δ notes found by an FRB falls under bank model #1; under bank models #2 and #3, it can rise when quality rises fast enough.*

The logic for this result builds on the last. An FRB finds a passed note when events (i)–(iii) hold, and then (iv)' the bank misses the counterfeit (chance $1 - \alpha$), and (v) transfers it to an FRB (chance ϕ). Unlike with commercial banks, the counterfeit buck stops at an FRB, and it is surely found. The reciprocal of the *FRB share* $\sigma[\Delta]$ is then:

$$\frac{1}{\sigma[\Delta]} = \frac{\kappa v + \kappa(1 - v)\beta\alpha + \kappa(1 - v)\beta(1 - \alpha)\phi}{\kappa(1 - v)\beta(1 - \alpha)\phi} = \frac{v}{(1 - v)\beta(1 - \alpha)\phi} + \frac{\alpha}{(1 - \alpha)\phi} + 1$$

Write the first two terms as the product of two factors: The first factor $1/\phi$ is rising. Under bank model #1, the second factor is an increasing term plus a constant, and thus the product is increasing. Under bank models #2 and #3, the second factor can decrease fast enough to swamp the first term: In bank model #2, α drops down, and so greater quality depresses the bank discovery rate α more than v rises, and both terms can drop. In bank model #3, α can continuously drop if quality quickly rises in the note.

This corollary makes sense of the data in Figure 6. At low denominations, notes are mostly made digitally, quality rises slowly, and the FRB share is falling. In this range,

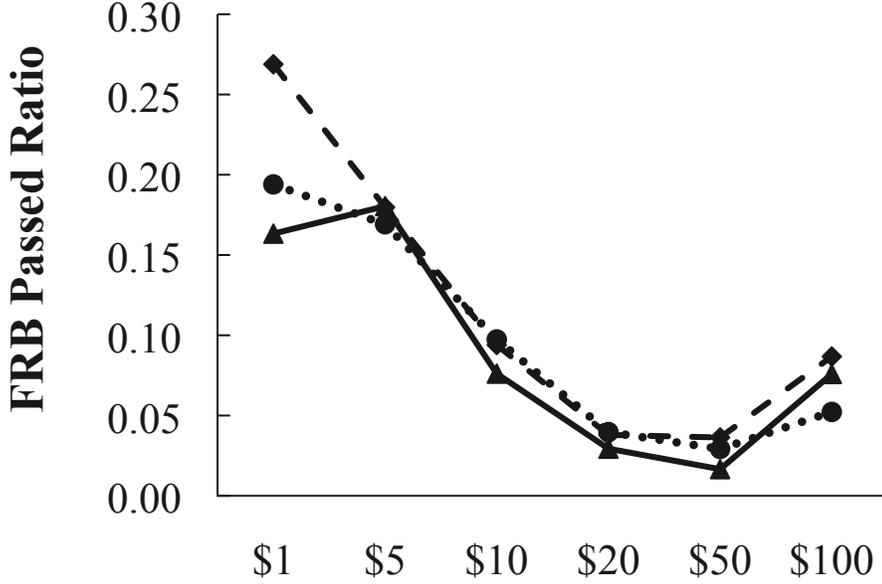


Figure 7: **Internal FRB Passed Rate / Passed Rate.** These are the ratios of the FRB passed money rate and the passed rate across denominations. The dashed line is 1998, the dotted line 2002, and the solid line 2005.

our costly verification story dominates, depressing the FRB passed note share. But at the \$50 and \$100 notes, quality jumps up, and the banks miss the counterfeits more often (bank models #2 and #3). The FRB share rises in years for which we have data.

The above exercises focused solely on the counterfeit notes. For a different lens on counterfeits in the banking system, let us consider the *internal bank passed rate*:

$$\xi[\Delta] = \frac{\text{passed notes hitting bank}}{\text{total notes hitting bank}} = \frac{\kappa(1-v)\beta\alpha}{(1-\kappa)\beta + \kappa(1-v)\beta\alpha} \approx \kappa(1-v)\alpha \quad (21)$$

The approximation is accurate within $\kappa \ll 0.0001$, or 0.01%. Likewise, the *internal FRB passed rate*, or fraction of passed notes hitting it that are counterfeit, is given by:

$$\zeta[\Delta] = \frac{\text{passed notes hitting FRB}}{\text{total notes hitting FRB}} \approx \kappa(1-v)(1-\alpha) \quad (22)$$

More passed notes hit a bank or FRB with a higher counterfeit rate. For instance, α can be identified as the ratio of the internal passed rates ξ/ζ . Thus motivated, we normalize (21) and (22) by the passed rate $p = \rho\kappa$, eliminating the counterfeit rate.

The bank share data in Figure 6 were influenced by the unmeasured but surely

falling FRB transfer rates ϕ . These new *passed rate ratios* below

$$\frac{\xi[\Delta]}{p[\Delta]} \approx \frac{(1 - v[\Delta])\alpha[\Delta]}{\rho[\Delta]} \quad \text{and} \quad \frac{\zeta[\Delta]}{p[\Delta]} \approx \frac{(1 - v[\Delta])(1 - \alpha[\Delta])}{\rho[\Delta]}$$

no longer suffer from this problem, but a new one. The discovery rate $\rho[\Delta]$ increases in the velocity, while the internal bank and FRB passed rates are unaffected by it. Since the velocity falls in the note, graphs of these ratios are biased upward in the denomination (versus a per transaction basis) — just like the passed rates in §7. Unlike in §7, we have adjusted the FRB passed rates by a simple velocity proxy, namely dividing them by the longevity measures in footnote 17. Absent this, the ratio instead rises from \$20 to \$50 and even more from \$50 to \$100, and is otherwise the same.

Corollary 9 (Passed Rates Ratios) *Assume velocity does not fall too fast in the note. The ratio $\xi[\Delta]/p[\Delta]$ of the internal bank passed rate and the overall passed rates falls in the note Δ under bank models #1–#3. The ratio of the internal FRB and overall passed rates $\zeta[\Delta]/p[\Delta]$ falls in Δ under bank model #1. Under bank models #2 and #3, it rises if quality rises enough. If velocity drops quickly, then either ratio may rise.*

Consistent with Corollary 9, for the only years with available data, 1998, 2002, and 2005, the ratio of the FRB and overall passed rates is falling monotonically only from the \$1 through the \$20 (Figure 7). But in each case, it turns up at the \$50 and further at \$100 — precisely the notes for which high quality circulars are common.

9 Conclusion

Counterfeiting is an interesting crime insofar as it induces two closely linked conflicts: counterfeiters against verifiers and law enforcement, and verifiers against verifiers. The focus on the first conflict in the small literature bypasses the key role of the second conflict in explaining passed counterfeit money. But since the late 1990s, seized money has only amounted to about 10% of counterfeit money, down from 90% in the 1970s.

We develop a novel strategic theory of counterfeiting subsuming both of the above conflicts. In our paper, bad guys wish to cheaply forge a counterfeit that passes for the real thing. A higher quality counterfeit is more costly, but better deceives good guys, and so passes more often. Good guys raise their guard with either dearer notes or greater counterfeit prevalence. Bad guys improve their quality with dearer notes or less careful good guys. As more bad guys enter, the counterfeiting rate rises. These three forces equilibrate in our large game. The endogenous verification effort explains the

rising counterfeit-passed ratio at low denominations, while variable quality counterfeit production justifies why this rise eventually tapers off. The model can capture changes in law enforcement, counterfeiting technology, or verification ease. It can explain a new set of stylized facts about counterfeiting across denominations that we identify.

On the normative side, we uncover a novel limit on the welfare losses of counterfeiting. We also predict that the unobserved counterfeiting rate is hill-shaped. We shed new light on the development of fiat currency — i.e. whose face value greatly exceeds its intrinsic cost: Since the counterfeiting rate is the ratio of verification to production costs, fiat currency required easily verified characteristics not easily reproduced.

The discovery chance of counterfeits depends on the verifiers' effort and counterfeit quality. Endogenous verification is a new assumption in this literature. Among the many possible functional forms for the verification rate, we have found an especially tractable one. Making a log-concavity assumption (possibly new for cost functions), we can rationalize the cross-sectional and time series properties of passed and seized money. This verification function should be useful in understanding counterfeit goods, or other economic settings where a conflict of wills determines a monitoring chance.

The passing game is a new use of supermodular games in monetary economics.²⁰

Finally, we return to the literature. The existing general equilibrium literature lets the price of money equilibrate the model. This is also done in the best papers on counterfeit goods Gene and Shapiro (1988). Our point of departure is thus to replace a priced asset with a new decision margin — individuals can continuously adjust their verification effort. We feel that a fixed value of notes is a good approximation for the USA now we examine where counterfeit notes are extremely rare. It agrees with the common observation that higher denominations may be declined if verification is too hard (“No \$100 bills accepted”), but are almost never discounted.²¹ Endogenizing the price of money cannot explain the *current* variation in seized or passed counterfeit levels across notes, since we have argued that one needs a variable verification effort. Not surprisingly, there has been no attempt by the existing literature to match the data.

One could imagine a general equilibrium setting — combining our insights and this literature — yielding a model where notes are both verified and discounted.²² *That* model would best capture runaway counterfeiting during say the Confederacy. It would also help understand counterfeit goods, where the face value price is endogenous.

²⁰Diamond (1982) developed a search-matching macroeconomics model that is supermodular in the production costs. Our monetary model is supermodular in a pairwise effort choice. Diamond studies multiple equilibria, while ours is nested with an entry game that forces a unique equilibrium.

²¹Notes a hardly ever discounted *domestically*. Older \$50 and \$100 bills may be declined abroad.

²²Our FOIA to the Secret Service asking for data on passed money in the banking sector was ignored.

A Appendix: Omitted Proofs

A.1 Existence and Uniqueness: Proof of Theorem 1

The existence proof proceeds in (q, v) space, and the uniqueness proof in (e, q) space.

STEP 1: EXISTENCE FOR $\Delta > \underline{\Delta}$. Assume $\Delta > \underline{\Delta}$. We exhibit a solution to the zero profit equation (6) and revised quality FOC (8). Since $f' < 0 < c'$, the zero profits condition (14) implicitly defines a continuous and decreasing function $q = Q_0(v)$. We must have $Q_0(0) > 0$, because $c(Q_0(0)) = \Delta x f(0) - \ell > 0$. Since $\Delta x f(0) > \ell$ and $f(1) = 0$, we may choose $\hat{v} < 1$ so that $\Delta x f(\hat{v}) = \ell$. Then $Q_0(v) \rightarrow 0$ as $v \rightarrow \hat{v}$. By the Implicit Function Theorem (IFT), because $q c'(q)$ is strictly increasing, the quality FOC (8) implicitly defines a differentiable function $q = Q_1(v)$. Since the limit $v \chi'(v)/\chi(v)$ exists and is positive as $v \rightarrow 0$, both sides of (8) vanish, and so $Q_1(0) = 0$. Easily, (8) is positive at $v = \hat{v}$, and thus $Q_1(\hat{v}) > 0$. Given $Q_1(0) = 0 < Q_0(0)$ and $Q_1(\hat{v}) > 0 = Q_0(\hat{v})$, the Intermediate Value Theorem yields $v \in (0, \hat{v})$ with $Q_0(v) = Q_1(v)$. But then $0 < v < 1$ and $0 < q = Q_1(v) = Q_0(v) < \infty$. So $\kappa > 0$ by (2). Finally, since $Q_0(v), Q_1(v)$ are differentiable in Δ , so is $q[\Delta]$ and $v[\Delta]$. (This conclusion would also have followed by applying the IFT on the system (6) and (8).)

STEP 2: UNIQUENESS. To see uniqueness, we return to (q, e) space. Assume two solutions (q_1, e_1) and (q_2, e_2) . Then $q_1 \neq q_2$, for if $q_1 = q_2$ then $e_1 = e_2$, since profits fall in effort. WLOG, let $q_1 < q_2$. Consider the line integral of profits $\Pi(q, e, \Delta)$ from (q_1, e_1) to (q_2, e_2) along the smooth curve $\mathcal{H} = \{(q, e) : \Pi_q(q, e, \Delta) = 0, q_1 \leq q \leq q_2\}$ where quality is optimal. Since $\Pi_e < 0$, we arrive at the contradiction:

$$0 - 0 = \Pi(q_2, e_2, \Delta) - \Pi(q_1, e_1, \Delta) = \int_{\mathcal{H}} (\Pi_q, \Pi_e) \cdot (dq, de) = \int_{e_1}^{e_2} \Pi_e de > 0 \quad \square$$

A.2 Initial Quality, Effort, and Verification: Proof of Theorem 2

By continuity of (6) and (7), the limits as $\Delta \downarrow \underline{\Delta}$ of e and q , and so v , exist.

STEP 1: QUALITY. If any limit $\underline{q} = \lim_{n \rightarrow \infty} q[\Delta_n] > 0$, then $\Pi(\underline{q}, \underline{v}, \Delta) = \Delta f(\underline{v}) - c(\underline{q}) - \ell \leq \Delta f(0) - \ell < 0$. But negative profits near $\Delta = \underline{\Delta}$ is impossible. So $\underline{q} = 0$.

STEP 2: EFFORT. If any limit $\underline{e} = \lim_{n \rightarrow \infty} e[\Delta_n] > 0$, then $\chi(v[\Delta]) = e[\Delta]/q[\Delta]$ must explode as $\Delta \downarrow \underline{\Delta}$. This is impossible because $\log \chi$ is concave.

STEP 3: VERIFICATION. If any limit $\underline{v} = \lim_{n \rightarrow \infty} v[\Delta_n] > 0$, where $\Delta_n \downarrow \underline{\Delta}$, then

$$\lim_{n \rightarrow \infty} \Pi_q(q[\Delta_n], e[\Delta_n], \Delta_n) = \Upsilon \frac{f(\underline{v})}{1 - \underline{v}} \cdot \frac{\chi(\underline{v})}{\underline{v} \chi'(\underline{v})} \cdot \underline{\Delta} \cdot \lim_{n \rightarrow \infty} \frac{v[\Delta_n]}{q[\Delta_n]} - c'(0)$$

by the quality FOC (8). Since $q[\Delta] \rightarrow 0$ as $\Delta \downarrow \underline{\Delta}$ by Step 2, and $c'(0) < \infty$ by cost convexity, it is impossible that $\Pi_q(q[\Delta_n], e[\Delta_n], \Delta_n) = 0$ at all Δ_n . Hence, $\underline{v} = 0$. \square

A.3 Quality Rises in the Denomination: Proof of Theorem 4

Claim 1 (Strict SOC) *The second order condition at an optimum is strict: $\Pi_{qq} < 0$.*

A strict second order condition is consistent with uniqueness proven in Theorem 1.

PROOF OF CLAIM: The SOC for maximizing $\Pi(q, e, \Delta)$ is locally necessary:

$$\Pi_{qq} = \Delta x f' V_{qq} + \Delta x f'' V_q^2 - c'' \leq 0 \quad (23)$$

The total derivative of the quality first order condition (7) is:

$$0 = \Pi_{qq} \dot{q} + \Pi_{qe} \dot{e} + \Pi_{q\Delta} \dot{\Delta} \quad (24)$$

For a contradiction, assume $\Pi_{qq} = 0$. Then (12) and (24) are linearly dependent. Since $\Pi_{qe} = \Delta (f' V_{qe} + f'' V_e V_q)$ and $\Pi_{q\Delta} = f' V_q$, this and Lemmas 1 and 2 give:

$$\frac{f' V_{qe} + f'' V_e V_q}{f' V_q} = \frac{f' V_e}{f} \quad \Rightarrow \quad 0 < \frac{V_{qe}}{V_q} = \left(\frac{f'}{f} - \frac{f''}{f'} \right) V_e$$

This is a contradiction, for $V_e > 0$ and $f'/f < f''/f'$ by strict log-concavity of f . \square

PROOF OF THEOREM 4: Since $\Pi_{qq} < 0$, $\dot{q} > 0$ follows from (24) if $\Pi_{qe} \dot{e} + \Pi_{q\Delta} \dot{\Delta} > 0$:

$$\begin{aligned} 0 < \Delta (f' V_{qe} + f'' V_e V_q) \dot{e} + f' V_q \dot{\Delta} &= -q (f' V_{qe} + f'' V_e V_q) f \chi' / f' + f' V_q \\ &= \left(f \frac{\chi}{\chi'} \left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right) + \frac{f'' f \chi}{f' \chi'} - f' \frac{\chi}{\chi'} \right) \frac{1}{q} \end{aligned}$$

This is positive, since by strict log-concavity of f and log-concavity of χ , we have

$$\left(\frac{-f' \chi}{f \chi'} \right)' = \frac{f''}{f'} - \frac{f'}{f} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} > 0 \quad (25)$$

A.4 Verification Bound in the Denomination: Proof of Theorem 5

Differentiating the zero profit condition (14) in Δ ,

$$c' \dot{q} = \Delta x f' \dot{v} + x f \quad (26)$$

Integrate (26) using $\dot{q} \geq 0$, the boundary condition $v[\underline{\Delta}] = 0$, and the definition (5) of Υ :

$$\frac{\Upsilon}{1-v} \dot{v} \leq 1/\Delta \quad \Rightarrow \quad -\Upsilon \log(1-v[\Delta]) \leq \log(\Delta/\underline{\Delta}) \quad \Rightarrow \quad 1-v[\Delta] \geq \left(\frac{\underline{\Delta}}{\Delta}\right)^{1/\Upsilon}$$

EXPLICIT FORMULA FORMULA FOR \dot{v} . Differentiating the first order condition (8) in Δ ,

$$(qc'' + c')\dot{q} + \Delta x f' \frac{\chi}{\chi'} \left(\frac{f''}{f'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right) \dot{v} = -x f' \frac{\chi}{\chi'}$$

Substituting for \dot{q} from the differentiated zero profit condition (26), we discover

$$(\Delta f' \dot{v} + f)(qc'' + c')/c' + \Delta f' \frac{\chi}{\chi'} \left(\frac{f''}{f'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right) \dot{v} = -f' \frac{\chi}{\chi'}$$

Multiplying by $-\chi'/(f\chi)$, using $-f'(v)/f(v) = \Upsilon/(1-v)$, and regrouping terms:

$$\frac{\Delta \Upsilon}{1-v} \dot{v} = \frac{\frac{f'}{f} + (qc''/c' + 1) \frac{\chi'}{\chi}}{\frac{f'}{f} + \frac{\chi'}{\chi} (qc''/c' + 1) + \left(\frac{f''}{f'} - \frac{f'}{f} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right)} \quad (27)$$

In light of inequality (25), $\dot{v} > 0$ if the above numerator is positive. This obtains iff

$$qc''/c' > -\frac{\chi}{\chi'} \left(\frac{\chi'}{\chi} + \frac{f'}{f} \right) = -\frac{\ell - [qc'(q) - c(q)]}{\Delta x f} \quad (28)$$

where the last term is (6) minus q times (7). This is positive when producer surplus $qc'(q) - c(q) < \ell$. Then (28) holds for all $\ell > 0$, since $(qc'/c)' \geq 0$ for all q implies:

$$\ell > \frac{q[(c')^2 - cc''] - cc'}{c' + qc''} = \frac{-c^2}{c' + qc''} \left(\frac{qc'}{c} \right)'$$

A.5 Proof that Quality Explodes: Proof of Theorem 6

Since $c(q) \geq 0$, we have $\Delta x f(0)(1-v)^\Upsilon \geq \ell$, and so by the FOC (8),

$$qc'(q) = \Upsilon \Delta x (1-v)^{\Upsilon-1} \frac{\chi(v)}{\chi'(v)} \geq \Upsilon \Delta x \left(\frac{\ell}{\Delta x f(0)} \right)^{1-1/\Upsilon} \frac{\chi(v)}{\chi'(v)} = O(\Delta^{1/\Upsilon}) \frac{\chi(v)}{\chi'(v)}$$

Since v increases in Δ by Theorem 5, and $\chi(v)/\chi'(v)$ is nondecreasing by log-concavity of χ , the right side explodes as $\Delta \uparrow \infty$. Thus $qc''(q) \uparrow \infty$, and so quality $q \rightarrow \infty$. \square

A.6 The Counterfeiting Rate Across Notes: Proof of Theorem 7

Substituting the formula for $c'(q)$ from (13) into counterfeiting rate (3), we find:

$$\kappa(v) = xf(0) \frac{(1-v)^{\Upsilon} \chi'(v)}{\rho(v)} \frac{q}{c(q) + \ell} \leq \frac{xf(0)\chi'(1)}{\alpha\beta c'(\bar{q})}$$

where $q = \bar{q}$ minimizes $(c(q) + \ell)/q$, i.e., with producer surplus $\bar{q}c'(\bar{q}) - c(\bar{q}) = \ell$. But if $c'(q) \geq c_0 q^\eta$ for all q , then producer surplus ℓ is at most $\eta c_0 \bar{q}^{\eta+1}/(\eta+1)$, and so:

$$\kappa(v) \leq \frac{xf(0)\chi'(1)}{\alpha\beta c_0} \left(\frac{\eta c_0}{\ell(\eta+1)} \right)^{\eta/(\eta+1)} < \frac{xf(0)\chi'(1)}{\alpha\beta(c_0\ell^\eta)^{1/(\eta+1)}}$$

A.7 Social Costs of Counterfeiting: Proof of Theorem 8

Since counterfeiters earn zero profits (6) in equilibrium, and $f(v) = (1-s(v))(1-v) \leq 1-v$, the average costs of counterfeiting a Δ note are at most $(1-v)\Delta$:

$$\Pi = 0 \Rightarrow [xc(q) + \ell]/x = f(v)\Delta \leq (1-v)\Delta$$

Next, since verifiers weakly prefer to choose v to no verification, the loss-reduction benefits of verifying exceed the verification costs in (1). So $\kappa(v)v\rho(v)\Delta \geq q\chi(v)$. Let $T(v)$ be the expected number of verifications of a circulating counterfeit note. Then the expected total verifying costs until a circulating counterfeit Δ note is found are:

$$q\chi(v)T(v) = q\chi(v)/\rho(v) \leq \kappa(v)v\Delta$$

where $T(v) = 1/\rho(v)$, since it is the mean of a geometric random variable.²³ \square

A.8 Technological Change: Proof of Theorem 9

PART I: Fix Δ . Abusing notation, write profits as $\Pi(q, e, \tau)$, where $\tau = 1/t$. Denote total derivatives in τ of any equilibrium variable z by \dot{z} . Note that all derivatives in t have the opposite sign of those in τ that we find below. We start at the knife-edge case of barely quality-reducing technological progress where costs have the form $c(\tau q)$. Assume $\tau = 1$. Profits are higher with a better technology, since $\Pi_\tau = -qc'(\tau q) < 0$.

STEP 1: EFFORT. Differentiate the zero profit condition $\Pi(q, e, \tau) \equiv 0$ in τ to get $\Pi_q \dot{q} + \Pi_e \dot{e} + \Pi_\tau = 0$. But $\Pi_q = 0$ by the quality FOC (7), and thus $\dot{e} = -\Pi_\tau/\Pi_e < 0$.

²³If we asked this question for an ex ante counterfeit note, then the expected number of verifications would be slightly greater, since we assume that counterfeiters do not try to pass their note in a bank.

STEP 2: QUALITY. Substitute the \dot{e} expression into the derivative of $\Pi_q \equiv 0$ in τ :

$$\frac{\Pi_{qq}}{\Pi_\tau} \dot{q} = -\frac{\Pi_{qe}}{\Pi_\tau} \dot{e} - \frac{\Pi_{q\tau}}{\Pi_\tau} = \frac{\Pi_{qe}}{\Pi_e} - \frac{\Pi_{q\tau}}{\Pi_\tau} \quad (29)$$

If $\tau = 1$, then $\Pi_{q\tau} = -c'(q) - qc''(q)$, and so $\Pi_{q\tau}/\Pi_\tau = 1/q + c''(q)/c'(q)$. Using $\Pi_{qe} = \Delta[f'V_{eq} + f''V_eV_q]$ and $\Pi_e = \Delta f'V_e$, and then Lemmas 1 and 2, we discover

$$\begin{aligned} \frac{\Pi_{qq}}{\Pi_\tau} \dot{q} &= \frac{V_{eq}}{V_e} + V_q \frac{f''}{f'} - \left(\frac{1}{q} + \frac{c''}{c'} \right) \\ &= V_q \left(\frac{f''}{f'} - \frac{f'}{f} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right) + V_q \frac{f'}{f} - \left(\frac{1}{q} + \frac{c''}{c'} \right) \end{aligned} \quad (30)$$

Dividing this by $V_q < 0$ yields the positive denominator of (27). Since $\Pi_{qq} < 0$ by Claim 1, and $\Pi_\tau = -qc' < 0$, we have $\dot{q} < 0$. So quality rises in the parameter t .

STEP 3: VERIFICATION. Since $\chi(v) = e/q$, the verification slope \dot{v} shares the sign of $q\dot{e} - e\dot{q}$. Substituting $\dot{e} = -\Pi_\tau/\Pi_e$ and \dot{q} from (29), \dot{v} shares the sign of

$$-q^2 \left(\frac{\Pi_{qq}}{\Pi_e} \right) + eq \left(\frac{\Pi_{qe}}{\Pi_e} - \frac{\Pi_{q\tau}}{\Pi_\tau} \right)$$

since $\Pi_{qq}, \Pi_\tau < 0$. Substituting from (23) and (24) for Π_{qq} and Π_{qe} , \dot{v} has the sign of:

$$-\frac{q^2[\Delta x f' V_{qq} + \Delta x f'' (V_q)^2 - c''(q)]}{x \Delta f' V_e} + eq V_q \left(\frac{f''}{f'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right) - e \left(1 + \frac{qc''}{c'} \right) \quad (31)$$

Since $c' = \Delta x f' V_q$, the terms in c'' cancel, and what remains vanishes. So $\dot{v} = 0$.

PART II: QUALITY-REDUCING OR QUALITY-AUGMENTING PROGRESS. Define the function $\mathcal{Q}(q, \tau) \equiv Q(q, 1/\tau)$. Consider the cost function family $c(\mathcal{Q}(q, \tau))$. At $\tau = 1$, strictly quality-reducing technological progress obeys $\mathcal{Q}_{q\tau}/\mathcal{Q}_\tau < \mathcal{Q}_q/\mathcal{Q} = 1/q$, since $\mathcal{Q}(q, 1) \equiv q$ implies $\mathcal{Q}_q(q, 1) = 1$. As in Step 1, $\dot{e} = -\Pi_\tau/\Pi_e < 0$, since $\Pi_\tau = -c'(q)\mathcal{Q}_\tau < 0$. The proof that $\dot{v} = 0$ using (31) with $\mathcal{Q}(q, \tau) = q\tau$ now yields $\dot{v} > 0$ since:

$$\frac{\Pi_{q\tau}}{\Pi_\tau} = \frac{c'(q)\mathcal{Q}_{q\tau} + c''(q)\mathcal{Q}_\tau}{c'(q)\mathcal{Q}_\tau} < \frac{1}{q} + \frac{c''(q)}{c'(q)}$$

If effort rises and verification falls in the parameter t , then quality must have risen.

Verification falls with quality-augmenting progress by analogy, while the proof in Step 2 that quality rises is more clearly true. \square

A.9 Changing Legal Costs: Proof of Theorem 10

Abusing notation, write profits as $\Pi(q, e, \ell)$, and the derivative in ℓ of a variable z as \dot{z} .

STEP 1: EFFORT. Differentiate the zero profit condition $\Pi(q, e, \ell) \equiv 0$ in ℓ to get $\Pi_e \dot{e} + \Pi_\ell = 0$. So $\dot{e} = 1/\Pi_e < 0$ as $\Pi_\ell = -1$.

STEP 2: THE LEAST NOTE $\underline{\Delta}$. Effort vanishes approaching the least counterfeit note $\underline{\Delta}$, and cannot drop further. To continue to earn zero profits, $\underline{\Delta}$ must rise.

STEP 3: THE VERIFICATION RATE. We modify the proof of Theorem 5. If we differentiate the zero profit condition $\Pi(q, q\chi(v), \Delta) = \Delta f(v) - c(q) - \ell = 0$ in ℓ , we get $c' \dot{q} = \Delta f' \dot{v} - 1$. Differentiating this identity once more in ℓ yields

$$(qc'' + c')\dot{q} + \Delta f' \frac{\chi}{\chi'} \left(\frac{f''}{f'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right) \dot{v} = 0 \quad (32)$$

Substituting $\dot{q} = (\Delta f' \dot{v} - 1)/c'$ as in Step 2 of Appendix A.4:

$$\Delta f'(v) \dot{v} = \frac{(qc''/c' + 1) \frac{\chi'}{\chi}}{\frac{f'}{f} + \frac{\chi'}{\chi} (qc''/c' + 1) + \left(\frac{f''}{f'} - \frac{f'}{f} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} \right)}$$

We proved after (27) that the denominator is positive, and so $\dot{v} < 0$.

STEP 4: QUALITY. If $\Upsilon = 1$, then $f'' = 0$, and \dot{q} and \dot{v} share the same negative sign, by (32): If $\Upsilon > 1$, then $f''/f' = (1 - \Upsilon)/(1 - v) < 0$ explodes near $v = 1$. The factor on \dot{v} in (27) is then negative at large $v < 1$ (it explodes to $-\infty$ as $v \uparrow 1$), and positive at small $v > 0$, since the χ difference explodes. So $\dot{q} > 0$ at large notes. \square

B Appendix: Discussion of Data

Our analysis has been graphical, as our claims about slopes are trivially statistically significant, given the massive numbers of notes (in the thousands). We have done pooled t -tests as well, but have excluded these, since all t statistics exceeded 5.

1. Data on Seized Money: Figure 3.

The Secret Service has given us data for 1995–2004 on Columbian counterfeits (family C-8094) passed and seized domestically by denomination, as well as an aggregate across all notes, including foreign and domestic passed and seized. Also, we have data for 2005–2007 that specifically separates foreign and domestic originated passed and seized. For 1995–2004, we have excised the Columbian counterfeit data — which are the largest portion of foreign counterfeits (especially for the \$100 note). But the

seizures for Colombian counterfeits are mostly in Colombia, while our data on passed notes is domestic. Since the vast majority of seizures are foreign, either in Columbia or en route to the USA, we have used these aggregate numbers year by year to scale each denomination's passed and seized ratio. We have verified that if all seized and passed data are included, the monotonic pattern in Figure 3 still holds for notes up to \$50, while the counterfeit-passed ratio at the \$100 dips slightly below that of the \$20.

We have no similar data for the Euro. Unlike the dollar, the Euro has no centralized tracking of seized money; this is separately handled by each member's police force.

2. Time Series Data: Figure 4.

The Secret Service was not offer us any pre-1990 data. We extracted aggregate values of passed and seized money in the 1970s and 1980s, and a wealth of data about the facts of counterfeiting, from the Annual Reports of the Secret Service on microfiche [USSS (1964) and USSS (1990)].

3. The Photocopying and Digital Counterfeiting Revolutions.

Photocopy "plants" suppressed by the USSS numbered: 11 from 1981–5, 30 in 1986, 345 from 1987–94, and finally 62 in each of 1995 and 1996 (the most recent year for which we have data). See the Annual Reports of the USSS until 1996, and then Table 6.8 in Treasury (2003).

Turning to the digital technological shift: There are no digital counterfeiting plants found until 1994. From 1995–2002, they grew from 19% to 95% of all plant seizures: 29, 101, 321, 547, 651, 527, 608, and 528. Since 1996, such counterfeits have risen from a small minority in 1995 to 98% of the \$5, \$10, and \$20 notes (Table 1). Our claims are consistent with findings in Chant (2004) of a digital revolution in the 1990's.

The digital counterfeiting is smaller scale is apparent in many ways. Eg., absent a major Secret Service Initiative, during the time span 1995–05 when total counterfeit production clearly fell (see Figure 4), counterfeiting arrests doubled from 1856 to 3717, and plant suppressions quadrupled from 153 to 611 Treasury (2006).

4. Data on Passed Money: Figure 5.

The USA passed money data is from the Secret Service, and the European data from the European Bank's web site. Apart from velocity, there is another problem with the annual passed over circulation data. First, we measure domestic passed notes, but circulation is worldwide. Also, the fraction of notes abroad likely rises in the denomination, possibly substantially. We then scaled circulation by established estimates. Judson and Porter (2003), eg., estimate that 3/4 of \$100 notes, and 2/3 of \$50 notes are abroad.

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