Debt Targeting in a Small Open Economy

(Preliminary and Incomplete)

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Abstract

This paper considers a debt targeting rule in a small open economy in the context of RBC model. First, the current account has a substantial impact on the optimal speed of fiscal adjustment. Under plausible calibration, a net lender can be better off under gradual fiscal adjustment, while a net borrower prefers rapid adjustment. The welfare difference depends on the risk premium. Second, it is always optimal to adjust labor tax than capital tax. In addition, the fiscal sustainability condition is analytically derived under the assumption of complete asset market. The condition depends on the speed of fiscal adjustment and the elasticity of tax revenue.

1 Introduction

Large public debt built-up in recent decades, coupled with the projection of aging population, has raised severe challenges in most countries. In response, fiscal rules are widely adopted in both developed and developing countries to safeguard budget sustainability, shown in Table(1) (Debrun, Epstein and Symansky (2008)). Well known is the rule of Maastricht Treaty in the European Union. The government deficit is limited to 3 percent of GDP, and the gross debt should not exceed 60 percent of GDP. Although the deepening global recession is currently deferring such fiscal rules, future adjustments will be inevitable to fill the huge holes left by bold fiscal stimulus.

When the government is unable or unwilling to cut government spending, taxes have to be adjusted to meet the fiscal restriction. However, the fluctuations of distortionary taxes deteriorate the welfare of households. The tradeoff between fulfilling fiscal discipline and smoothing taxation raises a question: what is the optimal speed of fiscal adjustment?

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Earlier works by Barro (1979), Lucas and Stockey (1983), and Aiyagari et al (2002) show that in a closed economy taxes should be smoothed across time or over state, and the government debt should be the shock absorber. Schmitt-Grohe and Uribe (2007), Kirsanova and Wren-Lewis (2007), and Kollman (2008) discuss the optimal simple and implementable fiscal policy joint with monetary policy. They find that the optimal fiscal feedback should be small.

However, it is still worthwhile to investigate the optimal simple fiscal policy in an open economy for two reasons. First, the discussion has been largely limited within a closed economy. The policy implication in a closed economy may or may not hold for small countries like Italy or New Zealand. For example, Kim and Kim (2006) find that a procyclical tax policy is optimal in an open economy, while a countercyclical tax policy is optimal in a closed economy. Second, active tax policies can play an important role to stabilize an economy when monetary policy can not be used, like the member countries of the European Union.

I use a standard RBC model with distortionary taxation to address the question of the optimal fiscal policy in a small open economy. As a first step on this topic, I assume the government adopts a linear debt targeting rule, instead of a nonlinear debt limit rule as many OECD countries are actually pursuing.

Three conclusions emerge from this paper. First, the choice of the optimal speed of fiscal adjustment hinges on the current account, which serves as a shock buffer in a small open economy. A net lender may prefer slow and smooth fiscal adjustment, while a net borrower may need more rapid fiscal action. A large risk premium can amplify the welfare difference. Second, it is always optimal to adjust labor tax more aggresively than capital tax. Finally, the fiscal sustainability conditions crucially depend on the speed of fiscal adjustment and the elasticity of tax revenue. Too sluggish adjustment leads to explosive debt path and violates the government intertemporal solvency, while too rapid adjustment destabilize the economy and leads to indeterminancy. The range of the speed of fiscal adjustment depends on the elasticity of tax revenue.

The bond market structure is important in the literature of optimal fiscal policy (Barro (1979), Lucas and Stokey (1983)). If the government can issue state-contingent bond, the financial market is complete and the household have perfect insurance; if the government can only issue risk-free bond, the financial market is incomplete. In Section 2, I discuss the optimal debt targeting rule by assuming that the asset market is incomplete. In Section 3, I investigate the fiscal sustainability conditions by allowing households to trade state-contingent bond in the international market. Such assumption largely simplifies the dynamics and allows me to derive the analytical results. Section 4 concludes.

2 Incomplete Asset Market

In an otherwise standard RBC small open economy, I give a non-trivial role to fiscal policy by modelling domestic households and fiscal authority separately. Households pay distortionary taxes, either labor tax or capital tax or both, to government. They can freely borrow from or lend to the international financial market at the constant world interest rate[1]. Government finances its exogenous spending by collecting tax revenue from domestic household and issuing non-state-contingent bond in the international market. In addition, government bond yield bears risk premium, meaning that increasing government indebtedness may keep the international market from purchasing government bond. Such risk premiums are widely identified in empirical studies (Bernoth et al (2006), Ardagna et al (2007)), although the literature is yet to agree on how large the risk premiums are. In this paper, I allow the size of risk premium to vary.

Several questions arise: Do a small open economy and an otherwise identical closed economy call for different debt-targeting rules? Is it optimal for small open economies with different debt burdens to pursue the debt targeting rule at the same speed? Does the persistence of the shock matters? What is the impact of the size of risk premium, if there is any? Does it matter which tax instrument, labor tax or capital tax, the fiscal authority uses to pursue the debt-targeting rule? To answer these questions, I start with a model without capital and then move to a model with both labor and capital.

2.1 Incomplete Asset Market without Capital

Household can trade bond b_t in the international asset market at fixed world interest rate R. Such a small open economy suffers the well-known non-stationarity problem due to the constant interest rate. I follow Schmitt-Grhoe and Uribe (2003b) and assume that household faces convex costs of holding bond in quantities different from some long-run level. In addition, with linear production technology, wage is normalized to 1. Household chooses consumption (c_t) and working hours (L_t) according to:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right) \tag{1}$$

$$s.t. \quad c_t + b_t + \frac{\psi_b}{2}(b_t - b)^2 = L_t\left(1 - \tau_t\right) + Rb_{t-1} \tag{2}$$

First-order conditions are straightforward:

$$-\frac{u_L(t)}{u_c(t)} = 1 - \tau_t \tag{3}$$

[1]This assumption may raise some concern as one can argue that the public and private bond yield may be positively correlated. Such concern is valid if households are borrowers in the international market. In Section(), I explore such possibilities and find no substantial changes to the results

$$\beta R E_t u_c(t+1) = u_c(t)\big(1 + \psi_b(b_t - b)\big) \tag{4}$$

Government finances its spending through collecting tax on labor income and issuing non-state-contingent debt d_t in the international market. I assume the risk premium is positively correlated with the public debt-GDP ratio S_t^d, defined as d_{t-1}/y_{t-1}. Mathematically, it is given by:

$$R_t^d = R \exp\left(\phi\left(S_{t-1}^d - S^d\right)\right) \tag{5}$$

R is the world interest rate, and ϕ is a country-specific interest rate premium. The government budget constraint is,

$$d_t + \tau_t L_t = R_{t-1}^d d_{t-1} + g_t \tag{6}$$

In addition, government pursues an explicit debt-GDP target at the long run level S^d.

$$\ln \frac{\tau_t}{\tau} = \gamma \ln \frac{S_{t-1}^d}{S^d} \tag{7}$$

The coefficient γ is the fiscal adjustment parameter. The larger γ is, the more aggresive fiscal adjustment is.

In a closed economy, the transversality on household's asset holding prohibits the government from running into Ponzi scheme. However, it no longer holds in an open economy. Benigno (2005) shows that the no-Ponzi-game constraint on housholds no longer guarantees that the government is not running a Ponzi scheme against the rest of the world. In this paper, I follow Schmitt-Grohe and Uribe (2003a) and assume that a prerequiste for the government to access to the international market is the satisfaction of the following borrowing limit:

$$\lim_{i\to\infty} \beta^i E_t \frac{u_c(t+i)}{u_c(t)} d_{t+i} = 0 \tag{8}$$

2.1.1 Method and Calibration

In the absence of a closed form solution, the equilibrium conditions are approximated around the deterministic steady states. A second-order solution is necessary, as conventional linearization can generate spurious welfare reversals (Kim and Kim (2003)). I use the perturbation method following Schmitt-Grohe and Uribe (2004). Let λ denote the welfare gain of adopting policy regime a compared with deterministic case conditional on the steady state in period zero[2].

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda)c, L) \tag{9}$$

[2] I follow Kollmann (2008) by comparing the welfare gain between under alternative debt-targeting rule and under deterministic steady state. Schmitt-Grohe and Uribe (2007) discuss another approach by comparing alternative fiscal regime with Ramsey regime

I assume the utility function is

$$U(c, L) = \frac{\left(c^{\chi}(1-L)^{1-\chi}\right)^{1-\sigma}}{1-\sigma} \tag{10}$$

σ is 2, which is widely used in business-cycle model. χ is calibrated to ensure the fraction of time spent working is 0.3. The household bond holding over GDP ratio is 0.5. The portfolio adjustment cost is very small. The coefficient ψ_b is 0.02, which implies that 10 percent change of household bond holding only costs about 0.01 percent (1 basis point) of consumption. The world interest rate is calibrated as 1.04, leaving the discount factor β to be 0.96.

The risk premium on government bond may vary so that it can cover the broad range of estimated sizes in the empirical literature. In the baseline model, the coefficient of risk premium ϕ is set to 0.2, such that 1 percent increase of government debt leads to 0.1 percent (or 10 basis point) increase of bond yield if the long run debt-GDP ratio is 0.5. For comparison, ϕ may vary from 0.05 to 0.87, an upper ceiling estimated by Bernoth et al (2006). That implies that 1 percent increase of government debt can raise government bond yield by as little as 2.5 basis point to as large as 43.5 basis point if the long run debt-GDP ratio is 0.5.

The long run government debt-GDP ratio is 0.5 in the baseline model. But it may vary from 0.3 to 0.8 to capture countries with different public burden. In addition, the government spending-GDP ratio is 0.2. The driving force g_t is parameterized to follow a univariate autoregressive process of the form,

$$\ln \frac{g_t}{\bar{g}} = \rho \ln \frac{g_{t-1}}{\bar{g}} + \epsilon_t \tag{11}$$

where $\bar{g}$ is a constant. The first-order autocorrelation ρ can either be 0 or 0.87, and the standard deviation of ϵ_t to 0.02.

2.1.2 Numerical Results

Several conclusions emerge. First, a small open economy and an otherwise identical closed economy call for different debt-targeting rule. Figure(1) shows that a small open economy's preference features an inverted-U shape, while a closed economy always prefer slower adjustment. Unable to borrowing from aboard, household consumption solely depends on the output, i.e. labor income in the economy with linear technology, Equation(12). Slower tax adjustment on labor helps household to smooth consumption.

$$c_t + g_t = L_t \tag{12}$$

On the other hand, households in open economies are less constrained as they can borrow from international market. There exists an optimal adjustment speed. Too sluggish

adjustment not only generates small tax revenue, but also bids up risk premium which raises interest payment and ultimately government debt burden. Too fast tax adjustment strongly discourges people from working which may limit people's ability to smooth consumption. In addition, the welfare difference is substainally larger in open economies (around 4 percent in the terms of consumption) than closed economy (less than 0.3 percent in the terms of consumption).

In addition, the optimal adjustment speeds may also depend on how willing the international market to hold Second, the optimal adjustment speeds depend on how willing the international market is to hold government bonds as the debt-GDP ratios surge. This point can be seen from two experiments.

The first one varies the government debt-GDP ratio while keeps the risk premium coefficient fixed at baseline calibration ($\phi = 0.2$). Figure(2) shows that the heavier the debt burden is, the faster the optimal adjustment speed becomes. This is intuitive. If the government debt-GDP ratio is as low as 0.3, meaning that 1 percent increase of government debt leads to only 6 basis point increase of bond yield, the optimal adjustment parameter should be 0.64 under a one-time government spending shock. On the contrast, the optimal parameter should be 1.72 if government debt-GDP ratio is 0.8, i.e. 1 percent increase of debt issurance leads to 16 basis point increase of bond yield. Since international investors charges higher risk premium to the government which bears higher public debt burden, relatively faster fiscal adjustment improves welfare by retiring debt faster. The interesting point is that, at their optimal adjustment parameters, all three governments will bring government debt-GDP back to the long run level within 10 periods.

The second experiment varies the risk premium coefficient while keeps the government debt-GDP ratio fixed at baseline calibration ($S^d = 0.5$). Figure(4) compares the welfare performance across different ϕ: the case of small risk premium, $\phi = 0.05$, implies that 1 percent increase of government debt leads to 2.5 basis point increase of bond yield; while the case of large risk premium, $\phi = 0.87$, implies that 1 percent increase of debt leads to 43.5 basis point increase of bond yield. If the international investors are less risk averse, fiscal authority have the luxury to raise tax revenue and retire debt slowly. In addition, the welfare difference is trivial, about 0.5 percent in terms of consumption, under a wide range of fiscal adjustment speeds. However, life is much harder for the government if investors are very risk averse. Not only does fiscal authority have to raise tax revenue swiftly, but also can the welfare cost be as high as 10 percent of consumption if it does not adjust appropriately.

Finally, the persistence of the shock matters a great deal. Figure(3) shows that a more persistent shock calles for more aggressive fiscal adjustment. Obviously, households are much worse off under persistent shocks, and the cost is a bit higher than 10 percent of consumption in the best scenario.

2.2 Incomplete Asset Market with Capital

In this section, I extend the baseline model by assuming that households can accumulate productive capital k_t. They have to pay capital adjustment cost, given by $\phi_k (k_t - k_{t-1})^2 /2$. Both the level and the slope of this cost function vanish in the long run. Small open economy models typically include capital adjustment cost to avoid excessive investment volatility (Schmitt-Grohe and Uribe (2003)). Household can still borrow from or lend to foreigners at the world interest rate. They pay tax on both labor and capital rental income.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \tag{13}$$

$$\begin{aligned} s.t. \qquad c_t + k_t + b_t &= w_t L_t \left(1 - \tau_t^L\right) + R_t^k k_{t-1} \left(1 - \tau_t^k\right) + R b_{t-1} - \frac{\phi_b}{2} (b_t - b)^2 \quad (14) \\ &\quad + (1-\delta) k_{t-1} - \frac{\phi_k}{2} (k_t - k_{t-1})^2 \end{aligned}$$

The first-order conditions are,

$$-\frac{u_L(t)}{u_c(t)} = w_t(1 - \tau_t^L) \tag{15}$$

$$u_c(t)\big(1 + \psi_b(b_t - b)\big) = \beta R E_t u_c(t+1) \tag{16}$$

$$u_c(t)\big(1 + \psi_k(k_t - k_{t-1})\big) = \beta E_t u_c(t+1) \left\{ R_{t+1}^k (1 - \tau_{t+1}^k) + 1 - \delta + \psi_k(k_{t+1} - k_t) \right\} \tag{17}$$

Firm maximizes its profit according to,

$$\max f(k_{t-1}, L_t) - R_t^k k_{t-1} - w_t L_t \tag{18}$$

The first-order conditions are $R_t^k = f_k(k_{t-1}, L_t)$ and $w_t = f_L(k_{t-1}, L_t)$.

Government finances its exogenous spending by issuing non-state-contingent bond and collecting taxes on labor and capital income. Again, its bond bears a risk premium that is positively correlated with the public debt-GDP ratio S_t^d. Government targets the debt-GDP ratio by adjusting labor and capital tax. Fiscal adjustment parameters γ^L and γ^k can be different.

$$d_t + \tau_t^k R_t^k k_{t-1} + \tau_t^L w_t L_t = R_{t-1}^d d_{t-1} + g_t \tag{19}$$

$$R_t^d = R \exp\left(\phi \left(S_{t-1}^d - S^d\right)\right) \tag{20}$$

$$\ln \frac{\tau_t^L}{\tau^L} = \gamma^L \ln \frac{S_{t-1}}{S} \tag{21}$$

$$\ln \frac{\tau_t^k}{\tau^k} = \gamma^k \ln \frac{S_{t-1}}{S} \tag{22}$$

2.2.1 Calibration

The utility function is calibrated similarly as the model without capital,

$$U(c, L) = \frac{\left(c^{\chi}(1-L)^{1-\chi}\right)^{1-\sigma}}{1-\sigma} \tag{23}$$

σ is 2. χ is calibrated to ensure the fraction of time spent working is 0.2. Household bond holding over GDP ratio is 0.5, and the consumption over GDP ratio is 0.8. The coefficient ψ_b is 0.08, which implies that 1 percent change of bond holding costs about 0.007 percent of consumption. The coefficient ψ_k is also 0.08, which implies that 1 percent change of capital stock costs about 0.0074 percent of consumption.

The production function is assumed to be of the Cobb-Douglas form,

$$f(k, L) = k^{\alpha} L^{1-\alpha} \tag{24}$$

α Capital depreciation rate δ is 0.1. α is calibrated such that capital ratio is 0.36.

In the baseline case, the government debt over GDP ratio is 0.5, and it may vary from 0.3 to 0.8 for comparison. The driving force g_t is parameterized the same as the model without capital. The world interest rate is still calibrated as 1.04 and the coefficient of risk premium ϕ is still 0.2.

2.2.2 Numerical Results

The key message from this model is that labor tax, compared with capital tax, is always a superior instrument for fiscal adjustment. Figure(5) shows the contour lines of λ, welfare gain in terms of consumption, on the plane of two fiscal adjustment parameters, γ^L and γ^k, in the caseline case. Warm-color lines implie higher welfare, compared with cool-color lines. Welfare gain decreases along the vertical axis, meaning that faster fiscal adjustment through capital tax incurs larger welfare loss. On the other hand, welfare gain first increases then decreases along the horizontal axis. It implies that there exists an optimal adjustment speed through labor tax, which is consistent with the model without capital. The optimal adjustment parameters are $\gamma^L = 1.25, \gamma^k = 0$. Fiscal authority should avoid raising capital tax, instead increase labor tax such that government debt-GDP ratio will return to the long-run level within 30 periods.

Figure(8) shows a closed economy which is calibrated identically to the baseline case. The optimal adjustment scheme is to keep capital tax fixed and raise labor tax as slowly as possible.

Figure(6) and Figure(7) show two open economies, one with lower government debt-GDP ratio ($S^d = 0.1$) and the other with higher government debt-GDP ratio ($S^d = 0.8$).

Again, the conclusion from the model without capital holds: the heavier the debt burden is, the faster the optimal adjustment speed should be.

2.3 Extension

Now consider the case where domestic households are borrowers, instead of savers as assumed the previous section, in the international asset market. It is possible that households may no longer be able to borrow freely at the world interest rate, and they may need to pay the same risk premium as their government does. This section extends the model without capital to such a scenario.

Household can borrow from in the international asset market at the rate R_t^d. In another word, $b < 0$ in this case.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right) \tag{25}$$

$$s.t. \quad c_t + b_t + \frac{\psi_b}{2}(b_t - b)^2 = L_t\left(1 - \tau_t\right) + R_{t-1}^d b_{t-1} \tag{26}$$

First-order conditions are straightforward:

$$-\frac{u_L(t)}{u_c(t)} = 1 - \tau_t \tag{27}$$

$$\beta R_t^d E_t u_c(t+1) = u_c(t)\big(1 + \psi_b(b_t - b)\big) \tag{28}$$

The risk premium is still given by:

$$R_t^d = R \exp\left(\phi\left(S_{t-1}^d - S^d\right)\right) \tag{29}$$

Government's budget constraint and debt-targeting rule are the same as the above model without capital.

It turns out that all the conclustions from Section(2.1.1) still hold.

3 Complete Asset Market

In a small open economy, the availability of state-contingent one-period real securities that span all the states of nature implies a constant marginal utility of consumption. If the utility function is separable, people can completely insure their consumption against any shock. The assumption of perfect insurance might sound unattractive, but it is often adopted in the discussion of optimal taxation in small open economies (Schmitte-Grohe and Uribe (2003a), Angyridis (2007)). I use this assumption in the following section to simplify the dynamics and to derive the fiscal sustainability conditions analytically.

3.1 Complete Asset Market without Capital

Let s_t be a random event that is an element of a finite set $\tilde{S}$. Let $s^t = (s_0, s_1, ..., s_t) \equiv \left(s^{t-1}, s_t\right)$. The state s^t is determined by the sequential shocks to government spending until time t. The probability at period 0 of any particular history s^t being realized is denoted by $\pi\left(s^t\right)$. The initial state is given.

At period t, the household can trade any one-period forward Arrow securities $b\left(s_{t+1}|s^t\right)$ at the market value of $p\left(s_{t+1}|s^t\right)$. They also allocate consumption and labor supply according to:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left(c\left(s^t\right), L\left(s^t\right)\right) \tag{30}$$

$$s.t. \quad c\left(s^t\right) + \sum_{s_{t+1}} p\left(s_{t+1}|s^t\right) b\left(s_{t+1}|s^t\right) = L\left(s^t\right)\left(1 - \tau\left(s^t\right)\right) + b\left(s^t\right) \tag{31}$$

It can be shown that the optimization is equivalent to the following question.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right) \tag{32}$$

$$s.t. \quad c_t + E_t q_{t+1} b_{t+1} = L_t\left(1 - \tau_t\right) + b_t \tag{33}$$

$$q_{t+1} \equiv \frac{p\left(s_{t+1}|s^t\right)}{\pi\left(s_{t+1}|s^t\right)} \tag{34}$$

where q_{t+1} denotes the period-t price of an asset that pays one unit of good in a particular state of period $t+1$ divided by the probability of occurrence of that state given information available in period t.

In the rest of the world, agents have access to the same array of financial assets as in the domestic economy (Schmitt-Grohe and Uribe (2003b)). Thus one can obtain similar first-order condition of the foreign household as domestic household. Let the starred letter denote foreign variables:

$$u_c^*\left(t\right) q_{t+1} = \beta u_c^*\left(t+1\right) \tag{35}$$

Since the domestic and foreign housholds share the same subjective discount factor, the domestic and foreign first-order conditions yield:

$$\frac{u_c\left(t\right)}{u_{c+1}\left(t\right)} = \frac{u_c^*\left(t\right)}{u_{c+1}^*\left(t\right)} \tag{36}$$

It implies that the domestic marginal utility of consumption is proportional to its foreign counterpart. It can be written as:

$$u_c\left(t\right) = \vartheta u_c^*\left(t\right) \tag{37}$$

where ϑ is a constant parameter that determines the wealth differences across countries. The domestic economy is assumed to be small, thus $u_c^*(t)$ is exogenous. To investigate the effects of domestic government spending shock, I assume $u_c^*(t)$ is constant throughout this paper. Therefore the interest rate is constant as well.

$$u_c(t) = \vartheta u_c^* \equiv \vartheta^* \tag{38}$$

$$q_{t+1} = \beta \tag{39}$$

3.1.1 Fiscal Sustainability Conditions

The existence of state-contingent bond in the international market allows households to insure against any shock and to maitain the marginal utility of consumption at constant level, i.e. $u_c = \vartheta^*$. With an explicit utility functional form, this model can be solved analytically. At each period t, income tax rate τ_t is predetermined due the debt-targeting rule, and consequently labor choice L_t is also predetermined. Thus the system can be nailed down to one single equation including one endogenous variable, the debt-GDP ratio S_t, and one exogenous variable, the government spending g_t:

$$S_t + \tau(S_{t-1}) = \frac{1}{\beta}\frac{L(S_{t-2})}{L(S_{t-1})} + \frac{g_t}{L(S_{t-1})} \tag{40}$$

$\tau(S_{t-1})$ and $L(S_{t-1})$ shows that tax rate and labor supply are determined by the debt-GDP ratio in previous period.

Equation(35) is log-linearized around the steady states in order to discuss the fiscal sustainability. Appendix C includes the details. It can be shown that the economy can stably converge back to steady state if the fiscal adjustment parameter γ satisfies the both following conditions:

$$\left(\left(\frac{1}{\beta} - 1\right)(1 - \epsilon^R) + \frac{T}{d}\epsilon^R\right)\gamma > \frac{1}{\beta} - 1 \tag{41}$$

$$\frac{1}{\beta} > \left(\frac{T}{d}\epsilon^R + \epsilon^R - 1\right)\gamma > \frac{1}{\beta} - 2 \tag{42}$$

where $T = \tau L$ is the tax revenue at steady state, $\epsilon^R = \widehat{T}/\widehat{\tau}$ is the elasticity of tax revenue with respect to tax rate. Intuitively, a very small γ implies very sluggish fiscal adjustment and may lead to an explosive path of government debt; on the other hand, a very large γ leads to a regime with the path oscillating around the steady state, and the economy may not be able to converge either.

In another word, the dynamic path starts to fluctuate if

$$\left(\frac{T}{d}\epsilon^R + \epsilon^R - 1\right)\gamma > \frac{1}{\beta} \tag{43}$$

The larger the elasticity ϵ^R is, the smaller the upper bound of γ is, and the easier to reach the regime with oscillation. The result is intuitive. Consider the case that the tax revenue is very responsive to tax change (large ϵ^R). Under a bad government spending shock, very aggresive adjustment (a very large γ) may cause the tax revenue in the coming period to jump up to such an extent that leads to government budget surplus. In response to the surplus, government would cut the tax again, if they take the debt-targeting rule serious. Such back-and-forth fiscal adjustments cause the economy dynamics to oscillate around the steady states and deteriorates welfare.

3.2 Complete Asset Market with Capital

I extend the previous model to include capital. Household chooses consumption and working hour according to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right) \tag{44}$$

$$s.t. \quad c_t + k_t + E_t q_{t+1} b_{t+1} = w_t L_t \left(1 - \tau_t^L\right) + r_t k_{t-1} \left(1 - \tau_t^k\right) + b_t + (1-\delta) k_{t-1} \tag{45}$$

The interest rate q_t is exogenously determined by the rest of the world. Due to the complete asset market, q_t is constant and equal to the discount rate β.

The specifications of firm, government and the rest of the world are the same as the simple model without capital in Section(3.1). All the details are included in Appendix D.

With explicit utility and production function, this model can be solved analytically. At each period t, the labor and capital tax rates, τ_t^L and τ_t^k, are predetermined due to the debt-targeting rules. The predetermined capital stock k_{t-1} and the income tax rate τ_t imply that labor supply L_t and output y_t are also predetermined. Therefore, there is no uncertainty in this model, and the capital stock at this period k_t is uniquely determined by the household first-order condition. The dynamic system is nailed down into to a single equation of the debt-GDP ratio S_t and the exogenous government spending g_t:

$$S_t + (1-\alpha)\tau^L\left(S_{t-1}\right) + \alpha\tau^k\left(S_{t-1}\right) = \frac{1}{\beta}\frac{y\left(S_{t-2}\right)}{y\left(S_{t-1}\right)} + \frac{g_t}{y\left(S_{t-1}\right)} \tag{46}$$

After log-linearing the system, the necessary conditions for fiscal sustainability are,

$$\frac{1}{\beta} > \left(\frac{T_L}{d} + \left(\frac{T}{d}+1\right)\left(\epsilon_L^R - 1\right)\right)\gamma^L + \left(\frac{T_k}{d} + \left(\frac{T}{d}+1\right)\left(\epsilon_k^R - 1\right)\right)\gamma^k \tag{47}$$

$$\left(\frac{T_L}{d} + \left(\frac{T}{d}+1-\frac{1}{\beta}\right)\left(\epsilon_L^R - 1\right)\right)\gamma^L + \left(\frac{T_k}{d} + \left(\frac{T}{d}+1-\frac{1}{\beta}\right)\left(\epsilon_k^R - 1\right)\right)\gamma^k > \frac{1}{\beta} - 1 \tag{48}$$

Again, if Equation(42) isn't met, the dynamic path oscillates around the steady states. The more elastically the tax revnue responds to tax rate, the easier for the dynamic system to slip into the regime with oscillation. This is a direct extension from the model without capital. Appedix D includes all detailed derivation.

4 Conclusion

This paper contributes to the literature through two lines. First, it reaffirms the conclusion from Kim and Kim (2006): extending the optimal fiscal policy from a closed economy to an open economy is not trivial. The access to international asset market may have fundamental implications for policy debate. Second, in the case of open economies, the current account plays a crucial role in determining the optimal debt targeting rule.

However, this paper *assumes* that government pursues the debt targeting rule. This leaves out a more interesting question: should government target or constrain debt at all? There is a stunning lack of consensus in academic debate as to whether these fiscal rules are beneficial.

The literature of optimal fiscal policy in closed economy does not favor stringent displines on debt accumulation. Seminal work by Barro (1979) and Lucas and Stockey (1983) embraced different assumption of bond structure market and discovered that tax should be smoothed across time or over state and government debt should be the shock absorber. By combining the two lines of work together in a RBC framework, Aiyagari et al (2002) shows that government debt follows a near-random walk under some pausible assumption, which is further confirmed by Schmitt-Grohe and Uribe (2004) in a New-Keysian model. However, it can neither explain the worldwide debt rise from 1970s to 1990s, nor the large cross-country differences of public debt. This theory may tell us what governments *should do*, but it does not tell us what governments actually *does*.

An alternative is to incorporate some political distortion, like self-interest politician or inefficient coalition government or political polarization. Since the inefficient political system fails to internalize the cost of issuing government debt, the fiscal discipline may be desirable. Several recent works have been attempting to fill in the gap, see Battaglini and Coate (2007, 2008a, 2008b), Azzimonti (2007), Debortoli and Nunes (2007), and Acemoglu, Golosov and Tsyvinski (2007, 2008). I plan to explore in the similar direction.

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Appendix A

Table 1: Fiscal Rules Across Countries

Country	Type of Rule/Objective	Statutory Base
Denmark	Structural budget balance	Political agreement
Estonia	Budget balance	Political agreement
Finland	Budget balance	Political agreement
	Debt in percent of GDP	Political agreement
France	Golden rule	Law
Lithuania	Ceiling on net borrowing	Law
Netherlands	Expenditure ceilings	Coalition agreement
Poland	Debt in percent of GDP	Constitution
Slovenia	Debt in percent of GDP	Coalition agreement
Spain	Budget balance	Law
Sweden	Budget surplus on average	Political agreement
Switzerland	Budget balance - Debt-Brake rule	Constitution
United Kingdom	Golden rule	Law
	Debt in percent of GDP	Law
Australia	Budget balance and debt ceiling	Law
Brazil	Debt in percent of revenues	Law
Colombia	Debt in percent of revenues	Law
India	Golden Rule	Law
New Zealand	Operational balance	Law

Source: Debrun, Epstein and Symansky (2008)

Appendix C: Proof of Fiscal Sustainability in Section 3.1

As explained, the model is nailed down into a single equation. It can be log-linearized around the steady states.

$$\widehat{s_t} + \alpha_1 \widehat{s_{t-1}} + \alpha_2 \widehat{s_{t-2}} = \alpha_3 \widehat{g_t} \tag{49}$$

where

$$\begin{aligned} \alpha_1 &= \tau L \left(1 - \frac{1}{\sigma}\frac{\tau}{1-\tau}\right) - \frac{1}{\beta} - \frac{\gamma}{\sigma}\frac{\tau}{1-\tau} \\ \alpha_2 &= \frac{\gamma}{\beta\sigma}\frac{\tau}{1-\tau} \\ \alpha_3 &= \frac{g}{d} \end{aligned} \tag{50}$$

As explained in the paper, both labor supply and tax rate are tied up to the debt-GDP ratio at previous period through the debt-targeting rule, so the system should be solved backward. Let B be the lag-operator, then

$$\widehat{s_t} = \frac{\alpha_3 \widehat{g_t}}{1 + \alpha_1 B + \alpha_2 B^2} \tag{51}$$

Define $\beta_1\beta_2 = \alpha_2$ and $\beta_1 + \beta_2 = -\alpha_1$, then

$$\widehat{\tau_t} = \left(\frac{\beta_1}{1-\beta_1 B} - \frac{\beta_2}{1-\beta_2 B}\right)\frac{\alpha_3 \widehat{g_t}}{\beta_1 - \beta_2} \tag{52}$$

The system has an explosive path if either $|\beta_1|$ or $|\beta_2|$ or both are larger than 1. The necessary and sufficient conditions for the existence of unique equilibrium are

- $\alpha_1^2 - 4\alpha_2 > 0$
- $\alpha_1 < 0$ and $\frac{-\alpha_1 + \sqrt{\alpha_1^2 - 4\alpha_2}}{2} < 1$; or $\alpha_1 > 0$ and $\frac{-\alpha_1 - \sqrt{\alpha_1^2 - 4\alpha_2}}{2} > -1$

The conditions are equivalent to

- $\alpha_1^2 - 4\alpha_2 > 0$
- $0 > \alpha_1 > -2$ and $\alpha_1 + \alpha_2 + 1 > 0$; or $2 > \alpha_1 > 0$ and $1 - \alpha_1 + \alpha_2 > 0$

If the above conditions are met, the system can be solved as

$$\widehat{s_t} = \sum_{i=0}^{t-1} \frac{\beta_1^{i+1} - \beta_2^{i+1}}{\beta_1 - \beta_2} \alpha_3 \widehat{g_{t-i}} \tag{53}$$

Assume the spending shock follows

$$\widehat{g_t} = \rho \widehat{g_{t-1}} + \eta_t \tag{54}$$

After one-time shock η_0, $\widehat{s_t}$ can be simplified as

$$\widehat{s_t} = \frac{\alpha_3}{\beta_1 - \beta_2}\left(\frac{\beta_1}{1-\beta_1\rho} - \frac{\beta_2}{1-\beta_2\rho}\right)\widehat{g_0} - \frac{\alpha_3}{\beta_1 - \beta_2}\left(\frac{\beta_1^{t+2}}{1-\beta_1\rho} - \frac{\beta_2^{t+2}}{1-\beta_2\rho}\right)\rho^{t+1}\widehat{g_0} \tag{55}$$

Note that α_2 is always positive, it implies the β_1 and β_2 are either both positive or both negative, depending on α_1. If α_1 is negative, the two roots are positive, and the path of τ_t steadily converge back to the steady state. On the other hand, with positive α_1, β_1 and β_2 are negative, and τ_t fluctuates around the steady states before it dies out eventually.

It can be shown that the necessary and sufficient conditions for the existence of unique and stable equilibrium are

$$\left(\left(\frac{1}{\beta} - 1\right)\left(1 - \epsilon^R\right) + \frac{T}{d}\epsilon^R\right)\gamma > \frac{1}{\beta} - 1 \tag{56}$$

$$\frac{1}{\beta} > \left(\frac{T}{d}\epsilon^R + \epsilon^R - 1\right)\gamma > \frac{1}{\beta} - 2 \tag{57}$$

$$\left(\sqrt{\frac{T}{d}\epsilon^R} - \sqrt{1-\epsilon^R}\right)\sqrt{\gamma} > \sqrt{\frac{1}{\beta}} \tag{58}$$

$$\epsilon^R > 0 \tag{59}$$

More specifically, the path starts to fluctuate if

$$\left(\frac{T}{d}\epsilon^R + \epsilon^R - 1\right)\gamma > \frac{1}{\beta} \tag{60}$$

where $T = \tau L$ is the tax revenue at steady state, $\epsilon^R = \frac{\widehat{T}}{\widehat{\tau}}$ is the uncompensated elasticity between tax revenue and tax rate. The larger the elasticity ϵ^R, the easier to reach the regime with fluctuation.

Appendix D: Proof of Fiscal Sustainability in Section 3.2

In the economy with capital and complete asset market,the first order conditions for household are straight-forward,

$$u_c(t)\, q_{t+1} = \beta u_c(t+1) \tag{61}$$

$$-\frac{u_L(t)}{u_c(t)} = \left(1-\tau_t^L\right) w_t \tag{62}$$

$$u_c(t) = E_t \beta u_c(t+1)\left(\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right) \tag{63}$$

Also, the allocations satisfy the transversality condition,

$$E_t \lim_{i\to\infty} Q_{t+i} b_{t+i} = 0 \tag{64}$$

$$E_t \lim_{i\to\infty} Q_{t+i} R_{t+j}^k k_{t+i-1} = 0 \tag{65}$$

where $Q_{t+i} = q_{t+1}q_{t+2}...q_{t+i}$ and $R_t^k = \left(1-\tau_t^k\right) r_t + 1 - \delta$. The firm's maximization problem implies that

$$r_t = f_k(k_{t-1}, L_t) \tag{66}$$

$$w_t = f_L(k_{t-1}, L_t) \tag{67}$$

Assume the following functional forms,

$$u(c, L) = \ln c - \chi \frac{L^{1+\sigma}}{1+\sigma} \tag{68}$$

$$f(k, L) = k^\alpha L^{1-\alpha} \tag{69}$$

The dynamic system can be written as

$$L_t^{\sigma+\alpha} = \frac{1-\alpha}{\chi c}\left(1-\tau_t^L\right) k_{t-1}^\alpha \tag{70}$$

$$y_t = k_{t-1}^\alpha L_t^{1-\alpha} \tag{71}$$

$$E_t\left(\left(1-\tau_{t+1}^k\right)\alpha k_t^{\alpha-1} L_{t+1}^{1-\alpha}\right) = \frac{1}{\beta} + \sigma - 1 \tag{72}$$

$$d_t = \frac{1}{\beta} d_{t-1} + g_t - (1-\alpha)\, y_t \tau_t^L - \alpha y_t \tau_t^k \tag{73}$$

$$\frac{\tau_t^L}{\tau^L} = \left(\frac{S_{t-1}^d}{S^d}\right)^{\gamma^L} \tag{74}$$

$$\frac{\tau_t^k}{\tau^k} = \left(\frac{S_{t-1}^d}{S^d}\right)^{\gamma^k} \tag{75}$$

After some substitutions, the path of capital can be shown as

$$k_t = a_4 \left(1 - a_2 \left(\frac{d_t}{y_t}\right)^{\gamma^k}\right)^{\frac{\sigma+\alpha}{\sigma(1-\alpha)}} \left(1 - a_3 \left(\frac{d_t}{y_t}\right)^{\gamma^L}\right)^{\frac{1}{\sigma}} \tag{76}$$

where

$$a_4 = \left(\frac{\alpha \left(\frac{1-\alpha}{\chi c}\right)^{\frac{1-\alpha}{\sigma+\alpha}}}{1/\beta + \delta - 1}\right)^{\frac{\sigma+\alpha}{\sigma(1-\alpha)}} \tag{77}$$

$$a_2 = \frac{\tau^k}{(S^d)^{\gamma^k}} \tag{78}$$

$$a_3 = \frac{\tau^L}{(S^d)^{\gamma^L}} \tag{79}$$

Substitute it into the household first-order condition for labor,

$$L_t = a_5 \left(1 - a_2 \left(\frac{d_t}{y_t}\right)^{\gamma^k}\right)^{\frac{\alpha}{\sigma(1-\alpha)}} \left(1 - a_3 \left(\frac{d_t}{y_t}\right)^{\gamma^L}\right)^{\frac{1}{\sigma}} \tag{80}$$

where

$$a_5 = \left(\frac{\alpha}{1/\beta + \delta - 1}\right)^{\frac{\alpha}{\sigma+\alpha}} \left(\frac{1-\alpha}{\chi c}\right)^{\frac{1}{\sigma}} \tag{81}$$

Therefore

$$y_t = a_6 \left(1 - a_2 \left(\frac{d_t}{y_t}\right)^{\gamma^k}\right)^{\frac{\alpha(1+\sigma)}{\sigma(1-\alpha)}} \left(1 - a_3 \left(\frac{d_t}{y_t}\right)^{\gamma^L}\right)^{\frac{1}{\sigma}} \tag{82}$$

where $a_6 = a_4^{\alpha} a_5^{1-\alpha}$.

The government budget constraint can be rewritten as

$$\frac{d_t}{y_t} = \frac{1}{\beta}\frac{d_{t-1}}{y_{t-1}}\frac{y_{t-1}}{y_t} + \frac{g_t}{y_t} - (1-\alpha)\,\tau_t^L - \alpha\tau_t^k \tag{83}$$

Define $S_t = \frac{d_t}{y_t}$. Substitute out y_t, y_{t-1}, τ_t^L and τ_t^k. The above equation shows how the path of S_t relates to exogenous shock g_t. It can be log-linearized around the steady state.

$$\widehat{s_t} + \alpha_1 \widehat{s_{t-1}} + \alpha_2 \widehat{s_{t-2}} = \frac{g}{d}\widehat{g_t} \tag{84}$$

where

$$\alpha_1 = -\alpha_2 \left(\frac{1}{\beta} + \frac{g}{d}\right) - \frac{1}{\beta} + (1-\alpha)\,\frac{a_3}{S}\gamma^L S^{\gamma^L} + \frac{a_2\alpha}{S}\gamma^k S^{\gamma^k} \tag{85}$$

$$\alpha_2 = \frac{1}{\beta}\left(\frac{\alpha}{1-\alpha}\frac{1+\sigma}{\sigma}\frac{a_2\gamma^k S^{\gamma^k}}{1-a_2 S^{\gamma^k}} + \frac{1}{\sigma}\frac{a_3\gamma^L S^{\gamma^L}}{1-a_3 S^{\gamma^L}}\right) \tag{86}$$

Follow the same strategy, S_t can be solved backward. The necessary condition for a unique equilibrium are

- $\alpha_1^2 - 4\alpha_2 > 0$
- $\alpha_1 < 0$ and $\frac{-\alpha_1+\sqrt{\alpha_1^2-4\alpha_2}}{2} < 1$; or $\alpha_1 > 0$ and $\frac{-\alpha_1-\sqrt{\alpha_1^2-4\alpha_2}}{2} > -1$

Or equivalent to

- $\frac{1}{\beta} > \lambda_1\gamma^L + \lambda_2\gamma^k > \frac{1}{\beta} - 2$;
- $\lambda_3\gamma^L + \lambda_4\gamma^k > \frac{1}{\beta} - 1$

where

$$\lambda_1 = \frac{T_L}{d} + \left(\frac{T}{d} + 1\right)\left(\epsilon_L^R - 1\right) \tag{87}$$

$$\lambda_2 = \frac{T_k}{d} + \left(\frac{T}{d} + 1\right)\left(\epsilon_k^R - 1\right) \tag{88}$$

$$\lambda_3 = \frac{T_L}{d} + \left(\frac{T}{d} + 1 - \frac{1}{\beta}\right)\left(\epsilon_L^R - 1\right) \tag{89}$$

$$\lambda_4 = \frac{T_k}{d} + \left(\frac{T}{d} + 1 - \frac{1}{\beta}\right)\left(\epsilon_k^R - 1\right) \tag{90}$$

Note the total tax revenue T is the sum of revenue from labor tax T_L and from capital tax T_k. $\epsilon_L^R = \frac{\widehat{T_L}}{\widehat{\tau_L}}$ is the elasticity of labor tax revenue, while $\epsilon_k^R = \frac{\widehat{T_k}}{\widehat{\tau_k}}$ is the elasticity of capital tax revenue.

In another word, if $\lambda_1\gamma^L + \lambda_2\gamma^k$ is larger than the $1/\beta$, there exists fluctuations.

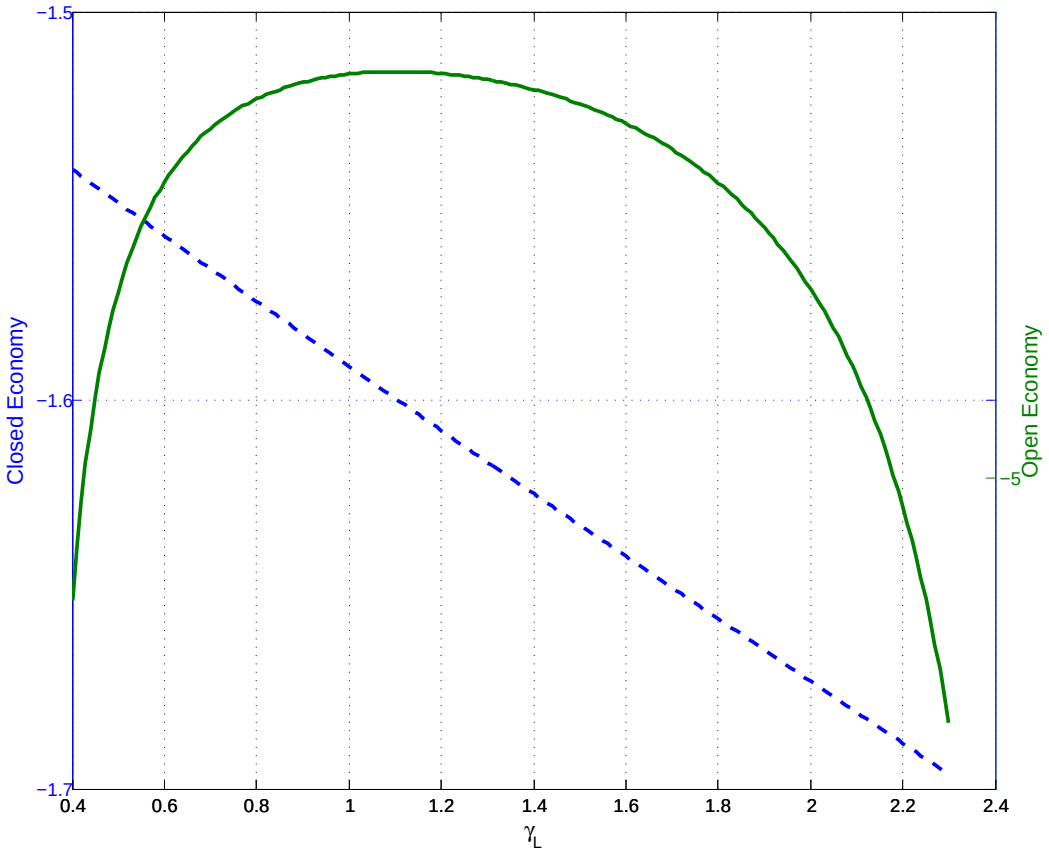

Figure 1: Welfare Comparison $\lambda * 100$ in the model without capital: closed economy vs. open economy (government debt-GDP ratio is 0.5)

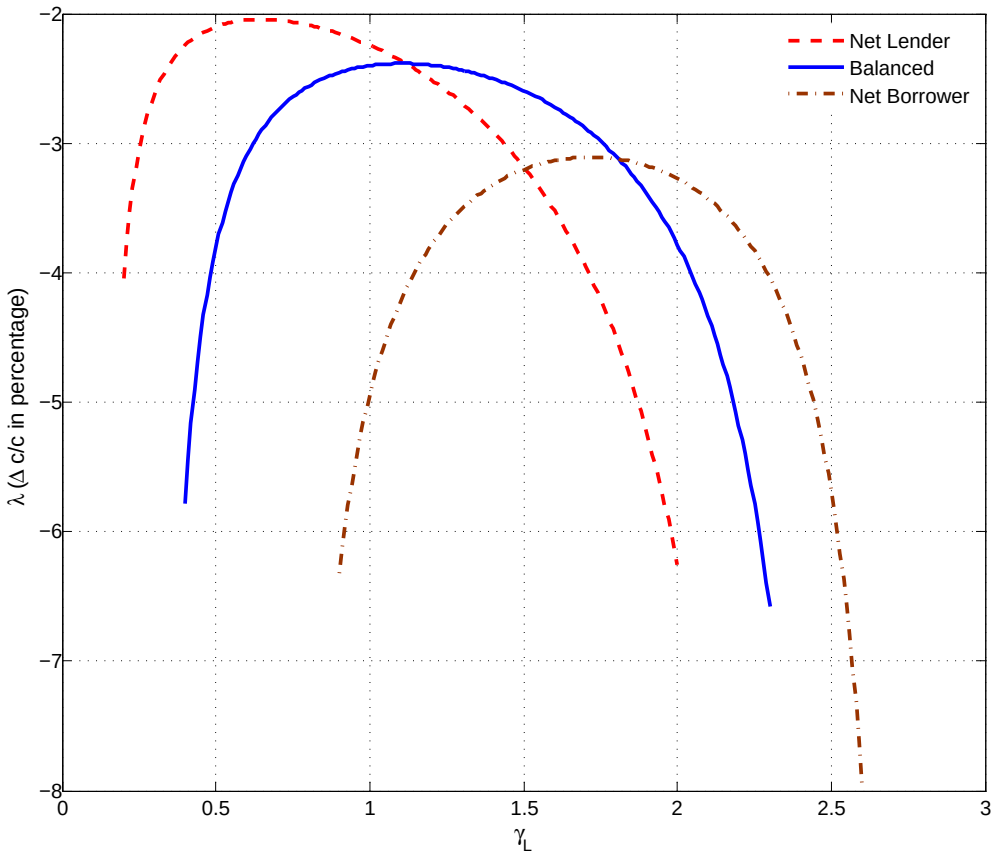

Figure 2: Welfare Comparison $\lambda * 100$ in the model without capital under i.i.d shock: net lender (government debt-GDP ratio is 0.3) vs. balanced (government debt-GDP ratio is 0.5) vs. net borrower (government debt-GDP ratio is 0.8)

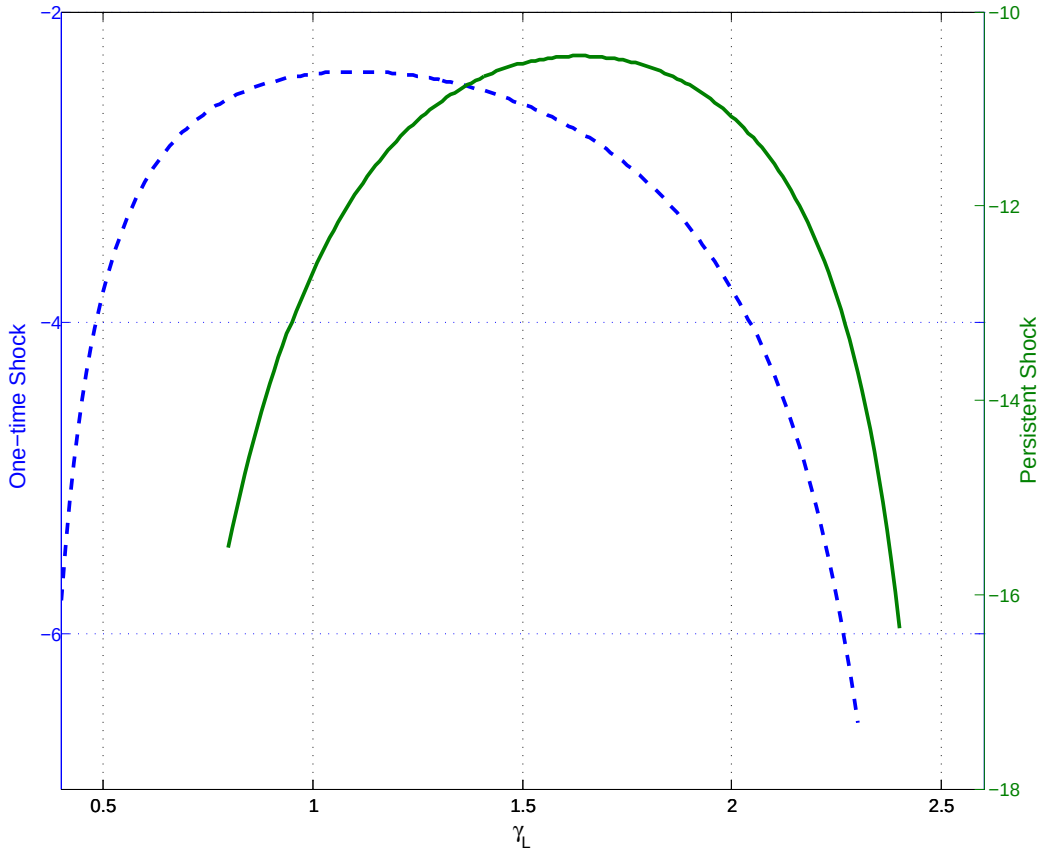

Figure 3: Welfare Comparison $\lambda * 100$ in the model without capital under i.i.d shock: closed economy vs. open economy (government debt-GDP ratio is 0.5)

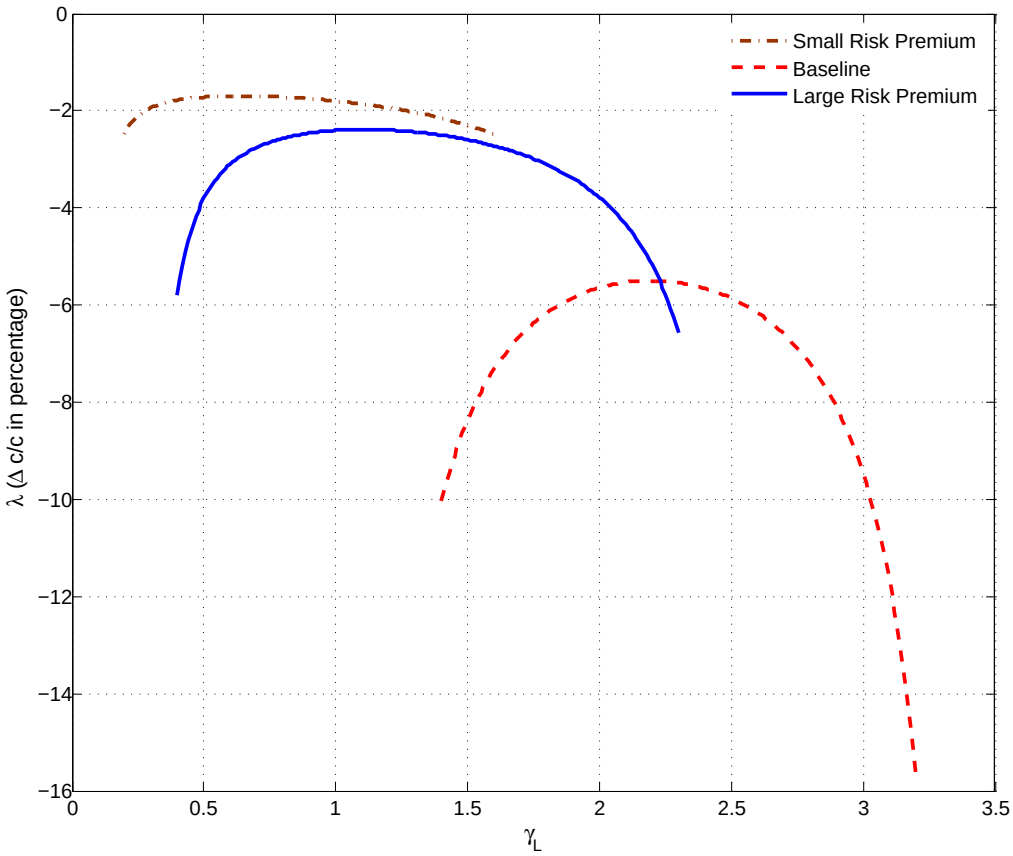

Figure 4: Welfare Comparison $\lambda * 100$ in the model without capital under i.i.d shock: closed economy vs. open economy (government debt-GDP ratio is 0.5)

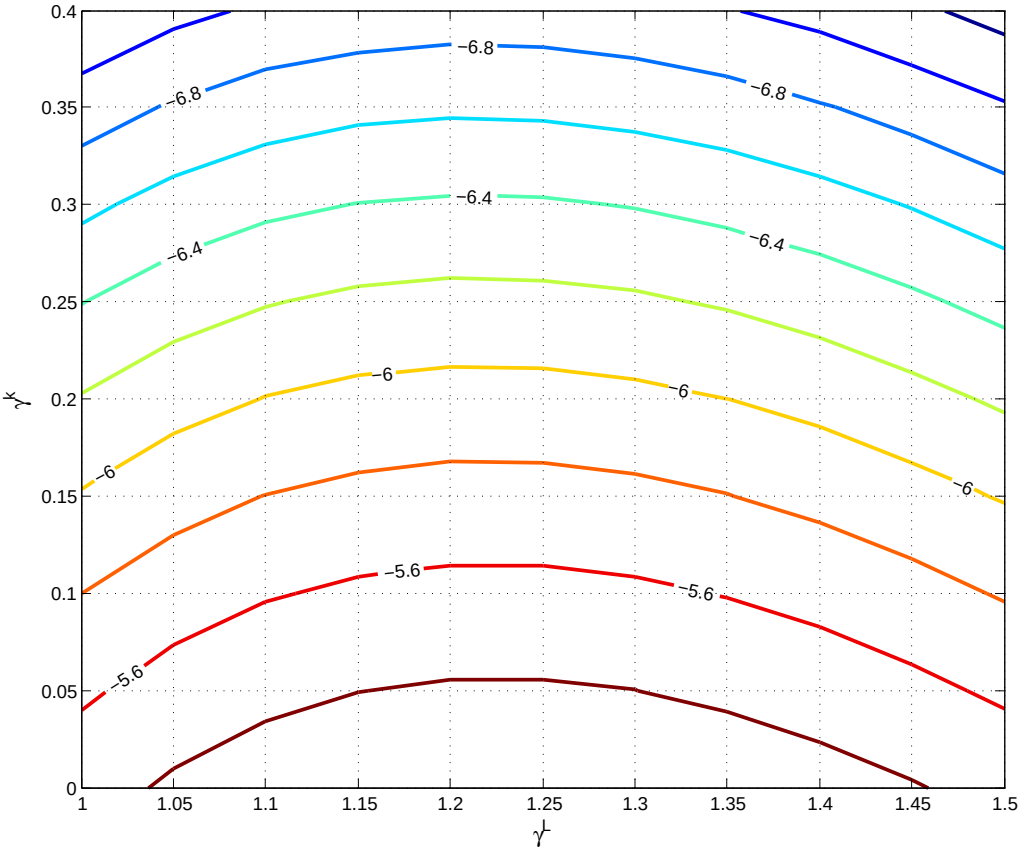

Figure 5: Welfare Comparison $\lambda * 100$ in the model with capital under i.i.d shock: government debt-GDP ratio is 0.5

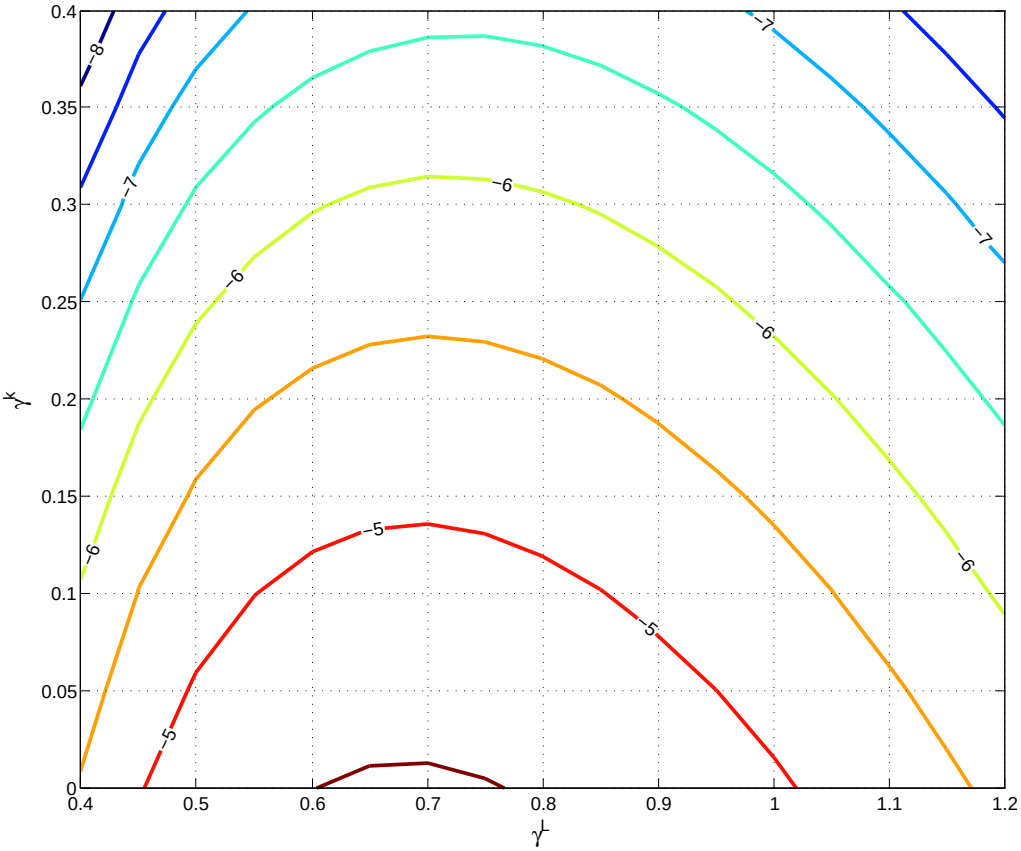

Figure 6: Welfare Comparison $\lambda * 100$ in the model with capital under i.i.d shock: government debt-GDP ratio is 0.3

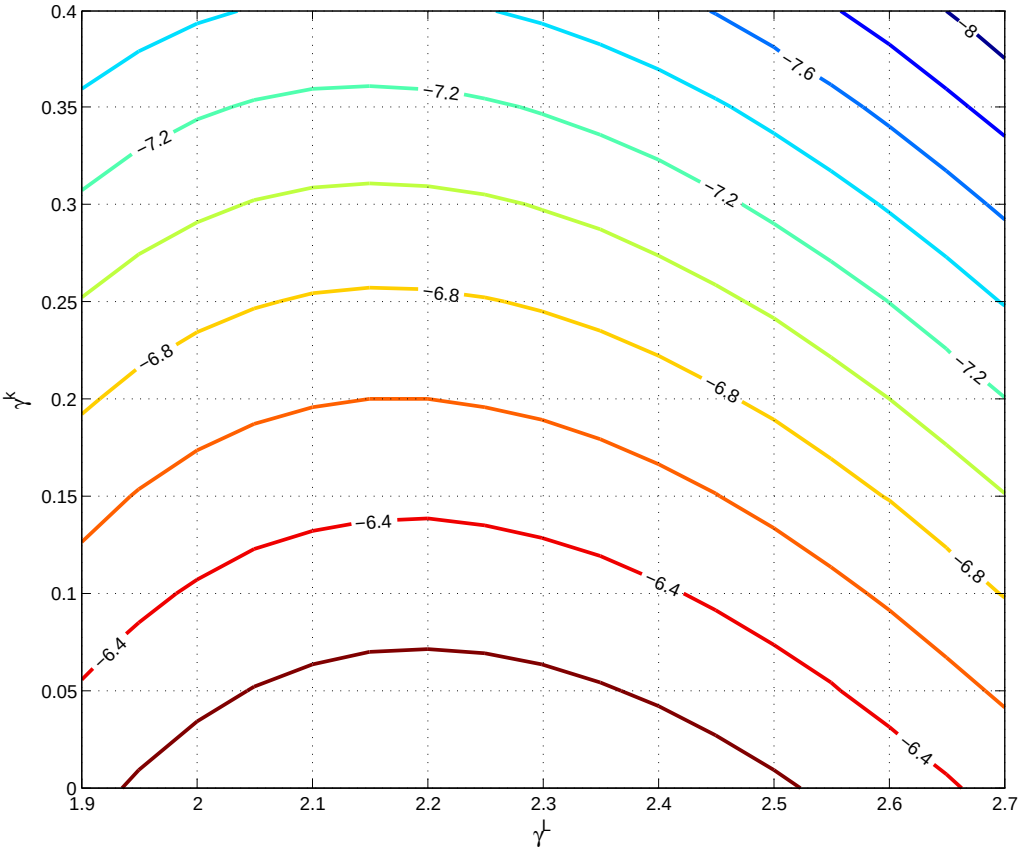

Figure 7: Welfare Comparison $\lambda * 100$ in the model with capital under i.i.d shock: government debt-GDP ratio is 0.8

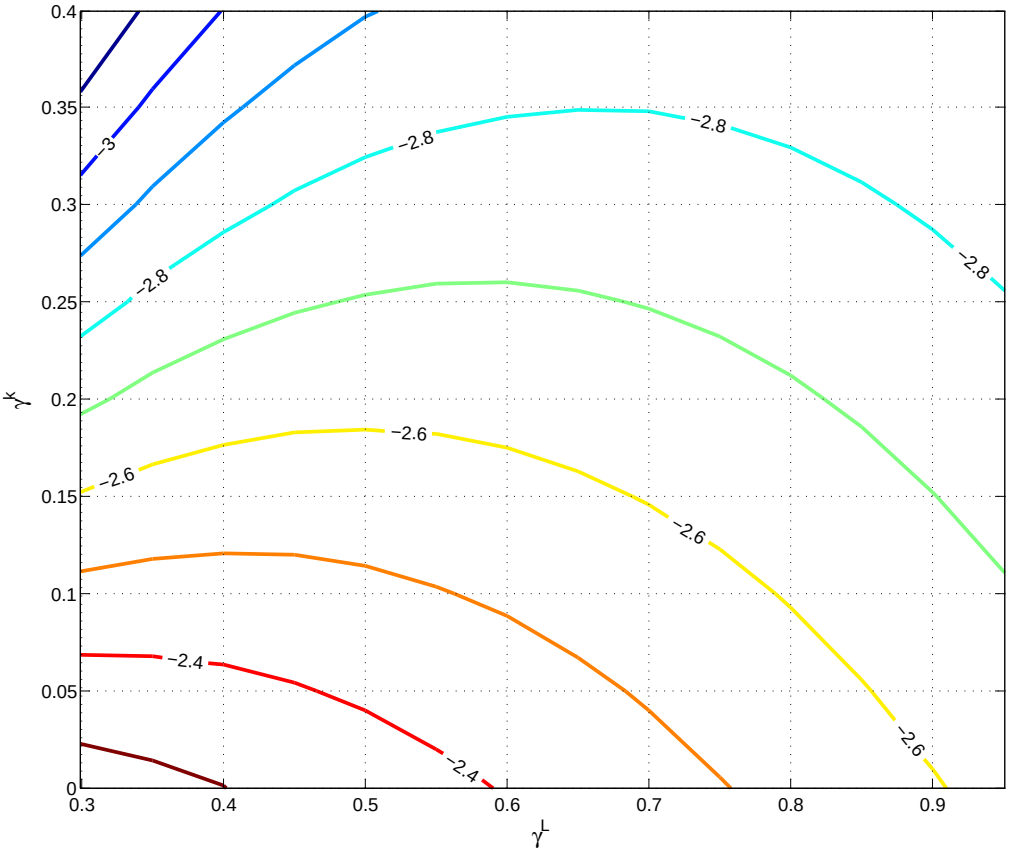

Figure 8: Welfare Comparison $\lambda * 100$ in the model with capital under i.i.d shock: closed economy