Learning, Structural Breaks, and Asset-Return Dynamics

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1 Introduction

This paper studies a representative-agent asset-pricing model of an endowment economy in which the agent has incomplete knowledge about exogenous stochastic endowment process and has incentive to learn about the process with adaptive learning rules. There is the well documented fact that when underlying economic environment is known and is common knowledge to investors, asset-pricing models under rational expectations with complete knowledge about model structure cannot account for such basic features of the data in the U.S. asset market as relative volatility between equity premium and risk-free rate, highly persistent excess return, and long-horizon predictability of asset returns. Although much effort to understand those aspects of asset-return dynamics have been done by relaxing underlying assumptions in Mehra and Prescott (1985) economy, for example more sophisticated forms of preference (Epstein and Zin, 1990, 1991; Weil, 1989; Kandel and Stambaugh, 1991; Abel, 1990; Constantinides, 1990; Heaton, 1995; Campbell and Cochrane, 1999), trading costs (Aiyagari and Gertler, 1991), and incomplete asset market (Weil, 1992), empirical performance has not been very successful.[1] Recently, one growing line of research takes an alternative approach that allows for departures from the assumption of common knowledge and fully rational agents. For example, Cecchetti, Lam, and Mark (2000) examine a representative agent who has distorted beliefs about transition probabilities in a two-state

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[1] For survey paper see Kocherlakota (1996).

Markov endowment process and show their model can generate the first and second moments of asset returns, persistence and predictability of equity premium that match the data. However, in their distorted beliefs model, the investor is not allowed to learn true economic environment over time. There have been extensive works that study implications of asset-pricing models when investors are capable of learning unknown economic structure. Timmermann (1996) and Brennan and Xia (2001) consider agents who have limited information about underlying exogenous stochastic dividend process learn parameter values of the process in the context of a present value model and a general equilibrium model, respectively.[2] Many alternative learning rules have been also investigated. Instead of learning an exogenous stochastic endowment process, Bullard and Duffy (2001) and Carceles-Poveda and Giannitsarou (2006) study the effect of self-referential learning model where agents fit a model to prices instead of a univariate endowment process.

In this paper, we consider some important alternative empirical specifications of the endowment process and investors' knowledge about the process so that we can investigate which parts of asset-return dynamics incomplete knowledge about the economic environment and learning can and cannot explain. First, we study whether specifications of endowment process play important role in explaining asset-return dynamics. We compare model-implications under the learning when the endowment process follows a two-state Markov switching process to those generated under the learning with a stationary autoregressive process which has been a standard model for the endowment process. Second, we ask whether the process of per capita consumption growth is stable in our sample period, 1890-2004. Investors, who set asset prices that become our data, do not know unanticipated structural changes until they observe them. We model unannounced structural changes in the endowment process so that investors who learn the structure of the economy contend with them. There have been many studies adopting constant gain learning rules, for example Orphanides and Williams (2005) and Branch and Evans (2006), among others, when investors are concerned about structural changes. In this paper, however, we do not employ constant learning rules since per capita consumption growth which is the endowment process in the asset-pricing model is usually believed to be quite stable even over long periods and

[2]Brock and Hommes (1998) also study the dynamics in a present value model with heterogenous beliefs.

thus frequent regime shifts with constant learning rules is unlikely to help understanding asset-return dynamics.[3] Therefore, instead of frequent changes in the process, we study a special case where agents learn the endowment process with least squares learning together with the possibility of unexpected infrequent large shocks to endowment process. Once investors know that there has been a structural break, they discard past observations which are no longer useful and start learning new endowment process with accumulated data points after the break. By doing so, we restrict our attention to least squares learning to focus on the effect of a structural change on matching statistics found in the data.

The remainder of the paper is organized as follows. The next section reports stylized facts about the excess return and the risk-free rate, and their dynamic relations that we seek to explain. Section 3 introduces a representative agent endowment economy and specifications of the exogenous stochastic endowment process. The asset return solutions under rational expectations and under least squares learning are presented in Section 4. A possible regime shift in the endowment process is also discussed. In Section 5, we compare the predictions of the model under adaptive learning to those generated under rational expectations in terms of its ability to account for the stylized facts in the U.S. asset market. Concluding remarks are contained in Section 6.

2 Stylized Facts

The list of features of the data in the U.S. asset market is presented in Table 1 through Table 2. The data are annual observations of equity and short-term returns, dividend, and per capita consumption used in Shiller (2003) and are obtained from his web site.

The first column of Table 1 reports our estimates of a mean level of the equity premium 5.4 percent, and the mean level of the risk-free rate somewhat below 3 percent. Estimates for the volatility of the equity premium and the risk-free rate, and the correlation between these returns are also presented. The sample standard deviations of the equity premium, which is over 18 percent per year, is excessively larger than the sample standard deviation of the risk-free rate, which is only 7 percent per year. We also report variance ratio statistics

[3] Carceles-Poveda and Giannitsarou (2006) show constant gain learning produces more volatile asset returns than recursive least squares learning, but constant gain rules do not contribute much to understand asset-return dynamics.

that provide a measure of the serial correlation properties of the data.[4] The variance ratio statistics for equity premium presented in panel B of Table 1 are greater than 1 over short horizon, but less than 1 and fall with the return horizon over long horizon. This suggests that the excess equity returns are negatively serially correlated over long horizon and thus exhibit mean reverting behavior because a positive change today is expected to be reversed in the future.

Finally, we present the predictability of the excess returns in Table 2. This can be examined by running regressions of k-period-ahead return on the log of dividend-price ratio as described in Campbell and Shiller (1988) and Fama and French (1988) or on the log of consumption-price ratio. Since high prices must be associated high expected future dividends, low expected future returns, or some combination of the two, the dividend-price ratio which is the most popular forecasting variable especially for long-horizon returns is positively associated with future returns. Panel A of Table 2 shows a typical pattern that slope coefficient, t-statistic and $\overline{R}^2$ increase with the return horizon.

3 The Economy

We begin with the Lucas (1978) endowment economy where a representative agent chooses consumption at each point in time so as to maximizes her lifetime utility. The instantaneous utility function which gives the agent's utility at a given period takes the form of constant-relative-risk-aversion (CRRA) utility defined over time-t consumption, $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, where $\gamma > 0$ is the coefficient of relative risk aversion. Let P be the price of the equity and assume that endowment is consumed in equilibrium, then the *Euler* equation is given by

$$P_t = \beta\ \tilde{\mathrm{E}}_t[(C_{t+1}/C_t)^{-\gamma}(P_{t+1} + C_{t+1})], \tag{1}$$

[4] Variance ratio statistics are widely used to measure the relative size of random walk component in a time-series. The variance ratio statistic for a time-series, s_t at horizon k is the variance of the k-period change of the variable divided by k times the one-period change, $\mathrm{VR}(k) = \frac{\mathrm{Var}(s_t - s_{t-k})}{k \cdot \mathrm{Var}(\Delta s_t)} = \frac{\mathrm{Var}(\Delta s_t + \cdots + \Delta s_{t-k+1})}{k \cdot \mathrm{Var}(\Delta s_t)}$. It is also represented by a linear combination of the first $j-1$ autocorrelation coefficients of $\{\Delta s_t\}$ with linearly declining weights, $\mathrm{VR}(k) = 1 + 2\sum_{j=1}^{k-1}(1 - \frac{j}{k})\rho(j)$, where $\rho(j)$ is the jth order autocorrelation coefficient of $\{\Delta s_t\}$. $\mathrm{VR}(k)$ exceeds one if Δs_t is positively serially correlated, and thus variances grow faster than linearly. Similarly, if Δs_t is negatively serially correlated, $\mathrm{VR}(k)$ is less than one and variances grow slower than linearly. Under the null hypothesis of random walk, the population value of variance ratio statistic $\mathrm{VR}(k)$ is one for all k since $\rho(j) = 0$ for all $j \geq 1$.

where β is the subjective discount factor and $\tilde{\mathrm{E}}_t$ is the agent's subjective expectation conditioned on available information at time t. As we discussed earlier, in our benchmark case, rational expectations, this subjective expectations formed at time t is the mathematical expectation conditional on observables at time t. That is, $\tilde{\mathrm{E}}_t = \mathrm{E}_t$. If the agent is assumed to have incomplete knowledge about the economic structure, the expectation operator takes an alternative form which is in general nonrational expectations. Let ω be the price-consumption ratio, then equation (1) becomes the stochastic difference equation,

$$\omega_t = \beta \; \tilde{\mathrm{E}}_t[e^{(1-\gamma)\xi_{t+1}}(\omega_{t+1} + 1)], \tag{2}$$

where ξ is the first difference of log of per capita consumption.

Equation (2), specification of expectations, together with the specification of functional form of the consumption growth rate completely determine solution and properties of the economy. Here we consider the standard practice of modelling empirical specification of ξ_t.

3.1 Case I: Markov Switching Process

We follow Cecchetti, Lam, and Mark (1990, 2000) who assume that consumption growth follows a Markov switching process with two-point Markov state variables

$$\xi_t = \alpha(S_t) + \varepsilon_t, \tag{3}$$

where ε is independently and identically distributed as $N(0, \sigma_\varepsilon^2)$. The process governing the state vector S is a two-point Markov chain with stationary probabilities. Each element of the state vector is allowed to be in either the good state of high-consumption growth ($S = 1$) or the bad state of low-consumption growth ($S = 0$). There exist four transition probabilities

$$\begin{aligned} &\mathrm{P}(S_{t+1} = 1 \mid S_t = 1) = p, \quad \mathrm{P}(S_{t+1} = 0 \mid S_t = 1) = 1 - p, \\ &\mathrm{P}(S_{t+1} = 0 \mid S_t = 0) = q, \quad \mathrm{P}(S_{t+1} = 1 \mid S_t = 0) = 1 - q. \end{aligned} \tag{4}$$

We estimate the transition probabilities and state-contingent mean of consumption growth. Maximum-likelihood estimates of this endowment process using per capita consumption growth rate as a percentage from 1890 to 2004 are reported Table 3. These estimates are not significantly different from the estimation results in Cecchetti, Lam, and

Mark (2000) who use the data from 1890 to 1994, but we find that the bad states are slightly less persistent and mean per capita consumption growth in the bad state is a bit lower as we extend the data.

3.2 Case II: Stationary AR(1) Process

Most attempting specification one may consider in this environment is a standard linear time-series model with Gaussian innovations. However, this specification may not be plausible due to its inability to account for higher moments properties of the data. Furthermore, if an autoregressive model does not contain substantial persistence, we do not expect very slow learning, so it is unlikely to help in understanding asset-returns dynamics. However, since the endowment process is subject to unanticipated changes in model structure that prevent the representative agent from learning the true economic structure, it is worth examining this specification in the presence of regime shifts. Therefore, we also adopt a conventional autoregressive model with normal error term.

We assume that ξ evolves according to the following AR(1) process:

$$\xi_t = \alpha + \rho\, \xi_{t-1} + \upsilon_t, \tag{5}$$

where $\rho < 1$ and υ is independently and identically distributed as $N(0, \sigma_\upsilon^2)$. Table 4 reports the OLS estimates of this process.

4 Asset Returns and Expectations

4.1 Asset Returns under Rational Expectations

In a rational expectations model, the agent is assumed to have complete knowledge about the fundamental process, the functional form of the stochastic process as well as parameter values. That is, the subjective expectations of the investor $\tilde{\mathrm{E}}_t$ coincide with the mathematical expectations E_t, taken with respect to the truth. Since model solutions depend on the specification of functional form of the consumption growth rate given the agent's expectations, we present asset returns under rational expectations for each case of exogenous endowment process.

4.1.1 Case I: Markov Switching Process

We now use the stochastic difference equation for the price-consumption ratio

$$\omega(S_t) = \beta \mathrm{E}([1 + \omega(S_{t+1})] \cdot e^{(1-\gamma)(\alpha(S_{t+1})+\varepsilon_{t+1})} \mid S_t). \tag{6}$$

to solve for equilibrium asset returns.[5] Since a random variable X that follows conditionally lognormal distribution implies $\ln \mathrm{E}_t(X) = \mathrm{E}_t(\ln X) + \frac{1}{2}\mathrm{Var}_t(\ln X)$, the difference equation for state-contingent price-consumption ratio is now given by

$$\omega(S_t) = \beta e^{(1-\gamma)^2(\sigma^2/2)} \mathrm{E}(e^{(1-\gamma)\alpha(S_{t+1})}[1 + \omega(S_{t+1})] \mid S_t). \tag{7}$$

Since each element of the state vector is allowed to be in either the good state of high-consumption growth ($S = 1$) or the bad state of low-consumption growth ($S = 0$), equation (7) can be written as a system of two linear equations in $\omega(0)$ and $\omega(1)$. Let $\hat{\beta}(S)$ be $\beta e^{(1-\gamma)^2(\sigma^2/2)+(1-\gamma)\alpha(S)}$ and solving the system of equations yields

$$\omega(0) = [q\hat{\beta}(0) + (1-q)\hat{\beta}(1) - (p+q-1)\hat{\beta}(0)\hat{\beta}(1)]/\Lambda, \tag{8}$$

$$\omega(1) = [p\hat{\beta}(1) + (1-p)\hat{\beta}(0) - (p+q-1)\hat{\beta}(0)\hat{\beta}(1)]/\Lambda \tag{9}$$

where $\Lambda = 1 - q\hat{\beta}(0) - p\hat{\beta}(1) + (1-p-q)\hat{\beta}(0)\hat{\beta}(1)$.

We use the state-contingent price-consumption ratios to derive the solution for state contingent one-period asset returns. First, state-contingent gross equity return R^e is

$$R^e(S_{t+1}, S_t) = \frac{\omega(S_{t+1}) + 1}{\omega(S_t)} \cdot \frac{C_t}{C_{t+1}} = \frac{\omega(S_{t+1}) + 1}{\omega(S_t)} \cdot e^{\alpha(S_{t+1})+\varepsilon_{t+1}}. \tag{10}$$

Next, state-contingent gross risk-free rate R^f is given by,

$$R^f(S_t) = \frac{1}{P^f(S_t)}, \tag{11}$$

where P^f is the price of a one-period risk-free asset which equals the expected intertemporal marginal rate of substitution (IMRS), $P^f(S_t) = \beta e^{\gamma^2(\sigma^2/2)} \mathrm{E}(e^{-\gamma\alpha(S_{t+1})} \mid S_t)$.

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[5] Note that $\tilde{\mathrm{E}}_t$ is now replaced by E_t.

4.1.2 Case II: Stationary AR(1) Process

When the endowment process is assumed to be a stationary AR(1) model, the stochastic difference equation for the price-consumption ratio is given by

$$\omega_t = \beta \ \mathrm{E}_t[e^{(1-\gamma)\xi_{t+1}}(\omega_{t+1} + 1)]. \tag{12}$$

Solving forward equation (12) yields

$$\omega_t = \sum_{j=1}^{\infty} \beta^j D_{t+j}, \tag{13}$$

and the series of D_{t+j} can be obtained from recursive formula with $D_{t+1} = e^{\theta+(1-\gamma)(\alpha+\rho\xi_t)}$,

$$D_{t+j+1} = D_{t+j} \ e^{\theta(\delta^j)^2+(1-\gamma)\alpha\delta^j+(1-\gamma)\rho^{j+1}\xi_t}, \tag{14}$$

where $\theta = (1-\gamma)^2(\sigma^2/2)$ and $\delta^j = \frac{1-\rho^{j+1}}{1-\rho}$. Next, gross equity return R^e is given by

$$R^e_{t+1} = \frac{\omega_{t+1}+1}{\omega_t} \cdot e^{\xi_{t+1}} = \frac{\omega_{t+1}+1}{\omega_t} \cdot e^{\alpha+\rho\xi_t+\upsilon_{t+1}} \tag{15}$$

and gross risk-free rate R^f is,

$$R^f_t = \frac{1}{P^f_t} = \frac{1}{\beta \cdot e^{\gamma^2(\sigma^2/2)-\gamma(\alpha+\rho\xi_t)}}, \tag{16}$$

where P^f is the price of a one-period risk-free asset implied by the model.

4.2 Asset Returns under Adaptive Learning

We have considered an important benchmark case, rational expectations, in which investors use all available information optimally to forecast the future values of price-consumption ratio. Next, we study a more plausible case where the investors initially face some limitations on knowledge about the underlying endowment process but they learn the true process over time using available information optimally at each period in time. A sensible strategy would be that investors act like applied econometricians when they forecast. Specifically, in our learning environment agents are assumed to believe that growth rate of consumption follows a stationary AR(1) process so that we can focus on the ability of least squares learning in explaining asset-return dynamics. On the other hand, the truth of the endowment process is either a Markov switching process or an AR(1) process. For case I, since agents' belief about the functional form of endowment process is assumed to be an AR(1) process while the truth is that consumption growth rate, ξ_t, follows a Markov switching process, they try to estimate and update parameter values over time as if they live in a linear environment.[6] Similarly, for case II, agents know that ξ follows a stationary AR(1) process, but not the parameter values, $\boldsymbol{\theta} = (\alpha \quad \rho)'$ in equation (5). In both cases, agents try to infer from new observations the parameter values and periodically update their estimates by employing least squares learning.

When investors know that true functional form of the growth rate of consumption is an AR(1) process and assess parameter values by least squares learning, they forecast future values of price-consumption ratio as if the economy were in an rational expectations equilibrium, except that they use time-t estimates of parameters which might be different from true values. Therefore, the functional form of time-t asset-return solutions under least-square learning is the same as that of the rational expectations case as shown in equations (15) and (16), but here parameter values are replaced with time-t least square estimates which evolve over time. On the other hand, for the Markov switching process case, since agents take the AR(1) process as the true endowment process and try to estimate parameter values $\boldsymbol{\theta} = (\alpha \quad \rho)'$ each period in time by employing recursive least squares, asset returns are set according to the functional form of equations (15) and (16) which is identical to the previous case. However, now agents use realizations of consumption growth generated by a Markov switching process, not by an AR(1) process, and thus parameter estimates that they assess at each period are not the same as those in the AR(1) case.

We follow Timmermann (1996) who suggests a recursive estimator when agents know the form of the endowment process but not the the value of parameters. Let $\boldsymbol{z}_t = (1 \quad \xi_{t-1})'$, $\boldsymbol{\xi}_t = (\xi_t \quad \xi_{t-1} \quad \cdots \quad \xi_1)'$, and $\mathbf{Z}_t = (\boldsymbol{\xi}_t \quad \boldsymbol{\xi}_{t-1} \quad \cdots \quad \boldsymbol{\xi}_1)'$, then the recursive estimator is given by

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \Theta(\xi_t - \boldsymbol{z}_t'\boldsymbol{\theta}_{t-1}), \tag{17}$$

$$\boldsymbol{S}_t = \boldsymbol{S}_{t-1} - \frac{\boldsymbol{S}_{t-1}\boldsymbol{z}_t\boldsymbol{z}_t'\boldsymbol{S}_{t-1}}{1 + \boldsymbol{z}_t'\boldsymbol{S}_{t-1}\boldsymbol{z}_t}, \tag{18}$$

[6] There may be a situation where investors do know the functional form of the endowment process is is a Markov switching process, but they do not have statistical device that estimate parameter values optimally because of the lack of computational ability. This is not this paper tries to look at. One may consider a non-optimal or a method that one can simply adopt such as the event-counting method to infer transition probabilities.

where $\Theta = \frac{\boldsymbol{S}_{t-1}\boldsymbol{z}_t}{1+\boldsymbol{z}_t'\boldsymbol{S}_{t-1}\boldsymbol{z}_t}$ and $\boldsymbol{S}_t = (\mathbf{Z}_t'\mathbf{Z}_t)^{-1}$.[7] The formula (17) shows that the updated estimator $\boldsymbol{\theta}_t$ is equal to the estimator used in the previous learning period $\boldsymbol{\theta}_{t-1}$ plus an adjustment factor which is proportional to the forecast error or innovation under least squares learning. Let $\boldsymbol{\theta}_t^A = (\alpha_t^A \quad \rho_t^A)'$ and $\boldsymbol{\theta}_t^M = (\alpha_t^M \quad \rho_t^M)'$ denote time-t estimates of parameter vector under this present-value learning when the data generating process is the AR(1) process and the Markov switching process, respectively.

Gross equity return implied by the model under least squares learning when the true endowment process is assumed to be the AR(1) process, $R^{e,A}$ is given by

$$R_{t+1}^{e,A} = \frac{\omega_{t+1}^A + 1}{\omega_t^A} \cdot e^{\xi_{t+1}^A}, \tag{19}$$

where ξ_t^A is time-t consumption growth rate which is exogenously generated by the AR(1) process and ω_t^A is the price-consumption ratio under least squares learning at time t when agents use $\boldsymbol{\theta}_t^A$ to determine price and consumption as described in equation (13) and (14). Similarly, let ξ_t^M denote time-t consumption growth rate generated by the Markov switching process and ω_t^A denote time-t price-consumption ratio under least squares learning with $\boldsymbol{\theta}_t^M$, then the implied gross equity return under the learning with the Markov switching endowment process is defined as

$$R_{t+1}^{e,M} = \frac{\omega_{t+1}^M + 1}{\omega_t^M} \cdot e^{\xi_{t+1}^M}. \tag{20}$$

Next, since model-implied gross risk-free rates under least squares learning also depends on the exogenous process of consumption growth, the model solution for risk-free rates takes the same form as (16). For the AR(1) process, the price of one-period risk-free asset at time t, $P_t^{f,A}$ is given by

$$P_t^{f,A} = \beta \cdot e^{\gamma^2((\sigma^A)^2/2) - \gamma(\alpha_t^A + \rho_t^A \xi_t^A)}, \tag{21}$$

where σ^A is the standard deviation of innovations of consumption growth rate for this case.[8] Time-t price of one-period risk-free asset under the learning with Markov switching

[7] Note that we use a general notation of recursive estimator when investors estimate the endowment process by recursive least squares. For the AR(1) case, $\boldsymbol{z}_t^A = (1 \quad \xi_{t-1}^A)'$, $\boldsymbol{\xi}_t^A = (\xi_t^A \quad \xi_{t-1}^A \quad \cdots \quad \xi_1^A)'$, and $\mathbf{Z}_t^A = (\boldsymbol{\xi}_t^A \quad \boldsymbol{\xi}_{t-1}^A \quad \cdots \quad \boldsymbol{\xi} A_1)'$. Analogous destinations hold for the Markov switching case.

[8] Note that agents are assumed to know true distribution of unobservable innovations of ξ, so they use standard deviation of unobservable component of consumption growth rate when they set asset prices.

endowment process, $P_t^{f,M}$, which is the inverse of gross risk risk-free rate is also defined in the same way.

4.3 Structural Changes in the Endowment Process

When investors set asset prices, their forecasts are based on the assumption of a stable process of endowment. However, this assumption becomes less plausible in the presence of structural changes in the process. The exogenous endowment process, like many other economic variables, are always subject to change for unexpected large shocks to the economy such as the Great Depressions, wars, and oil shocks. Therefore, the agents must contend with unannounced structural changes.

In order to model possible regime shifts, we investigate whether there has been a structural break in the endowment process using multiple structural changes model by Bai and Perron (2003). Of interest is the presence of structural changes in the mean of the consumption growth rate for the Markov switching process and constant (α) and ρ which is interpreted as measuring the persistence of the rate of consumption growth for a stationary AR(1) process as described in equation (5).[9] Both $UD\ max$ test and $WD\ max$ test indicate the presence of at least one break. Since overall performance of the sequential procedure is known to be better than information criteria, we used the sequential application of the $\sup F_T(l+1|l)$ test using the sequential estimates of the breaks and found one break estimate. The break date is estimated at 1933 when constant is subject to change and at 1929 when both constant and persistence parameter are used as regressors and both estimated dates are associated with the Great Depression.[10] Figures 1 and 2 plot actual per capita consumption growth rate and fitted values and we can clearly see that post-break consumption growth rates display higher mean value, less volatile, and more persistence than the rates of consumption growth before the Great Depression.

Tables 3 and 4 report parameter estimates of the endowment process for both one-regime and two-regime case. First, for the Markov switching process, we estimate the transition

[9] We allowed up to 5 breaks and used a trimming of $\varepsilon = 0.10$ so that each segment has at least 11 observations.

[10] We also applied the Quandt test for detecting a single unknown break point and the break date is estimated at 1930 which is also around the Great Depression for both cases. Note that our simulation results are not sensitive to the choice of a break date.

probabilities and state-contingent mean of consumption growth. Maximum-likelihood estimates of the endowment process for each regime suggest that state-contingent mean of consumption growth is relatively high in regime 2 and both the good and the bad states in regime 1 are much less persistent than regime 2. Next, Table 4 presents OLS estimates of the AR(1) process. We found that there is a change in the sign of the persistence parameter and the growth rate of consumption in regime 2 is more persistent than regime one, which ties in with the estimates of transition probabilities of the Markov switching process. Finally, for both endowment processes, that regime 1 has higher volatility of innovations in the fundamentals than regime 2 suggests the economic environment has been more stable.

Now we consider a very simple case that agents are assumed to know whether a break had in fact occurred right after the break. Once the agents know that there has been a break, they face uncertainty about new process unless they can instantly observe new functional form of the process and parameter values. We assume that agents discard past observations which are no longer useful and set asset prices using observations after the break. Specifically, before the break occurs, agents set prices according to their forecast rules and information availability as if they were in one-regime world. When they realized that there has been a regime shift, under rational expectations, they replace old parameters with new ones. On the other hand, if they were not able to obtain new parameters, which appears to be more realistic situation, they have to learn the new endowment process using post-break observations.

Introducing a structural break is likely to help to explain some important aspects of the data. First, the volatility of asset returns implied by the Lucas model would be higher. Even in the rational expectations monetary model, a one-time jump in the process generates more volatile returns.[11] On the other hand, under adaptive learning, since market participants struggle with small sample problems around the structural break point, the implied returns are expected to be highly volatile in comparison to the rational expectations case. Second, under adaptive learning, it is likely to help in explaining highly persistent excess returns over short horizons since an unanticipated regime shift or changes in parameter values of

[11] However, since we allow investors to access parameter values of new endowment process under rational expectations, it is unlikely to help generating other dimensions of the data.

the endowment process prevent investors from learning parameters in a new regime quickly that may result in very gradual adjustment of estimates assessed by the investors to the true values.

5 Simulation Results

In this section we compare predictions of the Lucas under adaptive learning to those generated under standard rational expectations. We examine how the simulated data are capable of replicating some important features of data in the U.S. asset market as described in section 2.

5.1 The Parameter Choices

We choose a set of reasonable model parameter values. First, we take Maximum-likelihood estimates of equation (3) for the Markov switching process in Table 3 and OLS estimates of equation (5) for the AR(1) process in Table 4 as the parameter values of data generating process for consumption growth rate. In the presence of a structural break in the endowment process, parameter values for both regimes are also presented in those tables. Second, since many features of asset-returns data can be explained by choosing both a large value of γ and a value of β in excess of one, we attempt to use values of β between 0 and 1,[12] and positive values of γ below 5, which predict a mean risk-free rate of 2.88 percent.

5.2 Statistics from Simulated Data

We simulate 2,000 of artificial data and calculate median values of model-implied statistics. We also consider both one-regime and two-regime endowment processes. Although statistical tests suggest that there is at least one structural change in the exogenous process, we study one-regime case to examine a possibility that agents do not believe there has been a regime shift in the process. More importantly, one-regime environment allows us to separate the ability of least squares learning in explaining the stylized facts from the presence of

[12] Note that we allowed a small violation of the value of discount factor that is a bit greater than one to examine whether equity premium implied by the Lucas model increases with the value of discount factor in both rational expectations as in Kocherlakota (1996) and adaptive learning.

structural beaks. First, we simulate the Lucas asset pricing model under rational expectations, where investors know parameter values of the data generating process that generates consumption growth rates and asset returns are determined according to equations (10) and (11) for the Markov switching endowment process and equations (15) and (16) for the AR(1) endowment process. Next, we introduce least squares learning to the Lucas model where investors believe that the growth rate of consumption follows a stationary autoregressive process, which may or may not be true as we describe in section 3, but not the parameter values, and asset returns are determined as if investors live in a fully rational economy, except the important ingredient that they use time-t estimates of parameters, not true parameter values.

We compare predictions of the Lucas asset-pricing model under least squares learning with those generated under standard rational expectations in its ability to account for the following aspects of the data shown in Tables 1 and 2: (1) the mean level of equity premium of 5.4 percent,[13] (2) the standard deviation of the equity premium that is around 18 percent, which is excessively volatile compared to the standard deviation of the risk-free rate which is 6.68 percent per year, (3) variance ratio statistics that decline from above one over short horizon to below one over long horizons, and (4) slope coefficients, t-statistics, and $\overline{R}^2$'s of long horizon excess return regressions on the log of consumption-price ratio that increase with return horizon. We present results for our two models, labelled RE and AE, in Table 5 through Table 12.

5.2.1 Model Implications under Rational Expectations

The implied behaviors of asset returns when investors have rational expectations are seen to perform poorly. The model-implied equity premium for both specifications of the endowment process is less than one percent per year for all plausible sets of γ and β and even for $\beta > 1$. The volatilities of equity premium and risk-free return are far below their sample values and excess returns do not display highly persistent behavior over the short horizon for both specifications of endowment process. However, asset returns with the Markov

[13]Note that we choose a set of γ and β that produces a mean risk-free rate of 2.88 percent. We found that sets of preference parameters, γ and β, that match the mean level of risk-free rate are not very sensitive to the choice of the endowment process.

switching process tend to exhibit higher variability than the autoregressive model by more than one percent for most cases. Long-horizon return regressions on current consumption-price ratio at horizons of 1, 2, 3, 5, and 8 years are also reported. The statistics obtained from the historical data show that slope coefficients $\hat{\beta}_k$ have all positive signs and increase linearly with horizon, and t-statistics and $\overline{R}^2$s start low but then rise to impressive values. The model under rational expectations does not yield the systematic pattern. Moreover, for some preference parameters choices, the consumption-price ratio predicts asset returns a wrong sign.[14] This anomalous simulation results may be due to the fact that, under rational expectations, the volatility of equity premium can be less than the volatility of consumption which is not true in the data.

As we mentioned earlier, the presence of structural break in the endowment process is unlikely help much to understand asset-return dynamics under rational expectations since investors are allowed to access parameter values of new endowment process. Tables 9 and 10 report model implications under rational expectations with a structural break for each endowment process. After introducing a regime shift in the endowment process, the generated volatility of returns under rational expectations is a bit higher than one-regime case, but still much lower than its sample value. Furthermore, the model under rational expectations with complete knowledge fails to produce highly persistent excess returns and generates the consumption-price ratio that predicts equity premium with a wrong direction for some cases. Therefore, we found that the model implications under rational expectations are not influenced by specifications of exogenous endowment process or presence of structural changes in the process and it is hard to save rational expectations asset pricing model without increasing model complexity or introducing other market frictions such as incomplete market and transaction costs.

5.2.2 Model Implications under Adaptive Learning

We now examine predictions of the Lucas model under adaptive learning and simulation results are presented in Tables 7 and 8. We found that the model under adaptive learning dominates the standard rational expectations with complete knowledge in its ability to

[14] Although the medium values of slope coefficients of excess return regressions are negative, the distribution of slope coefficients includes positive values.

account for some stylized facts in the U.S. asset market, although least squares learning in the Lucas asset pricing model does not contribute much to understand equity premium above 5 percent per year. First, model-implied volatilities of excess returns and risk-free returns are much higher than those generated under rational expectations. However, for reasonable sets of preference parameters, standard deviations of risk-free rates, especially in the autoregressive endowment process case, are less than one percent per year which is much smaller than its sample value. Note that the volatility of equity premium is higher as the values of preference parameters become larger for both endowment processes, and the volatility of risk-free rates exhibits the same pattern for the Markov switching case. Second, we also found that investors' incomplete knowledge about the fundamental process and their adaptive learning behavior are not enough to help to account for equity premium that display substantial persistence over the short horizon. Table 7 and Table 8 show that excess returns implied by the model under least squares learning are negatively serially correlated even over short horizons for the autoregressive process with all sets of preference parameters and for the Markov switching process with $\gamma < 1$. Third, the model under adaptive learning is now capable of accounting for why current consumption-price ratio predict excess asset returns over long horizon but not over short horizon. For both endowment processes, the model-implied slope coefficients and t–statistics are consistently positive and increase with return horizons, and the $\overline{R}^2$s start low but then rise to impressive values. This suggests that incomplete information about the endowment process causes the consumption-price ratio to deviate from its mean value over short horizon, but predicted returns tend to increase as the consumption-price ratio returns to its mean value.

Next, we study how implications of the Lucas asset-pricing model under least squares model change when we introduce a structural break in the growth rate of per capita consumption. First, we found that model-implied variance ratios that measure the serial correlation properties of the simulated data are dramatically changed in the presence of a structural break with learning. For both specifications of endowment process, the model under adaptive learning generates variance ratio statistics that are greater than 1 over short horizon, but less than 1 and fall with the return horizon over long horizon as found in the historical data.[15] Next, volatilities of asset returns with a regime shift are even higher now and nearly match for some sets of preference parameters while maintaining the predictability of asset returns that the predicted returns by the consumption-price ratio increase in the horizon. However, since the generated equity premium is still much less than 5 percent per year, the introduction of possible regime shifts in the endowment process is unlikely help much in explaining why stocks are not sufficiently riskier than risk-free asset to explain the spread in their returns.

6 Conclusion

We study a representative agent endowment economy where the agent, who has incomplete knowledge about the true structure of the economy, learns about the endowment process by employing adaptive learning rules. We compare the predictions of the model under least squares learning to those generated under standard rational expectations. We model unanticipated regime shifts in the endowment process that the agent must contend with. We also consider alternative empirical specifications of the endowment process to examine how the choice of the exogenous process change the model implications.

Although the Lucas asset pricing model in an endowment economy under least squares learning does not produce completely realistic description about how asset returns behave, our simulation results suggest that the model under the learning dominates standard rational expectations. Investors' adaptive learning behavior plays a key role in explaining why the consumption-price ratio predict excess returns over long horizons, for generating equity premium that is excessively volatile than risk-free rates. We also found that a Markov switching process for the endowment process generates more volatile asset returns than an autoregressive model which has been popular in learning literature. However, any of new ingredients we consider is unlikely aid in resolving the equity premium puzzle and the volatility puzzle. Finally, we found a apparent structural break in the U.S. per capita consumption growth rate which has been considered as one of stable time series. The introduction of a structural break in the exogenous endowment process produces much more volatile excess returns as well as positively serially correlated equity premium over short horizons, and this

[15] Note that excess returns are relatively more persistent in the case of the Markov process.

result provide another empirical support for constant or perpetual learning.

Our simulation results suggest some useful directions for future research. Some assumptions about our learning environment can be relaxed in more constructive ways. We consider investors who are allowed to detect a break when it occurs which is the first avenue we want to pursue. One may model a world in which investors are unsure of when the change has occurred. One reasonable way to model this is that at each point in time, investors do a test if a break has occurred. Once they have figured out that a break has occurred, they set the break point as marking the new regime and start learning new true parameter values with adaptive learning rules. Second, we introduce one statistically implied structural break point that represents a major economic event. One may model a world in which changes in regime due to relatively small events happen frequently. In this case, constant gain learning even with relatively small gain combined with structural changes in data generating processes is worth pursuing.

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A. First and Second Moments		B. Persistence (ep) Horizon	VR
μ_{ep}	5.38	1	1.0000
μ_{rf}	2.88	2	1.0765
σ_{ep}	18.11	3	0.9609
σ_{rf}	6.68	5	0.9091

Note: The data are annual observations from 1871-2004. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations. Variance ratio, $VR(k)$, is the variance of the k-year equity premium divided by k times the variance of the one-year equity premium for k=1,2,3, and 5.

Table 1: Stylized Facts of Asset Returns I: First and Second Moments and Persistence

A. Log Dividend-Price Ratio (1871-2004)			
Horizon	Slope	t-value	$\overline{R}^2$
1	0.039	1.007	0.000
2	0.116	1.955	0.021
3	0.163	2.293	0.031
5	0.340	3.845	0.085
8	0.471	3.879	0.093
B. Log Consumption-Price Ratio (1890-2004)			
Horizon	Slope	t-value	$\overline{R}^2$
1	0.131	3.792	0.071
2	0.274	5.545	0.158
3	0.375	6.223	0.222
5	0.586	8.554	0.345
8	0.761	8.948	0.444

Note: The regression slope, t-statistics, and $\overline{R}^2$ are for regressions of the k-year (k=1,2,3,5, and 8) ahead equity premium on the current log dividend-price ratio (panel A) and on the current log consumption-price ratio (panel B). Regressions are estimated by OLS with HAC standard errors using the automatic lag selection method of Newey and West (1994)

Table 2: Stylized Facts of Asset Returns II: Predictability

Table 3: Estimated Parameter Values: Markov Switching Process

p	q	$\alpha(S=1)$	$\alpha(S=0)$	σ_ε
A. One-Regime Case				
0.976	0.509	2.250	-5.866	3.071
(0.021)	(0.254)	(0.316)	(-1.737)	(1.191)
B. Two-Regime Case				
Regime 1				
0.237	0.198	3.603	-1.057	7.461
(0.179)	(0.152)	(0.626)	(-0.601)	(1.516)
Regime 2				
0.983	0.836	2.090	-0.847	5.377
(0.022)	(0.359)	(0.255)	(-0.573)	(0.759)

Note: Table entries are maximum-likelihood estimates of the Markov switching process, $\xi_t = \alpha(S_t) + \varepsilon_t$, and standard errors are in parenthesis.

Table 4: Estimated Parameter Values: AR(1) Process

A. One-Regime Case			B. Two-Regime Case					
α	ρ	σ_υ	α_1	ρ_1	$\sigma_{\upsilon 1}$	α_2	ρ_2	$\sigma_{\upsilon 2}$
2.249	-0.076	3.541	2.758	-0.437	4.149	1.527	0.326	2.704
(0.385)	(-0.094)		(0.731)	(-0.152)		(0.384)	(0.104)	

Note: Table entries are OLS estimates of the AR(1) Process, $\xi_t = \alpha + \rho\ \xi_{t-1} + \upsilon_t$, and standard errors are in parenthesis.

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
0.0	0.9716	μ_{ep}	0.29	1	1.0000	-0.0485	-0.0404	-0.0044
		μ_{rf}	2.88	2	0.9828	0.1155	0.0654	-0.0047
		σ_{ep}	3.53	3	0.9617	0.0792	0.0361	-0.0045
		σ_{rf}	0.00	5	0.9227	0.0524	0.0184	-0.0045
				8	0.8647	0.1689	0.0537	-0.0051
0.5	0.9817	μ_{ep}	0.33	1	1.0000	-1.2435	-0.5009	-0.0030
		μ_{rf}	2.88	2	0.9816	-0.8931	-0.2642	-0.0042
		σ_{ep}	3.52	3	0.9604	-0.8978	-0.2109	-0.0043
		σ_{rf}	0.13	5	0.9205	-1.0405	-0.1820	-0.0046
				8	0.8679	-0.8465	-0.1248	-0.0047
2.0	1.0106	μ_{ep}	0.47	1	1.0000	2.3125	1.8612	0.0185
		μ_{rf}	2.88	2	0.9494	2.0577	1.2102	0.0037
		σ_{ep}	3.66	3	0.9202	2.1218	0.9733	0.0001
		σ_{rf}	0.54	5	0.8731	2.1765	0.7748	-0.0020
				8	0.8177	2.1276	0.6162	-0.0033
3.0	1.0287	μ_{ep}	0.58	1	1.0000	1.7578	2.6683	0.0446
		μ_{rf}	2.88	2	0.9056	1.5972	1.7582	0.0158
		σ_{ep}	3.86	3	0.8660	1.6090	1.4491	0.0085
		σ_{rf}	0.81	5	0.8109	1.6640	1.1754	0.0031
				8	0.7529	1.6349	0.9405	-0.0002

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 5: Implications of RE Model: AR(1) Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
		μ_{ep}	-0.09	1	1.0000	0.0398	0.1566	0.0028
		μ_{rf}	2.88	2	0.9805	0.1853	0.5056	0.0056
0.0	0.9716	σ_{ep}	4.64	3	0.9494	0.2503	0.5447	0.0047
		σ_{rf}	0.00	5	0.8986	0.2893	0.4728	0.0019
				8	0.8369	0.3205	0.4523	0.0007
		μ_{ep}	0.00	1	1.0000	-0.1396	-0.3060	0.0005
		μ_{rf}	2.88	2	0.9989	-0.0607	-0.0924	0.0011
0.5	0.9815	σ_{ep}	4.13	3	0.9808	-0.0200	-0.0201	0.0006
		σ_{rf}	0.43	5	0.9517	-0.0024	0.0000	-0.0003
				8	0.8980	-0.0010	0.0000	-0.0008
		μ_{ep}	0.17	1	1.0000	0.5550	2.8704	0.0524
		μ_{rf}	2.88	2	0.9339	0.8143	2.9642	0.0565
2.0	1.0094	σ_{ep}	4.12	3	0.8824	0.9341	2.8050	0.0507
		σ_{rf}	1.75	5	0.8147	0.9982	2.4040	0.0363
				8	0.7395	1.0111	1.9930	0.0235
		μ_{ep}	0.17	1	1.0000	0.4473	3.6177	0.0845
		μ_{rf}	2.88	2	0.8562	0.6991	4.1853	0.1127
3.0	1.0263	σ_{ep}	5.74	3	0.7535	0.8130	4.1277	0.1106
		σ_{rf}	2.68	5	0.6323	0.8637	3.6489	0.0884
				8	0.5336	0.8945	3.1752	0.0683

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 6: Implications of RE Model: Markov Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
0.0	0.9716	μ_{ep}	0.32	1	1.0000	0.1198	3.1376	0.0637
		μ_{rf}	2.88	2	0.8969	0.2065	4.2788	0.1183
		σ_{ep}	8.69	3	0.7967	0.2836	5.0625	0.1614
		σ_{rf}	0.00	5	0.6722	0.3905	6.2613	0.2327
				8	0.5357	0.5128	7.5469	0.3127
0.5	0.9816	μ_{ep}	0.39	1	1.0000	0.1121	2.4568	0.0373
		μ_{rf}	2.88	2	0.9281	0.2048	3.3487	0.0734
		σ_{ep}	5.25	3	0.8493	0.2753	3.8222	0.0961
		σ_{rf}	0.16	5	0.7602	0.3987	4.7181	0.1444
				8	0.6713	0.5266	5.4198	0.1874
2.0	1.0104	μ_{ep}	0.44	1	1.0000	0.1230	3.0831	0.0614
		μ_{rf}	2.88	2	0.8934	0.1993	4.0430	0.1063
		σ_{ep}	7.86	3	0.7750	0.2653	4.7279	0.1430
		σ_{rf}	0.62	5	0.6532	0.3942	6.1081	0.2237
				8	0.5557	0.5226	7.1384	0.2888
3.0	1.0284	μ_{ep}	0.51	1	1.0000	0.1200	3.2720	0.0695
		μ_{rf}	2.88	2	0.8783	0.2072	4.3990	0.1245
		σ_{ep}	14.30	3	0.7676	0.2782	5.2447	0.1716
		σ_{rf}	0.93	5	0.6161	0.3959	6.9681	0.2740
				8	0.4954	0.5034	8.7736	0.3818

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 7: Implications of AL Model: AR(1) Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
		μ_{ep}	0.71	1	1.0000	0.1284	3.3132	0.0713
		μ_{rf}	2.88	2	1.0482	0.2418	4.1633	0.1124
0.0	0.9716	σ_{ep}	12.27	3	0.9731	0.3232	4.9031	0.1525
		σ_{rf}	0.00	5	0.8438	0.4886	6.1611	0.2268
				8	0.7192	0.6770	7.4792	0.3088
		μ_{ep}	0.57	1	1.0000	0.1505	3.1778	0.0654
		μ_{rf}	2.88	2	1.1290	0.2667	3.7860	0.0937
0.5	0.9811	σ_{ep}	7.16	3	1.0692	0.3668	4.4375	0.1274
		σ_{rf}	0.71	5	0.9710	0.5553	5.5579	0.1917
				8	0.8317	0.7846	6.4103	0.2458
		μ_{ep}	0.18	1	1.0000	0.0821	2.3503	0.0336
		μ_{rf}	2.88	2	0.6515	0.1518	3.9401	0.1012
2.0	1.0076	σ_{ep}	9.06	3	0.5368	0.2075	4.8234	0.1482
		σ_{rf}	2.80	5	0.4388	0.2976	6.5072	0.2471
				8	0.3593	0.4086	8.4404	0.3635
		μ_{ep}	0.05	1	1.0000	0.0794	2.2340	0.0298
		μ_{rf}	2.88	2	0.7824	0.1542	3.7889	0.0938
3.0	1.0261	σ_{ep}	15.41	3	0.6670	0.2069	4.6253	0.1374
		σ_{rf}	4.11	5	0.5645	0.3032	5.7136	0.2007
				8	0.4585	0.4478	7.7935	0.3269

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 8: Implications of AL Model: Markov Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
		μ_{ep}	0.49	1	1.0000	0.0135	0.4598	-0.0041
		μ_{rf}	2.88	2	0.9896	0.0278	0.6543	-0.0008
0.0	0.9716	σ_{ep}	4.16	3	0.9701	0.0426	0.8233	0.0029
		σ_{rf}	0.00	5	0.9389	0.0670	0.9888	0.0106
				8	0.8892	0.1091	1.3113	0.0247
		μ_{ep}	0.43	1	1.0000	-0.0010	-0.0160	-0.0041
		μ_{rf}	2.88	2	0.9470	0.0156	0.1919	-0.0008
0.5	0.9818	σ_{ep}	3.69	3	0.9277	0.0293	0.3052	0.0030
		σ_{rf}	0.75	5	0.8842	0.0612	0.5231	0.0102
				8	0.8330	0.1102	0.6889	0.0223
		μ_{ep}	0.38	1	1.0000	0.0509	0.8121	-0.0025
		μ_{rf}	2.88	2	0.5736	0.0369	0.5375	-0.0026
2.0	1.0113	σ_{ep}	4.14	3	0.5230	0.0465	0.5682	0.0007
		σ_{rf}	1.51	5	0.4341	0.0462	0.5107	0.0053
				8	0.3687	0.0441	0.3927	0.0143
		μ_{ep}	0.32	1	1.0000	0.0466	0.8828	-0.0016
		μ_{rf}	2.88	2	0.5224	0.0245	0.4549	-0.0036
3.0	1.0299	σ_{ep}	6.64	3	0.4653	0.0294	0.4551	-0.0020
		σ_{rf}	3.02	5	0.3692	0.0159	0.2107	0.0024
				8	0.3084	0.0059	0.0669	0.0089

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 9: Implications of RE Model with Structural Break: AR(1) Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
		μ_{ep}	0.17	1	1.0000	0.0122	0.6805	-0.0036
		μ_{rf}	2.88	2	0.9741	0.0253	0.9960	0.0009
0.0	0.9716	σ_{ep}	4.08	3	0.9606	0.0378	1.2335	0.0053
		σ_{rf}	0.00	5	0.9232	0.0623	1.5708	0.0144
				8	0.8700	0.0994	2.0220	0.0303
		μ_{ep}	0.08	1	1.0000	0.0102	0.3122	-0.0047
		μ_{rf}	2.88	2	0.8958	0.0233	0.5598	-0.0006
0.5	0.9790	σ_{ep}	3.33	3	0.8811	0.0351	0.6714	0.0031
		σ_{rf}	0.55	5	0.8339	0.0625	0.9607	0.0123
				8	0.7759	0.1046	1.3120	0.0243
		μ_{ep}	-0.06	1	1.0000	0.0370	0.6714	-0.0041
		μ_{rf}	2.88	2	0.7170	0.0186	0.3277	-0.0044
2.0	1.0003	σ_{ep}	5.54	3	0.7096	0.0207	0.2921	-0.0028
		σ_{rf}	2.21	5	0.6469	0.0113	0.1319	0.0014
				8	0.5918	-0.0014	-0.0134	0.0071
		μ_{ep}	-0.18	1	1.0000	0.0151	0.4795	-0.0053
		μ_{rf}	2.88	2	0.6143	0.0195	0.5540	-0.0025
3.0	1.0138	σ_{ep}	8.26	3	0.6104	0.0281	0.6711	0.0002
		σ_{rf}	3.33	5	0.5361	0.0408	0.7974	0.0071
				8	0.4784	0.0621	1.0394	0.0170

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 10: Implications of RE Model with Structural Break: Markov Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
		μ_{ep}	0.28	1	1.0000	0.0708	2.3018	0.0320
		μ_{rf}	2.88	2	1.0590	0.1525	3.5048	0.0804
0.0	0.9716	σ_{ep}	10.30	3	0.9736	0.2163	4.2807	0.1192
		σ_{rf}	0.00	5	0.8420	0.3259	5.4195	0.1838
				8	0.7196	0.4382	6.3343	0.2413
		μ_{ep}	0.33	1	1.0000	0.0697	1.9218	0.0203
		μ_{rf}	2.88	2	1.0414	0.1523	2.9795	0.0576
0.5	0.9815	σ_{ep}	6.06	3	0.9689	0.2198	3.5795	0.0845
		σ_{rf}	0.73	5	0.8701	0.3269	4.3617	0.1252
				8	0.7587	0.4266	4.9412	0.1599
		μ_{ep}	0.47	1	1.0000	0.0736	2.2819	0.0313
		μ_{rf}	2.88	2	1.0621	0.1579	3.5159	0.0809
2.0	1.0098	σ_{ep}	9.89	3	0.9818	0.2184	4.2601	0.1181
		σ_{rf}	2.92	5	0.8343	0.3147	5.3029	0.1771
				8	0.7127	0.4379	6.3787	0.2440
		μ_{ep}	0.63	1	1.0000	0.0762	2.3922	0.0351
		μ_{rf}	2.88	2	1.0539	0.1666	3.7215	0.0906
3.0	1.0273	σ_{ep}	18.76	3	0.9539	0.2197	4.4537	0.1283
		σ_{rf}	4.39	5	0.7738	0.3245	5.5449	0.1910
				8	0.6501	0.4377	6.8255	0.2704

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 11: Implications of AL Model with Structural Break: AR(1) Process

Preferences		First and Second		Persistence and Predictability				
γ	β	Moments		Horizon	VR	Slope	t-value	$\overline{R}^2$
		μ_{ep}	0.32	1	1.0000	0.0741	2.4410	0.0367
		μ_{rf}	2.88	2	1.1396	0.1422	3.2178	0.0676
0.0	0.9716	σ_{ep}	8.87	3	1.1670	0.2091	3.8312	0.0965
		σ_{rf}	0.00	5	1.0887	0.3183	4.7217	0.1446
				8	0.9552	0.4570	5.6946	0.2035
		μ_{ep}	0.28	1	1.0000	0.0777	2.3626	0.0340
		μ_{rf}	2.88	2	1.1204	0.1523	3.0401	0.0601
0.5	0.9806	σ_{ep}	4.89	3	1.1224	0.2214	3.5325	0.0823
		σ_{rf}	0.47	5	1.0312	0.3460	4.2919	0.1215
				8	0.8762	0.4845	5.0988	0.1689
		μ_{ep}	0.20	1	1.0000	0.0659	2.0907	0.0253
		μ_{rf}	2.88	2	1.1204	0.1382	3.1988	0.0668
2.0	1.0070	σ_{ep}	9.63	3	1.1028	0.1935	4.0758	0.1087
		σ_{rf}	1.86	5	0.9621	0.2878	5.1640	0.1692
				8	0.8066	0.3930	6.4634	0.2490
		μ_{ep}	0.25	1	1.0000	0.0748	2.2698	0.0309
		μ_{rf}	2.88	2	1.1054	0.1469	3.3869	0.0751
3.0	1.0237	σ_{ep}	19.42	3	1.0633	0.2170	4.2864	0.1195
		σ_{rf}	2.80	5	0.8984	0.3221	5.4681	0.1866
				8	0.7289	0.4407	6.7543	0.2662

Note: Table entries are the median values of 2,000 replications. μ_{ep} and μ_{rf} refer mean equity premium and mean risk-free rate, respectively. σ_{ep} and σ_{rf} are standard deviations as a measure for volatility.

Table 12: Implications of AL Model with Structural Break: Markov Process

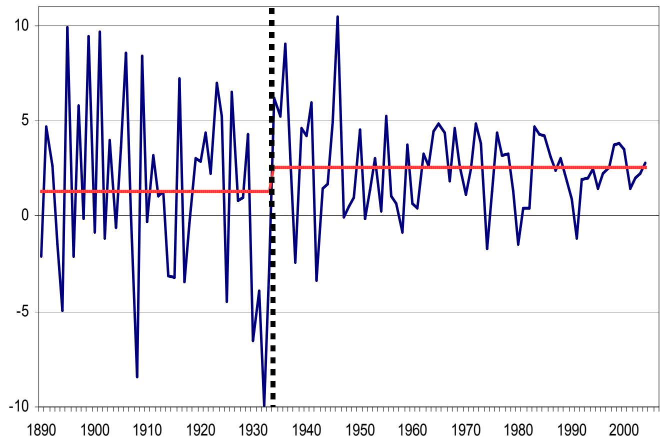

Figure 1: Data Generating Process with a Structural Break: Constant

Note: This figure plots actual consumption growth rates (solid line) and fitted values (dotted line) with a break point when constant (α) is subject to segment.

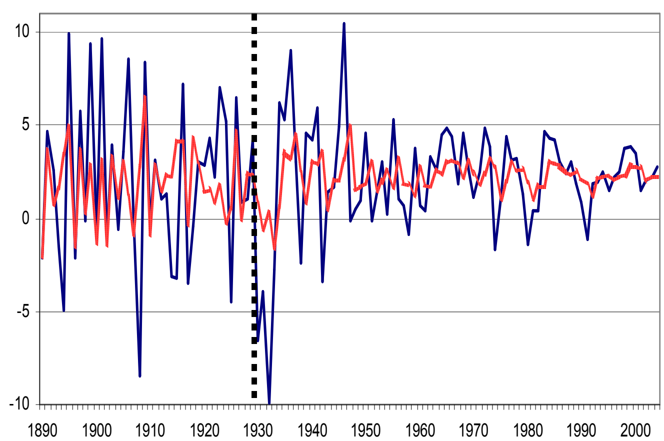

Figure 2: Data Generating Process with a Structural Break: Constant and Lagged Consumption Growth

Note: This figure plots actual consumption growth rates (solid line) and fitted values (dotted line) with a break point when both constant (α) and persistent parameter (ρ) are subject to segment.