Monetary policy and the asset risk taking channel*

Angela Abbate† Dominik Thaler‡

October 21, 2015

Abstract

Motivated by VAR evidence on the risk-taking channel in the US, we develop a New Keynesian model where low levels of the risk-free rate induce banks to grant credit to riskier borrowers. In the model banks are incentivized to take excessive risks by an agency problem between depositors and equity holders, that arises from limited liability and inobservable risk taking. As the real interest rate declines these incentives become stronger and risk taking increases. We estimate the model on US data using Bayesian techniques and assess how this novel channel affects optimal monetary policy conduct in the estimated model. Our results suggest that in a risk-taking channel environment, the monetary authority should seek to stabilize the path of the real interest rate, trading off more inflation volatility in exchange for less interest rate and output volatility.

J.E.L. Classification: E12 E44 E58

Keywords: Bank Risk; Monetary policy; DSGE Models;

*The authors would like to thank Arpad Abraham, Pelin Ilbas, Stephane Moyen, Evi Pappa, Raf Wouters, our colleagues at the Bundesbank, European University Institute and National Bank of Belgium, as well as participants to the XX Workshop on Dynamic Macroecomics in Vigo and to the 6th Bundesbank-CFS-ECB Workshop on Macro and Finance in Frankfurt for useful comments. We thank the NBB for funding Dominik Thaler's research stay in Brussels.

†Deutsche Bundesbank and European University Institute, Department of Economics, angela.abbate@bundesbank.de. This paper represents the authors' personal opinions and do not necessarily reflect the views of the Deutsche Bundesbank or its staff.

‡European University Institute, Department of Economics, dominik.thaler@eui.eu

1 Introduction

The recent financial crisis has marked the importance of monitoring the different types of risks to which the financial sector, and ultimately the real economy, are exposed. A relevant aspect is whether interest rates, and therefore monetary policy, can influence the risk-taking behavior of financial intermediaries. This transmission mechanism, known as the risk-taking channel of monetary policy,[1] could have contributed to the excessive levels of financial sector balance sheet risk which lead to the 2008 financial crisis. In the aftermath of the crisis interest rates have fallen considerably in many countries, raising concerns on whether financial market participants might be once again induced to reallocate portfolios towards riskier investments, creating the risk of yet another crisis.[2]

This paper addresses these concerns from a theoretical point of view, motivated by structural VAR evidence that expansionary monetary policy shocks indeed increase bank asset risk in the US. We model bank asset risk taking in a DSGE framework and assess how monetary policy should be conducted in the presence of a risk-taking channel. Building on Dell'Ariccia et al. [2014], we assume that banks can choose from a continuum of investment projects, each defined by different risk-return characteristics. Since depositors cannot observe this choice, and because bank owners are protected by limited liability, an agency problem emerges: banks are partially isolated from the downside risk of their investment and hence choose a risk level that exceeds what would be chosen if these frictions were absent. Because the importance of this distortion is proportional to the real rate of return, lower levels of the real risk-free rate induce banks to choose even riskier investments.

This connection between interest rates and asset risk raises the question of whether the monetary authority should take this channel into account when setting the interest rate. Since the answer to this question is of quantitative nature, we embed our banking sector in a medium scale Smets and Wouters [2007] type DSGE model, known to fit the data well along many dimensions, and estimate it on US data with Bayesian techniques. We then analyze optimal monetary policy in this model using simple rules. We find that, if the risk-taking channel is active, monetary policy should be less responsive to inflation and output fluctuations. In this way, the monetary authority allows more inflation volatility in exchange for stabilizing the real interest rate, which in turn reduces the welfare detrimental volatility of the banks' risk choice. The welfare gains from taking the risk-taking channel into account are significant.

Our work relates to a small but growing theoretical literature that links monetary policy to financial sector risk in a general equilibrium framework. Most of the existing works focus on *funding* risk, associating risk with leverage, and build on the financial accelerator framework of Bernanke et al. [1999].[3] Angeloni and Faia [2013] and Angeloni et al. [2015] instead construct

[1] The term was first coined by Borio and Zhu [2008].

[2] See for instance: (2014, 22 March). Staying Unconventional, *The Economist*.

[3] For example in Gertler et al. [2012] and de Groot [2014] a monetary expansion increases banking sector leverage, which in turn amplifies the financial accelerator and strengthens the propagation of shocks to the real economy.

a model where higher leverage induced by expansionary monetary policy does not just amplify other shocks but also translates into a higher fraction of inefficient bank runs.

In this paper by contrast we model *asset* risk, i.e. the riskiness of the assets on the banks' balance sheets, which is another aspect of financial sector risk that seems to have played an important role in the lead-up to the 2008 financial crisis. This type of risk has so far mainly been discussed in the literature on optimal regulation for example in Christensen et al. [2011] and Collard et al. [2012]. In these papers however, either the depositors or the financial regulator ensure that risk is always chosen optimally, so monetary policy has no influence on risk taking.[4] In contrast to the previous two papers we provide microfoundations for the asset risk taking channel by building on Dell'Ariccia et al. [2014], and focus on monetary policy while abstracting from regulation.[5] Adding to the literature on both types of risk taking, our goal is in fact to systematically explore how this novel channel affects optimal monetary policy in an estimated medium scale model, where the policy maker needs to trade off several conflicting frictions.

The lack of theoretical papers on the asset risk taking channel is not mirrored by lack of empirical evidence. On the contrary, several studies find a causal link between monetary policy and risk taking. Most of the existing research relies on loan or bank level panel data and identification is based on the assumption that monetary policy is exogenous. Jimenez et al. [2014] use micro data of the Spanish Credit Register from 1984 to 2006 and find that lower interest rates induce banks to make relatively more loans to firms that qualify as risky ex ante (firms with a bad credit history at time of granting the loan) as well as ex post (firms that default on the granted loan). They argue that this effect is economically significant and particularly strong for thinly capitalized banks. These findings are confirmed for Bolivia using credit register data in Ioannidou et al. [2014] and for the US using confidential loan level data from the Terms of Business Lending Survey in Dell'Ariccia et al. [2013].

Evidence based on aggregate, publicly available time series data, where identification is obtained through restrictions on the dynamic responses, is less clear cut, not least because of limited data availability. Buch et al. [2013] estimate a FAVAR on a large data set of US banking-sector variables from the public summary data of the Terms of Business Lending Survey. They find that an expansionary monetary policy shock induces small domestic banks to increase the fraction of risky new loans to total new loans, but this evidence is not confirmed for the entire banking system. Afanasyeva and Guentner [2015] show, using a similar FAVAR methodology, that a monetary policy expansion decreases the net percentage of banks reporting tighter lending standards in the Fed survey of business lending. This measure, which may be considered a possible proxy for asset risk-taking, has not been found to be impacted by monetary policy in the small scale VAR analysis[6] conducted in Angeloni et al. [2015]. We complement this result in the following

[4] Both papers feature ad hoc extension that relate risk to the amount of lending and hence indirectly to monetary policy.

[5] One could reinterpret our model as applying to an economy where regulation is not able to fully control risk taking

[6] Angeloni et al. [2015] use a recursive identification scheme on monthly US data between 1985 and 2008. They test three different measures for risk: funding risk (leverage), asset risk (percentage of loan officers reporting a

section, showing that further evidence of an asset risk-taking channel can be retrieved from a VAR analysis when using a more direct measure of asset risk taking.

An interesting detail on the asset risk-taking channel is found in Buch et al. [2013] and Ioannidou et al. [2014]. Both show that an increase in risk taking induced by low interest rates is not accompanied by an offsetting increase in the risk premium, indicating that the additional risk taking is inefficient.

Motivated by this comprehensive empirical evidence and own VAR analysis in section 2, we develop in section 3 a DSGE model of asset risk-taking, where banks respond to low interest rates by inefficiently taking more risk. Section 4 presents the results from the estimation of the model and discusses the steady-state and dynamic implications of bank risk taking. Section 5 analyses how monetary policy should be conducted if a risk-taking channel is present and section 6 concludes.

2 The asset risk-taking channel in the US

To motivate our subsequent theoretical analysis we provide in this section additional empirical evidence on the existence of the asset risk-taking channel in the US. Our starting point is a classical small scale VAR that includes inflation, output, a measure of bank risk taking and the effective federal funds rate, taken as the monetary policy instrument. Output is measured by real GDP growth, while inflation is defined as the log change in the GDP deflator. The transformation and units of the variables match those used for the DSGE estimation, and are described in appendix A. Identification of the monetary policy shock is achieved through sign restrictions that do not involve the measure of bank risk, and are in this sense agnostic. We find that an unexpected decrease in the risk-free interest rate causes a persistent increase in bank risk taking, a result robust to using a recursive identification scheme. Our positive findings contradict the insignificance result on asset risk in Angeloni et al. [2015], whose specification is identical apart from the risk measure used.[7]

Measuring bank risk-taking behavior There are many notions of asset risk. One can distinguish between ex-ante, ex-post and realized asset risk. The former is the risk perceived by the bank when making a loan or buying an asset. Banks can influence this class of risk directly, when making their investment decisions (the ex-ante risk choice). On the other hand, the ex-post risk of a bank's balance sheet is also affected by unforeseen changes in asset riskiness, that take place after origination and are largely outside the banks' influence. Lastly, the payoff ultimately

tightening of loan standards in the Fed survey of business lending) and overall risk (realized volatility of bank stock prices). They find (lagged) significant positive effects only for funding and overall risk.

[7] We believe that our measure of asset risk (weighted risk rating of new loans) is superior to theirs (percentage of loan officers reporting tightening of lending standards) in two ways: First, our measure provides level information while their measure indicates only the direction of change in risk. Second, our measure is based on the quantity and quality of loans granted (revealed preferences), whereas theirs is based on reported perceptions that are aggregated to an index in an arbitrary way.

paid by an asset is a materialization of the former two types of uncertainty (realized asset risk). In this paper we focus on active risk taking, that is the level of ex-ante risk that intermediaries choose. Ex-ante bank risk taking is however largely unobservable. Inferring it from realized risk (e.g. loan losses) is hardly possible with aggregate data. Inference from the spread between some measure of bank funding costs and loan rates neglects the fact that this spread not only reflects default risk but also incorporates a liquidity premium and the markup, which are likely to be affected by the same variables that influence the risk choice. Instead, we use a survey-based proxy for bank risk-taking, which is provided by the US Terms of Business Lending Survey and consists in the internal risk rating assigned by banks to newly issued loans. In this survey, available only from 1997-Q1 onwards, 400 banks report the volume of loans originated in the quarter prior to filling the survey, grouped by the internal risk rating. This rating varies between 1 and 5, with 5 being the maximum level of risk. Following Dell'Ariccia et al. [2014], we construct a weighted average loan risk series, using as weights the value of loans in each risk category[8]. We plot this measure of risk, together with the nominal risk-free interest rate, in figure 1. We can see that low levels of the monetary policy instrument tend to be associated with higher level of bank risk (lower or decreasing levels in the blue line). This is particularly evident in the period between 2000 and 2006, when the monetary policy stance in the US was deemed to be particularly lax.

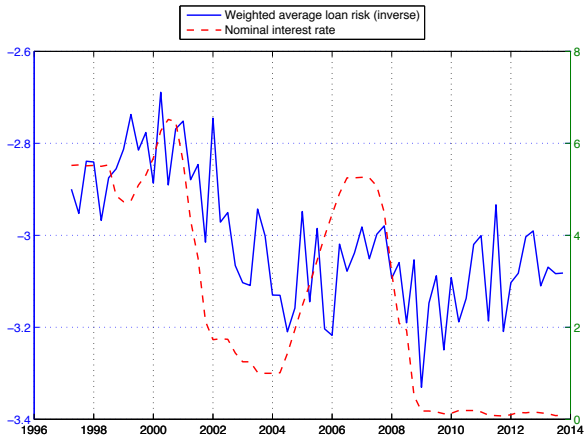
An increase in risk at times of low interest rates can be caused either by worsened macroeconomic conditions, giving banks no other choice than making riskier loans on average, or by an active choice of the banks to extend credit to riskier borrowers on average. We control for this effect, to the extent that it is reflected in inflation and output.

[8]The average loan risk is a perfect measure for bank risk taking if we assume that the volume of loans is constant. Else, banks could also minimize their risk exposure by reducing the quantity of loans as their average quality goes down. While the correlation between risk and loan volume growth is slightly negative, it is not significant at 10%. For a more in-depth discussion of the data we refer to Buch et al. [2013].

Table 1: **Sign restriction identification scheme**: restrictions are assumed to hold on impact and on the following period.

	variables		
SHOCK	y	π	R
MONETARY POLICY SHOCK	+	+	−
TOTAL FACTOR PRODUCTIVITY SHOCK	+	−	−

Figure 1: **Bank risk taking and nominal interest rate**: The risk measures (solid blue line, left axis) is defined such that a decrease can be associated to a higher risk-taking behavior of the banking sector. The nominal interest rate (dashed line, right axis) is the effective federal funds rate.



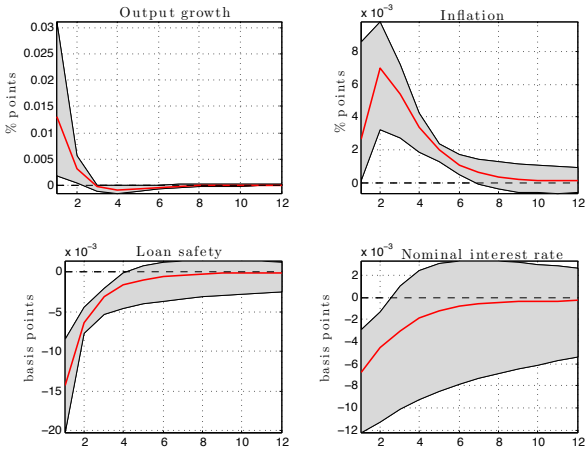
Identification strategy and results We estimate a VAR on US data from 1997:Q2, the start of the survey-based proxy for risk taking, to 2007:Q3, as described in appendix A[9]. The lag length is chosen to be 1, as indicated by the BIC information criterion.

We identify an unexpected monetary policy shock by using a set of sign restrictions that are robust across a variety of general-equilibrium models. In particular, we assume that an expansionary monetary policy shock decreases the nominal risk-free interest rate, and increases inflation and output, both at the time of the shock and in the quarter immediately after. Note that the response of inflation ensures that this shock is identified separately from a productivity shock, as summarized in table 1.

The response of bank asset risk to an expansionary monetary policy shock is shown in figure 2. An unexpected decrease in the monetary policy interest rate is followed by a moderate macroeconomic expansion: output growth increases for less than a year, while inflation displays a longer reaction of about two years. The results are compatible with the existence of a risk-

[9]We have decided to cut the zero-lower bound period, but our results still hold when the latest available data are used.

Figure 2: **Monetary policy shock on bank risk-taking:** Impulse responses over a 3-year horizon, identified through the sign restriction scheme in Table 1. Error bands correspond to 90% confidence intervals reflecting rotation uncertainty. Loan safety is defined as the inverse of the average loan risk rating, standarized to take values between 0 and 1. The transformation and units of the variables match those used for the DSGE estimation and are summarized in appendix A. See text for further details.



taking channel in the US: a fall in the nominal interest rate leads in fact to a decrease in the ex-ante proxy for the safety of banks' assets, i.e. as the interest rate falls banks issue riskier loans.

We assess the robustness of our results by employing a recursive identification scheme as in Angeloni et al. [2015] with two different variable orderings. Output and inflation are always ordered before the nominal interest rate, implicitly assuming that they do not react to interest rate shocks, nor to shocks coming from the banking sector. More controversial is the ordering of the nominal interest rate and of the loan risk rating. We experiment with both orderings and show in figure 4 in appendix B that results do not depend on the ordering chosen and confirm the results obtained using sign restrictions[10].

3 A Dynamic New Keynesian model with a bank risk-taking channel

In this section we build a general-equilibrium model where monetary policy can influence the risk-taking behavior of banks, thus providing an explanation for the risk-taking channel observed in the data. As a starting point we use a standard New Keynesian model with imperfect competition and price stickiness in the goods market, which implies a role for monetary policy. We augment

[10]Our measure of risk taking does not take into account loan quantities. For example, riskier lending may go hand in hand with lower loan volumes which may leave the expected loss volume unchanged. When adding loan volumes to the VAR we find that the response of volumes is not significantly different from zero with the mean showing no clear pattern in either direction, while the other IRFs remain unchanged.

this basic framework with an intermediation sector based on Dell'Ariccia et al. [2014]: competitive banks obtain funds from depositors and equity holders, which they invest into capital projects carried out by capital producers. Every bank chooses its investment from a continuum of available capital production technologies, each defined by a given risk-return characteristic. The risk choice of the representative bank is affected by the level of the real interest rate, and can be shown to be suboptimal. This model reproduces two features found in the data: risk taking depends on the contemporaneous interest rate and is inefficient.

While the aforementioned blocks are the necessary ingredients, in order to obtain a quantitatively more realistic model we add further elements, which are typically used in the DSGE literature. In particular we follow Smets and Wouters [2007] and allow for internal habits, investment adjustment costs and imperfect competition and wage stickiness in the labor market. Our model therefore features seven agents that are typical for DSGE models (households, unions, labor packers, capital producers, intermediate goods producers, final goods producers, central bank) and two agents that we introduce to model risk taking (banks, funds). Eight structural shocks hit the economy: these affect productivity, investment, time preferences, asset preferences, wage and price markups, as well as monetary and fiscal policy.

3.1 Households

The representative household chooses consumption c_t, working hours L_t and savings in order to maximize its discounted lifetime utility. Saving is possible through three instruments: government bonds s_t, deposit funds d_t, and bank equity funds e_t. The return on government bonds is safe and equal to the nominal interest rate R_t. The two funds enable the representative household to invest into the banking sector and pay an uncertain nominal return of $R_{d,t+1}$ and $R_{e,t+1}$.[11] Households maximize their lifetime utility function:

$$max_{d_t,e_t,s_t,c_t,L_t}\ E\left[\sum_{t=0}^{\infty}\beta^t\varepsilon_t^B\frac{(c_t-\iota c_{t-1})^{1-\sigma_C}}{1-\sigma_C}exp\left(\varphi L_t^{1+\sigma_L}\frac{\sigma_C-1}{1+\sigma_L}\right)\right] \tag{1}$$

subject to the per-period budget constraint in real terms:

$$c_t+d_t+e_t+s_t+T_t=L_tw_t+d_{t-1}\frac{R_{d,t}}{\pi_t}+e_{t-1}\frac{R_{e,t}}{\pi_t}+s_{t-1}\frac{R_{t-1}}{\pi_t}+\Pi_t\ \ , \tag{2}$$

where π_t is the inflation rate, while T_t and Π_t are taxes and profits form firm ownership, expressed in real terms. We allow for habits in consumption and furthermore assume that in each period a preference shock ε_t^B affects the time preference. This shock is assumed to be persistent with log-normal innovations, like all following shocks unless otherwise specified.... The households optimality conditions a given by the usual Euler equation and two no-arbitrage conditions:

[11] Note that in our notation the time index refers to the period that a variable is determined.

$$\Lambda_t = \beta E_t\big[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\big] \tag{3}$$

$$E_t\left[\Lambda_{t+1}\frac{R_{d,t+1}}{\pi_{t+1}}\right] = E_t\left[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right]\varepsilon_t^Q \tag{4}$$

$$E_t\left[\Lambda_{t+1}\frac{R_{e,t+1}}{\pi_{t+1}}\right] = E_t\left[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right]\varepsilon_t^Q \tag{5}$$

where $\Lambda_t = \varepsilon_t^B\left(c_t - \iota c_{t-1}\right)^{-\sigma_C} - \beta\iota E_t\left[\varepsilon_{t+1}^B\left(c_{t+1} - \iota c_t\right)^{-\sigma_C}\right]$ is the marginal utility of consumption. Note that as in Smets and Wouters [2007] we have included the ad-hoc shock ε_t^Q in the no arbitrage conditions, which drives a wedge between the government bond and real investments intermediated by banks.

3.2 Labor unions, final and intermediate good producers

The problems faced by final and intermediate good producers, as well as by labor unions, are standard. We discuss them only briefly and refer to Smets and Wouters [2007] and **?** for further details. The corresponding equilibrium conditions are listed in appendix C.

Final good producers assemble different varieties of intermediate goods through a Kimball [1995] aggregator with elasticity of substitution ϵ_p and Kimball parameter k_p, taking as given both the final good price and the prices of intermediate goods. Their optimization problem yields demand functions for each intermediate good variety as a function of its relative price.

A continuum of firms produces differentiated intermediate goods using capital K_{t-1} and "packed" labor l_t^d as inputs. The production function is Cobb-Douglas and is affected by a total factor productivity shock ε_t^A. Firms use their monopolistic power to set prices, taking as given their demand schedule. As in Calvo [1983], they can reset their prices in each period with probability λ^p, otherwise they index their prices to past inflation with degree γ^p and to steady state inflation with degree $(1-\gamma^p)$. Furthermore they are subject to a timevarying revenue tax ε_t^p that up to first order acts as a markup shock.

The labor market resembles the product market: Packed labor is produced by labor packers, who aggregate differentiated labor services using a Kimball [1995] aggregator with elasticity of substitution ϵ_p and Kimball parameter k_p.

Differentiated labor services are produced by a continuum of unions from the households labor supply. They use their monopolistic power to set wages. Wages are reset with probability λ^w, otherwise they are indexed to past inflation (with degree γ^w) and steady state inflation. Like intermediate firms, unions are subject to a stochastic wage tax ε_t^w. [12]

[12] Both ε_t^p and ε_t^w follow the standard shock process augmented by an moving average component.

3.3 Equity and deposit funds

As we explain in detail below, there is a continuum of banks which intermediate the households' savings using deposits and equity. Each bank is subject to a binary idiosyncratic shock which makes a bank fail with probability $1 - q_{t-1}$, in which case equity is wiped out completely and depositors receive partial compensation from the deposit insurance. The equity (deposit) funds' function is to diversify away the idiosyncratic bank default risk by buying a perfectly diversified portfolio of 1 period equity (deposits) of all banks.

The deposit fund works without frictions, that is it represents the depositors' interests perfectly. The deposit fund raises money from the households and invests it into d_t units of deposits[13]. Next period the fund receives the nominal deposit rate $r_{d,t}$ from each bank that does not fail. Deposits of failing banks are partially covered by deposit insurance. Most deposit insurance schemes around the world, including the US, guarantee all deposits up to a certain maximum amount per bank account.[14] We represent this capped insurance model by assuming that the deposit insurance guarantees deposits up to a fraction ψ of total bank liabilities l_t, which are the sum of deposits d_t and equity e_t. We assume that the deposit insurance cap is inflation adjusted, to avoid complicating the monetary policy trade-off by allowing an interdependence between monetary policy and deposit insurance. As we will show later, the deposit insurance cap is always binding in equilibrium, i.e. that the bank's liabilities exceed the cap of the insurance $r_{d,t}d_t > \psi(d_t + e_t)\pi_{t+1}$. Defining the equity ratio $k_t = \frac{e_t}{d_t+e_t}$, the deposit fund therefore receives a real return of $\psi/(1-k_t)$ per unit of deposits from each defaulting bank at t. The deposit fund hence pays a nominal return of:

$$R_{d,t+1} \equiv q_t r_{d,t} + (1 - q_t)\,\frac{\psi}{1-k_t}\pi_{t+1}\ . \tag{6}$$

Unlike the deposit fund, which is managed frictionlessly, the equity fund is subject to a simple agency problem. In particular, we assume that the fund manager faces two options. He can behave diligently and use the funds e_t, raised at t, to invest into e_t units of bank equity. A fraction q_t of banks will pay back a return of $r_{e,t+1}$ next period while the defaulting banks pay back nothing. Alternatively the fund manager can abscond with the funds, in which case he consumes a fraction ξ of the funds in the subsequent period and the rest is lost. Since he is a small member in the big family of the representative household his utility from doing so is $\Lambda_{t+1}\xi e_t$. To prevent the fund manager from absconding funds, the equity providers promise to pay him a premium p_t at time $t+1$ conditional on not absconding. This premium is rebated to the household in a lump sum fashion and the associated utility for the fund manager is $\Lambda_{t+1}p_t$. Equity providers pay the minimal premium that induces diligent behavior, i.e. $p_t = \xi e_t$. Once absconding is ruled out in equilibrium, the equity fund manager perfectly represents the interests of the equity providers. The nominal return on the bank equity portfolio is $q_t r_{e,t+1}$ per unit of

[13] We use deposits to refer to both units of deposit funds and units of bank deposits since they are equal. We do the same for equity.

[14] For a comprehensive documentation see, for instance, Demirgüç-Kunt et al. [2005].

equity, hence one share of the equity fund pays:

$$R_{e,t+1} \equiv q_t r_{e,t+1} - \xi \pi_{t+1} \ . \tag{7}$$

Note that, since bank equity is the residual income claimant, the return of the equity fund is affected by all types of aggregate risk that influences the surviving banks' returns.

The two financial distortions introduced so far have important implications. The agency problem implies an equity premium, i.e. a premium of the return on equity over the risk-free rate. Deposit insurance is instead a subsidy on deposits, which further reduces the cost of deposits relative to equity. As explained below, the difference in the costs of these two funding types induces a meaningful trade-off between bank equity and bank deposits under limited liability.

3.4 Capital producers

We assume that the capital production process is risky in a way that nests the standard capital production process in the New Keynesian model. In particular, capital is produced by a continuum of capital producers indexed by m. At period t they invest i_t^m units of final good into a capital project of size o_t^m . This project is successful with probability q_t^m, in which case the project yields $K_t^m = (\omega_1 - \frac{\omega_2}{2} q_t^m) o_t^m$ units of capital at $t+1$. Else, the project fails and only the liquidation value of θo_t^m units of capital can be recovered (where $\theta \ll a - \frac{b}{2} q_t^m$). Each capital producer has access to a continuum of technologies with different risk-return characteristics indexed by $q^m \in [0,1]$. Given a chosen technology q_t^m, the output of producer m therefore is:

$$K_t^m = \begin{cases} \left(\omega_1 - \frac{\omega_2}{2} q_t^m\right) o_t^m & \text{with probablity } q_t^m \\ \theta o_t^m & \text{else} \end{cases}$$

This implies that the safer the technology (higher q_t^m), the lower is output in case of success.

The bank orders the capital projects and requires the capital producer to use a certain technology, but this choice cannot be observed by any third party. Given the technology choice q_t and assuming that the projects of individual producers are uncorrelated, we can exploit the law of large numbers to derive aggregate capital:

$$K_t = o_t \left(q_t \left(\omega_1 - \frac{\omega_2}{2} q_t \right) + (1 - q_t)\theta \right) \ . \tag{8}$$

Furthermore we assume that capital, which depreciates at rate δ, becomes a project (of undefined q_t) at the end of every period. That is, existing capital may be destroyed due to unsuccessful reuse, and it can be reused under a different technology than it was originally produced[15].

[15]This assumption ensures that we do not have to keep track of the distribution of different project types. Think of a project as a machine that delivers capital services and that can be run at different speeds (levels of risk). In case it is run at higher speed, the probability of an accident that destroys the machine is higher. After each period the existing machines are overhauled by the capital producers and at this point the speed setting can

The total supply of capital projects by the capital producers is the sum of the existing capital projects $o_t^{old} = (1-\delta)K_{t-1}$, which they purchase from the owners (the banks) at price Q_t, and the newly created projects o_t^{new}, which are created by investing i_t units of the final good. We allow for investment adjustment costs and investment efficiency shocks, i.e. we assume that i_t units of investment yield $\varepsilon_t^I\left(1 - S(i_t/i_{t-1})\right)$ units of project, where $S = \kappa\left(\frac{i_t}{i_{t-1}} - 1\right)^2$. Hence $o_t = o_t^{new} + o_t^{old}$ and $o_t^{new} = \varepsilon_t^I\left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right)i_t$. Capital producers maximize their expected discounted profits taking as given the price Q_t and the households stochastic discount factor[16]:

$$max_{i_t, o_t^{old}} E_t \sum_0^\infty \beta^t \Lambda_t \left[Q_t \varepsilon_t^I \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t + Q_t o_t^{old} - i_t - Q_t o_t^{old}\right] .$$

While the old capital projects are always reused, the marginal capital project is always a new one[17]. Hence, the price of projects Q_t is determined by new projects according to the well known Tobin's q equation:

$$Q_t \varepsilon_t^I \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] - 1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}\varepsilon_{t+1}^I Q_{t+1} S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right] . \quad (9)$$

Note that our model of risky capital production boils down to the standard riskless setting of the New Keynesian model if we set $q_t = 1$ and $\omega_1 - \frac{\omega_2}{2} = 1$.

3.5 The Bank

The bank is the central agent of our model and is modeled similarly as in Dell'Ariccia et al. [2014]. Banks raise resources through deposits and equity and invest them into a risky project. Since depositors cannot observe the banks' risk choice and banks are protected from the downside risk of their investment by limited liability, an agency problem arises between them when the banks choose the risk level. The less equity a bank has, the higher the incentives for risk taking. Yet, since deposit insurance and the equity premium drive a wedge between the costs of deposits and equity, the banks' optimal capital structure comprises both equity and deposits, balancing the agency problem associated with deposits with the higher costs of equity. We will show that the equilibrium risk chosen by the banks is excessive, and that the interest rate influences the degree of its excessiveness.

We assume that there is a continuum of banks who behave competitively so that there is a representative bank (we therefore omit the bank's index in what follows). The bank is owned by the equity providers, and hence maximizes the expected discounted value of profits[18] using the household's stochastic discount factor. Every period the bank first raises deposits d_t and equity e_t from the respective funds (optimally choosing its liability structure). These resources are then

be changed.

[16]Their out of steady state profits are rebated lump sum to the household.

[17]We abstract from a non-negativity constraint on new projects.

[18]Profits in excess of the opportunity costs of equity.

invested into o_t capital projects, purchased at price Q_t. When investing into capital projects, the bank can choose the risk characteristic q_t of the technology that the capital producer applies. This risk choice is not observable for depositors. Each bank can only invest into one project and hence faces investment risk[19]: with probability q_t the bank receives a high pay-off from the capital project; with probability $1 - q_t$ the investment fails and yields only the liquidation value θ. Assuming a sufficiently low value for θ, a failed project implies that the bank, which is protected by limited liability, defaults. In that case equity providers get nothing and depositors get the deposit insurance benefit. In case of success the bank can repay its investors: depositors receive their promised return $r_{d,t}$ and equity providers get the state contingent return $r_{e,t+1}$.

It is useful to think of the bank's problem as a recursive two-stage problem. In the second stage, the bank chooses the optimal risk level q_t given a certain capital structure and a certain cost of deposits. In the first stage, the bank chooses the optimal capital structure, anticipating the implied solution for the second stage problem. Note that not only the bank but also the bank's financiers anticipate the second-stage risk choice and price deposits and equity accordingly, which is understood by the bank. Below we derive the solution for this recursive problem.

Before we do so, we need to establish the bank's objective function. Per dollar of funds raised (through deposits and equity) in period t the bank gets $1/P_t$ units of consumption good, where P_t is the current price level. These $1/P_t$ units of consumption goods are invested into a capital project of value $(1/P_t)/Q_t$ and type q_t, which are bought from the capital producer at price Q_t. If the project is successful it turns into $(\omega_1 - \frac{\omega_2}{2} q_t)/(Q_t P_t)$ capital goods. In the next period $t+1$, the bank rents the capital to the firm, who pays the real rental rate $r_{k,t+1}$ per unit of capital K_t. Furthermore the bank receives the depreciated capital, which becomes a capital project again, with real value of $(1-\delta)Q_{t+1}$ per unit of capital K_t. The bank's total nominal income, per dollar raised, conditional on success is therefore:

$$(\omega_1 - \frac{\omega_2}{2} q_t) \frac{r_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t} \frac{P_{t+1}}{P_t}$$

At the same time, the bank has to repay the deposit and equity providers $r_{d,t}$ and $r_{e,t+1}$. Using the equity ratio k_t, the total nominal payment to be made per dollar of funds in $t+1$ in case of success is $r_{e,t+1} k_t + r_{d,t} (1 - k_t)$.

The bank maximizes the expected discounted value of excess profits, i.e. revenues minus costs of funding, using the stochastic discount factor of the equity holders, that is the household. Given the success probability of q_t and the fact that the equity providers receive nothing in case of default, the bank's objective function is:

[19]The assumption that the bank can only invest into one project and can not diversify the project risk might sound stark. Yet three clarifications are in place: First, our setup is isomorphic to a model where the bank invests into an *optimally* diversified portfolio of investments but is too small to *perfectly* diversify its portfolio. The binary payoff is then to be interpreted as the portfolio's expected payoff conditional on default resp. repayment. Second, if the bank was able to perfectly diversify risk, then limited liability would become meaningless and we would have a model without financial frictions. Third, we don't allow the bank to buy the safe asset. Yet this assumption is innocuous: since the banks demand a higher return on investment than the households due to the equity premium, banks wouldn't purchase the safe asset even if they could.

$$max_{q_t,k_t}\ \beta E\left[\frac{\Lambda_{t+1}}{\pi_{t+1}}q_t\left(\left(\omega_1-\frac{\omega_2}{2}q_t\right)\frac{r_{k,t+1}+(1-\delta)Q_{t+1}}{Q_t}\pi_{t+1}-r_{d,t}(1-k_t)-r_{e,t+1}k_t\right)\right] \quad (10)$$

Note that we did not multiply the per-unit profits by the quantity of investment. In doing so we anticipate the equilibrium condition that the bank, whose objective function is linear in the quantity of investment, needs to be indifferent about the quantity of investment. The quantity will be pinned down together with the return on capital by the bank's balance sheet equation $e_t + d_t = Q_t o_t$, the market clearing and zero profit conditions.

The bank's problem can be solved analytically, yet the expressions get fairly complex. Therefore we derive here the solution of the banks problem for the case that $\psi = \theta = 0$, that is without deposit insurance and with a liquidation value of 0[20]. This simplifies the expressions but the intuition remains the same. Allowing ψ and θ to be nonzero on the other hand is necessary to bring the model closer to the data.

To make notation more tractable we rewrite the bank's objective function (10) in real variables expressed in marginal utility units[21]:

$$\omega_1 q_t \tilde{r}_{l,t} - \frac{\omega_2}{2}q_t^2\tilde{r}_{l,t} - q_t\tilde{r}_{d,t}(1-k_t) - q_t\tilde{r}_{e,t}k_t\ , \quad (11)$$

For later use we rewrite the household's no-arbitrage conditions (3) and (5) combined with the definition of the funds' returns (6) and (7) as $\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$ and $\tilde{r}_{e,t} = \frac{\tilde{R}_t+\tilde{\xi}_t}{q_t}$. Let us now solve the bank's problem recursively.

3.5.1 Second-stage problem:

At the second stage, the bank has already raised $e_t + d_t$ funds and now needs to choose the risk characteristic of her investment q_t, such that equity holders' utility is maximized. As already mentioned, we assume that the bank cannot write contracts conditional on q_t with the depositors at stage one, since q_t is not observable to them. Therefore at the second stage the bank takes the deposit rate as given. Furthermore, since the capital structure is already determined, maximizing the excess profit coincides with maximizing the profit of equity holders. The bank's objective function is therefore:

$$\max_{q_t}\omega_1 q_t\tilde{r}_{l,t} - \frac{\omega_2}{2}q_t^2\tilde{r}_{l,t} - q_t\tilde{r}_{d,t}(1-k_t)\ . \quad (12)$$

Deriving problem (12) with respect to q_t yields the following first-order condition[22]:

[20] The solution for the general case is provided in section 3.5.5.

[21] That is we use the following definitions: $\tilde{r}_{l,t} = E_t\left[\Lambda_{t+1}\left(\frac{r_{k,t+1}+(1-\delta)Q_{t+1}}{Q_t}\right)\right]$, $\tilde{r}_{d,t} = E_t\left[\Lambda_{t+1}\frac{r_{d,t}}{\pi_{t+1}}\right]$, $\tilde{r}_{e,t} = E_t\left[\Lambda_{t+1}\frac{r_{e,t+1}}{\pi_{t+1}}\right]$, $\tilde{R}_t = E_t\left[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right]$, $\tilde{\xi}_t = E_t\left[\Lambda_{t+1}\xi\right]$.

[22] We focus on interior solutions and choose the larger of the two roots, which is the closest to the optimum, as we will see below.

$$q_t = \frac{\omega_1 \tilde{r}_{l,t} - \tilde{r}_{d,t}\,(1-k_t)}{w_2 \tilde{r}_{l,t}} \ . \tag{13}$$

3.5.2 First-stage problem:

At the point of writing the deposit contract in stage one, depositors anticipate the bank's choices in stage two and therefore the depositors' no arbitrage condition $\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$ must hold in equilibrium[23]. Using this equation together with the previous first-oder condition (13) we can derive the optimal q_t as a function of k_t and $\tilde{r}_{l,t}$:

$$\hat{q}_t \equiv q_t(k_t) = \frac{1}{2\omega_2 \tilde{r}_{l,t}} \left(\omega_1 \tilde{r}_{l,t} + \sqrt{(\omega_1 \tilde{r}_{l,t})^2 - 4\omega_2 \tilde{r}_{l,t} \tilde{R}_t (1-k_t)} \right) \ . \tag{14}$$

We can now solve the first-stage problem of the banker. The bank chooses the capital structure k_t to maximize her excess profits, anticipating the $q_t(k_t)$ that will be chosen at the second stage:

$$\max_{k_t} \ \hat{q}_t \omega_1 \tilde{r}_{l,t} - \frac{\omega_2}{2} \tilde{r}_{l,t} \hat{q}_t^2 - q_t \tilde{r}_{d,t}(1-k_t) - q_t \tilde{r}_{e,t} k_t \ , \tag{15}$$

subject to the no-arbitrage condition for depositors ($\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$) and for equity providers ($\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t}$). Plugging these in and deriving, we obtain the first-order condition for k_t:

$$\omega_1 \tilde{r}_{l,t} \frac{\partial \hat{q}_t}{\partial k_t} - \tilde{\xi}_t - \frac{\omega_2}{2} \tilde{r}_{l,t} \frac{\partial \hat{q}_t^2}{\partial k_t} = 0 \ . \tag{16}$$

which (assuming an interior solution) can be solved for k_t as:

$$\hat{k}_t \equiv k_t\left(\tilde{r}_{l,t}\right) = 1 - \frac{\tilde{\xi}_t (\tilde{R}_t + \tilde{\xi}_t)(\omega_1 \tilde{r}_{l,t})^2}{\omega_2 \tilde{R}_t \tilde{r}_{l,t} \left(\tilde{R}_t + 2\tilde{\xi}_t^{\,2} \right)} \ . \tag{17}$$

3.5.3 Closing the bank model: the zero-profit condition

Since there is a continuum of identical banks, each bank behaves competitively and takes the return on investment $\tilde{r}_{l,t}$ as given. Prefect competition and free entry imply that banks will enter until there are no expected excess profits to be made. In the presence of uncertainty it is natural to focus on the case that banks make no excess profit in any future state of the world:

$$\left(\omega_1 - \frac{\omega_2}{2} q_{t-1}\right) \left(\frac{r_{k,t} + (1-\delta) Q_t}{Q_{t-1}} \right) - \frac{r_{d,t-1}}{\pi_t}(1-\hat{k}_{t-1}) - \frac{r_{e,t}}{\pi_t} \hat{k}_{t-1} = 0 \ . \tag{18}$$

Using the equity and deposit supply schedules and taking expectation over this equation we get:

$$\hat{q}_t \omega_1 \tilde{r}_{l,t} - \frac{\omega_2}{2} \tilde{r}_{l,t} \hat{q}_t^2 - \hat{k}_t \tilde{\xi}_t - \tilde{R}_t \ . \tag{19}$$

[23] Note that the agency problem arises from the fact that the bank does not consider this as a constraint of its maximization problem.

Combining (19) with the optimality conditions (14) and (17), we can derive analytical expressions for the equity ratio k_t, riskiness choice q_t (the last term in each row is an approximation under certainty equivalence and $R_t^r = R_t/E\left[\pi_{t+1}\right]$):

$$k_t = \frac{\tilde{R}_t}{\tilde{R}_t + 2\tilde{\xi}_t} \approx \frac{R_t^r}{R_t^r + 2\xi} \tag{20}$$

$$q_t = \frac{\omega_1(\tilde{\xi}_t + \tilde{R}_t)}{\omega_2(2\tilde{\xi}_t + \tilde{R}_t)} \approx \frac{\omega_1(\xi + R_t^r)}{\omega_2(2\xi + R_t^r)} \tag{21}$$

3.5.4 Properties of the banking sector equilibrium

These results for the banking sectors risk choice have four interesting implications that we first summarize in a proposition, before intuitively discussing them in turn.

Proposition 1: Be $[\tilde{r}_{l,t}, q_t, k_t]$ an equilibrium in the banking sector with interior bank choices under perfect competition. Denote the expected retun on investment in capital units by $f(q_t) \equiv \left(\omega_1 - \frac{\omega_2}{2} q_t\right) q_t$. Then:

(1) Risk decreases in the real interest rate: $\frac{\partial q_t}{\partial \tilde{R}_t} > 0$.

(2) The equity ratio increases in the real interest rate: $\frac{\partial k_t}{\partial \tilde{R}_t} > 0$.

(3) Risk taking is excessive: $q_t < \text{argmax } f(q_t)$.

(4) The expected return of an investment increases in the real interest rate: $\frac{\partial f(q_t)}{\partial \tilde{R}_t} > 0$.

(5) The expected return of an investment is a concave function of the real interest rate $\frac{\partial^2 f(q_t)}{\partial \tilde{R}_t^2} < 0$.

The proof can be found in apppendix D.

The first two results can be easily seen from equations (20) and (21). As the real risk-free rate R_t^r increases, the equity ratio k_t falls as banks substitute equity with deposits and the riskiness of the bank $(1 - q_t)$ decreases[24]. Whats the intuition behind this result? On one hand, a lower risk-free rate decreases the rate of return on capital projects, reducing the benefits of safer investments, conditional on repayment. This induces the bank to adopt a riskier investment technology. On the other hand, the lower risk-free rate reduces the cost of funding, leaving more resources available to the bank's owners in case of repayment: this force contrasts the first one, making safer investments more attractive. There is a third force: a lower risk-free interest rate means that the equity premium becomes relatively more important. As a result the bank shifts from equity to deposits, internalizing less the consequences of the risk decision and choosing a higher level of risk. The first and third effects dominate, and overall a decline in the real interest rate induces banks to choose more risk.

[24] At least up to a first order approximation, when the Λ_{t+1} terms contained in the tilde variables cancel out.

Since in a monetary model the real interest rate is a function of the nominal interest rate, which is the standard monetary policy tool, this mechanism provides an explanation for the asset risk-taking channel that we have discussed empirically in the first two sections.

The third result says that the bank's investment could have a higher expected return (in units of capital) if the bank were to choose a higher level of safety. In other words, risk taking is excessive, i.e. suboptimally high. This is due to the agency problem, which arises from limited liability and the lack of comittment/contractability of the banker regarding his risk choice. The importance of this friction can be assessed by comparing the solution of the imperfect markets bank model to the solution of the model without any frictions. The frictionless risk choice can be derived under any of the following alternative scenarios: Either both equity premium and deposit insurance are zero (which eliminates the cost disadvantage of equity and leads to 100% equity finance), or contracts are complete and deposit insurance is zero (which eliminates the agency problem and leads to 100% deposit finance), or liability is not limited and deposit insurance is zero (as before), or household invests directly into a diversified portfolio of capital projects (which eliminates the financial sector all together). Since in a frictionless model q_t is chosen to maximize the consumption value of the expected return:

$$max_{q_t} \tilde{r}_{l,t}(\omega_1 - \frac{\omega_2}{2} q_t) q_t \ ,$$

the optimal level of q_t trivially is $q_t^o = \frac{\omega_1}{\omega_2}$. Comparing the frictionless risk choice q_t^o and the choice given the friction q_t^f

$$q_t^f = q_t^o \frac{\tilde{\xi}_t + \tilde{R}_t}{2\tilde{\xi}_t + \tilde{R}_t} \approx q_t^o \frac{\xi + R_t^r}{2\xi + R_t^r} \ ,$$

we observe that the agency friction drives a wedge between the frictionless and the actually chosen risk level. This wedge has two important features. First, it is smaller than one[25], implying that under the agency problem the probability of repayment is too low, and hence banks choose excessive risk. Second, note that the wedge depends on R_t^r and that the derivative of the first order approximation of the wedge w.r.t. R_t^r is positive. This implies that the wedge increases, i.e. risk taking gets more excessive, as the real interest rate falls. As we move further away from the optimal level of risk the expected return on investments necessarily falls, which is the fourth result above.

Note that this feature of the model is consistent with the empirical finding of Ioannidou et al. [2014] and Buch et al. [2013] that the additional loan risk taking spurred by low interest rates is not fully compensated by a sufficient increase in the return on loans: As q_t decreases $(\omega_1 - \frac{\omega_2}{2} q_t)$ increases but not sufficient to avoid a drop in $(\omega_1 - \frac{\omega_2}{2} q_t) q_t$.

But not only the bank risk choice is suboptimal. Also the capital structure is chosen suboptimally. If banks could commit to choose the optimal level of risk, they would not need any skin in the game. Hence they would avoid costly equity and would finance themselves fully by

[25] This is true under certainty equivalence, i.e. up to first order approximation.

deposits: $k_t^o = 0$. Instead they choose $k_t^f = \frac{\tilde{R}_t}{\tilde{R}_t + 2\xi_t}$. The wedge between the frictionless and the actual equity ratio resembles the two features of the risk wedge. First, it is bigger than one, so there is excessive use of equity funding. Second, the equity wedge is increasing in R_t^r up to a first order approximation.

Both the risk and the capital structure choice have welfare implications. A marginal increase in q_t means a more efficient risk choice, i.e. a higher expected return, and should hence - ceteris paribus - be welfare improving. At the same time a marginal increase in k_t implies, due to the equity premium, a higher markup in the intermediation process, which distorts the consumption savings choice and hence - ceteris paribus - lowers welfare. Since both q_t and k_t are increasing functions of the real interest rate, this begs the question of whether an increase in the real rate alleviates or intensifies the misallocation due to the banking friction[26]. The answer to this question depends on the full set of general equilibrium conditions. Given the estimated model, we will later numerically verify that the positive first effect dominates, i.e. an increase in R_t^r has welfare improving consequences on the banking market[27]. The existence of these opposing welfare effects motivates our optimal policy experiments in section 5.

Finally, the last statement of the proposition implies that a mean preserving spread in the variance of the real interest rate decreases the mean of the expected return of the banks investment. As a result a policy maker who can affect the volatility but not the level if the real interest rate has an incentive to keep the real interest rate stable, at least as long as the opposing effect of the equity premium is negligible. We will discuss the implications of this property in the policy section.

3.5.5 Full model with deposit insurance and liquidation value

The simplified version of the bank's problem presented so far is useful to explain the basic mechanism. Yet deposit insurance and a non zero liquidation value are important to improve the quantitative fit of our model to the data.

The assumptions made about deposit insurance and the liquidation value imply that depositors get the maximum of the amount covered by deposit insurance and the value of the capital recovered from a failed project. That means that their return in case of default is:

$$min\left(\frac{r_{d,t}}{\pi_{t+1}}, max\left(\frac{r_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t(1-k_t)}\theta, \frac{\psi}{1-k_t}\right)\right) .$$

To make deposit insurance meaningful we assume that the liquidation value θ is small enough such that $\frac{r_{k,t+1}+(1-\delta)Q_{t+1}}{Q_t(1-k_t)}\theta < \frac{\psi}{1-k_t}$, which eliminates the inner maximum. As the following lemma, proven in the appendix, states, the outer maximum is unambigous in equilibrium[28].

[26] These two opposing forces are well known from the literature on bank capital regulation, where a raise in the capital requirements hampers efficient intermediation but leads to a more stable banks.

[27] The dominance of the risk-taking effect is intuitive for two reasons: First, while risk taking entails a real cost, the equity premium just entails a wedge but no direct real costs. Second, as the real interest rate moves up the equity premium becomes less important, so a more efficient allocation is intuitive.

[28] For this result, we abstract from the effect of uncertainty for simplicity.

Lemma: There can be no equilibrium such that the insurance cap is not binding, i.e. $\frac{r_{d,t}}{\pi_{t+1}} > \frac{\psi}{1-k_t}$.

Deposits therefore pay $\frac{\psi}{1-k_t}$ in case of default. Combining the nominal return on the the deposit funds (6) with the households no-arbitrage condition, and defining $\tilde{\psi}_t = E\left[\Lambda_{t+1}\right]\psi$, we can write the deposit supply schedule as:

$$q_t \tilde{r}_{d,t} + (1-q_t)\frac{\tilde{\psi}_t}{1-k_t} \quad = \quad \tilde{R}_t \ . \tag{22}$$

We assume that the deposit insurance scheme, which covers the gap between the insurance cap and the liquidation value for the depositors of failing banks, is financed through a variable tax on capital that is set ex post each period such that the insurance scheme breaks even. The return on loans $\tilde{r}_{l,t}$ can then be rewritten as:

$$\tilde{r}_{l,t} \equiv E_t\left[\frac{r_{k,t+1} + (1-\delta)Q_{t+1} - \tau_{t+1}}{Q_t}\right] \quad \text{where} \quad \tau_t = \frac{Q_{t-1}\frac{1-q_{t-1}}{q_{t-1}}\left(\psi - \theta\frac{r_{k,t}+(1-\delta)Q_t}{Q_{t-1}}\right)}{\omega_1 - \frac{\omega_2}{2}q_{t-1}}$$

This way the tax also perfectly offsets the distortion on the quantity of investment caused by the deposit insurance. Deposit insurance therefore influences only the funding decision of the bank and, through that, the risk choice. Hence, if q_t was chosen optimally (or was simply a parameter) the deposit insurance would not have any effect.

The same procedure as outlined above can be applied to obtain closed-form solutions[29] for the risk choice and the equity ratio. The solutions can be found in appendix C. Note that the equilibrium characterizations in subsection 3.5.4 remain valid. In particular note that the deviation of the chosen risk (equity ratio) from the optimal level decreases (increases) in the real interest rate. Given our estimation, the risk effect dominates in terms of welfare implications. The intuition for the risk taking channel is similar as before.

Deposit insurance makes deposits cheaper relative to equity: as a result, the bank will demand more deposits and choose a riskier investment portfolio. Deposit insurance furthermore strengthens the risk-taking channel, which is now affected not only by the importance of the equity premium relative to the real interest rate, but also by the importance the the deposit insurance cap relative to the real interest rate. On the other hand, the efficient risk level is not affected by the deposit insurance.

The liquidation value on the other hand is irrelevant for the banks' and investors' choice since it is assumed to be smaller than deposit insurance. Yet it eases the excessiveness of risk taking since it increases the optimal level of risk: $q_t^o = \frac{\omega_1-\theta}{\omega_2}$.

Finally, we would like to point out that none of the results in proposition 1 is due to the

[29] In this case, one needs to apply the adjusted deposit supply schedule (22) and to make sensible assumptions about the relative size of parameters and about the root when solving the zero-profit equation.

functional form for the risk return trade-off we have assumed. The statement continues to hold even for a generic expected return function $f(q_t)$ under realtively weak assumptions. These additional assumptions are sufficient but non neccessary. For a proof and a discussion of these assumptions see the appendix.

Proposition 2:

(1) Proposition 1 holds for the full bank model with deposit insurance as well.

(2) Statements (1)-(4) of proposition 1 hold for a generic expected return function $f(q_t)$ with deposit insurance under the additional assumptions that the expected return function $f(q_t)$ satisfies $f(q_t) \geq 0$, $f''(q_t) < 0$, $f'''(q_t) \leq 0$, $f''''(q_t) \leq 0$. Statement (5) holds if furthermore either the default probability is low relative to the parameters $\frac{q_t}{(1-q_t)}\tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t$ or there is no deposit insurance $\tilde{\psi}_t = 0$.

3.6 Monetary and fiscal policy

The central bank follows a nominal interest rate rule, targeting inflation and output deviations from the steady state:

$$R_t - \bar{R} = (1-\rho)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) + \rho\left(R_{t-1} - \bar{R}\right) + \varepsilon_t^R, \tag{23}$$

where ρ is a smoothing parameter, the hat symbol denotes percentage deviations from the steady state values, $\bar{R} = \frac{\pi_{ss}}{\beta}$ is the steady state nominal interest rate, and ε_t^R is an iid monetary policy shock.

In addition, the fiscal authority finances a stochastic expenditure stream $g_y \bar{Y} \varepsilon_t^G$:

$$ln\left(\varepsilon_t^G\right) = \rho_g ln\left(\varepsilon_{t-1}^G\right) + u_t^G + \rho_{GA} u_t^A \ ,$$

where we are allowing for a correlation between exogenous spending and innovations to total factor productivity[30]. For simplicity we rule out government debt ($s_t = 0$), implying that all expenditures are financed by lump sum taxes; i.e. $g_y \bar{Y} \varepsilon_t^G = T_t$.

4 Steady-state and dynamic implications of excessive risk taking in the estimated model

We have embedded our risk-taking channel in a medium-scale model which closely resembles the non-linear version of Smets and Wouters [2007][31] and we next estimate the model parameters

[30] This is a shortcut to take exports into account. Productivity innovations might rise exports in the data, and a way to capture it in a closed-economy model such as ours is to allow for $\rho_{GA} \neq 0$ as in Smets and Wouters [2007].

[31] Of all nominal and real frictions considered in Smets and Wouters [2007], we are missing the one concerning capital utilization, which is shown to be of secondary importance once wage stickiness is taken into account.

using Bayesian techniques. This serves two purposes. First, in order to perform a sound monetary policy evaluation we need a quantitative model that is able to replicate key empirical moments of the data. Second, it helps to understand whether our channel is quantitatively important compared to other monetary and real frictions that affect the monetary policy trade-off.

In this section we first discuss the estimation,and then examine the steady-state and dynamic macroeconomic implications of the risk-taking channel, before we turn to discussing optimal monetary policy in section 5.

4.1 Model estimation

We estimate a linearized version of the model with Bayesian techniques using eight US macroeconomic time series from 1985q1 to 2007q3. These include the seven series used by Smets and Wouters [2007], i.e. the federal funds rate, the log of hours worked, inflation and the growth rates in the real hourly wage and in per-capita real GDP, real consumption, and real investment. Furthermore we add a series of banking sector leverage, in order to improve the identification of the banking sector parameters. For a full description of the data we refer to appendix A and to the supplementary material of Smets and Wouters [2007]. The observation equations, linking the observed time series to the variables in the model, as well as the prior specifications can be found in appendix C. The priors of the non-bank parameters are standard, the priors for the banking sector are motivated by historical averages. Note that, instead of directly forming priors about ω_2 (risk return trade-off) and ψ (deposit insurance) we write the steady state values of the equity ratio $\bar{k}$ and the default rate $\bar{q}$ as functions of these parameters and form priors on the steady state values. Finally we normalize the units of capital versus final goods by setting ω_1 (return of most risky asset) such that one unit of final good is expected to produce one unit of capital good in steady state.

Table 2 summarizes the parameter values, which are broadly in line with existing empirical estimates for the US. The steady state inflation rate is estimated to be about 2.5% on an annual basis. The discount factor is estimated to be 0.996 implying an annual steady-state real interest rate of around 1.5%. Wages appear to be slower moving than prices: wages are reoptimized every year and a half, while prices are reoptimized approximately every six months. The coefficient of relative risk aversion σ_c is estimated to be 1.90, well above its prior mean. The posterior estimates of the Taylor rule parameters show a strong response to inflation and output (respectively 1.83 and 0.05), and a high degree of interest rate smoothing (0.85). The data are very informative on the parameters affecting the shock distribution, and no shock plays a predominant role in explaining output fluctuations, though the monetary policy shock and the two markup shocks are found to be relatively less important. Of the banking sector variables all but one are well identified. The equity ratio is estimated to be 8.3%, the posterior mean of the equity premium is around an annualized value of 6.6%, in line with the empirical estimates of Mehra and Prescott [1985] and the recovery rate is about 50%. While the steady state default rate is not well

Furthermore, since we will use one more time series we have added a time preference shock.

identified, sensitivity tests have shown that this parameter is only of very small quantitative relevance. For the following quantitative analysis we set the parameters to their posterior means.

Comparing our model to the Smets and Wouters model (i.e. the model without he banking sector) we find that the parameter estimates are rather similar. Furthermore, the fit of our model is not significantly different from theirs. The posterior odds ratio, not using the leverage data, is 10.63 which means that our additional channel is slightly more favoured by the data.

4.2 Steady-state and dynamic implication of excessive risk taking

In Table 3 we compare the non-stochastic steady state of the model with banking frictions (henceforth bank model) with that of the model without banking frictions. In the latter model the capital structure is undetermined and risk is equal to the socially optimal level. For the given set of estimated values, the optimum is a corner solution: $\overline{q}^o = 1$. In the bank model, the capital ratio is below one, implying that banks do not fully internalize the implications of their risk choice, and hence choose an excessive level of risk. This implies that the capital production technology is inefficient. Consequently, the bank economy is under-capitalized in the steady state, and output, consumption and welfare are inefficiently low.

To understand the dynamic effects of the risk taking channel, we now assess how the propagation mechanism of the model differs if a risk-taking channel is present. For illustration, we discuss an expansionary monetary policy shock. As we have just seen, the economy without financial frictions and the bank economy have different steady states. This makes dynamic comparisons of the two models difficult, since both the different behaviors of q_t and k_t as well as the different steady states imply different dynamics. In order not to mix the two effects, we focus on comparing models with the same steady state. For this purpose we alter the model without financial frictions by treating the risk choice q_t and the equity ratio k_t as parameters, which we set to the steady state values of the bank model. This model, henceforth benchmark model, has not only the same steady state as the bank model but also corresponds to a standard New Keynesian model with a small markup in capital markets.

In figure 3 we compare the dynamic responses in the bank model (solid red lines) and in benchmark model (dashed blue lines) to an expansionary monetary policy shock. A monetary policy expansion triggers a set of standard reactions, which are evident in the benchmark model. An unexpected fall in the nominal risk-free rate causes a drop in the real interest rate, since prices are sticky. Consequently, consumption is shifted forward, firms that can adjust the price do so causing an increase in inflation, while the remaining firms increase production. The risk-taking channel adds two further elements as both the risk level and the capital structure chosen by the bank respond to the real interest rate movement. On impact, the drop in the real interest rate cause banks to substitute equity for deposits, and to take more risk (lower loan safety). The risk choice therefore moves further away from the optimal level and the expected return on aggregate investment drops.[32] This implies that households would have to invest more to maintain the

[32] As the equity ratio is reduced the distortion through the equity premium is also reduced which per se reduces

Table 2: **Model estimation**: prior and posterior values

	parameter	prior shape	prior mean	prior std	post. mean	90% HPD interval	
			model parameters				
μ_y	trend growth	normal	0.4	0.1	0.4348	0.4012	0.4649
μ_l	labor normalization	normal	0	2	0.3821	-0.9944	1.7173
α	output share	normal	0.3	0.05	0.2029	0.1682	0.2423
$100\frac{1-\beta}{\beta}$	real rate in %	normal	0.25	0.1	0.4063	0.297	0.5297
$\bar{\varepsilon}^P$	price markup	normal	1.25	0.12	1.4907	1.3436	1.6451
$\bar{\pi}$	inflation in %	gamma	0.62	0.1	0.6352	0.5098	0.7865
ϕ_π	TR weight on inflation	normal	1.5	0.25	1.825	1.4869	2.1242
ϕ_y	TR weight on output	normal	0.12	0.05	0.0508	-0.0258	0.1231
ρ	TR persistence	beta	0.75	0.1	0.8493	0.8074	0.8912
κ	investment adj. costs	normal	4	1.5	6.6501	4.7567	8.654
ι	habits	normal	0.7	0.1	0.7277	0.6152	0.8462
σ_c	risk aversion	gamma	1.5	0.375	1.9071	1.4588	2.3804
σ_l	disutility from labor	gamma	2	0.75	2.2369	1.2607	3.3389
θ	liquidation value	normal	0.5	0.05	0.5021	0.4295	0.5717
Ψ	deposit insurance	normal	0.8	0.05	0.7881	0.7328	0.8489
ω_2	risk return trade-off	normal	0.41	0.05	0.4122	0.3335	0.4984
ξ	equity premium	normal	0.015	0.01	0.0166	0.0017	0.0307
λ_p	price calvo parameter	beta	0.5	0.1	0.5767	0.4967	0.6642
λ_w	wage calvo parameter	beta	0.5	0.1	0.8201	0.7807	0.864
γ_p	price indexation	beta	0.5	0.15	0.2912	0.1187	0.452
γ_w	wage indexation	beta	0.5	0.15	0.4881	0.2401	0.7229
			shocks				
σ_A	stdev TFP	uniform	0.5	0.2	0.9167	0.8572	0.9684
σ_B	stdev preference	uniform	0.5	0.2	0.2491	0.0647	0.4253
σ_G	stdev govt. spending	uniform	0.5	0.2	0.981	0.9688	0.9938
σ_I	stdev investment	uniform	0.5	0.2	0.689	0.5639	0.8077
σ_P	stdev price markup	uniform	0.5	0.2	0.9362	0.883	0.9905
σ_R	stdev monetary	uniform	0.5	0.2	0.5933	0.4898	0.7103
σ_W	stdev wage markup	uniform	0.5	0.2	0.8823	0.833	0.9406
ρ_A	persistence TFP	beta	0	1	0.3788	0.3266	0.4268
ρ_B	persistence preference	beta	0	10	3.1301	1.7896	4.3806
ρ_G	persistence gov. spending	beta	0	10	2.2331	1.9541	2.5321
ρ_I	persistence investment shock	beta	0	10	4.9205	2.9521	6.633
ρ_P	persistence price markup	beta	0	1	0.098	0.0747	0.1228
ρ_R	persistence monetary	beta	0	1	0.0977	0.0837	0.1144
ρ_W	persistence wage markup	beta	0	1	0.4952	0.4322	0.5692
$\rho_{G,A}$	correlation gov. spending & TFP	uniform	5	2.8868	2.3345	1.2085	3.4284
m_p	MA component of price markup	beta	0.5	0.2	0.5861	0.4103	0.7905
m_w	MA component of wage markup	beta	0.5	0.2	0.9707	0.946	0.9948

Table 3: **Steady state comparison**: The model without banking sector frictions features an undetermined equity ratio and risk equal to the socially optimal level; i.e. $\overline{q}^o = 1$. Parameters are fixed to the posterior mean estimates of the bank model reported in table 2.

VARIABLE	MODEL WITH BANKING FRICTIONS	MODEL WITHOUT BANKING FRICTIONS
q	0.954	1.000
k	0.181	-
C	0.632	0.770
K	5.690	15.364
I	0.142	0.157
L	1.000	0.987
Y	0.944	1.130
π	1.006	1.006
R	1.010	1.010
Welfare	-1947.876	-1626.163

same path of capital as in the benchmark case, which would entail a lower consumption path. Yet this would not be optimal because of consumption smoothing and because the expected return on investment is lower. Therefore investment rises by less then what would be needed to compensate the loss in investment efficiency, which makes the capital stock decline considerably. Overall, agents are worse off (in terms of welfare) in the bank model than in the benchmark economy.

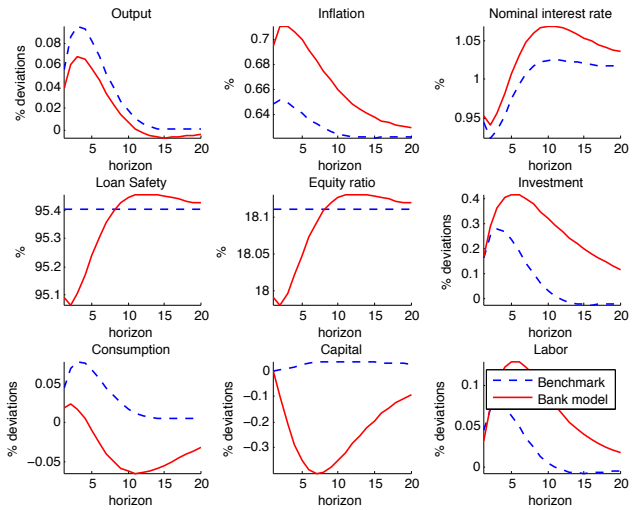
Lastly, note that the responses shown in figures 3 are in accordance with the observations in section 2, in particular with the empirical finding that the response of risk is proportional to that of the interest rate.

5 Monetary policy with a risk-taking channel

We have seen that the risk-taking channel has both static and dynamic effects. While monetary policy does not affect the nonstochastic steady state, it can influence the dynamics of the economy. In particular, it can influence the real rate and hence affect bank risk taking. But are these additional mechanisms implied by the risk-taking channel actually quantitatively significant for monetary policy? To answer this question we determine the *optimal simple implementable* monetary policy rule in the risk-taking channel model. We then compare this policy to the optimal policy in the benchmark economy with the same steady state but without the risk-taking channel. This comparison has an interesting interpretation. Suppose that the actual economy features the risk-taking channel (the bank model), but that the central bank is unaware of this channel and believes that risk is an irrelevant constant from her point of view. The central bank would then implement optimal policy based on a wrong model (the benchmark model). Our comparison

the cost of capital. Yet this effect is tiny relative to the increase in the cost of capital due to lower investment efficiency.

Figure 3: **Monetary policy shock in the bank and benchmark models:** dynamic responses in the bank model (solid red lines) and in the benchmark model (dashed blue lines) to an expansionary monetary policy shock, based on the estimated parameter values in Table 2.



then answers the question of how important understanding the risk-taking channel is, in terms of optimal policy and welfare.

Notice that in this paper we consider a central bank that has no policy tools besides the interest rate. With a second instrument, such as capital regulation, the central bank could do better or even eliminate the friction. Exploring optimal macroprudential regulation is however beyond the scope of the present paper[33].

In what follows, we first discuss the concept of the optimal simple implementable monetary policy rule, and then present our results.

5.1 The central bank problem

We follow Schmitt-Grohe and Uribe [2007] and characterize optimal monetary policy as the policy rule that maximizes welfare among the class of simple, implementable interest-rate feedback rules[34] given by:

$$R_t - \bar{R} = \phi_\pi \hat{\pi}_{t+s} + \phi_y \hat{y}_{t+s} + \phi_k \hat{k}_{t+s} + \rho \left(R_{t-1} - \bar{R} \right) \ . \tag{24}$$

[33]For a thorough analysis of macroprudential policy in an economy with bank risk-taking see Collard et al. [2012].

[34]The implementability criterion requires uniqueness of the rational expectations equilibrium, while simplicity requires the interest rate to be a function of readily observable variables. For a complete discussion, see Schmitt-Grohe and Uribe [2007]. Notice that we drop their second requirement for implementability which is that an implementable rule must avoid regular zero lower bound violations.

where the hat symbol denotes percentage deviations from the steady state, and the index s allows for forward- or contemporaneous-looking rules (respectively by setting $s = 1$ or $s = 0$). The policy rule specification (24) is chosen for its generality, as it encompasses both standard Taylor-type rules (setting $\phi_k = 0$), and the possibility that the central bank reacts to banking sector leverage, the inverse of the equity ratio k ($\phi_k \neq 0$). A fall in the equity ratio implies that banks increase their debt financing, i.e. they increase leverage. As a consequence banks internalize less the downside risk of their investments, and choose loans with a higher default probability. Hence, a fall in the equity ratio signals an increase in risk taking, to which the central bank may want to respond by increasing the interest rate. We choose not to let the interest rate depend on risk taking directly, because the latter is not a readily observable variable. We furthermore impose that the parameters on output and inflation have to be non-negative and that the inertia parameter ρ has to be between 0 and 1.

The welfare criterion, that defines the optimal parameter combination for rule (24), is the household's conditional lifetime utility:

$$V \equiv E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u(c_t, L_t) \ . \tag{25}$$

This measure is commonly used in the literature and yields the expected lifetime utility of the representative household, conditional on the economy being at the deterministic steady state. To be able to make meaningful comparisons of welfare levels we furthermore define the measure Ω as the fraction of the consumption stream that a household would need to receive as a transfer under the better rule to be equally well of as under the other rule. If o denotes the optimal and s another suboptimal rule, this fraction Ω is implicitly defined by the equation:

$$V^o = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u((1+\Omega)c_t^s, L_t^s) \ .$$

5.2 Findings

Using the welfare criterion just described we numerically determine the coefficients of the optimal simple implementable rules in the benchmark and in the bank model using second order approximations around the non-stochastic steady state. Table 4 reports the optimal coefficients for 5 different specifications of the monetary policy rule: contemporaneous and forward-looking, without inertia and with optimal inertia, without and with reaction to current leverage[35]. The coefficients of the optimal rules generally vary greatly between the two models. A set of results, which are robust across policy rule and estimation[36] specifications, are worth noticing.

First, the optimal coefficients on inflation and output deviations are smaller in the bank model

[35] Note that reacting to leverage is impossible in the benchmark model where risk and leverage are constant.

[36] We have experimented with different estimation samples and different fixed parameters: while the optimized parameters and transfers slightly change, the qualitative results discussed in the text are very robust. Results are available upon request.

Table 4: **Optimal simple rules**: optimal parameters for policy rules of the class $R_t - \bar{R} = \phi_\pi \hat{\pi}_{t+s} + \phi_y \hat{y}_{t+s} + \phi_y \hat{k}_{t+s} + \rho \left(R_{t-1} - \bar{R}\right)$. The hat symbol denotes percentage deviations from the steady state. Current- (forward-) looking rules let the interest rate react to current (future) deviations of variables from their steady state values. Ω is the welfare cost (in % of the consumption stream) associated to implementing in the bank model the optimal policy rule of the benchmark model. For the benchmark model the restriction $\phi_k = 0$ is irrelevant, since the equity ratio is a constant in the benchmark model. Entries without digits indicate restricted parameters.

		benchmark model			*bank model*				
s	rule	ρ	$\phi_{\pi_{t+s}}$	$\phi_{y_{t+s}}$	ρ	$\phi_{\pi_{t+s}}$	$\phi_{y_{t+s}}$	ϕ_{k_t}	Ω
0	$\phi_k, \rho = 0$	0	5.909	0.132	0	2.622	0.092	0	0.234
0	$\phi_k = 0$	0.000	5.909	0.132	1.000	0.110	0.009	0	0.407
0	$\rho = 0$	0	5.909	0.132	0	100.000	5.390	-12.505	0.240
1	$\phi_k, \rho = 0$	0	21.182	0.341	0	3.213	0.118	0	0.923
1	$\phi_k = 0$	0.000	21.182	0.341	1.000	0.175	0.014	0	1.108

compared to the benchmark model. For any given change in inflation and output, the nominal interest rate should move less if a risk-taking channel is present. Furthermore, if the central bank can optimize over its smoothing parameter, then full interest rate smoothing is optimal in the bank model. Given that the optimal output coefficient is close to zero, the optimal rule is close to a stable real interest rate rule. In doing so, the central bank limits fluctuations in the real interest rate and hence in risk taking and slightly raises the average level of q towards the efficient value, as it can be seen in table 5. At the same time inflation is significantly more volatile under the optimal rule. If a risk-taking channel is present, the central bank should accept higher inflation fluctuations in order to reduce the distortion stemming from risk taking.

What is the intuition behind this result? Monetary policy cannot affect the deterministic steady state, but it can control the real interest rate and therefore the fluctuations in excessive risk. Upward movements of the real interest rate are welfare enhancing since they lower the level of risk taking towards the efficient level, whereas downward changes of the real interest rate lead to even more excessive risk taking. But this does not mean that movements in the interest rate are irrelevant. Since the expected return on investments $f(q_t) = q_t(\omega_1 - \frac{\omega_2}{2} q_t)$ is convex in the real interest rate, as we have shown above, a mean preserving increase in the volatility of the real rate reduces the average expected return of investments $f(q_t)$. Therefore the risk-taking channel provides a motive for keeping the real interest rate constant. This adds a third dimension to the central bank problem: besides trading off inflation versus output stabilization the central bank would now also like to stabilize the real interest rate. As a result the optimal policy is tilted away from inflation stabilization.

Second, it is optimal for the central bank to respond to leverage. The reason is, that leverage signals risk taking (there is a one to one mapping from leverage to risk). Since leverage and risk taking do not only depend on the nominal interest rate (the instrument), but also on expected inflation, this allows the central bank to steer risk taking more precisely. Once risk taking is taken care of the monetary authority can again respond aggressively to inflation deviations from the steady state. Yet the additional welfare gains of augmenting the policy rule with a response

Table 5: **Differences in moments associated to the optimal simple rules in the benchmark and in the bank model**: main entries are the % differences in the mean and standard deviation associated to applying the different optimal rules in the bank model. The first entry, for example, indicates that under the optimal bank policy rule average risk would be 0.042% lower than if the rule, optimal for the benchmark model, had been applied.

		mean					*standard deviation*				
s	rule	q	R	π	y	c	q	R	π	y	c
0	$\phi_k, \rho = 0$	0.042	-0.001	-0.033	0.149	0.233	-47.618	-47.618	55.000	1.124	-2.112
0	$\phi_k = 0$	0.060	0.002	-0.063	0.209	0.319	-69.242	-69.242	89.646	0.474	-0.449
0	$\rho = 0$	0.043	-0.001	-0.039	0.156	0.243	-48.935	-48.935	55.415	1.101	-1.916
1	$\phi_k, \rho = 0$	0.124	-0.006	-0.046	0.551	0.859	-68.276	-68.276	100.578	1.106	-7.116
1	$\phi_k = 0$	0.142	-0.004	-0.075	0.617	0.953	-82.417	-82.417	134.242	0.896	-6.800

to leverage are quantitatively small.

To understand *how different* the equilibria associated to the two optimal rules are, and therefore how important it is for the central bank to take the risk-taking channel into account, we compute the cost Ω of applying the rule that is optimal for the benchmark model in the bank model. These costs, expressed in % of the lifetime consumption stream, are reported in the last column of table 4. Though the costs vary a lot across policy specifications, they are always non negligible. For the best performing policy (last row of table 4), the costs of applying the benchmark policy in the bank model is around 1.11% of the lifetime consumption stream in every quarter. Hence, internalizing the feedback effect that the nominal interest rate has on bank risk taking pays off in terms of welfare.

6 Conclusions

The recent financial crisis has highlighted the importance of monitoring the level of risk to which the financial sector is exposed. In this paper we focus on one aspect of financial sector risk, ex-ante bank asset risk, and on how the latter can be influenced by monetary policy.

First, we provide new empirical evidence of the impact of monetary policy on bank risk taking. We document that unexpected monetary policy shocks, identified through sign restrictions in a classical VAR framework, increase a measure for ex-ante bank risk taking in the US. This conclusion, robust to using a recursive identification scheme, is compatible with the monetary policy transmission mechanism in the theoretical model that we build to explain the effects of monetary policy on risk taking.

For this purpose, we extend the work of Dell'Ariccia et al. [2014] and build a dynamic general-equilibrium model where low levels of the risk-free interest rate induce banks to extend credit to riskier borrowers. At the core of this mechanism is an agency problem between depositors and equity providers: the latter choose the level of risk but are protected by limited liability. In general equilibrium, this friction leads to a steady state with excessive risk taking, and inefficiently

low levels of capital, output and consumption. Furthermore, risk taking alters the dynamic response of the economy to shocks. In particular, an expansionary monetary policy shock has unintended consequences: because banks choose a riskier investment strategy, the growth of capital, output and consumption will be lower than in the model without the risk-taking channel.

In order to assess the importance of the risk-taking channel and to study optimal monetary policy, we estimate the model on US data using Bayesian methods. We study optimal monetary policy in this environment, using optimal simple rules. Our results suggest that, if a risk-taking channel is present and the interest rate is the only instrument available to the monetary authority, the optimal rule should stabilize the path of the real interest rate more than without the risk-taking channel. This implies that the central bank should tolerate higher inflation volatility in order to reduce welfare detrimental fluctuations in risk taking. The welfare gains of taking the channel into account are found to be non negligible, yet a direct response to leverage is not of primary importance. Nevertheless, these results do not rule out that an alternative instrument could perform better at maximizing consumer welfare, an issue that deserves to be investigated in future work.

References

Elena Afanasyeva and Jochen Guentner. Lending Standards, Credit Booms, and Monetary Policy. Technical report, mimeo, July 2015.

Ignazio Angeloni and Ester Faia. Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, 60(3):311–324, April 2013.

Ignazio Angeloni, Ester Faia, and Marco Lo Duca. Monetary policy and risk taking. *Journal of Economic Dynamics and Control*, 52:285–307, March 2015.

Ben S. Bernanke, Mark Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. In John B. Taylor and Michael D. Woodford, editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier, 1999.

Claudio Borio and Haibin Zhu. Capital regulation, risk-taking and monetary policy: a missing link in the transmission mechanism? (268), Dec 2008.

Claudia Buch, Sandra Eickmeier, and Esteban Prieto. In search for yield? survey-based evidence on bank risk taking. Bundesbank working papers, Deutsche Bundesbank, March 2013.

Guillermo A. Calvo. Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics*, 12(3):383–398, September 1983.

Ian Christensen, Césaire Meh, and Kevin Moran. Bank Leverage Regulation and Macroeconomic Dynamics. Working Papers 11-32, Bank of Canada, 2011.

F. Collard, H. Dellas, B. Diba, and O. Loisel. Optimal Monetary and Prudential Policies. Technical report, 2012.

Oliver de Groot. The Risk Channel of Monetary Policy. *International Journal of Central Banking*, 10(2):115–160, June 2014.

Giovanni Dell'Ariccia, Luc Laeven, and Gustavo Suarez. Bank Leverage and Monetary Policy's Risk-Taking Channel: Evidence from the United States . Working Paper 13/143, IMF, June 2013.

Giovanni Dell'Ariccia, Luc Laeven, and Robert Marquez. Real interest rates, leverage, and bank risk-taking. *Journal of Economic Theory*, 149(0):65 – 99, 2014.

Asli Demirgüç-Kunt, Baybars Karacaovali, and Luc Laeven. Deposit Insurance Around the World: A Comprehensive Database. World Bank Policy Research Working Paper 3628, 2005.

Mark Gertler, Nobuhiro Kiyotaki, and Albert Queralto. Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics*, 59(S):S17–S34, 2012.

Vasso Ioannidou, Steven Ongena, and Jose-Lui Peydro. Monetary Policy, Risk-Taking and Pricing: Evidence from a Quasi-Natural Experiment. Discussion Paper 2009-04S, European Banking Center, June 2014.

Gabriel Jimenez, Steven Ongena, Jose Luis Peydro, and Jesus Saurina. Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking? *Econometrica*, 82(2):463–505, 2014.

Miles S Kimball. The Quantitative Analytics of the Basic Neomonetarist Model. *Journal of Money, Credit and Banking*, 27(4):1241–77, November 1995.

Rajnish Mehra and Edward C. Prescott. The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2):145–161, March 1985.

Stephanie Schmitt-Grohe and Martin Uribe. Optimal, Simple, and Implementable Monetary and Fiscal Rules . *Journal of Monetary Economics*, 54(6):1702–1725, September 2007.

Frank Smets and Rafael Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606, 2007. doi: 10.1257/aer.97.3.586.

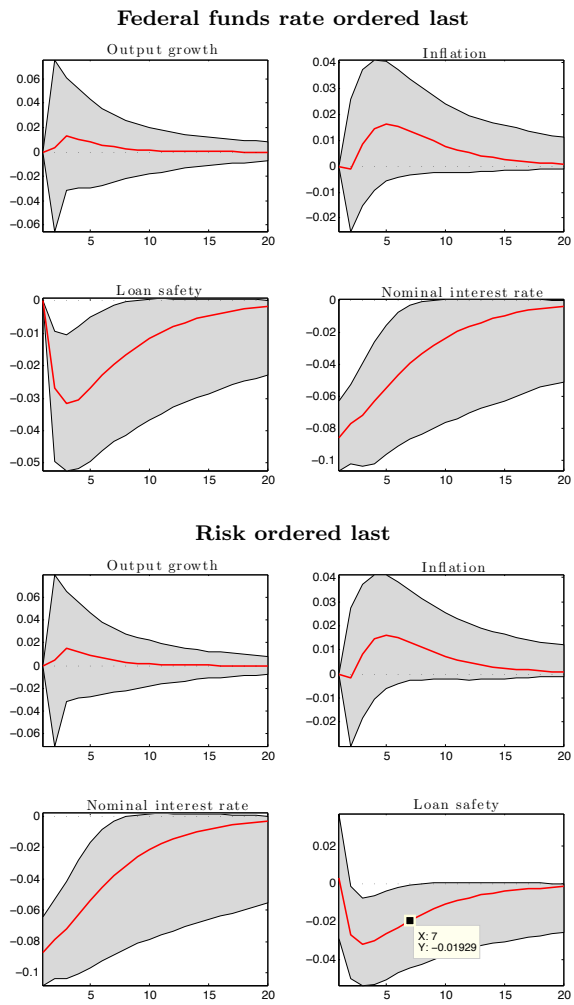
Appendix A: Data description

Table 6: **Data description**: All variables are expressed in per-capita terms (divided by N). Hours are measured as $H_1 \cdot H_2/N$ where H_1 is converted into an index. The nominal wage W is deflated by the GDP deflator. All indexes are adjusted such that $2009 = 100$. The estimation sample spans from 1988Q1 to 2007Q3 for the DSGE and from 1997Q2 to 2007Q3 for the VAR.

SYMBOL	SERIES	MNEMONIC	UNIT	SOURCE
Y	REAL GROSS DOMESTIC PRODUCT	GDPC96	BN. USD	FRED / BEA
P	GDP DEFLATOR	GDPDEF	INDEX	FRED / BEA
R	EFFECTIVE FEDERAL FUNDS RATE	FEDFUNDS	%	FRED / BOARD OF GOVERNORS
C	PERSONAL CONSUMPTION EXPENDITURE	PCEC	BN. USD	FRED / BEA
I	FIXED PRIVATE INVESTMENT	FPI	BN. USD	FRED / BEA
H_1	CIVILIAN EMPLOYMENT	CE16OV	THOUSANDS	FRED / BLS
H_2	NONFARM BUSINESS (..) HOURS	PRS85006023	INDEX	DEPARTMENT OF LABOR
W	NONFARM BUSINESS (..) HOURLY COMPENSATION	PRS85006103	INDEX	DEPARTMENT OF LABOR
N	CIVILIAN POPULATION	LNS1000000	0CE16OV	BLS
q	AVERAGE WEIGHTED LOAN RISK		%	BOARD OF GOVERNORS
k	TOTAL EQUITY TO TOTAL ASSETS FOR BANKS	EQTA	%	FRED

Appendix B: Empirical motivation - recursive identification scheme

Figure 4: **An expansionary monetary policy shock - Recursive identification scheme**. Error bands correspond to 90% confidence intervalsobtained by bootstrap. Loan safety is defined as the inverse of the average loan risk rating, standarized to take values between 0 and 1. The transformation and units of the variables match those used for the DSGE estimation and are summarized in appendix A. See text for further details.



Appendix C: The full model - Equilibrium and estimation details

Model summary: We report here the equations that enter the non-linear model, grouped by sector. Note that following Smets and Wouters [2007] we assume that different varieties of intermediate goods and of labor are assembled through a Kimball [1995] aggregator, rather than a Dixit-Stiglitz one. This latter assumption is introduced in order to obtain estimates of price and wage rigidity that are closer to micro estimates, but we do not derive the recursive formulation here (see e.g. **?**). Also note that the variables regarding the equity and deposit funds R^d and R^e have been substituted out.

Competitive equilbrium: The competitive equilibrium is a path of 40 variables (Λ, K, L, y, l, c, q, k, d, e, π, r_k, r_d, r_e, R, W, mc, o^{new}, o, $\pi^\star$, Z_{p1}, Z_{p2}, Z_{p3}, Z_{w1}, Z_{w2}, Z_{w3}, Δ_{p1}, Δ_{p2}, Δ_{p3}, Δ_{p4}, Δ_{w1}, Δ_{w2}, $\Delta_{w3}\Delta_{p4}$, $W^\star$, i, $\tilde{R}$, $\tilde{\xi}$, $\tilde{\psi}$, τ) that satisfy the following 40 equations at each point in time given initial conditions and the exogenous shock processes ε^A, ε^B, ε^G, ε^I, ε^P, ε^Q, ε^R, ε^W.

Household

$$\Lambda_t = \varepsilon_t^B \left(c_t - \iota c_{t-1}\right)^{-\sigma_C} - \beta\iota E_t\left[\varepsilon_{t+1}^B \left(c_{t+1} - \iota c_t\right)^{-\sigma_C}\right] \tag{26}$$

$$E_t\left[\Lambda_{t+1}\frac{q_t r_{d,t+1} + (1-q_t)\frac{\psi}{1-k_t}\pi_{t+1}}{\pi_{t+1}}\right] = E_t\left[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right] \tag{27}$$

$$E_t\left[\Lambda_{t+1}\frac{q_t r_{e,t+1} - \xi\pi_{t+1}}{\pi_{t+1}}\right] = E_t\left[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right] \tag{28}$$

$$\Lambda_t = \beta E_t\left[\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right] , \tag{29}$$

$$y_t = c_t + i_t + g_y\bar{Y}\varepsilon_t^G \tag{30}$$

Goods sector

$$\frac{L_t^d}{K_{t-1}}\frac{\alpha}{1-\alpha} = \frac{r_{k,t}}{w_t} \tag{31}$$

$$mc_t = \frac{1}{A_t}\alpha^{-\alpha} r_{kt}^{\alpha} w_t^{1-\alpha}\left(1-\alpha\right)^{\alpha-1} \tag{32}$$

$$\pi_t^\star = \frac{\epsilon_p\left(1+k_p\right)}{\epsilon_p\left(1+k_p\right)-1}\frac{Z_{p1,t}}{Z_{p2,t}} + \frac{k_p}{\epsilon_p - 1}\left(\pi_t^\star\right)^{1+\epsilon_p(1+k_p)}\frac{Z_{p3,t}}{Z_{p2,t}} \tag{33}$$

$$Z_{p1,t} = (1-\tau_{p,t})\Lambda_t mc_t y_t \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} + \beta\lambda_p E_t\left[\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{\epsilon_p(1+k_p)} Z_{p1,t+1}\right] \tag{34}$$

$$Z_{p2,t} = (1-\tau_{p,t})\Lambda_t y_t \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} + \beta\lambda_p E_t\left[\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{\epsilon_p(1+k_p)-1} Z_{p2,t+1}\right] \tag{35}$$

$$Z_{p3,t} = \Lambda_t y_t + \beta\lambda_p E_t\left[\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{-1} Z_{p3,t+1}\right] \tag{36}$$

$$\Delta_{p1,t} = (1-\lambda_p)\left(\pi_t^\star\right)^{1-\epsilon_p(1+k_p)} + \lambda_p\Delta_{p1,t-1}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{\epsilon_p(1+k_p)-1} \tag{37}$$

$$1 = \frac{1}{1+k_p}\Delta_{p1,t}^{1/(1-\epsilon_p(1+k_p))} + \frac{k_p}{1+k_p}\Delta_{p2,t} \tag{38}$$

$$\Delta_{p2,t} = (1-\lambda_p)\,\pi_t^\star + \lambda_p\Delta_{p2,t-1}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{-1} \tag{39}$$

$$\Delta_{p3,t} = \frac{1}{1+k_p}\Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))}\Delta_{p4,t} + \frac{k_p}{1+k_p} \tag{40}$$

$$\Delta_{p4,t} = (1-\lambda_p)\left(\pi_t^\star\right)^{-\epsilon_p(1+k_p)} + \lambda_p\Delta_{p4,t-1}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{\epsilon_p(1+k_p)} \tag{41}$$

$$A_t K_{t-1}^{\alpha}\left(\frac{L_t}{\Delta_{p3,t}}\right)^{1-\alpha} = \Delta_{p3,t} y_t \tag{42}$$

Labor sector

$$w_t^\star = \frac{\epsilon_w(1+k_w)}{\epsilon_w(1+k_w)-1}\frac{Z_{w1,t}}{Z_{w2,t}} + \frac{k_w}{\epsilon_w-1}\left(w_t^\star\right)^{1+\epsilon_p(1+k_p)}\frac{Z_{w3,t}}{Z_{w2,t}} \tag{43}$$

$$\begin{aligned} Z_{w1,t} \;=\; & \varepsilon_t^B \bar{L} L_t^{1+\sigma_L} w_t^{\epsilon_w(1+k_w)}\left(C_t-\iota C_{t-1}\right)^{1-\sigma_c}\exp\left(\bar{L}\frac{\sigma_c-1}{1+\sigma_l}L_t^{1+\sigma_L}\right)\Delta_{w1,t}^{\epsilon_w(1+k_w)/(1-\epsilon_w(1+k_w))} \\ & +\beta\lambda_w E_t\left[\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_w}\bar{\pi}^{1-\gamma_w}}\right)^{\epsilon_w(1+k_w)} Z_{w1,t}\right] \end{aligned} \tag{44}$$

$$Z_{w2,t} = (1-\tau_{w,t})\Lambda_t L_t\left[w\Delta_{w1,t}^{1/(1-\epsilon_w(1+k_w))}\right]^{\epsilon_w(1+k_w)} + \beta\lambda_w E_t\left[\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_w}\bar{\pi}^{1-\gamma_w}}\right)^{\epsilon_w(1+k_w)-1} Z_{w2,t+1}\right] \tag{45}$$

$$Z_{w3,t} = (1-\tau_{w,t})\Lambda_t L_t + \beta\lambda_w E_t\left[\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}\bar{\pi}^{1-\gamma_p}}\right)^{-1} Z_{w3,t+1}\right] \tag{46}$$

$$\Delta_{w1,t} = (1-\lambda_w)\left(\frac{w_t^\star}{w_t}\right)^{1-\epsilon_w(1+k_w)} + \lambda_w \Delta_{w1,t-1}\left(\frac{w_{t-1}}{w_t}\right)^{1-\epsilon_w(1+k_w)}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_w}\bar{\pi}^{1-\gamma_w}}\right)^{\epsilon_w(1+k_w)-1} \tag{47}$$

$$1 = \frac{1}{1+k_w}\Delta_{w1,t}^{1/(1-\epsilon_w(1+k_w))} + \frac{k_w}{1+k_w}\Delta_{w2,t} \tag{48}$$

$$\Delta_{w2,t} = (1-\lambda_w)\left(\frac{w_t^\star}{w_t}\right) + \lambda_w \Delta_{w2,t-1}\left(\frac{w_t}{w_{t-1}}\frac{\pi_{t+1}}{\pi_t^{\gamma_w}\bar{\pi}^{1-\gamma_w}}\right)^{-1} \tag{49}$$

$$\Delta_{w3,t} = \frac{1}{1+k_w}\Delta_{w1,t}^{\epsilon_w(1+k_w)/(1-\epsilon_w(1+k_w))}\Delta_{w4,t} + \frac{k_w}{1+k_w} \tag{50}$$

$$\Delta_{w4,t} = (1-\lambda_w)\left(\frac{w_t^\star}{w_t}\right)^{-\epsilon_w(1+k_w)} + \lambda_w \Delta_{w4,t-1}\left(\frac{w_t}{w_{t-1}}\frac{\pi_{t+1}}{\pi_t^{\gamma_w}\bar{\pi}^{1-\gamma_w}}\right)^{\epsilon_w(1+k_w)} \tag{51}$$

Government

$$R_t - \bar{R} = \phi_\pi \frac{\pi_{t+s}}{\bar{\pi}} + \phi_y \frac{y_{t+s}}{\bar{y}} + \phi_k \frac{k_{t+s}}{\bar{k}} + \rho\left(R_{t-1} - \bar{R}\right) \tag{52}$$

Capital producer

$$K_t = q_t\left(\omega_1 - \frac{\omega_2}{2}q_t\right)o_t + (1-q_t)\theta \tag{53}$$

$$o_t = o_t^{new} + (1-\delta)K_{t-1}$$

$$o_t^{new} = \varepsilon_T^I i_t\left(1 - \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) \tag{55}$$

$$Q_t\varepsilon_t^I\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] - 1 = \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\varepsilon_{t+1}^I Q_{t+1} S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]. \tag{56}$$

Bank

$$q_t = 1 - \frac{\tilde{R}}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2\left(\tilde{R}_t - \tilde{\psi}_t\right)(\tilde{R}_t + 2\tilde{\xi}_t)\left(2\omega_1\tilde{\psi}_t\left(\tilde{R}_t + \tilde{\xi}_t\right) + \omega_2\left(\tilde{R}_t - \tilde{\psi}_t\right)(\tilde{R}_t + 2\tilde{\xi}_t)\right)}}{\omega_2\tilde{\psi}_t(\tilde{R}_t + 2\tilde{\xi}_t)} \tag{57}$$

$$k_t = \frac{\tilde{R}_t - \tilde{\psi}_t}{\tilde{R}_t + 2\tilde{\xi}_t} \tag{58}$$

$$\left(\omega_1 - \frac{\omega_2}{2}q_{t-1}\right)\frac{r_{k,t} + (1-\delta)Q_t - \tau_{:t}}{Q_{t-1}} - \frac{r_{d,t}}{\pi_{t+1}}(1-k_t) - \frac{r_{e,t+1}}{\pi_{t+1}}k_t = 0 \tag{59}$$

$$\tau_t = \frac{Q_{t-1}\frac{1-q_{t-1}}{q_{t-1}}\left(\psi - \theta\frac{r_{k,t}+(1-\delta)Q_t}{Q_{t-1}}\right)}{\omega_1 - \frac{\omega_2}{2}q_{t-1}} \tag{60}$$

$$\tilde{\xi}_t = \xi E_t \left[\Lambda_{t+1}\right] \tag{61}$$

$$\tilde{R}_t = E_t \left[\Lambda_{t+1} \frac{R_t}{\pi_{t+1}}\right] \tag{62}$$

$$\tilde{\psi}_t = \psi E_t \left[\Lambda_{t+1}\right] \tag{63}$$

$$o_t Q_t = e_t + d_t \tag{64}$$

$$k_t = e_t / \left(e_t + d_t\right) \tag{65}$$

Shock processes

$$\log\left(\varepsilon_t^B\right) = \rho_P \log\left(\varepsilon_{t-1}^B\right) + \sigma^B u_t^B \tag{66}$$

$$\log\left(\varepsilon_t^Q\right) = \rho_I \log\left(\varepsilon_{t-1}^Q\right) + \sigma^Q u_t^Q \tag{67}$$

$$\log\left(\varepsilon_t^P\right) = (1-\rho_P)\log\left(\bar{\varepsilon}_{t-1}^P\right) + \rho_P \log\left(\varepsilon_{t-1}^P\right) + \sigma^P\left(u_t^P + m_p u_{t-1}^P\right) \tag{68}$$

$$\log\left(\varepsilon_t^W\right) = (1-\rho_W)\log\left(\bar{\varepsilon}_{t-1}^W\right) + \rho_W \log\left(\varepsilon_{t-1}^W\right) + \sigma^W\left(u_t^W + m_W u_{t-1}^W\right) \tag{69}$$

$$\log\left(\varepsilon_t^A\right) = \rho_A \log\left(\varepsilon_{t-1}^A\right) + \sigma^A u_t^A \tag{70}$$

$$\log\left(\varepsilon_t^R\right) = \rho_R \log\left(\varepsilon_{t-1}^R\right) + \sigma^R u_t^R \tag{71}$$

$$\log\left(\varepsilon_t^G\right) = \rho_G \log\left(\varepsilon_{t-1}^G\right) + \sigma^G u_t^G + \rho_{GA}\sigma^A u_t^A \tag{72}$$

Observational equations: The observation equations, linking the observed time series (left hand-side) to the variables in the non-linear model (right hand-side) are the following:

$$100\Delta \log\left(\frac{Y_t}{Y_{t-1}}\right) = 100\Delta \log\left(\frac{y_t}{y_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{C_t}{C_{t-1}}\right) = 100\Delta \log\left(\frac{c_t}{c_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{I_t}{I_{t-1}}\right) = 100\Delta \log\left(\frac{i_t}{i_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{W_t}{W_{t-1}}\right) = 100\Delta \log\left(\frac{w_t}{w_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{P_t}{P_{t-1}}\right) = 100\pi$$

$$100 \log\left(\frac{H_t}{\bar{H}}\right) = 100 \log\left(\frac{L_t}{\bar{L}}\right) + 100\mu_l$$

$$\left(\frac{R_t}{4}\right) = 100R$$

where $\bar{H}$ are hours worked in 2009 and μ_l is a shift parameter. Since there is no growth in the model, we estimate the mean growth rate in the data μ_y.

Prior specifications: We fix parameters that are not identified to values commonly used in the literature. In particular, we choose a depreciation rate δ of 0.025, a steady-state wage markup $\bar{\varepsilon}^W$ of 1.05, a steady-state spending to GDP ratio g_y of 18%, a weight of labor in the utility function $\bar{L}$ such that steady-state hours are equal to 1, and curvatures of the Kimball aggregator for goods and labor varieties of 10.

For all structural shocks, we employ a non-informative uniform distribution, with lower bound 0 and upper bound of 1 or 10 depending on the scale of the shock. The persistences of the shock processes are assumed to have a beta prior distribution centered at 0.5, and with standard deviation of 0.2. Following Smets and Wouters [2007], we further assume that the two markup shows have a moving average component.

The priors of the Taylor rule parameters are centered around very common values: the smoothing parameter has a Beta distribution with a mean of 0.75, while the responses to inflation and output are assumed to follow a Normal distribution with a mean of 1.5 and of $0.5/4 = 0.125$.

Since we use level data of the inflation rate and of the nominal interest rate, we choose the priors for the steady state of the inflation rate$\bar{\pi}$ and the real interest rate $1/\beta - 1$ to match the mean in the data, i.e. we assumed they follow a gamma distribution respectively centered around annualized values of 2.5% and 0.9.

The parameters affecting price and wage stickiness have a beta distribution centered at 0.5 with standard deviation of 0.1. Our prior is that prices and wages are reoptimized on average every 6 months, and that the degree of indexation to past inflation is only up to 50%. The steady-state price markup is assumed to be centered around 1.25, slightly above the steady-state wage markup.

We employ very common priors for all the parameters of the utility function. Habits are centered around 0.7, the intertemporal elasticity of substitution σ_c has a prior mean of 1.5, while the elasticity of labor supply σ_l has a prior mean of 2. The capital share in production has a prior mean of 0.3 while the investment adjustment costs parameter has a loose prior around 4.

The non-standard set of parameters are those pertaining to the banking sector. We center the prior for the equity premium ξ to 1.5%, matching the average real excess returns of stocks over the risk-free interest rate[37]. The recovery rate θ is centered at 0.5, broadly in line with empirical loan recovery rates reported by**?**. The equity ratio is centered around the data mean of around 8% and the default rate is centered around an annual loan default rate of 6%, which is in line with [XXXXX].

Appendix D: Proofs

The risk taking channel for a generic expected return function

Consider the bank problem discussed in section 3 with deposit insurance but with a generic expected return function $f(q_t)$.

Assume there exists a, equilibrium$[\tilde{r}_{l,t}, q_t, k_t]$ under perfect competition that satisfies the following conditions: (1) the banks choices are interior, i.e. $[k_{t,}q_t] \in [0,1]^2$, (2a) the default probability is low relative to the parameters $\frac{q_t}{(1-q_t)}\tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t$ or (2b) there is no deposit insurance $\tilde{\psi}_t = 0$, the expected return function $f(q_t)$ satisfies (3) $f(q_t) \geq 0$, $f''(q_t) < 0$ and (4) $f'''(q_t) \leq 0$, $f''''(q_t) \leq 0$.

Notice that assumption 2a), which is sufficient but by no means neccessary and only needed for claim (e), is weak if we consider the empirically relevant section of the parameter space with a low equity premium (around 0.0x), a real rate just above 1 (1.0x) and high deposit insurance (0.x) and high repayment probabilities (0.9x). Assumption 3 is straightforward as it guarantees a meaningful risk turn trade-off with an interior solution. Assumption 4 is another a sufficient but not neccessary condition.

We prove that, if such a solution exists, then

(a) risk taking is excessive: $q_t < \text{argmax } f(q_t)$,

(b) the safety of assets q_t is a positive function of $\tilde{R}_t$: $\frac{\partial q_t}{\partial \tilde{R}_t} > 0$,

(c) the equity ratio k_t is a positive function of $\tilde{R}_t$: $\frac{\partial k_t}{\partial \tilde{R}_t} > 0$,

(d) the expected return of an investment is a positive function of $\tilde{R}_t$: $\frac{\partial f\left(q_t\left(\tilde{R}_t\right)\right)}{\partial \tilde{R}_t} > 0$,

(e) the expected return of an investment is a concave function of $\tilde{R}_t$: $\frac{\partial^2 f\left(q_t\left(\tilde{R}_t\right)\right)}{\partial \tilde{R}_t^2} < 0$.

For a generic return function $f(q_t)$ the bank's objective function at the second stage is:

$$\max_{q_t} f\left(q_t\right)\tilde{r}_{l,t} - q_t\tilde{r}_{d,t}(1-k_t) \tag{73}$$

[37] A quarterly equity premium of 1.5% is compatible with the 6% historical annual equity premium found by Mehra and Prescott [1985]. Similar values for the equity premium are furthermore obtained by computing the excess return on bank equity over the risk-free interest rate.

Deriving this problem with respect to q_t yields the following first-order condition, which by concavity is neccessary and sufficient:

$$f'(q_t)\tilde{r}_{l,t} - \tilde{r}_{d,t}(1-k_t) = 0 \tag{74}$$

Since the deposit supply schedule must hold in equilibrium, we can rewrite this condition as

$$f'(q_t)\tilde{r}_{l,t} - \frac{\tilde{R}_t(1-k_t) + (1-q_t)\tilde{\psi}_t}{q_t} = 0 \tag{75}$$

Notice that this condition implies $f'(q_t) > 0$ (given that all parameters are positive and $\tilde{r}_{l,t} > 0$ by the zero profit condition). Notice further that in a frictionless world, e.g. without limited liability, the banks risk choice would satisfy $q_t^{opt} = \text{argmax } f(q_t)$, i.e. $f'(q_t^{opt}) = \theta$. Since we have assumed above that the recovery value θ is smaller than the deposit insurance cap, which in turn is smaller than the cost of deposits $\tilde{r}_{d,t}(1-k_t)$, we have $f'(q_t) > f'(q_t^{opt})$. Given $f''(q_t) < 0$ this implies excessive risk taking, i.e. $q_t < q_t^{opt}$ (claim(a)).

Equation (75) implicitly defines $\hat{q}_t(k_t)$. Using the implicit function theorem we find that,

$$\frac{\partial q_t}{\partial k_t} = \frac{-q_t\tilde{R}_t}{(1-k_t)\tilde{R}_t - \tilde{\psi}_t + q_t^2\tilde{r}_{l,t}f''(q_t)}$$

At the first stage the maximization problem is

$$\max_{k_t} f\left(q_t\right)\tilde{r}_{l,t} - q_t\tilde{r}_{d,t}(1-k_t) - q_t k_t \tilde{r}_{e,t} \ . \tag{76}$$

which, using the deposit and equity supply schedules $\tilde{r}_{d,t} = \frac{\tilde{R}_t - \frac{1-q_t}{1-k_t}\tilde{\psi}_t}{q_t}$ $\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t}$, can be written as

$$\max_{k_t} f\left(\hat{q}_t\right)\tilde{r}_{l,t} + (1-q_t)\tilde{\psi}_t - k_t\tilde{\xi}_t - \tilde{R}_t \ . \tag{77}$$

The corresponding FOC is

$$\left(f'\left(\hat{q}_t\right)\tilde{r}_{l,t} - \tilde{\psi}_t\right)\frac{\partial q_t}{\partial k_t} - \tilde{\xi}_t \ . \tag{78}$$

Finally, the zero profit condition can in expectations be written as

$$f\left(\hat{q}_t\right)\tilde{r}_{l,t} + (1-q_t)\tilde{\psi}_t - k_t\tilde{\xi}_t - \tilde{R}_t \ . \tag{79}$$

Equations (75), (78),(79) implicitly define q_t, k_t and $\tilde{r}_{l,t}$.

Soving the latter two equations for k_t and $\tilde{r}_{l,t}$ we obtain:

$$k_t = \frac{\left(-\tilde{\xi}_t\tilde{R}_t + q_t\tilde{R}_t\tilde{\psi}_t + \tilde{\xi}_t\tilde{\psi}_t\right) f(q_t) - q_t\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right)\left(\tilde{R}_t f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)}{-\tilde{\xi}_t\left(\tilde{R}_t f(q_t) - q_t\left(\tilde{R}_t f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)\right)} \tag{80}$$

$$\tilde{r}_{l,t} = \frac{\left(\tilde{R}_t + \tilde{\xi}_t\right)\left(\tilde{R}_t - \tilde{\psi}_t\right)}{\tilde{R}_t f(q_t) - q_t\left(\tilde{R}_t f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)} \tag{81}$$

Plugging these equations into (75) and rearranging we obtain the following equation, which implicitly defines q_t

$$\left(\tilde{R}_t + \tilde{\xi}_t\right)\frac{\tilde{R}_t\tilde{\psi}_t f(q_t) - \left(\tilde{R}_t\left(\tilde{R}_t + \tilde{\xi}_t\right) - \left((1-q_t)\tilde{R}_t + \tilde{\xi}_t\right)\tilde{\psi}_t\right) f'(q_t) - q_t\tilde{\xi}_t\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right) f''(q_t)}{-\tilde{\xi}_t\left(\tilde{R}_t f(q_t) - q_t\left(\tilde{R}_t f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)\right)} = 0$$

We can simplify this condition further by multiplying with the denominator and deviding by $\left(\tilde{R}_t + \tilde{\xi}_t\right)\tilde{R}_t$

$$F(q_t, R_t) \equiv \frac{\tilde{R}_t\tilde{\psi}_t f(q_t) - \left(\tilde{R}_t\left(\tilde{R}_t + \tilde{\xi}_t\right) - \left((1-q_t)\tilde{R}_t + \tilde{\xi}_t\right)\tilde{\psi}_t\right) f'(q_t) - q_t\tilde{\xi}_t\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right) f''(q_t)}{\tilde{R}_t} = 0 \tag{82}$$

Using the implicit function theorem on equation (82) we find that

$$\frac{\partial q_t}{\partial R_t} = -\frac{\frac{\partial F}{\partial R_t}}{\frac{\partial F}{\partial q_t}} \tag{83}$$

where

$$\frac{\partial F}{\partial R_t} = \frac{\left(\tilde{R}_t^2 + \tilde{\xi}_t\tilde{\psi}_t\right) f'(q_t) + (1-q_t)q_t\tilde{\xi}_t\tilde{\psi}_t f''(q_t)}{-\tilde{R}_t^2}$$

$$\frac{\partial F}{\partial q_t} = \frac{\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right)\left(\left(\tilde{R}_t + 2\tilde{\xi}_t\right) f''(q_t) + q_t\tilde{\xi}_t f'''(q_t)\right)}{-\tilde{R}_t}$$

Using our assumptions on f, the parameters and assuming an interior solution it is obvious that $\frac{\partial F}{\partial q_t} > 0$. How about the $\frac{\partial F}{\partial R_t}$?

To get at the sign of $\frac{\partial F}{\partial R_t}$, we solve 82 for $f(q_t)$

$$f(q_t) = \frac{\left(\tilde{R}_t\left(\tilde{R}_t + \tilde{\xi}_t\right) - \left((1-q_t)\tilde{R}_t + \tilde{\xi}_t\right)\tilde{\psi}_t\right) f'(q_t) + q_t\tilde{\xi}_t\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right) f''(q_t)}{\tilde{R}_t\tilde{\psi}_t}$$

and plug this expression into the equations (80) and (81) for k_t and $\tilde{r}_{l,t}$:

$$k_t = \frac{f'(q_t)(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t) + f''(q_t)q_t\tilde{\xi}_t(\tilde{R}_t - (1-q_t)\tilde{\psi}_t)}{\tilde{R}_t\left(\left(\tilde{R}_t + \tilde{\xi}_t\right)f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)^2} \tag{84}$$

$$\tilde{r}_{l,t} = \frac{\left(\tilde{R}_t + \tilde{\xi}_t\right)\tilde{\psi}_t}{\left(\tilde{R}_t + \tilde{\xi}_t\right)f'(q_t) + q_t\tilde{\xi}_t f''(q_t)} \tag{85}$$

Since in equilibriumd $\tilde{r}_{l,t} > 0$ and since the numerator of $\tilde{r}_{l,t}$ is obviously positive it must hold that its denominator is also positive:

$$\left(\tilde{R}_t + \tilde{\xi}_t\right)f'(q_t) + q_t\tilde{\xi}_t f''(q_t) > 0 \tag{86}$$

Similarly, since $k_t > 0$ and since the denominator of k_t is obviously positive, the numerator must be positive too:

$$f'(q_t)(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t) + f''(q_t)q_t\tilde{\xi}_t(\tilde{R}_t - (1-q_t)\tilde{\psi}_t > 0 \tag{87}$$

Since $f' > 0$ and $f'' < 0$ we can conclude that for any $[x_1, x_2] \in \mathbb{R}^2$ it must hold that $f'(q_t)x_1 + f''(q_t)x_2 > 0$ if

$$\frac{x_1}{x_2} \geq \frac{\left(\tilde{R}_t + \tilde{\xi}_t\right)(\tilde{R}_t - \tilde{\psi}_t)}{q_t\tilde{\xi}_t(\tilde{R}_t - (1-q_t)\tilde{\psi}_t)} \tag{88}$$

We now test this condition for the numerator of $\frac{\partial F}{\partial R_t}$

$$\frac{\tilde{R}_t^2 + \tilde{\xi}_t\tilde{\psi}_t}{(1-q_t)q_t\tilde{\xi}_t\tilde{\psi}_t} \lessgtr \frac{\left(\tilde{R}_t + \tilde{\xi}_t\right)\left(\tilde{R}_t - \tilde{\psi}_t\right)}{q_t\tilde{\xi}_t\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right)}$$

Rearranging, multiplying only with positive values, yields

$$0 \lessgtr -\tilde{R}_t(\tilde{R}_t - (1-q_t)\tilde{\psi}_t) - q_t\tilde{\psi}_t\tilde{\xi}_t - \left(\tilde{R}_t(1-q_t)\tilde{\psi}_t + (1-q_t)\tilde{\psi}_t\tilde{\xi}_t\right)\frac{q_t\tilde{\psi}_t}{\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right)}$$

The RHS is obviously negative since from the proposition that every deposit insurance cap will be exceeded it follows that $\tilde{R}_t > \tilde{\psi}_t$. Hence the condition $\frac{\tilde{R}_t^2 + \tilde{\xi}_t\tilde{\psi}_t}{(1-q_t)q_t\tilde{\xi}_t\tilde{\psi}_t} \geq \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t\tilde{\xi}_t}$ is satisfied and we can conclude the the numerator of $\frac{\partial F}{\partial R_t}$ is positive. Hence $\frac{\partial F}{\partial R_t} < 0$ and therefore $\frac{\partial q_t}{\partial R_t} > 0$ (claim (b)).

Equation (84) defines $k_t = \mathfrak{K}(q_t, \tilde{R}_t)$. Its derivative is given by

$$\frac{\partial k_t}{\partial \tilde{R}_t} = \frac{\partial \mathfrak{K}}{\partial \tilde{R}_t} + \frac{\partial \mathfrak{K}}{\partial q_t}\frac{\partial q_t}{\partial \tilde{R}_t}$$

where

$$\frac{\partial \mathfrak{K}}{\partial \tilde{R}_t} = \frac{\left(\left(f'(q_t)\right)^2 \tilde{R}_t^2 + 2f'(q_t)\left(f'(q_t) + f''(q_t)(1-q_t)q_t\right)\tilde{R}_t\tilde{\xi}_t + \left(f'(q_t) + f''(q_t)q_t\right)\left(f'(q_t) - f''(q_t)(1-q_t)q_t\right)\tilde{\xi}_t^2 \right)\tilde{\psi}_t}{\tilde{R}_t^2\left(f''q\tilde{\xi}_t + f'(\tilde{R}_t + \tilde{\xi}_t)\right)^2}$$

$$\frac{\partial \mathfrak{K}}{\partial q_t} = \frac{q_t\tilde{\xi}_t f''(q_t)\tilde{\psi}_t\left(2\left(\tilde{R}_t + \tilde{\xi}_t\right)f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)}{\tilde{R}_t\left(f''q\tilde{\xi}_t + f'(\tilde{R}_t + \tilde{\xi}_t)\right)^2}$$

From (88) it is immediatley obvious that the numerator of $\frac{\partial \mathfrak{K}}{\partial q_t}$ is negative, hence $\frac{\partial \mathfrak{K}}{\partial q_t} < 0$. After division by $\tilde{\psi}_t$, the numerator of $\frac{\partial \mathfrak{K}}{\partial \tilde{R}_t}$ can be rewritten as

$$\left(\left(\tilde{R}_t + \tilde{\xi}_t\right)f'(q_t) + q_t\tilde{\xi}_t f''(q_t)\right)^2 - \left(f''(q_t)\right)^2 q_t^3\tilde{\xi}_t^2 - f''(q_t)f'(q_t)q_t^2\tilde{\xi}_t(2\tilde{R}_t + \tilde{\xi}_t)$$

Since the first term is positive and bigger then the absolute value of the second termm we can see that $\frac{\partial \mathfrak{K}}{\partial \tilde{R}_t} > 0$. Hence we have shown that $\frac{\partial k_t}{\partial \tilde{R}_t} > 0$ (claim (c)).

Applying the implifit function theorem a second time on equation (82) we can find the following expression for the second derivative of q_t

$$\frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = \frac{\left(\frac{\partial^2 F}{\partial R_t \partial q_t} + \frac{\partial^2 F}{\partial q_t^2}\frac{\partial q_t}{\partial R_t}\right)\frac{\partial F}{\partial R_t} - \left(\frac{\partial^2 F}{\partial R_t \partial q_t}\frac{\partial q_t}{\partial R_t} + \frac{\partial^2 F}{\partial R_t^2}\right)\frac{\partial F}{\partial q_t}}{\left(\frac{\partial F}{\partial q_t}\right)^2} \tag{89}$$

where

$$\frac{\partial^2 F}{\partial \tilde{R}_t \partial q_t} = \frac{\left(\tilde{R}_t^2 + 2(1-q_t)\tilde{\xi}_t\tilde{\psi}_t\right)f''(q_t) + q_t(1-q_t)\tilde{\xi}_t\tilde{\psi}_t f'''(q_t)}{-\tilde{R}_t}$$

$$\frac{\partial^2 F}{\partial q_t^2} = \frac{\tilde{\psi}_t\left(q_t\tilde{\xi}_t f'''(q_t) + \left(\tilde{R}_t + 2\tilde{\xi}_t\right)f''(q_t)\right) + \left(f''''(q_t)q_t\tilde{\xi}_t + f'''(q_t)\left(\tilde{R}_t + 3\tilde{\xi}_t\right)\right)\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right)}{-\tilde{R}_t}$$

$$\frac{\partial^2 F}{\partial R_t^2} = \frac{2\left(f'(q_t) + f''(q_t)(1-q)q\right)\tilde{\xi}_t\tilde{\psi}_t}{\tilde{R}_t^3}$$

since $f'' < 0$ and $f''' \leq 0$ $f'''' \leq 0$ and all parameters are positive it is obvious that $\frac{\partial^2 F}{\partial R_t \partial q_t} > 0$ and $\frac{\partial^2 F}{\partial q_t^2} > 0$. The term$\frac{\partial^2 F}{\partial R_t^2}$ is less straight forward. A sufficient condition for $\frac{\partial^2 F}{\partial R_t^2} > 0$ can be found using again condition (88).

$$\frac{1}{(1-q_t)q_t} \geq \frac{\left(\tilde{R}_t + \tilde{\xi}_t\right)\left(\tilde{R}_t - \tilde{\psi}_t\right)}{q_t\tilde{\xi}_t\left(\tilde{R}_t - (1-q_t)\tilde{\psi}_t\right)}$$

Which simplifies to

$$\frac{q_t}{(1-q_t)}\tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t$$

Given the signs of the terms in (89) we have finally verified that

$$\frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = \frac{((+)+(+)(+))(-)-((+)(+)+(+))(+)}{(+)} < 0$$

Under alternative assumption (2b) the expression for $\frac{\partial^2 q_t}{\partial \tilde{R}_t^2}$ simplifies to

$$\frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = -\frac{f'(q_t)\left(-2f''(q_t)f'''(q_t)q_t\tilde{\xi}_t - 2\left(f''(q_t)\right)^2\left(\tilde{R}_t+2\tilde{\xi}_t\right) + f'(q_t)\left(f''''(q_t)q_t\tilde{\xi}_t + f'''(q_t)\left(\tilde{R}_t+3\tilde{\xi}_t\right)\right)\right)}{\left(f'''(q_t)\tilde{\xi}_t + f''(q_t)\left(\tilde{R}_t+2\tilde{\xi}_t\right)\right)^3}$$

which is negative without further conditions.

Finally, we can determine the slope and curvature of the expected return of the banks investment.

$$\frac{\partial f(q_t)}{\partial \tilde{R}_t} = \underbrace{f'(q_t)}_{+}\underbrace{\frac{\partial q_t}{\partial \tilde{R}_t}}_{+} > 0$$

$$\frac{\partial^2 f(q_t)}{\partial \tilde{R}_t^2} = \underbrace{f'(q_t)}_{+}\underbrace{\frac{\partial^2 q_t}{\partial \tilde{R}_t^2}}_{+} + \underbrace{f''(q_t)}_{-}\underbrace{\frac{\partial q_t}{\partial \tilde{R}_t}}_{+} < 0$$

This completes the proof of claims (d) and (e).

Notice that the functional form we assumed for $f(q_t)$ in the model section satisfies assumption (3) and (4). Therefore claims (a), (b), (c) and (d) hold. To see that claim (e) holds independent of assumption (2a) and (2b), consider the solution for q_t

$$q_t = 1 - \frac{\tilde{R}}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2\left(\tilde{R}_t-\tilde{\psi}_t\right)(\tilde{R}_t+2\tilde{\xi}_t)\left(2\omega_1\tilde{\psi}_t\left(\tilde{R}_t+\tilde{\xi}_t\right)+\omega_2\left(\tilde{R}_t-\tilde{\psi}_t\right)(\tilde{R}_t+2\tilde{\xi}_t)\right)}}{\omega_2\tilde{\psi}_t(\tilde{R}_t+2\tilde{\xi}_t)}$$

The second derivative of this expression is

$$\frac{\partial^2 q}{\partial \tilde{R}_t^2} = -\frac{\omega_1\omega_2\left\{2\omega_2\left(\tilde{R}_t-\tilde{\psi}_t\right)^3\tilde{\xi}_t\left(\tilde{R}_t+2\tilde{\xi}_t\right)+\omega_1\tilde{\psi}_t\left[\tilde{R}_t^4+2\tilde{R}_t\left(4\tilde{R}_t^2-3\tilde{R}_t\tilde{\psi}_t+2\tilde{\psi}_t^{\,2}\tilde{\xi}_t\right)+\left(12\tilde{R}_t-4\tilde{R}_t\tilde{\psi}_t+5\tilde{\psi}_t^{\,2}\right)\right.\right.}{\left(\tilde{R}_t+2\tilde{\xi}_t\right)\left\{\omega_2\left(\tilde{R}_t-\tilde{\psi}_t\right)\left(\tilde{R}_t+2\tilde{\xi}_t\right)\left[2\omega_1\tilde{\psi}_t\left(\tilde{R}_t+\tilde{\xi}_t\right)+b\left(\tilde{R}_t-\tilde{\psi}_t\right)\left(\tilde{R}_t+2\tilde{\xi}_t\right)\right]\right.}$$

Both numerator and denominator are obviously positive, so $\frac{\partial^2 q}{\partial \tilde{R}_t^2} < 0$. Hence $\frac{\partial^2 f(q_t)}{\partial \tilde{R}_t^2} < 0$. ■

Deposits in excess of insurance

The proof is by contradiction: Assume that there exists an equilibrium with no excess profits where the bank would issue so little deposits that the promised repayment $r_{d,t}$ would be lower than the cap on deposit insurance $\psi/(1-k_t)\pi_{t+1}$.[38] In this case the deposit rate $r_{d,t}$ would be equal to the risk free rate R_t.

The second stage maximization problem of the bank would then be

$$\max_{q_t \in [0,1]} \; f(q_t) - q_t \tilde{R}_t (1-k_t)$$

and its solution $\hat{q}_t$ is implied by

$$f'(q_t) = \tilde{R}_t (1-k_t)$$

The first stage maximization problem would be

$$\max_{k_t \in [0,1]} \; V(k) = f(\hat{q}_t) - \hat{q}\tilde{R}_t(1-k_t) - (\tilde{\xi}_t + \tilde{R}_t)k_t$$

$\hat{q}_t$ can either be a corner or an interior solution. If $\hat{q}_t$ is a corner solution, the first stage objective function of the bank is obviously decreasing in k_t, hence $k_t = 0$ is optimal. If $\hat{q}_t$ is an interior solution, the first derivative of the first stage objective function is

$$\tilde{R}_t - \tilde{\xi}_t - \tilde{R}_t(1-\hat{q}_t)$$

Since $\hat{q}_t \in [0,1]$ this derivative is negative for all $k_t \in [0,1]$, i.e. the objective function again is decreasing in k. Hence the solution to the first stage problem is $k_t = 0$. Optimality with full insurance therefore requires that the bank uses only deposits. This contradicts our initial assumption. This result implies that any insurance cap smaller than 100% would be exceeded by the deposit liabilities in case of default. Depositors are therefore never fully insured.

Notice that for a cap to be effective in the sense of ruling out full insurance equilibria, the cap has to be low enough. Formally speaking, it needs to hold that$r_{d,t}(1-k_t) > \tilde{\psi}_t$ even under full insurance, i.e. $\tilde{R}_t > \tilde{\psi}_t$.■

[38] For simplicity, we abstract from the possibility that the cap is binding for some states of the future but not for others, which would be possible due to the inconsistency between the timing of inflation and the nominal deposit rate. Note that this distinction disappears under certainty equivalence or first order approximation.