Dynamic Factor Models with Time-Varying Parameters *

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Abstract

We develop a dynamic factor model with time-varying coefficients. The coefficients that exhibit time variation are the factor loadings, or exposures of the observable time series to the common dynamic factors. This formulation of the dynamic factor model is capable of capturing both changes in volatility and changes in co-movement in a set of time series variables. Furthermore, these changes may vary in both magnitude and timing across variables in the dataset. We use the model to study the evolution of international business cycle dynamics using data on in a panel of 18 countries.

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1 Introduction

Over the past two decades dynamic factor models have become a standard econometric tool for both measuring comovement in and forecasting macroeconomic time series. The popularity of these models has risen as methods have been developed to perform factor analysis on the large datasets that these models naturally apply to (e.g. Stock and Watson 1998, Forni, Hallin, Lippi, Reichlin 1998, Kose, Otrok and Whiteman 2003). The motivation underlying these models is that there are a few common factors that drive fluctuations in large cross sections of macroeconomic time series. One goal of this literature has been to extract information from large datasets that is useful in forecasting exercises. A second goal, and the one we focus on this paper, is the use of the factor models to quantify both the extent and nature of comovement in a set of time series data. For example, Forni et. al. (1998) study the role of sector specific and aggregate technological shocks on disaggregated industry level output data in the United States. Stock and Watson (1999) use their model to study the dynamics of inflation in the US using sectoral inflation data. In the international context Forni and Reichlin (2001) study comovement of regional output in European countries while Kose, Otrok and Whiteman (2003) quantify the importance of world, regional and country specific cycles in international macroeconomic data.

An assumption of these models is that the relationships between variables has not changed over time. Recent empirical work shows that the assumption of structural stability is invalid for many macroeconomic datasets. For example, the break in volatility in the United States documented by McConnell and Perez-Quiros (2000) suggests a change in the nature of US business cycles. Cogley and Sargent (2000) study and document the changing nature of inflation dynamics in the United States during the post-War period. In the international arena Heathcote and Perri (2002), Doyle and Faust (2002) and Kose, Prasad and Terrones (2003) document that cross country correlations have changed over time while Kose, Otrok and Whiteman (2003) find that the importance of a common dynamic factor in G7 data varies by sample period

In this paper we develop a dynamic factor model with time-varying coefficients to bridge the literature on factor models with the literature on instability in the time series properties of macroeconomic data. Specifically, the model allows us to measure and quantify the changing nature of relationships between individual time series through their dependence on common factors. Our model is general enough that it allows for changing relationships among variables in the dataset without imposing that these changes have occurred or assuming a date for the changes. Additionally, we allow the dates and extent of these changing relationships to vary across the variables in the dataset. Whether or not changes have occurred, when these changes took place, and to which variables these changes affect are all dictated by the data

The application of our econometric model is to the study of international business cycle dynamics and is motivated by the apparently large structural changes that the world has faced in the past three decades. In this period the world has experienced a series of trade agreements, large macroeconomic shocks and changes in monetary and exchange rate regimes. What has been the impact of these changes on the nature of the international business cycle? Has the degree of comovement increased or decreased? Have common fluctuations become more or less volatile? Has the impact of the cycle on individual countries evolved over time? These questions are of interest to both policy makers and academic economists. The issues of whether the volatility of the cycle has increased or declined, and whether countries have become more or less symmetric, are central to monetary and fiscal policy issues. These questions are also of interest to academics, who have been debating the effects of trade, monetary, and financial integration on cross-country business cycle synchronization. [1] Since economic theory does not give us unambiguous predictions for the effects of these changes, empirical work on these issues is particularly valuable.

[1] Kose and Yi (2002) argue trade theory predicts that increased trade increases synchronization of macroeconomic aggregates. Frankel and Rose (2002) argue that monetary unions spur trade which increases comovement across countries. However, trade (Krugman 1991) and financial integration (Kalemli-Ozcan, Sørensen, and Yosha 2003) can also increase the degree of specialization, and higher specialization can make business cycles less synchronized (Kalemli-Ozcan, Sørensen, and Yosha 2001).

2 Measuring and Estimating Models of Macroeconomic Comovement

Part of the reason behind the resurgence in interest in dynamic factor models are the new techniques for estimating such models that have become available, such as the time-domain approach of Stock and Watson (1989, 1998) and the frequency-domain approach of Forni and Reichlin (1998) and Forni, Hallin, Lippi, Reichlin (2000, 2001).[2] The work of Otrok and Whiteman (1998) and Kim and Nelson (1998) provide a Bayesian alternative to the classical approaches. The motivation for developing new estimation techniques for these models is that they are the favored econometric tools for many researchers who wish to characterize comovement in macroeconomic variables. Dynamic factor models have the advantage over observable index models, such as weighted aggregates, that one does not need to take a stand on the weighting scheme used in aggregation. Furthermore, in a dynamic factor model the sheer size of a country does not imply that one country is naturally more important than smaller countries in characterizing the common comovement.[3]

Our papers builds on the existing dynamic factor literature by developing a procedure that allows for time variation in the factor loadings. The current literature on dynamic factor models generally assumes no time variation in the relationships in the data.[4] One approach to studying changes in comovement has been to study subsamples. For example, Stock and Watson (2003) and Kose, Otrok and Whiteman (2003b) use factor models to investigate changes in international business cycles.[5] Since neither of their factor models have an intrinsic time-varying feature, changes in comovement are based on sub-sample estimation. While these studies provide valuable information using relatively parsimonious models the limitations of this approach are twofold. First, one must take a stand on which subsamples to study. Second, this approach imposes that changes in

[2] Forni and Reichlin's (2001) study of comovements among regional output in Europe is an application of these techniques.

[3] This follows from the fact that a factor model is a decomposition of the second moments of a time series.

[4] Chauvet and Potter 2001 represents an exception.

[5] The sub-sample approach is also used by those who study comovement using correlations.

comovement across variables occurs at the same time. By allowing for time variation in the factor loadings we capture changes in comovement without imposing a date, much less a common date, for those changes. [6]

The time-variation in the exposure of macroeconomic aggregates to the common factors captures two evolving features of the business cycle. First, we are able to study changes in the volatility of macroeconomic aggregates. Second, we study the evolution in the country specific sensitivity to common cycles. The current theoretical literature suggests that this sensitivity may have changed following trade and financial integration. In addition, this sensitivity may have evolved at different dates from country to country, depending on the initial conditions, or the country's monetary arrangements. We model the dynamics of the time-varying exposures as drift-less random walks, given our view that the nature of the business cycle has evolved slowly over time.[7] The econometric procedure is a Gibbs sampling procedure that generates a set of draws from the posterior distribution of the parameters and factors.

The estimation procedure we develop for our factor model is explicitly Bayesian. What characterizes the Bayesian approach in the existing literature is not so much its Bayesian nature - not a lot of emphasis is placed on the role of the priors, which are often loose - but rather the fact that Gibbs sampling techniques make it computationally feasible to draw from the exact finite sample distribution of the parameters and factors of interest. The fact that the Otrok and Whiteman approach does not rely on asymptotics in either the cross-sectional (N) or the time (T) dimension makes it particularly attractive for cases where only one dimension (N or T) is large. At the same time, the Otrok and Whiteman approach is feasible from a computational standpoint for datasets where N or T are large or there are a large number of factors. For instance, Kose, Otrok, and

[6] Artis, Krolzig and Toro (1999) study changes in the European cycle at the business cycle frequency using a Markov-switching framework . Their research is designed to investigate how exposure to the common cycle changes between booms and recessions, not the long-run evolution of cross-country co-movement. Work more closely related to ours is that of Artis and Zhang, who look at the change in the correlation in industrial production pre and post ERM.

[7] In introducing time-variation in the parameters we follow the recent work of Cogley and Sargent (2002, 2003), who have time-varying coefficients in vector autoregressions. As in their model, our coefficients evolve according to a random walk process.

Whiteman (2003) apply the approach to study the properties of world business cycles ($N = 180$, 68 factors), while Otrok, Silos, and Whiteman (2003) extend the Otrok-Whiteman (1998) estimation procedure to allow for large T. They use the procedure to forecast US post-war inflation ($N = 100$, $T = 480$). For datasets that are large in only one dimension and hence where asymptotic results may not apply, the most direct competitors to Bayesian dynamic factor models - the maximum likelihood techniques based on the Kalman filter or the EM algorithm (Quah and Sargent, 1993) - are hard to apply from a computational point of view. The issue often arises when studying models where there is a combination of factors which load on many variables simultaneously with factors which load only on a few variables. This is the case for us, where we have all variables loading on a common factor as well as subsets of variables loading on country and industry factors.

3 The Model

We begin the description of the model with the equation describing a n-dimensional vector of observable variables denoted $y_{i,t}$ at time t ($t = 1, .., T$). The observables evolve as:

$$y_{i,t} = a_i + b_{i,t}^0 f_t^0 + \sum_{c=1}^{n} b_{i,t}^k f_t^k + \epsilon_{i,t} \tag{1}$$

where the $a_i' s$ are constant terms, f_t^0 is a factor common to all observables and $f_t^1, \ldots, f_t^n$ are factors that we will restrict to affecting a subset of the observable variables. The factor loadings, $b_{i,t}^0$ and $b_{i,t}^k$ capture the exposure of the observables to the common factors. Note that the factor loadings depend on the date. Finally, $\epsilon_{i,t}$ denotes the idiosyncratic components. For example, in our dataset f_t^0 will be the world factor while the factors restricted to a subset of the data will be country or region specific factors.

We assume that the factors evolve as independent AR(q) processes that are invariant over time:

$$f_t^k = \phi_1^k f_{t-1}^k + .. + \phi_q^k f_{t-q}^k + u_t^k, \; k = 0, 1, \ldots, n \tag{2}$$

where u_t^k is an i.i.d. innovation, uncorrelated across factors:

$$\mathbb{E}[u_t^k u_{t-s}^l] = \begin{cases} 1 \text{ for } s = 0, \ k = l \\ 0 \text{ otherwise} \end{cases} . \tag{3}$$

Note that the variance of the innovation in the law of motion for the factor, $u_{0,t}$, is normalized to one. The motivation for this normalization, which is standard in the factor literature, can be seen by studying equation (1). If one increases the standard deviation of f_t^k by a factor of κ and at the same time divides all the $b^k_{...}$'s by κ, one obtains exactly the same value for the likelihood function. This identification problem is solved in the literature by normalizing the standard deviation of the factor innovation to one, and letting the factor loading be unconstrained.

Following Stock and Watson (1989) and others we assume that the idiosyncratic terms $\epsilon_{i,t}$ follow independent AR(p) processes, which are also time invariant:[8]

$$\epsilon_{i,t} = \phi_{i,1}\epsilon_{i,t-1} + \ldots + \phi_{i,p}\epsilon_{i,t-p} + u_{i,t}, \tag{4}$$

where the $u_{y_i,t}$ are i.i.d.innovations, uncorrelated with each other across variables (i).

The law of motion for the factor loading coefficients follows a driftless random walk:

$$b_{i,t}^k = b_{i,t-1}^k + \eta_{i,t}^k, \tag{5}$$

for $k = 0, 1, \ldots, n$. The disturbances $\eta_{i,t}^k$ are i.i.d. and uncorrelated with each other. Additionally, the factor innovations, factor loading innovations, and idiosyncratic shocks are all independent of each other. Finally, we assume that all disturbances are normally distributed.

Of course, there are alternative ways to model changes in economic time series. For example, time variation in the innovation variances is one obvious alternative. By modeling the time variation in the factor loadings we are able to capture two of the

[8]The literature on dynamic factor analysis has made different assumptions in regard to the law of motion of the idiosyncratic component. For instance, Otrok, Silos, and Whiteman (2002) follow Kim and Nelson (1998) and Quah and Sargent (1993) and assume that it depends on the past p values of the observables.

features that we are most interested in: the change in the volatility of macroeconomic aggregates due to global, country and region specific factors, and the change in the comovement across the variables due to these factors. While it seems clear that the factor loadings will capture changes in comovement it is less clear that the model will capture changes in volatility. To show that our model does in fact capture this aspect of the data we will work through an example using the global factor only. In order to see how the time-varying factor loadings capture the change in the volatility, notice that equation (1) could have the alternative specification:

$$y_{i,t} = a_i + b_i^0 \kappa_{,t}^0 f_t^0 + \ldots + \epsilon_{i,t}. \tag{6}$$

Since the volatility of the innovations to f_t^0 is normalized to one, the shift parameter κ_t^0 would essentially allow for changes in the standard deviation of the observable variables over time. Of course, specification (6) is embedded in specification (1) under the restriction $b_{i,t}^0 = b_i^0 \kappa_t^0$. Specification (1) is more general in that it allows for country or industry specific evolution in the factor loadings as well as changes in the world factor.

4 Estimation

To estimate the model we use a Gibbs sampler to draw from the joint posterior distribution of parameters and factors[9]. A Gibbs sampler is a method to obtain a set of draws from a joint distribution by successively drawing from a sequence of conditional distributions. It has been shown elsewhere that such a procedure will yield a set of draws from the desired joint distribution.

Our approach to partitioning the joint posterior of the factors and parameters for our model begins with the observation that conditional on the factors the equations (1) for $i = 1, .., n$ are n independent regression models with AR(p) errors (Otrok and Whiteman 1998). Conditional on the realization for the factors (and in our case, conditional also on the realization for the $b_{i,t}^k$'s) one can then use the distribution theory developed

[9]One step of our procedure use a Metroplis-Hastings algorithm to draw from one of the conditional distributions, so technically we employ a Metroplis-in-Gibbs procedure.

in Chib and Greenberg (1994) in order to draw the parameters $\{a_i, \phi_{i,1}, .., \phi_{i,p}\}$. Since this is done equation by equation, the size of n does not affect the feasibility of these computations. Conditional on the factor realizations, the Chib and Greenberg procedure can also be applied to equation (2) to obtain draws for the autoregressive parameters of the factors $\{\phi_{0,1}^k, .., \phi_{0,q}^k\}$.

In the second block of the procedure we draw the factors conditional on the parameters and factor loadings. We begin by writing model (1), in stacked form, as:

$$\tilde{y}_t = \tilde{a}_y + B_{y,t}\tilde{f}_t + \tilde{\epsilon}_t, \tag{7}$$

where $\tilde{y}_t = (y_{1,t}, \ldots, y_{n,t})'$, $\tilde{a}_y = (a_{y_1}, \ldots, a_{y_n})'$, $\tilde{f}_t = (f_t^0, \ldots, f_t^n)'$, and $\tilde{\epsilon}_t = (\epsilon_{y_1,t}, \ldots, \epsilon_{y_n,t})'$. The matrix $B_{y,t}$ is of dimension $n \times (n+1)$ and its $(ik)^{th}$ element is $b_{y_i,t}^0$. The law of motion of the factors (2) expressed in terms of $\tilde{f}_t$ is:

$$\tilde{f}_t = \Phi_1\tilde{f}_{t-1} + .. + \Phi_q\tilde{f}_{t-q} + \tilde{u}_t. \tag{8}$$

where $\tilde{u}_t = (u_{0,t}, \ldots, u_{n,t})'$, and where the Φ_s matrices are diagonal with elements ϕ_s^k on the diagonal. One can obtain draws of the factors $\{\tilde{f}_1, .., \tilde{f}_T\}$ applying the procedure of Carter and Kohn (1994) using equation (7) as the measurement equation and equation (8) as the transition equation. The algorithm in Carter and Kohn, which is described more fully in the appendix, consists of applying the Kalman filter to obtain the mean and the variance of the conditional distribution of $\tilde{f}_T$, and then proceed "backward" to obtain the distribution of $\tilde{f}_{T-1}$ conditional on the realization of $\tilde{f}_T$, et cetera. The fact that the idiosyncratic shocks are autocorrelated, however, makes direct application of this natural formulation of the model computationally expensive, as one would have to include the $2n$ idiosyncratic shocks (and their lags) as part of the state vector. The approach of Kim and Nelson (1998), also applied in Chauvet, Juhn, and Potter (2001), offers a more computationally efficient formulation of this problem when there are many observable variables. The law of motion of the idiosyncratic shocks (4), also written in stacked form, is:

$$\tilde{\epsilon}_{y,t} = \Phi_{y,1}\tilde{\epsilon}_{y,t-1} + .. + \Phi_{y,p}\tilde{\epsilon}_{y,t-p} + \tilde{u}_{y,t}, \tag{9}$$

where $\tilde{u}_{y,t} = (u_{y_1,t}, \ldots, u_{y_n,t})'$, and where the $\Phi_{y,s}$ matrices are diagonal with elements $\phi_{y_i,s}$. Let $\Phi_y(L) = \sum_{s=1}^{p} \Phi_{y,s} L^s$. Now premultiply 7 by $I_n - \Phi_y(L)$ and obtain the system:

$$\tilde{y}_t^* = \tilde{a}_y^* + \Phi_{y,1} B_{y,t} \tilde{f}_t - \sum_{s=1}^{p} \Phi_{y,s} B_{y,t-s} \tilde{f}_{t-s} + \tilde{u}_{y,t}, \tag{10}$$

where $\tilde{y}_t^* = (I_n - \Phi_y(L))\tilde{y}_t$, $\tilde{a}_y^* = (I_n - \Phi_y(L))\tilde{a}_y$. Now the disturbances in the transformed measurement equation (10) are i.i.d. One can then apply the approach of Carter and Kohn to equation (10) and (8) using the vector $(\tilde{f}_t', \ldots, \tilde{f}_{t-q}')'$ as state vector.[10] Note that the size of the state vector in this approach, and hence the feasibility of the approach for large datasets, does not increase with T or n.

In our application it is critical to not condition on the initial observations when drawing the factor. As we will see subsequently, due to the non-stationarity of the factor loadings the initial condition itself needs to be derived. The mean and variance for the initial state $\tilde{f}_p = (f_p, \ldots, f_{p-q+1})'$, conditional on the first p observations can be derived as follows. Define:

$$\bar{B}_t = \begin{bmatrix} b_{1,t} & \\ \vdots & 0_{n,q-1} \\ b_{n,t} & \end{bmatrix},$$

$\tilde{y}^{p..1} = (\tilde{y}_p', \ldots, \tilde{y}_1')'$, and $\tilde{f}^{p..1} = (\tilde{f}_{p-1}', \ldots, \tilde{f}_1')'$. Note that

$$\tilde{y}_t = \tilde{a} + \bar{B}_t \Phi^t \tilde{f}_0 + \bar{B}_t \sum_{j=0}^{t-1} \Phi^j \tilde{u}_{0,t-j} + \tilde{\epsilon}_t.$$

Hence we can write the first p observations as:

$$\tilde{y}^{p..1} = \begin{bmatrix} \tilde{a} + \bar{B}_p \Phi^p \tilde{f}_0 \\ .. \\ \tilde{a} + \bar{B}_2 \Phi^2 \tilde{f}_0 \\ \tilde{a} + \bar{B}_1 \Phi \tilde{f}_0 \end{bmatrix} + \begin{bmatrix} \bar{B}_p & .. & \bar{B}_p \Phi^{p-2} & \bar{B}_p \Phi^{p-1} \\ .. & & & \\ 0 & .. & \bar{B}_2 & \bar{B}_2 \Phi^1 \\ 0 & .. & 0 & \bar{B}_1 \end{bmatrix} \begin{bmatrix} \tilde{u}_{0,p} \\ .. \\ \tilde{u}_{0,2} \\ \tilde{u}_{0,1} \end{bmatrix} + \begin{bmatrix} \tilde{\epsilon}_p \\ .. \\ \tilde{\epsilon}_2 \\ \tilde{\epsilon}_1 \end{bmatrix}. \tag{11}$$

[10]Equation (8) needs to be appropriately rewritten in companion form.

and $\tilde{f}^{p..1}$:

$$\tilde{f}^{p..1} = \begin{bmatrix} \Phi^p \tilde{f}_0 \\ .. \\ \Phi^2 \tilde{f}_0 \\ \Phi \tilde{f}_0 \end{bmatrix} + \begin{bmatrix} I & .. & \Phi^{p-2} & \Phi^{p-1} \\ .. & & & \\ 0 & .. & I & \Phi^1 \\ 0 & .. & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{u}_{0,p} \\ .. \\ \tilde{u}_{0,2} \\ \tilde{u}_{0,1} \end{bmatrix}. \tag{12}$$

Call $\mathcal{U}_y$ and $\mathcal{U}_f$ the two upper triangular matrices in equations 11 and 12, respectively. Also define

$$\mathcal{I}_p = \begin{bmatrix} I_p \\ .. \\ I_p \\ I_p \end{bmatrix}, \; \mathcal{B}_p^y = \begin{bmatrix} \bar{B}_p \Phi^p \\ .. \\ \bar{B}_2 \Phi^2 \\ \bar{B}_1 \Phi \end{bmatrix}, \; \mathcal{B}_p^f = \begin{bmatrix} \Phi^p \\ .. \\ \Phi^2 \\ \Phi \end{bmatrix}.$$

Finally, call Σ_0 and $\Sigma_{\epsilon^{p..1}}$ the variance covariance matrix of $\tilde{u}_{0,t}$ and $(\tilde{\epsilon}_p' , \ldots , \tilde{\epsilon}_1')'$, respectively . This is a matrix of zeros except for the $(1,1)$ element, which is σ_0^2. From our distributional assumptions (see also section **??**) we have that

$$\begin{bmatrix} \tilde{y}^{p..1} \\ \tilde{f}^{p..1} \end{bmatrix} = \mathcal{N}\left(\begin{matrix} I_p \tilde{a} + B_p^y \tilde{f}_{0,0} \\ B_p^y \tilde{f}_{0,0} \end{matrix} , \right. \\ \left. \begin{matrix} \mathcal{B}_p^y \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_y (I_p \otimes \Sigma_0) \mathcal{U}_y' + \Sigma_{\epsilon^{p..1}} & \cdots \\ \mathcal{B}_p^f \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_f (I_p \otimes \Sigma_0) \mathcal{U}_y' & \mathcal{B}_p^f \tilde{s}_{0,0} \mathcal{B}_p^{f\,\prime} + \mathcal{U}_f (I_p \otimes \Sigma_0) \mathcal{U}_f' \end{matrix} \right) \tag{13}$$

where $\tilde{f}_{0,0}$ and $\tilde{s}_{0,0}$ are the unconditional mean and variance of $\tilde{f}_t$. Therefore the conditional mean and the variance of $\bar{b}_{i,1}$ are given by:

$$\begin{aligned} E_{p_i}[\tilde{f}^{p_i..1}] = \; & \mathcal{B}_p^y \tilde{f}_{0,0} + \left(\mathcal{B}_p^f \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_f (I_p \otimes \Sigma_0) \mathcal{U}_y' \right) \\ & \left(\mathcal{B}_p^y \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_y (I_p \otimes \Sigma_0) \mathcal{U}_y' + (\sigma_i^2 \Sigma_i \otimes I_n) \right)^{-1} \left(\tilde{y}^{p..1} - \mathcal{I}_p \tilde{a} - \mathcal{B}_p^y \tilde{f}_{0,0} \right) \\ V_{p_i}[\tilde{f}^{p_i..1}] = \; & \mathcal{B}_p^f \tilde{s}_{0,0} \mathcal{B}_p^{f\,\prime} + \mathcal{U}_f (I_p \otimes \Sigma_0) \mathcal{U}_f' - \left(\mathcal{B}_p^f \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_f (I_p \otimes \Sigma_0) \mathcal{U}_y' \right) \\ & \left(\mathcal{B}_p^y \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_y (I_p \otimes \Sigma_0) \mathcal{U}_y' + (\sigma_i^2 \Sigma_i \otimes I_n) \right)^{-1} \left(\mathcal{B}_p^f \tilde{s}_{0,0} \mathcal{B}_p^{y\,\prime} + \mathcal{U}_f (I_p \otimes \Sigma_0) \mathcal{U}_y' \right). \end{aligned} \tag{14}$$

In the cases $q-1 > p$ one simply needs to add $q-1-p$ columns of zeros to the matrix B^*. Vice versa, in the the case $q-1 < p$ one adds $p-q+1$ columns of zeros to the matrix Φ_0.

The third block of the Gibbs sampler consists of drawing from the distribution of the $b^k_{i,t}$'s, conditional on all other parameters and the factor realizations. Following Kim and Nelson (1999) and Cogley and Sargent (2002), we derive this conditional distribution of the Gibbs sampler by framing the problem in the language of Carter and Kohn (1994). This is accomplished by treating equation (5) as the transition equation and equation (1) as the measurement equation. The fact that the errors in the measurement approach are autocorrelated can again be addressed using the quasi-differencing procedure described above. Given our assumption that the shocks to the law of motion of the factor loadings (5) are uncorrelated across variables, this block of the Gibbs sampler is applied equation by equation, so that it is feasible even for large n.

One issue we face is that law of motion for the factor loadings (5) is non-stationary. The initial condition for the Kalman Filter, which is typically the unconditional expectation of the state vector, is not well defined here. In our case, the initial condition needs to be treated as a hyperparameter. Recent work on VARs with time-varying parameters (Cogley and Sargent 2002, 2003) treats the initial condition as a known parameter.[11] This approach is not appropriate in our setting. If the loadings do not vary much over time, having a degenerate prior on the initial condition implies having a degenerate prior on the loadings themselves. Instead we estimate the initial condition, denoted $b_{i,0}$. We will return to this issue after describing the procedure for drawing the factor loadings given an initial condition

Conditional on the factors, parameters other than the factor loadings and on the initial condition $b_{i,0}$, the distribution of the parameters $b_{i,t},\ t = 1, \ldots, T$ can be obtained by applying the Carter-Kohn approach equation by equation. This can be done as follows. Consider pre-multiplying equation 1 by the quantity $1 - \phi_{i,1}L^1 - .. - \phi_{i,p_i}L^{p_i}$ where L is the lag operator. One obtains the equation:

$$y^*_{i,t} = a^*_i + \omega^*_t \tilde{b}_{i,t} + \tilde{u}_{i,t} \text{ for } t = p_i{+}1, .., T \tag{15}$$

where $y^*_{i,t} = (1 - \phi_{i,1}L^1 - .. - \phi_{i,p_i}L^{p_i})y_{i,t}$, $a^*_i = (1 - \phi_{i,1} - .. - \phi_{i,p_i})a$, $\omega^*_t = (f_t - \phi_{i,1}f_{t-1} .. - \phi_{i,p_i}f_{t-p_i})$, $\tilde{b}_{i,t} = (b_{i,t}, \ldots, b_{i,t-p_i})'$, $\tilde{u}_{i,t} = (u_{i,t}, 0, \ldots, 0)'$. Equation 15 is the

[11]Cogley and Sargent (2002, 2003) use pre-sample information to pin down the initial condition.

measurement equation. The transition equation is simply 5 written to accommodate $\tilde{b}_{i,t}$:

$$\tilde{b}_{i,t} = \Xi \tilde{b}_{i,t-1} + \tilde{\eta}_{i,t} \tag{16}$$

where $\tilde{\eta}_{i,t} = (\eta_{i,t}, 0, \ldots, 0)$ and

$$\Xi = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ & I_{p_i} & & 0 \end{bmatrix}.$$

Note that the first t one considers in eq. 15 is $t = p_i + 1$. Once again, a key ingredient in the procedure is the mean and variance for the initial state $\tilde{b}_{i,p_i} = (b_{i,p_i}, \ldots, b_{i,1}, b_{i,0})'$. For expositional simplicity, the initial condition $b_{i,0}$ is treated as a constant for the derivation of this distribution. In the next section we derive the distribution for drawing the initial condition. To draw $b_{i,p_i}, \ldots, b_{i,1}$ we need to obtain the mean and variance of $b_i^{p_i..1} = (b_{i,p_i-1}, \ldots, b_{i,1})'$ given the first p_i observations $y_i^{p_i..1} = (y_{i,p_i}, \ldots, y_{i,1})'$, the initial condition $b_{i,0}$, and the factors. Note that

$$y_{i,t} = a_i + b_{i,0} f_t + (\sum_{j=1}^{t} \eta_{i,j})\ f_t + \epsilon_{i,t}.$$

We can write the first p_i observations as:

$$y_i^{p_i..1} = \begin{bmatrix} a_i + b_{i,0} f_{p_i} \\ .. \\ a_i + b_{i,0} f_2 \\ a_i + b_{i,0} f_1 \end{bmatrix} + \begin{bmatrix} f_{p_i} & .. & f_{p_i} & f_{p_i} \\ .. & & & \\ 0 & .. & f_2 & f_2 \\ 0 & .. & 0 & f_1 \end{bmatrix} \begin{bmatrix} \eta_{i,p_i} \\ .. \\ \eta_{i,2} \\ \eta_{i,1} \end{bmatrix} + \begin{bmatrix} \epsilon_{i,p_i} \\ .. \\ \epsilon_{i,2} \\ \epsilon_{i,1} \end{bmatrix} \tag{17}$$

and $b_{i,p_i..1}$:

$$b_i^{p_i..1} = \begin{bmatrix} b_{i,0} \\ .. \\ b_{i,0} \\ b_{i,0} \end{bmatrix} + \begin{bmatrix} 1 & 1 & .. & 1 \\ .. & & & \\ 0 & .. & 1 & 1 \\ 0 & .. & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_{i,p_i} \\ .. \\ \eta_{i,2} \\ \eta_{i,1} \end{bmatrix} \tag{18}$$

Let U_y and U_b denote the two upper triangular matrices in equations 17 and 18, respectively. From our distributional assumptions (see also section 3) we have that

$$\begin{bmatrix} y_i^{p_i..1} \\ b_i^{p_i..1} \end{bmatrix} = \mathcal{N} \begin{pmatrix} a_i\ 1_{p_i} + b_{i,0} \bar{f}_1 & , & \sigma_{\eta_i}^2 U_y\ U_y' + \sigma_i^2 \Sigma_i & \ldots \\ b_{i,0}\ 1_{p_i} & & \sigma_{\eta_i}^2 U_b\ U_y' & \sigma_{\eta_i}^2 U_b\ U_b' \end{pmatrix} \tag{19}$$

where $f^{p_i..1} = (f_{p_i}, \ldots, f_1)'$ and 1_{p_i} is a unitary vector of length p_i. Therefore the conditional mean and the variance of $b_i^{p_i..1}$ are given by:

$$\begin{aligned} E_{p_i}[b_i^{p_i..1}] &= b_{i,0}\, 1_{p_i} + \sigma^2_{\eta i} U_b\, U_y'\, (\sigma^2_{\eta i} U_y\, U_y' + \sigma_i^2 \Sigma_i)^{-1}\, (y_i^{p_i..1} - a_i\, 1_{p_i} - b_{i,0} f^{p_i..1}) \\ V_{p_i}[b_i^{p_i..1}] &= \sigma^2_{\eta i} U_b\, U_b - \sigma^2_{\eta i} U_b\, U_y'\, (\sigma^2_{\eta i} U_y\, U_y' + \sigma_i^2 \Sigma_i)^{-1}\, \sigma^2_{\eta i} U_y\, U_b'. \end{aligned} \tag{20}$$

Conditional on the loadings $(b_{i,1}, \ldots, b_{i,T})$ the posterior distribution for the loadings' innovation $\sigma^2_{\eta i}$ is an inverted gamma distribution $IG(\frac{\bar{\nu}_{\eta i}+T}{2}, \bar{\delta}^2_{\eta i} + d_i^2)$ where $d_i^2 = \sum_{t=1}^{T} (b_{i,t} - b_{i,t-1})^2$.

4.0.1 The initial condition

To estimate the initial condition,l $b_{i,0}$ we begin with a non degenerate prior for $b_{i,0}$, which is $N(\bar{b}_{i,0}, \bar{v}_{i,0})$. Conditional on $(b_{i,1}, \ldots, b_{i,T})$, the posterior distribution of $b_{i,0}$ is $N(\underline{b}_{i,0}, \underline{v}_{i,0})$. The mean and the variance of the posterior, $\underline{b}_{i,0}$ and $\underline{v}_{i,0}$, are obtained (as in Carter and Kohn) by updating $\bar{b}_{i,0}$ and $\bar{v}_{i,0}$ using

$$b_{i,1} = b_{i,0} + \eta_{i,1}$$

as measurement equation. An additional hyperparamter in the variance of the initial condition in the Kalman filter. Here we follow Cogley and Sargent (2002) and use a fixed (large) number (see also Harvey). Our rationale for this choice is that the variance of the initial condition matters less than the initial condition itself for the outcome of the Kalman filter. Hence it is not as dangerous to have a degenerate prior.

To summarize our econometric procedure, the Gibbs sampler partitions the model into 3 blocks. In the first block we draw the constant, the innovation variances as well as the autoregressive parameters for the factors and the idiosyncratic terms conditional on the factor loadings and factors. In the second block we condition on the factor loadings and the parameters drawn in the first block and draw the factors. In the third block we condition on the factors and parameters drawn in the first block, and draw the time-varying factor loadings.

We conclude the section by specifying the (conjugate) priors. These are:

$$a_{y_i} : N(\bar{a_{y_i}}, \bar{A}_{y_i}^{-1}),\ \phi^k : N(\bar{\phi}^k, \bar{V}^{k\ -1})I_{S\phi^k},\ \phi_{y_i} : N(\bar{\phi}_{y_i}, \bar{V}_{y_i}^{-1})I_{S\phi_{y_i}},$$
$$b_{i,0} : N(\bar{b}_{i,0}, \bar{v}_{i,0}), \sigma_{y_i}^2 : IG(\tfrac{\bar{\nu}_i}{2}, \bar{\delta}_i^2),\ \sigma_{b^k,y_i}^2 : IG(\tfrac{\bar{\nu}_{b^k,y_i}}{2}, \bar{\delta}_{b^k,y_i}^2),$$

where ϕ^k and ϕ_{y_i} are the vectors $\{\phi_1^k, .., \phi_q^k\}$ and $\{\phi_{y_i,1}, .., \phi_{y_i,p_i}\}$, N is the Normal distribution,IG is the inverted gamma distribution, $I_{S\phi_i}$ is an indicator function that equals zero if the roots of the ϕ_i polynomial lie outside the unit circle. One issue with our model is that if the factor loadings are left unconstrained the model will have too many free parameters. We address this issue by imposing relatively tight priors on some of the innovation variances. In particular, we want to ensure that the factors dynamics capture cyclical relationships and the factor loadings capture more slowly evolving relationships. Hence we use a prior on the variance of the innovations in factor loading equations (5) that is relatively tight. [12]

5 The Data

The data consist of Output growth for an 15 country sample of developed economies. The data are in real per capita terms. Growth rates are 4 quarter growth rates.

6 Empirical Results

There is a large literature that has studied the properties of international business cycle dynamics. A prominent strand of this literature has used dynamic factor models to characterize these dynamics. Here we apply our time varying parameter dynamic factor

[12]The other priors are fairly loose. The prior on a_i is very loosely centered around zero. The priors on the AR coefficients follows Otrok and Whiteman (1998).

model described by equations (1)-(6) to some of this same international data to demonstrate the dimensions on which our model can reveal new insights. We allow for one global factor and one factor common to all European countries. That is, in equation (1) n is set equal to one and only the countries in Europe have a non-zero factor loading on this factor. We focus on the World and Europe for a number of reasons. First, Europe has undergone many well known changes in terms of financial and trade integration over this period. Furthermore, there has been heterogeneity in terms of when and to what extent each country has chosen to participate in this integration. We hope to exploit the flexibility of our econometric model to help characterize these changes in terms of each European countries exposure to the World and European factors. Second, over this period there has been a moderation in volatility global. Our model can shed some light on the source of this change, whether global or regional in nature. Third, there is some dissension on whether or not there is in fact a truly European cycle rather than simply a common cycle among all developed economies (e.g. Canova et. al.). Given that the European cycle may have changed significantly over time we wish to investigate the existence of such a distinct cycle in the context of a model that allows for the variation we expect to see in Europe as it integrates.

We first turn to the estimated factors themselves. The top panel of Figure 1 depicts the world factor along with an average growth rate across all variables in our dataset. Note that the world factor departs in some cases from the average growth rate and tends to be a bit smoother-demonstrating a feature of most factor models that they are not overly influenced by occasional large idiosyncratic jumps in one or two large countries.

Of primary interest to us is the evolution in the sensitivity of each country to the factors depicted in Figure 1. In Figure 2 we plot the factor loadings of each country to the world factor over time. We also include 90 percent posterior coverage intervals for the factor loadings. Despite the sometimes wide posterior coverage intervals in some cases we find that for a number of countries the posterior distribution of the factor loadings varies significantly over time. For example, the factor loading for Japan is positive and significant at the beginning of the sample and then declines to nearly zero by the 1990s, indicating the significant break of Japan with the rest of the world. On the other hand, Sweden and Finland began a period of rapid integration in the mid 1980s. Even among these two similar countries there is variation in the manner in which this integration took place. In Sweden the change was gradual while in Finland the change happened sharply over a short period of time. In some cases, there is no time variation, such as Belgium. The heterogeneity depicted in this figure demonstrates a nice feature of our model in that it allows for considerable flexibility in the timing and size of changes in comovement and volatility. In some cases where the data dictate rapid movement in factor loadings are model can capture this.

In Figure 3 we plot the factor loadings for Europe. The figures show that there is a decline in the sensitivity of the UK and Switzerland to the European cycle from the 1970s to the later period. In the case of the UK this changed occurred gradually throughout the 1980s and was complete by 1990. In the case of Switzerland the change was essentially complete by 1980. For the remaining countries there seems to be little evidence of changes in the sensitivity to the European factor.

We use the model to decompose movements in output growth into components attributed to world, European, and idiosyncratic shocks. For each period we compute the variance for each component during the previous ten years. We do this for each posterior draw and report the mean. We therefore have a rolling window estimate of the importance of each component in determining the volatilities of the series. Table 1 provides similar information, except that there we focus on three equally-spaced sub-periods (19770:2-81:2, 1981:3-1992:4,1993:1-2004:1). For each sub-period, Table 1 also shows the variance of the actual data. Note that although in sample the three component are not by construction orthogonal, for most countries/sub-periods the sum of their variance roughly adds up to the total variance, suggestion that our decomposition is not too far-fetched. We find: 1) for most countries output volatility has decreased over time, as found elsewhere. The magnitude of the decrease has been substantial for some countries (US, Canada, UK, Italy,...) but much smaller for others. 2) Idiosyncratic shocks have been the main driving force of the decline in volatility. Idiosyncratic shocks capture country-specific shocks but also (especially for some countries like Spain and possible Germany) measurement error. 3) For a number of countries (US, Canada, France,..) the decline in the volatility of the component due to the world cycle has been significant. 4) The patterns do not seem to be affected by business cycles. For instance, the recent recession does not seem to have resulted in increased volatility for the world component.

Finally, Table 2 shows the relative importance of the three components during the three equally-spaced sub-periods (19770:2-81:2, 1981:3-1992:4,1993:1-2004:1).

7 Conclusions

We develop a dynamic factor model allowing for time variation in the factor loadings. We estimate the model using a Gibbs sampling procedure to draw from the joint posterior distribution of the model parameters and latent factors. An important feature of our procedure is the derivation of the distribution of the initial conditions. In models with time variation the initial conditions can have important implications for estimates of the model parameters and hence it is important to let the data dictate these initial conditions by including these conditions in the likelihood function for the model. [FOOTNOTE: Sims (1996) discusses some of the problems associated with conditioning in time series data.]

We apply our econometric model to the study of international business cycles in developed economies. The model has wide applicability to other issues. In the international business cycle literature the model had natural applications to understanding the impact of integration in developing countries as well. The short samples typically available for these countries are not an issue for our procedure. Factor models have also been used extensively as powerful forecasting tools. Apply our time-varying model may result in more accurate forecasts as the model parameters adapt to changing structure in the world. Factor models have also been used extensively in the finance literature and its is natural to consider time variation in these models in future work.

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Table 1: Variance Attributable to World, European, and Idiosyncratic Shocks: Three Subperiods

	Data			World			Europe			Country		
periods:	1	2	3	1	2	3	1	2	3	1	2	3
U.S.	21.44	10.67	4.52	7.13	7.04	1.45				13.66	4.56	3.42
Japan	18.64	12.54	11.84	4.14	1.31	0.68				13.56	12.52	11.48
Canada	15.76	16.68	4.09	5.00	9.70	2.18				10.94	6.27	2.69
Australia	33.19	17.19	6.45	2.32	5.93	0.78				31.98	10.00	6.40
U.K.	36.13	8.02	1.99	5.49	4.82	1.06	1.63	1.34	0.34	27.84	6.38	1.96
Belgium	14.42	7.74	6.56	4.88	6.11	2.18	4.53	4.37	1.72	6.37	4.34	4.16
Denmark	18.81	21.40	10.79	5.11	3.97	1.41	1.55	2.48	0.98	11.91	20.67	8.83
France	7.44	4.03	4.09	3.58	3.34	1.48	2.68	2.80	1.32	2.49	2.09	1.94
Germany	16.48	46.82	6.90	5.92	12.66	3.47	3.81	4.19	1.64	10.82	36.45	5.58
Italy	20.13	7.89	5.59	5.24	4.55	1.02	3.72	2.37	1.35	11.88	5.64	4.34
Netherlands	11.67	16.45	5.32	3.09	6.62	1.61	2.66	2.91	1.47	7.20	11.84	3.48
Sweden	38.34	21.45	11.63	0.91	5.53	2.65	1.29	1.85	1.14	36.38	17.06	9.98
Switzerland	57.43	11.11	5.50	3.55	5.15	1.72	3.93	2.12	1.30	50.94	7.72	4.62
Finland	27.24	31.47	18.93	1.90	10.34	4.70	1.90	2.42	1.23	25.46	19.19	15.96
Spain	9.37	13.85	3.63	1.95	2.08	0.66	1.27	2.27	0.82	6.04	11.12	3.05

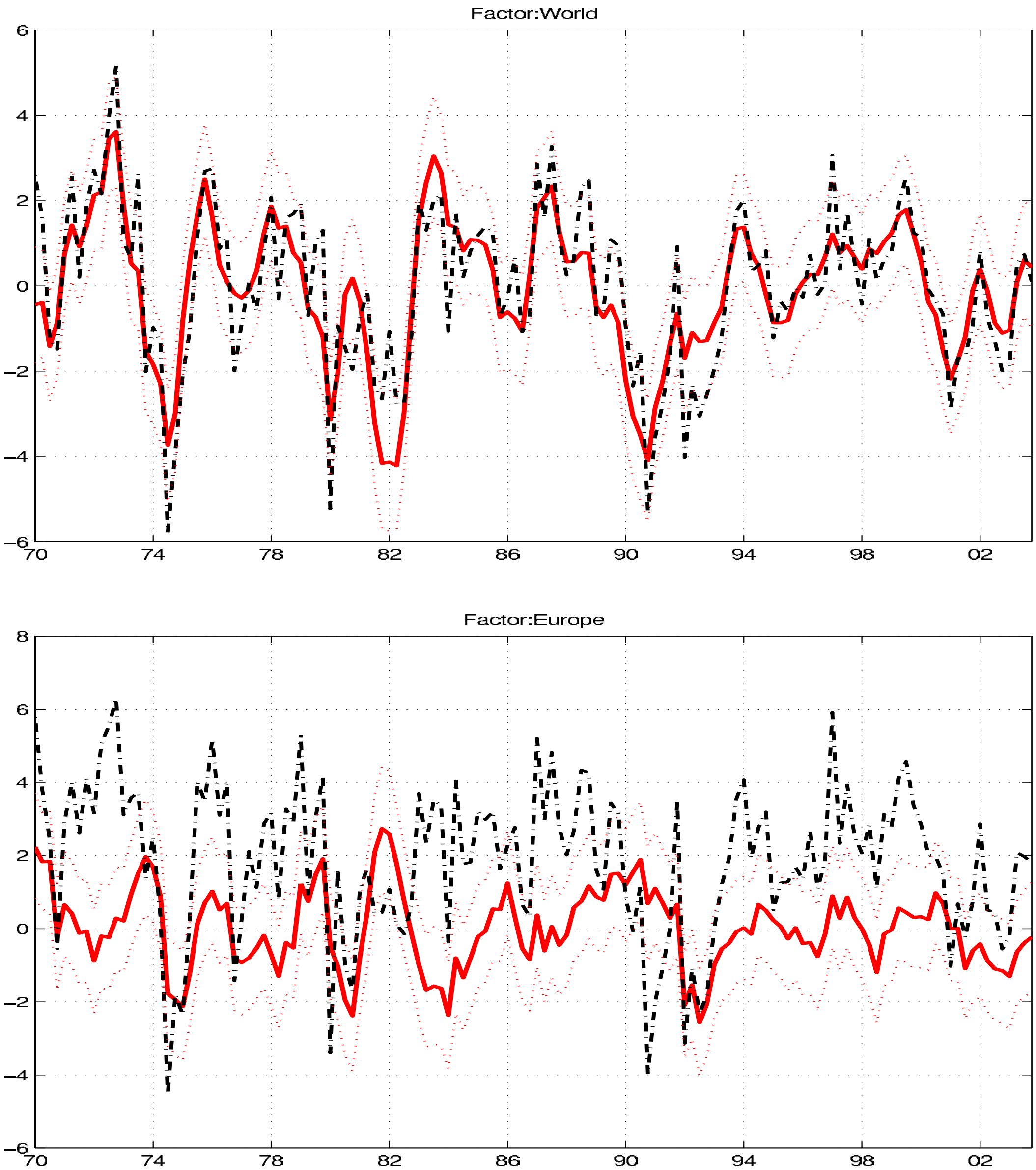
Notes: subperiod 1: 70-Q2 – 81-Q2 ; subperiod 2: 81-Q3 – 92-Q4 ; subperiod 3: 93-Q1 – 04-Q1

Table 2: Relative Importance of World, European, and Idiosyncratic Shocks: Three Subperiods

	World			Europe			Country		
periods:	1	2	3	1	2	3	1	2	3
U.S.	0.34	0.60	0.28				0.66	0.40	0.72
Japan	0.23	0.09	0.05				0.77	0.91	0.95
Canada	0.31	0.60	0.43				0.69	0.40	0.57
Australia	0.07	0.36	0.10				0.93	0.64	0.90
U.K.	0.15	0.37	0.29	0.05	0.10	0.09	0.80	0.53	0.61
Belgium	0.30	0.40	0.26	0.28	0.29	0.21	0.41	0.31	0.53
Denmark	0.27	0.14	0.12	0.08	0.09	0.08	0.65	0.77	0.80
France	0.40	0.39	0.31	0.30	0.33	0.27	0.29	0.28	0.42
Germany	0.28	0.23	0.31	0.18	0.08	0.15	0.55	0.69	0.55
Italy	0.25	0.35	0.14	0.17	0.18	0.19	0.58	0.48	0.66
Netherlands	0.23	0.30	0.24	0.20	0.13	0.22	0.57	0.57	0.55
Sweden	0.02	0.22	0.19	0.03	0.07	0.08	0.94	0.71	0.74
Switzerland	0.06	0.33	0.21	0.07	0.13	0.16	0.87	0.54	0.63
Finland	0.06	0.31	0.21	0.06	0.07	0.05	0.88	0.62	0.74
Spain	0.20	0.13	0.14	0.13	0.14	0.17	0.66	0.73	0.69

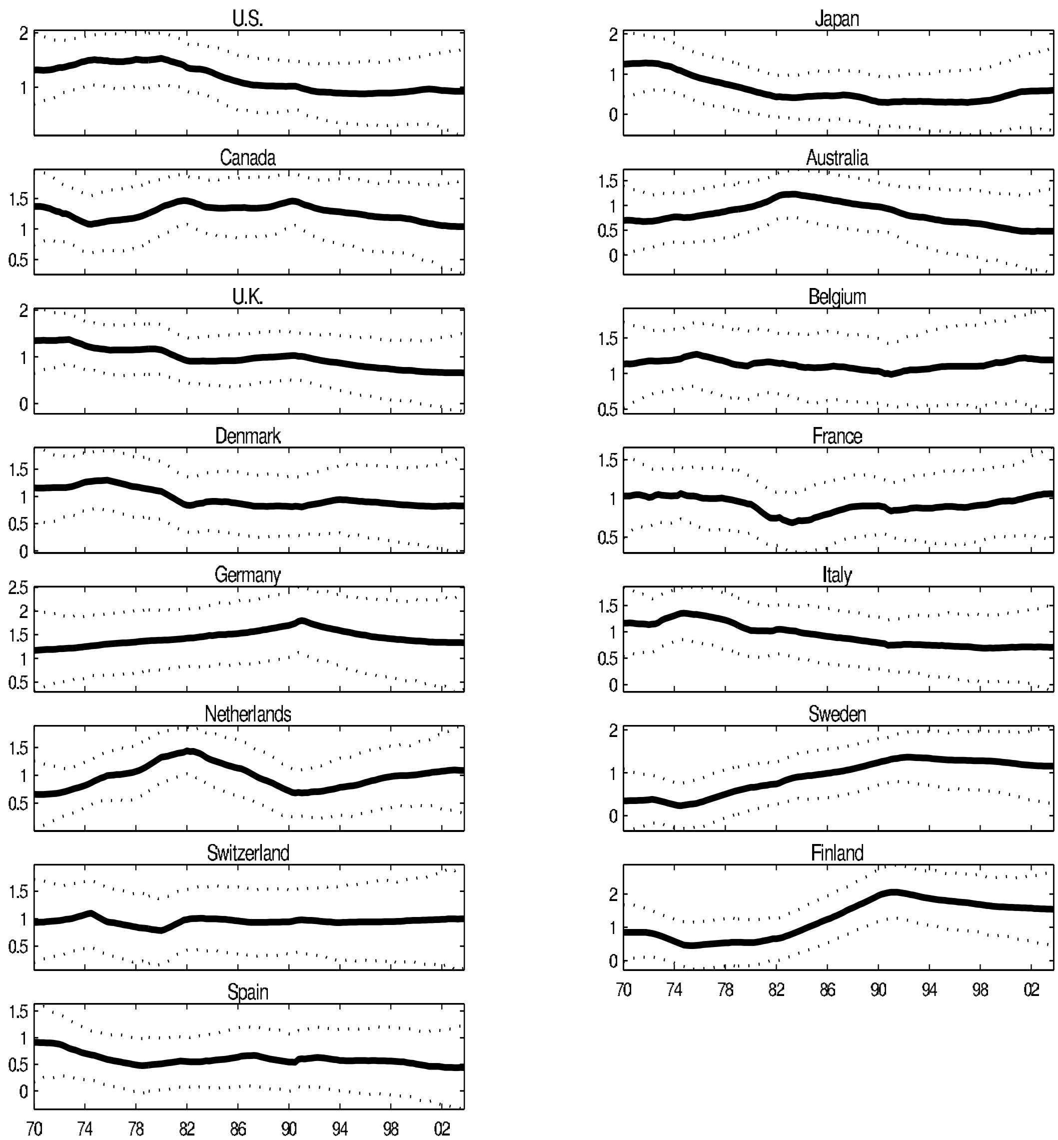
Notes: subperiod 1: 70-Q2 – 81-Q2 ; subperiod 2: 81-Q3 – 92-Q4 ; subperiod 3: 93-Q1 – 04-Q1

Figure 1: Factors



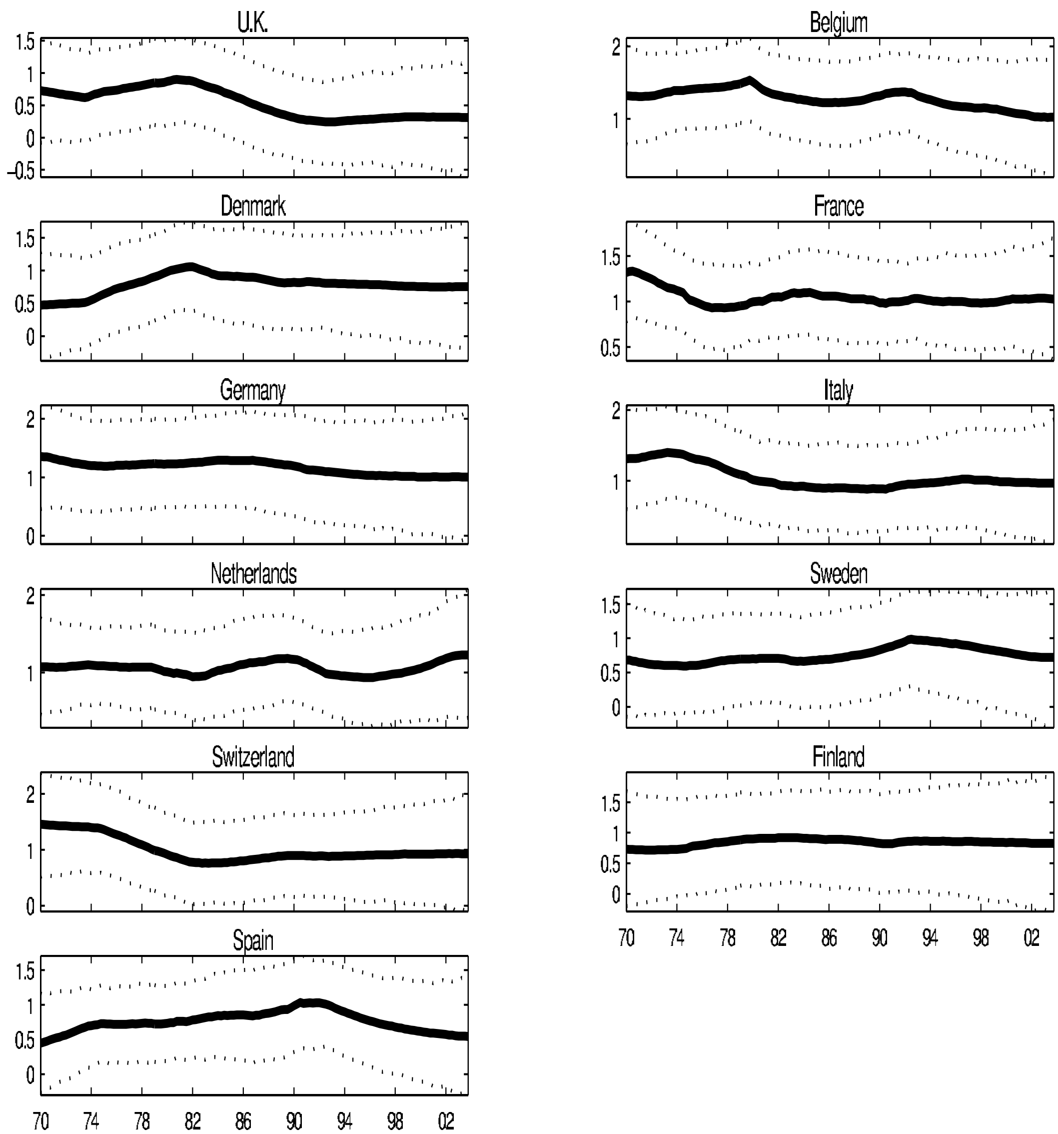
Notes:

Figure 2: Time-Varying Betas: World Factor



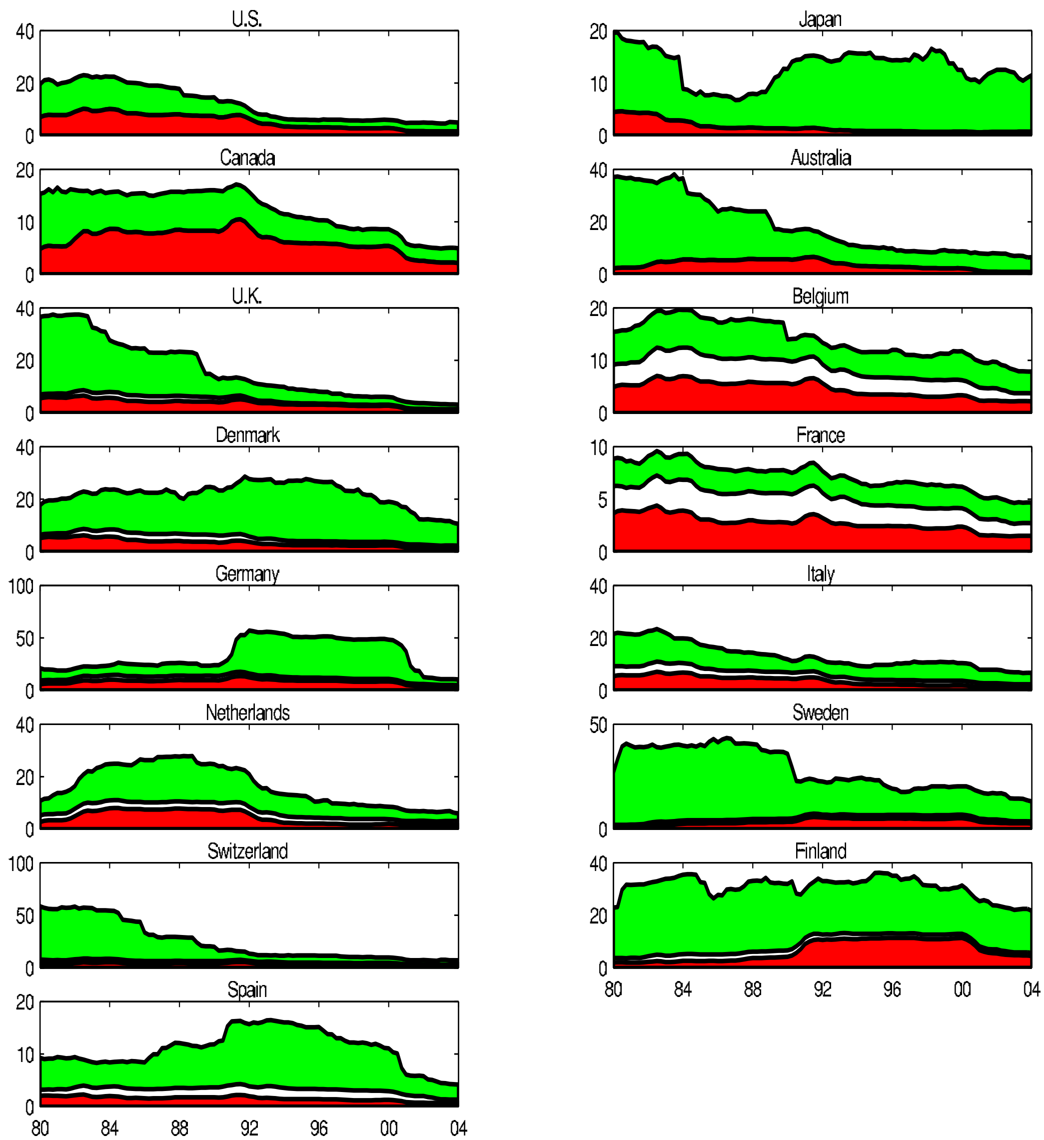
Notes:

Figure 3: Time-Varying Betas: European Factor



Notes:

Figure 4: Variance Attributable to World (Red), European (White), and Idiosyncratic (Green) Shocks Over Time: Ten Years Rolling Window



Notes: