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Rehypothecation*

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Abstract

Rehypothecation refers to the practice of re-using (selling or pledging as collateral) an asset that has already been pledged as collateral for a loan. We develop a dynamic general equilibrium monetary model where an “asset shortage” motivates the rehypothecation of assets. We find that in high inflation-high interest rate economies, rehypothecation improves economic welfare, but that there is generally too much of it. We find that regulatory constraints that limit the practice can improve economic welfare.

Keywords: rehypothecation, money, collateral.
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1 Introduction

Hypothecation refers to the practice of pledging an asset as collateral for a loan, that is, in the event of default, title to the asset passes from pledger to pledgee. Rehypothecation is a right which a pledger can give, at his discretion, to a pledgee to re-use the collateral as he wishes. When the rehypothecation right is exercised, the pledge is replaced with a right to the return of the same (or similar) asset. If the pledgee defaults, the pledger is secured only up to the value of the loan. The rehypothecation right is a standard clause in most client-broker margin account agreements.

What is the economic rationale for rehypothecation? Essentially, it is the same as hypothecation—it is a mechanism to expand liquidity when liquidity constraints bind. The source of a liquidity constraint is a lack of commitment/trust (Gale, 1978). In a world where unsecured credit is free from strategic default, there would be no need for assets to serve as exchange media. Assets would, in this case, be valued solely for their income-generating properties. When commitment is lacking, however, an unsecured pledge becomes illiquid. This motivates the accumulation of liquid assets, like money and pledgeable securities. A highly-rated tranche of mortgage-backed securities, for example, is now valued not only for its income-generating properties, but also for its ability to be sold or pledged for cash on short notice.¹

But if the supply of good collateral assets remains scarce (i.e., if hypothecation is not sufficient to relax all liquidity constraints), then rehypothecation can “stretch” a given amount of collateral over multiple transactions.² Consider, for example, a first and second party to a loan agreement. The first party borrows cash from the second party, pledging an asset as collateral (hypothecation). Suppose that in the interim, the second party meets a third party offering a profitable consumption or investment opportunity. If the second party is credit constrained, then one way to relax this constraint is to sell (or pledge) the first party’s asset to the third party. This asset is then re-acquired by the second party at or before the time the initial loan is to be settled. Rehypothecation essentially permits an asset to circulate as a

¹The ability to rapidly sell and re-purchase a security also makes it an ideal shorting/hedging instrument. Of course, sometimes things do not work out so well. For example, in 2008, Lehman Brother’s used client U.S. treasury securities to purchase subprime MBS. Going long subprime and short treasuries in 2008 turned out to be a bad bet.
²Caballero (2006) discusses the macroeconomic implications of “asset shortages.”
form of money (purchasing power). To the extent that added liquidity helps facilitate exchanges that might not otherwise take place, one should expect rehypothecation to improve market efficiency.

Of course, any private activity that relates to liquidity creation is bound to attract the attention of financial regulators. And indeed, since the financial crisis of 2008 in which rehypothecation was blamed for the Lehman collapse, the practice has come under intense regulatory scrutiny. The Dodd-Frank Act, for example, limits rehypothecation in credit derivatives markets—swap contracts must now be cleared by central counterparties holding collateral in segregated accounts. The Financial Stability Board is, as of this writing, considering various proposals to limit the rehypothecation and re-use of assets. But even prior to the crisis, various regulators had placed limits on rehypothecation. In the United States, for example, SEC Rule 15c3-3 permits a broker to rehypothecate client collateral held in a margin account up to 140% of the associated cash loan. These restrictions, however, appear to vary across jurisdictions. For example, no such limit exists in the United Kingdom while, to the best of our knowledge, the practice appears to be prohibited in Canada.

One goal of this paper is to explore the theoretical implications of a legal restriction that limits (or precludes) the practice of rehypothecation. Of course, to answer this question we need a theory of rehypothecation. To the best of our knowledge, no such theory exists—at least, not one that has been written down in the context of a monetary general equilibrium model.\textsuperscript{3}

We consider a situation in which two agents are in a trusting relationship—a partnership (we relax the assumption of commitment between these two agents later on). While trust is present within the partnership, it does not exist among potential trading partners outside the relationship. Thus, intertemporal trades that occur outside the relationship can only occur through the use of an exchange medium. We assume two possible exchange media, one of which is more liquid than the other. That is, in a subset of transactions, only cash can be used—call this the \textit{cash market}. In another subset of transactions, both cash and an interest-bearing security (a Lucas tree) can be used—call this the \textit{(secured) credit market}.

At the beginning of a trading day, partners are allocated to one of the two

\textsuperscript{3}Maurin (2014) presents a finite horizon general equilibrium model of rehypothecation, but without money.
markets. Common sense suggests that their combined wealth, consisting of money and securities, should be temporarily reassigned—cash should flow to the partner who is travelling to the cash market, and securities should flow to the partner travelling to the credit market. From an outsider’s point of view, the transaction looks like a repo—a collateralized cash loan. Of course, it is not at all clear who is lending what here—one might alternatively interpret the transaction as a loan of securities, collateralized by cash. In the model, however, collateral plays no role in securing trade within the partnership because we assume the existence of trust.

But if the security plays no role in securing debt within a partnership, what accounts for the delivery of securities in exchange for a cash loan? The answer is that the security may play some useful role in a future transaction where securities are an acceptable exchange medium. To realize this potential, it is absolutely essential that security holder is granted the right of rehypothecation. In this manner, the security enhances the commitment power for the agent engaged extra-partnership trades where securities are acceptable exchange media.

We use a quantitative version of our model to answer three questions. First, what are the positive and normative effects of permitting rehypothecation relative to a world in which it is prohibited? Second, what are the positive and normative effects of introducing an SEC Rule 15c3-3 type regulation relative to a world in which rehypothecation is unregulated? Third, what are the positive and normative effects of higher rates of inflation on rehypothecation?

In our model, like most monetary models, the optimal policy is the Friedman rule. At a low enough inflation rate (high enough rate of return on money), rehypothecation plays no economic role. For monetary policies away from the Friedman rule, however, cash-only transactions are constrained. Transactions that are supported with collateral assets may or may not be constrained, depending on the supply of available assets. These two features are familiar in many monetary models. What is new here is the demand for rehypothecation when inflation is away from the Friedman rule. Rehypothecation increases welfare because it permits cash to flow where it is needed, and the asset to flow where it can potentially be used to support a future transaction. Moreover, the welfare benefits of rehypothecation are increasing in the rate of inflation (the nominal interest rate).
For our numerical examples, we find that restrictions on the extent of rehypothecation are generally welfare improving. From the perspective of the theory of the second-best, this result is not surprising. The exact mechanism by which the result emerges, however, is interesting. Evidently, the SEC Rule 15c3-3 type regulatory constraint we consider has the effect of increasing the demand for cash (there is an incentive to accumulate additional cash to overcome the regulatory restriction). This, in turn, has the effect of increasing the value of cash, which relaxes the cash constraint in the cash market. The cost of this is reduced trade in the securities market but this latter effect is more than offset by the former.\footnote{The models of Berentsen, Huber, and Marchesiani (2014) and Geromichalos and Herrenbrueck (2014) also have the property that liquidity restrictions can improve economic welfare. See also Andolfatto (2011).}

The paper is organized as follows. We describe the economic environment in Section 2 and then describe market structure in Section 3. In Section 4, we formalize individual decision-making and characterize stationary equilibria for different cases in which debt and regulatory constraints may or may not bind. In Section 4, we relax the assumption of full commitment within partnerships and discuss the implications. We offer our conclusions in Section 6.

## 2 Environment

The model is based on Lagos and Wright (2008) and Geromichalos, Licari and Lledo (2007). Time is discrete and the horizon is infinite, $t = 0, 1, 2, \ldots, \infty$. Each date $t$ is divided into three subperiods. The first subperiod permits asset exchange. In the second subperiod, there are two segmented goods markets where output is exchanged for various types of assets. In the third subperiod, credit arrangements made in the previous two subperiods are settled (or not).

The economy is populated by two types of infinitely-lived agents labeled bankers and workers. There is a continuum of each type of agent, with the population mass of each normalized to unity.

In the first subperiod of each date, bankers and workers are assigned to one of the two segmented goods markets that operate in the second subperiod. For simplicity, we assume that a half of all bankers and workers are allocated
to each of the two second subperiod goods markets. At the individual level, this assignment is viewed as an \textit{i.i.d.} idiosyncratic shock in the first subperiod (there is no aggregate risk).

Let \((x_k, y_k)\) denote the per capita output consumed and produced in goods market \(k = 1, 2\) at date \(t\). Output is nonstorable. Let \(u(c)\) denote the flow utility payoff from consuming \(c\) and let \(-h(y)\) denote the utility cost of producing \(y\). Assume \(u'' < 0 < u', u(0) = 0\) and \(h' > 0, h'' \geq 0\) with \(h'(0) = h(0) = 0\). In the settlement subperiod, all agents have linear preferences defined over a nonstorable good, \(x \in \mathbb{R}\) (negative values of \(x\) are interpreted as production). Thus, our model adopts the quasilinear preference and timing structure of Lagos and Wright (2005).

Preferences for banker \(i \in [0, 1]\) are given by

\[
E_0^i \sum_{t=0}^{\infty} \beta^t \left[ 0.5u(c_{1,t}(i)) + 0.5u(c_{2,t}(i)) + x_t^b(i) \right] \tag{1}
\]

and preferences for worker \(i \in [0, 1]\) are given by

\[
E_0^i \sum_{t=0}^{\infty} \beta^t \left[ -0.5h(y_{1,t}(i)) - 0.5h(y_{2,t}(i)) + x_t^w(i) \right] \tag{2}
\]

where \(0 < \beta < 1\). Finally, there is a single fixed productive asset—a Lucas tree that generates a constant nonstorable income flow \(\omega \geq 0\) at the beginning of each night.

A Pareto optimal allocation is a feasible allocation that maximizes a weighted sum of \textit{ex ante} utilities (1) and (2). If \(u\) is strictly concave, then efficiency dictates \(c_{k,t}(i) = c_t\) for all bankers. If \(h\) is strictly convex, then efficiency dictates \(y_{k,t}(i) = y_t\) for all workers. Since there are equal numbers of bankers and workers in each segmented goods markets, a resource constraint implies \(c_t = y_t\) for all \(t\). Clearly, the efficient goods market allocation is stationary and satisfies \(u'(y^*) = h'(y^*)\).

The resource constraint at night is given by

\[
\int_0^1 x_t^b(i) di + \int_0^1 x_t^w(i) di = \omega \tag{3}
\]

One may, without loss treat bankers and workers symmetrically, so let \(x = x_t^b(i)\) and \(z = x_t^w(i)\) for all \(i\) and \(t\). The resource constraint may therefore
be written as $x + z = \omega$. The choice of $x$ (and $z$) serves to only distribute utility. Since the total surplus is proportional to $[u(y^*) - h(y^*)]$, the ex ante participation constraints are satisfied for any $x$ such that $h(y^*) \leq x \leq u(y^*)$.

3 Market structure

Bankers are paired off in durable relationships. For now, we assume that partnerships are fully cooperative.\textsuperscript{5} To make things even simpler than they need to be, assume that the idiosyncratic “location shock” is perfectly negatively correlated within a partnership. Thus, in the first subperiod, partners learn of their itinerary: each banker will be travelling to a different segmented goods market in the second subperiod. All agents travel together to the third subperiod, where debts are repaid and/or wealth portfolios are rebalanced.

Workers act individually and, given the structure of our model, have no reason to operate in the first subperiod. In the second subperiod, workers, like bankers, are divided equally across the two segmented goods markets. In these markets, workers have an opportunity to produce and bankers have a desire to consume. Since workers and bankers are anonymous to each other, unsecured credit arrangements are not feasible, so that exchange media are necessary.

We assume the existence of two assets: money and securities. Money takes the form of cash–zero interest fiat liabilities issued by the government. Securities consist of equity claims in the Lucas tree. Only money is acceptable for payment in one of the segmented goods market—the cash market. In the other goods market—the (secured) credit market—both money and securities can be used to finance payments. That is, in the credit market, bankers can acquire consumption with cash and securities that are either sold outright or used as collateral against an obligation to be settled in the third subperiod.

Apart from exchanges that may occur in the first subperiod (between bank partners), all markets are competitive. We assume throughout that the supply of fiat money grows at a constant gross rate $\mu > \beta$ and that new money is injected (or withdrawn) via lump-sum transfers (or taxes) to the bankers in the third subperiod. Let $\tau$ denote the real transfer per banker.

\textsuperscript{5}We will later relax this assumption and discuss its implications.
4 Decision-making

Let \((p_1, p_2)\) denote the price of output, measured in units of money, in the afternoon cash and credit markets, respectively. Let \(q\) denote the real price of securities in the second subperiod credit market and let \(\phi\) denote the ex-dividend real price of securities in the third subperiod. Finally, let \(p\) denote the nominal price of output (transferable utility) in the third subperiod.

4.1 Workers

Let \(\omega(\mu, \alpha)\) denote the value of worker who enters the second subperiod cash market \((\mu = 1)\) and credit market \((\mu = 2)\) with money balances \(\mu\) and securities \(\alpha\). Let \(N(m', a')\) denote the value of a worker who enters the third subperiod with money balances \(m'\) and securities \(a'\). Assume that \(W^k, N\) are increasing and strictly concave in each of their arguments. Let \(W^k_j, N_j\) denote the first derivatives of these functions with respect to arguments \(j = m, a\).

4.1.1 Second subperiod

A worker in the cash market sells product for cash. Since securities are illiquid in the cash market \((a' = a)\), his choice problem is given by:

\[
W^1(m, a) \equiv \max_{y_1} \{ -h(y_1) + N(m + p_1 y_1, a) \}
\]

(4)

where \(m'_1 = m + p_1 y_1 \geq 0\). Product supply satisfies the first-order condition:

\[
(1/p_1)h'(y_1) = N_m(m'_1, a)
\]

(5)

By the envelope theorem:

\[
W^1_m(m, a) = (1/p_1)h'(y_1)
\]

\[
W^1_a(m, a) = N_a(m'_1, a)
\]

(6)

(7)

In the credit market, workers supply product for money and/or assets:\(^6\)

\[
W^2(m, a) \equiv \max_{y_2, \alpha_2} \{ -h(y_2) + N(m'_2, \alpha'_2) \}
\]

\(^6\)Again, workers may acquire assets contemporaneously, or issue loans against the equivalent market value of assets put up as collateral.
where \( m'_2 = m + p_2 q a + p_2 y_2 + p_2 q a'_2 \). While there are non-negativity constraints to consider here \((m'_2, a'_2 \geq 0)\), we anticipate that these will not bind since workers are at this stage wishing to accumulate assets, not spend them. Consequently, optimal behavior is characterized by:

\[
(1/p_2)h'(y_2) = N_m(m'_2, a'_2) \tag{8}
\]

\[
p_2 q N_m(m'_2, a'_2) = N_a(m'_2, a'_2) \tag{9}
\]

By the envelope theorem:

\[
W_m^2(m, a) = (1/p_2)h'(y_2) \tag{10}
\]

\[
W_a^2(m, a) = p_2 q N_m(m'_2, a'_2) \tag{11}
\]

### 4.1.2 Third subperiod

Let \( W(m, a) \) denote the value of entering the first subperiod as a worker with portfolio \((m, a)\); i.e.,

\[
W(m, a) \equiv 0.5 W^1(m, a) + 0.5 W^2(m, a) \tag{12}
\]

For a worker that enters the evening with portfolio \((m', a')\), the choice problem is given by:

\[
N(m', a') \equiv \max_{m^+, a^+} \left\{ \begin{array}{c}
[(\phi + \omega) a' + (1/p)(m' - m^+) - \phi a^+] \\
+ \beta W(m^+, a^+) + \zeta m^+ + \xi a^+
\end{array} \right\} \tag{13}
\]

We anticipate that workers will want to dispose of their money and asset holdings in the evening, so here we make explicit the non-negativity constraints \( m^+, a^+ \geq 0 \). Optimality requires:

\[
(1/p) = \beta W_m(m^+, a^+) + \zeta \tag{14}
\]

\[
\phi = \beta W_a(m^+, a^+) + \xi \tag{15}
\]

By the envelope theorem:

\[
N_m(m', a') = (1/p) \tag{16}
\]

\[
N_a(m', a') = (\phi + \omega) \tag{17}
\]
4.1.3 A simplifying assumption: linear cost

For the remainder of the paper, we assume $h(y) = y$. In this case, the first-order conditions above should be re-interpreted to be equilibrium no-arbitrage conditions. These conditions will then be imposed when we characterize the decisions of bankers. The linear cost structure permits us to proceed in this manner and greatly aids exposition without detracting from the key results we report below.

Since $h'(y) = 1$, conditions (5) and (16) imply $p_1 = p$. Likewise, conditions (8) and (16) imply $p = p_2$. Finally, conditions (9), (16) and (16) imply $q = (\phi + \omega)$.

4.2 Bankers

4.2.1 Morning

Because bankers acquire money and securities in the third subperiod, they will be carrying assets into the first subperiod of the next period. Consider a bank partnership that enters the first subperiod with a combined wealth portfolio $(m, a)$. Ex ante, individuals in a bank partnership do not know whether they will be travelling to the cash market or the credit market. This location shock is realized in the first subperiod and at this point, bank partners have an opportunity to reallocate their combined assets between them.

Each banker is assumed to own an equal share of the coalition portfolio $(m/2, a/2)$. The value of the partnership lies in the opportunity for “asset trades” in the morning, conditional on type realizations. Let $(m_2, a_2)$ denote the portfolio allocated to the banker travelling to the credit market, where:

$$m \geq m_2 \geq 0$$  \hspace{1cm} (18)

$$a \geq a_2 \geq 0$$  \hspace{1cm} (19)

Given our setup, a natural exchange would have cash flowing to the cash-
banker and assets flowing to the credit-banker; i.e.,

\[ m_2 < m/2 \]
\[ a_2 > a/2 \]

If \( m_2 < m/2 \), then the credit-banker is in effect sending \([m/2 - m_2]\) dollars to the cash-banker. If \( a_2 > a/2 \), then the cash-banker is in effect sending \( p(\phi + \omega)[a_2 - a/2]\) dollars worth of assets to the credit-banker, where \( p(\phi + \omega) \) is the nominal price of the security in the second subperiod. If the value of what is exchange is equated, then the transaction can be considered either as an outright purchase of assets by the credit-banker, or as a fully collateralized cash loan to the cash-banker. (Alternatively, one might view the transaction as a fully collateralized asset loan to the credit-banker.)

Of course, there is no reason to believe \textit{a priori} that a “balanced” trade \([m/2 - m_2]\) = \( p(\phi + \omega)[a_2 - a/2]\) is necessarily optimal in this partnership. In fact, we anticipate that an optimal intrabank allocation will sometimes have the property \( 0 < [m/2 - m_2] < p(\phi + \omega)[a_2 - a/2] \). In this case, the transaction looks like an overcollateralized cash loan (from credit-banker to cash-banker) or, equivalently, an undercollateralized asset loan (from cash-banker to credit-banker).

It is the undercollateralized asset loan that seems to trouble regulators concerned with the rehypothecation of securities. While the rehypothecation of borrowed cash is viewed as natural (what else is one \textit{supposed} to do with borrowed cash?), the rehypothecation of borrowed securities is not.\(^8\) As such, some jurisdictions place restrictions on the rehypothecation of borrowed securities. One such real-world restriction takes the following form:

\[ \theta [m/2 - m_2] \geq p(\phi + \omega)[a_2 - a/2] \]  \hspace{1cm} (20)

for some policy parameter \( \theta \geq 1 \). Think of \( \theta \) as the largest leverage ratio permissible in any under collateralized asset loan (in which the collateral can be rehypothecated). In what follows, we will be interested in examining the economic consequences of this regulatory restriction.\(^9\)

\(^8\)An image sometimes offered of the practice is that of borrowing money against a vehicle, and then discovering that your vehicle has unwittingly been sold under your nose to another agent who takes possession of it. In reality, the rehypothecation right is something a debtor must agree to beforehand.

\(^9\)Note that we assume that there are no outright sales of securities between bank part-
4.3 Second subperiod

Recall that \((m, a)\) represents the partnership’s combined assets in the morning and when entering the afternoon. Recall as well that \((m_2, a_2)\) denotes the portfolio allocated to the banker travelling to the credit market in the afternoon. Let \((m', a')\) denote the partnership’s combined asset portfolio entering the evening.

The afternoon flow budget constraint for the partnership is given by:

\[
m - m' + p(\phi + \omega)(a - a') - py_1 - py_2 \geq 0
\]  

(21)

The individual bankers are subject to liquidity constraints depending on their itinerary. The cash-banker is subject to the following liquidity constraint:

\[
m - m_2 - py_1 \geq 0
\]  

(22)

while the credit-banker is subject to:

\[
m_2 + p(\phi + \omega)a_2 - py_2 \geq 0
\]  

(23)

There are a number of non-negativity constraints that need to be made explicit. Individual banker asset holdings must be non-negative. Since these constraints hold individually, they will automatically hold at the partnership level; i.e., \(m', a' \geq 0\).

Condition (22) guarantees that the cash-banker’s asset holdings are non-negative when entering the evening. Condition (23) for the credit-banker only imposes a non-negativity restriction on his combined asset holdings. We need to impose the additional conditions that restrict his cash and security holdings be non-negative, respectively.

Since \(m'\) is the total cash brought into the evening by the partnership and \([m - m_2 - py_1]\) is the cash brought into the evening by the cash-banker, the difference between these two objects, representing the cash brought into the evening by the credit-banker, must be non-negative:

\[
m' - [m - m_2 - py_1] \geq 0
\]  

(24)

It would be straightforward to embed assumptions that motivated the choice to pledge rather than sell assets; see Duffie, Gârleanu, and Pederson (2008) or Geanakoplos (2010). Doing so would not alter the qualitative nature of the conclusions we report below.
A similar argument applied to the credit-banker’s securities holdings implies:

\[ a' - [a - a_2] \geq 0 \]  \hspace{1cm} (25)

Recall that \( a' \) represents the partnership’s total security holdings entering the evening. The difference \( a - a_2 \) represents the (unspent) securities held by the cash-banker. Thus (25) restricts the credit-banker’s security holdings to be non-negative.

We will now show that if the liquidity constraint on the cash-bankers holds with equality (which is always the case when \( \mu > \beta \) and wlog when \( \mu = \beta \)), then restrictions (21), (22), (24) and (25) imply (23). Given that (25) implies \( a_2 \geq a - a' \), the flow budget constraint (21) implies \( m - m' + p(\phi + \omega)a_2 - py_1 - py_2 \geq m - m' + p(\phi + \omega)(a - a') - py_1 - py_2 \geq 0 \). Note that (22) and (24) imply \( m' \geq 0 \). Thus, \( m + p(\phi + \omega)a_2 - py_1 - py_2 \geq 0 \). Assuming (22) holds with equality, \( m - py_1 = m_2 \), which yields (23).

4.3.1 Evening

Let \((m^+, a^+)\) denote the partnership’s combined assets carried into the next period. The partnership is subject to the following budget constraint:

\[ x = (\phi + \omega)a' + (1/p)(m' - m^+) - \phi a^+ - 2\tau \]  \hspace{1cm} (26)

4.4 Bank partnership problem

Let \( B(m, a) \) denote the value of the bank partnership entering the morning with combined assets \((m, a)\). Let \( V(m', a') \) denote the value of the partnership entering the evening with combined assets \((m', a')\). These value functions must satisfy the recursion:

\[ B(m, a) \equiv \max_{y_1, y_2, m_2, a_2 m', a'} \{u(y_1) + u(y_2) + V(m', a')\} \]

subject to (20), (21), (22) and the non-negativity constraints (18), (19), (24), (25). Note that since we anticipate morning cash flowing to the cash-banker and morning securities flowing to the credit-banker, the non-negativity constraints (18), (19) simplify to \( m_2 \geq 0 \) and \( a - a_2 \geq 0 \). Recall that (23) is implied by the other constraints.
The necessary first-order conditions for an optimum are:

\[ u'(y_1) - p[\psi + \lambda - \zeta_1] = 0 \]  
\[ u'(y_2) - p\psi = 0 \]  
\[ -\chi \theta - \lambda + \zeta_1 + \zeta_3 = 0 \]  
\[ -p(\phi + \omega)\chi + \zeta_2 - \zeta_4 = 0 \]  
\[ V_m(m', a') - \psi + \zeta_1 = 0 \]  
\[ V_a(m', a') - \psi p(\phi + \omega) + \zeta_2 = 0 \] 

By the envelope theorem:

\[ B_m(m, a) = \chi \theta / 2 + \psi + \lambda - \zeta_1 \]  
\[ B_a(m, a) = p(\phi + \omega) [\chi / 2 + \psi] - \zeta_2 + \zeta_4 \] 

In the evening, the choice problem solves:

\[ V(m', a') \equiv \max_{m^+, a^+} \{ (\phi + \omega) a' + (1/p)(m' - m^+) - \phi a^+ - 2\tau + \beta B(m^+, a^+) \} \]

There are also the non-negativity constraints \( m^+, a^+ \geq 0 \), but we anticipate that these will not bind for bankers in the evening.\(^{10}\) The money and asset demands in the evening must satisfy:

\[ (1/p) = \beta B_m(m^+, a^+) \]  
\[ \phi = \beta B_a(m^+, a^+) \] 

By the envelope theorem:

\[ V_m(m', a') = (1/p) \]  
\[ V_a(m', a') = (\phi + \omega) \] 

### 4.5 Gathering restrictions

We restrict attention to stationary allocations where all real variables are constant over time and nominal variables grow at rate \( \mu \). Combine (35), (33) and (27) to form:

\[ \mu = \beta[u'(y_1) + \theta p \chi / 2] \]

\(^{10}\)Bankers will want to rebuild their asset positions in order to finance their consumption expenditures the next day.
When the regulatory constraint is slack ($\chi = 0$), we get the standard result that $\mu > \beta$ implies $y_1 < y^*$. 

Now combine (36), (34), (28), (30) and (32) to form:

$$\phi = \beta(\phi + \omega) [u'(y_2) - p\chi/2]$$

(40)

Conditions (27), (31) and (37), we have

$$p\lambda = u'(y_1) - 1$$

(41)

which implies $\lambda > 0$ iff $y_1 < y^*$. From (28) we have

$$p\psi = u'(y_2)$$

(42)

Clearly, $p\psi \geq 1$, i.e., $\psi > 0$, so that the afternoon flow budget constraint (21) binds. Using (28), (31), (32), (37) and (38) we get

$$p\zeta_1 = u'(y_2) - 1$$

(43)

$$(\phi + \omega)^{-1} \zeta_2 = u'(y_2) - 1$$

(44)

There are two cases to consider: $y_2 = y^*$ and $y_2 < y^*$. If $y_2 = y^*$, then $u'(y_2) = 1$ and so (43)-(44) imply $\zeta_1 = \zeta_2 = 0$. If $y_2 < y^*$, then $u'(y_2) > 1$ which again, from (43)-(44) implies $\zeta_1 > 0$, and $\zeta_2 > 0$.

Finally, using (29), (30), (41)--(44) we get

$$p\zeta_3 = u'(y_1) - u'(y_2) + \theta p\chi$$

$$\zeta_4 = u'(y_2) - 1 - p\chi$$

(45)

(46)

**Lemma 1** $y_2 = y^*$ only if $\chi = 0$.

**Proof.** Follows from (46) and $\zeta_4 \geq 0$. 

We now invoke the market-clearing condition $m = 2M$. Cash-bankers spend all of their cash (at the Friedman rule, they weakly prefer to do so). Thus, (22) holds with equality. Together with the market-clearing condition, we have:

$$2M - m_2 = py_1$$

(47)

Finally, the regulatory constraint needs to be satisfied in equilibrium. Thus,

$$\theta[M - m_2] \geq p(\phi + \omega)(a_2 - 1)$$

(48)
where here, we have invoked the equilibrium conditions $m = 2M$ and $a = 2$. If this constraint remains slack, then $\chi = 0$; otherwise $\chi > 0$ and the condition above holds with equality.

To characterize the different possible configurations for stationary equilibria, we begin by assuming that the regulatory constraint is slack.

### 4.6 Regulatory constraint is slack

Assume that the regulatory constraint is slack ($\chi = 0$). Then from (39) we have a condition that determines $y_1$ solely as a function of inflation $\mu$; i.e., $\mu = \beta u'(y_1)$. Once again, the Friedman rule is consistent with first-best implementation, at least, as far as the cash-market is concerned. Let us assume (and then verify) that $y_2 = y^*$, so that $u'(y_2) = 1$. Then from (40) we have the standard asset-pricing formula for the Lucas tree; i.e, $\phi^* \equiv \beta (\phi^* + \omega)$. Assume that $\mu > \beta$, so that $u'(y_1) > 1$. Since $\chi = 0$, from (45) we have $\zeta_3 > 0$. This, in turn, implies $m_2 = 0$, so that (47) determines the evening price-level $p = 2M/y_1$. Note, as well that $\zeta_4 = 0$ from (46) but that without loss, we can assign all securities to the credit-banker, i.e., $a_2 = 2$.

We now identify the circumstances under which the assumption $y_2 = y^*$ is legitimate. Essentially, we need condition (23) to hold at $y_2 = y^*$. That is, since $m_2 = 0$, we need $(\phi + \omega) a_2 \geq y^*$. With the asset priced at fundamental value $\phi^*$ and with all assets allocated to the credit-banker ($a_2 = a = 2$), this latter condition implies

$$\omega \geq 0.5 (1 - \beta) y^* \equiv \hat{\omega} \quad \text{(49)}$$

Let us now fix $\omega \geq \hat{\omega}$ and identify what is needed to ensure that the regulatory constraint does not bind. Recall that $p = 2M/y_1$, $m_2 = 0$, $a_2 = 2$ and $\phi = \phi^*$. Combining these conditions with (48), we derive:

$$\theta \geq \left[ \frac{2\omega}{(1 - \beta) y_1} \right] \equiv \Theta(\omega, \mu) \quad \text{(50)}$$

These results are summarized in the following proposition.

**Proposition 2** Assume $\mu > \beta$. If $\omega \geq \hat{\omega}$ and $\theta \geq \Theta(\omega, \mu)$, then $y_1 < y^*$, $y_2 = y^*$ and $\chi = 0$. The critical regulatory parameter $\Theta(\omega, \mu)$ is increasing in $\omega$ and $\mu$. 

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Recall that higher values of $\theta$ imply that it is easier to satisfy the regulatory restriction on rehypothecation. Larger values of $\omega$ imply a higher value for securities and hence, a higher value for the securities that may potentially be rehypothecated. For the regulatory constraint to remain slack as $\omega$ is increased, the parameter $\theta$ must be increased to relax the regulatory restriction.

The critical value $\Theta(\omega, \mu)$ is increasing in $\mu$ because higher rates of inflation reduce the demand for real money balances $y_1$. Since $p = 2M/y_1$, the price-level increases which again, from (48), makes the regulatory constraint more difficult to satisfy. Higher $\mu$ reduces the real value of cash balances or, equivalently, raises the nominal value of securities that may be rehypothecated.

4.7 Regulatory constraint binds

Consider an $\omega \geq \hat{\omega}$ so that $y_2 = y^*$ is implementable when $\theta \geq \Theta(\omega, \mu)$. Now assume that for this $\omega$ we have $\theta < \Theta(\omega, \mu)$ so that the regulatory constraint can no longer be satisfied when $y_2 = y^*$.

Lemma 3 If $\omega \geq \hat{\omega}$ and $\theta < \Theta(\omega, \mu)$, then $\chi > 0$ and $y_2 < y^*$.

Proof. If $y_2 = y^*$ and $\chi = 0$, then condition (50) cannot be satisfied for $\theta < \Theta(\mu, \omega)$. From Lemma 1, we have $y_2 = y^*$ only if $\chi = 0$. Thus, when $\theta < \Theta(\omega, \mu)$ we either have: (i) $y_2 < y^*$ and $\chi = 0$; or (ii) $y_2 < y^*$ and $\chi > 0$. We will show that (i) leads to a contradiction so that (ii) must be true.

Suppose $y_2 < y^*$ and $\chi = 0$. Condition (49) implies that when $\omega \geq \hat{\omega}$, $(\phi^* + \omega)2 \geq y^*$, i.e., the credit banker can purchase the first-best when securities are priced at their fundamental value and $a_2 = a = 2$. When $\chi = 0$, (40) implies that $\phi > \phi^*$ (the real value of the Lucas tree is above fundamental) and (46) implies that $\zeta_4 > 0$ so that $a_2 = a = 2$ (all the partnership’s securities are allocated to the credit-banker). Thus, $(\phi + \omega)2 \geq y^*$, i.e, the credit-banker can more than afford to purchase $y^*$, a contradiction with $y_2 < y^*$. ■

From (43) and (44) we see that $y_2 < y^*$ implies $\zeta_1 > 0$ and $\zeta_2 > 0$, which in turn implies $m' = 0$ (since 22 holds with equality) and $a' = a - a_2$. In other
words, the partnership carries no cash into the evening and the credit-banker spends all of his assigned securities.\footnote{Note that the credit-banker may not possess all of the partnership’s securities when the restriction on rehypothecation binds. The credit-banker will spend all securities at his disposal and the partnership carries the unspent securities $a - a_2$ into the evening.}

Using (39), we can express $p\chi$ in terms of $y_1$,

$$p\chi = (2/\theta)[\mu/\beta - u'(y_1)]$$

which allows us to solve the asset price $\phi$ from (40) as a function of $(y_1, y_2)$, i.e.,

$$\phi = \left[\frac{\beta L(y_1, y_2)}{1 - \beta L(y_1, y_2)}\right] \omega$$

where

$$L(y_1, y_2) \equiv (1/\theta)[u'(y_1) - \mu/\beta] + u'(y_2)$$

Note that $L(y_1, y_2) < 1/\beta$ can be interpreted as a liquidity premium on the security.

Recall that $\zeta_3$ and $\zeta_4$ are the Lagrange multipliers associated with the constraints $m_2 \geq 0$ and $a - a_2 \geq 0$, respectively. Recall that when $\chi = 0$ and $y_2 = y^*$, we showed that $m_2 = 0$ and $a_2 = a$. From (45) and (46), we see that $\chi > 0 \Rightarrow y_2 < y^*$ does not necessarily imply that one or both of these constraints bind. Combine (51) and (52) with conditions (45) and (46) to form:

$$p\zeta_3 = \frac{\omega \theta \zeta_4}{1 - \beta L(y_1, y_2)} = \left[u'(y_1) - \mu/\beta\right] + \theta[L(y_1, y_2) - 1] \geq 0$$

Next, we combine the market-clearing conditions $m = 2M$ and $a = 2$ with the liquidity constraints (22) and (23) which, together with the regulatory constraint (48) and (52), implies

$$y_1 = \frac{2M/p - m_2/p}{a_2 \omega}$$

$$y_2 = \frac{m_2/p + - \frac{a_2 \omega}{1 - \beta L(y_1, y_2)}}{\theta[M/p - m_2/p]} = \frac{(a_2 - 1) \omega}{1 - \beta L(y_1, y_2)}$$
With \( \phi \) determined by (52), conditions (53)-(57), together with the non-negativity constraints \( m_2 \geq 0, a-a_2 \geq 0 \), characterize the equilibrium values \( y_1, y_2, m_2, a_2, \rho, \zeta_3, \zeta_4 \).

**Proposition 4** Assume \( \mu > \beta, \omega \geq \hat{\omega}, \) and \( \theta < \Theta(\omega, \mu) \). Then, the following is true: (i) \( u'(y_1) < \mu/\beta \); (ii) \( y_2 < y^* \); (iii) \( a_2 < 2 \) so that \( \zeta_4 = 0 \). Furthermore if \( \theta > 1 \) then: (iv) \( m_2 = 0 \) with \( \zeta_3 > 0 \); and (v) \( a_2 > 1 \).

**Proof.** Lemma 3 implies \( \chi > 0 \).

(i) \( u'(y_1) < \mu/\beta \) follows from (51) and \( \chi > 0 \).

(ii) \( y_2 < y^* \), follows from Lemma 3.

(iii) First, we need to show that \( L(y_1, y_2) \in (1, 1/\beta) \). \( L(y_1, y_2) < 1/\beta \) follows from (52) and \( \phi > 0 \) in equilibrium. To show \( L(y_1, y_2) > 1 \) we use (54). A simple rearrangement implies \( L(y_1, y_2) \geq 1 + (1/\theta)[\mu/\beta - u'(y_1)] > 1 \), where the last inequality follows from (i).

To show \( a_2 < 2 \) (and hence, \( \zeta_4 = 0 \)), note that (56) can be rearranged as
\[
\frac{[1 - \beta L(y_1, y_2)]y_2}{1 - \beta} = \frac{[1 - \beta L(y_1, y_2)]m_2/\rho}{1 - \beta} + \frac{a_2\omega}{1 - \beta}.
\]

Given \( m_2 \geq 0 \) and \( \omega \geq \hat{\omega} \) we obtain, using (49)
\[
\frac{[1 - \beta L(y_1, y_2)]y_2}{1 - \beta} \geq \frac{a_2\omega}{1 - \beta} \geq 0.5a_2y^*.
\]

Suppose \( a_2 = 2 \). Since \( L(y_1, y_2) \in (1, 1/\beta) \), we get \( y_2 > y^* \), a contradiction with (ii). Thus, \( a_2 < 2 \) and \( \zeta_4 = 0 \).

(iv) Suppose \( \zeta_3 = 0 \) and impose \( \zeta_4 = 0 \) from (iii). Then, (53) and (54) imply
\[
\begin{align*}
    u'(y_1) - 1 &= \frac{2(\theta - 1)}{(\theta - 2)}(\mu/\beta - 1) \\
    u'(y_2) - 1 &= \frac{2}{(2 - \theta)}(\mu/\beta - 1).
\end{align*}
\]

If \( \theta > 1 \), we need both \( \theta < 2 \) and \( \theta > 2 \), so this cannot be an equilibrium. Thus, \( \zeta_3 > 0 \) with implies \( m_2 = 0 \).
(v) \( a_2 > 1 \) follows from (57), \( m_2 = 0 \) and \( L(y_1, y_2) < 1/\beta \). □

We can use the results in Proposition 4 to characterize the equilibrium more sharply. Given \( \zeta_4 = 0 \), (54) implies \( L(y_1, y_2) = \ell(y_1) \) where:

\[
\ell(y_1) \equiv 1 + (1/\theta)[\mu/\beta - u'(y_1)].
\]

When parameters are such that \( m_2 = 0, \zeta_3 > 0 \), the equilibrium allocation \((y_1, y_2)\) is characterized by:

\[
\begin{align*}
y_2 - 0.5\theta y_1 &= \frac{\omega}{1 - \beta \ell(y_1)} \\
\mu/\beta - u'(y_1) &= 0.5\theta[u'(y_2) - 1].
\end{align*}
\]

Proposition 4 states that \( \theta > 1 \) is sufficient to guarantee \( \zeta_3 > 0 \). However, it is easy to construct example where \( \theta < 1 \) and \( \zeta_3 > 0 \).

In addition, given the allocation \((y_1, y_2)\) solved above, we get

\[
\begin{align*}
p &= \frac{2M}{y_1} \\
a_2 &= \frac{y_2[1 - \beta \ell(y_1)]}{\omega}.
\end{align*}
\]

With a binding regulatory constraint, the cash-banker is disposing of all the coalition’s cash, while the credit-banker is unable to utilize all the coalition’s securities. Thus, the market value of the cash loan to the cash-banker exceeds the market value of the securities loan to the credit-banker. From an outsider’s perspective, this transaction would look like an under-collateralized cash loan or, equivalently, an over-collateralized securities loan.

Assuming this is the case, then the effect of a binding regulatory constraint is to *increase* \( y_1 \) *closer to its first-best level* at the cost of lowering \( y_2 \). This suggests the possibility that the regulatory constraint might improve welfare due to consumption smoothing across types. When the regulatory constraint binds, the bank partnership attaches additional value to cash accumulation because more cash helps to overcome the regulatory constraint on future rehypothecation. The additional cash helps finance more consumption in the early afternoon, at the expense of the late afternoon.
4.8 Comparative statics with a binding regulatory constraint

Suppose \( u(c) = \ln c \). Then, we can solve the model analytically. The expression are, however, rather complex and cannot be easily interpreted. Consider instead expressing \( y_1 \) in terms of \( y_2 \). We get

\[
 y_1 = \frac{2\beta}{2\mu - \beta \theta (1/y_2 - 1)}.
\]

Since \( y_2 < y^* = 1 \) when \( \chi > 0 \), we have that \( y_1 \) is decreasing in \( y_2 \). Therefore, if we start in an equilibrium where \( y_2 = y^* \) and the regulatory constraint is just satisfied, i.e., where \( \omega = \hat{\omega} \) and \( \theta = \Theta(\hat{\omega}, \mu) \), then any parameter perturbation that makes the regulatory constraint bind implies \( y_1 \) increases and \( y_2 \) decreases. Since bankers’ preferences are strictly concave, any small such perturbation improves their ex-ante welfare (i.e., before their afternoon type is revealed). Specifically, define \( W(y_1, y_2) \equiv u(y_1) - y_1 + u(y_2) - y_2 \) and we get

\[
 \frac{dW(y_1, y_2)}{dy_2} \bigg|_{y_2 = y^*} = -\frac{\beta \theta (\mu - \beta)}{2\mu^2} < 0.
\]

The intuition for the welfare result derived above is that the regulatory constraint corrects the inefficiency generated by being away from the Friedman rule. Starting from an equilibrium with \( \mu > \beta \) and \( y_1 < y^*, y_2 = y^* \), imposing a binding regulatory constraint increases the rate of return on cash, since it is now valued higher than before due its ability to relax the regulatory constraint.

4.9 Numerical analysis

We now show the effects of tightening regulation and increasing inflation. Throughout this section, we assume \( U(c) = \ln c, \beta = 0.96 \) and \( \omega = 0.015 \). When we vary regulation tightness \( \theta \), we assume \( \mu = 1 \). When we vary inflation, we assume: \( \theta = 1.05 \).

Figures 1-3 show the effects of tightening the regulatory constraints, i.e., reducing \( \theta \). The x-axis of each figure is in terms of \( 1/\theta \), so that we are tightening the constraint as we move to the right. The welfare calculations in Figure 3 correspond to equivalent consumption compensation, relative to
the case when banks act unilaterally (i.e., when there is no partnership). As we can see, allowing for rehypothecation (unregulated case) is welfare improving, but for moderate levels of \( \theta \) we can improve welfare even further by restricting the practice.

Figures 4-6 show the effects of increasing inflation. As we can see, the effects of increasing inflation are similar to those of tightening the regulatory constraint. Figure 6 shows welfare for a given inflation rate relative to the first best (i.e., at the Friedman rule). It compares the cases of no partnership, unregulated partnership and regulated partnership. Allowing for rehypothecation is especially useful for high inflation rates. The gains from a binding regulation are also increasing in inflation, as the regulation mitigates the inefficiency of a low rate or return on cash.

5 Limited commitment

In this section, we relax the assumption of full commitment between members of a bank partnership. We focus on verifying when the equilibrium described in the previous section can be supported under limited commitment and how default incentives vary with regulation tightness, inflation and securities valuation. In what follows, we assume a parameterization such that \( m_2 = 0 \).

In the equilibrium described above, bankers dispose of their assets in the afternoon and are required to re-build their asset positions in the evening. In some cases, bankers will have different incentives to rebalance the partnership’s portfolio. One case in which this happens is when the regulatory constraint binds so that the cash banker is carrying securities into the evening. There is an incentive for the cash banker to “run away” with these securities, instead of pooling them for the benefit of the partnership.

Suppose now that banks in partnership do not trust each other and that some penalty can be enforced if a bank fails to fulfill its obligation in the evening. Specifically, we assume that the punishment for default is autarky, i.e., a bank operates on its own after defaulting. Let \( W \) be the ex-ante value of the equilibrium allocation. The evening value for a cash-banker remaining in the partnership is

\[
\begin{align*}
& \left( \phi + \omega \right)(2 - a_2) + \phi(a_2 - 1) - \frac{M}{p} - \phi - \mu\frac{M}{p} + \tau + \beta W \\
& \quad \text{non-lent assets} \quad \text{repayment} \quad \text{new cash & assets}
\end{align*}
\]
whereas the evening value for a credit-banker remaining in the partnership is

\[ -\phi(a_2 - 1) + \frac{M}{p} \quad -\phi - \mu \frac{M}{p} + \tau + \beta W \]

repayment \quad new cash & assets

Clearly, tightening rehypothecation and/or varying inflation affects the value of remaining in the partnership.

The evening values of defaulting are

\[ (\phi + \omega)(2 - a_2) + \max_{\hat{m}', \hat{a}'} - \hat{m}' / p - \phi \hat{a}' + \tau + \beta D(\hat{m}', \hat{a}') \]

\[ \max_{\hat{m}', \hat{a}'} - \hat{m}' / p - \phi \hat{a}' + \tau + \beta D(\hat{m}', \hat{a}') \]

for the cash-banker and credit-banker, respectively. The function \( D(m, a) \) is the value of autarky (omitted here to save space). Note that the cash-banker has a higher default payoff than the credit-banker when \( a_2 < 2 \). Thus, tightening rehypothecation and/or varying inflation affects the value of defaulting on the part of the cash-banker.

Figure 7 shows how tighter regulation and lower inflation increase default incentives for the cash-banker. Both lower \( \theta \) and lower \( \mu \) increase the rate or return on cash, which makes autarky more attractive. Figure 8 shows that lower dividends (lower fundamental value for securities) also increase the default incentives for the cash-banker, as the partnership becomes less valuable.

6 Conclusions

Rehypothecation turns collateral into exchange media, which is desirable when liquidity is scarce. The practice is generally improves welfare, especially in high inflation (interest rate) environments. However, there is some reason to believe that there is generally too much of it. We find that limited interventions can improve welfare by enhancing the demand for cash, which is too low when monetary policy is away from the Friedman rule.

It would be of some interest to extend the model developed here to incorporate aggregate uncertainty over asset returns when commitment between bankers is limited. Imagine, for example, a “news shock” arriving in the
afternoon that leads to a rational revision in the conditional forecast of the asset’s payoff in the evening. A good news shock may increase the value of collateral to a point where short positions (held by the credit banker) may not be covered. It would then be possible to investigate how policy might be designed to accommodate spikes in “repo failures” in times of financial crisis (see Hördahl and King 2008).
7 References


