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Endogenous Borrowing Constraints and Stagnation in Latin America

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Preliminary and Incomplete

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Abstract

Latin America has had striking changes in economic performance over time. Following the recession and debt crises of the early 1980's, consumption declined for about ten years and consumption per-capita in the year 2004 was roughly the same as it was in 1980. This paper studies consumption stagnation in Latin America using a small open economy real business cycle model with endogenous borrowing limits, capitalistic production and domestic productivity and international interest rate shocks. I find that the model does an excellent job matching the observed behavior of per-capita consumption, and that the interaction of both productivity and international interest rate shocks with the borrowing limit is key.

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Figure 1: Per-capita consumption in Latin America

1 Introduction

It is well known that in Latin America fluctuations are bigger and recessions last longer than in developed countries. Major recessions tend to occur at the same time as debt crises, and borrowing capacity changes significantly around the time of recessions.

Furthermore, Figure 1 shows that following the recession and debt crises of the early 1980's (shaded grey region) per-capita consumption declined for about ten years (blue line) and that per-capita consumption in 2004 was not very different from that in 1980.

Due to the coincidence between debt crises and stagnation, there is interest in studying these events within a single framework to understand how debt crises impact recessions, and how recessions feed back on international borrowing.

In this paper I study Latin America's per-capita consumption stagnation in a real business cycle model with endogenous borrowing limits and capitalistic production. I concentrate in understanding

the medium to low frequencies of the data: the consumption slowdown of the 80's and the slower pick up of the 90's. As such I will focus in understanding the Hodrick and Prescott trend of the data (black line in Figure 1).

In the model, the borrowing limits arise endogenously due to limited commitment as in Kehoe and Levine (1993, [8]), Alvarez and Jermann (1996, [3]) and Kehoe and Perri (2002, [7]) between others. The existence of limited commitment is relevant for Latin America, as their willingness to pay is often questioned and restricts their access to international markets. Furthermore, limited commitment models are widely used in theory but we have little or no idea about how they perform when we want to explain the equilibrium path of fundamentals.

The model has two stochastic elements: international interest rate shocks and domestic productivity shocks. Incorporating the productivity and international interest rate shocks allows me to study the impact of domestic and external factors that are often cited in analyzing fluctuations in emerging economies¹ and evaluate the effect that these shocks have on the medium to long term economic performance of Latin America when there are limits to international borrowing.

I infer the productivity shock using Latin data, and I measure the external interest rate shock using data from rich OECD countries. I feed these shocks into the model and compare the Hodrick and Prescott trend of the equilibrium path generated by the model with that of the observed data to assess how endogenous variations in borrowing capacity have affected Latin America.

I find that the model does an excellent job matching the observed behavior of per-capita consumption. More specifically, from Figure 1 it can be seen that per-capita consumption in Latin America grew at a rate of about 2.8% prior to 1980, it had a slightly negative growth rate in the 80's and it picked up again at a rate of around 1.4% from the 90's to 2005. The model is able to generate the fast growth at the beginning of the period, the decade long slowdown, and the slower pick up after the 90's. Furthermore, the interaction of both productivity and interest rate shocks with the borrowing limit is key for this result. Nevertheless, the model does less well at accounting for domestic investment and hence for net-exports.

My results also have some implications for theory. The duration of the constrained spells is smaller than in a model without interest rate shocks. As a result, the outside option in the participation

¹See Aguiar and Gopinath (2007, [2]) and Neumeyer and Perri (2005, [13]) for example.

constraint does not have to be penalized to avoid spending too much time constrained in equilibrium. Also, the participation constraint binds not only in good times—as is known for limited commitment models—, but it can also be triggered in prolonged bad times. This feature is not specific to my model, this would be the case in every limited commitment model, but this feature is unknown given that this is the first paper to examine the equilibrium path followed by a participation constraint given a time path for productivity and interest rate shocks.

The model consists of a small open economy (SOE) that interacts with a risk neutral lender. I solve a dynamic contracting problem between these two parties. The SOE produces a single good using labor and capital as inputs, and can not commit to repaying its debt (one sided limited commitment model). Finally, the international interest rate shock is a shock to the discount factor of the lender.

The limited commitment implies that international loans are feasible only to the extent to which they can be enforced by the threat of exclusion from future intertemporal and interstate trade (autarky). So whether the constraint is binding or not, depends on how the value of being in autarky given the actual capital stock and productivity level compares to the value of being in the contract. Whenever the value of autarky exceeds the value of the contract, the borrowing constraint binds. At this point the risk neutral lender has to raise the value of the contract by increasing the consumption and capital assigned to the SOE, so that the participation constraint holds with equality.

Prior to 1980 productivity was high in Latin America and international interest rates were low. High productivity increases the value of autarky and the low interest rate lowers the cost of capital, making its accumulation cheaper and hence increasing the value of autarky further. These two effects trigger the constraint and hence the risk neutral lender has to increase consumption to keep Latin America in the contract. After 1980 productivity in Latin America starts going down relative to trend and world interest rates go up, decreasing the value of autarky. These effects loosen the constraint. Productivity does not recover and interest rates remain high until the year 2000. Given that the model is not a two country model and hence there are no marginal product of capital differentials, it is standard in the literature to assume that the borrower (SOE) is more impatient than the lender. When there is relative impatience and the borrowing constraint is not binding, the SOE front-loads consumption. As a result consumption decreases over time and its asset position depreciates. Nevertheless, the SOE

will only run down consumption until the value of being in the contract is equal to the value of being in autarky. At that point the constraint binds and the risk neutral lender increases the consumption of the SOE to keep it in the contract. This means that prolonged bad times will eventually trigger the constraint.

Summarizing, the model is able to capture the behavior of the observed data because the low productivity and high interest rates after 1980 loosen the constraint and consumption and assets decrease over time. By 1990, the decrease in consumption has diminished the value of the contract so much that the value of autarky is higher despite the continuously low productivity and high interest rates. This forces the risk neutral lender to raise the consumption of the SOE back again. Hence we observe a decline in per-capita consumption during the 80's and an increase in the 90's as in the data.

The interest rate shock plays an important role for the results. It implies that the relative impatience of the SOE is stochastic. This means that the rate at which consumption is runned down when the borrowing constraint is not binding depends directly on the interest rate shock. A high interest rate shock makes the lender more impatient and hence the SOE will run down consumption at a slower rate. On the other hand a low interest rate shock makes the lender more patient and the SOE will run down consumption faster. In the model, the high interest rates observed after the 80's prevent the model from overshooting the decline in consumption observed in the data.

A problem that has been identified in this models, is that in equilibrium, they spend too much time constrained. This implies that in equilibrium the outcome of a limited commitment model is not very different from the closed economy outcome (when the constraint is binding the value of the contract is the same as in autarky). The way that the literature has dealt with this is that in the model there is not only a threat of exclusion from future and interstate trade, but the threat also includes an output loss of some fixed percentage. When there are interest rate shocks, high interest rates slow down the speed at which the constraint is approached and hence the duration of the constrained spells is smaller, making it unnecessary to penalize the outside option.

Finally, As capital accumulation plays such an important role in determining the binding pattern of the borrowing constraint, a planner can use investment as an instrument to prevent the constraint from binding. More specifically, when solving the planner's problem, one can see that limited commitment

models endogenously generate a distortion to investment that is isomorphic to an investment tax. Whenever the planner believes that the constraint is more likely to bind, it will “tax” investment at a higher rate to prevent capital accumulation and prevent the value of autarky from going up. As a result it is natural for this type of model to generate a lower investment to output ratio than the one observed in the data.

Consequently, as net-exports are a residual and the model underestimates investment, the model does less well at accounting for net-exports. It captures the capital flows reversal prior to the debt crisis of the early eighties, but it does not capture the decade long period of capital outflows observed in Latin America in the 80’s.

Puzzling enough, I have found that in limited commitment models, the same feature (the binding pattern of the participation constraint) that helps explain consumption, prevents the model from capturing domestic investment and hence net-exports. The binding pattern of the participation constraint depends on the value of autarky (the outside option for the SOE). This means that autarky is presumably not the appropriate outside option to consider when one wants to explain consumption and investment at the same time. Exploring new alternatives for the outside option is left for future work.²

As mentioned above, I follow the literature on international debt that relies on the willingness to pay as well as the literature on debt-constrained asset markets. This literature studies the theoretical implications of limited commitment constraints, but mostly in pure exchange, closed economy setups. On the international debt literature I follow, among others, Eaton and Gersovitz (1981, [6]) where a debtor who defaults faces permanent exclusion from international capital markets, and Atkeson (1991, [4]) who considers an environment where the participation constraint interacts with a moral hazard problem.

Kehoe and Perri (2002, [7]) go a step further and extend the work of Kehoe and Levine (1993, [8]) and Kocherlakota (1996, [9]), to a full-blown international business cycle model with production. They study business cycle co-movements across industrial countries, their paper is one of the few quantitative applications of this type of model. My paper follows Kehoe and Perri (2002, [7]), in the sense that it

²Apart from exploring different alternatives for the outside option, it is also likely that the model has a better fit to investment if capital is made non-contractable. In other words, if capital is not chosen by the planner. Exploring this alternative is also left for future work.

is an open economy setup and it has capitalistic production. Their model has productivity shocks but does not have international interest rate shocks as mine has.

The problem with introducing capitalistic production into limited commitment models, is that capital introduces a non-convexity to the outside option. This generates serious complications when it comes to solving the model because the solution can be infeasible and/or suboptimal (see Messner and Pavoni (2004, [12])). Feasibility can be checked, but dealing with the sub-optimality is much more complicated. I suggest a new methodology that uses an idea very similar to a homotopy to achieve optimality.

The closest model to mine is that of Aguiar et al (Forthcoming, [1]). They consider a small open economy, where the government cannot commit to policy and seeks to insure a risk averse domestic constituency. The setup coincides in that the limited commitment is one-sided in the context of a small open economy. My model differs in that the small open economy is subject to international interest rate shocks and labor supply is elastic.

Finally, this paper also relates to the international real business cycles literature. This literature has shown that the excess volatility of consumption in Latin America can be explained by productivity shocks (see Aguiar and Gopinath (2007, [2])) and by shocks to the international interest rate (see Neumeyer and Perri (2005, [13])). In this paper I show the medium and long term importance of these two types of shocks when they affect borrowing constraints.

The rest of the paper proceeds as follows: Section 2 summarizes the limited commitment model, Section 3 describes the methodology, Section 4 explains the model dynamics, Section 5 explains the calibration and solution method, Section 6 shows the results and Section 7 summarizes my findings.

2 The model economy

The model economy consists of a small open economy (SOE) that interacts with a risk neutral lender, and there is a dynamic contracting problem between these two parties. The small country produces a good using domestic labor and capital, and production is subject to a country-specific productivity shock. Output is used for domestic consumption, domestic investment and to make a transfer to the lender. This transfer can be positive or negative (depending on whether the SOE is a net lender or

borrower), but the SOE can not commit to repaying its debt, in other words the SOE has limited commitment.

The risk neutral lender has linear utility on the transfers received from the SOE, and faces a stochastic discount factor. The discount factor is the inverse of the world interest rate. Note that in the model, there are two stochastic elements driving business cycle fluctuations: the productivity shock for the SOE and the shock to the discount factor of the lender (or international interest rate shock).

Time is discrete and runs to infinity. In each period t , the state of the world s_t , determined by the productivity and the stochastic discount factor shocks, is realized. I denote by $s^t = (s_0, \dots, s_t)$ the history of events up to and including period t . The probability of any particular history s^t as of period 0, is given by $\pi(s^t)$, and the initial realization s_0 is such that $\pi(s_0) = 1$. In period t , a good is produced in the SOE using inputs of capital $k(s^{t-1})$ and domestic labor $h(s^t)$. Production is affected by a productivity shock $A(s^t)$ which follows an exogenous process. Output at history s^t in the SOE is given by

$$A(s^t) F(k(s^{t-1}), h(s^t)), \quad (1)$$

where F is a standard constant returns to scale production function. Consumers in the SOE have preferences given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), h(s^t)), \quad (2)$$

where $c(s^t)$ denotes consumption at s^t and β denotes the discount factor. The resource constraint of the SOE is given by

$$c(s^t) + x(s^t) + T(s^t) \leq A(s^t) F(k(s^{t-1}), h(s^t)), \quad (3)$$

where $T(s^t)$ denotes the transfers received/made from/to the lender at s^t and $x(s^t)$ denotes investment at s^t . Note that the transfers $T(s^t)$, can also be interpreted as the net-exports of the SOE.

Investment is determined by the following capital-accumulation equation:

$$x(s^t) = k(s^t) - (1 - \delta) k(s^{t-1}). \quad (4)$$

The risk neutral lender faces the following utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \pi(s^t) T(s^t), \quad (5)$$

where $Q(s^t)$ denotes the stochastic discount factor, $Q(s^t) = q(s^0)q(s^1) \dots q(s^t)$, and $q(s^t)$ is the realization of the stochastic discount factor shock at s^t . I assume that the SOE is more impatient than the lender, $Q(s^t) > \beta^t$. This is a common assumption in the literature and it prevents the SOE from infinitely accumulating assets over time, it also reflects the fact that in developing countries political uncertainty is higher and politicians are short sighted.

If we solve the model in a centralized manner, in the absence of limited commitment the planner would maximize a weighted sum of the expected discounted utilities of the risk neutral lender and the SOE

$$\max_{\{k(s^t), h(s^t), c(s^t)\}} \left[\sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \pi(s^t) T(s^t) + \mu \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), h(s^t)) \right] \quad (6)$$

subject to the resource constraint (3) and the capital accumulation equation (4) for the SOE.

Now consider a situation where the SOE can not commit to repaying its debt, it has limited commitment. In this case, apart from the resource and capital accumulation constraints, the SOE faces a participation constraint. By enforcing this constraint the planner guarantees that the SOE will stay in the contract with the risk neutral lender and repay its debt. The participation constraint states that at every point in time and every state of the world, the SOE has to weakly prefer the allocation it receives by being in a contract with the risk neutral lender (and ship $T(s^t)$ units of output to the lender), to the allocation it could attain if it were in autarky from then on. The participation constraint is of the form

$$E_t \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r | s^t) u(c(s^r), h(s^r)) \geq V^A(k(s^{t-1}), s^t) \quad \forall r, s^r \quad (7)$$

where $\pi(s^r | s^t)$ denotes the conditional probability of s^r given s^t , $\pi(s^t | s^t) = 1$, and $V^A(k(s^{t-1}), s^t)$ denotes the value of autarky from s^t onward. The value of autarky corresponds to the utility delivered

by the following problem:

$$V^A(k(s^{t-1}), s^t) = \max_{\{k(s^r), h(s^r), c(s^r)\}} \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r | s^t) u(c(s^r), h(s^r))$$

subject to

$$c(s^r) + k(s^r) \leq A(s^r) F(k(s^{r-1}), h(s^r)) + (1 - \delta) k(s^{r-1}) \quad \forall r, s^r$$

where $r \geq t$, and $k(s^{r-1})$ is given.

In the context of limited commitment, the planner maximizes the weighted sum of the expected discounted utilities of the big and small country (6), subject to the participation constraint (7), the resource constraint (3) and the capital accumulation constraint (4) of the SOE

$$\max_{\{k(s^t), h(s^t), c(s^t)\}} \left[\sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \pi(s^t) T(s^t) + \mu \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), h(s^t)) \right] \quad (8)$$

subject to

$$E_t \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r | s^t) u(c(s^r), h(s^r)) \geq V^A(k(s^{t-1}), s^t) \quad \forall r, s^r,$$

$$c(s^t) + T(s^t) + x(s^t) \leq A(s^t) F(k(s^{t-1}), h(s^t)),$$

and

$$x(s^t) = k(s^t) - (1 - \delta) k(s^{t-1}).$$

Notice that the model with limited commitment has incomplete markets in the sense that there is a limit to the amount of contingent claims of a particular type that can be sold. The limit is determined by the amount the SOE is willing to repay according to the participation constraint.

3 Methodology

Solving problem (8) in Section 2 is difficult because it has an infinite number of participation/enforcement constraints, which can have complicated binding patterns. Furthermore, given that consumption and leisure enter the current enforcement constraint, the standard dynamic programming approach cannot

be used.

Kydland and Prescott (1980, [10]) show that when this feature is present, the state space can be expanded to include an extra state variable, in this way the problem has a solution that is stationary in the new expanded state space. Marcet and Marimon (1999, [11]) follow Kydland and Prescott (1980, [10]) and extend their approach to different applications.

To solve the limited commitment model, I extend the recursive contract approach of Marcet and Marimon (1999, [11]) in a similar way to Kehoe and Perri (2002, [7]). The added state variable is the current relative weight of the small country in the planning problem. Adding this state variable, and assuming that the shocks to productivity and the stochastic discount factor are Markovian, allows me to write a recursive problem.

I can write the Lagrangian for problem (8) as follows

$$\begin{aligned} \max_{\{k(s^t), h(s^t), c(s^t)\}} & \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \pi(s^t) \{A(s^t) F(k(s^{t-1}), h(s^t)) - c(s^t) - k(s^t) + (1 - \delta) k(s^{t-1})\} \\ & + \mu \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), h(s^t)) \\ & + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \lambda(s^t) \left\{ \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r | s^t) u(c(s^r), h(s^r)) - V^A(k(s^{t-1}), s^t) \right\}, \end{aligned}$$

where $\beta^t \pi(s^t) \lambda(s^t)$ denote the multipliers on the participation constraint. Note that the set up of this Lagrangian is standard, I only substituted transfers (T) in the lenders utility by their definition from the resource constraint.

Marcet and Marimon (1999, [11]) point out that given that we know that $\pi(s^r) = \pi(s^r | s^t) \pi(s^t)$ then we can define $\sum_{t=0}^{\infty} \beta^t \lambda_t \sum_{r=t}^{\infty} \beta^{r-t} u(c_r) = \sum_{t=0}^{\infty} \beta^t M_t u(c_t)$, where $M_t = M_{t-1} + \lambda_t$, $M_{-1} = 0$ and $\lambda(s^t)$ is the Lagrange multiplier associated with the participation constraint.

Given this, the participation constraint can be written as

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \{M(s^t) u(c(s^t), h(s^t)) - \lambda(s^t) V^A(k(s^{t-1}), s^t)\}$$

where $M_t = M_{t-1} + \lambda(s^t)$ and $M(s^{-1}) = \mu$. Hence, the Lagrangian can be re-written as

$$\begin{aligned} & \max_{\{k(s^t), h(s^t), c(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \pi(s^t) \{A(s^t) F(k(s^{t-1}), h(s^t)) - c(s^t) - k(s^t) + (1 - \delta) k(s^{t-1})\} \\ & + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \{M(s^{t-1}) u(c(s^t), h(s^t)) + \lambda(s^t) (u(c(s^t), h(s^t)) - V^A(k(s^{t-1}), s^t))\} \end{aligned} \quad (9)$$

where $M(s^t)$ is defined as the original planner weight for the small country μ , plus the sum of the past multipliers on the enforcement constraint along history s^t .

The optimality conditions for consumption, hours worked and capital from (9) are given by

$$u_{c(s^t)} = \frac{Q(s^t)}{\beta^t (M(s^{t-1}) + \lambda(s^t))}, \quad (10)$$

$$-u_{h(s^t)} = u_{c(s^t)} A(s^t) F_{h(s^t)}, \quad (11)$$

$$u_{c(s^t)} = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) \left[u_{c(s^{t+1})} \frac{M(s^{t+1})}{M(s^t)} (A(s^{t+1}) F_{k(s^t)} + 1 - \delta) - \frac{\lambda(s^{t+1})}{M(s^t)} V_{k(s^t)}^A \right], \quad (12)$$

and the complementary slackness condition.

For computational convenience and following Kehoe and Perri (2002, [7]), I normalize the multiplier by defining $v(s^t) = \frac{\lambda(s^t)}{M(s^t)}$. I also denote the right-hand side of equation (10) as

$$\frac{Q(s^t)}{\beta^t (M(s^{t-1}) + \lambda(s^t))} = \frac{Q(s^t)}{\beta^t M(s^t)} = \hat{M}(s^t). \quad (13)$$

By doing this transformation I do not have to keep track of all past realizations of q . A transition law for $\hat{M}(s^t)$ is enough to determine its evolution. This transition equation is given by

$$\hat{M}(s^t) = \frac{q(s^t)}{\beta^t} (1 - v(s^t)) \hat{M}(s^{t-1}). \quad (14)$$

Using Equation (13), the first-order conditions can be re-written and summarized by (11),

$$\hat{M}(s^t) = u_{c(s^t)} \quad (15)$$

instead of (10),

$$u_{c(s^t)} = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[\frac{u_{c(s^{t+1})}}{1 - v(s^{t+1})} (A(s^{t+1}) F_{k(s^t)} + 1 - \delta) - \frac{v(s^{t+1})}{1 - v(s^{t+1})} V_{k(s^t)}^A \right] \quad (16)$$

instead of (12), (13) and the complementary slackness condition.

Note that (14) can be written in terms of consumption by using (15)

$$u_{c(s^t)} = \frac{q(s^t)}{\beta^t} (1 - v(s^t)) u_{c(s^{t-1})}. \quad (17)$$

This substitution changes the nature of consumption within the model and transforms it from a control variable to a state variable. Hence the state is given by $x_t = (c(s^{t-1}), k(s^{t-1}), s_t)$, where $s = (A(s^t), q(s^t))$

I assume that the underlying shocks for productivity $A(s^t)$ and the stochastic discount factor $q(s^t)$ are Markov. This assumption implies that the conditional probability $\pi(s^t|s^{t-1})$ can be written as $\pi(s_t|s_{t-1})$, and the solution to the programming problem in (8) can be characterized recursively by policy rules for $k(s^t)$, $c(s^t)$, $h(s^t)$ and $v(s^t)$, where the state is x_t .

The policy rules satisfy the first-order conditions (11), (16), (17), the participation constraint (7) and the complementary slackness condition on the multiplier.

To calculate the policy functions I use a version of policy function iteration, and modify it to handle enforcement constraints in a similar way to Kehoe and Perri (2002, [7]). Specifically, I define a grid X on the state space. I restrict the search to functions that take arbitrary values for every $x \in X$ and are completely characterized over the state space when their value for every $x \in X$ is identified.

I define a value function for each party. $W(x)$ for the SOE, and $P(x)$ for the risk neutral lender. These value functions satisfy the first-order conditions (11), (16), (17), the participation constraint (7)

and the complementary slackness condition on the multiplier, and are of the form

$$W(x) = u(c(x), h(x)) + \beta \sum_{s'} \pi(s'|s) W(x'), \quad (18)$$

$$P(x) = T(x) + \sum_{s'} \pi(s'|s) q(x') W(x'). \quad (19)$$

I start with the solution to the planner's problem when there is no limited commitment (6). This guarantees that the initial value functions $W^0(x)$ and $P^0(x)$ are uniformly greater than or equal to the value of the true solution. This condition is needed for the algorithm to converge to the right solution.

Given the first-order conditions and the initial guess for labor, the normalized multiplier and the value functions $(h^0(x), v^0(x), W^0(x), P^0(x))$ I find a new set of policy functions

$$(k^1(x), c^1(x), h^1(x), v^1(x), W^1(x), P^1(x)).$$

To do so, I first assume that the participation constraint does not bind and find a set of allocations (h, c', k') . When the participation constraint does not bind $v = 0$, and the set of allocations has to satisfy (11),

$$u_{c(s^t)} = \frac{q(s^t)}{\beta^t} u_{c(s^{t-1})}, \quad (20)$$

and

$$u_{c(s^t)} = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) u_{c(s^{t+1})} (A(s^{t+1}) F_{k(s^t)} + 1 - \delta). \quad (21)$$

After finding this set of allocations, I check if they satisfy the participation constraint

$$u(c, h) + \beta \sum_{s'} \pi(s'|s) W^0(x') \geq V^A(k, s), \quad (22)$$

if they do then I define them to be the new set of allocations for x , and $v^1(x) = 0$. If they don't satisfy the participation constraint (22) then I solve for a set of allocations (h, v, c', k') , that satisfy (11), (16), (17) and the participation constraint (22). This new set of allocations then becomes

$$(k^1(x), c^1(x), h^1(x), v^1(x), W^1(x), P^1(x)).$$

4 Model dynamics

The model dynamics depend on whether the participation constraint is binding or not, and the binding patterns of the constraint depend on the productivity and stochastic discount factor shock.

When the participation constraint is not binding $v = 0$. Assuming that the utility function is logarithmic in consumption, and from (17), then

$$c(s^t) = \frac{\beta^t}{q(s^t)} c(s^{t-1}). \quad (23)$$

Given that the SOE is more impatient than the risk neutral lender $\frac{\beta^t}{q(s^t)} < 1$, the relative impatience of the SOE leads to declining consumption. Furthermore, due to the discount factor shocks, consumption declines at a stochastic rate. High discount factor shocks (which correspond to a decrease in the international interest rate) make consumption decline at a faster rate, low discount factor shocks make consumption decline slower.

When the constraint is not binding the Euler equation is equal to that of a frictionless economy:

$$1 = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \frac{q(s^{t+1})}{\beta} (A(s^{t+1}) F_{k(s^{t+1})} + 1 - \delta). \quad (24)$$

When the participation constraint is binding $v \neq 0$. From (17) and keeping the assumption that the utility function is logarithmic in consumption, we know

$$c(s^t) = \underbrace{\frac{\beta}{q(s^t)}}_A \underbrace{\frac{1}{1-v(s^t)}}_B c(s^{t-1}). \quad (25)$$

Equation (25) states that there are two competing forces leading the long-run properties of consumption. Part A , represents the relative impatience of the SOE and drives consumption down. On the other hand, part B drives consumption up. As a result, when the constraint is binding, consumption might increase, decrease or remain constant, depending on which one of the two forces is dominating.

When the participation constraint is binding the Euler equation becomes

$$u_{c(s^t)} = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[\frac{u_{c(s^{t+1})}}{1 - v(s^{t+1})} (A(s^{t+1}) F_{k(s^t)} + 1 - \delta) - \underbrace{\frac{v(s^{t+1})}{1 - v(s^{t+1})} V_{k(s^t)}^A}_C \right]. \quad (26)$$

Note that in this case, the Euler equation depends on the marginal product of capital tomorrow (as usual), but it has an extra term that depends on the expected impact of capital on next periods participation constraint. In other words the return to capital tomorrow is now affected by the change in the value of autarky when there is an extra unit of capital. If having one more unit of capital tomorrow increases the value of autarky, then it is more likely that the SOE will want to walk away from the contract tomorrow. As a result, the optimal strategy for the planner is to discourage capital accumulation by “taxing” the return to capital by c in Equation (26). This term is analogous to an investment “wedge”, see Chari, Kehoe and McGrattan (2007, [5]), and as I said it is isomorphic to a tax on investment. Note that this feature is not exclusive to the model presented here but all limited commitment models endogenously generate a “tax” on investment.

Summarizing, consumption shrinks if the participation constraint does not bind. If the participation constraint binds, the dynamics of consumption depend on the tightness of the constraint (i.e. whether A or B dominates in Equation (25)). History matters through $c(s^t)$ and $k(s^t)$.³ When consumption and capital are low, the participation constraint is more likely to bind and generate a higher investment tax. When consumption and capital are high, the participation constraint is less likely to bind and the investment tax is lower.

Up to now I have not discussed the effects that the productivity and stochastic discount factor shocks have on the constraints. Productivity shocks have a different effect in the short and in the long run. In the short run, a high productivity shock tightens the constraint because it increases the value of autarky. When the value of autarky is higher the need for the insurance provided by the contract goes down and the SOE is more likely to want to walk away from the contract. In the long-run the situation can change. It is still true that a long sequence of high productivity shocks will keep the constraint tight, but it is also the case that a long sequence of low productivity shocks will trigger the

³Unlike the endowment economy where history doesn't matter.

constraint. This is true because when productivity goes down, so does the value of autarky. When the value of autarky goes down, the constraint loosens and the planner assigns less consumption and capital to the SOE in that period. If this happens repeatedly the SOE will depreciate its asset position until the insurance provided by the contract is no longer better than being in autarky, at this point the participation constraint will be triggered again.

On the other hand, shocks to the stochastic discount factor have the same effect in the long and in the short-run. An increase in the international interest rate, which is the same as a decrease in the stochastic discount factor, increases the cost of capital, as a result it is more costly to accumulate and the value of autarky goes down. This loosens the constraint.

Finally, hours worked are determined by (11), (28) and (29)

$$h(s^t) = \left(\frac{1}{\psi c(s^t)} A(s^t) (1 - \alpha) k(s^t)^\alpha \right)^{\frac{1}{\gamma + \alpha}}, \quad (27)$$

independent of whether the constraint is binding or not.

5 Parametrization, functional forms and application

To be able to compare the equilibrium path of the model generated data with that of the observed data, I parametrize the model to compute the policy functions, and I pin down the productivity and discount factor shocks to feed into the model.

The model has six structural parameters. The structural parameters define the preferences, the production function and the capital accumulation equation.

I assume that the preferences for the SOE are given by

$$U(c, h) = \log c_t - \left(\frac{\psi}{1 + \gamma} (h_t)^{1 + \gamma} \right), \quad (28)$$

and output is determined by a standard constant returns to scale, Cobb-Douglas production function

$$F(k, h) = k^\alpha h^{1 - \alpha}. \quad (29)$$

Table (1) summarizes the parameter values for the utility and production function. I set $\alpha = 0.36$, as is standard in the literature. ψ is set such that the level of hours worked in the model matches the data, and γ is set to minus one, which means that the utility function is logarithmic in hours as well as consumption.

Table 1: Preferences and Production Parameters

δ	α	γ	ψ
0.10	0.36	-1	1.5

δ is set by the depreciation value generated by the first order condition with respect to capital in the no limited commitment problem, when it is evaluated at the sample means of the data. I use aggregate annual data for consumption, investment, output and hours worked for all the main Latin American countries and generate a regional aggregate.⁴ The countries in the sample are all the main Latin American countries except for Chile and Venezuela. Chile and Venezuela have a different behavior to that of the rest of the region. Venezuela's per-capita consumption has been going down since the 70's. Chile, on the other hand, has had a very fast consumption growth since the 90's.

For the parametrization, I define the lender to be an aggregate of the developed world. Mainly the United States, Canada, Western Europe, Australia and New Zealand.

The discount factor of the SOE (β) is set to 0.85. The level of this parameter on its own is not that important. What really matters for the dynamics of the model is the relative impatience of the SOE to lender, the difference between β and $\bar{q}$. Where $\bar{q}$ is the mean of the stochastic discount factor of the lender.

The path for q is the inverse of the international interest rate. For the international interest rate I use the nominal lending rate free of inflation as calculated by the GDP deflator. The lending rate is the base rate charged by banks on short-term business loans. The monthly rate is the average of rates of all calendar days and is posted by a majority of the top 25 insured chartered commercial banks. $\bar{q}$

⁴The data for Latin America is obtained from the World Bank *Global Development Indicators* and supplemented by Mitchell (2001) and other country specific sources.

Figure 2: Time series for exogenous processes

is 0.96, an average discount factor of 0.96 implies an annual international interest rate of 4% which is close to the average rate of return on capital over the past hundred years.

The lower panel in Figure (2) shows the time path for the stochastic discount factor that is feeded into the model. Interest rates were low prior to the 80's. At the beginning of the 1980's, after the oil shock, interest rates go up and remain high until the year 2000, when they decrease but do not return to the level they were prior to the 80's.

I assume that the logarithm of the stochastic discount factor follows a standard autorregressive process of order one

$$\log(q_{t+1}) = (1 - \rho_q) \log(\bar{q}) + \rho_q \log(q_t) + \sigma_q \varepsilon_{qt}. \quad (30)$$

To estimate ρ_q and σ_q I use OLS, and find $\rho_q = 0.27$ and $\sigma_q = 0.0134$.

To recover the time series for the productivity shock, I calculate the Solow residual by using the production function (29) and aggregate data on output, hours worked and investment for Latin

Figure 3: Discretized productivity and discount factor shocks

America. To recover capital I use the perpetual inventory method according to (4).

I assume that the logarithm of productivity follows a standard autorregressive process of order one

$$\log(A_{t+1}) = (1 - \rho_A) \log(A_{ss}) + \rho_A \log(A_t) + \sigma_A \varepsilon_{At}. \quad (31)$$

To estimate ρ_A and σ_A I use OLS, and find $\rho_A = 0.99$ and $\sigma_A = 0.025$. The upper panel of Figure (2) shows the recovered time series for productivity.

In order to solve the model, I have to discretize the stochastic discount factor and productivity shocks. To do so, I follow the methodology suggested by Tauchen and Hussey (1991). I discretize A to 5 states and q to 3 states, for a total of 15 states. Figure (3) shows the time path for the shocks and their realisations when they are discretized.

Figure 4: Observed and model generated per-capita consumption

6 Results

This section, compares the model generated data with the observed data for Latin America. As mentioned earlier the aggregate for Latin America consists of all the main Latin American countries except for Chile and Venezuela. To produce the model generated data, I feed the time series for productivity and stochastic discount factor to the policy functions.

Figure (4) shows the Hodrick and Prescott (HP) trend for the observed per-capita consumption data (black line) and the HP trend for the model generated consumption (blue line). The shaded areas in the figure represent the periods in which the constraint was binding. The limited commitment model is able to generate the initial fast growth, the lost decade of the 80's and the slower recovery observed from the 90's onward. The shaded regions show that the participation constraint was binding prior to 1976, it binds again in 1980 (at the beginning of the debt crises), and for most part of the 90's.

Figure (5), is the same as Figure (4) but now the time series paths for productivity and discount factor are over imposed on the per-capita consumption graph. The purpose is to compare the binding patterns of the participation constraint with the changes in productivity and the discount factor. Note that the shocks are not to scale in the graph, what matters is to see their qualitative behavior.

From Figure (5) we can see that when productivity is high and interest rates are low (1970 to 1976), the value of autarky is high. When state contingent insurance is not needed, there is a threat of default to increase the value of being in the contract and the participation constraint binds.

When state contingent insurance is needed (when productivity goes down or interest rates increase), the constraint is non-binding. Latin America front-loads consumption due to impatience. This is what we observe in the 80's. Nevertheless, note that the participation constraint starts to bind around 1992, when neither productivity or the discount factor have changed. This happens because in prolonged bad times (repeated low productivity draws), the degree of insurance provided by the contract goes to zero (consumption is declining and asset position is depreciating) and there is a threat of default to keep insurance at a minimum.

Note that this feature implies that the constraint can bind in good times (as it does in 1980). A good productivity shock increases the value of autarky triggering the constraint, but the constraint can also bind in prolonged bad times as is the case of the 1990's. Given that productivity has been low for some time, and interest rates stay high, Latin America has been running down consumption and depreciation its assets until the value of being in the contract is no longer greater than that of autarky.

It is also important to note that the equilibrium path generated by the model spends a good amount of time outside the constraint. In other words, the solution is considerably different from that of the closed economy (the constraint is not constantly binding). This is important because a common concern about this type of model is that the constraint binds too much. The existing literature avoids this problem by penalising the outside option, they fix the outside option to the value of autarky minus some percentage of output which would be lost in case of walking away from the contract. In my model this is not necessary due to the discount factor shocks. The discount factor of the risk neutral lender

Figure 5: Per-capita consumption: data versus model

determines the speed at which the constraint is approached. High interest rate shocks, imply that consumption goes down at a slower rate, and hence, the equilibrium path is unconstrained for longer.

Figure (6) shows the equilibrium path generated by the model when only the productivity shock is feeded in and the discount factor is fixed at its unconditional mean and when only the discount factor shock is feeded in and productivity is fixed at its unconditional mean. When only productivity is feeded in (dashed pink line), the model does a good job qualitatively but misses much more in terms of the level. This shows us that productivity governs the pattern of the constraint and through it affects the behavior of consumption.

On the other hand when only the discount factor shock is feeded in (the dotted green line), we see that the model can not reproduce the observed pattern for per-capita consumption. In this case the constraint is very tight and the consumption achieved by being in the contract is constantly increasing. So we observe that productivity governs the pattern of the constraint and the interest rate governs the tightness of the constraint.

Figure 6: Effect of stochastic discount factor versus productivity shock on per-capita consumption

Figure (7) shows the observed versus model generated investment to output ratio. The black line represents the observed data and the blue line represents the model generated data. We observe that even though the model is able to explain consumption to a great extent, the model's investment is lower than in the observed data. This is due to the distortion that is endogenously generated by limited commitment models. As this distortion is isomorphic to an investment tax, it deters investment from happening in equilibrium.

Figure (8) shows the observed and model generated data for net-exports. The model is able to generate the sharp reversal of capital flows around 1980—although it comes a little sooner than it does in the data—nevertheless the model is not able to capture the prolonged debt crisis. This is expected because in the model the intent to default is corrected immediately (there is no default in

Figure 7: Investment to output ratio

Figure 8: Equilibrium path of net-exports

equilibrium), while in reality we know that the debt crisis took around a decade to overcome. In this type of model, there are capital inflows only when the constraint is not binding and debt is being accumulated, as a result, before 1976 the model generates positive net-exports, and only until 1977 that the constraint relaxes there are capital inflows (negative net-exports). Immediately after this, in 1980 a high productivity shock hits together with a high stochastic discount factor shock making the constraint bind and net-exports become positive. This situation changes after 1984 because the stochastic discount factor shock goes back to around its unconditional mean and productivity starts going down making autarky less attractive. The model only generates negative net-exports in the periods where the constraint is not binding.

Overall, we learn several things from this exercise. We learn that a limited commitment model like the one presented above is able to generate the observed behavior of Latin American consumption. We learn that given that the model misses on investment also misses on net-exports. We know that the model misses on investment because it endogenously generates an investment tax which is presumably too high.

7 Conclusion

In this paper I show that following the recession and debt crises of the early 1980's per-capita consumption declined for about ten years and that per-capita consumption in 2004 was not very different from that in 1980. Motivated by the fact that this decline in per-capita consumption coincides with the start of the Latin American debt crisis, this paper examines a real business cycle model with limited commitment and capitalistic production where the access to international capital markets depends on domestic productivity shocks and international interest rate shocks.

I find that a model with limited commitment and productivity and international interest rate shocks can account for the behavior of per-capita consumption in Latin America. Furthermore, I find that the productivity shock governs the pattern of the constraint and that the international interest rate shock governs the tightness of the constraint.

My results have some implications for theory as well. The interest rate shock determines the speed at which the constraint is approached (unlike [7] and [1]). The lower relative impatience generated by

high international interest rate shocks buys more time outside the constraint, in this way the outside option does not have to be penalized with loss of output to prevent the equilibrium from being mostly on the constraint. Also, the constraint can bind in good and prolonged bad times. This last feature was not known because previous work does not study medium and long run properties.

Finally, I find that in limited commitment models the same feature that helps explain consumption, also prevents the model from capturing domestic investment and hence net-exports. One alternative to solve this issue is to make capital non-contractable. In this way the planner can not discourage capital accumulation to prevent the constraint from binding. This extension is left for future work.

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