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Lending to Uncreditworthy Borrowers

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Abstract

How might a low lending costs prompt lenders to include uncreditworthy borrowers in their portfolio? This paper presents a theoretical study into how lender competition can affect borrower quality, especially in a low interest rate setting. I study equilibria where lenders compete aggressively by poaching on rivals' clients. Poaching is impeded because rivals not only have superior information about the quality of existing (creditworthy) clients but also uncreditworthy types in the borrower population. Screening is costly, and the uninformed lender's ability to use collateral as a screening mechanism depends on its cost advantage over its informed rival (i.e., *relative levels of lending costs*). Importantly, the uninformed lender can pool uncreditworthy borrowers with creditworthy types in low interest rate settings (i.e., for low *absolute level of lending cost*). Therefore, while a secular decline in lending costs leaves the uninformed lender's ability to screen uncreditworthy borrowers unchanged, it opens the opportunity for them to pool these borrowers with creditworthy types. This not only facilitates entry of outside lenders into "high-risk" credit markets, but also makes it optimal for them to poach borrowers from rivals by including uncreditworthy borrowers in their loan portfolio. Equilibrium lending behavior in this setting can also explain the phenomenon of *cream-skimming on entry* by outside (foreign) lenders.

Keywords: Bank competition; Credit allocation, Lending standards
JEL: G21; D43.

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1 Introduction

> Financial institutions reacted to the surplus of available funds by competing aggressively for borrowers, and, in the years leading up to the crisis, credit to both households and businesses became relatively cheap and easy to obtain… Unfortunately, much of this lending was poorly done, involving, for example, little or no down payment by the borrower or insufficient consideration by the lender of the borrower's ability to make the monthly payments.[1]

How might "aggressive" lender competition in a low cost environment prompt lenders to include uncreditworthy borrowers in their loan portfolio? How does strategic lending behavior interact with information asymmetries in credit markets to produce a deterioration of loan quality? These questions lie at the heart of how events and policy might conspire to precipitate financial crises. Although the importance of these questions is widely appreciated, less is understood by way of which this causal link can be established.

This paper is a theoretical study of how lender competition in a low interest rate setting can affect lending standards. It studies an environment in which lenders lend aggressively, poaching borrowers from rivals, in a bid to increase profits by gaining market share. The impediment for lenders that seek to poach clients of rival lenders is that rivals have superior information about the quality of existing clients. Informed lenders typically gain knowledge about borrower quality from previous lending relationships (see Boot, 2000 and references therein). Consequently, their lending rates to existing customers are adjusted according to the customer's credit risk.[2] In addition, the informed lender is likely to have identified a section of the borrower population as bad risk: borrowers whose likelihood of default is so high that it is not profitable to lend to them at any rate.[3] Although this implies that such borrowers are likely to be denied loans from their current (informed) lenders, they may choose to apply for loans from other (uninformed)

[1]Remarks by Chairman Ben Bernanke, in a speech titled Four Questions about the Financial Crisis at the Morehouse College, Atlanta, Georgia, April 14, 2009

[2]Under adverse selection, riskiness is an exogenous and unobservable characteristic of agents. Accordingly, the characterization of risk throughout this article refers to unobservable risk (i.e., risk conditional on observables).

[3]Even though existing customers are deemed creditworthy, they can differ in their credit risk. Accordingly, the paper will distinguish between two types of good-risk or creditworthy borrowers: high-risk and low-risk borrowers. A third category of borrowers will be classified as bad-risk or uncreditworthy borrowers.

lenders in the future (Sharpe, 1990). This has important implications for borrower poaching and lender competition in credit markets.

In this paper, I examine the problem of competition between an informed lender and an uninformed lender, in which the informed lender's information advantage not only includes its own clients but extends to uncreditworthy borrowers as well.[4] Accordingly, I assume that the informed lender has knowledge about prospective uncreditworthy borrowers from previous transactions. Therefore, not only does the uninformed lender have to sort creditworthy borrowers of different risk quality, but it also has to avoid lending to uncreditworthy borrowers.[5] Following Besanko and Thakor (1987), I assume that the uninformed lender uses collateral to screen out bad risks and to sort high-risk borrowers from low-risk ones. The use of collateral is a costly and inefficient screening mechanism because the value of the collateralized asset in use is higher than the salvage value of the collateral to the lender when the borrower defaults.

The results of this paper are summarized as follows. Equilibrium contract offers depend on three features of the model, namely, (i) the distribution of types in the borrower population, (ii) the relative levels of the lending cost of the uninformed and the informed lender and, most notably, (iii) the absolute level of the lending cost. The use of collateral as a screening device is a costly process and the informed lender does not require borrowers to post collateral. However, the uninformed lender's use of collateral as a screening device involves this additional cost, which it must recover to compete with its informed rival. Consequently, screening equilibria in which the uninformed lender is able to attract borrowers away from the informed lender as well as sort borrower types depends on the *cost advantage of the uninformed lender* (i.e., *the relative levels of lending costs*). In contrast, the uninformed lender's ability to pool borrowers depends, in addition to its having the cost advantage, on *the absolute level of its lending cost.* When the uninformed lender pools superior types with inferior types, it expects to offset the losses from the latter with profits from the former. The uninformed lender's offers to superior types under pooling are bounded above by borrowers' reservation payoff, namely, the offers made by

[4] Alternatively, this problem can be viewed as problem of entry into credit markets, wherein the informed lender is an inside lender (incumbent who already exist in this market) that have arguably better information on borrower quality than outside lenders (entrants or new lenders).

[5] Sengupta (2007) examines a problem of entry in which lenders compete over the incumbent's clients only. The question addressed in this paper takes on greater relevance because it relaxes the restrictive assumption that all borrowers are known to be creditworthy.

its informed rival. This implies that for a given cost advantage of the uninformed lender, its profits from creditworthy types under pooling do not change with the absolute level of lending costs. In contrast, the expected losses in lending to uncreditworthy types decline with decreases in lending cost. At very low costs of lending, these expected losses are significantly less, making it optimal for the uninformed lender to include uncreditworthy types in its portfolio.

The highlight of these results is that there exist equilibria wherein a low lending cost makes it optimal for the uninformed lender to include uncreditworthy borrowers in its portfolio.[6] A cost advantage for the uninformed lender is a necessary but not a sufficient condition for successfully poaching creditworthy clients from the informed lender. In any separating equilibrium wherein it successfully secures creditworthy types, the uninformed lender needs this cost advantage to be sufficiently large. Otherwise, it cannot poach clients from the informed lender because its cost advantage cannot cover for the costs of screening uncreditworthy borrowers. A small cost advantage precludes screening equilibria; but in a low rate environment—as would prevail if there was a significant decline in the lending cost—it opens the opportunity for pooling creditworthy borrowers with non-creditworthy types.[7]

In addition to the principal result above, the model offers an equilibrium wherein the informed lender retains high-risk clients while the uninformed lender is able to secure borrowers of the highest quality (lowest risk) despite its information disadvantage. This occurs when the uninformed lender's offer of a significantly lower rate combined with a sufficiently high collateral requirement attracts borrowers of the highest quality only. This "cream-skimming" result finds support in some of the evidence on foreign lenders in poor countries (see Dietragiache et al, 2008 and references therein). While this literature has attributed this phenomenon to differences in lending technologies between foreign and domestic lenders, this model shows that such a result can be derived from a general model of asymmetrically informed lenders (see Section 5 for more details).

[6] Naturally, a low lending cost implies an increase in the pool of creditworthy borrowers, because borrowers deemed uncreditworthy at the higher rate are now creditworthy. However, the pooling equilibrium considered here involves the inclusion of borrowers who are uncreditworthy even at the lower rate.

[7] The result is similar to the "overlending" problem in De Meza and Webb (1987). However, the lack of borrower screening in De Meza and Webb (1987) renders vacuous any discussion of lending standards (see Section 5 for details). Notably, such pooling equilibria do not exist in traditional competitive screening models (Rothschild and Stiglitz, 1976 and Wilson, 1977). However, they have been shown to exist under fairly elaborate settings in Dubey and Geanakoplos (2002) and Martin (2007).

There is a growing literature on how lender competition affects lending standards. For example, Ruckes (2004) and Dell'Ariccia and Marquez (2006) attribute changes in bank lending to exogenous changes in the demand for credit during the upward phase of the credit cycle. In contrast, this paper attributes such changes to the supply of funds, in particular, a decline in lending costs. This feature finds support in recent empirical work on risk-taking by lenders (Jiménez et al. 2007, Ioannidou et al. 2009, and Maddaloni and Peydro, 2009). In a similar vein, Rajan (1994) studies how supply side factors, like reputational concerns of bank managers, affect lending behavior of banks. In credit booms, reputation concerns can lead bank managers to hide losses and adopt a liberal credit policy to improve the market's assessment of the bank performance. However, because markets are more forgiving in times of a systemic adverse shock, managers coordinate to tighten credit policy. Although the mechanism outlined in this paper is significantly different, the results provide support for Rajan's hypothesis. For example, a low lending cost assists a liberal credit policy by allowing bank managers to absorb realized and expected losses, whereas a higher lending cost would prompt more managers to screen borrowers. Finally, in contrast to this paper and the previous work mentioned above, Gorton and He (2008) study fluctuations in bank lending behavior that are driven by strategic interactions and not exogenous changes in the economic environment.

The work closely related to this paper is Dell'Ariccia and Marquez (2006), in which the uninformed lender is unable to distinguish between "lemons" rejected by the incumbent and new borrowers shopping around for lower interest rates (Dell'Ariccia et. al, 1999, Dell'Ariccia and Marquez, 2004). An interesting feature of these models is that the informed lender successfully retains all of its creditworthy clients, and therefore, lenders effectively compete for new borrowers only.[8] In this setting, equilibrium behavior depends on the proportion of new (unknown) borrowers in the population. Yet, at any given time, the number of new entrepreneurs seeking credit could be small when compared with the number of existing firms in the market. In contrast, this paper models the information problems faced by banks in lending to existing borrowers, such as that described in the conventional bank lending channel (Bernanke

[8] DellArricia et al. (1999), DellArricia and Marquez (2004) also model a similar setting that precludes borrower poaching.

and Blinder, 1988).[9]

Also related to this paper, are two strands of literature outlined in Bernanke and Gertler (1999) that discuss how loan quality is adversely affected at the upward phase of a credit cycle. One strand explores how concerns about financial instability arise when financial liberalization (e.g., deregulation in the banking sector) is not well coordinated with the regulatory safety nets (e.g., deposit insurance and lender-of-last-resort provisions as in Keeley (1990) and Besanko and Thakor (2004)). While much of this literature relies on the agency problems that arise out of deposit insurance, this paper shows that such excessive risk-taking can occur even in the absence of such insurance. In this sense, the implications of the model have a broader appeal to all financial intermediaries and not just depository institutions. A second strand of literature studies how credit quality deteriorates during the upward phase of the credit cycle through the asset-based lending channel. Competitive leveraged bidding can raise asset prices, which in turn encourages further lending against these assets, increasing demand and asset prices through a dynamic multiplier effect (Kiyotaki and Moore, 1997). Finally, the deterioration of loan quality comes about as lenders become less concerned about the ability of the borrowers to repay loans and instead rely on further appreciation of the asset to shield themselves from losses (Mishkin, 2008).

The causal link described in the paper is independent of the traditional asset-based lending mechanism. The emphasis here lies in the role of lender competition and information asymmetry in establishing the causal link between a low lending cost and a decline in borrower quality. Nevertheless, it is important to mention that the two mechanisms are not in conflict. In fact, it is not difficult to view the two mechanisms as reinforcing each other in practice to exacerbate this decline in average borrower quality. I describe this in greater detail in Section 6, arguing that the deterioration of borrower quality during the recent housing boom in the United States could be explained in terms of this model. The model puts forward a theoretical explanation as to how the decline in lending costs can be significant factor in explaining the acceptance of

[9] Bernanke and Gertler (1995, p. 40) describe this channel as follows. "Banks, which remain the dominant source of intermediated credit in most countries, specialize in overcoming informational problems and other frictions in credit markets. If the supply of bank loans is disrupted for some reason, bank-dependent borrowers (small and medium-sized businesses, for example) may not be literally shut off from credit, but they are virtually certain to incur costs associated with finding a new lender, establishing a credit relationship and so on."

higher default risk when extending loans.[10] In addition to theoretical work, there is a significant volume of empirical studies that examine how bank lending standards change (from tightness to laxity) over the credit cycle.[11] In Section 6, I discuss how the results obtained in the paper find support in recent empirical work.

An important concern here is whether the results emphasized in this paper can be established in a more parsimonious model. In Section 5, I put forward the reasons as to why this concern is misplaced. The rest of this paper is organized as follows. Section 2 provides the basic setup for the model. Section 3 describes the set of candidate equilibria in which borrowers go to the uninformed lender for loans. Section 4 uses numerical methods to solve for equilibria of the model. In particular, this section determines the conditions under which the candidates listed in Section 3 emerge as the equilibria of the game. Section 5 provides a discussion of the results. Section 6 discusses the implications of this result in the context of the current crisis in the subprime mortgage market in the United States and Section 7 concludes.

2 Preliminaries

The basic setup of this paper is similar to that of Besanko and Thakor (1987). Entrepreneurs (also called borrowers) can borrow a dollar from a lender and invest in a project. The project returns x if it succeeds (with probability $1-\theta$) and zero if it fails (with probability θ). Lenders' loan contracts consist of a repayment R and a collateral requirement C. Borrowers' reservation utility and lenders' lending cost are denoted by V^0 and ρ, respectively. A lender can recover only a fraction, β, of the collateral, which the borrower loses if she defaults on the loan ($0 < \beta < 1$). Thus, the parameter β is a measure of the disparity in the borrower and lender valuation of collateral. Both lenders and borrowers are risk neutral. Lenders' profits from the loan contract (R, C) is given by $\pi(R, C, \theta) = (1-\theta)R + \beta\theta C - \rho$, while a borrower's payoff under the same contract is $V(R, C, \theta) = (1-\theta)(x-R) - \theta C$. Therefore, a loan contract (R, C)

[10] The proximate cause of the low cost of lending is attributed to what Bernanke (2005) described as the Global Savings Glut (see Caballero et al. 2008). Some observers have also attributed this in part to the easing of monetary policy by the Federal Reserve which lowered the target federal funds rate from 6.5 percent in July 2000 to 1 percent in April 2004.

[11] This includes earlier empirical work by Asea and Blomberg (1998), Lown and Morgan (2003), Berger and Udell (2004) and some more recent studies by Jimenez et al. (2007) and Ioannidou et al. (2009).

generates a social surplus of $[(1-\theta)x-\rho-V^0]-(1-\beta)\theta C$. Notably, a strictly positive collateral requirement entails a deadweight loss of $(1-\beta)\theta C$, implying that, *ceteris paribus*, zero-collateral loan contracts are first-best.

The model assumes a fixed pool of borrowers indexed by their risk parameter, θ, the probability of default. The fraction ν_l of entrepreneurs are low-risk ($\theta = \theta_l$), the fraction ν_h of borrowers are high-risk ($\theta = \theta_h$), and the fraction ν_b are bad-risk types ($\theta = \theta_b$), with $0 < \theta_l < \theta_h < \theta_b < 1$ and $\nu_h + \nu_l + \nu_b = 1$. Bad-risk borrowers are uncreditworthy in that the surplus generated on loans to them is strictly negative (i.e., $(1-\theta_b)x < \rho + V^0$, for all ρ). Both high-risk and low-risk borrowers are creditworthy (or "good"-risk) in that all loan contracts generate a positive social surplus (i.e., $(1-\theta_k)x > \rho + V^0$, where $k = g = h, l$). Stated differently, a lender with complete information would always extend loans to good risks and deny them to bad risks. Throughout, we will impose the boundary condition $[(1-\theta_g)(1-\theta_b)x - (1-\theta_g)V^0 - (1-\theta_b)\rho] > 0$ for $g = h, l$. This condition ensures that uncreditworthy types do not find the lenders' competitive offers to creditworthy types unattractive.

In this setting, this paper analyzes competition between an informed (incumbent) lender that has complete information about borrower creditworthiness and an uninformed lender (new or outside lender) that is unable to distinguish between borrowers' risk types. The informed lender (or Lender-I) is (pre-entry) a price-setting monopolist whose lending cost is ρ^I. The uninformed lender (or Lender-U) is a new or outside lender whose lending cost is ρ^U. Lender-I's private information here extends not only to its existing (and therefore) creditworthy clients but also to other "prospective" uncreditworthy borrowers that Lender-U would like to avoid. Lender j's offer to borrower k is denoted by (R_k^j, C_k^j), where $j = I, U$ and $k = b, h, l$. The lender's profits from this offer are given by $\pi_k^j = (1-\theta_k)R_k^j + \beta\theta_k C_k^j - \rho^j$ if the borrower accepts the loan contract and zero otherwise. Also, V_k^j denotes borrower k's payoff from contract (R_k^j, C_k^j).

The timing of the game can be described as follows. Nature selects borrower types. The informed lender can distinguish borrower types and offers one contract for each type. The uninformed lender cannot distinguish between types and therefore offers a menu of contracts. Lenders offer contracts simultaneously. Finally, borrowers either accept or reject contracts.

After eliminating a set of dominated strategies for the informed lender, I describe the set of contracts that Lender-I can offer in equilibrium in terms of the following lemma:

Lemma 1 *For borrowers of type $k = b$, the informed lender denies credit. For borrowers of type $k = g = h, l$ the informed lender offers a contract from the set $Z_g^I(\rho^I) = \{(R_g^I, 0) : R_g^I \in [\underline{R}_g^I(\rho^I), \bar{R}_g]\}$, where $\underline{R}_g^I(\rho^I) = \frac{\rho^I}{1-\theta_g}$ and $\bar{R}_g = x - \frac{V^0}{1-\theta_g}$ are the first-best (zero-collateral) minimum and maximum repayments, respectively.*

Proof: See appendix.

Since the informed lender denies credit to bad-risk types, they continue to receive their reservation payoff V^0. Also, the contract $(\underline{R}_g^I(\rho^I), 0)$ yields borrower $g = h,\ l$, the maximum utility Lender-I can provide, denoted $\bar{V}_g^I(\rho^I)$, and is defined by

$$\bar{V}_g^I(\rho^I) \equiv (1-\theta_g)x - \rho^I, g = h, l.$$

The uninformed lender faces borrowers with two different types of participation constraints. For good risks ($k = g = h, l$), the participation constraints are determined by the payoffs that the borrowers receive from loan contracts offered by the informed lender, V_g^I. For bad risks ($k = b$), however, the participation constraint is given by V^0, the reservation utility of the borrower (the opportunity cost of her time). In the same vein, the informed lender's equilibrium offer from the set $Z_g^I(\rho^I)$ depends on Lender-U's offer in equilibrium. Stated differently, Lender-I's offer in set $Z_g^I(\rho^I)$ seeks to match the highest payoff that Lender-U can provide a creditworthy borrower.

I focus exclusively on pure strategy equilibria. Without loss of generality, I hold the uninformed lender's lending cost constant at ρ^U and vary the informed lender's lending cost ρ^I. As is standard in the principal agent literature, I will assume that if the borrower is indifferent between two loan contracts *offered by the same lender*, the contract that the lender prefers is chosen. Also, if a borrower is indifferent between contracts offered by either lender, in equilibrium, she chooses to borrow from the lender that makes higher profits from the contract.[12]

[12] Suppose a borrower is indifferent between loan contracts offered by two lenders with one of the lenders

3 Candidate Equilibria

This section provides an intuitive discussion of the (candidate) equilibria of the model. A rigorous treatment of the results referred to in this section, is provided in the appendix.[13] I begin with a description of the candidate equilibria for a particular case of this model—namely, the situation in which the uninformed lender screens all borrowers. Following this, I describe other candidate equilibria of the model. In the next section, a solution to the model is provided showing how each of the candidates (described below) emerge as the (final) equilibria of the model under different sets of parameter values.

How might the uninformed lender successfully sort all creditworthy types? Lender-U can successfully sort all borrowers only if its incentive scheme yields each borrower at least as much payoff as that from contracts offered by Lender-I. Consequently, Lender-U faces borrowers whose reservation utilities are determined by the maximum payoff that Lender-I can offer borrowers. These reservation utilities for the high-risk and low-risk borrower—namely, $\bar{V}_h^I$ and $\bar{V}_l^I$—are shown by the indifference curves through $(\underline{R}_h^I, 0)$ and $(\underline{R}_l^I, 0)$ in Figure 1. Figure 1 illustrates the (candidate) equilibria in (R, C) space. Borrowers' payoffs increase as one moves southwest, while lenders' profits increase to the northeast. Because the informed lender denies credit to the bad risks, their reservation utility is V^0. This is shown in Figure 1 by the indifference curve of the bad-risk type through points **A** and **C.**

A *first candidate (screening) equilibrium*, denoted as *Screen-1*, is one where the uninformed lender screens all borrower types (see Proposition 7 in appendix). The uninformed lender offers contract menu $\{(R_h^U, C_h^U); (R_l^U, C_l^U)\}$ as shown in Figure 1.[14] The informed lender offers $(\underline{R}_h^I, 0)$ to high risks, $(\underline{R}_l^I, 0)$ to low risks, and denies credit to bad-risk types. The bad-risk borrowers reject offers of the uninformed lender. The good-risk types (both h and l) borrow from the uninformed lender, selecting loan contracts with strictly positive collateral requirements. The contract offers to high and low risks are shown as points **A** and **B** in Figure 1, respectively.

making zero profits. The other lender making higher (positive) profits can make offers that increase borrower payoff by reducing its profits. The lender making zero profits cannot do so and still break even.

[13] Statements of the lemmas and propositions referred to in this discussion and elsewhere in the paper are provided in the appendix.

[14] The closed form expressions for R_h^U, C_h^U, R_l^U, and C_l^U are provided in the appendix.

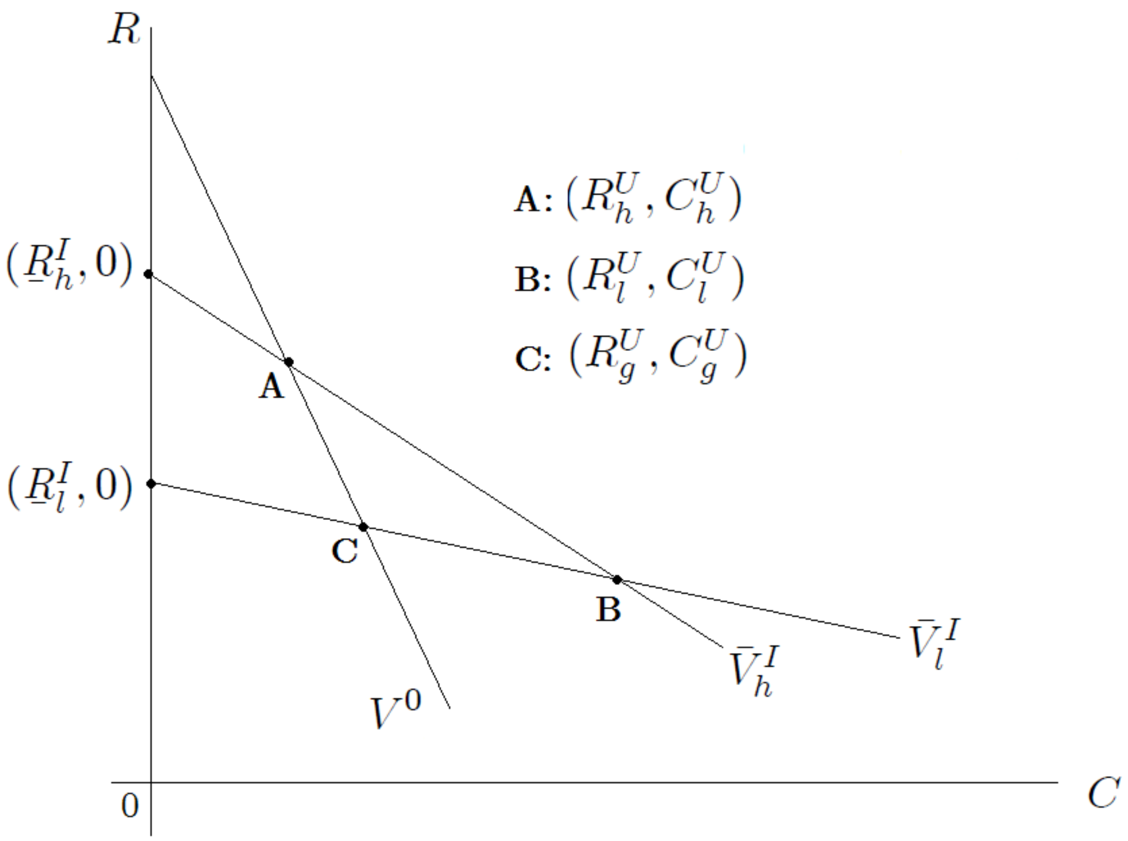

Figure 1: Lender-U's contract offers under different equilibria in (R, C) space. Borrowers' payoffs increase as one moves southwest, while lenders' profits increase to the northeast. Lender-U offers $(\underline{R}_l^I, 0)$ and $(\underline{R}_h^I, 0)$ in Pool-1 and Pool-2, respectively. For Hybrid-1, the Lender-U pools bad risks and high risks at $(\underline{R}_h^I, 0)$ and sorts low risks at (R_l^U, C_l^U). For Hybrid-2, Lender-U screens out bad risks and pools good risks at (R_g^U, C_g^U). Finally, Lender-U offers (R_h^U, C_h^U) for Screen-2 and (R_l^U, C_l^U) for Screen-3.

The contract for low-risk borrowers has a lower repayment and a higher collateral requirement than that for high-risk borrowers. Since inferior types try to mimic the superior types in this competitive setting, the uninformed principal always chooses to offer incentive schemes that are just as good as the inferior agent's outside alternative. It follows that $V_b(R_h^U, C_h^U) = V^0$ and $V_h(R_h^U, C_h^U) = V_h(R_l^U, C_l^U) = \bar{V}_h^I$. Evidently, *local* incentive constraints (i.e., incentive constraints for adjacent types) bind in this screening equilibrium.

The cost of screening is borne only by the uninformed lender. Therefore, to successfully screen or sort borrower types, the uninformed lender needs to have a large cost advantage over its informed rival. Stated differently, this candidate equilibrium is feasible only if the uninformed lender's cost advantage is sufficiently large so that two screening cutoffs (one for each pair of adjacent types) are satisfied. The first cutoff is $\tilde{\rho}_S^{b,h}$ for screening the bad-risk types from the high-risk types, and the second is $\tilde{\rho}_S^{h,l}$ for screening the high-risk types from the low-risk types. Also, as shown in the appendix, the screening cutoffs are independent of the distribution of borrower types in the population and are given by

$$\tilde{\rho}_S^{h,l} = \frac{1}{1-(1-\beta)\theta_l}\rho^U \tag{1}$$

$$\tilde{\rho}_S^{b,h} = \frac{\theta_b - \theta_h}{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}\rho^U + \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}[(1-\theta_b)x - V^0]. \tag{2}$$

Therefore, to sort all borrower types the uninformed lender's cost advantage needs to be sufficiently large; that is, it must be true that $\rho^I > \max(\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l}) > \rho^U$.

However, if for example, $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h} > \rho^U$, the uninformed lender cannot sort good-risk borrowers into high-risk and low-risk types, but it can still screen out bad risks. This gives us a *second candidate (screening) equilibrium,* denoted as *Screen-2*, where the uninformed lender secures the high-risk types by screening out bad risks. Since, $\tilde{\rho}_S^{h,l} > \rho^I$, it follows that, if offered (R_l^U, C_l^U) would yield losses for Lender-U. Therefore, if we denote (R_l^0, C_l^0) to be the best offer that Lender-U can make to low-risk types, then $\pi_l^U(R_l^0, C_l^0) = 0$. Consequently, the informed lender offers $(R_l^I, 0)$ such that $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$ and $V_h(R_l^0, C_l^0) = V_h(\underline{R}_h^I, 0)$. This offer just matches the payoff from the best offer that the uninformed lender can provide low-risk types. Here, the uninformed lender's offer is given by $\{(R_h^U, C_h^U); (R_l^0, C_l^0)\}$. Also, the informed lender offers $(\underline{R}_h^I, 0)$ to high-risks and denies loans to bad risks. Except for low-risk types, the equilibrium behavior of agents in *Screen-2* is similar to that in *Screen-1.* In *Screen-2,* low risks borrow from the informed lender whose profits from low-risk types are strictly positive.

TABLE 1. Lender-U's offers under different candidate equilibria

Candidate equilibria	*Profit*	*Customer types accepting Lender-U*'s offer	*Menu of contracts offered by Lender-U*	*Breakeven cutoff*
Screen-1	Π_S^1	$(h); (l)$	$(R_h^U, C_h^U); (R_l^U, C_l^U)$	$\tilde{\rho}_S^{b,h}$, $\tilde{\rho}_S^{h,l}$
Screen-2	Π_S^2	(h)	$(R_h^U, C_h^U); (R_l^0, C_l^0)$	$\tilde{\rho}_S^{b,h}$
Screen-3	Π_S^3	(l)	(R_l^U, C_l^U)	$\tilde{\rho}_S^{h,l}$
Hybrid-1	Π_Y^1	$(b, h); (l)$	$(\underset{\sim}{R}_h^I, 0); (R_l^U, C_l^U)$	$\tilde{\rho}_P^1(\nu_h, \nu_l)$, $\tilde{\rho}_S^{h,l}$
Hybrid-2	Π_Y^2	(h, l)	(R_g^U, C_g^U)	$\tilde{\rho}_Y(\nu_h, \nu_l)$
Pool-1	Π_P^1	(b, h, l)	$(\underset{\sim}{R}_l^I, 0)$	$\tilde{\rho}_P^1(\nu_h, \nu_l)$
Pool-2	Π_P^2	(b, h)	$(\underset{\sim}{R}_h^I, 0)$	$\tilde{\rho}_P^2(\nu_h, \nu_l)$

On the other hand, if $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l} > \rho^U$, the uninformed lender cannot screen out bad-risk types. However, Lender-U can sort good-risks into high-risk and low-risk types. This gives us a *third candidate (screening) equilibrium,* denoted as *Screen-3*, where the uninformed lender secures the low-risk types while the informed lender retains high-risk types. In this equilibrium, Lender-U simply offers (R_l^U, C_l^U). Lender-I offers $(\underset{\sim}{R}_h^I, 0)$ even though the payoff to the high-risk types from this contract cannot be matched by Lender-U. Lender-I cannot raise $\underset{\sim}{R}_h^I$ because high-risks are just indifferent between Lender-U's offer of (R_l^U, C_l^U) and $(\underset{\sim}{R}_h^I, 0)$. Again, lenders split the market with only low-risk types borrowing from Lender-U.

In addition, if $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l} > \rho^U$, then *a fourth candidate (hybrid) equilibrium* is possible in this situation, denoted as *Hybrid-1.* In general, a hybrid equilibrium can be described as one in which the uninformed principal pools or bunches offers to adjacent types while sorting (or screening out) the third type. In *Hybrid-1*, the uninformed lender seeks to pool bad risks with high risks while sorting them from low-risk types. It offers the menu $\{(\underset{\sim}{R}_h^I, 0); (R_l^U, C_l^U)\}$ while the informed lender's offers are the same as that in *Screen-1.* In equilibrium, all borrowers would go to the uninformed lender whose aggregate profits would depend on the distribution of bad risks in the population. With $\rho^I \geq \tilde{\rho}_S^{h,l}$, its profits from loans to low risks are non-negative. However, by pooling bad risks with high risks, the uninformed lender can no longer ensure strictly positive profits from its offer of $(\underset{\sim}{R}_h^I, 0)$ unless the proportion of bad risks in the population is sufficiently small. I return to this point below in my discussion of pooling equilibria.

The *fifth candidate (hybrid) equilibrium,* denoted as *Hybrid-2*, involves bunching good risks and screening out bad risks. Here, the uninformed lender offers (R_g^U, C_g^U), where (R_g^U, C_g^U) is shown by the point **C** in Figure 1.[15] Since, the equilibrium involves screening out uncreditworthy types while lending to the low-risk type, it follows that $V_b(R_g^U, C_g^U) = V^0$ and $V_l(\underline{R}_l^I, 0) = V_l(R_g^U, C_g^U)$. Again, the informed lender's offers are the same as those in *Screen-1*. Lender-U's offer is rejected by uncreditworthy type but accepted by both creditworthy types. Evidently, the uninformed lender's offer in *Hybrid-2* involves pooling and therefore is feasible only if the proportion of high risks in the population is sufficiently small. *Hybrid-2* is feasible for the uninformed lender only if $\rho^I \geq \tilde{\rho}_Y(\nu_h, \nu_l)$, where $\tilde{\rho}_Y(\nu_h, \nu_l)$ denotes the hybrid cutoff for the uninformed lender.

The last two candidate equilibria involve pooling contracts. In *Pool-1*, the uninformed lender pools all borrowers by offering $(\underline{R}_l^I, 0)$. This subsidizes losses from bad risks and high risks with profits from low risks. Therefore, *Pool-1* is feasible only when the proportion of low risks in the population is high; this is denoted by the breakeven cutoff $\tilde{\rho}_P^1(\nu_h, \nu_l)$. The uninformed lender can also pool bad risks with high risks. This is given by *Pool-2*, where the uninformed lender offers $(\underline{R}_h^I, 0)$ and the breakeven cutoff for such a contract is given by $\tilde{\rho}_P^2(\nu_b, \nu_h)$. The contract offers by the uninformed lender for each of the seven candidate equilibria are given in Table 1.

In summary, there are three categories of candidate equilibria: pooling, screening, and hybrid. Within each category, candidate-1 has a larger number of customer types going to the uninformed lender for loans than candidate-2 or candidate-3. For example, in candidate equilibrium *Hybrid-2*, the uninformed lender screens out the bad risks but in *Hybrid-1* it pools them with high risks. In fact, if the uninformed lender can screen the low-risk borrowers (i.e., if $\rho^I \geq \tilde{\rho}_S^{h,l}$), then the uninformed lender's profits from offers in *Screen-1* dominate those from offers in *Screen-2*. Similarly, the uninformed lender's offers in *Hybrid-1* dominate those in *Pool-2*. Finally, the informed lender dominates if ρ^I is strictly lower than all of the breakeven cutoffs given in the last column of Table 1.

[15] The closed form expressions for R_g^U and C_g^U are provided in the appendix.

4 Model Solution

The set of pure strategy (candidate) equilibria for the model are given in Table 1. The next step is to determine which of these candidates emerge as the (final) equilibrium of the model for a given set of parameter values. The closed-form solutions to the lenders' offers in each equilibria are given in the appendix. These solutions are used to derive Lender-U's profits from its offers in each candidate equilibria as a function of parameters. From the set of profit functions, one for each of the equilibrium offers, Lender-U selects one that yields the maximum (non-negative) profits. This gives us the optimal lending behavior and the optimal contract offers of each lender for a given set of parameter values. Since the maximum is obtained over a finite set of values, each a function of the parameters, it provides us with a solution to the model.

I define parameters x, V^0 and the three different values of θ, namely θ_b, θ_h, and θ_l to be the *primitives* of the model. For a given set of primitives, the screening cutoffs $\tilde{\rho}_S^{h,l}$ and $\tilde{\rho}_S^{b,h}$ in (1)-(2) vary with β. Since there can be infinite variations in the set of parameter values, the exposition here is selective. The aim is to illustrate how institutional features and market conditions affect lending behavior. This is done by using a numerical example showing how the equilibria change with changes in parameter values of ν_b, β, and ρ^U respectively. Recall that, the solution to the model is obtained by fixing the value of ρ^U. Nevertheless, it is interesting to understand how different values of ρ^U yield different solutions to the model. Therefore, changes in ρ^I (for a given ρ^U) yield variations in the *cost advantage* of Lender-U whereas changes in ρ^U denote changes in its *absolute level of lending costs*.[16]

A closer look at the candidate equilibria in Table 1 reveals two types of costs for Lender-U. The first type are screening costs that arise because of the expected deadweight losses in liquidating collateral. This cost increases with decreases in β.[17] Consequently, the smaller the proportion β that the lender can recover on default, the higher the cost of screening. Since, the Lender-I does not require borrowers to post collateral, the screening costs impinge only on

[16]Needless to say, the entire exercise can be replicated by interchanging the roles of ρ^I and ρ^U to yield similar results.

[17]As mentioned earlier, the cost of screening is given by the expected deadweight loss $(1-\beta)\theta C$ from the collateral requirement on the loan.

Lender-U. Consequently, the uninformed lenders' ability to screen borrower types depends on its *cost advantage* over Lender-I.

The second type arises from costs of cross-subsidization in pooling different borrower types. In pooling equilibria, profits from superior types are used to offset losses from inferior ones. Pooling costs increase with the proportion of inferior types in the pool, and therefore, these equilibria prevail only when the borrower population has a sufficiently large proportion of superior types. More importantly, Lender-U's offers to superior types is bounded above by Lender-I's offers to this type. Therefore, profits from superior types are determined by the relative levels of lending costs; profits remain unchanged with changes in the absolute levels of lending costs. In contrast, losses from inferior types decrease with the absolute levels of lending cost. As a result, the Lender-U's ability to pool borrowers depends, in addition to its having the cost advantage, on *the absolute level of lending costs.* Therefore, in describing the equilibria of the model, it is critical to distinguish between the Lender-U's *cost advantage* ($\rho^I - \rho^U$) and *the level of lending cost* (the numerical value of ρ^U). As will be illustrated below, the former determines the Lender-U's ability to screen borrowers, whereas the latter determines its success in pooling borrower types.

I begin with a discussion of the equilibria for primitives of $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$.[18] For the given set of primitives, Figures 2, 3 and 4 describe solutions to the model in (ν_h, ρ^I) space for variations in parameter values of ν_b, β, and ρ^U respectively. The dotted lines in the graphs denote Lender-U's lending cost, ρ^U. The colored regions denote equilibria in which Lender-U is able to secure at least one creditworthy borrower type. If Lender-U's cost advantage is sufficiently small, Lender-I dominates (i.e., all borrowers go to Lender-I for loans), as shown by the white region above the dotted line in all Figures.[19] Evidently, Lender-I dominates in this region because Lender-U is unable to screen out bad risks despite its cost advantage. On the other hand, if Lender-U's cost advantage is very large, the lender can successfully screen borrower types. Therefore, with a large cost advantage, Lender-U screens borrower types as shown by the equilibria Hybrid-2 for low ν_h and Screen-1 for high ν_h.

[18] The choice of parameter values is motivated purely in terms of exposition; the aim here is to illustrate the conditions under which each of the candidates emerge as equilibria in the model.

[19] Domination below the dotted line is trivial since, in that case, the informed lender has both cost and information advantage.

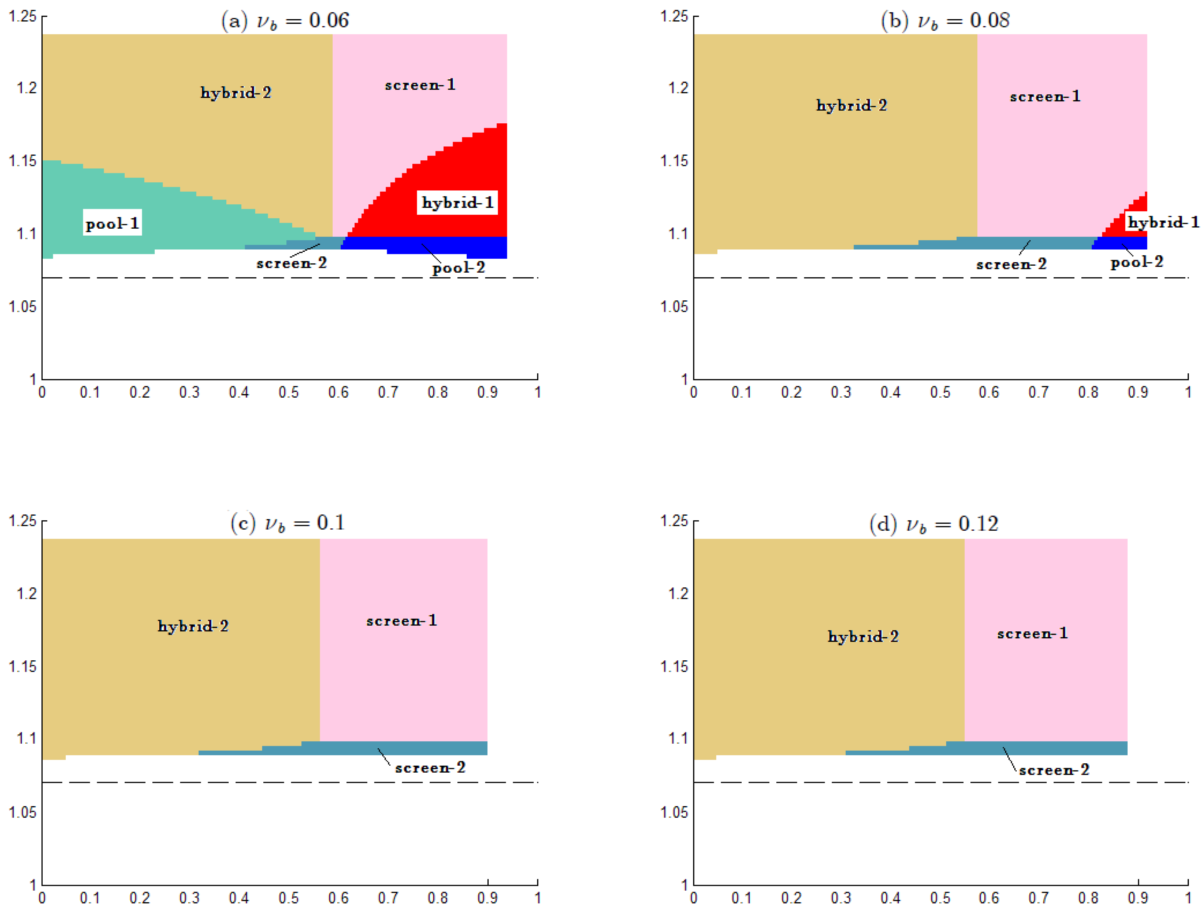

Figure 2: Solution in (ν_h, ρ^I) space with variations in ν_b. The dotted lines in the graphs denote the Lender-U's lending cost, ρ^U. The plots are drawn to parameter values $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$ for $\rho^U = 1.07$ and $\beta = 0.5$. The value of ν_b varies from 0.06 in (a) to 0.12 in (d).

As will be demonstrated below, most of the changes in the solution to the model are observed for moderate cost advantage of Lender-U. I begin with four subplots in Figure 2 that describe the solutions to the model for different values of ν_b for given values of $\beta(= 0.5)$ and $\rho^U(= 1.07)$. For $\nu_b = 0.06$, Figure 2(a) shows that the equilibria for a low cost advantage are dominated by *Pool-1*, *Pool-2* and *Hybrid-1*. However, for $\nu_b = 0.1$, Figure 2(c) shows that equilibria for a similar range of cost advantage is largely dominated by *Screen-1*, *Screen-2* and *Hybrid-2* wherein Lender-U screens out the bad-risk types. Naturally, the lower the proportion of bad-risks in the population, the higher is the profitability of adopting pooling strategies as opposed to screening.

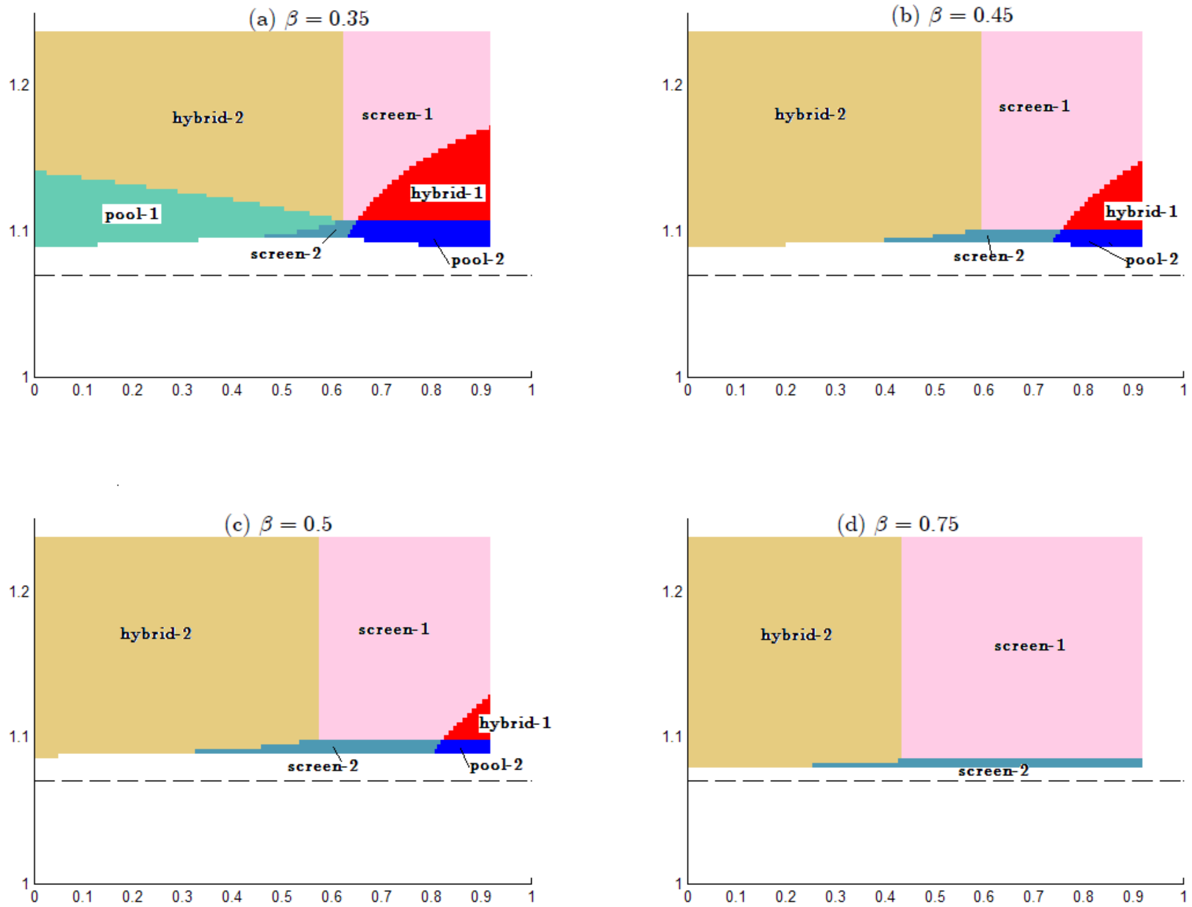

Figure 3: Solution in (ν_h, ρ^I) space with variations in β. The dotted lines in the graphs denote the Lender-U's lending cost, ρ^U. The plots are drawn to parameter values $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$ for $\rho^U = 1.07$ and $\nu_b = 0.08$. The value of β varies from 0.35 in (a) to 0.75 in (d).

Next, parameter values $\nu_b = 0.08$ and $\rho^U = 1.07$ are used to generate Figure 3, which describe solutions to the model in (ν_h, ρ^I) space for different values of β. The four subplots in Figure 3 are solutions to the model for different values of β. With a moderate cost advantage, pooling yields higher profits for Lender-U at lower values of β, as shown in Figure 3(a). However, screening replaces pooling at higher values of β. This follows from the fact that a higher β implies a greater salvage value for the collateral used in screening. Therefore, a higher β implies a lower cost of screening increasing the likelihood that screening yields higher profits to Lender-U.

In addition, $\nu_b = 0.08$ and $\beta = 0.5$ yields Figure 4, which describe solutions to the model

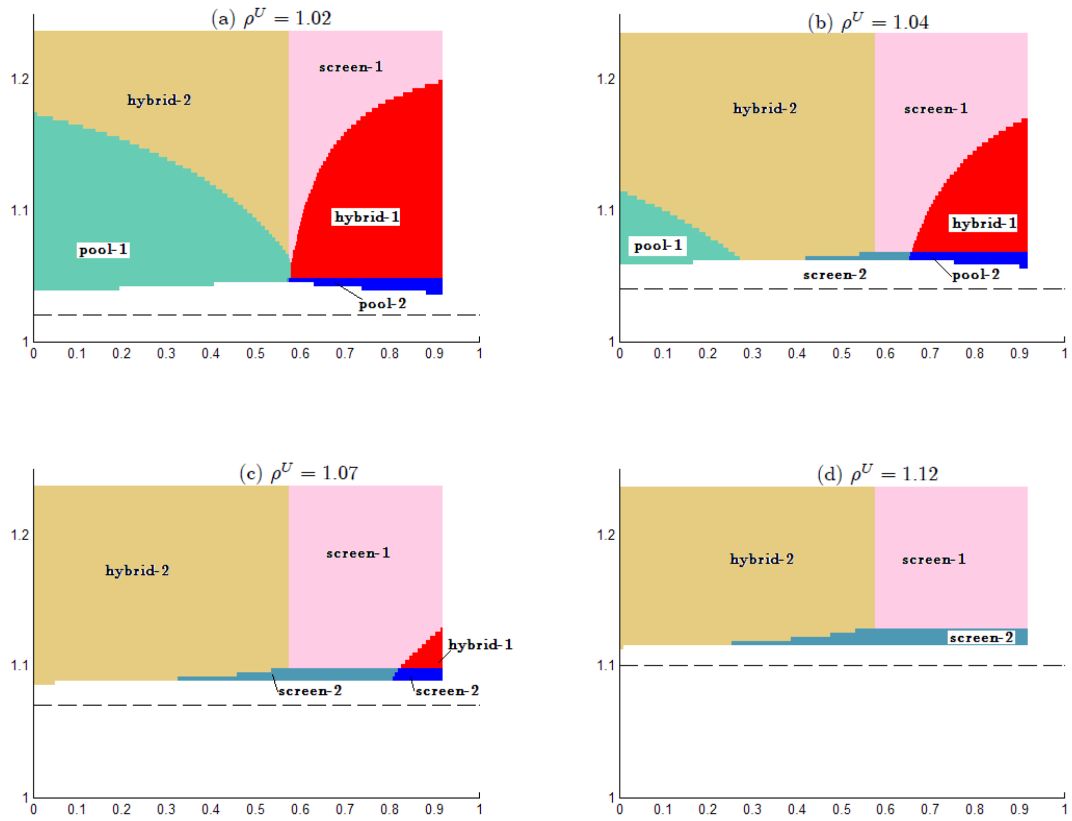

Figure 4: Solution in (ν_h, ρ^I) space with variations in ρ^U. The dotted lines in the graphs denote the Lender-U's lending cost, β. The plots are drawn to parameter values $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$ for $\nu_b = 0.08$ and $\beta = 0.5$. The value of ρ^U varies from 1.02 in (a) to 1.12 in (d).

in (ν_h, ρ^I) space for different values of ρ^U. For a given ρ^U, distance above the dotted line denotes the cost advantage of Lender-U ; determining the relative levels of the lending costs. However, variations in ρ^U as shown across the different plots show changes in the absolute level of lending costs. For lower values of ρ^U, pooling borrowers yields higher profits for Lender-U than screening uncreditworthy types (Figure 4(a)). However, Figure 4(d) shows that screening equilibria dominate at higher ρ^U. This result is described in greater detail below.

In the numerical example given above, the model does not describe a scenario in which *Screen-3* emerges as the equilibrium of the model. This is because for the numerical examples considered above, $\tilde{\rho}_S^{h,l} > \rho^I > \tilde{\rho}_S^{b,h} > \rho^U$. However, for a different numerical example where $\tilde{\rho}_S^{b,h} > \rho^I > \tilde{\rho}_S^{h,l} > \rho^U$, *Screen-3* is shown an equilibrium of the model. Using primitives $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.062$, $\theta_l = 0.04$, the set of parameter values $\nu_b = 0.06$ and

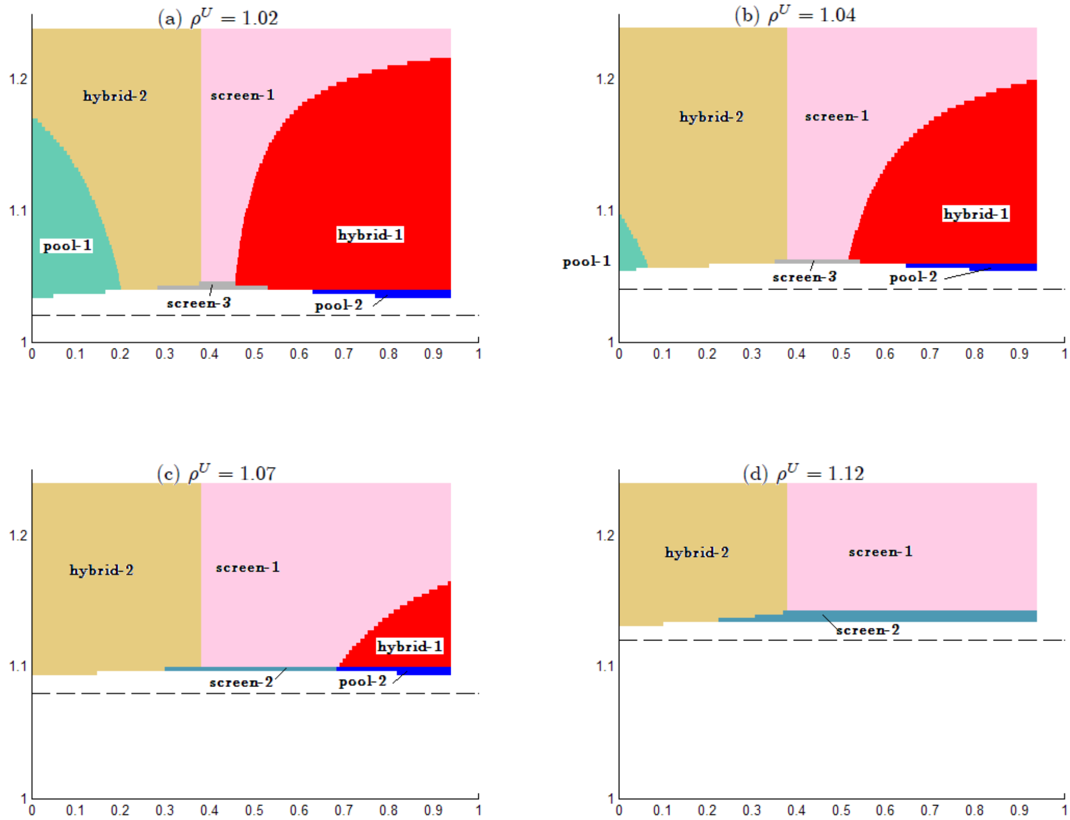

Figure 5: Solution in (ν_h, ρ^I) space with variations in ρ^U. The dotted lines in the graphs denote the Lender-U's lending cost, β. The plots are drawn to parameter values $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.062$, $\theta_l = 0.04$ for $\nu_b = 0.06$ and $\beta = 0.5$. The value of ρ^U varies from 1.02 in (a) to 1.12 in (d).

$\beta = 0.5$, yields $\tilde{\rho}_S^{b,h} > \tilde{\rho}_S^{h,l}$ for low values of ρ^U, namely $\rho^U = 1.02$.[20] In this scenario, Screen-3 emerges as an equilibrium in Figure 5(a). However, as ρ^U increases, the equilibrium in Figure 5(d) is similar to that shown in the numerical example discussed above.

In what follows, I discuss features of the equilibrium for both numerical examples, in terms of the four cases given below.

Case (i) $\min\{\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l}\} > \rho^I > \rho^U$**.** Since Lender-$U$ cannot screen adjacent types, the discussion here will focus on the following three candidates: *Pool-1*, *Pool-2*, and *Hybrid-2*. Figures

[20] It is follows from (2) and (1) that there exists a $\hat{\rho}^U$ such that $\rho^U \gtreqless \hat{\rho}^U \Leftrightarrow \tilde{\rho}_S^{h,l} \gtreqless \tilde{\rho}_S^{b,h}$. The closed form solution is $\hat{\rho}^U \equiv \frac{\frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}[(1-\theta_b)x-V^0]}{(\frac{1}{1-(1-\beta)\theta_l}-\frac{\theta_b-\theta_h}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)})}$. Strictly speaking, this cutoff $\hat{\rho}^U$ is an artefact of keeping ρ^U constant while varying ρ^I. Intuitively, this implies that the nature of equilibria are different for absolute values of lending costs.

2-5 show that Lender-I can dominate in regions even if Lender-U has the cost advantage. This holds true partly because, as mentioned earlier, screening out bad risks is costly. On the other hand, pooling borrowers can be costly as well, and this cost depends on the proportion of inferior types in the borrower population. In the absence of bad-risk types in the borrower population, any cost advantage is sufficient for Lender-U to secure creditworthy types. However, as long as there are uncreditworthy types, Lender-I can dominate the market even when its rival has the cost advantage. This is shown by the clear (white) region just above the dotted lines in Figures 2-5. However, if the proportion of bad-risk types (ν_b) is sufficiently small or the salvage rate (β) of collateral is low (or both), pooling contracts are available to Lender-U as shown in Figures 2(a) and (b) and 3(a) and (b).

In addition, if ν_h is low, then Lender-U either pools all borrowers (under *Pool-1*) or pools the good risks while screening them from bad risks (under *Hybrid-2*). The cutoff for *Hybrid-2*, $\tilde{\rho}_Y(\nu_h, \nu_l)$, is increasing and convex in ν_h. A higher cost advantage is needed for pooling a larger proportion of high risks in the population. Candidate *Hybrid-2* dominates *Pool-1* for higher ν_b because a larger proportion of bad risks implies that it is now more profitable to screen them out than it is to pool them with good risks.

On the other hand, if ν_h is high, Lender-U can pool them with bad risks under *Pool-2*. An interesting feature of the equilibrium in these regions is that, while pooling is feasible for high or low values of ν_h, Lender-I dominates for intermediate values of ν_h. This happens in situations where the proportion of high risk is neither too large to be pooled with bad risks nor too small to be pooled with low risks. In these regions Lender-U would ideally like to screen out bad risks, but is unable to do so since $\min\{\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l}\} > \rho^I$.

Case (ii) $\tilde{\rho}_S^{h,l} > \rho^I > \tilde{\rho}_S^{b,h} > \rho^U$. This situation is best illustrated in terms of Figure 2. For low values of ν_b, the equilibrium is similar to that in the previous case: *Pool-1* and *Hybrid-2* emerge as the equilibria for low ν_h, whereas *Pool-2* is the equilibrium at high ν_h. But whereas earlier Lender-I dominated at intermediate values of ν_h, Lender-U can now capture the high-risk market by screening high risks from bad risks. This is shown by the region labeled *Screen-2* in Figure 2(a). Note that the size of this region increases (at the expense of *Pool-2*) with increases in ν_b because pooling higher proportions of bad risks is no longer profitable as it increases costs of cross-subsidization. Therefore for higher values of ν_b, pooling equilibria are replaced by *Screen-2* (for high ν_h) and *Hybrid-2* (for low ν_h) as shown in Figure 2(b)-(d).

Case (iii) $\tilde{\rho}_S^{b,h} > \rho^I > \tilde{\rho}_S^{h,l} > \rho^U$**.** This case is illustrated in Figure 5(a) and (b). Lender-U cannot screen out bad risks from good-risks but can sort among good risk types. For high ν_h, *Hybrid-1* replaces *Pool-2*, because with $\rho^I > \tilde{\rho}_S^{h,l}$ Lender-U sorts low-risk types while bunching bad risks with high-risk types. For low ν_h, the equilibrium is given by *Pool-1* or *Hybrid-2*, just as in the previous case. However, unlike the previous case, Lender-U cannot screen high risks from bad risks for the intermediate values of ν_h. Nor can it pool high risks, either with low risks or with bad risks. Interestingly, because it's screening offer to low-risk types are rejected by bad-risks, it can secure only low-risks under *Screen-3*. In this equilibrium, the high-risks accept offers from Lender-I.

Case (iv) $\rho^I > \max(\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l}) > \rho^U$**.** This implies that the complete set of contracts listed in Table 1 yields strictly positive profits to Lender-U. Among them, Lender-U's offers in *Pool-2* are dominated by those in *Hybrid-1*, and its offers in *Screen-2* and *Screen-3* by those in *Screen-1*. Consequently, Lender-U chooses among contract offers in the four alternatives: *Pool-1, Screen-1*, *Hybrid-1,* and *Hybrid-2*. For large cost advantages of Lender-U, *Hybrid-2* and *Screen-1* dominate because they not only screen out the bad risks but also include low-risk types. Note, however, that for low ρ^U, Lender-U's offers in *Hybrid-1* continue to dominate those in *Screen-1* for high ν_h despite the fact that Lender-U can now screen high risks from bad risks. This is because the cost of pooling for low ρ^U is still less than the costs of screening.

Another way of illustrating the full scope of possible equilibria is in terms of Figure 6, which replicates the equilibria in Figure 3 in terms of three simplex diagrams. These diagrams illustrate how the equilibria change with changes in the cost advantage of Lender-U.

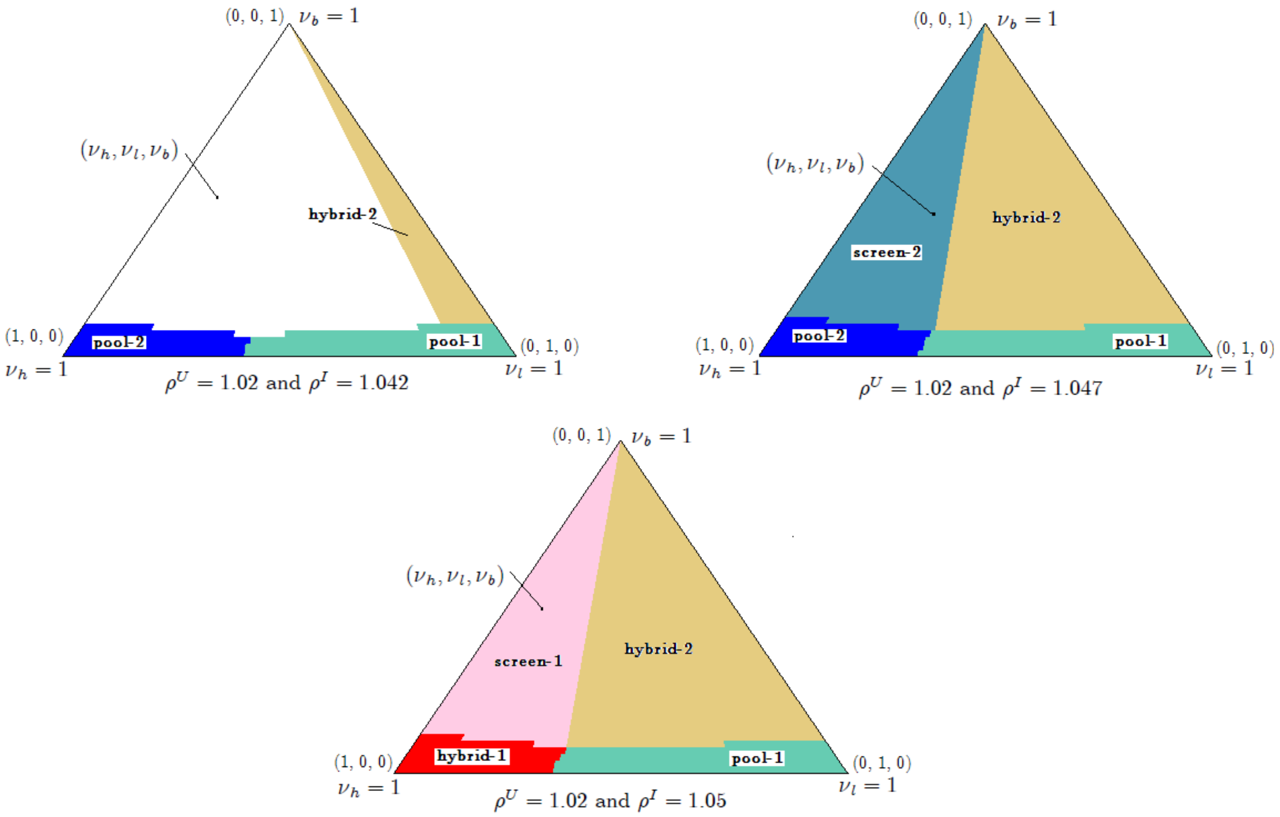

Figure 6: Three simplexes drawn to parameter values $x = 3.99$, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$. The simplexes show the solution to the model for $\rho^U = 1.02$ and $\rho^I = 1.042, 1.047$ and 1.05 respectively. The clear regions (in white) denote parameter values for which Lender-I dominates. Note that for the parameter values under consideration $\tilde{\rho}_S^{h,l} > \tilde{\rho}_S^{b,h}$.

5 Discussion of Results

5.1 Lending to Uncreditworthy Borrowers

In this section, I discuss equilibria in which it is optimal for Lender-U to extend credit to uncreditworthy types. As before, I restrict this discussion to the interesting case where Lender-U has the cost advantage. In the equilibrium plots shown above, this is given by the distance above the dotted line. From the discussion above, it is not difficult to see that screening equilibria are more likely to prevail in market segments where (i) the proportion of bad-risk types is not sufficiently small or (ii) the deadweight loss from collateral use is not sufficiently large, or both. Conversely, low β and low ν_b are favorable to the use of pooling strategies. The numerical solutions presented above show that even under conditions which are otherwise generally favorable to screening equilibria, the solution to the model can change with the absolute value of lending costs.

To illustrate this, I begin with a description of the equilibria (the solution to the model) for the same set of parameter values $\beta = 0.5$, $\nu_b = 0.08$ and $\rho^U = 1.07$ as shown in plots 2(b), 3 (c) and 4 (c). Except for significantly high values of ν_h, the model solution predominantly comprises of screening equilibria wherein Lender-U screens out uncreditworthy types. Interestingly, this occurs even for situations where Lender-U's cost advantage is not significantly high. Therefore, the market equilibria is either dominated by Lender-I (as shown by the clear regions above the dotted line) or Lender-U screens out bad-risk types and either pools creditworthy types (in *Hybrid-2* for low ν_h) or captures just the high risk type (in *Screen-2* for high ν_h).

Starting with these market conditions, lowering lending costs can change the solution significantly as shown in Figure 4. A low cost environment makes it optimal for Lender-U to pool borrowers as shown in plots 4(a)-(b). In 4(a), a significantly low lending cost is shown to facilitate pooling for almost all values of ν_h. Moreover, pooling is optimal even for a very high cost advantage of Lender-U—that is, for high cost advantages that would enable it to screen all borrower types. In sum, lowering lending costs significantly increase the range of values over which pooling becomes the optimal strategy for Lender-U.

The intuition behind this result can be described as follows. Pooling equilibria allow lenders to offset losses from inferior (uncreditworthy) types with profits from superior (creditworthy) types. As described in Section 3, Lender-U's offers under pooling are bounded above by borrowers' reservation payoff, namely, the offers made by its informed rival Lender-I. This implies that for a given cost advantage of Lender-U, its profits from superior (creditworthy) types under pooling do not change with the absolute level of lending costs. In contrast, the expected losses in lending to inferior (uncreditworthy) types decline with decreases in lending cost. At very low costs of lending, these expected losses are significantly less when compared to costs of screening, making it optimal for Lender-U to include uncreditworthy types in its portfolio.

Whereas Lender-U's ability to screen borrowers depends on its cost advantage over its rival (and hence *the relative levels of lending costs*), its ability to pool borrowers depends on the *absolute level of its lending costs*. Herein lies the causal link between a low cost of lending and the extension of credit to uncreditworthy borrowers. A significantly low value of the lending cost allows uninformed lenders to poach borrowers by lowering collateral requirements for all borrowers. Starting from a point where the lending costs are (moderately) high, consider a situation where a significant increase in the supply of funds exogenously lowers the cost of lending in the economy. This does not change Lender-U's cost advantage and hence leaves its ability to screen borrowers unaltered. However, it can significantly enhance Lender-U's ability to poach borrowers from Lender-I by pooling creditworthy borrowers with uncreditworthy ones.

5.2 Satisfying Occam's Razor

In this section, I address the concern that the main result of the paper could be obtained in a simpler model. Let us begin with the simple case of a monopolist lender. Would such a lender adopt a pooling strategy that includes uncreditworthy borrowers in its portfolio? It is important to remember that a monopolist lender would seek to extract the entire surplus generated from the loan contract. Since creditworthy borrowers generate a higher (positive) surplus than uncreditworthy types, a monopolist lender can always charge a significantly high rate on loans so as to price out uncreditworthy types from the market. Therefore, one can restrict attention to competitive markets for the desired result.

As mentioned before, the pooling equilibria described above have similarities to the overlending result under competition in De Meza and Webb (1987). However, the lack of screening provisions in De Meza and Webb (1987) renders vacuous any discussion of lending standards. Turning our attention to models which include the provision for borrower screening, we find that for competition under asymmetric information, Nash equilibria are never pooling. This is a fairly well established result in the literature on competition under asymmetric information (Rothschild and Stiglitz, 1976, Wilson 1977). Importantly, Besanko and Thakor (1987) demonstrate that the overlending problem of De Meza and Webb (1987) disappears if one allows for the provision of borrower screening. Separation can be induced as borrowers with lower risk of default choose contracts with lower interest rates and higher collateral requirements whereas borrowers with higher risk of default choose contracts with higher interest rates and lower collateral requirements. Modifying the framework in Besanko and Thakor (1987) to asymmetrically informed lenders, this paper shows that for a significantly low value of the lending cost, uninformed lenders can poach borrowers using pooling contracts. Moreover, these pooling contracts attract non-creditworthy types as well. To conclude, it is difficult to argue that a simpler model allowing for screening provisions would illustrate equilibria linking a low lending costs with lending to uncreditworthy borrowers.[21]

5.3 Cream Skimming on Entry

The only provision in the model that does not satisfy the dictum of Occam's razor is the inclusion of two creditworthy borrower types. Here the deviation from brevity allows us to illustrate an interesting result that has its parallels in observed lending patterns. In terms of the model, this result is illustrated as the equilibrium in *Screen-3*, where the uninformed lender secures the low-risk borrower. The lenders split the market, with the uninformed lender securing low-risk types, whereas the informed lender is saddled with high-risk types despite its information advantage.

Recent empirical and theoretical studies suggest that the entry of outside lenders into credit markets can lead to "cream-skimming", whereby outside lender obtains a safer loan portfolio on entry, leaving inside lenders with the riskier clients (see Dietragiache et al. (2008) and references therein). Dietragiache et al. (2008) provide evidence on this phenomenon for the entry of foreign banks into developing countries, also showing how this effect can be welfare reducing. Their model shows that cream skimming arises primarily out of differences in lending technologies between foreign and domestic lenders.

From an information perspective however, the phenomenon of cream skimming seems counterintuitive. After all, how might an outside lender compete with a better informed inside lender and yet, be successful in securing the most creditworthy clients in the borrower pool?

[21] This does not imply that competitive pooling equilibria cannot exist under adverse selection. Existence of pooling equilibria in competitive screening models has been developed under more elaborate settings (see Dubey and Geanakoplos (2002) and Martin (2007) for further details).

Arguably, the inside lender should be able to use its information advantage to retain clients of the highest quality.

The model presented here shows how this counterintuitive result may hold in equilibrium. An uninformed lender's cost advantage can help in offering a lower rate. When combined with a sufficiently high collateral requirement, this low rate helps it secure only the low risk types while screening bad-risks and high-risks. Even though the high-risks are creditworthy, the uninformed lender doesn't include them in its portfolio because it cannot sort them from uncreditworthy types. Therefore, the market is split between the informed (inside) lender and the uninformed (outside) lender, with the high-risk types borrowing from the informed lender and the uninformed lender cream-skimming borrowers of the highest quality.

6 Implications of the Model

6.1 Bank Lending Cycles and lending costs

The model relates to a significant volume of research on the cyclicality of bank lending. The traditional central bank response to increase rates to curb inflation can bring about a drop in bank lending from its peak of the credit cycle. These changes could occur either by affecting the supply of bank credit, relative to other forms of credit as modeled in the traditional bank lending channel (Kashyap and Stein, 2000) or by a change in lending standards, as demonstrated above. Interestingly, Kashyap and Stein (2000) observe that tightening monetary policy has a greater impact on smaller (local) lenders as opposed to larger banks.[22] To the extent that smaller local banks have the information advantage and their larger rivals have the cost advantage, one could interpret this finding as affecting the cost of the informed lender more than that of the uninformed lender. In terms of this model, a tightening of monetary policy leads to an increase in the uninformed lender's cost advantage, which in turn increases the likelihood of screening equilibria and a reduction in lending volumes.

Although there has been considerable work done on the cyclicality of bank lending, fewer

[22] Kashyap and Stein (2000) attribute this to smaller banks having less liquid balance sheets where liquidity is measured as the ratio of securities to assets.

studies have focused on the cyclicality of lending standards by banks. In recent work, Jimenez et al. (2007) use the Spanish credit registry of more than 23 million bank loans to show how changes in short term rates influence credit risk taking by lenders. In broad support of the results of this model, they find that low lending costs prior to origination create excessive risk taking by lenders. In a related study, Ioannidou et al. (2009) find evidence in support of this result for the Bolivian credit market.[23] Moreover, Ioannidou et al. (2009) find that risk pricing is inadequate in times of lax lending because spreads do not reflect the additional risk taken. This phenomenon can be explained in terms of the model: Lending to uncreditworthy types involves pooling equilibria, which, unlike borrower screening, does not allow for adequate risk pricing of individual loans.

In their extensive panel study of loan contract terms, Asea and Blomberg (1998) find that the environment supporting lax lending standards predate the peak of the credit cycle.[24] As to whether this occurs in the early stages of the upward phase or closer to the peak, is an open research question. However, there is considerable evidence that points to the sticky response of bank lending to market rates either due to credit rationing (Berger and Udell, 1994) or due to institutional memory (Berger and Udell, 2002). Therefore, it is not inconceivable that a low-rate environment, as the one that prevailed in the early part of this decade, contributes significantly to the lax lending standards in the years that followed.[25]

A low interest rate environment is identified as a major factor in banks optimally lending to uncreditworthy types. Admittedly, a sustained low rate environment as prevailed in the early part of this decade was unprecedented in recent monetary history. However, the fallout in terms of institutional lending was no less remarkable. For the first time in recent economic history mainstream lenders penetrated "subprime" markets making loans to borrowers who were until

[23] These studies exploit different institutional arrangements for econometric identification. To establish the exogeneity of monetary policy, Jimenez et al. (2009) utilize Spain's membership to the European Monetary Union while Ioannidou et al. (2009) exploit the "dollarization" of Bolivia's banking system.

[24] Asea and Blomberg (1998, p. 92) argue that "during booms asymmetric information in credit markets may cause good projects to draw in bad ones. . . . bank lending standards have a more profound effect . . . during expansions—when the seeds of a future recession are sown—than during contractions" This is contrast to the traditional view that "cyclical changes in firm financing are dominated by changes in bank lending, especially at the peak and during the downward phase of the credit cycle."

[25] The Federal Reserve lowered the target federal funds rate from a high of 6.5 percent in early January 2001 to just one percent in January 2002. The FOMC statement released on August 2003 announced that "policy accommodation can be maintained for a considerable period" and the low rate environment continued well into 2004.

now denied conventional sources of funds. The best example of this phenomenon is the entry of mainstream lenders to the subprime mortgage market in the US. I discuss this example in greater detail below. A lesser known phenomenon has been the entry of private banks into the microfinance market in developing countries during this period.[26]

The driving forces behind changes in the lending patterns in terms of our model are the lending costs of banks, independent of the source. The model also allows for a broader interpretation of lending costs than suggested by deposit rates. Asymmetries in lending costs can arise from differences in operating cost, interest expenses or even cost of inefficiencies that arise due to deviations from best-practices.[27] Consequently, while deposit insurance can mitigate asymmetry in the costs of funds for different lenders, it cannot completely eliminate it.

Another feature of this model implies that lenders which increase market share by lending aggressively during booms should ex post display higher default rates. This prediction seems at odds with Ruckes (2004) and Dell'Ariccia and Marquez (2006) where portfolio quality is similar across banks and each bank has similar acceptance of default risk when extending loans. Interestingly, the model's predictions are more in line with Rajan (1994) even though the mechanism described in this model is different. In the vein of Rajan (1994), differences in loan quality across banks is most pronounced for those that successfully poach borrowers during times of expansion. In the example below, I argue that this prediction seems consistent with the anecdotal evidence documenting the significant growth and subsequent demise of all top lenders in the subprime market.

6.2 An Example: Subprime Mortgage Market

Previous theoretical work on mortgage finance, such as Brueckner (2000), extend the seminal work on competitive screening models by Rothschild and Stiglitz (1976) and Besanko and Thakor (1987) to a model of adverse selection in the mortgage market. Brueckner argues that competition can induce separation in the mortgage market because riskier borrowers agree to a

[26] Reille and Forster (2008) record that, between 2004 and 2006, the stock of foreign capital investment—covering both debt and equity—more than tripled to US$4 billion.

[27] See Berger and Mester (2003) for a more formal treatment of the lending costs of banks.

price premium for high loan-to-value (LTV) mortgages. Using Brueckner's insight, this model of borrower poaching and lender competition among asymmetrically informed principals can be extended to explain the current turmoil in the mortgage market. Needless to say, recent events are more complex than the mechanisms outlined in terms of this stylized model. Accordingly, the aim here is somewhat modest: I discuss the causal link established above in light of some of the evidence on recent events in financial markets. What follows is a simple description of the intuition and some anecdotal evidence in support of the arguments.

While prime loans are made to borrowers with strong credit histories and a demonstrated capacity to repay, loans to subprime borrowers involve elevated credit risk. In the early years, a majority of subprime lenders were a combination of non-depository finance companies, specialized subprime mortgage lenders, and local depository institutions (Temkin et. al, 2002). Subsequently, subprime originations increased at a high rate of 25 percent per year from 1994 to 2003.[28] Of course, other factors like changes in the regulatory structure, intense competition over profits in the prime market and the house price appreciation in the U.S. since 1996 significantly influenced the increase in subprime lending (Gramlich, 2004).

Interestingly, a significant majority of subprime mortgages are refinances, emphasizing the role of repeated interaction(s) between borrowers and lenders in this market. Using First American Loan Performance data on more than nine million securitized loans, Bhardwaj and Sengupta (2009) find that between 60 to 72 percent of first-lien subprime originations between 1998 and 2007 were refinances. As the subprime market grew, poaching borrowers became increasingly relevant because major players increased market share at the expense of other informed lenders, including local lending companies.[29]

Translating the more general credit market framework to the case of the mortgage market is simpler if we abstract from asset price movements. To describe the lender competition and equilibrium in the context of the mortgage market, we use the screening mechanism described in Brueckner (2000).[30] It is important to point out that the loan amount in this model is

[28] *Mortgage Market Statistical Annual*, Inside Mortgage Finance Publications, various issues.

[29] It is important to point out that, of the largest and most notable subprime lenders in 2004 and 2005 such as Ameriquest, New Century, Countrywide, and Wells Fargo, only Countrywide ranked among the top 10 lenders in 2000 (*Mortgage Market Statistical Annual*, Inside Mortgage Finance Publications, various issues).

[30] A complete and formal description of the details is available in Brueckner (2000). The model abstracts from

one dollar. Therefore, in the context of mortgage financing, the collateral requirement in the model turns out to be the inverse of the LTV ratio on mortgages. In terms of the mechanism described in the paper, competition would require uninformed lenders to screen borrowers conditional on observable risk. Borrowers with higher unobservable risk self-select into contracts with higher LTV (lower collateral requirement) and higher repayment. Lenders wary about unobservable risk characteristics of subprime borrowers would ideally want borrowers to satisfy a higher downpayment requirement before approving the mortgage. This unobservable risk component assumes greater importance because a significant proportion of subprime borrowers had incomplete or impaired documentation on loans. Interestingly, however, there has been sharp increase in LTV on first-lien subprime mortgage originations from 2003 to 2005 (Bhardwaj and Sengupta, 2009).

A plausible explanation for this increase in LTV on mortgage contracts can be given in terms of the pooling equilibria described in the model. Screening equilibria in the model require both a significant cost advantage over the informed lender to offset screening costs and the low-risk borrower's ability to post the required collateral. The extent to which either condition was met on subprime mortgage lending is not fully known. It is even less certain if such a cost advantage could be used as an effective means of poaching borrowers. In this scenario, a large secular decline in lending costs, as witnessed in the first half of this decade, would allow for pooling equilibria. Accordingly, lenders could successfully attract borrowers away from local competition by lowering downpayment (collateral) requirements (i.e., allowing higher LTV in lieu of higher mortgage rates) on mortgage contracts.[31] By doing so, they could effectively be pooling creditworthy borrowers with uncreditworthy ones. Generally, lenders would be less inclined to raise interest rates and relax LTV requirements because of the fear that such loans are attractive to bad-risk types. However, in an environment where credit is cheap, lenders expect to offset these losses from bad-risks with profits from good-risk types.[32]

choice of housing consumption and prepayment risk.

[31] This would be especially true for borrowers seeking to refinance mortgages and/or extracting equity on their homes. It is interesting to note that the proportion of first-lien subprime mortgages in the (cumulative) LTV range of 90+ increased from 10 percent in 2000 to over 50 percent in 2006 (Bhardwaj and Sengupta, 2009)..

[32] As to whether this was a conscious forethought strategy by subprime lenders or whether such pooling was viewed as a viable option under the circumstances as the events developed is an open research question. Ex post, it is difficult to argue that lenders did not err in their estimates on the proportion of bad risks in the borrower population.

7 Conclusion

This paper presents a simple theoretical model as to how low lending costs prompts lenders to include uncreditworthy borrowers in their loan portfolio. A secular decline in lending costs does not help uninformed lenders to screen uncreditworthy borrowers, but it does allow them to pool these borrowers with creditworthy types. This not only facilitates entry of outside lenders into high-risk credit markets, but also makes it optimal for them to poach borrowers from rivals by including non-creditworthy borrowers in their loan portfolio. The framework is particularly relevant in explaining how an easing of monetary policy at the early part of this decade could be a significant factor in lending to uncreditworthy borrowers in the mortgage market.

In conclusion, it is crucial to mention that the choice of the screening model is driven by two considerations. The first is to illustrate how lender competition in the face of a low lending costs can adversely affect borrower quality. The second is to demonstrate that this effect can occur in the absence of an asset price boom. It is important to illustrate that the causal link laid out in the model is independent of the traditional asset-based lending channels typically used to describe this link. However, it is equally important to emphasize that this link can occur in the presence of, or perhaps even reinforce, the traditional asset-based lending channel.

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Appendix

Throughout, we will impose the boundary conditions $[(1-\theta_b)x - V^0 - \rho] < 0$ and $[(1-\theta_g)(1-\theta_b)x - (1-\theta_g)V^0 - (1-\theta_b)\rho] > 0$ for $g = h, l$. The first condition also ensures that lenders' offers to screen out uncreditworthy borrowers do not have the undesirable property that $C > R$. The second condition ensures that uncreditworthy types do not find the lenders' competitive offers to creditworthy types unattractive.

Lemma 1 *For borrowers of type-b, the Lender-I denies credit. For borrowers of types $g = h, l$ the Lender-I offers a contract from the set $Z_g^I(\rho^I) = \{(R_g^I, 0) : R_g^I \in [\underline{R}_g^I(\rho^I), \bar{R}_g]\}$, where $\underline{R}_g^I(\rho^I) = \frac{\rho^I}{1-\theta_g}$ and $\bar{R}_g = x - \frac{V^0}{1-\theta_g}$, $g = h, l$ are the first-best (zero-collateral) minimum and maximum repayments, respectively.*

Proof. Since $\pi_b^I < 0$, the Lender-I denies credit to the b-type. Since Lender-I knows borrower type, we can focus our attention without loss of generality to either creditworthy type $g = h, l$. We first show that Lender-I will always offer zero-collateral contracts.

We prove by contradiction. Suppose not, that is, there exists an offer with a positive collateral requirement from Lender-U, (R_g^1, C_g^1), that yields non-negative profits, $\pi_g^I(R_g^1, C_g^1) \geq 0$. Consider offer (R_g^2, C_g^2) with $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, $g = h, l$. From $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, it follows that $(1-\theta_g)(R_g^2 - R_g^1) = \theta_g(C_g^1 - C_g^2)$. Therefore, $\pi_g(R_g^2, C_g^2) - \pi_g(R_g^1, C_g^1) = \theta_g(1-\beta)(C_g^1 - C_g^2) > 0$. As long as $C_g^1 > 0$, Lender-I can reduce the collateral requirement to C_g^2, with offer (R_g^2, C_g^2) which provides borrower g with the same payoff as (R_g^1, C_g^1). All such deviations yield higher profits for Lender-I. Therefore, Lender-I will always choose a contract that sets its collateral requirement to zero. The first-best (zero-collateral) minimum repayment is obtained by setting $\pi_g^I = 0$, $g = h, l$. The first-best (zero-collateral) maximum is obtained by setting $V_g(R_g^I, 0) = V^0, g = h, l$. ■

Three categories of equilibria are characterized in terms of Lender-U's offers. For example, any equilibria wherein Lender-U's offers successfully screen borrower types are characterized as separating equilibria of the model.[33] A second category involves pooling equilibria wherein Lender-U's offer of a single contract is accepted by two or more borrower types. Finally, there is a third category of equilibria

[33]The terms "sorting", "screening" and "separating" are used interchangeably. Also, the terms "bunching" and "pooling" are used interchangeably. A successful sorting occurs when, conditional on each borrower type

wherein Lender-U's offers involve the bunching (or pooling) of adjacent borrower types while screening the non-adjacent borrower type. This occurs, when, for example, Lender-U bunches the creditworthy borrowers (the l-type and the h-type) while screening the uncreditworthy borrower (b-type). Since Lender-U's equilibrium offer involves both pooling and screening, we characterize this third category of equilibria as hybrid equilibria. It is important to mention here that these are candidate equilibria which emerge as the final equilibria of the model for different values of the model parameters.

The three categories of candidate equilibria: pooling, screening, and hybrid are summarized in Table 1. Within each category, candidate-1 has a larger number of borrower types accepting offers from Lender-U than candidate-2 or candidate-3. For example, in candidate equilibrium Hybrid-2, Lender-U screens out the b-type, but in Hybrid-1 it pools them with h-types. If Lender-U can screen the b-type from the h-type, but not sort between the h-type and the l-type, then its offers in Screen-2 would be accepted by the h-types. Conversely, if Lender-U can sort creditworthy types, but not screen out the b-type (or even pool them with h-types), Lender-U's offer in Screen-3 would be accepted by the l-types. However, if Lender-U can sort between all borrower types, it can offer Screen-1 whose profits dominate those of Screen-2 and Screen-3. Similarly, for a given distribution of borrower types, the Lender-U's offers in Hybrid-1 dominate those in Pool-2. There is no equilibrium in which Lender-U bunches the non-adjacent, b- and l-types.

Lender-U's offer in equilibria

The case for which Lender-U successfully captures creditworthy borrower types is discussed first. In order to do so, the payoff from Lender-U's offers to each borrower type must be at least as good as those from Lender-I. Therefore, Lender-U's optimization problem for the case where it successfully sorts all borrower types can be written as follows:[34]

$$\max \quad \Pi^U \equiv \nu_b \pi_b^U + \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{3}$$

accepting Lender-U's offers, Lender-U makes non-negative profits overall. Unless otherwise mentioned, we will restrict attention to successful sorting (and pooling) in what follows.

[34] In this section, for the most part, we drop the superscript denoting offers by borrower U. Therefore, unless otherwise mentioned, we are considering profits and offers of Lender-U only. We will re-introduce superscripts below.

where $\pi_k^U = (1-\theta_k)R_k^U + \beta\theta_k C_k^U - \rho^U$, $k = b, h, l$ subject to the following participation constraints

$$
\begin{aligned}
V_b(R_b, C_b) &\leq V^0 && (4)\\
V_h(R_h, C_h) &\geq \bar{V}_h^I && (5)\\
V_l(R_l, C_l) &\geq \bar{V}_l^I && (6)
\end{aligned}
$$

and the following incentive compatibility constraints

$$
\begin{aligned}
V_b(R_b, C_b) &\geq V_b(R_h, C_h) && (7)\\
V_b(R_b, C_b) &\geq V_b(R_l, C_l) && (8)\\
V_h(R_h, C_h) &\geq V_h(R_b, C_b) && (9)\\
V_h(R_h, C_h) &\geq V_h(R_l, C_l) && (10)\\
V_l(R_l, C_l) &\geq V_l(R_b, C_b) && (11)\\
V_l(R_l, C_l) &\geq V_l(R_h, C_h). && (12)
\end{aligned}
$$

Note that since $\pi_b^U < 0$, Lender-U does not offer contract (R_b, C_b) in equilibrium. Therefore, (4), (9) and (11) are redundant. Moreover, one may replace $V_b(R_b, C_b)$ with the b-type's reservation utility V^0 on the left-hand side of (7) and(8). Consequently, the above maximization problem in (3)-(12) reduces to

$$\max \quad \Pi^U \equiv \nu_b \pi_b^U + \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{13}$$

where $\pi_k^U = (1-\theta_k)R_k^U + \beta\theta_k C_k^U - \rho^U$, $k = b, h, l$ subject to the following participation constraints

$$
\begin{aligned}
V_h(R_h, C_h) &\geq \bar{V}_h^I && (14)\\
V_l(R_l, C_l) &\geq \bar{V}_l^I && (15)
\end{aligned}
$$

and the following incentive compatibility constraints

$$\begin{aligned} V^0 &\geq V_b(R_h, C_h) &(16)\\ V^0 &\geq V_b(R_l, C_l) &(17)\\ V_h(R_h, C_h) &\geq V_h(R_l, C_l) &(18)\\ V_l(R_l, C_l) &\geq V_l(R_h, C_h). &(19)\end{aligned}$$

Lemma 2 *In any equilibirum offer by Lender-U, its expected profits from loans to l-types are non-negative, $\pi_l^U(R_l, C_l) \geq 0$.*

Proof. Suppose not, that is there exist equilibria in which $\pi_l^U(R_l, C_l) < 0$. It follows that $\pi_b^U(R_l, C_l) < \pi_h^U(R_l, C_l) < \pi_l^U(R_l, C_l) < 0$. Lender-$U$ can always drop this contract and increase profits. Therefore, in any equilibrium, it is always the case that $\pi_l^U(R_l, C_l) \geq 0$. ■

Lemma 3 *In any equilibirum offer by Lender-U that is accepted by the l-type, the IR constraint of the l-type, (15), must bind.*

Proof. Case 1: Lender-U does not sort h-type from l-type

We prove by contradiction. Suppose not, that is, there exists a solution to (13)-(17)[35] characterized by (R_g^1, C_g^1) such that

$$V_l(R_g^1, C_g^1) > \bar{V}_l^I \tag{20}$$

We will show that there exists an offer (R_g^2, C_g^2) such that it satisfies (15) but yields higher profit. We begin by characterizing this contract. Consider contract (R_g^2, C_g^2), where $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that

$$V_b(R_g^1, C_g^1) = V_b(R_g^2, C_g^2) \tag{21}$$

[35] Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (18) and (19) are trivially satisfied.

It follows that

$$
\begin{aligned}
V_h(R_g^2, C_g^2) &< V_h(R_g^1, C_g^1) \\
V_l(R_g^2, C_g^2) &< V_l(R_g^1, C_g^1)
\end{aligned} \tag{22}
$$

Following the last inequality, we focus our attention to (R_g^2, C_g^2) such that

$$
V_l(R_g^1, C_g^1) > V_l(R_g^2, C_g^2) \geq \bar{V}_l^I \tag{23}
$$

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2) in (21) and (23) satisfies all other constraints as well. Constraints (15), (16) and (17) are satisfied by construction.[36]

It remains to be shown that (14) is satisfied. We prove by contradiction. Suppose not, then $V_h(R_g^1, C_g^1) \geq \bar{V}_h^I > V_h(R_g^2, C_g^2)$. It follows that $\rho^I < (1-\theta_h)R_g^2 + \theta_h C_g^2$. From (23), we obtain $\rho^I \geq (1-\theta_l)R_g^2 + \theta_l C_g^2$. Combining the two, we have $(1-\theta_h)R_g^2 + \theta_h C_g^2 > (1-\theta_l)R_g^2 + \theta_l C_g^2$ or

$$
(\theta_h - \theta_l)(C_g^2 - R_g^2) > 0
$$

Since $\theta_h > \theta_l$, it must be the case that $C_g^2 > R_g^2$. This is impossible given our intial assumption $[(1-\theta_b)x - V^0 - \rho^I] < 0$, so that all lender offers to creditworthy types have the property $R \geq C$. We have a contradiction. It must be the case that (14) is satisfied.

Finally, since $R_g^2 > R_g^1$, $C_g^2 < C_g^1$, we have using (21) that $\pi_h(R_g^2, C_g^2) > \pi_h(R_g^1, C_g^1)$. Also, $\pi_l(R_g^2, C_g^2) > \pi_l(R_g^1, C_g^1)$. It follows that $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. Therefore, (R_g^1, C_g^1) cannot be an equilibrium because Lender-U can offer an alternative contract of the form (R_g^2, C_g^2) and increase profits. Such profitable deviations are not possible only if $V_l(R_g, C_g) = \bar{V}_l^I$. Therefore (15) must bind in equilibrium.

Case 2: when Lender-U sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (13)-(19) characterized

[36] Since Lender-U has only one offer for all creditworthy types, (16) and (17) are the same constraint.

by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that

$$V_l(R_l^1, C_l^1) > \bar{V}_l^I \tag{24}$$

We will show that there exist a menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (15) but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2), where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_b(R_l^1, C_l^1) = V_b(R_l^2, C_l^2) \tag{25}$$

This implies

$$\begin{aligned} V_h(R_l^2, C_l^2) &< V_h(R_l^1, C_l^1) \\ V_l(R_l^2, C_l^2) &< V_l(R_l^1, C_l^1) \end{aligned} \tag{26}$$

Following this, we can choose (R_l^2, C_l^2) such that either of the following are true

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \geq V_l(R_h, C_h) \geq \bar{V}_l^I \tag{27}$$

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \geq \bar{V}_l^I > V_l(R_h, C_h) \tag{28}$$

First, we show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (25) and either (27) or (28) satisfies all other constraints as well. Constraints (14) and (16) are trivially satisfied. Constraint (15), (17) and (19) are satisfied by construction. Lastly, (18) is satisfied because $V_h(R_h, C_h) \geq V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2)$.

Next, we show that by offering (R_l^2, C_l^2) instead of (R_l^1, C_l^1), Lender-U increases profits. Since $R_l^2 > R_l^1$, $C_l^2 < C_l^1$, we have using (25)[37],

$$\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1).$$

Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits.

Note that this holds for either (27) or (28). For (27), all deviations from (R_l^1, C_l^1) to (R_l^2, C_l^2) yield

[37] This follows from $\pi_l(R_l^2, C_l^2) - \pi_l(R_l^1, C_l^1) = \frac{\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b)}{(1-\theta_b)}(C_l^1 - C_l^2)$ and $\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b) > (1-\beta)\theta_l(1-\theta_b) > 0$.

higher profits, unless $V_l(R_l, C_l) = V_l(R_h, C_h)$; in which case, Lender-$U$ bunches h-types and l-types and we follow the proof as given in Case 1 above. Alternatively, for (28) such profitable deviations are not possible only if $V_l(R_l, C_l) = \bar{V}_l^I$. Therefore (15) must bind in equilibrium. ■

Lemma 4 *In any equilibrium wherein Lender-U screens out the b-type, the IC constraint of the b-type w.r.t the h-type, (16), must bind.*

Proof. Case 1: when Lender-U sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (13)-(19) characterized by $\{(R_h^1, C_h^1), (R_l, C_l)\}$ such that

$$V^0 > V_b(R_h^1, C_h^1)$$

We will show that there exist menus such as $\{(R_h^2, C_h^2), (R_l, C_l)\}$ that satisfy (16) all the other constraints but yield higher profits. We begin by characterizing such contracts. Consider contract (R_h^2, C_h^2), where $R_h^2 > R_h^1$, $C_h^2 < C_h^1$ such that

$$V_h(R_h^1, C_h^1) = V_h(R_h^2, C_h^2). \tag{29}$$

It follows that

$$\begin{aligned} V_l(R_h^2, C_h^2) &< V_l(R_h^1, C_h^1) \\ V_b(R_h^2, C_h^2) &> V_b(R_h^1, C_h^1) \end{aligned} \tag{30}$$

Therefore, if $V^0 > V_b(R_h^1, C_h^1)$, we can choose (R_h^2, C_h^2) so that

$$V^0 \geq V_b(R_h^2, C_h^2) > V_b(R_h^1, C_h^1).$$

Since $\{(R_h^1, C_h^1), (R_l, C_l)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that menus $\{(R_h^2, C_h^2), (R_l, C_l)\}$ satisfy all the constraints as well. Constraints (15) and (17) are trivially satisfied. Constraint (14) and (18) are satisfied by construction. Lastly, (19) is satisfied because $V_l(R_l, C_l) \geq V_l(R_h^1, C_h^1) > V_l(R_h^2, C_h^2)$.

But, since $R_h^2 > R_h^1$ and $C_h^2 < C_h^1$, we use (29) to obtain

$$\pi_h(R_h^2, C_h^2) > \pi_h(R_h^1, C_h^1)$$

Therefore, $\{(R_h^1, C_h^1), (R_l, C_l)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h^2, C_h^2), (R_l, C_l)\}$ and increase profits. Such profitable deviations are not possible only if $V^0 = V_b(R_h, C_h)$. Therefore (16) must bind in equilibrium.

Case 2: Lender-U does not sort h-type from l-type

This holds for either creditworthy type, yielding two sets of equilibria where Lender-U screens out just the b-type. The first occurs when $g = h$, and Lender-U captures only the h-type by screening them from the b-type, as described in the candidate equilibria Screen-2. The second occurs when $g = l$ and Lender-U captures both h- and l-types by bunching them and screening them from the b-type, as described in the candidate equilibria Hybrid-2.

We prove by contradiction for Hybrid-2. Suppose not, that is, there exists a solution to (13)-(17)[38] characterized by (R_g^1, C_g^1), where (R_g^1, C_g^1) is the offer to both the h-type and the l-type, and $V^0 > V_b(R_g^1, C_g^1)$. Note that one can find an alternative contract (R_g^2, C_g^2) with $R_g^2 > R_g^1$ and $C_g^2 < C_g^1$, such that

$$V_l(R_g^1, C_g^1) = V_l(R_g^2, C_g^2). \tag{31}$$

It follows that

$$\begin{aligned} V_h(R_g^2, C_g^2) &> V_h(R_g^1, C_g^1) \\ \text{and } V_b(R_g^2, C_g^2) &> V_b(R_g^1, C_g^1). \end{aligned} \tag{32}$$

Following this, we can restrict our attention to contracts such that

$$V^0 \geq V_b(R_g^2, C_g^2) > V_b(R_g^1, C_g^1). \tag{33}$$

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2)

[38] Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (18) and (19) are trivially satisfied.

in (31) and (33) satisfies all other constraints as well. Constraints (15), (16) and (17) are satisfied by construction.[39] Lastly, (14) is satisfied from (32).

In offering (R_g^2, C_g^2) instead of (R_g^1, C_g^1), Lender-U's change in profits from the l-type and h-type are given by

$$\begin{aligned}\Delta\pi_l^U &= (1-\beta)\theta_l(C_g^1 - C_g^2) > 0 \\ \Delta\pi_h^U &= \frac{\theta_l(1-\theta_h) - \beta\theta_h(1-\theta_l)}{(1-\theta_l)}(C_g^1 - C_g^2)\end{aligned}$$

Since bunching is only feasible for $\nu_l \geq \nu_h$ and

$$(1-\beta)\theta_l > \frac{(1-\beta)\theta_l(1-\theta_h)}{(1-\theta_l)} > \frac{\theta_l(1-\theta_h) - \beta\theta_h(1-\theta_l)}{(1-\theta_l)}$$

Therefore Lender-U's offer of (R_g^2, C_g^2) to both h-type and l-types yields higher profits than (R_g^1, C_g^1). That is, $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. This implies that (R_g^1, C_g^1) cannot be an equilibrium. Such deviations are no longer possible if $V_b(R_g, C_g) = V^0$.

Proceeding exactly as above, we can prove the same for candidate equilibria Screen-2, where Lender-U's offers are accepted by the h-types only. ■

Lemma 5 *In any equilibrium wherein Lender-U sorts the h-types from the l-type, the IC constraint of the h-type w.r.t the l-type,(18), must bind.*

Proof. We prove by contradiction. Suppose not, that is, there exists a solution to (13)-(19) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that

$$V_h(R_h, C_h) > V_h(R_l^1, C_l^1)$$

We will show that there exist a menus of contracts $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (18) and all the other constraints but yields higher profit. We begin by characterizing this contract. Consider

[39] Since Lender-U has only one offer for all creditworthy types, (18) and (19) are the same constraint.

contract (R_l^2, C_l^2), where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_l(R_l^1, C_l^1) = V_l(R_l^2, C_l^2) = \bar{V}_l^I \tag{34}$$

It follows that

$$\begin{aligned} V_h(R_l^2, C_l^2) &> V_h(R_l^1, C_l^1) \\ V_b(R_l^2, C_l^2) &> V_b(R_l^1, C_l^1) \end{aligned}$$

Therefore, if $V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$, we can choose (R_l^2, C_l^2) so that

$$V_h(R_h, C_h) \geq V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1) \tag{35}$$

Since $\{(R_h, C_h), (R_l^1, C_l^1)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (34) and (35) satisfies all the constraints as well. Constraints (14) and (16) are trivially satisfied. Constraint (15) and (19) are satisfied by construction.

For (17), we prove by contradiction. Suppose not, then $V_b(R_l^2, C_l^2) > V^0$. In addition, Lemma 4 implies $V_b(R_l^2, C_l^2) > V^0 = V_b(R_h, C_h)$. That is, $(1-\theta_b)(R_h - R_l^2) > \theta_b(C_l^2 - C_h)$. Also, because (35) holds, it follows that $\theta_h(C_l^2 - C_h) \geq (1-\theta_h)(R_h - R_l^2)$. Combining both,

$$\begin{aligned} (\frac{1-\theta_b}{\theta_b})(R_h - R_l^2) &> (C_l^2 - C_h) \geq (\frac{1-\theta_h}{\theta_h})(R_h - R_l^2) \\ (\frac{1-\theta_b}{\theta_b} - \frac{1-\theta_h}{\theta_h})(R_h - R_l^2) &> 0 \end{aligned}$$

Since the first expression is negative, this would imply $R_l^2 > R_h$. Similarly we can show that $C_l^2 < C_h$. However, since (19) is satisfied, we get $(1-\theta_l)(R_h - R_l^2) > \theta_l(C_l^2 - C_h)$. Again, using (35) it follows that $\theta_h(C_l^2 - C_h) \geq (1-\theta_h)(R_h - R_l^2)$. Combining both,

$$\begin{aligned} (\frac{1-\theta_l}{\theta_l})(R_h - R_l^2) &> (C_l^2 - C_h) \geq (\frac{1-\theta_h}{\theta_h})(R_h - R_l^2) \\ (\frac{1-\theta_l}{\theta_l} - \frac{1-\theta_h}{\theta_h})(R_h - R_l^2) &> 0 \end{aligned}$$

Now, since the first expression is positive, this would imply $R_l^2 < R_h$. Similarly we can show that $C_l^2 > C_h$. We have a contradiction. Therefore, it cannot be the case that $V_b(R_l^2, C_l^2) > V^0$ and (17) is satisfied.

Since $R_l^2 > R_l^1$, $C_l^2 < C_l^1$, using (34), we get

$$\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1)$$

Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits. Such profitable deviations are not possible only if $V_h(R_h, C_h) = V_h(R_l, C_l)$. Therefore (18) must bind in equilibrium. ■

Lender-U's offers in screening equilibria

Consequently, the above maximization problem in (3)-(12) reduces to

$$\max\ \Pi^U \equiv \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{36}$$

where $\pi_k^U = (1-\theta_k) R_k^U + \beta\theta_k C_k^U - \rho^U$, subject to the following participation constraints

$$\begin{aligned} V_h(R_h, C_h) &\geq \bar{V}_h^I & (37) \\ V_l(R_l, C_l) &= \bar{V}_l^I & (38) \end{aligned}$$

and the following incentive compatibility constraints

$$\begin{aligned} V^0 &= V_b(R_h, C_h) & (39) \\ V^0 &\geq V_b(R_l, C_l) & (40) \\ V_h(R_h, C_h) &= V_h(R_l, C_l) & (41) \\ V_l(R_l, C_l) &\geq V_l(R_h, C_h). & (42) \end{aligned}$$

Using the equations (37)-(42), we obtain expressions for (R_h, C_h) and (R_l, C_l) as follows:

$$\begin{aligned} R_g^U &\geq R_l \geq R_l^U, \\ R_h^U &\geq R_h \geq R_g^U \\ C_g^U &\leq C_l \leq C_l^U \\ C_h^U &\leq C_h \leq C_g^U \end{aligned}$$

where

$$R_l^U = C_l^U = \rho^I, \tag{43}$$

$$R_g^U = \frac{\theta_b}{\theta_b - \theta_l}\rho^I - \frac{\theta_l}{\theta_b - \theta_l}[(1-\theta_b)x - V^0] \tag{44}$$

$$C_g^U = -\frac{1-\theta_b}{\theta_b - \theta_l}\rho^I + \frac{1-\theta_l}{\theta_b - \theta_l}[(1-\theta_b)x - V^0] \tag{45}$$

$$R_h^U = \frac{\theta_b}{\theta_b - \theta_h}\rho^I - \frac{\theta_h}{\theta_b - \theta_h}[(1-\theta_b)x - V^0] \tag{46}$$

$$C_h^U = -\frac{1-\theta_b}{\theta_b - \theta_h}\rho^I + \frac{1-\theta_h}{\theta_b - \theta_h}[(1-\theta_b)x - V^0]. \tag{47}$$

Since we assume $[(1-\theta_b)x - V^0 - \rho^I] < 0$ and $[(1-\theta_g)(1-\theta_b)x - (1-\theta_g)V^0 - (1-\theta_b)\rho^I] > 0$ for $g = h, l$ all of the offers given by (43)-(47) are strictly positive.

From (37)-(42), note that if the offer to h-types is $(R_h, C_h) = (R_h^U, C_h^U)$, then the offer to l-types is $(R_l, C_l) = (R_l^U, C_l^U)$ and vice-versa. Likewise, $(R_h, C_h) = (R_l, C_l) = (R_g^U, C_g^U)$ is an offer satisfying (37)-(42) in which Lender-U bunches both h-types and l-types.

Lemma 6 *In any equilibrium wherein Lender-U sorts the h-types from the l-type, the IR constraint of the l-type, (37) must bind.*

Proof. First note that $V_h(R_h^U, C_h^U) = \bar{V}_h^I$. Also lemmas 3-5 hold.

We prove by contradiction. Suppose not. That is, there exists a menu $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ which satifies (37)-(42) where $R_h^2 < R_h^U$ and $C_h^2 > C_h^U$ such that $V_b(R_h^2, C_h^2) = V_b(R_h^U, C_h^U)$. It follows that

$$V_h(R_h^2, C_h^2) > V_h(R_h^U, C_h^U) = \bar{V}_h^I$$

or (37) is slack. Also, from (38) and (41) it follows that $R_l^2 > R_l^U$, $C_l^2 < C_l^U$. Therefore, replacing $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$ by $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ would increase profits from the l-types but decrease profits from the h-types as follows:

$$\begin{aligned}\Delta\pi_l^U &= (1-\beta)\theta_l \frac{(1-\theta_l)}{(1-\theta_b)}(\frac{\theta_b - \theta_h}{\theta_h - \theta_l})(C_h^2 - C_h^U) > 0 \\ \Delta\pi_h^U &= -\frac{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}{(1-\theta_b)}(C_h^2 - C_h^U) < 0\end{aligned}$$

For given values of ν_h and ν_l, the effect on total profits Π^U is either monotonically increasing or monotonically decreasing with the magnitude of the deviation $(C_h^2 - C_h^U)$.[40] In particular, if the effect on $\Pi^U$is positive, all deviations with menus of the type $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ yield higher profits than $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$.That is, profits are maximized by offering (R_g^U, C_g^U) which does not induce separation. Therefore, in an equilibrium that induces separation of the h-types and l-types, (37) must bind. ■

We are now in a position to list the candidate equilibria of the model. These are given below in Propositions 7-14. They are candidate equilibria of the model. In what follows, we hold the value of Lender-U's lending costs fixed at ρ^U and vary the value of Lender-I's lending costs, ρ^I. Depending on the distribution of borrower types ν_b and ν_h, and the value of ρ^I we can determine which of the following candidate equilibria will emerge as the final equilibrium of the model. Hereafter, we reintroduce the superscripts I and U for lenders' offers and profits.

Candidate equilibrium: Screen-1

We can now state Lender-U's offers under Screen-1 in terms of the following proposition.

Proposition 7 *A pure strategy equilibrium wherein Lender-U separates all borrower types occurs when* $\rho^I \geq \max(\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l})$ *and is characterized as follows:*

(a) Lender-U offers menu $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$ *given by (43), (46) and (47).*

(b) Lender-I offers $(\underline{R}_h^I, 0)$ *to h-types and* $(\underline{R}_l^I, 0)$ *to l-types.*

(c) b-types types reject offers from either lender. Both creditworthy types accept offers from Lender-U,

[40] Strictly speaking, if $\Delta\Pi^U \equiv \nu_h\Delta\pi_l^U + \nu_l\Delta\pi_l^U = 0$, then all points satisfying (37)-(42) can be supported as equilbira. However, we restrict our attention to the screening offer given in lemma 6.

and

$$\tilde{\rho}_S^{h,l} = \frac{1}{1-(1-\beta)\theta_l}\rho^U$$
$$\tilde{\rho}_S^{b,h} = \frac{\theta_b-\theta_h}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}\rho^U + \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}[(1-\theta_b)x - V^0].$$

Proof. In any screening equilibrium wherein Lender-U separates all borrower types, Lemmas 1-6 must hold. This implies that Lender-U offers menu $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$. Moreover, in equilibrium, Lender-I gives the break-even contract offers to each type, that is $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types, which the borrowers reject. Finally, for Lender-U to screen the l-type from the h-type, profits $\pi_l^U(R_l^U, C_l^U) \geq 0$. This occurs when $\rho^I \geq \tilde{\rho}_S^{h,l} = \frac{1}{1-(1-\beta)\theta_l}\rho^U$. Similarly, Lender-$U$ can screen the h-type from the b-type, if $\pi_h^U(R_h^U, C_h^U) \geq 0$. That is if $\rho^I \geq \tilde{\rho}_S^{b,h}$, where

$$\tilde{\rho}_S^{b,h} = \frac{\theta_b-\theta_h}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}\rho^U + \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}[(1-\theta_b)x - V^0].$$

■

In order to screen all borrower types, we must have $\rho^I > \max(\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l})$. However, if $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$ or $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$, Lender-$U$ still has the option to just screen one creditworthy type, as given by the following propositions.

Candidate Equilibrium: Screen-2

Proposition 8 *A pure strategy equilibrium wherein Lender-U screens out just the b-type and lends to the h-types only, occurs when $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$ and is characterized as follows:*
(a) Lender-U offers menu $\{(R_h^U, C_h^U); (R_l^0, C_l^0)\}$, with (R_h^U, C_h^U) given by (46) and (47) and (R_l^0, C_l^0) given as in (b) below
(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(R_l^I, 0)$ to l-types where $\pi_l^U(R_l^0, C_l^0) = 0$, $V_h(\underline{R}_h^I, 0) = V_h(R_l^0, C_l^0)$, and $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$.
(c) b-types reject offers from either lender. h-types accept offers from Lender-U, but l-types accept the offer from Lender-I.

Proof. The maximization problem for Screen-2 is the same as that of Screen-1, except for the fact that now $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$. Therefore, in Screen-2, all the results of Screen-1 hold except for the lenders' offers to the l-type. This implies that since $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$, (R_l^U, C_l^U) as given in (43), if offered, would yield negative profits for Lender-U. Also, so that h-types do not find the offer to the l-types attractive, any such offer would have to satisfy (41). Therefore, the best contract (highest borrower payoff) Lender-U can offer to the l-type is given by (R_l^0, C_l^0), where $\pi_l^U(R_l^0, C_l^0) = 0$. For any such offer, (38) is not satisfied: $V_l(R_l^0, C_l^0) < \bar{V}_l^I$. This implies that Lender-I does not need to offer $(\underline{R}_l^I, 0)$ to retain l-types. Lender-I can offer $(R_l^I, 0)$ instead, so that $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$ and still retain the l-type with higher profits. ■

Candidate equilibrium: Screen-3

Proposition 9 *A pure strategy equilibrium wherein Lender-U separates only the l-types occurs when $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$ and $\tilde{\rho}^h \geq \rho^I \geq \tilde{\rho}_S^{h,l}$, where $\tilde{\rho}^h = \frac{1}{1-(1-\beta)\theta_h}\rho^U$, is characterized as follows:*

(a) Lender-U offers menu (R_l^U, C_l^U) given by (43).

(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types,

(c) Only the l-type accepts the offer from Lender-U. The h-type is retained by Lender-I.

Proof. The maximization problem for Screen-3 is the same as that of Screen-1, except for the fact that now $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$. Lender-U can simply make and offer to (R_l^U, C_l^U) as given in (43). This implies that since $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$, (R_h^U, C_h^U) as given in (46)-(47), if offered, would yield negative profits for Lender-U. Therefore, Lender-U cannot screen out the b-types. Lender-I continues to offer $(\underline{R}_h^I, 0)$ as before. Note that while Lender-U cannot match this offer, Lender-I cannot raise R_h^I because the h-types are just indifferent between its offer of $(\underline{R}_h^I, 0)$ and Lender-U's offer of (R_l^U, C_l^U). It is important, therefore, that Lender-U's profits from h-types are non-positive. That is $\tilde{\rho}^h \geq \rho^I$, where $\tilde{\rho}^h \equiv \frac{1}{1-(1-\beta)\theta_h}\rho^U$. ■

Equilibria with Lender-U pooling the b-type

When the proportion of the b-type is sufficiently small, Lender-U can choose to pool them with creditworthy types. There are two such types of equilibria; one, where b-types are pooled with the h-type and the other where all types are pooled together.

Lemma 10 *In any equibrium where the Lender-U pools the bad-risk type, it offers a zero collateral contract.*

Proof. The proof is by contradiction. Suppose not, that is, there exists a pooling equilibrium where Lender-U's offers (R_P^1, C_P^1) and pools all borrowers. Since l-types accept this contract, we must have from Lemma 3 that $V_l(R_P^1, C_P^1) = \bar{V}_l^I$. Consider alternative offer (R_P^2, C_P^2) with $R_P^2 > R_P^1$ and $C_P^2 < C_P^1$, such that $V_l(R_P^2, C_P^2) = V_l(R_P^1, C_P^1)$. It follows that

$$\begin{aligned} V_h(R_P^2, C_P^2) &> V_h(R_P^1, C_P^1) \\ V_b(R_P^2, C_P^2) &> V_b(R_P^1, C_P^1) \end{aligned}$$

Therefore, both b-types and h-types accept this new contract as it yields them higher payoff. But (R_P^2, C_P^2) yields Lender-U higher profits than (R_P^1, C_P^1).[41] Proceeding just as in the cases above, this implies that in equilibrium where Lender-U pools all borrowers, $C_P = 0$.

Note that in the same way, we can show that this result holds in an equilibrium where Lender-U pools just the h- and b-types, but not the l-types. ■

Candidate Equilibrium: Pool-1

Proposition 11 *A pure strategy equilibrium wherein Lender-U pools all borrowers occurs if $\rho^I \geq \tilde{\rho}_P^1 (\equiv (\frac{1-\theta}{1-\mathbb{E}(\theta)})\rho^U)$ and is characterized as follows:*

(a) Lender-U offers menu $(\underline{R}_l^I, 0)$.

(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types.

(c) All borrowers go to the Lender-U for loans.

Proof. First, from lemma 10, it follows that the pooling offer is of the form $(R_P, 0)$. Since the

[41] One can show this proceeding in a similar way as in Case 2 for Lemma 4. Also note that pooling is feasible only if $\nu_l \geq \nu_b + \nu_h$.

pooling offer must yield non-negative profits, it must satisfy

$$[1 - \mathbb{E}(\theta)]R_P \geq \rho^U \tag{48}$$

where $E(\theta) \equiv \nu_b\theta_b + \nu_h\theta_h + \nu_l\theta_l$ is the expected value of θ. This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \geq \rho^U/[1 - E(\theta)]$. Third, for a pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that l-types accept its offer. Therefore, it must be true that $R_P \leq \underline{R}_l^I$, that is $\rho^I \geq (\frac{1-\theta_l}{1-\mathbb{E}(\theta)})\rho^U \equiv \tilde{\rho}_P^1$. Since increasing R_P increases profits, Lender-U offers $(\underline{R}_l^I, 0)$ such that the participation constraint of l-types just bind. ■

Candidate Equilibrium: Pool-2

Proposition 12 *A pure strategy equilibrium wherein Lender-U pools b-types and h-types occurs if $\rho^I \geq \tilde{\rho}_P^2$ and is characterized as follows:*
(a) Lender-U offers menu $(\underline{R}_h^I, 0)$.
(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(R_l^I, 0)$ to l-types where $\pi_l^U(R_l^0, C_l^0) = 0$, $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$ and $V_l(\underline{R}_h^I, 0) = V_l(R_l^0, C_l^0)$.
(c) Both b-type and h-type borrowers accept Lender-U's offer, while l-types accept Lender-I's offer.

Proof. Following the same procedure as above, we know that the pooling offer must yield non-negative profits. So it must satisfy

$$[1 - (\nu_b\theta_b + \nu_h\theta_h)]R_P \geq \rho^U \tag{49}$$

This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \geq \rho^U/[1 - (\nu_b\theta_b + \nu_h\theta_h)]$. For this pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that h-types accept its offer. Therefore, it must be true that $R_P \leq \underline{R}_h^I$, that is $\rho^I \geq (\frac{1-\theta_h}{1-(\nu_b\theta_b+\nu_h\theta_h)})\rho^U \equiv \tilde{\rho}_P^2$. Since increasing R_P increases profits, Lender-U offers $(\underline{R}_h^I, 0)$ such that the participation constraint of the h-types just bind. Likewise, Lender-I's offers $(R_l^I, 0)$ where $R_l^I \geq \bar{R}_l^I$ to l-types is exactly as given in Screen-2, and yields the best payoff that Lender-U could offer them. ■

Hybrid Equilibria

Hybrid equilibria has elements of pooling and screening. This occurs when Lender-U pools or bunches adjacent types but screens the non-adjacent types. The hybrid equilibria can be described as follows.

Candidate Equilibrium: Hybrid-1

Proposition 13 *A pure strategy equilibrium wherein Lender-U separates only the l-types and bunches (pools) the b-types and the h-types occurs when $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$ and $\rho^I \geq \tilde{\rho}_P^2$, and is characterized as follows:*

(a) Lender-U offers menu $\{(\underline{R}_h^I, 0); (R_l^U, C_l^U)\}$ given by (43).

(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types.

(c) The l-type accepts the offer (R_l^U, C_l^U) from Lender-U. The b-type and the h-type accept Lender-U's offer $(\underline{R}_h^I, 0)$.

Proof. Since $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$, Lender-$U$ cannot sort the h-types from b-types but can sort l-types from h-types. Lender-U offers (R_l^U, C_l^U) as given in (43). Just as in Screen-1, this is accepted by l-types and rejected by b-types and the h-types. Also, as was the case for Pool-2, for a sufficiently low ν_b and $\rho^I \geq \tilde{\rho}_P^2$, Lender-$U$ offers $(\underline{R}_h^I, 0)$, which is accepted by the b-type and the h-type and yields non-negative profits.[42] ■

Candidate Equilibrium: Hybrid-2

Proposition 14 *A pure strategy equilibrium wherein Lender-U screens out the b-types and bunches the l-type and the h-type occurs when $\rho^I \geq \tilde{\rho}_Y$ is characterized as follows:*

(a) Lender-U offers menu (R_g^U, C_g^U) given by (44) and (45)

(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types.

(c) The h-type and the l-type accept the offer (R_g^U, C_g^U). The b-types reject this offer.

Proof. First, as l-types accept Lender-U's offer Lemma 3 must hold. Also, because Lender-U

[42] Note that, since $\rho^I > \rho^U$, Lender-U can make offers with $R_h^I < \underline{R}_h^I$, but $(\underline{R}_h^I, 0)$ maximizes the Lender-U's profits from this bunching.

screens out the b-types, Lemma 4 must hold. Therefore, $V_l(R, C) = \bar{V}_l^I$ and $V^0 = V_b(R, C)$. Solving these two equations for (R, C), we get Lender-U's offers to be (R_g^U, C_g^U). Note that $V_h(R_g^U, C_g^U) > \bar{V}_h^I$, and therefore, h-types borrow from Lender-U. ■