The Expectation Hypothesis of the Term Structure of Very Short-Term Rates: Statistical Tests and Economic Value

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Abstract

This paper re-examines the validity of the Expectation Hypothesis (EH) of the term structure of US repo rates ranging in maturity from overnight to three months. We extend the work of Longstaff (2000a) in two directions: (i) we implement statistical tests designed to increase test power in this context; (ii) more importantly, we assess the economic value of departures from the EH based on criteria of profitability and economic significance in the context of a simple trading strategy. The EH is rejected throughout the term structure examined on the basis of the statistical tests. However, the results of our economic analysis are favorable to the EH, suggesting that the statistical rejections of the EH in the repo market are economically insignificant.

Keywords: Expectation Hypothesis; Term Structure of Interest Rates; Vector Autoregression; Economic Value.

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1 Introduction

Ever since Fisher (1896) postulated the Expectation Hypothesis (EH) of the term structure of interest rates, this simple and intuitively appealing theory has attracted an enormous amount of attention in financial economics. Many authors have argued that interest rates at different maturities move together because they are linked by the EH and a number of studies have addressed the empirical validity of this theory. However, this literature, using a variety of tests and data, generally rejects the EH (e.g. Roll, 1970; Fama, 1984; Fama and Bliss, 1987; Frankel and Froot, 1987; Stambaugh, 1988; Froot, 1989; Campbell and Shiller, 1991; Bekaert, Hodrick and Marshall, 1997; Bekaert and Hodrick, 2001; Clarida, Sarno, Taylor and Valente, 2006; Sarno, Thornton and Valente, 2007).

An important exception is provided by Longstaff (2000a), who finds that the EH is supported by the data. Longstaff (2000a) presents the first tests of the EH at the extreme short end of the term structure, using repurchase (repo) rates with maturities measured in days or weeks. There are two reasons why Longstaff's study is important. First, if the EH cannot explain the term structure at this extreme short end, it seems unlikely that it can be of value at longer maturities. Second, the use of repo rates is especially appropriate for investigating the EH because repo rates represent the actual cost of holding riskless securities. Hence, repo rates provide potentially better measures of the short-term riskless term structure than other interest rates commonly used by the relevant literature, such as Treasury bill rates.

This paper revisits the EH using an updated data set of repo rates from the same source as Longstaff (2000a). Our motivation is twofold. First, the literature on testing the EH has made much progress in recent years by developing increasingly sophisticated testing procedures that are particularly useful in this context. Given the statistical problems afflicting conventional tests of the EH, in this paper we employ a test that was originally proposed in Campbell and Shiller (1987) and made operational in Bekaert and Hodrick (2001).[1] Bekaert and Hodrick (2001) develop a procedure for testing the parameter restrictions that the EH imposes on a vector autoregression (VAR) of the short- and long-term interest rates. The procedure's size and power properties have been thoroughly investigated by Bekaert and Hodrick (2001) and Sarno, Thornton and Valente (2007). We apply this test to US repo rates ranging in maturity from overnight to three months over the sample period 1991-2005.

Second, we move beyond testing the validity of the EH from a purely statistical perspective and

[1] It is well known that tests that are commonly used to investigate the EH may generate paradoxical results due to finite sample biases, size distortions and power problems (e.g. see Campbell and Shiller, 1991; Bekaert, Hodrick and Marshall, 1997; Thornton, 2005, 2006).

provide evidence on whether deviations from the EH are economically significant. Distinguishing between statistical analysis and economic evaluation is crucial for at least three reasons: in general statistical rejections of a hypothesis do not necessarily imply economic rejections (e.g. Leitch and Tanner, 1991); statistical VAR tests of the EH do not allow for transactions costs, which are critical for exploiting departures from the EH in real-world financial markets; and very powerful statistical tests may reject virtually any null hypothesis in large samples, without necessarily being informative about the size of departures from the hypothesis tested (Leamer, 1978). All these reasons suggest that an economic assessment of the deviations from the EH is desirable to complement the statistical tests.

In a mean-variance framework, we compare the performance of a dynamic portfolio strategy consistent with the EH to a dynamic portfolio strategy that exploits the departures from the EH. We use a utility-based performance criterion to compute the fee a risk-averse investor would be willing to pay to switch from the EH to a strategy that exploits departures from the EH to forecast interest rates. As an alternative economic measure, we also employ the risk-adjusted return of these two strategies. In short, we provide an economic test of the EH by evaluating the incremental profitability of an optimal (mean-variance efficient) strategy which relaxes the restrictions implied by the EH statement.

To anticipate our results, we find that the EH is statistically rejected for all pairs of repo rates in our sample throughout the maturity spectrum from overnight to three months. Our results differ from Longstaff's (2000a) presumably because the VAR test is more powerful and our sample period is somewhat longer than his. However, the results of our economic analysis lend support to the EH as we find no tangible economic gain to an investor who exploits departures from the EH relative to an investor who allocates capital simply on the basis of the predictions of the EH. Specifically, the evidence in this paper shows that the economic value of departures from the EH is modest and generally smaller than the costs that an investor would incur if he were to trade to exploit the mispricing implied by EH violations. Hence, despite the statistical rejections of the EH, we conclude that the EH provides a fairly reasonable approximation to the repo rates term structure, consistent with Longstaff's interpretation of the functioning of the repo market.

The outline of the paper is as follows. Section 2 briefly describes the data and preliminary statistics on repo rates. Section 3 introduces the EH and the VAR framework within which the empirical work is carried out, with a description of the essential ingredients of the VAR testing procedure proposed by Bekaert and Hodrick (2001). We report the results from the VAR tests of the EH in Section 4. In Section 5, we outline the framework for measuring the economic value of departures from the EH in a mean-variance setting and describe the performance measures used to assess the economic significance of EH violations. Section 6 reports the results on the validity of the EH using economic value measures. The conclusions are presented in Section 7. The Appendix provides technical details on the VAR framework and estimation issues, in addition to further empirical results.

2 Data

The data set comprises daily observations of the closing overnight i_t, 1-week $i_t^{(1w)}$, 2-week $i_t^{(2w)}$, 3-week $i_t^{(3w)}$, 1-month $i_t^{(1m)}$, 2-month $i_t^{(2m)}$, and 3-month $i_t^{(3m)}$ general collateral government repo rates, from May 21, 1991 to December 9, 2005. The data are obtained from Bloomberg and the source of the data is Garban, a large Treasury securities broker. Repo rates are quoted on a 360-day basis and the rate quotations in Bloomberg are given in increments of basis points (*bps*). The total number of daily observations available is 3,625 and is essentially an update of the data set used by Longstaff (2000a).[2]

Table 1A reports the summary statistics for repo rates, in level and first difference. All variables are expressed in percentage points per annum. The data display similar properties to those described by Longstaff (2000a) for a shorter sample. The mean of the repo rates displays a mild smile effect across the term structure. In particular, the mean overnight rate of 3.9600 is slightly higher than the mean one-week rate of 3.9492, which turns out to be the lowest mean across the different maturities. The mean three-month rate is 3.9924, which is approximately 3 *bps* higher than the mean overnight rate. Table 1A also reports the mean repo rates for the different maturities by day of the week and shows a number of calendar regularities in the data. The mean repo tends to increase from Monday to Tuesday and to decrease afterwards, while the mean on Monday is always higher than the mean on Friday. For example, the mean overnight rate on Monday is 3.9718, which is about 5 *bps* higher than the mean overnight rate on Friday, equal to 3.9260. A similar pattern is observed for all other rates. However, it is important to note that these unconditional means are all very close to one another, and the differences are much smaller than the differences typically observed on other interest rates typically used in empirical research on the EH. For example, it is interesting

[2]Professor Longstaff kindly checked the consistency of our data set with the data used in Longstaff (2000a), which covered the sample from May 21 1991 to October 15 1999. Notice that only days for which a complete set of rates for all maturities are available are included in the sample. This resulted in 42 days being dropped from the sample. Finally, the period September 11, 2001 through September 30, 2001 is not available.

to compare the means of repo rates to the means of Treasury bill (T-bill) rates. For comparison purposes, in Table 1B we report descriptive statistics on daily 1-month and 3-month US T-bill rates, also obtained from Bloomberg, both for a long sample from 1961 to 2005 and for the same sample as the repo rates data. The differences in the unconditional means between the 1–month and 3-month T-bill rates over the 1991-2005 sample are often about 15 *bps*, approximately five times larger than the maximum difference observed in repo markets for the same maturities. The differences in unconditional means for the full sample are even larger, up to 25 *bps*. Before embarking in our econometric analysis designed to test the EH, it is therefore worthwhile to note that the tiny differences in the unconditional means of repo rates at different maturities suggest that risk premia in repo markets are unlikely to be of particular economic importance. Put another way, these descriptive statistics are clearly indicative that the EH is more likely to hold on repo rates than T-bill rates.

We also report the standard deviations of daily changes in repo rates in Table 1. The overnight rate displays a standard deviation higher than the rates at other maturities. The standard deviation of daily changes in the overnight rate is about 18 *bps*, while the standard deviations for the other rates range from 5 to 6 *bps* per day. The standard deviations vary somewhat across days. The corresponding figures for T-bill rates, given in Table 1B, indicate that changes in T-bill rates display a substantially higher dispersion than repo rates, with a standard deviation of about 16 *bps* for both 1-month and 3-month rates. However, it is worth mentioning that the standard deviation of the raw variables (annualized percentage returns) is not the standard deviation associated with an annual holding period. Therefore, we also report the annualized volatility $\sigma(a)$.[3] This battery of descriptive statistics confirms Longstaff's (2000a) argument that repo rates are smaller in magnitude and less volatile than T-bills.[4]

[3] Following Lo (2002), we compute the annualized volatility as $\sigma(a) = \sqrt{Var\left[i_t(a)\right]}$, where $i_t(a) = \sum_{k=0}^{a-1} i_{t-k}(d)$ is the sum of the daily returns, and $a = 250$ is the average number of trading days. Notice that the raw data are quoted on a 360-day basis and expressed in percentage points per annum. Hence, we determine the daily return as $i_t(d) = \frac{i_t}{360 \times 100}$ for a given raw repo rate i_t. We also report the product of the unconditional mean times the annualized volatility, $Mean \times \sigma(a)$, since this may be interpreted as the commonly used Black's volatility for caps under the assumption of log-normality.

[4] Notice also that the autocorrelation coefficients indicate a high level of persistence for all interest rates examined.

3 The Expectation Hypothesis

The EH of the term structure of interest rates relates a long-term n-period interest rate $i_t^{(n)}$ to a short-term m-period interest rate $i_t^{(m)}$. In the case of pure discount bonds, the EH can be stated as:

$$i_t^{(n)} = \frac{1}{k}\sum_{i=0}^{k-1} E_t[i_{t+mi}^{(m)}] + c^{(n,m)} \tag{1}$$

where $c^{(n,m)}$ is the term premium between the n- and m-period bonds (and may vary with the maturity of the rates); $k = n/m$ and is restricted to be an integer; and E_t denotes the mathematical expectation conditional on information set I_t available at time t.

In a market where expectations are formed rationally, an investor may either invest funds in a long-term n-period discount bond and hold it until maturity, or buy and roll over a sequence of short-term m-period discount bonds over the life of the long-term bond. Under the EH, these strategies should only differ by a constant term. As result, the long-term rate should be determined by a simple average of the current and expected future short-term rates plus a time-invariant term premium.[5] If the term premium $c^{(n,m)}$ is zero, the resulting form of the EH is often termed the 'pure' EH.

While much of the relevant literature relies on single equation tests of the EH, derived by reparameterizing equation (1), a number of scholars reconsider the EH in a linear VAR framework and test the set of nonlinear restrictions which would make the VAR model consistent with the EH (Campbell and Shiller, 1991; Bekaert and Hodrick, 2001; Sarno, Thornton and Valente, 2007).[6] However, while the EH postulated in equation (1) is only a statement about how longer-term rates are related to expected short term rates, the VAR setting further assumes a joint linear stochastic process for the dynamics of the long-term and short-term interest rates. This is a convenient assumption to extract predictions of future short-term rates by using current and past values of interest rates as information set. The VAR model is also inspired by the affine term structure literature in which conditional means are linear in a set of Markovian state variables (Duffie and Singleton, 1999; Dai and Singleton, 2000; Jagannathan, Kaplin and Sun, 2003; Ahn, Dittmar and Gallant, 2002; Bansal and Zhou, 2002;

[5] Fama (1984) derives equation (1) by assuming that the expected continuously compounded yields to maturity on all discount bonds are equal, up to a constant, while Shiller, Campbell, and Schoenholtz (1983) show that equation (1) is exact in some special cases and that it can be derived as a linear approximation to a number of nonlinear expectation theories of the term structure. For coupon bonds and consols with $n = \infty$, Shiller (1979) derives a similar linearized model where the long-term rate is a weighted average of expected future short-term rate plus a constant liquidity premium. Finally, note that, as showed by Longstaff (2000b), all traditional forms of the EH can be consistent with absence of arbitrage if markets are incomplete.

[6] The VAR methodology has been popular in the context of formulating and estimating dynamic linear rational expectations models since the 1970s, starting from Sargent (1977), Hansen and Sargent (1980), Sims (1980) and Wallis (1980).

Clarida, Sarno, Taylor and Valente, 2006). This literature generally documents that affine specifications are unable to simultaneously match conditional means and conditional variances, leading to term premium puzzles.[7] Therefore, the linear VAR framework is rooted in a literature that has the potential to inherit some of the challenges faced by more traditional affine term structure models. This means that one cannot rule out that the impact of these issues on EH tests based on the VAR framework is substantial. For example, potential biases of the EH tests would arise if the interest rates data are generated by a process that is not encompassed within the VAR framework due to nonlinearities or time-varying covariances. In short, EH tests based on a VAR context are only valid under the maintained hypothesis that a linear VAR accurately describes the process of the short- and long-term interest rates and the relationship between them. This maintained assumption is questionable due to the well-documented limitations of affine specifications in matching the level and term premium in bonds simultaneously with the volatility of interest rates.

These caveats notwithstanding, in this paper we rely on the VAR testing framework developed by Bekaert and Hodrick (2001) because of its desirable power properties in presence of highly nonlinear restrictions. Specifically, we implement the Generalized Method of Moments (GMM) to estimate a constrained VAR which forces the data to yield the relationship postulated by the EH and, then, test the validity of these restrictions by using the Lagrange Multiplier (LM) and Distance Metric (DM) statistics.[8]

3.1 The VAR Framework

Consider a bivariate VAR representation for the short- and long-term interest rates measured as deviations from their respective means:

$$i_t^{(m)} = a(L)i_{t-1}^{(m)} + b(L)i_{t-1}^{(n)} + u_{1,t} \tag{2}$$

$$i_t^{(n)} = c(L)i_{t-1}^{(m)} + d(L)i_{t-1}^{(n)} + u_{2,t} \tag{3}$$

where $a(L)$, $b(L)$, $c(L)$, and $d(L)$ are polynomials in the lag operator of order p, and $u_{1,t}$ and $u_{2,t}$ are error terms. For the sake of notational convenience and without loss of generality, we set $c^{(n,m)} = 0$ in equation (1) and use demeaned data in our analysis. This implies that we cannot discriminate

[7] Another stream of the literature also documents that affine structures cannot capture what is termed 'unspanned stochastic volatility' (e.g. Collin-Dufresne and Goldstein, 2002; Collin-Dufresne, Goldstein and Jones, 2007).

[8] A simple alternative would be to estimate the model without restrictions by least squares and to apply a Wald test. However, Bekaert and Hodrick (2001) provide simulation evidence that the Wald test has poor finite sample properties in presence on nonlinear restrictions relative to test statistics constrained under the null. Specifically, Bekaert and Hodrick (2001) show that the LM test has very satisfactory size properties and reasonable power. The DM test displays less satisfactory size and power properties than the LM test, whereas the Wald test shows the worst properties among these three test statistics.

between the standard formulation of the EH and the pure EH with a zero average term premium, but we focus on testing whether the term premium is constant over time.

The above formulation can be interpreted as a system where the forecasting equation (2) is used to generate the expected future short-term rate and the equation (3) determines the current long-term rate. Simultaneously, the system determines endogenously both sides of the EH statement given in equation (1), and allows joint estimation of the parameters. This improves efficiency by incorporating contemporaneous cross-correlation in the errors (Pagan, 1984; Mishkin, 1982).

The EH implies a set of nonlinear restrictions on the parameters of the above system. To define these restrictions, let us simplify the notation by translating the above p-order system into a first-order VAR companion form as

$$\begin{bmatrix} i_t^{(m)} \\ i_t^{(n)} \\ i_{t-1}^{(m)} \\ i_{t-1}^{(n)} \\ \vdots \\ i_{t-p+1}^{(m)} \\ i_{t-p+1}^{(n)} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & \cdots & a_{p-1} & b_{p-1} & a_p & b_p \\ c_1 & d_1 & \cdots & c_{p-1} & d_{p-1} & c_p & d_p \\ 1 & & & & & & \\ & 1 & & & & & \\ & & \ddots & & & & \\ & & & 1 & & & \\ & & & & 1 & & \end{bmatrix} \begin{bmatrix} i_{t-1}^{(m)} \\ i_{t-1}^{(n)} \\ i_{t-2}^{(m)} \\ i_{t-2}^{(n)} \\ \vdots \\ i_{t-p}^{(m)} \\ i_{t-p}^{(n)} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \\ \\ \\ \\ \end{bmatrix} \tag{4}$$

where the blank elements are zeros. In compact form, this VAR can be expressed as

$$Y_t = \Gamma Y_{t-1} + \nu_t \tag{5}$$

where Y_t has $2p$ elements, Γ is a $2p$ square companion matrix, and v_t is the vector of innovations orthogonal to the information set available at time t, with zero mean and covariance matrix Σ_ν. Then, the EH subjects equation (5) to the following set of nonlinear cross-equation restrictions

$$e_2' = e_1' k^{-1} \left(I - \Gamma^m\right)^{-1} \left(I - \Gamma^n\right) \tag{6}$$

where $e_1 = (1, 0, \ldots, 0)'$ and $e_2 = (0, 1, 0, \ldots, 0)'$ are $2p$ dimensional indicator vectors.[9] Although equation (6) does not have a straightforward intuition, it gives a $2p$ dimensional vector of restrictions, nonlinear in the underlying parameters of Γ, such that the predictions of future short-term rates are consistent with the EH and the resulting constrained VAR collapses to equation (1). We can interpret these restrictions as a concise summary of the main implications stated by the theory. First, the constrained VAR defines the theoretical long-term rate we would observe in a world where expectations about future short-term rates are formed rationally. Second, under these restrictions

[9] Appendix A.1 provides further technical details on the restrictions implied by the EH in the VAR model.

the long-term rate contains all relevant information required by the market participants to predict future short-term rates. Put another way, the long-term rate provides optimal predictions of future short-term rates and deviations of the actual long-term rate from the theoretical long-term rate are unsystematic and unpredictable. Then, by rewriting the $2p$ dimensional vector of restrictions as

$$a(\theta) = e_2' - e_1' k^{-1} \left(I - \Gamma^m\right)^{-1} \left(I - \Gamma^n\right) \tag{7}$$

we can define the null hypothesis of rational expectations and constant term premium as

$$H_0 : a(\theta) = 0 \tag{8}$$

where θ is formed by collecting the relevant parameters of the companion matrix Γ.[10]

3.2 The VAR Tests

Bekaert and Hodrick (2001) propose a feasible method based on the GMM to estimate the VAR model under the hypothesis that the EH holds, defined by the nonlinear cross-equation restrictions on the parameters θ.[11]

Let $y_t \equiv [i_t^{(m)}, i_t^{(n)}]$ be the vector of data available at time t, u_t be the vector of orthogonal errors defined by the model, and x_{t-1} be the vector of instruments available at time $t-1$, formed by stacking lagged values of y_t (and possibly a constant term). Next, define the vector $z_t \equiv (y_t', x_{t-1}')'$, the vector-valued function of the data and the parameters $g(z_t, \theta) \equiv u_t \otimes x_{t-1}$, and the set of orthogonality conditions $E\left[g(z_t, \theta)\right] \equiv 0$. Using the corresponding sample moment conditions $g_T(\theta) \equiv T^{-1} \sum_{t=1}^{T} g(z_t, \theta)$ for a sample of size T, the parameters, θ, are estimated by minimizing the GMM criterion function

$$Q_T(\theta) \equiv g_T(\theta)' \Omega_T^{-1} g_T(\theta) \tag{9}$$

where Ω_T^{-1} is a positive semidefinite weighting matrix (Hansen, 1982).[12] To estimate the parameters, θ, subjected to the nonlinear restrictions defined by equation (6), we define the Lagrangian as

$$L(\theta, \gamma) = -\frac{1}{2} g_T(\theta)' \Omega_T^{-1} g_T(\theta) - a_T(\theta)' \gamma \tag{10}$$

where γ is a vector of Lagrange multipliers, and $a_T(\theta)$ is the sample counterpart of $a(\theta)$. While direct maximization of the Lagrangian is difficult as the constraints are nonlinear, Bekaert and Hodrick

[10] Specifically, the vector of parameters θ is defined as $\theta = (a_1, \cdots, a_p, b_1, \cdots, b_p, c_1, \cdots, c_p, d_1, \cdots, d_p)'$.

[11] Full maximum likelihood estimation of the restricted model is generally considered as cumbersome (e.g. Bekaert and Hodrick, 2001; Melino, 2001).

[12] When Ω_T is chosen optimally, $\widehat{\theta}$ is asymptotically distributed as $\sqrt{T}(\widehat{\theta} - \theta_0) \rightarrow N(0, G_T' \Omega_T G_T)^{-1}$, where θ_0 denotes the true parameters, $\widehat{\theta}$ the parameter estimates, $G_T \equiv \nabla g_T(\theta)$ the gradient of the orthogonality conditions, and the symbol $\rightarrow$ denotes convergence in distribution.

(2001) develop a recursive algorithm which extends the estimator proposed by Newey and McFadden (1994).[13]

If the restrictions have a significant impact on parameter estimation, then the value of the Lagrange multipliers is significantly different from zero and the null hypothesis that the EH holds is rejected. The hypothesis that the multipliers are jointly zero can be tested using the LM statistic

$$T\bar{\gamma}\left(A_T B_T^{-1} A_T'\right)\bar{\gamma} \longrightarrow \chi^2_{(2p)} \tag{11}$$

or the DM statistic

$$T g_T(\overline{\theta})' \Omega_T^{-1} g_T(\overline{\theta}) \longrightarrow \chi^2_{(2p)} \tag{12}$$

where $\overline{\theta}$ denotes the constrained estimates, and $2p$ is the number of restrictions implied by the EH.

3.3 Small Sample Properties

Tests of the EH null hypothesis have been known to suffer severely from problems related to finite sample bias estimation errors. In essence, the sampling distribution in finite sample may be significantly different from the asymptotic distribution (e.g. Bekaert, Hodrick and Marshall, 1997; Bekaert and Hodrick, 2001, Thornton, 2005, 2006). Thus, before estimating the unconstrained and constrained VARs, we follow Bekaert and Hodrick (2001) and use two different data generating processes (DGPs). Specifically, from the original data set, we simulate via bootstrap two bias-corrected data sets of 70,000 observations, with homoskedastic innovations and GARCH innovations, and use them throughout the econometric analysis. See Appendix A.3 for technical details on the procedure to account for small-sample bias in our analysis.

4 Empirical Results I: The VAR Test of the EH

In the empirical analysis, we obtain the unconstrained parameter estimate of θ, denoted $\widehat{\theta}$, by least squares and its constrained estimate $\overline{\theta}$ by the constrained GMM scheme for all possible pairwise combinations of short- and long-term rates such that $k = n/m$ is an integer. To take into account the day-of-the-week regularities in the short-term repo rates, documented in Table 1A, we follow Longstaff (2000a) and set the VAR lag length to be $p = 5$.

Tables 2 and 3 report bias-corrected coefficients for the unconstrained VARs and the constrained VARs that satisfy the EH, respectively, when the DGP used to bias correct the parameters assumes

[13]Notice that the GMM estimation is applied to the VAR defined in equations (2) and (3), whereas the companion VAR is exclusively used to simplify the derivation of the cross-equation restrictions. We refer to Appendix A.2 for further technical details on the GMM procedure.

homoskedastic innovations. Comparing the coefficients in Tables 2 and 3, we note that there are sharp differences in the constrained and unconstrained estimated dynamics. In particular, for each pairwise comparison, we find that the standard errors are quite large in the constrained VAR. Also, the absolute size of the constrained coefficients is much larger than the corresponding unconstrained ones, and, perhaps more importantly, the constrained coefficients measuring the response of the short-term rate to the long-term rate sometimes have a different sign from the corresponding unconstrained estimates. This is *prima facie* evidence that the EH restrictions may be inconsistent with the data, although this evidence does not constitute a formal statistical test.

For robustness, we also carry out estimation of the VAR-GARCH model, reported in Tables B1, B2 and B3 in Appendix B. Table B1, Panel A reports the factor loadings, which are found to be statistically significant at standard significance levels, indicating the presence of GARCH effects. In Panel B, we also notice that the conditional variance turns out to be persistent for the overnight repo and moderately persistent for the spreads. Hence, departing from the assumption of homoskedasticity is likely to yield more accurate estimates of the VAR parameters and, consequently, more precise tests of the EH.

Tables B2 and B3 report bias-corrected coefficients for the unconstrained VARs and the constrained VARs that satisfy the EH, respectively, when the DGP used to bias correct the parameters assumes GARCH innovations. These results are quantitatively different from but qualitatively identical to the results for the VAR with homoskedastic innovations given in Tables 2-3. Specifically, the standard errors of parameters estimates in the constrained VAR are large, the absolute size of the constrained coefficients is larger than the corresponding unconstrained ones, and the constrained coefficients measuring the response of the short-term rate to the long-term rate have sometimes a different sign from the corresponding estimates in the unconstrained VAR.

4.1 LM and DM Tests of the EH

The LM and DM tests results are presented in Table 4, where we report the p-values for the null hypothesis that the EH holds for all possible repo rates combinations of the integer $k = n/m$. The results in Table 4 indicate that the EH is rejected for each rate pair with p-values that are well below standard significance levels. Table 4 also reports the p-values from the J-test, which provides a specification test of the validity of the overidentifying moment conditions. The p-values are comfortably larger than conventional significance levels, validating the GMM estimation and, hence, the LM and DM tests.

These findings differ from Longstaff (2000a), who does not reject the EH using conventional tests, because the VAR test is particularly powerful – and, hence, more likely to detect fine departures from the null hypothesis in finite sample – and because our sample is larger than Longstaff's (2000a). However, despite this statistical evidence, a legitimate and unanswered concern is whether the rejection of the EH may be due to small departures from the null hypothesis (or tiny data imperfections) which are not economically meaningful but appear statistically significant given the powerful test statistics and the very large sample size employed.[14] Moreover, the VAR tests are not designed to incorporate the fact that if one wanted to trade on departures from the EH – rather than assuming that the EH holds in a simple buy-and-hold allocation strategy – transactions costs create a wedge between returns from an active strategy exploiting departures from the EH and a simple buy-and-hold strategy. Finally, while the VAR tests rely on the ability of the VAR to capture the time-series properties of the term structure of repo rates, we are aware that the simple VAR tests, inspired by the literature on affine term structure models, is in fact unable to satisfactorily explain conditional means and volatility of interest rates. Hence, potential model misspecification and model uncertainty could play an important role in determining the rejection of the EH recorded in Table 4. In order to address these issues and to shed light on the economic significance of the statistical rejections of the EH recorded in this section, we proceed to an economic evaluation of the EH departures.

5 Measuring the Economic Value of Deviations from the EH

We wish to measure whether departures from the EH provide information that is economically valuable, regardless of whether or not they are statistically significant on the basis of econometric tests. This section discusses the framework we use to evaluate the impact of allowing for deviations from the EH on the performance of dynamic allocation strategies in the repo market. We employ mean-variance analysis as a standard measure of portfolio performance assuming quadratic utility. Ultimately, we aim at measuring how much an investor is willing to pay for switching from a strategy that assumes that the EH holds ($\mathcal{EH}$ strategy) to a dynamic strategy which conditions on departures from the EH ($\mathcal{DEH}$ strategy). The $\mathcal{EH}$ strategy uses the outcome from the constrained VAR to determine the portfolio allocation, whereas the $\mathcal{DEH}$ strategy is based on the unconstrained VAR. The allocation strategy we consider is simple and intuitive. It consists of taking a position (either long or short) in a long-term repo, and then hedging it with an offsetting rolling position in a series of

[14] Leamer (1978, Chapter 4) points out that classical hypothesis testing will lead to rejection of any null hypothesis with a sufficiently large sample: 'Classical hypothesis testing at a fixed level of significance increasingly distorts the interpretation of the data against a null hypothesis as the sample size grows. The significance level should consequently be a decreasing function of sample size' (p. 114).

short-maturity repos. If the EH governs the relation between the long-term and short-term rates and an investor takes long positions in long-term repos and short rolling positions in short-term repos, then following this strategy over time allows the investor to earn the unconditional term premium, denoted as $c^{(n,m)}$ in equation (1). However, if one thinks of all repo rates in deviations from their unconditional mean (i.e. setting $c^{(n,m)} = 0$), as we do in our setting below, then this strategy should earn a return of zero before costs.

Regardless of the EH rejections recorded in Table 4, the tiny differences in unconditional means of repo rates at different maturities observed in Table 1A suggest the possibility that the economic value of trading on deviations from the EH in the repo market may not be as appealing as the statistical rejections from the VAR tests may imply. The investor using the constrained VAR is effectively using the simple strategy described above based upon the belief that there is no differences in the returns from investing in the longer repo rate and from investing in a series of shorter repo rates. However, if the investor does not believe in the EH and hence uses the unconstrained VAR, the resulting allocation strategy will be the outcome of the predictions of the model with respect to whether the longer-term rate is under- or over-valued relative to the series of shorter repo rates over the maturity of the longer rate. This may be seen as the implementation of the popular carry trade strategy that attempts to exploit mispricing along the term structure of interest rates. In other words, using the unconstrained VAR is tantamount to exploiting the deviations from the EH which we have recorded in the earlier statistical analysis. If the unconstrained VAR model gives predictions of short-term repo rates consistent with the EH, the results from the $\mathcal{EH}$ strategy should be equal to the results from the $\mathcal{DEH}$ strategy.[15] From this setting we can calculate directly a variety of common performance measures, in the form of performance fees $\mathcal{F}$ (Fleming, Kirby and Ostdiek, 2001) and risk-adjusted abnormal returns $\mathcal{M}$ (Modigliani and Modigliani, 1997).

We realize that a portfolio consisting only of repo rates is unlikely to be a realistic portfolio managed by a US investor. The repurchase agreements involving US Treasury securities are mainly used by banks in order to manage the quantity of reserves on a short-term basis and, hence, play an important role in the Federal Reserve's implementation of monetary policy. Moreover, the repo market plays a fundamental role in dealers' hedging activities and repos are used by investment managers who sell short Treasury securities in order to hedge the interest rate risk in other securities. Our main objective is not to design a realistic (executable) asset allocation strategy, but rather to measure the economic significance of deviations from the EH. Our measures of economic value

[15] Nevertheless, when incorporating transactions costs, this equality will not hold exactly, and therefore incorporating transactions costs is a further relevant issue in the construction of a measure of economic value.

complement the LM and DM tests for statistical significance of the EH by showing whether the constraints imposed on the VAR by the EH have economic value. On the one hand, departures from the EH may be statistically insignificant, and yet provide considerable value to an investor. On the other hand, the departures might be statistically significant, but be of little or no economic value to a repo market investor.[16] This economic evaluation is easier to carry out and assess by focusing exclusively on a VAR where the only assets being modelled are repo rates at various maturities, because the only source of risk in the resulting repo portfolio is interest rate risk.

5.1 The EH in a Dynamic Mean-Variance Framework

In mean-variance analysis, the maximum expected return strategy leads to a portfolio allocation on the efficient frontier. Specifically, consider the trading strategy of an investor who has a k-period horizon and constructs a daily dynamically rebalanced portfolio that maximizes the conditional expected return subject to achieving a target conditional volatility. Computing the time-varying weights of this portfolio requires predictions of the k-period ahead forecast of the conditional mean and the conditional variance-covariance matrix.

Let r_{t+k} denote the $N \times 1$ vector of risky asset returns; $\mu_{t+k|t} = E_t\left[r_{t+k}\right]$ is the conditional expectation of r_{t+k}; and $\Sigma_{t+k|t} = E_t[(r_{t+k} - \mu_{t+k|t})(r_{t+k} - \mu_{t+k|t})']$ is the conditional variance-covariance matrix of r_{t+k}.[17] At each period t, the investor solves the following problem:

$$\begin{aligned} &\max_{w_t}\{\mu_{p,t+k} = w_t'\mu_{t+k|t} + \left(1 - w_t'\iota\right) r_f\} \\ &\text{s.t.} \quad \left(\sigma_p^*\right)^2 = w_t'\Sigma_{t+k|t}w_t \end{aligned} \tag{13}$$

where w_t is the $N \times 1$ vector of portfolio weights on the risky assets, $\mu_{p,t+k}$ is the conditional expected return of the portfolio, σ_p^* is the target conditional volatility of the portfolio returns, and r_f is the return on the riskless asset.[18] The solution to this optimization problem delivers the following risky asset weights:

$$w_t = \frac{\sigma_p^*}{\sqrt{C_t}}\Sigma_{t+k|t}^{-1}(\mu_{t+k|t} - \iota r_f) \tag{14}$$

where $C_t = (\mu_{t+k|t} - \iota r_f)'\Sigma_{t+k|t}^{-1}(\mu_{t+k|t} - \iota r_f)$. The weight on the riskless asset is $1 - w_t'\iota$.

By design, in this setting the optimal weights will vary across models only to the extent that predictions of the conditional moments will vary, which is precisely what the empirical models provide.

[16] See Leitch and Tanner (1991) for an early treatment of the relationship between statistical significance and economic value.

[17] We use the subscript $t + k$ to indicate an investment horizon of k periods ahead, where $k = n/m$ is an integer which depends on the long- and short-term interest rates.

[18] For simplicity, we drop the subscript t from the riskless return r_f.

In our setting, we carry out the economic value analysis comparing the outcome from the $\mathcal{DEH}$ strategy – a strategy that exploits deviations from the EH – to the $\mathcal{EH}$ strategy which assumes that the EH holds. We compute the calculations for both cases with homoskedastic and GARCH innovations in the bias-correction DGPs. In short, our objective is to determine whether there is economic value in using the unconstrained VAR which relaxes the constraints imposed by the EH.

5.2 Quadratic Utility

We rank the performance of the competing repo rate models using the West, Edison, and Cho (1993) methodology, which is based on mean-variance analysis with quadratic utility. The investor's realized utility in period $t+k$ can be written as:

$$U\left(W_{t+k}\right) = W_{t+k} - \frac{\lambda}{2} W_{t+k}^2 = W_t R_{p,t+k} - \frac{\lambda W_t^2}{2} R_{p,t+k}^2 \tag{15}$$

where W_{t+k} is the investor's wealth at $t+k$, λ determines his risk preference, and

$$R_{p,t+k} = 1 + r_{p,t+k} = 1 + \left(1 - w_t' 1\right) r_f + w_t' r_{t+k} \tag{16}$$

is the period $t+k$ gross return on his portfolio.

We quantify the economic value of deviations from the EH by setting the investor's degree of relative risk aversion (RRA), $\delta_t = \lambda W_t / \left(1 - \lambda W_t\right)$, equal to a constant value δ. In this case, West, Edison, and Cho (1993) demonstrate that one can use the average realized utility, $\overline{U}\left(\cdot\right)$, to consistently estimate the expected utility generated by a given level of initial wealth. Specifically, the average utility for an investor with initial wealth W_0 is equal to:

$$\overline{U}\left(\cdot\right) = W_0 \sum_{t=0}^{T-1} \left\{ R_{p,t+k} - \frac{\delta}{2\left(1+\delta\right)} R_{p,t+k}^2 \right\}. \tag{17}$$

We standardize the investor problem by assuming he allocates \$1 in every time period. Average utility depends on taste for risk. In the absence of restrictions on δ, quadratic utility exhibits increasing degree of RRA. This is counterintuitive since, for instance, an investor with increasing RRA becomes more averse to a percentage loss in wealth when his wealth increases. As in West, Edison and Cho (1993) and Fleming, Kirby and Ostdiek (2001), fixing the degree of RRA, δ, implies that expected utility is linearly homogeneous in wealth: double wealth and expected utility doubles. Furthermore, by fixing δ rather than λ, we are implicitly interpreting quadratic utility as an approximation to a non-quadratic utility function, with the approximating choice of λ dependent on wealth. The estimate of expected quadratic utility given in Equation (17) is used to implement the

Fleming, Kirby and Ostdiek (2001) framework for assessing the economic value of the $\mathcal{DEH}$ and $\mathcal{EH}$ strategies.[19]

5.3 Performance Measures

At any point in time, one set of estimates of the conditional moments is better than a second set if investment decisions based on the first set lead to higher average realized utility, $\overline{U}$. Alternatively, a better model requires less wealth to yield a given level of $\overline{U}$ than the alternative model. Following Fleming, Kirby, and Ostdiek (2001) we measure the economic value of the interest rate strategies by equating the average utilities for selected pairs of portfolios. Suppose, for example, that holding a portfolio constructed using the optimal weights based on the $\mathcal{EH}$ strategy yields the same average utility as holding the portfolio implied by the $\mathcal{DEH}$ strategy. The latter portfolio is subject to daily management expenses $\mathcal{F}$, expressed as a fraction of wealth invested in the portfolio. Since the investor would be indifferent between these two strategies, we interpret $\mathcal{F}$ as the maximum performance fee the investor would be willing to pay to switch from the $\mathcal{EH}$ to the $\mathcal{DEH}$ strategy. In general, this utility-based criterion measures how much an investor with a mean-variance utility function is willing to pay for conditioning on the deviations from the EH, as modelled in the unconstrained VAR model.[20]

The performance fee depends on the investor's degree of risk aversion and is a measure of the economic significance of violations of the EH. To estimate the fee, we find the value of $\mathcal{F}$ that satisfies

$$\sum_{t=0}^{T-1}\left\{\left(R_{p,t+k}^{\mathcal{DEH}}-\mathcal{F}\right)-\frac{\delta}{2\left(1+\delta\right)}\left(R_{p,t+k}^{\mathcal{DEH}}-\mathcal{F}\right)^{2}\right\}=\sum_{t=0}^{T-1}\left\{R_{p,t+k}^{\mathcal{EH}}-\frac{\delta}{2\left(1+\delta\right)}\left(R_{p,t+k}^{\mathcal{EH}}\right)^{2}\right\} \tag{18}$$

where $R_{p,t+k}^{\mathcal{DEH}}$ denotes the gross portfolio return constructed using the predictions from the unconstrained VAR model, and $R_{p,t+k}^{\mathcal{EH}}$ is the gross portfolio return implied by the constrained VAR model. In the absence of transactions costs, under the EH $\mathcal{F}=0$, while if the EH is violated $\mathcal{F}>0$. However, when allowing for transactions costs, it is also possible that $\mathcal{F}<0$ if the positive gain from

[19] A critical aspect of mean-variance analysis is that it applies exactly only when the return distribution is normal *or* the utility function is quadratic. Hence, the use of quadratic utility is not necessary to justify mean-variance optimization. For instance, one could instead consider using utility functions belonging to the constant relative risk aversion (CRRA) class, such as power or log utility. However, quadratic utility is an attractive assumption because it provides a high degree of analytical tractability. Quadratic utility may also be viewed as a second order Taylor series approximation to expected utility. In an investigation of the empirical robustness of the quadratic approximation, Hlawitschka (1994) finds that a two-moment Taylor series expansion "may provide an excellent approximation" (p. 713) to expected utility and concludes that the ranking of common stock portfolios based on two-moment Taylor series is "almost exactly the same" (p. 714) as the ranking based on a wide range of utility functions.

[20] For studies following this approach see also Fleming, Kirby and Ostdiek (2003), Marquering and Verbeek (2004) and Han (2006).

trading on the information provided by the EH violation is lower than the loss incurred by the more costly dynamic rebalancing of the $\mathcal{DEH}$ strategy.

We also consider the Modigliani and Modigliani (1997) measure $\mathcal{M}$, which defines the abnormal return that the $\mathcal{DEH}$ strategy would have earned over the $\mathcal{EH}$ strategy if it had the same risk as the $\mathcal{EH}$ strategy

$$\mathcal{M} = \sigma[\varkappa^{\mathcal{EH}}](\mathcal{SR}^{\mathcal{DEH}} - \mathcal{SR}^{\mathcal{EH}}) \tag{19}$$

where $\mathcal{SR} = E[\varkappa]/\sigma[\varkappa]$ is the Sharpe Ratio, and $E[\varkappa]$ and $\sigma[\varkappa]$ are the expected value and standard deviations of the excess return, $\varkappa$, of a selected strategy. The $\mathcal{DEH}$ strategy is leveraged downwards or upwards, so that it has the same volatility as the $\mathcal{EH}$ strategy. Therefore, the risk-adjusted abnormal return, $\mathcal{M}$, measures the outperformance of the $\mathcal{DEH}$ strategy with respect to the $\mathcal{EH}$ strategy while matching the same level of risk.[21]

5.4 Dynamic Strategies, Transaction Costs and Short Selling

Consider a US investor who allocates his wealth between a long–term n-period discount bond and a sequence of k short-term m-period discount bonds. The long-term bond price is known with certainty and implies a riskless return, whereas the rolling combination of short-term bonds generates a risky return, since $k-1$ future short-term bond prices are not known. Hence, on the basis of riskless return, r_f, and the forecasts of the conditional moments of risky return, $r_{t+k|t}$, the investor will define his portfolio optimization problem at time t.

We consider two alternative trading strategies. The $\mathcal{EH}$ strategy assumes that EH holds exactly, and hence the investor takes a position using forecasts based on the constrained VAR. In this case, the investor effectively trades assuming that equation (1) holds and, in the absence of transactions costs, he is indifferent between investing in the long rate or a series of short rates. However, if transactions costs are positive and equal for short- and long-rates, the investor will prefer investing in the long rate as this minimizes costs. The $\mathcal{DEH}$ strategy uses the forecasts based on the unconstrained VAR. Specifically, each strategy comprises two steps at time t. First, the investor uses the selected VAR model to generate the conditional moments of the rolling strategy, $\mu_{t+k|t}$ and $\Sigma_{t+k|t}$. Second, conditional on the predictions of this model and given the riskless return r_f, he dynamically rebalances his portfolio by computing optimal weights. He repeats this process every day until the end of the sample period.[22]

[21] We also compute a measure that allows for downside risk. However, since the results are qualitatively identical to the performance fees and risk-adjusted abnormal returns, we do not report them here to conserve space.

[22] Since we consider a single risky return, $\Sigma_{t+k|t}$ simply reduces to a variance term.

This setup determines whether using one particular conditional specification affects the performance of a short-horizon allocation strategy in an economically meaningful way. The predictions are all in-sample predictions, since our focus is not to provide forecasting models of the repo term structure but to evaluate the measured departures from the EH as determined by the unconstrained VAR model.

With daily rebalancing, transaction costs play an important role in evaluating the relative performance of different strategies. In particular, we assume that transaction costs at time t equal a fixed proportion τ of the value traded in long-term and short-term repos (Marquering and Verbeek, 2004; Han, 2006). We also assume that the costs are the same for trading short and long rates. This is consistent with the fact that the bid-ask spread is fairly constant across maturities in the repo market, in the order of 2 to 5 *bps*. We report results both with and without transactions costs, and also study the impact of short selling constraints. In the case of limited short selling we constrain the portfolio weights to be bounded between -1 and 2 (assuming that the investor can borrow no more than 100% of his wealth), while in the case of no short selling, the portfolio weights are constrained between 0 and 1.

6 Empirical Results II: The Economic Value of EH Departures

Given the parameter estimates reported in Tables 3-4 and B2-B3, we assume that a US investor dynamically updates his portfolio weights daily after reestimating the VAR model with the latest available data. The key question is whether the dynamic strategy that allows for departures from the EH generates economic gains relative to a benchmark dynamic strategy that assumes that the EH holds. We assess the economic value of conditioning on departures from the EH by analyzing the performance of the dynamically rebalanced portfolio constructed using pairwise combinations of repo rates.[23]

We compute the performance fee $\mathcal{F}$ and the risk-adjusted abnormal return $\mathcal{M}$ for (i) two target annualized portfolio volatilities, $\sigma_p^* = \{1\%, 2\%\}$, which are in a range that includes the observed annualized standard deviation of the data reported in Table 1A; (ii) a degree of relative risk aversion $\delta = 5$;[24] (iii) for each pair of repo maturities where the long maturity is an exact multiple of the short maturity; (iv) two different DGPs for the parameter estimates, with homoskedastic and heteroskedastic innovations. Furthermore, we also exploit the impact of transaction costs and short

[23] For weekends and holidays we consider the rate on the previous business day for which a rate was reported.

[24] We investigated different values of δ in the range between 2 and 10 but found no qualitative difference in our results (not reported but available from the authors upon request).

selling by considering four different scenarios. In *case* 1 transaction costs are ignored and the weights are unrestricted; *in case* 2 the weights are unrestricted but we introduce transaction costs with $\tau = 4$ *bps*, a realistic cost on the basis of the observed bid-ask spread in the repo market; in *case* 3 we also add a limited short selling constraint by restricting the weights to be between -1 and 2; and finally in *case* 4 we do not allow short selling so that the weights are between 0 and 1. The performance measures, $\mathcal{F}$ and $\mathcal{M}$, are reported in annualized basis points.[25]

6.1 Performance Measures

Table 5 presents the in-sample performance fees $\mathcal{F}$ and the risk-adjusted abnormal returns $\mathcal{M}$ for the $\mathcal{DEH}$ strategy against the $\mathcal{EH}$ strategy when the bootstrap experiment for bias correction assumes homoskedastic innovations. Panel A reports the results for a target volatility $\sigma_p^* = 1\%$, and Panel B for $\sigma_p^* = 2\%$.

The results in Table 5 suggest that the performance fees for switching from a model that assumes the EH holds to a model that exploits departures from the EH is generally fairly modest when we do not consider transaction costs and the portfolio weights are unrestricted (*case* 1). For example, if we set the target volatility at $\sigma_p^* = 1\%$, the annual performance fee a risk-averse investor would be willing to pay to switch from the $\mathcal{EH}$ strategy to the $\mathcal{DEH}$ strategy is at most 1.34 *bps*. If we calibrate the target volatility to be $\sigma_p^* = 2\%$, the largest annual performance fee reaches 2.70 *bps* and occurs when the overnight repo rate is the short-term rate and the 1-week repo rate is the long-term rate.

However, when we introduce transaction costs (*case* 2), the performance fees $\mathcal{F}$ become even smaller and are slightly negative at the shorter end of the maturity spectrum. For instance, given $\sigma_p^* = 1\%$ and the overnight repo rate versus the 3-week repo rate, the $\mathcal{DEH}$ strategy has a negative annual performance fee of about 3 *bps*. This suggests that the higher transactions costs incurred in the $\mathcal{DEH}$ strategy outweigh the benefit of conditioning on EH violations, with the performance fee generally decreasing in $k = m/n$ due to the larger number of trades needed in the rolling strategy. In other words, the EH violations are not economically significant after costs are taken into account.

When we move at the longer spectrum of the maturity and consider 1-month versus 3-month repo rates for $\sigma_p^* = 1\%$, we notice a performance fee of 0.49 *bps*. Interestingly, when we combine transaction costs and limited short-selling (*case* 3), the performance measures remain virtually the same as in *case 2*, suggesting that the weights are in the range from -1 and 2. In the fourth scenario,

[25] We experimented with slightly different values of transactions costs in the range between 2 and 5 *bps*, and found qualitatively similar results (not reported to conserve space). Note that the transactions costs are virtual identical across maturities in the repo market, possibly only slightly smaller on one-day repos by some 0.5 *bps*.

we consider dynamic strategies without short selling and with transaction costs (*case* 4). In this case the fees decrease moderately in absolute values confirming that the short selling constraints are now binding on the profitability of the strategies but their impact is modest. The risk-adjusted abnormal returns $\mathcal{M}$, are of very similar magnitude as (in some columns identical to) the performance fees $\mathcal{F}$, leading therefore to the same conclusions.

For robustness purposes, Table 6 reports the same performance criteria, $\mathcal{F}$ and $\mathcal{M}$, when we assume GARCH innovations for the bias correction procedure. The results are qualitatively identical to the case of the VAR with homoskedastic errors discussed in Table 5, providing evidence that EH violations are economically unimportant. However, quantitatively the results in Table 6 provide evidence of even smaller gains from the $\mathcal{DEH}$ strategy, with the performance fee $\mathcal{F}$ never reaching 2 *bps*.

7 Conclusions

The EH plays an important role in economics and finance and, not surprisingly, has been widely tested using a variety of tests and data. Much of the empirical literature has struggled to find evidence supporting the validity of the EH across a variety of data sets and countries, and employing increasingly sophisticated testing procedures. This paper re-examines an important exception in this literature: the result that the EH appears to fit the behavior of US repo rates at the shortest end of the term structure, measured at daily frequency from overnight to the 3-month maturity (Longstaff, 2000a). We innovate in this context on two grounds. First, we extend this research by testing the restrictions implied by the EH on a VAR of the long- and short-term repo rate using the test proposed by Bekaert and Hodrick (2001). These results are not encouraging for the EH, which is statistically rejected across the term structure considered.

Second, we move beyond statistical tests and provide complementary evidence on the validity of the EH using some economic value calculations. We assess the economic value of exploiting departures from the EH – i.e. using empirical models which condition on information contained in EH deviations – relative to the economic value of using a model that assumes the EH holds. The empirical results indicate that the economic value of departures from the EH is modest and generally smaller than the costs that an investor would incur to exploit the mispricing implied by EH violations. These findings are consistent with the thrust of Longstaff's (2000a) original conclusion.

The results from economic value calculations are in contrast with the results from VAR tests reported earlier. This difference confirms that statistical rejections of a hypothesis do not always imply economic rejections and raises doubts about the ability of the simple linear VAR framework to capture the relationship between repo rates at different maturities. Activities in the repo market at maturities of days or weeks are largely driven by liquidity considerations and by the attempts of banks to manage the quantity of reserves and to hedge interest rate risk on a short-term basis, rather than to speculate in search of excess returns. Hence, it seems unlikely that investors would be actively exploiting EH departures on a very short-term basis. Our main conclusion is that, even though the EH may be rejected statistically, it still provides a very reasonable approximation to the term structure of repo rates and constitutes a useful theory for practitioners in the repo market.

Table 1A

Descriptive Statistics for Daily Repo Rates

	Panel A: Percent Values							*Panel B: Percent Daily Changes*						
	i_t	$i_t^{(1w)}$	$i_t^{(2w)}$	$i_t^{(3w)}$	$i_t^{(1m)}$	$i_t^{(2m)}$	$i_t^{(3m)}$	Δi_t	$\Delta i_t^{(1w)}$	$\Delta i_t^{(2w)}$	$\Delta i_t^{(3w)}$	$\Delta i_t^{(1m)}$	$\Delta i_t^{(2m)}$	$\Delta i_t^{(3m)}$
$Mean$	3.9600	3.9492	3.9521	3.9544	3.9623	3.9752	3.9924	−0.0004	−0.0005	−0.0005	−0.0005	−0.0005	−0.0005	−0.0004
$Mean_{Mon}$	3.9718	3.9433	3.9420	3.9428	3.9483	3.9599	3.9764	−0.0360	−0.0106	−0.0057	−0.0051	−0.0016	−0.0004	0.0000
$Mean_{Tue}$	3.9728	3.9628	3.9657	3.9672	3.9757	3.9890	4.0051	−0.0040	−0.0064	−0.0037	−0.0029	−0.0036	−0.0035	−0.0030
$Mean_{Wed}$	3.9616	3.9496	3.9544	3.9571	3.9650	3.9784	3.9952	0.0036	−0.0022	−0.0049	−0.0039	−0.0037	−0.0022	0.0002
$Mean_{Thu}$	3.9683	3.9492	3.9526	3.9563	3.9642	3.9780	3.9969	−0.0330	−0.0028	−0.0006	−0.0010	0.0004	−0.0016	−0.0027
$Mean_{Fri}$	3.9260	3.9403	3.9445	3.9474	3.9565	3.9692	3.9866	0.0643	0.0191	0.0123	0.0103	0.0060	0.0055	0.0033
$Std\ Dev$	1.6998	1.6944	1.6973	1.6990	1.7003	1.7007	1.7013	0.1738	0.0648	0.0524	0.0517	0.0488	0.0507	0.0567
$Std\ Dev_{Mon}$	1.7039	1.7008	1.7017	1.7032	1.7023	1.7019	1.7010	0.1533	0.0672	0.0584	0.0621	0.0550	0.0517	0.0656
$Std\ Dev_{Tue}$	1.6951	1.6927	1.6959	1.6978	1.6999	1.6997	1.7009	0.1818	0.0640	0.0486	0.0498	0.0516	0.0540	0.0609
$Std\ Dev_{Wed}$	1.7115	1.7015	1.7054	1.7069	1.7072	1.7075	1.7074	0.2081	0.0621	0.0484	0.0448	0.0466	0.0484	0.0492
$Std\ Dev_{Thu}$	1.6975	1.6884	1.6927	1.6948	1.6978	1.6993	1.7009	0.1383	0.0447	0.0447	0.0463	0.0471	0.0518	0.0570
$Std\ Dev_{Fri}$	1.6953	1.6935	1.6956	1.6969	1.6991	1.6996	1.7006	0.1580	0.0779	0.0590	0.0535	0.0426	0.0470	0.0500
Min	0.8400	0.8900	0.8800	0.8700	0.8600	0.8300	0.8300	−1.5500	−0.8200	−0.8300	−0.8400	−0.8600	−0.8100	−0.8600
Max	6.7500	6.7000	6.5000	6.4900	6.4700	6.5000	6.5800	3.4000	1.1000	0.4100	0.6300	0.2900	0.3700	0.6200
ρ_1	0.9948	0.9993	0.9995	0.9995	0.9996	0.9996	0.9994	−0.3226	−0.0308	−0.1077	−0.1885	−0.1806	−0.2354	−0.2882
ρ_2	0.9929	0.9986	0.9991	0.9992	0.9993	0.9993	0.9992	−0.0921	−0.0150	0.0420	0.0399	0.0209	−0.0158	0.0467
ρ_3	0.9920	0.9979	0.9987	0.9989	0.9990	0.9991	0.9989	−0.0287	−0.0650	−0.0112	−0.0200	−0.0449	0.0123	−0.0345
ρ_4	0.9914	0.9973	0.9983	0.9986	0.9988	0.9989	0.9987	−0.0041	−0.1112	0.0101	0.0388	0.0491	0.0500	0.0494
ρ_5	0.9909	0.9969	0.9979	0.9983	0.9985	0.9986	0.9984	−0.0350	−0.0270	−0.0022	−0.0097	0.0276	−0.0225	0.0019
$\sigma(a)$	1.1640	1.1625	1.1654	1.1669	1.1681	1.1687	1.1691							
$Mean \times \sigma(a)$	4.6093	4.5909	4.6057	4.6142	4.6282	4.6460	4.6676							

The table summarizes the descriptive statistics for the daily repo rates (Panel A), and daily changes in repo rates (Panel B), from overnight to 3-month maturity. The data set consists of 3,625 daily observations of the indicated term government general collateral repo rates from May 21, 1991 to December 9, 2005, quoted on a 360-day basis and expressed in percentage points per annum. The period September 10, 2001 to September 30, 2001 is not included. The daily change in repo rate for the indicated weekday is measured from the indicated day to the next business day. ρ_i denotes the i-th order serial correlation coefficient. $\sigma(a) = \sqrt{Var[i_t(a)]}$ is the annualized volatility, where $i_t(a) = \sum_{k=0}^{a-1} i_{t-k}(d)$ is the sum of the daily returns, $a = 250$ is the average number of trading days, and $i_t(d) = \frac{i_t}{360 \times 100}$ is the daily return for a given raw repo rate i_t. All statistics are measured in percentage points per annum.

Table 1B
Descriptive Statistics for Daily T-bill Rates

	Panel A: 1961-2005				*Panel B: 1991-2005*			
	$Tb_t^{(1m)}$	$Tb_t^{(3m)}$	$\Delta Tb_t^{(1m)}$	$\Delta Tb_t^{(3m)}$	$Tb_t^{(1m)}$	$Tb_t^{(3m)}$	$\Delta Tb_t^{(1m)}$	$\Delta Tb_t^{(3m)}$
$Mean$	5.5130	5.7597	0.0002	0.0001	3.6823	3.8358	−0.0005	−0.0006
$Mean_{Mon}$	5.5339	5.7754	0.0004	0.0018	3.7016	3.8508	−0.0034	−0.0057
$Mean_{Tue}$	5.5337	5.7798	−0.0044	−0.0103	3.7046	3.8584	−0.0057	−0.0102
$Mean_{Wed}$	5.5424	5.7864	−0.0176	−0.0079	3.6982	3.8483	−0.0111	−0.0048
$Mean_{Thu}$	5.5152	5.7694	0.0063	0.0012	3.6831	3.8405	0.0055	0.0029
$Mean_{Fri}$	5.4428	5.6900	0.0160	0.0158	3.6285	3.7851	0.0113	0.0137
$Std\ Dev$	2.7856	2.8567	0.1305	0.0933	1.5764	1.6112	0.0693	0.0416
$Std\ Dev_{Mon}$	2.8002	2.8709	0.1197	0.0805	1.5863	1.6217	0.0761	0.0466
$Std\ Dev_{Tue}$	2.7946	2.8591	0.1249	0.0725	1.5769	1.6200	0.0811	0.0386
$Std\ Dev_{Wed}$	2.7979	2.8678	0.1206	0.0818	1.5904	1.6219	0.0643	0.0386
$Std\ Dev_{Thu}$	2.7693	2.8501	0.1248	0.0953	1.5797	1.6124	0.0627	0.0431
$Std\ Dev_{Fri}$	2.7685	2.8382	0.1555	0.1238	1.5536	1.5855	0.0585	0.0369
Min	0.7360	0.7900	−1.8830	−1.3080	0.7360	0.7900	−1.1120	−0.8570
Max	17.926	17.682	2.0760	1.5090	6.4290	6.2970	0.9880	0.4490
ρ_1	0.9989	0.9995	0.0449	0.2000	0.9990	0.9997	−0.0239	0.0917
ρ_2	0.9977	0.9987	0.0344	0.0606	0.9981	0.9993	−0.0485	−0.0130
ρ_3	0.9964	0.9979	0.0187	0.0180	0.9973	0.9989	−0.0023	−0.0125
ρ_4	0.9951	0.9971	0.0270	0.0598	0.9965	0.9985	0.0026	0.0492
ρ_5	0.9938	0.9962	0.0718	0.0556	0.9956	0.9981	0.0305	0.0361
$\sigma(a)$	1.8488	1.9144			1.0682	1.0988		
$Mean \times \sigma(a)$	10.194	11.027			3.9310	4.2120		

The table summarizes the descriptive statistics for daily T-bill rates, Tb_t, and daily changes in T-bill rates, ΔTb_t, for the 1-month ($1m$) and 3-month ($3m$) maturity, respectively. The data are measured in percentage points per annum. Panel A reports the statistics for the period June 14, 1961 to December 30, 2005 and consists of 11110 daily observations. Panel B reports the statistics for the period May 21, 1991 to December 9, 2005 and consists of 3,568 daily observations. The daily change in the T-bill rate for the indicated maturity is measured from the indicated day to the next business day. ρ_i denotes the i-th order serial correlation coefficient. $\sigma(a) = \sqrt{Var[i_t(a)]}$ is the annualized volatility, where $i_t(a) = \sum_{k=0}^{a-1} i_{t-k}(d)$ is the sum of the daily returns, $a = 250$ is the average number of trading days, and $i_t(d) = \frac{i_t}{360 \times 100}$ is the daily return for a given raw repo rate i_t. All statistics are measured in percentage points per annum.

Table 2

Unconstrained VAR Dynamics and Bias-Correction with Homoskedastic Innovations

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
Panel A: overnight $i_t^{(m)}$ vs. 1–week $i_t^{(n)}$										
$i_t^{(m)}$	0.2662	0.7015	−0.0455	0.0347	−0.0146	0.0423	−0.0093	−0.0219	−0.0087	0.0548
	(0.0040)	(0.0086)	(0.0041)	(0.0115)	(0.0041)	(0.0115)	(0.0041)	(0.0115)	(0.0040)	(0.0087)
$i_t^{(n)}$	0.0462	0.9267	−0.0238	0.0305	−0.0034	−0.0438	−0.0128	−0.0427	0.0008	0.1219
	(0.0018)	(0.0040)	(0.0019)	(0.0053)	(0.0019)	(0.0053)	(0.0019)	(0.0053)	(0.0018)	(0.0040)
Panel B: overnight $i_t^{(m)}$ vs. 2–week $i_t^{(n)}$										
$i_t^{(m)}$	0.3258	0.4357	−0.0210	0.2842	−0.0094	−0.0262	−0.0048	−0.0774	−0.0223	0.1135
	(0.0039)	(0.0104)	(0.0041)	(0.0137)	(0.0041)	(0.0138)	(0.0041)	(0.0137)	(0.0039)	(0.0106)
$i_t^{(n)}$	0.0241	0.8714	−0.0194	0.1566	−0.0176	−0.0164	−0.0071	0.0026	0.0095	−0.0042
	(0.0014)	(0.0039)	(0.0015)	(0.0051)	(0.0015)	(0.0051)	(0.0015)	(0.0051)	(0.0014)	(0.0040)
Panel C: overnight $i_t^{(m)}$ vs. 3–week $i_t^{(n)}$										
$i_t^{(m)}$	0.3544	0.3889	0.0036	0.2738	0.0122	−0.0763	0.0060	−0.0514	−0.0023	0.0884
	(0.0039)	(0.0108)	(0.0041)	(0.0137)	(0.0041)	(0.0139)	(0.0041)	(0.0138)	(0.0038)	(0.0111)
$i_t^{(n)}$	0.0126	0.7984	−0.0162	0.1985	−0.0190	0.0083	−0.0096	0.0466	−0.0024	−0.0177
	(0.0014)	(0.0039)	(0.0015)	(0.0049)	(0.0015)	(0.0050)	(0.0015)	(0.0049)	(0.0014)	(0.0040)
Panel D: overnight $i_t^{(m)}$ vs. 1-month $i_t^{(n)}$										
$i_t^{(m)}$	0.4106	0.3061	0.0346	0.1944	0.0362	−0.1804	0.0221	0.1004	0.0123	0.0611
	(0.0038)	(0.0116)	(0.0041)	(0.0149)	(0.0041)	(0.0150)	(0.0041)	(0.0149)	(0.0038)	(0.0119)
$i_t^{(n)}$	0.0179	0.8146	−0.0164	0.1630	−0.0118	−0.0162	−0.0085	0.0848	−0.0038	−0.0240
	(0.0013)	(0.0038)	(0.0014)	(0.0049)	(0.0014)	(0.0049)	(0.0013)	(0.0049)	(0.0012)	(0.0039)
Panel E: overnight $i_t^{(m)}$ vs. 2-month $i_t^{(n)}$										
$i_t^{(m)}$	0.4539	0.2259	0.0589	0.1228	0.0596	−0.1476	0.0455	0.1073	0.0451	0.0262
	(0.0038)	(0.0116)	(0.0042)	(0.0143)	(0.0042)	(0.0144)	(0.0042)	(0.0144)	(0.0038)	(0.0117)
$i_t^{(n)}$	0.0293	0.7349	−0.0302	0.1841	−0.0095	0.0742	−0.0095	0.0686	−0.0124	−0.0301
	(0.0012)	(0.0038)	(0.0014)	(0.0047)	(0.0014)	(0.0047)	(0.0014)	(0.0047)	(0.0012)	(0.0039)

(continued)

Table 2 *(continued)*

Panel F: overnight $i_t^{(m)}$ *vs. 3-month* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.4780	0.2409	0.0786	0.0645	0.0743	−0.1137	0.0592	−0.0243	0.0706	0.0699
	(0.0038)	(0.0106)	(0.0042)	(0.0129)	(0.0042)	(0.0131)	(0.0042)	(0.0129)	(0.0037)	(0.0108)
$i_t^{(n)}$	0.0226	0.6935	−0.0184	0.2493	−0.0104	0.0231	−0.0068	0.0756	−0.0160	−0.0131
	(0.0014)	(0.0038)	(0.0015)	(0.0046)	(0.0015)	(0.0047)	(0.0015)	(0.0046)	(0.0013)	(0.0039)

Panel G: 1-week $i_t^{(m)}$ *vs. 2-week* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6103	0.3270	−0.0389	0.0793	−0.1263	0.1010	−0.1026	0.0647	0.0132	0.0706
	(0.0048)	(0.0055)	(0.0056)	(0.0067)	(0.0056)	(0.0067)	(0.0056)	(0.0067)	(0.0047)	(0.0059)
$i_t^{(n)}$	0.0377	0.8525	−0.0320	0.1683	−0.0916	0.0476	−0.0220	0.0249	0.0311	−0.0171
	(0.0043)	(0.0049)	(0.0050)	(0.0059)	(0.0050)	(0.0060)	(0.0050)	(0.0060)	(0.0042)	(0.0052)

Panel H: 1-week $i_t^{(m)}$ *vs. 3-week* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.7264	0.1871	−0.0187	0.0822	−0.1119	0.1230	−0.0749	0.0284	0.0732	−0.0164
	(0.0046)	(0.0054)	(0.0056)	(0.0066)	(0.0056)	(0.0067)	(0.0056)	(0.0066)	(0.0045)	(0.0058)
$i_t^{(n)}$	0.0201	0.7837	−0.0437	0.2176	−0.0392	0.0331	−0.0629	0.0876	0.0167	−0.0138
	(0.0039)	(0.0046)	(0.0048)	(0.0056)	(0.0048)	(0.0057)	(0.0048)	(0.0056)	(0.0038)	(0.0049)

Panel I: 1-month $i_t^{(m)}$ *vs. 2-month* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6411	0.1791	0.1533	−0.0186	−0.0528	0.0345	0.0745	−0.0047	−0.0159	0.0090
	(0.0054)	(0.0052)	(0.0061)	(0.0058)	(0.0062)	(0.0058)	(0.0061)	(0.0058)	(0.0053)	(0.0054)
$i_t^{(n)}$	0.1690	0.6119	0.0047	0.1651	−0.1572	0.1768	−0.0625	0.1200	−0.0500	0.0219
	(0.0057)	(0.0054)	(0.0064)	(0.0060)	(0.0064)	(0.0061)	(0.0064)	(0.0061)	(0.0055)	(0.0056)

Panel J: 1-month $i_t^{(m)}$ *vs. 3-month* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6952	0.1253	0.1603	−0.0171	−0.0267	−0.0066	0.0688	0.0136	−0.0051	−0.0080
	(0.0047)	(0.0041)	(0.0055)	(0.0047)	(0.0056)	(0.0048)	(0.0055)	(0.0047)	(0.0045)	(0.0041)
$i_t^{(n)}$	0.1163	0.6336	0.0127	0.2296	−0.0956	0.0671	0.0210	0.0745	−0.0977	0.0382
	(0.0053)	(0.0047)	(0.0063)	(0.0053)	(0.0064)	(0.0054)	(0.0063)	(0.0053)	(0.0052)	(0.0048)

The table presents the unconstrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes homoskedastic innovations. $i_t^{(n)}$ is the n-period (long-term) rate and $i_t^{(m)}$ is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-term repo rates such that $k = n/m$ is an integer. Standard errors are reported in parenthesis.

Table 3

Constrained VAR Dynamics and Bias-Correction with Homoskedastic Innovations

Panel A: overnight $i_t^{(m)}$ vs. 1–week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.2286	0.8225	−0.5215	0.6936	−0.2054	−0.1770	−0.2616	−0.0859	0.1196	0.3863
	(0.1751)	(0.3164)	(0.1746)	(0.4348)	(0.1817)	(0.4436)	(0.1711)	(0.4316)	(0.1773)	(0.3267)
$i_t^{(n)}$	−0.1347	1.2090	−0.0385	0.0089	−0.0320	−0.0419	−0.0141	0.0079	0.0084	0.0272
	(0.0894)	(0.1305)	(0.0926)	(0.1884)	(0.0987)	(0.1886)	(0.0959)	(0.1871)	(0.0842)	(0.1456)

Panel B: overnight $i_t^{(m)}$ vs. 2–week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.7535	0.9498	−0.5332	2.1630	−0.0431	−2.7820	0.2823	−0.5580	0.3245	0.4431
	(0.2055)	(0.5970)	(0.2291)	(0.8370)	(0.2345)	(0.8645)	(0.2224)	(0.9049)	(0.2017)	(0.6167)
$i_t^{(n)}$	−0.0633	1.2590	−0.0745	0.0563	−0.0104	−0.2213	0.0280	−0.0241	0.0214	0.0293
	(0.0546)	(0.1538)	(0.0576)	(0.2237)	(0.0587)	(0.2347)	(0.0587)	(0.2277)	(0.0566)	(0.1723)

Panel C: overnight $i_t^{(m)}$ vs. 3–week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.5064	2.1260	0.4321	−0.5844	0.1015	−0.7700	0.2523	−1.2270	0.1721	−0.0069
	(0.2210)	(0.6373)	(0.2362)	(0.9172)	(0.2336)	(0.8275)	(0.2351)	(0.8132)	(0.2133)	(0.6715)
$i_t^{(n)}$	−0.0898	0.9670	−0.0293	0.0775	−0.0160	0.0661	−0.0145	0.0446	−0.0062	0.0002
	(0.0543)	(0.1397)	(0.0566)	(0.1853)	(0.0592)	(0.1886)	(0.0570)	(0.1921)	(0.0497)	(0.1507)

Panel D: overnight $i_t^{(m)}$ vs. 1-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.9455	1.2760	−0.3140	1.1240	0.5022	−2.8020	0.0288	0.8825	0.1478	−0.7888
	(0.1866)	(0.5981)	(0.2179)	(0.7717)	(0.2246)	(0.8165)	(0.2080)	(0.8894)	(0.1735)	(0.6722)
$i_t^{(n)}$	−0.0707	0.9640	−0.0082	0.0393	−0.0211	0.0867	−0.0052	−0.0062	−0.0049	0.0260
	(0.0444)	(0.1205)	(0.0480)	(0.1619)	(0.0476)	(0.1657)	(0.0480)	(0.1691)	(0.0402)	(0.1396)

Panel E: overnight $i_t^{(m)}$ vs. 2-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.7475	0.9542	0.1531	0.3933	0.4843	−0.3941	−0.1387	−0.4929	−0.0651	−0.6399
	(0.1841)	(0.5065)	(0.2052)	(0.5847)	(0.1991)	(0.5977)	(0.2048)	(0.6083)	(0.1945)	(0.4921)
$i_t^{(n)}$	−0.0483	0.9502	−0.0123	0.0237	−0.0087	0.0387	0.0056	0.0305	0.0019	0.0183
	(0.0452)	(0.1608)	(0.0468)	(0.1838)	(0.0494)	(0.1776)	(0.0469)	(0.1737)	(0.0447)	(0.1550)

(continued)

Table 3 *(continued)*

Panel F: overnight $i_t^{(m)}$ vs. 3-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.7324	0.7945	0.1271	0.2549	0.4525	0.2989	−0.1255	−0.9856	−0.0764	−0.4717
	(0.1788)	(0.5197)	(0.1994)	(0.5724)	(0.1871)	(0.6151)	(0.2020)	(0.5764)	(0.1812)	(0.4872)
$i_t^{(n)}$	−0.0310	0.9698	−0.0070	0.0133	−0.0050	0.0193	0.0037	0.0265	0.0014	0.0088
	(0.0512)	(0.1415)	(0.0538)	(0.1703)	(0.0558)	(0.1651)	(0.0544)	(0.1673)	(0.0489)	(0.1429)

Panel G: 1-week $i_t^{(m)}$ vs. 2-week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6383	0.7791	−0.7206	0.8372	0.9606	−1.0300	−1.2080	0.4947	0.6610	−0.4139
	(0.2936)	(0.3515)	(0.3955)	(0.4998)	(0.3996)	(0.5078)	(0.3849)	(0.4794)	(0.2983)	(0.4074)
$i_t^{(n)}$	−0.2584	1.4060	0.0007	0.1942	0.1562	−0.3139	−0.4770	0.1818	0.2953	−0.1849
	(0.1946)	(0.2209)	(0.2292)	(0.2792)	(0.2279)	(0.2946)	(0.2244)	(0.2747)	(0.1950)	(0.2586)

Panel H: 1-week $i_t^{(m)}$ vs. 3-week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6635	0.3950	−0.2646	0.3161	0.3033	−0.2055	−0.6291	0.5706	0.1752	−0.3269
	(0.2598)	(0.2815)	(0.3411)	(0.3520)	(0.3318)	(0.3629)	(0.3085)	(0.3373)	(0.2474)	(0.3081)
$i_t^{(n)}$	−0.0064	1.0090	−0.1196	0.1536	0.0726	−0.0245	−0.2311	0.2025	0.0699	−0.1272
	(0.2071)	(0.2002)	(0.2615)	(0.2228)	(0.2481)	(0.2341)	(0.2513)	(0.2137)	(0.1665)	(0.1855)

Panel I: 1-month $i_t^{(m)}$ vs. 2-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.7047	0.1758	0.1700	−0.0967	−0.5617	0.3737	0.4135	−0.1273	−0.2036	0.1514
	(0.2284)	(0.1910)	(0.2531)	(0.2134)	(0.2592)	(0.2085)	(0.2603)	(0.2124)	(0.2246)	(0.2008)
$i_t^{(n)}$	−0.1482	1.0880	0.0859	−0.0489	−0.2815	0.1872	0.2070	−0.0638	−0.1019	0.0758
	(0.2451)	(0.1911)	(0.2794)	(0.2258)	(0.2925)	(0.2083)	(0.2933)	(0.2144)	(0.2214)	(0.2041)

Panel J: 1-month $i_t^{(m)}$ vs. 3-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6712	0.2068	0.2236	−0.1122	−0.4692	0.2558	0.3072	−0.0090	−0.0269	−0.0472
	(0.1905)	(0.1495)	(0.2260)	(0.1677)	(0.2270)	(0.1647)	(0.2292)	(0.1733)	(0.1762)	(0.1594)
$i_t^{(n)}$	−0.1101	1.0690	0.0747	−0.0375	−0.1565	0.0853	0.1024	−0.0030	−0.0090	−0.0157
	(0.2277)	(0.1742)	(0.2659)	(0.1925)	(0.2753)	(0.1930)	(0.2715)	(0.2000)	(0.2165)	(0.1809)

The table presents the constrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes homoskedatsic innovations. $i_t^{(n)}$ is the n-period (long-term) rate and $i_t^{(m)}$ is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-term repo rates such that $k = n/m$ is an integer. Standard errors are reported in parenthesis.

Table 4

Test Statistics

$i^{(n)}/i^{(m)}$	$i_t^{(1w)}/i_t$	$i_t^{(2w)}/i_t$	$i_t^{(3w)}/i_t$	$i_t^{(1m)}/i_t$	$i_t^{(2m)}/i_t$	$i_t^{(3m)}/i_t$	$i_t^{(2w)}/i_t^{(1w)}$	$i_t^{(3w)}/i_t^{(1w)}$	$i_t^{(2m)}/i_t^{(1m)}$	$i_t^{(3m)}/i_t^{(1m)}$
Panel A: Bias-Correction with Homoskedastic Innovations										
LM	0.0001	0	0	0	0	0.0055	0	0.0001	0.0037	0.0023
DM	0	0	0	0	0	0	0	0	0	0
$J-Test$	0.34	0.40	0.18	0.18	0.59	0.81	0.16	0.16	0.48	0.78
Panel B: Bias-Correction with GARCH Innovations										
LM	0	0	0	0	0	0	0	0.0001	0.0001	0.0005
DM	0	0	0	0	0	0	0	0	0	0
$J-Test$	0.63	0.37	0.38	0.30	0.91	0.71	0.16	0.58	0.43	0.96

The table reports the p-values for the Lagrange Multiplier (LM) and distance metric (DM) statistics under the null hypothesis that the EH is validated by the data for each pairwise combination of short-term and long-term repo rates such that $k = n/m$ is an integer. $i_t^{(n)}$ is the n-period (long-term) rate and $i_t^{(m)}$ is the m-period (short-term) rate. The p-values are calculated by bootstrap as described in the text. Panel A reports the results when the data generating process (DGP) used for bias-correction assumes homoskedastic innovations. Panel B reports the results when the DGP used for bias-correction assumes GARCH innovations. 0 denotes p-values below 10^{-5}. The $J-Test$ is the test for the overidentifying moment conditions in the GMM estimation, and figures reported are p-values.

Table 5
Economic Value Results

Panel A: $\sigma_p^* = 1\%$

	Case 1		*Case* 2		*Case* 3		*Case* 4	
$i_t^{(m)} - i_t^{(n)}$	$\mathcal{F}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{M}$
$i_t - i_t^{(1w)}$	1.34	1.34	−1.01	−0.95	−1.01	−0.95	−0.57	−0.52
$i_t - i_t^{(2w)}$	0.47	0.47	−2.62	−2.50	−2.62	−2.50	−1.41	−1.28
$i_t - i_t^{(3w)}$	0.20	0.20	−3.33	−3.15	−3.33	−3.15	−1.77	−1.56
$i_t - i_t^{(1m)}$	0.44	0.44	−4.70	−4.46	−4.70	−4.46	−2.79	−2.45
$i_t - i_t^{(2m)}$	0.92	0.92	−7.19	−7.23	−7.19	−7.23	−4.63	−4.11
$i_t - i_t^{(3m)}$	1.51	1.51	−12.29	−12.34	−12.29	−12.34	−6.19	−6.40
$i_t^{(1w)} - i_t^{(2w)}$	0.34	0.34	0.31	0.31	0.31	0.31	0.05	0.05
$i_t^{(1w)} - i_t^{(3w)}$	0.49	0.49	0.47	0.47	0.47	0.47	0.11	0.11
$i_t^{(1m)} - i_t^{(2m)}$	0.40	0.40	0.39	0.39	0.39	0.39	0.20	0.21
$i_t^{(1m)} - i_t^{(3m)}$	0.60	0.60	0.49	0.50	0.49	0.50	0.32	0.32

Panel B: $\sigma_p^* = 2\%$

	Case 1		*Case* 2		*Case* 3		*Case* 4	
$i_t^{(m)} - i_t^{(n)}$	$\mathcal{F}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{M}$
$i_t - i_t^{(1w)}$	2.70	2.67	−1.11	−1.17	−1.11	−1.17	−0.54	−0.41
$i_t - i_t^{(2w)}$	0.95	0.94	−3.32	−3.04	−3.32	−3.04	−2.82	−2.39
$i_t - i_t^{(3w)}$	0.39	0.39	−7.73	−7.35	−7.73	−7.35	−3.55	−3.80
$i_t - i_t^{(1m)}$	0.88	0.88	−9.52	−9.00	−9.52	−9.00	−4.58	−4.39
$i_t - i_t^{(2m)}$	1.83	1.83	−17.29	−17.32	−17.29	−17.32	−8.27	−9.33
$i_t - i_t^{(3m)}$	3.02	3.02	−22.49	−22.54	−22.49	−22.54	−11.40	−11.37
$i_t^{(1w)} - i_t^{(2w)}$	0.68	0.68	0.62	0.63	0.62	0.63	0.10	0.10
$i_t^{(1w)} - i_t^{(3w)}$	0.99	0.99	0.95	0.96	0.95	0.96	0.21	0.21
$i_t^{(1m)} - i_t^{(2m)}$	0.80	0.80	0.76	0.76	0.76	0.76	0.41	0.41
$i_t^{(1m)} - i_t^{(3m)}$	1.20	1.20	1.10	1.10	1.10	1.10	0.63	0.63

The table reports the in-sample performance fees $\mathcal{F}$ and the risk-adjusted abnormal returns $\mathcal{M}$ for the $\mathcal{DEH}$ strategy against the $\mathcal{EH}$ strategy when the data generating process used for bias-correction assumes homoskedastic innovations. Panel A (B) reports the performance measures when the target portfolio volatility is set to 1% (2%) for all pairwise combinations of short-term $i_t^{(m)}$ and long-term $i_t^{(n)}$repo rates such that $k = n/m$ is an integer. Each strategy is consistent with an optimizing investor allocating capital in two assets: the long-term repo rate, known with certainty at the time of trading, and a risky return generated by rolling the short-term asset for k periods. The $\mathcal{EH}$ strategy assumes that he EH holds exactly and uses the conditional forecasts implied by the constrained VAR. The $\mathcal{DEH}$ strategy conditions on the departures from the EH and uses the conditional forecasts implied by the unconstrained VAR. The performance fees $\mathcal{F}$ denote the amount an investor with quadratic utility and a degree of relative risk aversion equal to 5 is willing to pay for switching from the benchmark strategy $\mathcal{EH}$ to the alternative strategy $\mathcal{DEH}$. The risk-adjusted abnormal return, $\mathcal{M}$, defines the outperformance of the $\mathcal{DEH}$ strategy over the $\mathcal{EH}$ strategy if they had the same level of risk. We consider four different scenarios: *case* 1 (zero transaction costs and no short selling constraints); *case* 2 (non-zero transaction costs and no short selling constraints); *case* 3 (non-zero transaction costs and limited short-selling between -1 and 2); and *case* 4 (non-zero transaction costs and no short-selling). All the performance measures are reported in annual basis points.

Table 6
Economic Value Results

Panel A: $\sigma_p^* = 1\%$

$i_t^{(m)} - i_t^{(n)}$	*Case* 1 $\mathcal{F}$	*Case* 1 $\mathcal{M}$	*Case* 2 $\mathcal{F}$	*Case* 2 $\mathcal{M}$	*Case* 3 $\mathcal{F}$	*Case* 3 $\mathcal{M}$	*Case* 4 $\mathcal{F}$	*Case* 4 $\mathcal{M}$
$i_t - i_t^{(1w)}$	0.55	0.55	−1.35	−1.35	−1.35	−1.35	−0.94	−0.92
$i_t - i_t^{(2w)}$	0.02	0.02	−2.44	−2.42	−2.44	−2.42	−1.20	−1.11
$i_t - i_t^{(3w)}$	0.02	0.02	−3.81	−3.63	−3.81	−3.63	−1.89	−1.68
$i_t - i_t^{(1m)}$	0.52	0.52	−5.76	−5.50	−5.76	−5.50	−2.17	−2.82
$i_t - i_t^{(2m)}$	0.57	0.57	−8.66	−8.68	−8.66	−8.68	−3.06	−3.02
$i_t - i_t^{(3m)}$	0.86	0.86	−11.87	−11.91	−11.87	−11.91	−5.54	−5.65
$i_t^{(1w)} - i_t^{(2w)}$	0.23	0.23	0.21	0.21	0.21	0.21	0.02	0.02
$i_t^{(1w)} - i_t^{(3w)}$	0.26	0.26	0.23	0.24	0.23	0.24	0.05	0.05
$i_t^{(1m)} - i_t^{(2m)}$	0.23	0.23	0.20	0.19	0.20	0.19	0.12	0.12
$i_t^{(1m)} - i_t^{(3m)}$	0.26	0.26	0.23	0.24	0.23	0.24	0.14	0.14

Panel B: $\sigma_p^* = 2\%$

$i_t^{(m)} - i_t^{(n)}$	*Case* 1 $\mathcal{F}$	*Case* 1 $\mathcal{M}$	*Case* 2 $\mathcal{F}$	*Case* 2 $\mathcal{M}$	*Case* 3 $\mathcal{F}$	*Case* 3 $\mathcal{M}$	*Case* 4 $\mathcal{F}$	*Case* 4 $\mathcal{M}$
$i_t - i_t^{(1w)}$	1.10	1.10	−3.78	−3.79	−3.78	−3.79	−1.44	−1.38
$i_t - i_t^{(2w)}$	0.04	0.04	−6.26	−6.27	−6.26	−6.27	−2.39	−2.11
$i_t - i_t^{(3w)}$	0.03	0.03	−7.79	−7.39	−7.79	−7.39	−3.78	−3.07
$i_t - i_t^{(1m)}$	1.03	1.03	−11.67	−11.10	−11.67	−11.10	−5.35	−5.12
$i_t - i_t^{(2m)}$	1.14	1.14	−17.28	−17.27	−17.28	−17.27	−6.12	−6.15
$i_t - i_t^{(3m)}$	1.72	1.71	−21.71	−21.69	−21.71	−21.69	−11.07	−11.84
$i_t^{(1w)} - i_t^{(2w)}$	0.45	0.45	0.39	0.40	0.39	0.40	0.03	0.03
$i_t^{(1w)} - i_t^{(3w)}$	0.52	0.52	0.48	0.48	0.48	0.48	0.10	0.10
$i_t^{(1m)} - i_t^{(2m)}$	0.46	0.46	0.39	0.39	0.39	0.39	0.24	0.24
$i_t^{(1m)} - i_t^{(3m)}$	0.51	0.51	0.46	0.46	0.46	0.46	0.28	0.28

The table reports the in-sample performance fees $\mathcal{F}$ and the risk-adjusted abnormal returns $\mathcal{M}$ for the $\mathcal{DEH}$ strategy against the $\mathcal{EH}$ strategy when the data generating process used for bias-correction assumes GARCH innovations. Panel A (B) reports the performance measures when the target portfolio volatility is set to 1% (2%) for all pairwise combinations of short-term $i_t^{(m)}$ and long-term $i_t^{(n)}$repo rates such that $k = n/m$ is an integer. Each strategy is consistent with an optimizing investor allocating capital in two assets: the long-term repo rate, known with certainty at the time of trading, and a risky return generated by rolling the short-term asset for k periods. The $\mathcal{EH}$ strategy assumes that he EH holds exactly and uses the conditional forecasts implied by the constrained VAR. The $\mathcal{DEH}$ strategy conditions on the departures from the EH and uses the conditional forecasts implied by the unconstrained VAR. The performance fees $\mathcal{F}$ denote the amount an investor with quadratic utility and a degree of relative risk aversion equal to 5 is willing to pay for switching from the benchmark strategy $\mathcal{EH}$ to the alternative strategy $\mathcal{DEH}$. The risk-adjusted abnormal return, $\mathcal{M}$, defines the outperformance of the $\mathcal{DEH}$ strategy over the $\mathcal{EH}$ strategy if they had the same level of risk. We consider four different scenarios: *case* 1 (zero transaction costs and no short selling constraints); *case* 2 (non-zero transaction costs and no short selling constraints); *case* 3 (non-zero transaction costs and limited short-selling between -1 and 2); and *case* 4 (non-zero transaction costs and no short-selling). All the performance measures are reported in annual basis points.

A Appendix

A.1 The EH Restrictions in the VAR Framework

In this section we derive the restrictions implied by the EH in the VAR framework. Define the indicator vectors $e_1 = (1, 0, \ldots, 0)'$ and $e_2 = (0, 1, 0, \ldots, 0)'$ with dimension $2p$ and select from the companion VAR the long-term rate and expected future short-term rates as $i_t^{(n)} = e_1' Y_t$ and $E_t[i_{t+i}^{(m)}] = e_1' \Gamma^i Y_t$, respectively.[26] Hence, the general statement in equation (1)

$$i_t^{(n)} = k^{-1} \left\{ i_t^{(m)} + E_t[i_{t+m}^{(m)}] + E_t[i_{t+2m}^{(m)}] + \ldots + E_t[i_{t+m(k-1)}^{(m)}] \right\} \tag{A.1}$$

can be rewritten, under the maintained assumption that the joint process of the short- and long-term interest rates is accurately described by a linear VAR, as

$$e_2' Y_t = e_1' k^{-1} \left[I + \Gamma^m + \Gamma^{2m} + \ldots + \Gamma^{m(k-1)} \right] Y_t \tag{A.2}$$

which converges, if the eigenvalues λ_i of Γ are such that $|\lambda_i| < 1$, to the following compact form

$$e_2' Y_t = e_1' k^{-1} \left(I - \Gamma^m \right)^{-1} \left(I - \Gamma^n \right) Y_t. \tag{A.3}$$

Notice that right-hand-side of equation (A.3) gives the sum of the current and expected short-term rates implied by the predictions of the VAR representation, while the left-hand-side of equation (A.3) gives the current long-term rate. In order to satisfy this equality and, hence, makes equation (A.3) consistent with equation (A.1), equation (A.3) implies the following system of nonlinear equations

$$e_2' = k^{-1} e_1' \left(I - \Gamma^m \right)^{-1} \left(I - \Gamma^n \right) \tag{A.4}$$

whose solution implies a $2p$ dimensional vector of highly nonlinear restrictions in the underlying parameters of the VAR. In the case where $m = 1$, the system of equation in (A.4) has a simple analytical solution (see Campbell and Shiller, 1987), but in the general case analyzed in this paper and in Bekaert and Hodrick (2001) we have to rely on the numerical outcome of the GMM maximization.

A.2 GMM Iterative Procedure

In this section we present the iterative procedure used for the constrained GMM maximization. The first-order conditions for the Lagrangian problem in equation (10) can be written as

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -G_T' \Omega_T^{-1} \sqrt{T} g_T(\overline{\theta}) - A_T' \sqrt{T} \bar{\gamma} \\ -\sqrt{T} a_T(\overline{\theta}) \end{bmatrix} \tag{A.5}$$

[26] As elsewhere in the paper, the expectation is with respect to the information set of the VAR.

where $A_T \equiv \nabla_\theta a_T(\theta)$ and $G_T \equiv \nabla_\theta g_T(\theta)$. By using the Taylor's expansion of $g_T(\theta)$ and $a_T(\theta)$ around the true parameter value, θ_0, and substituting into the first-order conditions, Newey and McFadden (1994) derive an approximate asymptotic solution under the null hypothesis $a_T(\theta_0) = 0$ as

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -G_T'\Omega_T^{-1}\sqrt{T}g_T(\theta_0) \\ 0 \end{bmatrix} - \begin{bmatrix} B_T & A_T' \\ A_T & 0 \end{bmatrix} \begin{bmatrix} \sqrt{T}\left(\overline{\theta} - \theta_0\right) \\ \sqrt{T}\overline{\gamma} \end{bmatrix}. \tag{A.6}$$

Next, the formula for a partitioned inverse implies that

$$\begin{bmatrix} B_T & A_T' \\ A_T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} B_T^{-1/2}M_T B_T^{-1/2} & B_T^{-1}A_T'\left(A_T B_T^{-1}A_T'\right)^{-1} \\ \left(A_T B_T^{-1}A_T'\right)^{-1}A_T B_T^{-1} & -\left(A_T B_T^{-1}A_T'\right)^{-1} \end{bmatrix} \tag{A.7}$$

where $M_T = I - B_T^{-1/2}A_T'\left(A_T B_T^{-1}A_T'\right)^{-1}A_T B_T^{-1/2}$ is an idempotent matrix, and $B_T \equiv G_T'\Omega_T^{-1}G_T$. Hence, the asymptotic distribution for the constrained estimator and the Lagrange multiplier turns out to be $\sqrt{T}[\overline{\theta} - \theta_0] \to N[0, B_T^{-1/2}M_T B_T^{-1/2}]$ and $\sqrt{T}\overline{\gamma} \to N[0, \left(A_T B_T^{-1}A_T'\right)^{-1}]$, respectively. Then, given an initial consistent unconstrained estimate $\widehat{\theta}$, by deriving $g_T(\overline{\theta}) \approx g_T(\widehat{\theta}) + G_T(\overline{\theta} - \widehat{\theta})$ and $a_T(\overline{\theta}) \approx a_T(\widehat{\theta}) + A_T(\overline{\theta} - \widehat{\theta})$, and substituting into the first-order conditions, Bekaert and Hodrick (2001) define the following iterative scheme

$$\overline{\theta} \approx \widehat{\theta} - B_T^{-1/2}M_T B_T^{-1/2}G_T'\Omega_T^{-1}g_T(\widehat{\theta}) - B_T^{-1}A_T'\left(A_T B_T^{-1}A_T'\right)^{-1}a_T(\widehat{\theta}) \tag{A.8}$$

$$\overline{\gamma} \approx -\left(A_T B_T^{-1}A_T'\right)^{-1}A_T B_T^{-1}G_T'\Omega_T^{-1}g_T(\widehat{\theta}) + \left(A_T B_T^{-1}A_T'\right)^{-1}a_T(\widehat{\theta}) \tag{A.9}$$

To obtain the constrained parameters $\overline{\theta}$, we iterate on equations (A.8) and (A.9), substituting the first constrained estimate for the initial consistent unconstrained estimate to derive a second constrained estimate and so forth. The iterative process continues until the constrained estimate satisfies the constraints, that is $a_T(\overline{\theta}) = 0$.

A.3 Small Sample bias correction

Let $Z_t = [i_t, S_t^{(1w)}, S_t^{(2w)}, S_t^{(3w)}, S_t^{(1m)}, S_t^{(2m)}, S_t^{(3m)}]'$, where $S_t^{(j)}$ denotes the spread between repo rate $i_t^{(j)}$ and the overnight repo rate i_t, and assume a $VAR(p)$ dynamics

$$Z_t = \varphi + \sum_{j=1}^{p} \Phi_j Z_{t-j} + \varepsilon_t \tag{A.10}$$

where φ is a vector of constant and Φ_j is a square matrix. Under the assumption of homoskedastic innovations, we proceed as follows. Estimate equation (A.10) on the original data set and simulate $100,000$ artificial data sets of $3,625$ by using an i.i.d. bootstrap of ε_t. Next, reestimate equation (A.10) for each replication and determine bias as the difference between the parameter estimates of the initial data set and the average of the parameter estimates of the artificial data sets. Then, correct the original parameters, simulate 70,000 observations, and add the simulated i_t to each simulated spread $S_t^{(j)}$. This bias corrected data set is, hence, subjected for each pairwise combination of short-term and long-term rate to the analysis described in Section 3.

In the second DGP, reparameterize $\varepsilon_t = F\eta_t$, to capture the effects of temporal heteroskedasticity, where η_t is a vector of idiosyncratic innovations and F is a 7×7 factor loadings matrix defined as

$$F = \begin{bmatrix} 1 & & & 1 \\ f_{21} & \ddots & & f_{27} \\ \vdots & & \ddots & \vdots \\ f_{71} & & & 1 \end{bmatrix} \tag{A.11}$$

where the blank elements are zero. Define $E_{t-1}[\eta_t'\eta_t] = V_t$, and $E_{t-1}[\varepsilon_t'\varepsilon_t] = FV_tF'$, where V_t is a diagonal matrix and each element is assumed to follow an GARCH(1,1) process augmented with square root of overnight rate, $h_{jt} = \omega_j\sqrt{i_{t-1}} + \beta_j h_{jt-1} + \alpha_j\eta_{jt-1}^2$ with $j \in \{1,\ldots,7\}$, as in Gray (1996), Bekaert and Hodrick (2001), Longstaff (2000a), and Ang and Bekaert (2002), in order to accommodate shifts in the short-rate volatility. Hence, estimate equation (A.10) and proceed with bias correction as in the previous experiment, Next, compute the residual vector ε_t, estimate the factor GARCH parameters via quasi-maximum likelihood, and simulate a second bias corrected data set as in the previous experiment. Finally, we always generate additional 1,000 discarding values to avoid any dependence on the starting values.

B Appendix: Table B1

Factor GARCH Model

Panel A: Factor Loadings							
Maturity	f_1	f_2	f_3	f_4	f_5	f_6	f_7
i_t	1	0	0	0	0	0	0
$S_t^{(1w)}$	−0.8436 (0.00693)	1	0	0	0	0	0.2447 (0.01446)
$S_t^{(2w)}$	−0.8933 (0.00638)	0	1	0	0	0	0.3058 (0.01298)
$S_t^{(3w)}$	−0.9116 (0.00616)	0	0	1	0	0	0.3786 (0.01231)
$S_t^{(1m)}$	−0.9463 (0.00615)	0	0	0	1	0	0.4794 (0.01172)
$S_t^{(2m)}$	−0.9559 (0.00635)	0	0	0	0	1	0.6792 (0.00924)
$S_t^{(3m)}$	−0.9653 (0.00718)	0	0	0	0	0	1
Panel B: GARCH (1,1)							
Parameter	h_{1t}	h_{2t}	h_{3t}	h_{4t}	h_{5t}	h_{6t}	h_{7t}
ω_j	0.0001 (0.00001)	0.0006 (0.00004)	0.0003 (0.00003)	0.0004 (0.00004)	0.0005 (0.00012)	0.0001 (0.00003)	0.0008 (0.00006)
β_j	0.9014 (0.00888)	0.1995 (0.03783)	0.5305 (0.04272)	0.2212 (0.05774)	0.1121 (0.0614)	0.4940 (0.06973)	0.1427 (0.04294)
α_j	0.0886 (0.00888)	0.0795 (0.00773)	0.0240 (0.00340)	0.0300 (0.00411)	0.0144 (0.00298)	0.0091 (0.00159)	0.0650 (0.00833)

The table reports the volatility parameters, estimated by quasi-maximum likelihood, for the Factor-GARCH model. i_t denotes overnight repo rate and $S_t^{(j)}$ the spread between the j-period repo rate $i_t^{(j)}$ and overnight repo rate; F is the factor loadings matrix governing the structure given in equation (16) with overnight repo rate and 3-month spread as the factors, denoted as f_1 and f_7 respectively. The idiosyncratic innovations are assumed to follow an Augmented GARCH (1,1) process $h_{jt}{=}\,\omega_j\sqrt{i_{t-1}}{+}\beta_j h_{kt-1}{+}\alpha_j v_{j,t-1}$ with $j \in \{1, 2, \ldots, 7\}$. Asymptotic standard errors are reported below the parameter estimates.

B Appendix: Table B2

Unconstrained VAR Dynamics and Bias-Correction with GARCH Innovations

Panel A: overnight $i_t^{(m)}$ vs. 1–week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.2754	0.6997	−0.0456	0.0491	−0.0179	0.0445	−0.0285	−0.0368	−0.0182	0.0763
	(0.0044)	(0.0128)	(0.0045)	(0.0168)	(0.0045)	(0.0168)	(0.0045)	(0.0168)	(0.0044)	(0.0119)
$i_t^{(n)}$	0.0301	0.9498	−0.0332	0.0416	−0.0110	−0.0308	−0.0206	−0.0348	−0.0108	0.1192
	(0.0015)	(0.0044)	(0.0015)	(0.0058)	(0.0016)	(0.0058)	(0.0016)	(0.0058)	(0.0015)	(0.0041)

Panel B: overnight $i_t^{(m)}$ vs. 2–week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.3455	0.4422	−0.0186	0.2476	−0.0036	−0.0154	−0.0182	−0.0881	−0.0252	0.1302
	(0.0041)	(0.0144)	(0.0043)	(0.0188)	(0.0043)	(0.0189)	(0.0043)	(0.0189)	(0.0041)	(0.0139)
$i_t^{(n)}$	0.0247	0.8858	−0.0197	0.1562	−0.0196	−0.0212	−0.0139	0.0081	0.0031	−0.0040
	(0.0012)	(0.0041)	(0.0012)	(0.0054)	(0.0012)	(0.0054)	(0.0012)	(0.0054)	(0.0012)	(0.0040)

Panel C: overnight $i_t^{(m)}$ vs. 3–week $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.3719	0.3720	0.0038	0.2369	0.0160	−0.0575	−0.0092	−0.0512	−0.0119	0.1248
	(0.0040)	(0.0143)	(0.0042)	(0.0181)	(0.0042)	(0.0182)	(0.0042)	(0.0181)	(0.0040)	(0.0140)
$i_t^{(n)}$	0.0218	0.8077	−0.0140	0.1948	−0.0187	0.0039	−0.0143	0.0504	−0.0033	−0.0290
	(0.0011)	(0.0040)	(0.0012)	(0.0051)	(0.0012)	(0.0051)	(0.0012)	(0.0051)	(0.0011)	(0.0039)

Panel D: overnight $i_t^{(m)}$ vs. 1-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.4114	0.2689	0.0329	0.1777	0.0383	−0.1292	0.0009	0.0825	0.0053	0.1068
	(0.0039)	(0.0142)	(0.0042)	(0.0182)	(0.0042)	(0.0184)	(0.0042)	(0.0183)	(0.0039)	(0.0142)
$i_t^{(n)}$	0.0240	0.8122	−0.0103	0.1622	−0.0118	−0.0133	−0.0083	0.0787	−0.0041	−0.0297
	(0.0010)	(0.0039)	(0.0011)	(0.0049)	(0.0011)	(0.0050)	(0.0011)	(0.0049)	(0.0011)	(0.0038)

Panel E: overnight $i_t^{(m)}$ vs. 2-month $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.4470	0.1867	0.0565	0.1016	0.0606	−0.1241	0.0228	0.0967	0.0344	0.1136
	(0.0038)	(0.0150)	(0.0042)	(0.0188)	(0.0042)	(0.0190)	(0.0042)	(0.0188)	(0.0039)	(0.0149)
$i_t^{(n)}$	0.0336	0.7628	−0.0259	0.1750	−0.0097	0.0664	−0.0074	0.0621	−0.0126	−0.0449
	(0.0010)	(0.0038)	(0.0011)	(0.0048)	(0.0011)	(0.0048)	(0.0011)	(0.0048)	(0.0010)	(0.0038)

(continued)

B Appendix: Table B2 *(continued)*

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
Panel F: overnight $i_t^{(m)}$ vs. 3-month $i_t^{(n)}$										
$i_t^{(m)}$	0.4692	0.1991	0.0739	0.0364	0.0744	−0.0822	0.0385	−0.0270	0.0624	0.1512
	(0.0038)	(0.0136)	(0.0042)	(0.0167)	(0.0042)	(0.0170)	(0.0042)	(0.0168)	(0.0038)	(0.0137)
$i_t^{(n)}$	0.0268	0.7143	−0.0158	0.2416	−0.0133	0.0174	−0.0022	0.0712	−0.0184	−0.0224
	(0.0011)	(0.0038)	(0.0012)	(0.0047)	(0.0012)	(0.0047)	(0.0012)	(0.0047)	(0.0011)	(0.0038)
Panel G: 1-week $i_t^{(m)}$ vs. 2-week $i_t^{(n)}$										
$i_t^{(m)}$	0.6555	0.3337	−0.0672	0.0616	−0.1200	0.0931	−0.1038	0.0540	0.0168	0.0743
	(0.0054)	(0.0061)	(0.0064)	(0.0074)	(0.0064)	(0.0074)	(0.0063)	(0.0074)	(0.0053)	(0.0064)
$i_t^{(n)}$	0.0500	0.8743	−0.0608	0.1650	−0.1000	0.0472	−0.0226	0.0309	0.0272	−0.0118
	(0.0048)	(0.0054)	(0.0057)	(0.0066)	(0.0057)	(0.0066)	(0.0057)	(0.0066)	(0.0048)	(0.0057)
Panel H: 1-week $i_t^{(m)}$ vs. 3-week $i_t^{(n)}$										
$i_t^{(m)}$	0.7871	0.1764	−0.0575	0.0738	−0.1066	0.1144	−0.0790	0.0231	0.0840	−0.0175
	(0.0048)	(0.0055)	(0.0060)	(0.0067)	(0.0060)	(0.0068)	(0.0060)	(0.0067)	(0.0047)	(0.0058)
$i_t^{(n)}$	0.0672	0.7766	−0.0786	0.2205	−0.0517	0.0352	−0.0513	0.0830	0.0124	−0.0142
	(0.0042)	(0.0048)	(0.0052)	(0.0059)	(0.0052)	(0.0059)	(0.0052)	(0.0059)	(0.0041)	(0.0051)
Panel I: 1-month $i_t^{(m)}$ vs. 2-month $i_t^{(n)}$										
$i_t^{(m)}$	0.6449	0.2023	0.1449	−0.0272	−0.0467	0.0257	0.0751	−0.0102	−0.0098	0.0004
	(0.0054)	(0.0055)	(0.0061)	(0.0062)	(0.0061)	(0.0062)	(0.0061)	(0.0062)	(0.0053)	(0.0056)
$i_t^{(n)}$	0.1784	0.6350	−0.0067	0.1523	−0.1542	0.1733	−0.0631	0.1177	−0.0470	0.0139
	(0.0053)	(0.0054)	(0.0059)	(0.0060)	(0.0059)	(0.0060)	(0.0059)	(0.0060)	(0.0052)	(0.0055)
Panel J: 1-month $i_t^{(m)}$ vs. 3-month $i_t^{(n)}$										
$i_t^{(m)}$	0.7057	0.1360	0.1508	−0.0221	−0.0258	−0.0107	0.0687	0.0120	−0.0046	−0.0107
	(0.0046)	(0.0043)	(0.0055)	(0.0049)	(0.0055)	(0.0050)	(0.0055)	(0.0049)	(0.0045)	(0.0044)
$i_t^{(n)}$	0.1337	0.6479	−0.0023	0.2213	−0.1025	0.0650	0.0203	0.0745	−0.0923	0.0338
	(0.0049)	(0.0046)	(0.0059)	(0.0053)	(0.0059)	(0.0054)	(0.0059)	(0.0053)	(0.0048)	(0.0047)

The table presents the unconstrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes GARCH innovations. $i_t^{(n)}$ is the n-period (long-term) rate and $i_t^{(m)}$ is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-term repo rates such that $k = n/m$ is an integer. Standard errors are reported in parenthesis.

B Appendix: Table B3

Constrained VAR Dynamics and Bias-Correction with GARCH Innovations

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
Panel A: overnight $i_t^{(m)}$ vs. 1–week $i_t^{(n)}$										
$i_t^{(m)}$	0.3251	1.7610	−0.4489	1.2170	−0.6072	−0.4146	0.4183	−2.3480	0.2903	0.8060
	(0.2008)	(0.7283)	(0.2135)	(0.9923)	(0.2034)	(0.9415)	(0.1991)	(0.9386)	(0.1912)	(0.5594)
$i_t^{(n)}$	−0.0542	1.1950	−0.1091	0.2705	−0.0905	−0.1230	0.0480	−0.2598	0.0325	0.0903
	(0.0407)	(0.1287)	(0.0421)	(0.1788)	(0.0430)	(0.1801)	(0.0425)	(0.1865)	(0.0427)	(0.1129)
Panel B: overnight $i_t^{(m)}$ vs. 2–week $i_t^{(n)}$										
	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.3725	4.2410	−0.5280	−0.1566	−0.1095	−3.3550	0.9648	−3.5840	0.2606	2.8940
	(0.2266)	(1.1830)	(0.2552)	(1.4590)	(0.2489)	(1.4720)	(0.2654)	(1.5570)	(0.2092)	(1.0480)
$i_t^{(n)}$	−0.0026	0.8769	−0.0323	0.3286	−0.0610	0.0505	−0.0046	−0.1853	0.0024	0.0273
	(0.0300)	(0.1282)	(0.0340)	(0.1724)	(0.0340)	(0.1791)	(0.0323)	(0.1742)	(0.0321)	(0.1151)
Panel C: overnight $i_t^{(m)}$ vs. 3–week $i_t^{(n)}$										
	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	1.0480	1.5620	−0.1600	1.5160	−0.7218	−1.3830	0.6094	−2.6810	0.5887	0.6252
	(0.2449)	(1.1850)	(0.2659)	(1.4880)	(0.2751)	(1.5010)	(0.2651)	(1.5000)	(0.2298)	(1.0660)
$i_t^{(n)}$	−0.0036	0.9236	−0.0326	0.1892	−0.0622	0.0377	−0.0116	−0.0574	0.0080	0.0085
	(0.0280)	(0.1101)	(0.0301)	(0.1367)	(0.0307)	(0.1476)	(0.0299)	(0.1424)	(0.0298)	(0.1042)
Panel D: overnight $i_t^{(m)}$ vs. 1-month $i_t^{(n)}$										
	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.9087	2.6450	−0.1655	0.3231	−0.5085	−1.2960	0.5701	−2.7540	0.5065	0.7747
	(0.1952)	(1.1030)	(0.2189)	(1.3950)	(0.2154)	(1.4340)	(0.2073)	(1.5210)	(0.1961)	(1.0260)
$i_t^{(n)}$	−0.0137	0.8960	−0.0093	0.1264	−0.0336	0.0706	−0.0198	−0.0099	−0.0029	−0.0043
	(0.0281)	(0.1044)	(0.0329)	(0.1266)	(0.0317)	(0.1290)	(0.0325)	(0.1352)	(0.0300)	(0.1052)
Panel E: overnight $i_t^{(m)}$ vs. 2-month $i_t^{(n)}$										
	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.9254	1.1460	0.1275	0.2946	−0.4429	0.9320	0.3702	−2.5520	0.2059	−0.0044
	(0.1606)	(0.9667)	(0.1857)	(1.1410)	(0.1850)	(1.1700)	(0.1725)	(1.2540)	(0.1631)	(0.8935)
$i_t^{(n)}$	−0.0339	0.9697	−0.0032	0.0165	−0.0012	0.0229	−0.0090	0.0409	−0.0033	0.0001
	(0.0300)	(0.1347)	(0.0355)	(0.1697)	(0.0365)	(0.1665)	(0.0358)	(0.1675)	(0.0308)	(0.1296)

(continued)

B Appendix: Table B3 *(continued)*

Panel F: overnight $i_t^{(m)}$ *vs.* 3-*month* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.8504	1.3090	0.1023	−0.2667	−0.3512	1.1180	0.3024	−2.2630	0.1693	0.0312
	(0.1403)	(0.9041)	(0.1688)	(1.1020)	(0.1670)	(1.0980)	(0.1566)	(1.1480)	(0.1445)	(0.7819)
$i_t^{(n)}$	−0.0186	0.9923	−0.0014	0.0093	−0.0007	0.0076	−0.0034	0.0162	−0.0012	−0.0002
	(0.0307)	(0.1233)	(0.0378)	(0.1327)	(0.0371)	(0.1366)	(0.0369)	(0.1329)	(0.0319)	(0.1137)

Panel G: 1-week $i_t^{(m)}$ *vs.* 2-*week* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.9011	0.8882	−0.1610	−0.2609	−0.3622	0.4408	−0.0055	−0.4077	0.5327	−0.5652
	(0.2267)	(0.2760)	(0.2827)	(0.3567)	(0.2752)	(0.3508)	(0.2945)	(0.3640)	(0.2282)	(0.3030)
$i_t^{(n)}$	0.0226	1.2010	−0.1065	−0.0195	−0.2151	0.2979	−0.0620	−0.1045	0.2194	−0.2328
	(0.1540)	(0.1775)	(0.1839)	(0.2253)	(0.1815)	(0.2253)	(0.1785)	(0.2116)	(0.1470)	(0.1995)

Panel H: 1-week $i_t^{(m)}$ *vs.* 3-*week* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	1.4440	−0.4138	−0.8129	0.7940	−0.1750	0.4348	−0.4230	0.0078	0.5080	−0.3659
	(0.2134)	(0.2671)	(0.3113)	(0.3196)	(0.3124)	(0.3207)	(0.3037)	(0.3227)	(0.2319)	(0.2912)
$i_t^{(n)}$	−0.2060	1.2570	−0.2410	0.1966	0.0251	0.0161	−0.0307	−0.0559	0.1335	−0.0961
	(0.1840)	(0.1944)	(0.2428)	(0.2277)	(0.2354)	(0.2305)	(0.2398)	(0.2304)	(0.1784)	(0.1962)

Panel I: 1-month $i_t^{(m)}$ *vs.* 2-*month* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.6018	0.2211	−0.0800	0.1783	−0.2035	0.0155	0.2014	0.1004	−0.0995	0.0638
	(0.1682)	(0.1672)	(0.1766)	(0.1894)	(0.1694)	(0.1823)	(0.1722)	(0.1891)	(0.1535)	(0.1722)
$i_t^{(n)}$	−0.1992	1.1110	−0.0400	0.0892	−0.1018	0.0077	0.1007	0.0502	−0.0498	0.0319
	(0.1764)	(0.1637)	(0.1897)	(0.1801)	(0.1851)	(0.1704)	(0.1915)	(0.1840)	(0.1558)	(0.1647)

Panel J: 1-month $i_t^{(m)}$ *vs.* 3-*month* $i_t^{(n)}$

	$i_{t-1}^{(m)}$	$i_{t-1}^{(n)}$	$i_{t-2}^{(m)}$	$i_{t-2}^{(n)}$	$i_{t-3}^{(m)}$	$i_{t-3}^{(n)}$	$i_{t-4}^{(m)}$	$i_{t-4}^{(n)}$	$i_{t-5}^{(m)}$	$i_{t-5}^{(n)}$
$i_t^{(m)}$	0.7011	0.2237	0.0444	0.0875	−0.2645	0.0844	0.1491	−0.0607	−0.0083	0.0431
	(0.1649)	(0.1458)	(0.1933)	(0.1680)	(0.1896)	(0.1686)	(0.1885)	(0.1718)	(0.1522)	(0.1574)
$i_t^{(n)}$	−0.0997	1.0750	0.0148	0.0292	−0.0882	0.0281	0.0497	−0.0202	−0.0028	0.0144
	(0.1701)	(0.1377)	(0.1994)	(0.1690)	(0.2064)	(0.1697)	(0.2123)	(0.1659)	(0.1646)	(0.1461)

The table presents the constrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes GARCH innovations. $i_t^{(n)}$ is the n-period (long-term) rate and $i_t^{(m)}$ is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-term repo rates such that $k = n/m$ is an integer. Standard errors are reported in parenthesis.

References

Ahn, D.H., R.F. Dittmar, and A.R. Gallant (2002). "Quadratic Term Structure Models: Theory and Evidence," *Review of Financial Studies*, **15**, 243-288.

Ang, A., and G. Bekaert (2002). "Regime Switches in Interest Rates," *Journal of Business and Economic Statistics*, **20**, 163-182.

Bekaert, G., and R.J. Hodrick (2001). "Expectations Hypotheses Tests," *Journal of Finance*, **56**, 1357-1394.

Bekaert, G., R.J. Hodrick, and D.A. Marshall (1997). "On Biases in Tests of the Expectations Hypothesis of the Term Structure of Interest Rates," *Journal of Financial Economics*, **44**, 309-348.

Bansal, R., and H. Zhou (2002). "Term Structure of Interest Rates with Regime Shifts," *Journal of Finance*, **57**, 1997–2043.

Campbell, J.Y., and R.J. Shiller (1987). "Cointegration and Tests of Present Value Models," *Journal of Political Economy*, **95**, 1062-1088.

Campbell, J.Y., and R.J. Shiller (1991). "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies*, **58**, 495-514.

Clarida, R.H., L. Sarno, M.P. Taylor, and G. Valente (2006). "The Role of Asymmetries and Regime Shifts in the Term Structure of Interest Rates," *Journal of Business*, **79**, 1193-1225.

Collin-Dufresne, P., and R.S. Goldstein (2002). "Do Bonds Span the Fixed Income Markets? Theory and Evidence for Unspanned Stochastic Volatility," *Journal of Finance*, **57**, 1685-1730.

Collin-Dufresne, P., R.S. Goldstein, and C.S. Jones (2007). "Identification of Maximal Affine Term Structure Models," *Journal of Finance*, forthcoming.

Dai, Q., and K.J. Singleton (2000). "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, **55**, 1943-1978.

Duffie, D., and K.J. Singleton (1999). "Modeling Term Structures of Defaultable Bonds," *Review of Financial Studies*, **12**, 687-720.

Fama, E.J. (1984). "The Information in the Term Structure," *Journal of Financial Economics*, **13**, 509-528.

Fama, E.J., and R.R. Bliss (1987). "The information in Long-Maturity Forward Rates," *American Economic Review*, **77**, 680-692.

Fisher, I. (1896). "Appreciation and Interests," *Publications of the American Economic Association*, **11**, 21-29.

Fleming, J., C. Kirby, and B. Ostdiek (2001). "The Economic Value of Volatility Timing," *Journal of Finance*, **56**, 329-352.

Fleming, J., C. Kirby, and B. Ostdiek (2003). "The Economic Value of Volatility Timing Using 'Realized' Volatility," *Journal of Financial Economics*, **67**, 473-509.

Frankel, J.A., and K.A. Froot (1987). "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations," *American Economic Review*, **77**, 133-153.

Froot, K.A. (1989). "New Hope for the Expectations Hypothesis of the Term structure of Interest Rates," *Journal of Finance*, **44**, 283-305.

Gray, S. F. (1996). "Modeling the Conditional Distribution of Interest Rates as a Regime Switching Process," *Journal of Financial Economics*, **42**, 27-62.

Han, Y. (2006). "Asset Allocation with a High Dimensional Latent Factor Stochastic Volatility Model," *Review of Financial Studies*, **19**, 237-271.

Hansen, L.P. (1982). "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, **50**, 1029-1054.

Hansen, L.P., and T.J. Sargent (1980). "Formulating and Estimating Dynamic Linear Rational Expectations Models," *Journal of Economic Dynamics and Control,* **2**, 7-46.

Hlawitschka, W. (1994). "The Empirical Nature of Taylor-Series Approximations to Expected Utility," *American Economic Review*, 1994, **84**, 713-719.

Jagannathan, R., A. Kaplan, and S.G. Sun (2003). "An Evaluation of Multi-Factor CIR Models Using LIBOR, Swap Rates, and Cap and Swaption Prices," *Journal of Econometrics*, **116**, 113-146.

Leamer, E.E. (1978). *Specification Searches: Ad Hoc Inference with Nonexperimental Data*. New York, NY: John Wiley and Sons.

Leitch, G., and J. Tanner (1991). "Economic Forecast Evaluation: Profit Versus the Conventional Error Measures," *American Economic Review*, **81**, 580-590.

Lo, A.W. (2002). "The Statistics of Sharpe Ratios," *Financial Analyst Journal*, **58**, 36-52.

Longstaff, F. (2000a). "The Term Structure of Very Short Term Rates: New Evidence for the Expectations Hypothesis," *Journal of Financial Economics*, **58**, 397-415.

Longstaff, F. (2000b). "Arbitrage and the Expectations Hypothesis," *Journal of Finance*, **55**, 989-994.

Marquering, W., and M. Verbeek (2004). "The Economic Value of Predicting Stock Index Returns and Volatility," *Journal of Financial and Quantitative Analysis*, **39**, 407-429.

Melino, A. (2001). "Estimation of a Rational Expectations Model of the Term Structure," *Journal of Empirical Finance*, **8**, 639-668.

Mishkin, F.S. (1982). "Does Anticipated Monetary Policy Matter? An Econometric Investigation," *Journal of Political Economy*, **90**, 22-51.

Modigliani, F., and L. Modigliani (1997). "Risk-Adjusted Performance," *Journal of Portfolio Management,* **23**, 45-54.

Newey, W. K., and D.L. McFadden (1994). "Large Sample Estimation and Hypothesis Testing," in Engle, R.F., and D.L. McFadden (eds.), *Handbook of Econometrics,* **4,** Amsterdam: Elsevier.

Pagan, A. (1984). "Econometric Issues in the Analysis of the Regressions With Generated Regressors," *International Economic Review,* **25**, 221-247.

Roll, R. (1970). *The Behavior of Interest Rates: An Application of the Efficient Market Model to U.S. Treasury Bills.* New York, NY: Basic Books.

Sargent, T.J. (1977). "The Demand for Money during Hyperinflation under Rational Expectations," *International Economic Review,* **18**, 59-82.

Sarno, L., D.L. Thornton, and G. Valente (2007). "The Empirical Failure of the Expectations Hypothesis of the Term Structure of Bond Yields," *Journal of Financial and Quantitative Analysis,* **42**, 81-100.

Shiller, R.J. (1979). "The Volatility of Long-Term Interest Rates and Expectation Models of the Term Structure," *Journal of Political Economy*, **87**, 1190-1219.

Shiller, R., J.Y. Campbell, and K.L. Schoenholtz (1983). "Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates," *Brookings Papers on Economic Activity*, **1**, 173-217.

Sims, C.A. (1980). "Macroeconomics and Reality," *Econometrica*, **48**, 1-48

Stambaugh, R.F. (1988). "The Information in Forward Rates: Implications for Models of the Term Structure," *Journal of Financial Economics*, **21**, 41–70.

Thornton, D.L. (2005). "Tests of the Expectations Hypothesis: Resolving Some Anomalies When the Short-Term Rate is the Federal Funds Rate," *Journal of Banking and Finance*, **29**, 2541-2556.

Thornton, D.L. (2006). "Tests of the Expectations Hypothesis: Resolving the Campbell-Shiller Paradox," *Journal of Money, Credit, and Banking*, **38**, 511-542.

Wallis, K.F. (1980). "Econometric Implications of the Rational Expectations Hypothesis," *Econometrica*, **48**, 49-72.

West, K.D., H.J. Edison, and D. Cho (1993). "A Utility-Based Comparison of Some Models of Exchange Rate Volatility," *Journal of International Economics*, **35**, 23-45.