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Volatility, Growth, and Welfare

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Volatility, Growth, and Welfare*

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Abstract

This paper constructs an endogenous growth model to explain the stylized fact that the growth rate of GDP is negatively related to its standard deviation. We also show that the welfare gain from further stabilizing the U.S. economy can be several orders larger than that calculated by Lucas (1987) because policies designed to reduce fluctuations can generate permanently higher rates of growth.

Keywords: Endogenous Growth, Welfare Cost of Business Cycle, Stabilization Policy, Sunspots, Imperfect Competition, Coordination Failures.

JEL codes: E12, E32, O40.

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1 Introduction

Business cycles and growth are undoubtedly the two most important issues in macroeconomics, yet traditionally they have been treated as separate areas of research, as if fluctuations and growth are unrelated. This dichotomy is illustrated most clearly by the independent development of the neoclassical growth model (Solow 1956) and the Keynesian IS-LM model (Hicks 1937). The modern real business cycle (RBC) theory, developed by Kydland and Prescott (1982) and Long and Plosser (1983), intends to end this dichotomy by adopting a common general-equilibrium framework and hypothesizing a common driving force for both growth and fluctuations. Nonetheless, RBC theory maintains a fundamental assumption that the mean growth rate of technology is independent of the volatility of shocks to the economy.[1] Based on this fundamental assumption, temporary fluctuations may have permanent effects on the level of output, but they do not affect the mean growth rate of output (i.e., an *ad hoc* distinction between a level effect and a growth effect). Thus, long-run growth and short-run fluctuations are still viewed as unrelated and determined by fundamentally different forces. Therefore, by merely postulating a common driving force for growth and business cycles, the RBC theory has not succeeded in ending the dichotomy.[2] This is further highlighted by the popularity of the Hodrick-Prescott filter widely used by macroeconomists to decompose aggregate output into a trend (growth) component and a cyclical component, of which only the cyclical component is analyzed seriously by RBC models (see, Hodrick and Prescott 1997). The underlying assumption behind this practice is that growth and fluctuations can be understood in isolation.

One of the most far-reaching implications of this classical dichotomy between growth and fluctuations is that the welfare gains of eliminating fluctuations are trivial compared to that of stimulating long-run growth (Lucas, 1987). This influential calculation made by Lucas has survived numerous robustness analyses and has been a major challenge to the old Keynesian belief that stabilization policies are desirable (see Lucas 2003, and the references therein). In fact, the policy implication of the Lucas calculation is even more robust than its welfare implication because even if one can find models in which the welfare costs of fluctuations are large, the gains from stabilization policy may still be small (see, e.g., Kiley 2003, and Barlevy 2004a). The reason is that general-equilibrium business-cycle models typically imply volatile consumption as optimal allocation under exogenous

[1] For example, RBC theory assumes that technology shocks can follow a random walk with a constant drift, where the drift is independent of the innovations to technology.

[2] The RBC literature views business cycles as temporary deviations around a long-run steady state. Although the requirement for balanced steady-state growth places restrictions on the structure of RBC models, the steady state itself cannot be affected by business cycle volatility. For a clear presentation of the underling dichotomy in the RBC theory, see King, Plosser, and Rebelo (1988).

shocks.

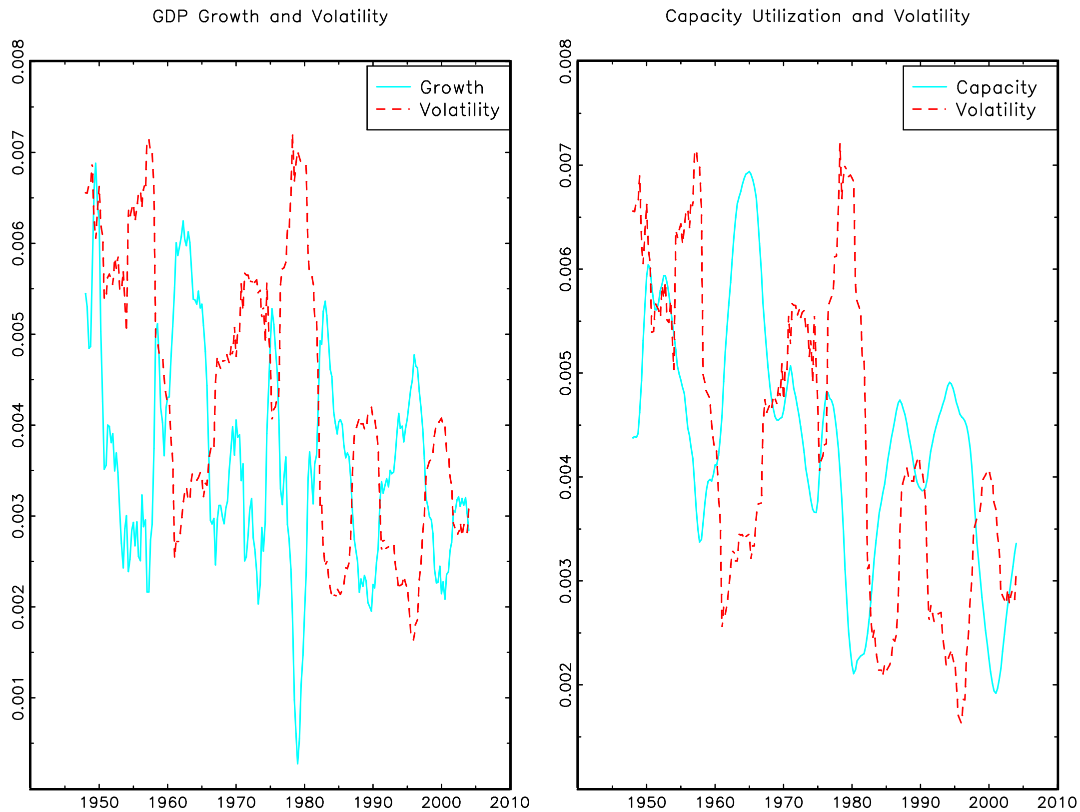

Figure 1. U.S. Time Series (1948:1 - 2007:4).

Yet there is a growing awareness that the dichotomy between volatility and growth is hard to square with the facts. For example, Ramey and Ramey (1995) present evidence of a negative relationship between business cycle volatility and long-run growth: countries with higher output volatility tend to have lower output growth. This negative relationship has also been validated by other empirical studies.[3] This paper presents additional evidence showing that the same negative relationship between the business cycle and growth also exists in the U.S. time series. In particular, for the post-war period, the rate of capacity utilization is positively correlated with the mean growth rate and negatively with its standard deviation. This suggests that periods of higher capacity utilization are associated with periods of faster growth and smaller fluctuations. Figure 1 shows movements of the U.S. manufacturing sector's capacity utilization rate, the real GDP growth rate, as well as the standard deviation of GDP growth based on a 16-quarter rolling-window moving average.[4] It is clear from the graph that the average growth rate and the capacity utilization rate

[3] See, e.g., Aghion, Angeletos, Banerjee, and Manova (2005), Easterly, Islam, and Stiglitz (2000), Hnatkovska and Loayza (2004), Kroft and Lloyd-Ellis (2002), Martin and Carol Ann (2000), and Mobarak (2005), among others. Barlevy (2004a) documents a negative relationship between growth and volatility using time series data from the U.S.

[4] The definition of the variables are as follows: the average 4-year capacity utilization rate is $\bar{u}_t = \frac{1}{16}\sum_{j=0}^{15} u_{t+j}$,

are both negatively related to the standard deviation of GDP growth. This has important welfare and policy implications, suggesting that stabilization policies may be able to increase the average growth rate by raising the capacity utilization rate.

This paper constructs an endogenous growth model with variable capacity utilization and imperfect competition. Because of endogenous growth, fluctuations in capacity utilization can affect the mean growth rate. Because of imperfect competition, firms opt to charge higher prices with higher average markup during periods of larger volatility. Higher markup leads to lower capacity utilization and lower rate of return to capital. Consequently, low capacity utilization is associated with low growth and high volatility. Time varying markups are a crucial link among capacity utilization, mean growth, and volatility in our model. To generate a time varying markup, we use the mechanism of Svensson (1986), Ireland (1996), Carlstrom and Fuerst (1998a, 1998b), Adão, Correia, and Teles (2003), and Wang and Wen (2007), among others. In these papers firms set prices one-period in advance and hence may face extrinsic uncertainty regarding other firms' price-setting behavior and the level of aggregate demand (even in the absence of fundamental shocks). Due to strategic complementarity among firms' actions, which arises from imperfect substitutability of firms' output in the goods market, extrinsic uncertainty can be self-fulfilling and, consequently, the economy can suffer from coordination failures and endogenous fluctuations. It is shown that a stochastic growth path driven by firms' speculations about aggregate demand yields a strictly lower mean growth rate than a constant equilibrium growth path. Under reasonable parameter values, the model predicts a negative relationship between volatility and growth that is quantitatively similar to the U.S. data.

Because volatility and growth is negatively related in the model, the welfare cost of business cycles can be hundreds of times larger than that calculated by Lucas. Since expectations-driven fluctuations are inefficient, the welfare gain from eliminating such fluctuations by stabilization policies is equally large.

Our approach are related to the work of Francois and Lloyd-Ellis (2003) and Barlevy (2004a). Francois and Lloyd-Ellis (2003) show growth and business cycles can be intimately linked via a Shumpeterian process of creative destruction. In particular, they show volatility and growth can be negatively related across cycling economies. However, our approach differs from theirs in at least three aspects: 1) the mechanisms for generating endogenous growth cycles are different; 2) growth cycles in their model are deterministic whereas in our model growth cycles are stochastic; and 3) we conduct quantitative analysis on the welfare cost of business cycles and study optimal

the average 4-year GDP growth rate is $\bar{g}_t = \frac{1}{16}\sum_{j=0}^{15} g_{t+j}$, and the standard deviation of GDP growth is $\sigma_t = \left[\frac{1}{16}\sum_{j=0}^{15}(g_{t+j} - \bar{g}_t)^2\right]^{\frac{1}{2}}$. A rolling window is used in the estimation. The sample rage is 1948:1 - 2007:4. The capacity series u is based on the U.S. manufacturing sector and is normalized so that it has similar magnitude as the other series in the graph. The results are robust for window sizes longer than 3 years.

stabilization policies, whereas such analyses are not conducted by Francois and Lloyd-Ellis.[5] Using an AK endogenous growth model featuring adjustment costs in investment, Barlevy (2004a) shows that volatility (due to technology shocks) can reduce long run growth. Consequently, the welfare cost of fluctuations can be potentially large. However, the policy implication of Barlevy's model fundamentally diverges from ours. In Barlevy's model, little scope exists for stabilization policies despite the potentially large welfare gains from eliminating fluctuations, because fluctuations in his model are optimal responses to exogenous technology shocks. Thus, there is no gain from stabilizing the economy unless the source of fluctuations lies in government policy itself. In our model, endogenous fluctuations are caused by coordination failures and self-fulfilling expectations, and are intrinsically inefficient regardless of fundamental shocks.[6]

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 calibrates the model and examines its dynamic properties. Section 4 discusses the welfare cost of business cycles. Section 5 studies optimal stabilization policies; and Section 6 concludes the paper.

2 The Model

2.1 Firms

There is a final good in the economy. The final good producers behave competitively and households buy the final good for both consumption and investment. The final good is produced by using intermediate goods according to the Dixit-Stiglitz technology:

$$Y = \left(\int_0^1 y(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where $\epsilon > 1$ measures the elasticity of substitution among intermediate goods $y(i)$. The price of the final good is normalized to one and the price of intermediate good i is denoted $p(i)$. Profit maximization in the final good sector yields the demand function for intermediate goods, $y(i) = p(i)^{-\epsilon} Y$. Substituting this into the production function yields the aggregate price index, $\int_0^1 p(i)^{1-\epsilon} di = 1$.

The economy has a continuum of monopolistic intermediate good producers of measure one,

[5] Stabilization policy is always an important issue in macroeconomics. Ironically, the sunspots literature has largely bypassed this question with only a few exceptions. For example, Shleifer (1986) shows that while an informed stabilization policy can sometimes raise welfare, stabilization policy can stop technological progress and harm the economy if large booms are necessary to cover fixed costs of innovation. Christiano and Harrison (1999) show that stabilizing sunspots fluctuations is desirable in an economy featuring production externalities. They identify an automatic stabilizer income tax-subsidy schedule with two properties: (i) it specifies the tax rate to be an increasing function of aggregate employment, and (ii) earnings are subsidized when aggregate employment is at its efficient level.

[6] For comprehensive literature reviews on the issue of welfare cost of business cycles and the benefits of stabilization, see Lucas (2003) and Barlevy (2004b). For previous works that evaluate welfare cost by linking endogenous growth to exogenous fluctuations, see Blackburn and Pelloni (2005), de Hek (1999), Epaulard and Pommeret (2003), Jones, Manuelli, Siu, and Stacchetti (2005), and Krebs (2003), among others.

each producing a single differentiated good $y(i)$. Intermediate goods are produced by using capital (k). The production function for intermediate goods is identical across firms and is given by:

$$y(i) = Au(i)k(i), \tag{2}$$

where A denotes the level of technology common to all firms and $u(i)$ denotes the rate of capacity utilization for firm i. Intermediate good producers are assumed to be price takers in the input market. Let r denote the market interest rate, and let $\delta(i)$ denote the rate of capital depreciation for firm i. Following Greenwood et al. (1988), the rate of capital depreciation is assumed to depend on its usage rate:

$$\delta(i) = \frac{\alpha}{1+\theta} u(i)^{1+\theta}, \quad \theta > 0. \tag{3}$$

Hence the user's cost of capital facing firm i is $r + \delta(i)$.[7]

The cost function of an intermediate firm can be found by minimizing $[r + \delta(i)]\, k(i)$ subject to $Au(i)k(i) \geqslant y(i)$. Denoting ϕ as the Lagrangian multiplier for the above constraint, which is also the marginal cost, cost minimization yields the relationship, $r + \delta(i) = \phi Au(i)$ and $\alpha u(i)^{\theta} = \phi A$.These first-order conditions imply $\delta(i) = \delta = \frac{1}{1+\theta}\alpha^{-\frac{1}{\theta}} (\phi A)^{\frac{\theta+1}{\theta}}$ and

$$r = \theta\delta = \frac{\theta}{1+\theta} (\alpha)^{-\frac{1}{\theta}} (\phi A)^{\frac{\theta+1}{\theta}}. \tag{4}$$

Since the production technology has constant returns to scale and firms face the same market interest rate, the marginal cost ϕ is the same across all firms. Consequently, the optimal rates of capital utilization and depreciation are also the same across firms. Thus, firms' output differ from each other only if their capital stocks differ in the absence of idiosyncratic shocks.

Each intermediate good firm faces a downward sloping demand curve, $y(i) = p(i)^{-\epsilon} Y$, and sets prices to maximize profits. Since firms have no influence on the aggregate quantity Y, there exists a strategic complementarity among firms' actions, in the language of Cooper and John (1988). Namely, every firm will opt to produce more if they all anticipate that the other firms will produce more. This strategic complementarity, however, is only a necessary but not sufficient condition for multiple Nash equilibria in this model.

Another key condition for multiple equilibria is one-period information lag as explained by Svensson (1986), Ireland (1996), Carlstrom and Fuerst (1998a, 1998b), Adão, Correia, and Teles (2003), and Wang and Wen (2007), among others. That is, intermediate good firms each must choose a price one period in advance without knowing the aggregate economic conditions (such

[7] Since the capital stock is fixed in period t, in order for sunspots shocks to affect aggregate output, a second production factor (such as capacity utilization or hours worked) that can adjust instantaneously is needed. The importance of capacity utilization in understanding business cycles and growth has been emphasized by Greenwood et al. (1988), King and Rebelo (1999), Wen (1998), and Chatterjee (2004), among others.

as aggregate demand or aggregate marginal cost) that may prevail in period t. These aggregate economic conditions depend crucially on the actions of the other firms over which an individual firm has no influence. Thus, each individual firm must form expectations for the level of aggregate demand (Y) when setting prices. Such expectations can become self-fulfilling as explained below.[8]

Without loss of generality, assume there are no fundamental shocks in the economy; then the only type of uncertainty, if any, is extrinsic uncertainty in the language of Cass and Shell (1983) (i.e., due to sunspots). An intermediate good firm's objective function is then to solve

$$\max_{p_t(i)} E_{t-1}\left[\left(p_t(i)-\phi_t\right) y_t(i)\right] \tag{5}$$

subject to the demand function $y_t(i) = p_t(i)^{-\epsilon} Y_t$.[9] The optimal price is given by $p_t(i) = \frac{\epsilon}{\epsilon-1}\frac{E_{t-1}(\phi_t Y_t)}{E_{t-1}Y_t}$. Assuming firms are rational and have the same information sets, then they all set the same prices. Thus, $p(i) = p = 1$ and

$$E_{t-1}(\phi_t Y_t) = \frac{\epsilon-1}{\epsilon} E_{t-1}(Y_t). \tag{6}$$

In the limiting case where $\epsilon \to \infty$, the model converges to a perfectly competitive economy. Notice that without the information lag, equation (6) implies the marginal cost is constant, $\phi_t = \frac{\epsilon-1}{\epsilon}$. In this case, the equilibrium is unique, as in standard Dixit-Stiglitz models.

2.2 Households

There is a continuum of infinitely lived identical households of measure one. The representative agent chooses paths of consumption ($\{C_t\}_{t=0}^{\infty}$) and capital holdings ($\{K_t\}_{t=1}^{\infty}$) to solve

$$\max E_0 \sum_{t=0}^{\infty} \log(C_t) \tag{7}$$

subject to $K_0 > 0$ given and the budget constraint,

$$C_t + K_{t+1} = (1+r_t)K_t + D_t, \tag{8}$$

where D_t denotes real profits distributed from intermediate good firms. The first-order condition is given by $\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}(1+r_{t+1})$, plus the transversality condition, $\lim_{T\to\infty} \beta^T \frac{K_{T+1}}{C_T} = 0$.

[8] See Wang and Wen (2007) for a more general approach of incomplete information.

[9] Modifying the firm's profit function to include the marginal utility of income, $Eu'(c)\left[(p(i)-\phi)y(i)\right]$, where $u'(c)$ is household's marginal utility, has no effect on the existence of sunspots equilibria in the model, although the particular distribution of the sunspots process may differ.

2.3 Symmetric Rational Expectations Equilibrium

The economy's technology is symmetric with respect to all the intermediate inputs. This paper restricts attention to symmetric equilibria where $y(i) = Y$ and $k(i) = K$ for all $i \in [0,1]$. Notice that in the absence of extrinsic uncertainty, Equation (6) implies the marginal cost is constant, $\phi = \frac{\epsilon-1}{\epsilon}$. Given the value of ϕ, the value of interest rate is then fully determined, as is the balanced growth rate. However, as will be shown shortly, constant marginal cost is not the only possible equilibrium in this model. There are also multiple Nash-sunspots equilibria that feature stochastic marginal cost and stochastic interest rate.

The equilibrium conditions in this economy can be summarized by the following equations:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left(1 + \frac{\theta}{1+\theta} \alpha^{-\frac{1}{\theta}} \left(\phi_{t+1} A\right)^{\frac{\theta+1}{\theta}}\right), \tag{9}$$

$$C_t + K_{t+1} = Y_t + (1-\delta_t)k_t = \left[1 + \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t{}^{\frac{1}{\theta}} \left(1 - \frac{\phi_t}{1+\theta}\right)\right] K_t, \tag{10}$$

$$E_{t-1}\phi_t^{\frac{1+\theta}{\theta}} = \frac{\epsilon-1}{\epsilon} E_{t-1}\phi_t^{\frac{1}{\theta}}; \tag{11}$$

where the last equation is derived from Equation (6).[10] These three equations, in conjunction with a transversality condition, fully determine the equilibrium paths of the marginal cost, consumption, and the capital stock. In particular, given any path of the marginal cost (ϕ) as specified by Equation (11), Equations (9) and (10) fully determine the paths of consumption and the capital stock.

Notice that Equation (11) implies $E\phi^{\frac{1}{\theta}}(E\phi - \frac{\epsilon-1}{\epsilon}) = -cov(\phi^{\frac{1}{\theta}}, \phi) \leq 0$, hence any stochastic process $\{\phi_t\}_{t=0}^{\infty}$ satisfying $E\phi \leq \frac{\epsilon-1}{\epsilon}$ and $cov(\phi^{\frac{1}{\theta}}, \phi) = E\phi^{\frac{1}{\theta}}(\frac{\epsilon-1}{\epsilon} - E\phi)$ constitutes a rational expectations equilibrium path for the marginal cost.[11] The fundamental equilibrium (in the absence of extrinsic uncertainty or sunspots) corresponds to the case where $cov(\phi^{\frac{1}{\theta}}, \phi) = 0$ and $\phi = \frac{\epsilon-1}{\epsilon}$, and it is clearly unique.[12] But there also exists multiple sunspots equilibria. To construct such sunspots equilibria, consider the process

$$\phi_t = \frac{\epsilon-1}{\epsilon}\varepsilon_t, \tag{12}$$

where ε denotes sunspots shocks. Equation (11) implies

$$E_{t-1}\varepsilon_t^{\frac{1+\theta}{\theta}} = E_{t-1}\varepsilon_t^{\frac{1}{\theta}}. \tag{13}$$

[10] Note that K_t is a state variable known to firms in the end of period $t-1$.

[11] To avoid complex values, the condition $E\phi \geq 0$ must be imposed.

[12] Notice that the uniqueness is regardless of fundamental shocks. For example, suppose the technology A is a stochastic process, then in the fundamental equilibrium, we still have $\phi = \frac{\sigma-1}{\sigma}$.

Clearly, any random variable satisfying the distribution,

$$E_{t-1}\varepsilon_t \in [0,1], \quad cov(\varepsilon_t^{\frac{1}{\theta}}, \varepsilon_t) = E_{t-1}\varepsilon_t^{\frac{1}{\theta}} \left(1 - E_{t-1}\varepsilon_t\right), \tag{14}$$

constitutes an equilibrium. This paper restricts attention to *i.i.d.* sunspots shocks with mean $E\varepsilon = \bar{\varepsilon} \in [0,1]$.

Defining a balanced growth path in the model as an equilibrium path along which consumption, the capital stock, and output all grow at the same expected rate, we have the following propostions.

Proposition 1 *For any and each i.i.d. sunspots shock process, there always exists a balanced growth path along which the stochastic growth rates of consumption and capital are both given by* $\ln[s(1+\varphi_t)]$*, and the growth rate of output is given by* $\ln\left[\left(\frac{\phi_t}{\phi_{t-1}}\right)^{1/\theta} s(1+\varphi_t)\right]$*; where* $\varphi_t \equiv \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}} \left(1 - \frac{\phi_t}{1+\theta}\right)$ *and* $s \equiv \beta E_t \frac{1+r_{t+1}}{1+\varphi_{t+1}}$.

Proof. Since ϕ_t is *i.i.d.*, any function of ϕ_t is also *i.i.d.* An educated guess of the equilibrium paths of consumption and the capital stock is given by

$$C_t = (1-s)(1+\varphi_t)K_t, \tag{15}$$

$$K_{t+1} = s(1+\varphi_t)K_t, \tag{16}$$

where $s = \beta E_t \frac{1+r_{t+1}}{1+\varphi_{t+1}}$ denotes the optimal rate of savings, which is a constant under the *i.i.d.* assumption and is derived from the intertemporal Euler equation

$$\frac{1}{(1-s)(1+\varphi_t)K_t} = \beta E_t \frac{1+r_{t+1}}{(1+\varphi_{t+1})(1-s)s(1+\varphi_t)K_t}. \tag{17}$$

Using Equations (15) and (16), it can be shown that $\frac{C_{t+1}}{C_t} = s(1+\varphi_{t+1})$. Hence the balanced growth rates of consumption and capital are both given by $g = \ln[s(1+\varphi_t)]$. The growth rate of output is given by $g_y = \ln \frac{u_t K_t}{u_{t-1}K_{t-1}} = \frac{1}{\theta}\left(\ln\phi_t - \ln\phi_{t-1}\right) + \ln[s(1+\varphi_t)]$, which has the same (unconditional) expected value as g. ■

Proposition 2 *In the absence of extrinsic uncertainty, the model has a unique balanced growth path with its growth rate determined by*

$$g = \ln s(1+\varphi) = \ln\left[\beta\left(1 + \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}\left(\frac{\epsilon-1}{\epsilon}A\right)^{\frac{\theta+1}{\theta}}\right)\right]. \tag{18}$$

Proof. In the absence of extrinsic uncertainty, Equation (6) implies the marginal cost is constant, $\phi = \frac{\epsilon-1}{\epsilon}$. Hence r and φ are all constant. Consequently, the fundamental (no-sunspots) growth rate in the economy is uniquely determined by $\ln\beta(1+r(\frac{\epsilon-1}{\epsilon}))$. ■

Proposition 3 *If $\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}$, the mean growth rate of a stochastic growth path is strictly less than the deterministic growth rate without uncertainty ($\phi_t = \frac{\epsilon-1}{\epsilon}$), i.e., $E\left[s(1+\varphi(\phi_t))\right] < \beta(1+r(\frac{\epsilon-1}{\epsilon}))$.*

Proof. See the **Appendix**. ■

As an example, consider the limiting case where $\epsilon = \infty$. In this case, the deterministic (gross) growth rate is given by $g^* = \beta(1+r) = \beta\left(1 + \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\right)$, and the price equation (11) becomes

$$E\phi_t^{\frac{1}{\theta}+1} = E\phi_t^{\frac{1}{\theta}}. \tag{19}$$

Since we restrict our attention to the interval, $0 \leq \phi \leq 1$, the only distribution that can satisfy the above relationship for the marginal cost is the binary distribution, $\phi_t = \{0, 1\}$ with probability $\{1-p, p\}$. Under this distribution, we have $r_t = \varphi_t$, hence $s = \beta E\frac{1+r_{t+1}}{1+\varphi_{t+1}} = \beta$ and $E\varphi_t = p\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}$. The mean (gross) growth rate is hence given by

$$\bar{g} = \beta\left(1 + p\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}\right), \tag{20}$$

which is strictly less than the deterministic (gross) growth rate g^* for any $p \in (0,1)$. In this limiting case, the condition, $\epsilon > \frac{1+\theta}{\theta} + 2A^{\frac{1}{\theta}+1}$, is trivially satisfied.

Notice that if the source of uncertainty comes from shocks to the aggregate technology (A), the relationship between volatility and growth is then strictly positive. However, if a sufficiently large adjustment cost of investment is assumed, then this relationship can become negative (see Barlevy 2004a).

3 Model Simulation

The simulation exercise is meant to be a way of assessing the model's quantitative implications. Let the time period be a year and the time discounting rate $\beta = 0.98$. For the U.S. economy, the markup is approximately $10\% \sim 20\%$. This implies that $\phi = 0.9 \sim 0.8$ or $\epsilon = 10 \sim 6$. Let the real annual interest rate be $r = 6\%$ and the annual rate of depreciation be $\delta = 10\%$ in the deterministic economy without sunspots.[13] Hence Equation (4) implies $\theta = r/\delta = 0.6$. Since $\alpha u^{\theta} = \phi A$ and $\delta = \frac{1}{1+\theta}\alpha^{-\frac{1}{\theta}}(\phi A)^{\frac{\theta+1}{\theta}}$, these two equations can help pin down the values of $\{\alpha, A\}$ once the value of the utilization rate (u) is given. Let $u = \phi = 0.9$ in the deterministic economy (which implies $\epsilon = 10$); then the above two relationships imply $\alpha = 0.189\,38$ and $A = 0.19753$. Given these values, the condition required in Proposition 3, $\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}$ (≈ 3.1), is more than satisfied. It can easily be shown that this condition is still satisfied under other plausible parameter configurations, such as when the annual real interest rate in the deterministic equilibrium is as low as 1.5%. Figure 2 indicates that the condition can be satisfied for a wide range of parameter values. In particular, the higher the interest rate, the easier the condition can be satisfied. For example, when $\delta = 0.1$, $\phi = 0.9$ (implying $\epsilon = 10$), the condition is satisfied for $r > 1.2\%$; when $\delta = 0.1$, $\phi = 0.8$ (implying $\epsilon \approx 6$), the condition is satisfied for $r > 2.4\%$.

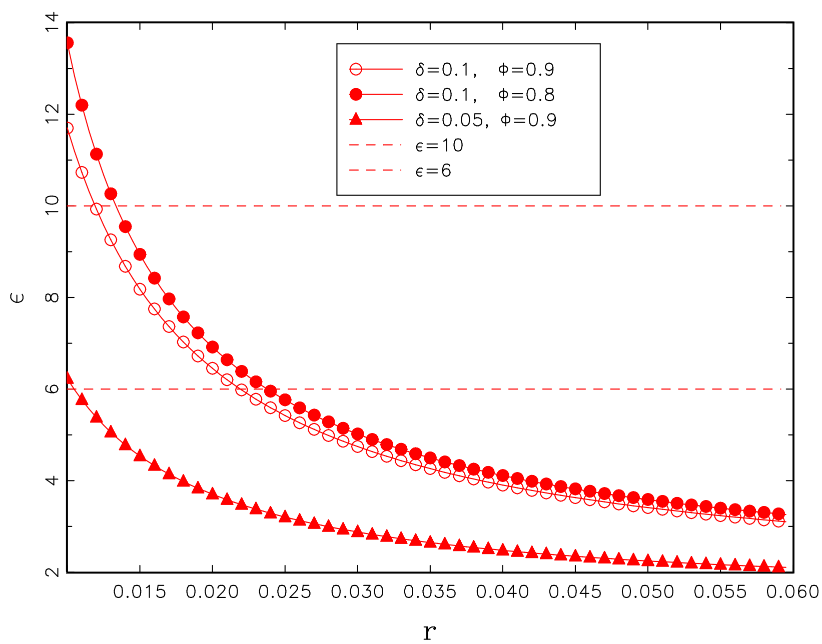

Figure 2. Parameter Region for $\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}$.

Based on the calibrated parameter values, the deterministic growth rate is given by $\ln s(1+\bar{\varphi}) = \ln \beta(1+r) \simeq 0.0381$; in other words, the fundamental growth rate is about 4% a year. To compute the mean growth rate of a stochastic growth path, we generate a time series for $\phi_t = \frac{\epsilon-1}{\epsilon}\varepsilon_t$, where the sunspots shock (ε) has the log-normal distribution $\ln \varepsilon \sim N(\mu, \sigma^2)$ with

$$e^{\frac{\sigma^2}{\theta}} E\varepsilon_t = 1. \tag{21}$$

[13] The average interest rate in the model can be significantly lower under the influence of sunspots than it is in the deterministic equilibrium.

Notice that this distribution satisfies Equation (13) and the condition, $0 < E\varepsilon_t < 1$.

Based on these calibrated parameter values, Table 1 shows the statistical relationship between volatility and mean growth rate for the range of σ that yields empirically plausible mean growth rates. The statistics reported in the table are estimates based on simulated time series with sample size of 10^6. The table shows that, as the standard deviation of the sunspots shock (σ) increases, the standard deviation of the stochastic growth rate (σ_g) also increases, while the mean growth rate of the economy ($\bar{g}$) tends to decrease. Table 2 shows the same result is also confirmed for a uniform distribution of sunspots shocks.[14] This prediction of a negative relationship between volatility and growth is consistent with the empirical regularity documented by Ramey and Ramey (1995) in cross-country data.

Table 1. Predicted Volatility and Growth (Log-Normal Distribution)

σ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\bar{g}(\%)$	3.81	3.68	3.32	2.83	1.83	1.54	0.62	−0.11	−1.01	−1.22	−2.79
σ_g	0	0.003	0.01	0.02	0.03	0.05	0.07	0.09	0.11	0.15	0.32

Table 2. Predicted Volatility and Growth (Uniform Distribution)

σ	0	0.029	0.058	0.087	0.12	0.14	0.17	0.23	0.29	0.35	0.40
$\bar{g}(\%)$	3.81	3.79	3.77	3.70	3.63	3.54	3.41	3.10	2.69	2.17	1.70
σ_g	0	0.001	0.002	0.003	0.004	0.005	0.007	0.011	0.016	0.022	0.026

Figure 3 shows simulations of the stochastic growth paths of consumption and the implied log consumption levels for each of the distributions considered above. In particular, the simulation under the log-normal distribution is presented in the first row windows (A and B), and the simulation based on uniform distribution is presented in the second row windows (C and D). The growth rate series are graphed in the left column windows (A and C) and the log output level series are graphed in the right column windows (B and D). In windows showing the growth series (Window A and C), the horizontal line is the deterministic growth rate in the absence of sunspots shocks, the solid lines represent a growth rate series under the influence of a particular sunspots process, and the dashed lines represent the annual consumption growth of the U.S. economy for the period 1947-2005. Clearly, the model is able to generate similar volatility in growth rate to the U.S. data. Since the mean growth rate in the model under a particular sunspots process is lower than that

[14] Even with the large sample size, the standard deviation of the growth rate (σ_g) is quite large for the log-normal distribution, suggesting that the estimated mean growth rate can have large standard errors. Despite this, the tendency for the mean growth rate to decline as the growth volatility increases is clear. When a uniform distribution is assumed instead for sunspots shocks, the standard error of the growth rate (σ_g) is much smaller and the mean growth rate is more tightly estimated, which makes the negative relationship between volatility and growth even clearer (see Table 2). Note that under the uniform distribution the growth rate of the model is always positive when the parameters of the distribution (mean and variance) of sunspots shocks satisfy Equation (12).

of the U.S. data, the implied consumption level (Window B or D) is stochastically dominated by the U.S. consumption level. Notice that a mean growth rate similar to the actual U.S. data can also be generated from the model by using sunspots shocks with a smaller variance than the one represented by the solid lines. As suggested by Windows B and D, along a lower consumption growth path due to a higher volatility, the loss in consumption is irreversible (unrecoverable) even if the mean growth rate later recovers to the previous level due to a decrease in volatility. The fact that such a large and ever increasing gap in consumption levels, in sharp contrast to the Okun's gap and the random walk phenomenon, can be caused by business cycles (volatility) alone is striking. The lesson: when growth is endogenous, fluctuations can affect not only the consumption level permanently, but also its long-run growth rate.

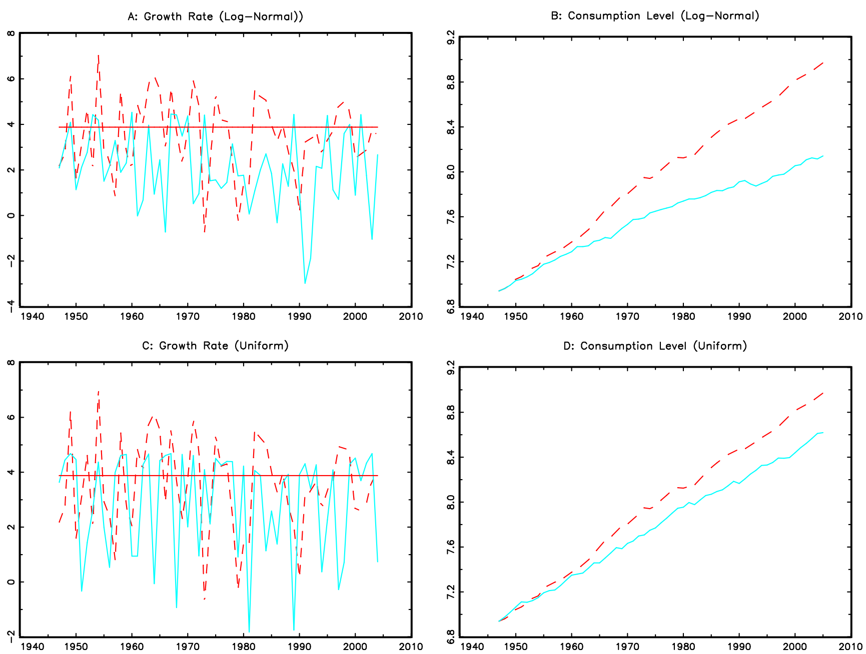

Figure 3. Volatility and Growth (—— Model; - - - Data).

4 Welfare Cost of Fluctuations

4.1 The Lucas Calculation

The Lucas calculation of the cost of business cycles is based on a simple yet fundamental assumption: volatility and growth are unrelated. Given this dichotomy and the fact that the aggregate consumption series is smooth, Lucas (1987 and 2003) concludes that the welfare cost of fluctuations is trivial in terms of consumption goods. Suppose a representative consumer is endowed with the stochastic consumption stream,

$$c_t = Ae^{ut}e^{-(1/2)\sigma^2}\varepsilon_t, \tag{22}$$

where u is a deterministic growth rate and $\ln(\varepsilon_t)$ is a normally distributed random variable with zero mean and variance σ^2. Hence $Ee^{-(1/2)\sigma^2}\varepsilon_t = 1$. The preference over consumption is assumed to be $E\sum_{t=0}^{\infty}\left(\beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right)$. The welfare gain can be computed as the percentage increase in consumption one would get by eliminating all the volatility, namely:

$$E\sum_{t=0}^{\infty}\left[\beta^t \frac{\left((1+\lambda)c_t\right)^{1-\gamma}}{1-\gamma}\right] = \sum_{t=0}^{\infty}\left[\beta^t \frac{\left(Ae^{ut}\right)^{1-\gamma}}{1-\gamma}\right], \tag{23}$$

where λ measures the welfare gain. Since growth and fluctuations are unrelated, λ can be computed easily by comparing the utilities in a single period:

$$E\left((1+\lambda)c_t\right)^{1-\gamma} = \left(Ae^{ut}\right)^{1-\gamma}, \tag{24}$$

which implies $\lambda \approx \frac{1}{2}\gamma\sigma^2$. The annual U.S. real consumption growth in the period of 1947-2005 is about 3.5% with a standard deviation of 1.65%. Assuming log utility ($\gamma = 1$), the welfare cost is estimated to be $\lambda \approx \frac{1}{2}(0.0165)^2 \approx 0.014\%$. This is less than 1.5¢ for every \$100 of annual consumption.[15]

4.2 Calculation based on Hall's (1978) Random Walk

A crucial feature of the Lucas calculation is that random shocks to consumption have no permanent effect on the consumption level. According to the permanent income theory, however, consumption follows a random walk, hence transitory shocks can have permanent effects (Hall, 1978). Adopting the random walk framework, the consumption path can be described by

$$c_t = c_{t-1}(e^{u-\frac{\sigma^2}{2}}\varepsilon_t), \tag{25}$$

where u is a drift term in the random walk specification of log consumption, which determines the average growth rate of consumption. This characterization of consumption is also an implication of the RBC theory where technology shocks follow random walks. Suppose the initial consumption level is given by $c_0 = A$. Equation (25) implies that in the absence of uncertainty (i.e., $\varepsilon_t = e^{\sigma^2/2}$ for all t), consumption grows at the rate $u : c_t = Ae^{ut}$. It also implies that under random shocks consumption evolves according to

$$c_t = Ae^{\left(u-\sigma^2/2\right)t}\varepsilon_1\varepsilon_2...\varepsilon_t. \tag{26}$$

[15] Of course, a higher γ can increase the estimation. Micro evidence suggests that $\gamma \in [1, 4]$. But even with $\gamma = 100$, the annual cost of business cycle is still less than 1.5 percent of consumption.

The welfare cost of fluctuations can then be computed as the solution (λ) to Equation (23) based on the random-walk consumption in (26). Again assuming log utility ($\gamma = 1$) and $\ln \varepsilon_t \sim N(0, \sigma^2)$, Equation (23) implies

$$\frac{\ln(1+\lambda)}{1-\beta} - \frac{\sigma^2}{2}\left(\beta + 2\beta^2 + 3\beta^3 + ...\right) = 0 \tag{27}$$

Solving for λ we get

$$\lambda \approx \frac{\sigma^2}{2}\frac{\beta}{(1-\beta)}. \tag{28}$$

Notice that the welfare measure under the random walk assumption is a multiplier ($\frac{\beta}{1-\beta}$) times the welfare measure of Lucas. This is the result obtained by Obstfeld (1994).[16] This multiplier exists because a one dollar increase in consumption today is translated into $\sum_{t=1}^{\infty} \beta^t = \frac{\beta}{1-\beta}$ dollars increase in life-time consumption. This suggests that when shocks to consumption have permanent effects, the welfare cost of business cycles can be potentially much larger. Letting $\beta = 0.98$ and $\sigma = 0.0165$, we get $\lambda \approx 0.67\%$, which is more than 47 times larger than the welfare gain under the Lucas specification of the consumption path. However, it is still small in absolute magnitude: less than one dollar for every \$100 of annual consumption. Notice that this calculation is still based on the assumption that volatility and growth are unrelated. Namely, even if shocks have permanent effects on the level of consumption, they have no effects on the average growth rate of consumption. Consequently, the welfare cost of fluctuations is still small.

4.3 Calculation based on Ramey and Ramey (1995)

According to the empirical studies of Ramey and Ramey, volatility and growth are negatively related. Hence eliminating volatility should increase the growth rate, which implies a large welfare cost of business cycles, consistent with Lucas's (1987) analysis on the welfare effect of long-run growth. But Lucas did not relate business cycle to growth, hence he failed to appreciate the welfare cost of fluctuations. To illustrate this, consider a counterfactual experiment where completely removing uncertainty can increase the growth rate by π percent from u to $u(1+\pi)$. Then Equation (23) becomes

$$E\sum_{t=0}^{\infty}\left[\beta^t \frac{\left((1+\lambda)c_t\right)^{1-\gamma}}{1-\gamma}\right] = \sum_{t=0}^{\infty}\left[\beta^t \frac{\left(Ae^{u(1+\pi)t}\right)^{1-\gamma}}{1-\gamma}\right]. \tag{29}$$

Under the random-walk consumption path (26), Equation (28) then becomes

[16] Also see Reis (2005) for a more general ARMA specification of the consumption process.

$$\frac{\ln(1+\lambda)}{1-\beta} - \frac{\sigma^2}{2}\left(\beta + 2\beta^2 + 3\beta^3 + ...\right) = \pi u \left(\beta + 2\beta^2 + 3\beta^3 + ...\right), \tag{30}$$

which implies $\lambda \approx \left(\frac{\sigma^2}{2} + \pi u\right)\frac{\beta}{1-\beta}$. According to Ramey and Ramey (1995, p1141), one standard deviation of the volatility in growth rate of output translates into about one-third of a percentage point of the mean growth rate. Applying this estimate to consumption, it means that by decreasing the consumption volatility from $\sigma = 0.0165$ to zero, the gain in growth rate is about $\frac{0.0165}{3} = 0.55\%$, which is about 16% of the current mean consumption growth rate for the U.S. economy ($u = 3.5\%$). This implies that $\pi = 16\%$ and $\pi u = 0.55\%$. Assuming $\beta = 0.98$, we have $\lambda \approx 28\%$. This is an enormous welfare gain: more than a quarter of total annual consumption.[17]

4.4 Calculation Based On Our Model

Consumption in our model follows the path $c_t = c_{t-1}\left[s(1+\varphi_t)\right]$, where $c_0 = (1-s)(1+\varphi_0)k_0$. Notice that since the sunspots shocks are *i.i.d.*, we have $E_0 g(\varphi_1) = E_0 g(\varphi_2) = \cdots = E_0 g(\varphi_t)$ for all $t > 0$. Hence the expected life-time utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t(1+\lambda)) = \frac{\ln(1+\lambda)}{1-\beta} + \frac{\ln c_0}{1-\beta} + \frac{\beta\left(\ln s + E\ln(1+\varphi)\right)}{(1-\beta)^2}. \tag{31}$$

In the absence of uncertainty, the model implies $\phi = \frac{\sigma-1}{\sigma}$, and the fundamental growth rate of consumption is given by $\ln\beta(1+r) = 3.81\%$. The life-time value of the deterministic consumption path is given by

$$\sum_{t=0}^{\infty} \beta^t \ln\left(c_0\,(1.0388)^t\right) = \frac{\ln c_0}{1-\beta} + \frac{\beta\ln(1.0388)}{(1-\beta)^2}. \tag{32}$$

Comparing the two expressions in (31) and (32) gives the welfare gain:

$$\lambda \approx \frac{\beta}{1-\beta}\left(0.0381 - E\ln s(1+\varphi)\right), \tag{33}$$

Notice that the welfare gain is the multiplier $(\frac{\beta}{1-\beta})$ times the difference between the maximum sustainable growth rate under full information and the mean of the stochastic growth rate under sunspots shocks. As Proposition 3 shows, the mean growth rate of a stochastic growth path is strictly less than the fundamental growth rate. Hence λ is always positive. Furthermore, as Table 1 and Table 2 both show, when the volatility of sunspots shocks increases in the model, the mean of the

[17] Interestingly, this estimate is very close to the estimate obtained by Alvarez and Jermann (2004) using a non-parametric asset-pricing approach.

stochastic growth rate, $E \ln s(1+\varphi)$, decreases, which increases the value of λ. For example, under the assumption of a log-normal distribution (Table 1), a standard deviation of 0.3 for sunspots shocks (ε_t) implies a stochastic consumption growth path with a standard deviation $\sigma_g = 0.02$, similar to the U.S. consumption data. Under this volatility, the mean consumption growth is 2.83%. Substituting this number into Equation (33) implies $\lambda = 24\%$. Under the assumption of a uniform distribution (Table 2), a standard deviation of 0.29 for sunspots shocks implies a stochastic growth path with a standard deviation $\sigma_g = 0.016$, which almost exactly matches the U.S. consumption data. Under this volatility, the mean consumption growth is 2.69%. Substituting this number into Equation (33) implies $\lambda = 27\%$.[18] Thus, based on our endogenous growth model, the welfare cost of business cycles with volatility similar to the U.S. data is about a quarter of annual consumption. The estimates are quite consistent with the estimate based on Ramey and Ramey's empirical studies and the estimates obtained by Alvarez and Jermann (2004) based on a nonparametric asset-pricing approach. Although our quantitative estimates of the welfare cost depend on the calibrated parameter values of the model (such as β), their qualitative scales are robust to small changes in the parameter values because fluctuations in the model can significantly decrease the average growth rate for a wide range of plausible parameter values and the welfare cost of a small decrease in growth is very large (as realized by Lucas, 1987).

An important caveat is that the welfare-cost estimates implied by our model only represent possible upper bonds. This is not only because the AK model is a highly stylized model without labor and human capital but also because there are no fundamental shocks in the model except sunspots. Hence, a correct reading of our welfare estimates is that the cost of business cycles can be as large as one fourth of annual consumption if all fluctuations are driven by sunspots. But how much of the business cycle in reality is driven by sunspots and how much by fundamental shocks are yet to be determined by empirical studies.

5 Robustness

In this section we add endogenous labor supply to the model so as to show that the negative relationship between growth and volatility does not depend on our choice of the production technology in (2). Let the intermediate good producers' production function be given by

$$y(i) = AK^{1-\alpha} k(i)^{\alpha} n(i)^{1-\alpha}, \tag{34}$$

where K stands for the average capital stock among intermediate good producers and n is labor. The production externality will not generate sunspot equilibrium in our setup. It is just a conve-

[18] Suppose we use the actual U.S. consumption growth rate ($u = 3.5\%$) instead. Equation (32) implies that the welfare cost of volatility is about 7% of annual consumption.

nient way to generate endogenous growth. For simplicity, we assume full capacity utilization and depreciation of capital, $u(i) = \delta(i) = 1$, and let the household's period-utility function be give by $\log(C) - N$.

The household's problem is then to solve

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{\log(C_t) - N_t\right\} \tag{35}$$

subject to $C_t + K_{t+1} = w_t N_t + (1 + r_t) K_t + D_t$. Cost minimization for intermediate good firms implies that $w_t = (1-\alpha)\phi_t \frac{Y_t}{N_t}$ and $r_t = \alpha\phi_t \frac{Y_t}{K_t} - 1$. Hence, the first order conditions of the household are given by

$$\frac{1}{C_t} = \beta E_t \frac{\alpha\phi_{t+1} Y_{t+1}/K_{t+1}}{C_{t+1}}, \tag{36}$$

$$(1-\alpha)\phi_t \frac{Y_t}{C_t} = N_t. \tag{37}$$

Because of full depreciation of the capital stock, the household's resource constraint in equilibrium becomes

$$C_t + K_{t+1} = Y_t. \tag{38}$$

Assuming that sunspots are *i.i.d*, it can be shown that in equilibrium the optimal consumption follows

$$C_t = (1 - s) Y_t, \tag{39}$$

where $s = \alpha\beta E_t \phi_{t+1}$ is the optimal saving rate; the optimal labor supply is determined by

$$N_t = \frac{1-\alpha}{1-s}\phi_t; \tag{40}$$

and the aggregate output is given by

$$Y_t = A \left(\frac{1-\alpha}{1-s}\phi_t\right)^{1-\alpha} K_t. \tag{41}$$

The price equation for the intermediate good producers is the same as before:

$$E_{t-1}\left(\phi_t Y_t\right) = \frac{\epsilon - 1}{\epsilon} E_{t-1} Y_t. \tag{42}$$

Substituting out output by (41), equation (42) becomes

$$E_{t-1}\phi_t^{2-\alpha} = \frac{\epsilon-1}{\epsilon}E_{t-1}\phi_t^{1-\alpha}, \tag{43}$$

which is similar to equation (11).

Clearly, sunspot equilibria still exist as in the previous model with capacity utilization. In order to show the negative relationship between growth and volatility, assume $\phi_t = \frac{\epsilon-1}{\epsilon}\varepsilon_t$, where the sunspots ($\varepsilon$) follow the log-normal distribution, $\ln(\varepsilon_t) \sim N(\mu, \sigma^2)$. Equation (43) then implies the following relationship between the mean and the variance of sunspots,

$$E\ln\varepsilon_t = -\frac{3-2\alpha}{2}\sigma^2. \tag{44}$$

It follows that $E\varepsilon_t = e^{(\alpha-1)\sigma^2} \leq 1$. The mean growth rate in this economy can be determined by

$$\bar{g} = E\ln(K_{t+1}/K_t) = \ln(A) + E\ln(s) + (1-\alpha)E\ln(N_t). \tag{45}$$

Notice the saving rate is given by

$$s = \beta\alpha E_t\phi_t = \beta\alpha\frac{\epsilon-1}{\epsilon}e^{(\alpha-1)\sigma^2}, \tag{46}$$

which is a decreasing function of σ^2. According to equation (40),

$$E\ln(N_t) = -\ln(1-s) + \ln(1-\alpha) + \ln\left(\frac{\epsilon-1}{\epsilon}\right) + E\ln(\varepsilon_t), \tag{47}$$

where the last term is decreasing in σ^2. Since s is a decreasing function of σ^2, the first term is also an decreasing function of σ^2. Hence, $E\ln(N_t)$ is decreasing in σ^2 and, consequently, the average growth rate is a decreasing function of the volatility.

6 Optimal Stabilization Policy

A large welfare cost of fluctuations by no means implies an equally large welfare gain from stabilization policy because volatile consumption can itself be optimal. For example, Barlevy (2004a) provides a model in which the welfare cost of fluctuations can be at least as large as $7-8$ percent of annual consumption. But, since volatile consumption is an optimal response to technology changes in his model, there are no gains from reducing or eliminating consumption fluctuations. Thus, despite the large welfare cost of business cycles, the policy implication of Barlevy's model is the same as the Lucas calculation: stabilization policy is counter-productive and hence undesirable. However, fluctuations in the real world may be highly inefficient as suggested by our model. In this case, the welfare gain from stabilizing consumption is as large as the welfare cost of fluctuations. This section discusses how to design stabilization policies to eliminate sunspots.

6.1 Pareto Optimal Allocation

Consider the Pareto optimal allocation first. The Pareto allocation is determined by solving the following social planner problem,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \tag{48}$$

subject to

$$C_t + K_{t+1} = Au_tK_t + (1-\delta_t)\, K_t, \tag{49}$$

and

$$\delta_t = \frac{1}{1+\theta} u_t^{1+\theta}. \tag{50}$$

Notice that we have assumed $\alpha = 1$ for equation (3) for simplicity. Define $\varphi_t \equiv Au_t - \delta_t$. It can be shown that under optimal capacity utilization we have $\delta_t = \frac{A^{(1+\theta)/\theta}}{1+\theta}$ and

$$\varphi_t = \frac{\theta}{1+\theta} A^{(1+\theta)/\theta}. \tag{51}$$

Thus, in the absence of technology change, δ_t and φ_t are constant. The optimal allocation is thus given by

$$C_t = (1-\beta)\,(1+\varphi)K_t, \tag{52}$$

$$K_{t+1} = \beta(1+\varphi)K_t, \tag{53}$$

where the balanced growth rate $\beta(1+\varphi)$ is given by $\beta(1+\frac{\theta}{1+\theta}A^{(1+\theta)/\theta})$. The result also holds for the case where A is stochastic.

6.2 Optimal Policy without Sunspots

Under imperfect competition and in the absence of extrinsic uncertainty (i.e., no sunspots), the Pareto optimal allocation can be achieved by subsidizing monopolistic firms for production, which is a standard result in the literature. To see this, consider that the government subsidizes the intermediate producers by the amount τ for each unit of good it sells. The profit maximization problem for each intermediate good producer becomes

$$\max\,(p+\tau-\phi))\, p^{-\epsilon} y. \tag{54}$$

The optimal price is given by $p = \frac{\epsilon}{\epsilon-1}(\phi-\tau)$, which is lower than the monopolistic price $\frac{\epsilon}{\epsilon-1}\phi$. In equilibrium, $p=1$, hence the optimal rate of subsidy must satisfy $\tau = \phi - \frac{\epsilon-1}{\epsilon}$. Since Pareto allocation requires $\phi = 1$, the optimal subsidy is given by $\tau = \frac{1}{\epsilon}$. Notice that a positive price requires $\tau < 1$, which is satisfied since $\epsilon > 1$. The equilibrium allocation of consumption and capital is given by

$$C_t = (1 - \beta) \left(1 + \frac{\theta}{1 + \theta} A^{\frac{1+\theta}{\theta}} \right) K_t, \tag{55}$$

$$K_{t+1} = \beta \left(1 + \frac{\theta}{1 + \theta} A^{\frac{1+\theta}{\theta}} \right) K_t, \tag{56}$$

which is Pareto optimal.

Notice that the optimal policy allows monopolist firms to make positive profits that are the same as the amount they would make without subsidies. To finance the amount of subsidies, τY, the government can use a non-distortionary lump-sum tax (T) on household income. A balanced budget implies $T_t = \tau Y_t$.

6.3 Optimal Policy under Sunspots

When there exists incomplete information, there is an additional source of inefficiency in the economy - the decrease of the average growth rate due to sunspots-driven fluctuations. Hence, we can design two separate policies to deal with the two source of inefficiency: one is the subsidizing policy (τ) discussed above, and another is a stabilization policy (ω) to deal with volatility specifically. Since fluctuations arise from firms' expectation about other firms' production levels, the stabilization policy can focus on stabilizing firms' output level via subsidizing capacity utilization. Consider a policy that subsidizes a firm's marginal cost of production by the amount, ω_t, for each additional unit of output produced or for each unit-increase in capacity utilization. The cost minimization problem of the intermediate good producer becomes to minimize $(r + \delta(i) - \omega A u(i))\, k(i)$ subject to $Au(i)k(i) \geqslant y(i)$. The first order conditions are

$$r + \delta(i) - \omega A u(i) = \phi A u(i) \tag{57}$$

$$u(i)^{\theta} - \omega A = \phi A. \tag{58}$$

These equations imply $u_t = (\phi_t + \omega_t)^{\frac{1}{\theta}} A^{\frac{1}{\theta}}$ and $r_t = \theta \delta_t = \frac{\theta}{1+\theta} \left[(\phi_t + \omega_t) A \right]^{\frac{\theta+1}{\theta}}$. The output level is given by

$$y(i) = (\phi_t + \omega_t)^{\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} k(i). \tag{59}$$

The price-setting problem for an intermediate good firm is the same as before, which is to maximize the expected profits $E\left[(p(i)+\tau-\phi)\,p(i)^{-\epsilon}Y\right]$. The optimal monopoly price is given by

$$p(i)=\frac{\epsilon}{\epsilon-1}\frac{E(\phi_t-\tau)Y_t}{EY_t}. \tag{60}$$

In a symmetric equilibrium, $p(i)=1$, $y(i)=Y$, and $k(i)=K$. Substituting output from Equation (59) into the above price equation gives

$$1=\frac{\epsilon}{\epsilon-1}\frac{E(\phi_t-\tau)\left(\phi_t+\omega_t\right)^{\frac{1}{\theta}}}{E\left(\phi_t+\omega_t\right)^{\frac{1}{\theta}}}. \tag{61}$$

As before, we can set $\tau=\frac{1}{\epsilon}$. Hence the pricing rule (61) implies

$$E\left(\phi_t+\omega_t\right)^{\frac{1}{\theta}}=\frac{1}{\epsilon-1}E\left(\phi_t+\omega_t\right)^{\frac{1}{\theta}}\left(\epsilon\phi_t-1\right). \tag{62}$$

Clearly, the optimal stabilization policy is given by

$$\omega_t=1-\phi_t. \tag{63}$$

Under this policy, the paths of consumption and capital are given by

$$C_t=(1-\beta)\left(1+\frac{\theta}{1+\theta}A^{\frac{1+\theta}{\theta}}\right)K_t, \tag{64}$$

$$K_{t+1}=\beta\left(1+\frac{\theta}{1+\theta}A^{\frac{1+\theta}{\theta}}\right)K_t, \tag{65}$$

which were shown to be Pareto optimal previously. Equation (61) implies that the marginal cost is given by $E\phi_t=1$. Although the marginal cost can be stochastic in equilibrium, its volatility has no consequence on the real variables in the economy under the stabilization policy ω_t. In order to have a balanced budget for the government, the government can simultaneously impose a lump-sum income tax on households such that:

$$\begin{aligned} T_t &= (\tau+\omega_t)Y_t \\ &= \left(\frac{1}{\epsilon}+(1-\phi_t)\right)Y_t. \end{aligned} \tag{66}$$

We can also combine the two policies together into one single policy that eliminates both types of inefficiencies simultaneously. Clearly, it is still required that $\omega_t=1-\phi_t$ so as to ensure constant growth rate. Setting $\tau=\omega$ and replacing τ by ω in Equation (61) gives $E\phi_t=\frac{2\epsilon-1}{2\epsilon}$. In this case, the expected profit is the same as before but the markup is smaller than the monopolistic markup but greater than zero.

6.4 Optimal Policy under Imperfect Information for the Government

6.4.1 Case 1:

The stabilization policy, $\omega_t = 1 - \phi_t$, requires that the government have full information about the marginal cost; namely, ϕ_t is observable to the Government. In reality, the marginal cost is difficult to observe directly. Here we discuss optimal stabilization policies which do not depend on the observability of the marginal cost. The only assumption required in this policy is that the government can observe the aggregate utilization rate of capital.

Without loss of generality, assume $\epsilon = \infty$ so that the only source of inefficiency is from sunspots-driven fluctuations.[19] Denote $u = \int_0^1 u(i)di$ as the aggregate (average) capacity utilization rate, and denote $\omega_t = \omega(u_t)$ as the optimal subsidy to each firm's marginal cost of production via capacity utilization. A firm's total cost of production is given by $(r + \delta(i) - \omega(u)Au(i))k(i)$. In a symmetric equilibrium, the first-order condition from cost minimization is given by

$$u_t^\theta - \omega(u_t)A = \phi_t A, \tag{67}$$

where ϕ is the marginal cost. Notice the Pareto optimal allocation is given by a constant capacity utilization rate, $u^* = A^{\frac{1}{\theta}}$. The key of the policy design is to find an incentive compatible subsidy policy $\omega(u_t)$ such that all firms choose $u_t = u^*$ in equilibrium.

Proposition 4 *The subsidy policy,*

$$\omega(u) = \begin{cases} 0 & \text{if } u_t > u^* \\ \frac{u^\theta}{A} - \frac{A^{\frac{1}{\theta}}}{u} & \text{if } u_t \leq u^* \end{cases}. \tag{68}$$

achieves the Pareto allocation.

Proof. Equation (67) implies

$$\phi = \frac{u^\theta - \omega(u)A}{A}. \tag{69}$$

The monopolist price is determined by the equation $E\phi_t Y_t = EY_t$. Substituting out ϕ_t and Y in the price equation gives

$$E\left[u_t^{1+\theta} - \omega(u_t)Au_t - Au_t\right] = 0, \tag{70}$$

the firm's profit maximization condition or incentive compatibility condition. Define the function $P(u) \equiv u^{\theta+1} - \omega(u)Au - Au$. Substituting the subsidy policy into $P(u)$ gives $P(u) > 0$ for $u \neq u^*$

[19] Namely, we consider stabilization policies that are separate from τ.

and $P(u) = 0$ for $u = u^*$. Since $EP(u) \neq 0$ is not optimal (or incentive compatible), firms will never choose $u_t \neq u^*$ under the above subsidy policy. Note that under the optimal capacity utilization u^*, the marginal cost is given by $\phi_t = 1$. Hence the allocation under $\omega(u)$ is Pareto optimal. ■

Clearly the functional form of the optimal policy is not unique. In fact, any policy function $\omega(u)$ such that $P(u) = 0$ if $u_t = u^*$ and $P(u) \neq 0$ if $u \neq u^*$ is optimal. Whatever the optimal policy is, it must provide incentives to induce firms to choose u^* and penalize them when $u \neq u^*$. Otherwise the policy is ineffective. For example, let $\omega(u) = \frac{u^\theta - A}{A}$. This policy is derived by setting $\phi = 1$ in equation (67). This policy is not effective in eliminating sunspots equilibria because under this policy, $P(u) = 0$ regardless of u. Hence it cannot eliminate sunspots-driven fluctuations.

6.4.2 Case 2:

The previous analyses have assumed away any fundamental shocks in order to simplify the exposition. Although allowing for fundamental shocks in the model will not change the results, it does complicate the issue of policy design when information is imperfect for the government. For example, since sunspots shocks to the marginal cost behave very much like technology shocks (i.e., sunspots shocks affect the marginal product of capital by affecting capacity utilization), it may not be possible for the government to distinguish where the shocks are coming from if neither sunspots shocks nor technology shocks are directly observable to the government. In this case, policies that completely stabilize the growth rate are no longer optimal if technology shocks dominate.

We show it is still possible to find stabilization policies completely eliminating the undesirable effects of sunspots, provided that the government has access to certain types of information. Assume the public information available to the government include firms' output (y), capital stock (k) and the market interest rate r. To prevent the government from deducing the level of technology (A_t) from the production function, assume the government cannot observe the rate of capacity utilization (u). Define a firm's output-capital ratio as $z(i) = y(i)/k(i) = Au(i)$ and the aggregate (average) output-capital ratio as $z = \int z(i)di$. Because $z = Y/K$ is observable to the government, it can be used as the basis for designing stabilization policy. However, note that since $z = Au$, the government cannot differentiate whether movements in z are caused by technology or by capacity utilization driven by sunspots.

Under technology shocks, the Pareto optimal allocation is given by $u_t = A_t^{1/\theta}$, $Y_t = A_t^{(1+\theta)/\theta} K_t$, and $r_t = \frac{\theta}{1+\theta} A_t^{(1+\theta)/\theta}$. This implies the Pareto optimal output-capital ratio is given by $z_t^* = \frac{1+\theta}{\theta} r_t$. If there is influence from sunspots, however, $z_t = \frac{1+\theta}{\theta} \frac{r_t}{\phi_t}$. The key of the policy design is to find an incentive compatible subsidy policy where it is in the best interest of all firms to choose $z_t = z_t^*$ in equilibrium.

Denote $\omega_t = \omega(r, z)$ as the optimal subsidy to each firm's marginal cost of production, which individual firms take as given. A firm's total cost of production is then given by $(r + \delta(i) - \omega(r, z)Au(i))k(i)$. In a symmetric equilibrium, the first-order conditions can be expressed as

$$r + \delta = (\phi + \omega(r, z))z \tag{71}$$

$$u^{1+\theta} = (\phi + \omega(r, z))z \tag{72}$$

These imply $r = \theta\delta$ and

$$\phi = \frac{(1+\theta)}{\theta}\frac{r}{z} - \omega(r, z). \tag{73}$$

The optimal monopoly price is still determined by the pricing rule, $E\phi Y = EY$. Since $Y = zK$ and K is known to firms in the beginning of each period, substituting out ϕ and Y in the pricing rule gives

$$E\left[\frac{1+\theta}{\theta}r - \omega(r, z)z - z\right] = 0. \tag{74}$$

Define the function $P(r, z) \equiv \frac{(1+\theta)}{\theta}r - \omega(z)z - z$. Since $EP(r, z) \neq 0$ is not optimal (or incentive compatible) to firms, the key of the policy design is to set the subsidy rate $\omega(r, z)$ such that firms will never choose an output-capital ratio $z_t \neq z_t^*$ under the subsidy policy. Hence, any policy function $\omega(r, z)$ such that it makes $P(r, z) = 0$ if $z = z^*$ and $P(r, z) \neq 0$ if $z \neq z^*$ can achieve the Pareto allocation. For example, it is easy to check that the following policy can achieve the Pareto allocation:

$$\omega(r, z) = \begin{cases} 0 & \text{if } z_t \geq \frac{1+\theta}{\theta}r \\ \frac{2(1+\theta)}{\theta}\frac{r}{z} - 2 & \text{if } z < \frac{1+\theta}{\theta}r \end{cases}. \tag{75}$$

Under this policy, we have $EP(r, z) = 0$ if and only if $z = \frac{1+\theta}{\theta}r$. When $z = \frac{1+\theta}{\theta}r$, we can show $\phi_t = 1$, $u_t = A_t^{1/\theta}$, and $r_t = \frac{\theta}{1+\theta}A_t^{(1+\theta)/\theta}$. Hence the allocation under the policy $\omega(r, z)$ is Pareto optimal.

7 Conclusion

Ordinary market imperfections (i.e., imperfect competition and incomplete information) can lead to endogenous fluctuations in firms' markup, which can directly translate into fluctuations in output growth and adversely affect its mean growth rate. Consequently, the welfare cost of business cycle and the associated gain from stabilization can be potentially large. In particular, the welfare gain from further stabilizing the U.S. economy can be hundreds of times larger than that calculated by Lucas (1987), in the order of around 25% of annual consumption. Although this figure is likely to be an upper bond, it is too large to ignore. The optimal stabilization policy we found is consistent with those in practice: stabilizing output growth (or capacity utilization) around a potential target.

Appendix: Proof of Proposition 3.

Proof. The key of the proof is showing that $E\phi_t^{\frac{1}{\theta}} \leq \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}$ and that the average growth rate is a strictly increasing function of $E\phi_t^{\frac{1}{\theta}}$ under certain conditions. In such a case, the maximum growth rate is achieved when $\phi_t = \frac{\epsilon-1}{\varepsilon}$.

The growth rate of the model is given by

$$g_t = s(1+\varphi_t), \tag{76}$$

where $\varphi_t = \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t{}^{\frac{1}{\theta}}(1-\frac{\phi_t}{1+\theta})$, $s = \beta E\frac{1+r_t}{1+\varphi_t}$, and $r_t = \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}(\phi_t A)^{\frac{1+\theta}{\theta}}$. The monopolistic price follows the rule, $E\phi_t{}^{\frac{1+\theta}{\theta}} = \frac{\epsilon-1}{\epsilon}E\phi_t^{\frac{1}{\theta}}$. Since $Ex^{1+\theta} \geq (Ex)^{1+\theta}$, we have $\frac{\epsilon-1}{\epsilon}E\phi_t^{\frac{1}{\theta}} = E\phi_t{}^{\frac{1+\theta}{\theta}} \geq \left(E\phi_t^{\frac{1}{\theta}}\right)^{1+\theta}$. It follows that

$$E\phi_t^{\frac{1}{\theta}} \leq \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}. \tag{77}$$

Given the definition of φ_t, we have

$$\begin{aligned} E\varphi_t &= \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \left(E\phi_t^{\frac{1}{\theta}} - \frac{1}{1+\theta}E\phi_t{}^{\frac{1+\theta}{\theta}}\right) \\ &= \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} E\phi_t^{\frac{1}{\theta}} \left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right) \\ &\leq \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}} \left(1-\frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right). \end{aligned} \tag{78}$$

Notice that s can be approximated as

$$\begin{aligned} s &\simeq \beta E(1+r_t-\varphi_t) \\ &= \beta E\left(1+\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}(\phi_t A)^{\frac{1+\theta}{\theta}} - \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t{}^{\frac{1}{\theta}}\left(1-\frac{\phi_t}{1+\theta}\right)\right) \\ &= \beta\left(1+\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\left(E\phi_t{}^{\frac{\theta+1}{\theta}} - E\phi_t^{\frac{1}{\theta}}\right)\right) \\ &= \beta\left(1-\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\frac{1}{\epsilon}E\phi_t^{\frac{1}{\theta}}\right). \end{aligned} \tag{79}$$

Denoting $\bar{x} \equiv E\phi_t^{\frac{1}{\theta}}$, the mean growth rate is then given by

$$\begin{aligned} \bar{g} &= s\left(1 + E\varphi_t\right) && (80) \\ &= \beta\left[1 - \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \frac{1}{\epsilon}\bar{x}\right]\left[1 + \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\left(1 - \frac{\epsilon - 1}{\epsilon}\frac{1}{1+\theta}\right)\bar{x}\right]. \end{aligned}$$

Differentiating with respect to $\bar{x}$, it can be shown that if the condition,

$$\bar{x} < \frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}}, \tag{81}$$

is satisfied, then $\bar{g}$ is a strictly increasing function of $\bar{x}$. Since $\bar{x} \leq \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}$ according to (77), hence the maximum growth rate will be achieved by the certainty equilibrium where $\phi_t = \frac{\epsilon-1}{\epsilon}$, provided that Condition (81) holds.

But a sufficient condition for (81) to hold is the condition,

$$\left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{\theta}} < \frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}}. \tag{82}$$

Notice that $\left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1} \leq \frac{1+\theta}{\theta}$ since $\epsilon \leq \infty$, hence we have the following inequality for the right-hand side of (82):

$$\frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-1/\theta} A^{(1+\theta)/\theta}} \geq \frac{\epsilon - \frac{1+\theta}{\theta}}{2\alpha^{-1/\theta} A^{(1+\theta)/\theta}}. \tag{83}$$

If the right-hand side of the above equation is greater than one, namely, if

$$\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}, \tag{84}$$

we then have

$$\frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}} \geq \frac{\epsilon - (1+\theta)/\theta}{2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}} > 1 \geq \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{\theta}}. \tag{85}$$

Hence, (84) is a sufficient condition for the inequality (81) to hold. ■

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