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# The Dynamic Relationship Between Permanent and Transitory Components of U.S. Business Cycles

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## Abstract

This paper investigates the dynamic relationship between permanent and transitory components of post-war U.S. business cycles. We specify a time-series model for real GNP and consumption in which the two share a common stochastic trend and transitory component, and Markov-regime switching is used to model business cycle phases in these components. The timing of switches between business cycle phases is allowed to differ across the permanent and transitory components. We find strong evidence of a lead-lag relationship between the switches in the two components. Specifically, switches in the permanent component leads switches in the transitory component when entering recessions.

*Key words:* Asymmetry, Business Cycle, Markov-Switching, Fluctuations

*J.E.L classification:* C32, E32

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The decomposition of aggregate measures of output into permanent and transitory components, with the components often used as measurements of “trend” and “cycle”, is a primary tool for modern analysis of the business cycle. The unobserved-components (UC) approach of Harvey (1985) and Clark (1987) is a popular methodology for performing this decomposition. The vast majority of the literature applying UC models to measures of economic activity has adopted two assumptions. First, linear time-series models such as ARMA processes are used to describe the unobserved components. Second, the permanent and transitory components are assumed to be independent.

Recently, Kim and Murray (2002), using a multivariate framework of monthly economic indicators, extended the UC model to allow for nonlinear dynamics in both the permanent and transitory components.<sup>1</sup> Using Markov-switching techniques, these authors allow for two distinct business cycle phases, expansion and recession, over which the time-series dynamics of the permanent and transitory components differ.<sup>2</sup> However, the assumption of independent unobserved components is maintained. Morley, Nelson and Zivot (2003), working with a linear UC model of real GDP, relax the assumption of independent unobserved components, and document substantial contemporaneous correlation between the shocks to the permanent and transitory components.

In this paper we estimate a multivariate UC model of U.S. real GNP and consumption, which, following Kim and Murray (2002), incorporates regime-switching in both the permanent and transitory components. The primary contribution is to allow the regime shifts in the unobserved components to be correlated, both contemporaneously and at lags. We uncover a

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<sup>1</sup> Building on work by Diebold and Rudebusch (1996), Chauvet (1998) and Kim and Yoo (1995) incorporate nonlinear dynamics into the common factor of a multivariate system. However, these authors do not decompose the time series in the system into permanent and transitory components.

surprising, and very strong, temporal pattern to recessions: the permanent component leads the transitory component when entering recessions. We also find that both the transitory and permanent components contribute to short-run fluctuations in both series.

The details of our empirical model are as follows. We specify real GNP and consumption as a cointegrated system with a common, random walk, stochastic trend. The deviation from the common stochastic trend is the transitory component of each series, which is modeled as arising partly from common shocks and partly from shocks idiosyncratic to each series. To capture recession and expansion phases, we allow for regime shifts in the mean growth rate of the common stochastic trend as in Hamilton (1989), and in the mean of the transitory component as in Kim and Nelson (1999a), with separate regime indicator variables used for the two components. We then investigate what dependence might exist, both contemporaneously and at lags, between the regime shifts in the permanent component and regime shifts in the transitory component. We accomplish this by modeling the evolution of the two Markov-switching state variables as driven by a single, four-state Markov-switching process.

The results suggest that the historical record of NBER recessions can be usefully characterized by a typical pattern: Recessions begin with a switch to the recession state in the permanent component, characterized by a reduction in the mean growth rate of the common stochastic trend. During most recessions, following the reduction in trend growth rate, a corresponding switch to the recession state in the transitory component occurs, characterized by large negative reductions to its level. The effects of the regime shift in the transitory component contribute more to movements in real GNP during recessions than the slowdown in the growth

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<sup>2</sup> The Kim and Murray (2002) model is extended to a cointegrated system in Kim and Piger (2002), which is the framework used in this paper.

rate of the common stochastic trend. The recession then ends and the economy gradually asymptotes to its new growth path.

In this paper we are primarily interested in documenting stylized facts regarding the dynamic relationship between permanent and transitory components of the business cycle. However, the result that recessions begin with a switch in the permanent component, rather than a switch in the transitory component, may suggest sources underlying the recessions. In particular, permanent and transitory components of business cycles are often interpreted as “trend” and “cycle”. To the extent that variation in trend and cycle are due to different sources, such as technology vs. demand shocks, our empirical results may suggest a prominence of one of these sources in triggering recessions.

In the following section we formally present the empirical model. Section 3 reports and interprets the estimation results. Section 4 concludes.

## 2. Model Specification

### 2.1 A Time-Series Model of the Business Cycle

Consider the following unobserved-components model of business cycle fluctuations:

$$\begin{bmatrix} y_t \\ c_t \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha \end{bmatrix} + \begin{bmatrix} 1 \\ \gamma_x \end{bmatrix} x_t + \begin{bmatrix} 1 \\ \gamma_z \end{bmatrix} z_t + \begin{bmatrix} e_{y,t} \\ e_{c,t} \end{bmatrix} \quad (1)$$

Here, the log of real GNP ( $y_t$ ) and the log of real consumption of non-durable goods and services ( $c_t$ ) are divided into a common stochastic trend  $x_t$ , a common transitory component,  $z_t$ , and idiosyncratic transitory components  $e_{y,t}$  and  $e_{c,t}$ . This specification is based on simple neoclassical growth models such as that in King, Plosser and Rebelo (1988) suggesting that output and consumption exhibit balanced stochastic growth, that is they are cointegrated with

cointegrating vector  $(1, -\gamma_x)$ , where  $\gamma_x$  is equal to one. Here we will estimate  $\gamma_x$  rather than impose this theoretical value. The transitory components,  $z_t$ ,  $e_{y,t}$  and  $e_{c,t}$  capture transitory deviations from the shared common stochastic trend, which may arise from a variety of sources such as the propagation of supply-side shocks, as in Kydland and Prescott (1982), or more traditional demand shocks.

We model the common stochastic trend component as in Hamilton (1989):

$$x_t = \mu_1^* S_t^P + \mu_0^* (1 - S_t^P) + x_{t-1} + v_t \quad (2)$$

where  $v_t \sim N(0, \sigma_v^{2*})$ , and  $S_t^P = \{0, 1\}$  indicates the state of the economy for the trend component. Labeling  $S_t^P = 1$  as the recession state, the average growth rate of  $x_t$  is given by  $\mu_0^*$  during expansions and  $\mu_1^*$  during recessions. Thus, the average growth rate of the trend is reduced by the discrete amount  $\mu_0^* - \mu_1^*$  during each quarter that  $S_t^P = 1$ . This reduction in trend leaves output and consumption permanently lower than if the recession had never occurred.

Each series contains two sources of transitory variation. The first is the common transitory component,  $z_t$ , which evolves according to the following stationary autoregression:

$$\phi(L)z_t = \varepsilon_t \quad (3)$$

where  $\phi(L)$  has all roots outside the unit circle and  $\varepsilon_t \sim N(0, \sigma_\varepsilon^{2*})$  is uncorrelated with  $v_t$ . The second is the idiosyncratic transitory components,  $e_{y,t}$  and  $e_{c,t}$ . These are assumed to evolve according to a regime-switching, stationary autoregressive “plucking model” as in Kim and Nelson (1999a).

$$\begin{aligned} \psi_y(L)e_{y,t} &= \tau_y S_t^T + \omega_{y,t} \\ \psi_c(L)e_{c,t} &= \tau_c S_t^T + \omega_{c,t} \end{aligned} \quad (4)$$

where  $\psi_y(L)$  and  $\psi_c(L)$  have all roots outside the unit circle and  $\omega_{y,t} \sim N(0, \sigma_{\omega_y}^2)$ ,

$\omega_{c,t} \sim N(0, \sigma_{\omega_c}^2)$  are uncorrelated with each other and with  $\varepsilon_t$  and  $v_t$ .

$S_t^T = \{0, 1\}$  indicates the state of the economy for the transitory component. Labeling  $S_t^T = 1$  as the recession state,  $e_{y,t}$  and  $e_{c,t}$  are reduced by the discrete amounts,  $\tau_y$  and  $\tau_c$ , during each quarter that  $S_t^T = 1$ .<sup>3</sup> However, when the economy returns to normal times, that is  $S_t^T = 0$ , the effects of past  $\tau_y$  and  $\tau_c$  wear off in accordance with the transitory autoregressive dynamics and the economy reverts back to the stochastic trend. The farther the economy is plucked down, the faster the growth of the economy as it “bounces back” or “peak-reverts” to trend. Note that this sort of pattern is consistent with Friedman’s (1964, 1993) “plucking” model of business cycles.<sup>4</sup>

The last 30 years of U.S. macroeconomic data are problematic for the estimation of UC models, as it contains two well documented sources of structural change in the model parameters. First, there is a large literature suggesting that the growth rate of productivity slowed at some point in the postwar sample, with the predominant view that this slowdown roughly coincides with the first OPEC oil shock. For example, Perron (1989) identifies 1973 as the date of a break in the trend growth of U.S. quarterly real GNP. Using multivariate techniques, Bai, Lumsdaine and Stock (1998) find evidence of a reduction in the growth rate of the common stochastic trend shared by real GNP and consumption, dating the break to the late

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<sup>3</sup> Note that the two idiosyncratic components share the same Markov-switching state variable, introducing a source of common dynamics into these “idiosyncratic” components. The model could be modified so that the regime shifts enter the common transitory component instead. We make the former modeling choice to avoid having the loading factor on the common transitory component scale both the variance of shocks to the common transitory component and the size of the effect of the regime shifts.

<sup>4</sup> See also Beaudry and Koop (1993) and Sichel (1994).

1960's. To account for this productivity slowdown we allow for a reduction in the average growth rate of trend beginning in 1973.<sup>5</sup> This is accomplished by defining:

$$\begin{aligned}\mu_0^* &= \mu_0 + \mu^k DU1_t \\ \mu_1^* &= \mu_1 + \mu^k DU1_t\end{aligned}\tag{5}$$

where  $DU1_t$  is 0 before the first quarter of 1973 and 1 thereafter. The second structural change we consider is in the volatility of U.S. real GNP, which has seen a marked reduction in the last 20 years. Kim and Nelson (1999b) and McConnell and Perez-Quiros (2000) both date this break to 1984. To account for this volatility reduction we define:

$$\begin{aligned}\sigma_\varepsilon^* &= \sigma_\varepsilon(1 - DU2_t) + \sigma_\varepsilon^k DU2_t \\ \sigma_v^* &= \sigma_v(1 - DU2_t) + \sigma_v^k DU2_t\end{aligned}\tag{6}$$

where  $DU2_t$  is 0 before the first quarter of 1984 and 1 thereafter.<sup>6</sup>

## 2.2 Modeling the Relationship between Regime Shifts in the Permanent and Transitory Components

In this subsection we discuss the methodology used to allow the timing of regime shifts in the permanent and transitory components to be correlated. Note that each of  $S_t^P$  and  $S_t^T$  can take on one of two values, 0 or 1, corresponding to expansion or recession. Therefore,  $S_t^P$  and  $S_t^T$  as a pair can take on one of four different combinations. It will be useful to think in terms of this four combination, or four-state model:

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<sup>5</sup> Preliminary estimation suggested that if a productivity slowdown is not incorporated the autoregressive dynamics of  $z_t$  are very persistent. This is consistent with Perron's (1989) finding that unit root tests are biased towards non-rejection if a break in mean growth has occurred and is not allowed. Our results are robust to dating the structural break to the late 1960's, as suggested by Bai, Lumsdaine and Stock (1998).

<sup>6</sup> We could also include a structural break in the variances of the shocks to the idiosyncratic components. However, based on a likelihood ratio test, we can not reject the null hypothesis that these variances are stable at the 10% level. By contrast, the structural break in the variances of the shocks to the common trend and transitory components are highly statistically significant.

Value of $S_t^P$	Value of $S_t^T$	Interpretation
0	0	Expansion
0	1	Recession State for Transitory Component Only
1	0	Recession State for Permanent Component Only
1	1	Recession State for Both Components

We assume that the four states above evolve according to a first-order Markov process with the following sixteen transition probabilities:

$$P(S_t^P = i, S_t^T = j | S_{t-1}^P = k, S_{t-1}^T = q), \quad i, j, k, q = 0,1 \quad (7)$$

For particular realizations of  $S_t^P$  and  $S_t^T$  these can be represented with the notation,  $p_{S_t^P S_t^T | S_{t-1}^P S_{t-1}^T}$ .

For example,  $p_{10|01}$  would correspond to  $P(S_t^P = 1, S_t^T = 0 | S_{t-1}^P = 0, S_{t-1}^T = 1)$ . These transition

probabilities are summarized in the following table in which the  $m, n$ 'th element is the

probability of moving to the value of  $S_t^P$  and  $S_t^T$  specified in row  $m$  given that the values of  $S_{t-1}^P$

and  $S_{t-1}^T$  were as in column  $n$ :

	$(S_{t-1}^P = 0, S_{t-1}^T = 0)$	$(S_{t-1}^P = 0, S_{t-1}^T = 1)$	$(S_{t-1}^P = 1, S_{t-1}^T = 0)$	$(S_{t-1}^P = 1, S_{t-1}^T = 1)$
$(S_t^P = 0, S_t^T = 0)$	$p_{00 00}$	$p_{00 01}$	$p_{00 10}$	$p_{00 11}$
$(S_t^P = 0, S_t^T = 1)$	$p_{01 00}$	$p_{01 01}$	$p_{01 10}$	$p_{01 11}$
$(S_t^P = 1, S_t^T = 0)$	$p_{10 00}$	$p_{10 01}$	$p_{10 10}$	$p_{10 11}$
$(S_t^P = 1, S_t^T = 1)$	$p_{11 00} = 1 - p_{00 00} - p_{01 00} - p_{10 00}$	$p_{11 01} = 1 - p_{00 01} - p_{01 01} - p_{10 01}$	$p_{11 10} = 1 - p_{00 10} - p_{01 10} - p_{10 10}$	$p_{11 11} = 1 - p_{00 11} - p_{01 11} - p_{10 11}$

These transition probabilities allow for two kinds of interdependence between  $S_t^P$  and  $S_t^T$ . The first is that the evolution of  $S_t^P$  and  $S_t^T$  depends on both  $S_{t-1}^P$  and  $S_{t-1}^T$ , so that lagged

values of both states influence a state's current value. Second,  $S_t^P$  and  $S_t^T$  are allowed to be contemporaneously correlated conditional on lagged values of the states.

### **3. Empirical Results**

#### *3.1 Data*

The data are quarterly observations on 100 times the logarithm of U.S. real GNP and U.S. real consumption of non-durables and services. The latter series was constructed from total consumption and consumption of durable goods using the Tornqvist approximation to the ideal Fisher index described in Whelan (2000). The data span from the first quarter of 1952 to the second quarter of 2003.

#### *3.2 Evidence on Integration and Cointegration*

The model in Section 2 imposes a common stochastic trend in the logarithm of output and consumption. Thus, we are interested in testing for a unit root in each of these series, and for cointegration between the series. Table 1 presents details of such tests. Based on the Augmented Dickey-Fuller (ADF) test developed by Dickey and Fuller (1979) and Said and Dickey (1984), we fail to reject the null hypotheses that the logarithm of real GNP and consumption are integrated at the 10% level. With regards to cointegration, the neoclassical growth theory that motivates the cointegration of the logarithms of real GNP and consumption gives a theoretical cointegrating vector of (1,-1), suggesting the difference between these series will be stationary. In this case, one approach to test for cointegration, advocated by Stock (1994), is simply to apply ADF tests to the difference between the logarithm of real GNP and real consumption of non-durable goods and services. Based on this test, we reject the null

hypothesis of no cointegration at the 1% level. This is consistent with the results of other investigations of the cointegration properties of output and consumption, such as King, Plosser, Stock and Watson (1991), Bai, Lumsdaine, and Stock (1998) and Stock and Watson (1999).<sup>7</sup>

### *3.3 Maximum Likelihood Estimation*

The model described in Section 2 is estimated via Kim's (1994) approximate maximum likelihood algorithm, implemented in Gauss 6.0 using the "Optmum" numerical optimization procedure. It is well known that maximum likelihood estimation of regime-switching models is plagued by complicated likelihood functions with numerous local maxima. To provide some reassurance that our estimates represent the global maximum, we estimated the model with 100 different sets of starting values for the model parameters.<sup>8</sup> These starting values were determined as follows: Preliminary investigation indicated that the ability of the numerical optimization routine to converge was very sensitive to starting values for  $\mu_0$ , the expansion growth rate for the common stochastic trend, and  $\gamma_x$ , the loading factor for consumption on the common stochastic trend. In particular, starting values for these parameters far from "reasonable" values almost always resulted in a failure to converge. Thus, for each set of starting values, these parameters are set equal to  $\mu_0 = 0.8$ , which is very close to the mean quarterly growth rate for real GNP and real consumption of non-durables and services over the sample, and  $\gamma_x = 1$ , which is the value implied by theory. Also, because we are interested in regime switching related to the business cycle, each set of starting values for the transition

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<sup>7</sup> Evans and Lewis (1993) show that cointegration tests can be biased in favor of the null hypothesis if a series in the cointegrating equation undergoes Markov regime switching. Since we reject the null hypothesis this does not seem to be a significant problem in this case.

probabilities imply an expected length of expansion and recession equal to the average length of post-war NBER expansion and recession phases. The value of each remaining parameter is then drawn from a  $N(0,1)$  distribution for each set of starting values. The parameter estimates corresponding to the highest converged likelihood value from these 100 estimations are then taken as our maximum likelihood estimates.

### *3.4 Estimation Results and the Relationship between Permanent and Transitory Components*

In this subsection we describe the estimation results for the model described in Section 2. We present results for the estimation in which the lag orders of  $\phi(L)$ ,  $\psi_y(L)$  and  $\psi_c(L)$  are each set equal to two. This choice was based on likelihood ratio tests suggesting that higher order lags are statistically insignificant. Table 2 presents the maximum likelihood parameter estimates. We are particularly interested in the estimates of the model's regime-switching parameters. The estimates of  $\tau_y$  and  $\tau_c$  are -1.5 and -0.9, implying that, when  $S_t^T = 1$ , the transitory components of GNP and consumption are reduced by 1.5 and 0.9 percent each quarter respectively. In the permanent component,  $\mu_1$  is estimated to be less than  $\mu_0$  by 0.5, suggesting high and low (although still positive) growth phases for the trend component. In sum, these parameter estimates suggest that the occurrence of  $S_t^P = 1$  or  $S_t^T = 1$  is characterized by a large reduction in the level of real GNP from what would have obtained had the state not occurred.

We turn next to the timing of the regime switches in  $S_t^P$  and  $S_t^T$ . Figure 1 shows the filtered probability that either  $S_t^P$  or  $S_t^T$  is one, given by  $P(S_t^P = 1 \cup S_t^T = 1 | t) =$

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<sup>8</sup> Recall, the model presented in Section 2 has several parameter constraints pertaining to variances, autoregressive parameters and probabilities. The values of the starting values described here are for unconstrained parameters, which are then converted to constrained values before evaluating the likelihood function.

$P(S_t^P = 1, S_t^T = 0 | t) + P(S_t^P = 0, S_t^T = 1 | t) + P(S_t^P = 1, S_t^T = 1 | t)$ , along with shading indicating NBER recession phases. From the figure,  $P(S_t^P = 1 \cup S_t^T = 1 | t)$  is close to 1 during every NBER recession. However, the probability is also high during many NBER expansion quarters. Figure 2 plots the filtered probability that  $S_t^T = 1$ , given by  $P(S_t^T = 1 | t) = P(S_t^P = 0, S_t^T = 1 | t) + P(S_t^P = 1, S_t^T = 1 | t)$ , while Figure 3 plots the filtered probability that  $S_t^P = 1$ , given by  $P(S_t^P = 1 | t) = P(S_t^P = 1, S_t^T = 0 | t) + P(S_t^P = 1, S_t^T = 1 | t)$ . These figures demonstrate that  $P(S_t^T = 1 | t)$  is highly correlated with NBER recession and expansion dating, while  $P(S_t^P = 1 | t)$  is high during some NBER expansion quarters. This demonstrates that  $P(S_t^P = 1, S_t^T = 0 | t) = 1$ , the probability that only  $S_t^P = 1$ , is responsible for the high values of  $P(S_t^P = 1 \cup S_t^T = 1 | t)$  outside of NBER recession phases. Recall from Table 2 that the growth rate for the common stochastic trend component is still positive when  $S_t^P = 1$ , that is  $\mu_1^* > 0$ , suggesting that these non-NBER recession episodes for the common stochastic trend are consistent with “growth recessions”.

We turn now to an examination of the dynamic relationship between switches in the permanent and transitory component from expansion to recession, that is between  $S_t^P$  and  $S_t^T$ . Our first task is to evaluate the statistical significance of the correlation between these state variables. To do so, we compare the estimated model to two alternative models that make opposite assumptions regarding the correlation between  $S_t^P$  and  $S_t^T$ . In the first,  $S_t^P$  and  $S_t^T$  are assumed to be independent, so that the stochastic process for  $S_t^P$  and  $S_t^T$  can be completely

described based on their own lagged values. That is, we estimate transition probabilities of the form:

$$P(S_t^r = w | S_{t-1}^r = l), \quad r = P, T; \quad w, l = 0, 1 \quad (8)$$

Here, there are eight transition probabilities (four that must be estimated), which can be used to recover the 16 transition probabilities in (7) as follows:

$$P(S_t^P = i, S_t^T = j | S_{t-1}^P = k, S_{t-1}^T = q) = P(S_t^P = i | S_{t-1}^P = k) * P(S_t^T = j | S_{t-1}^T = q) \quad (9)$$

The log likelihood for this restricted model is -323.5, which yields a likelihood ratio test statistic for the null hypothesis that  $S_t^P$  and  $S_t^T$  are independent of 20.4. Given the 8 additional parameters in the unrestricted model, this test statistic has a p-value of 0.01, suggesting the null hypothesis of the independent model is strongly rejected.

The second comparison model assumes that  $S_t^P$  and  $S_t^T$  are perfectly correlated, so that  $S_t^P = S_t^T = S_t$ . In this case, there are four transition probabilities (two that must be estimated), given by:

$$P(S_t = w | S_{t-1} = l); \quad w, l = 0, 1 \quad (10)$$

The log-likelihood for this restricted model is -325.6, which yields a likelihood ratio statistic of 24.8 when compared to the model with unrestricted probabilities. If we assume a  $\chi^2$  distribution, this test statistic has a p-value less than 0.01. However, an asymptotic  $\chi^2$  distribution is not valid as this test is subject to the Davies (1977) “problem” of unidentified nuisance parameters under the null hypothesis. Specifically, under the null hypothesis of perfect correlation, any transition probability  $p_{ijklq}$  for which  $i \neq j$  or  $k \neq q$  is not identified.

Nevertheless, we take the large value of this test statistic as suggestive of substantial evidence in favor of the model with unrestricted correlations.

Given this statistically significant dependence, what is the dynamic relationship between  $S_t^P$  and  $S_t^T$  that is captured by the model? To begin, Table 3 shows the estimated four-state transition probability matrix, which can be used to trace out a pattern for  $S_t^P$  and  $S_t^T$  over the business cycle. The first column of Table 3 shows how recessions begin. When the economy was in an expansion last period, that is  $S_{t-1}^P = S_{t-1}^T = 0$ , the economy tends to stay in the expansion:  $S_t^P = S_t^T = 0$  with probability 0.88 ( $p_{0000} = 0.88$ ). The probability that a recession begins with both the transitory and permanent component switching at the same time ( $p_{1100}$ ), or just the transitory component switching ( $p_{0100}$ ), are both estimated to be zero to the third decimal place. Therefore, recessions begin with a switch of the permanent component to its recession state. In other words, recessions begin with a reduction in the average growth rate of the stochastic trend shared by output and consumption.

While this result is quite striking, the parameter estimates in Table 3 say nothing about the statistical significance of the result. To evaluate this significance, we estimate a model in which the three paths by which a recession can begin are restricted to be equal, so that  $p_{0100} = p_{1000} = p_{1100}$ .<sup>9</sup> The maximized likelihood value for this restricted model is -316.8, which yields a likelihood ratio statistic of 7.1 and a p-value of 0.029. Thus, it appears that the evidence for the permanent component leading the transitory component into recessions is fairly strong.

We now trace out the remainder of a recession episode once the permanent component growth slowdown has begun. The third column of Table 3 indicates that the  $S_t^P = 1$ ,  $S_t^T = 0$  state has a 57% chance of persisting ( $p_{1010} = 0.57$ ), while there is a 19% chance that the

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<sup>9</sup> We thank an anonymous referee for suggesting this test.

economy moves back into an expansion ( $p_{00|10} = 0.19$ ). Note that it is possible for the permanent component to switch into and out of its recession state without a corresponding shift in the transitory component, a feature apparent in Figures 1-3. Finally, there is a 24% chance that economy moves into the regime in which only the transitory component is in recession ( $p_{01|10} = 0.24$ ). When this occurs, from the second column we can see that there is then a 13% chance of this state persisting, a 67% chance of moving back into the state in which only the permanent component is in recession, and a 20% chance of moving to the state in which both components are in the recession state. When this occurs, the fourth column of Table 3 indicates that the state moves back to the expansion regime after one quarter with probability one ( $p_{00|11} = 1.00$ ).

### *3.5 Evidence on the Relative Importance of the Permanent and Transitory Components*

The above discussion characterized the correlation between the two recession state variables,  $S_t^P$  and  $S_t^T$ . In this subsection we use the estimated model to obtain measures of the relative importance of the permanent and transitory components for explaining fluctuations in real GNP.

First, we investigate the relative importance of the regime shifts in the permanent and transitory components for explaining output losses in real GNP during recessions. To do so, we perform a simulation experiment in which 1000 recession episodes are generated from the transition probabilities in Table 3. In the simulation we focus only on those recessions for which both the permanent and transitory components enter their recession state. For each recession episode, three counterfactual GNP series are simulated. The first is based on the estimated

parameters from Table 2, with the exception that  $\mu_1 = \mu_0$  and  $\tau_y = 0$ , so that there are no effects of regime shifts in the permanent or transitory components. The second and third simulated GNP series are generated similarly, except that one of either the permanent or transitory component experiences effects from the regime shifts. We then compare the level of real GNP in the last quarter of a recession from the second and third series to that from the first series. This gives the amount of the output loss in the level of real GNP from what it would have been if the recession hadn't occurred that can be attributed to the permanent and transitory component. This simulation experiment suggests that the average output loss resulting from the permanent component during a recession episode is 3.2 percent. Similarly, the average output loss resulting from the transitory component is 4.5 percent, larger than for the permanent component.

This calculation is an average across the historical record of recessions. To analyze the role that the regime shifts in the permanent and transitory components have had in specific recessions, we can view the graphs of the filtered probabilities  $P(S_t^P = i, S_t^T = j | t)$   $i, j = 0, 1$ . Again, Figure 2 plots the filtered probability that the transitory component has shifted into its recession state,  $P(S_t^T = 1 | t)$ , while Figure 3 plots the filtered probability that the permanent component has shifted into its recession state,  $P(S_t^P = 1 | t)$ . These figures demonstrate that both the permanent and transitory component have played a role in most post-war recession, with  $P(S_t^P = 1 | t)$  and  $P(S_t^T = 1 | t)$  each rising above 50%. The only exception is for the 2001 recession, for which only the permanent component appears to enter its recession state.<sup>10</sup>

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<sup>10</sup> Note that for the 1990-1991 recession, the transitory component briefly enters the recession regime, a result that is inconsistent with Kim and Murray (2002), who find that this recession is entirely accounted for by the permanent component. The model in Kim and Murray (2002) differs in several dimensions from that considered here. They consider a four variable system of monthly variables without cointegration whereas we consider a two variable system of quarterly NIPA variables with cointegration. Their sample period also differs from ours. Finally, the state variables in the permanent and transitory components are forced to be independent in their specification. In

What is the relative importance of the permanent and transitory components in explaining the variability in real GNP growth? To answer this question, we simulated 1000 real GNP series from the parameter estimates in Table 2 and the transition probabilities in Table 3. We find that the standard deviation of the growth rate of the permanent component,  $\Delta x_t$ , is 0.45, while the standard deviation of the growth rate of the sum of the common and idiosyncratic transitory components,  $\Delta(z_t + e_{y,t})$  is 0.83. Thus the transitory component is quite important in explaining overall variability in real GNP.

In sum, the evidence from these various measures suggest that both the permanent and transitory component play a role in explaining fluctuations in real GNP both over the business cycle and during recessions, with the transitory component the more important of the two. Note that this stands in contrast to the evidence presented by Beveridge and Nelson (1981), Nelson and Plosser (1982) and Campbell and Mankiw (1987), who find, using linear time series models, that the majority of output fluctuations in the United States are due to permanent shocks. Instead, our results are consistent with recent studies using nonlinear models to investigate this question, such as Kim and Murray (2002) and Kim and Piger (2002).

### *3.6 Evidence on the Dynamics of Consumption*

Cochrane (1994) and Fama (1992) have both argued that aggregate consumption of non-durable goods and services is close to a random walk process, consistent with the permanent income hypothesis (PIH). This, along with the cointegration of consumption and real GNP, suggests that consumption is close to the common stochastic trend shared with real GNP. Are

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unreported work, we estimate a version of our model in which the state variables are assumed independent over the same sample period as Kim and Murray. With this specification we still find that the transitory component enters its

the parameter estimates in Table 3 consistent with this finding? Contrary to the PIH, the loading coefficient on the common transitory component shared with real GNP,  $\gamma_z$ , is statistically significant, although it is relatively small and negative. The variance of idiosyncratic shocks to consumption, given by  $\sigma_{\omega_c}$ , is estimated to be close to zero, a result consistent with the PIH.

The most significant departure from the PIH comes from the  $\tau_c$  parameter, which is estimated to be negative and large in absolute value, suggesting that consumption undergoes substantial transitory shocks during recessions. Overall, these results suggest that consumption contains an important transitory component, particularly during recessions.

#### **4. Conclusion**

In this paper we have investigated the relationship between permanent and transitory components of U.S. recessions in a model that explicitly incorporates business cycle asymmetry. In particular we specify a cointegrated model of real GNP and consumption that separates both series into permanent and transitory components, the dynamics of which are allowed to undergo regime shifts between expansion and recession states. The timing of switches from expansion to recession in the permanent component is allowed to be correlated with those in the transitory component. We find strong evidence of a lead-lag relationship between the switches in the two components. Specifically, the permanent component leads switches in the transitory component when entering recessions.

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recession state during the 1990-1991 recession, suggesting that the difference in results is likely traced to differences in the data used to measure the business cycle.

## References

- Bai, J., R.L. Lumsdaine and J.H. Stock (1998), 'Testing for and dating common breaks in multivariate time series', *Review of Economic Studies*, 65, 395-432.
- Beaudry, P. and G. Koop (1993), 'Do recessions permanently change output?', *Journal of Monetary Economics*, 31, 149-163.
- Beveridge, S. and C.R. Nelson (1981), 'A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle', *Journal of Monetary Economics*, 7, 151-174.
- Campbell, J.Y. and G.N. Mankiw (1987), 'Are output fluctuations transitory?', *Quarterly Journal of Economics*, 102, 857-880.
- Chauvet, M. (1998), 'An econometric characterization of business cycle dynamics with factor structure and regime switching', *International Economic Review*, 39, 969-996.
- Clark, P.K. (1987), 'The cyclical component of U.S. economic activity', *Quarterly Journal of Economics*, 102, 797-814.
- Cochrane, J.H. (1994), 'Permanent and transitory components of GNP and stock prices', *Quarterly Journal of Economics*, 109, 241-263.
- Davies, R.B. (1977), 'Hypothesis testing when a nuisance parameter is present only under the alternative', *Biometrika*, 64, 247-254.
- Dickey, D.A. and W.A. Fuller (1979), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, 74, 427-31.
- Diebold, F.X. and G.D. Rudebusch (1996), 'Measuring business cycles: A modern perspective', *The Review of Economics and Statistics*, 78, 67-77.
- Evans, M.D.D. and K.K. Lewis, 1993, Trend in excess returns in currency and bond markets, *European Economic Review*, 37, 1005-1019.
- Fama, E.F. (1992) 'Transitory variation in investment and output', *Journal of Monetary Economics*, 30, 467-480.
- Friedman, M. (1964), *Monetary Studies of the National Bureau, the National Bureau Enters its 45<sup>th</sup> Year, 44<sup>th</sup> Annual Report*, 7-25, NBER, New York; Reprinted in Friedman, M. (1969), *The Optimum Quantity of Money and Other Essays*, Aldine, Chicago.
- Friedman, M. (1993), 'The "plucking model" of business fluctuations revisited', *Economic Inquiry*, 31, 171-177.

- Hamilton, J.D. (1989), 'A new approach to the economic analysis of nonstationary time series and the business cycle', *Econometrica*, 57, 357-384.
- Harvey, A.C. (1985), 'Trends and cycles in macroeconomic time series' *Journal of Business and Economic Statistics*, 3, 216-227.
- Kim, C.-J. (1994), 'Dynamic linear models with Markov-switching', *Journal of Econometrics* 60, 1-22.
- Kim, C.-J. and C.J. Murray (2002), 'Permanent and transitory components of recessions', *Empirical Economics* 27, 163-183.
- Kim, C.-J. and C.R. Nelson (1999a), 'Friedman's plucking model of business fluctuations: tests and estimates of permanent and transitory components', *Journal of Money, Credit and Banking*, 31, 317-34.
- Kim, C.-J. and C.R. Nelson (1999b), 'Has the U.S. economy become more stable? A Bayesian Based Approach Based on a Markov Switching Model of the Business Cycle', *Review of Economics and Statistics*, 81, 608-616.
- Kim, C.-J. and J. Piger (2002), 'Common stochastic trends, common cycles, and asymmetry in economic fluctuations', *Journal of Monetary Economics*, 49, 1189-1211.
- Kim, M.-J. and J.-S. Yoo (1995), 'New index of coincident indicators: A multivariate Markov switching factor model approach', *Journal of Monetary Economics*, 36, 607-630.
- King, R.G., C.I. Plosser, J.H. Stock and M.W. Watson (1991), 'Stochastic trends and economic fluctuations', *American Economic Review*, 81, 819-840.
- King, R.G., C.I. Plosser and S.T. Rebelo (1988), 'Production, growth and business cycles: II. new directions', *Journal of Monetary Economics*, 21, 309-341.
- Kydland, F.E. and E.C. Prescott, 1982, Time to build and aggregate fluctuations, *Econometrica*, 50, 1345-1370.
- McConnell, M.M. and G.P. Quiros (2000), 'Output fluctuations in the United States: What has changed since the early 1980s?', *American Economic Review*, 90, 1464-1476.
- Morley, J., C.R. Nelson and E. Zivot (2003), 'Why are the Beveridge-Nelson and unobserved-component decompositions of GDP so different?', *Review of Economics and Statistics*, 85, 235-243.
- Nelson, C.R., and Plosser, C.I. (1982), 'Trends and random walks in macroeconomic time series: some evidence and implications', *Journal of Monetary Economics*, 10, 139-162.

- Perron, P. (1989), 'The great crash, the oil shock and the unit root hypothesis', *Econometrica*, 57, 1361-1401.
- Said, S.E., and Dickey, D.A. (1984), "Testing for unit roots in autoregressive-moving average models of unknown order," *Biometrika*, 71, 599-607.
- Sichel, D. E. (1994), 'Inventories and the three phases of the business cycle', *Journal of Business and Economic Statistics*, 12, 269-277.
- Stock, J.H. (1994), 'Unit roots, structural breaks, and trends', in R. Engle and D. McFadden (eds), *Handbook of Econometrics, Volume IV*, Elsevier, Amsterdam, 2740-2843.
- Stock, J.H. and M.W. Watson (1999), 'Business cycle fluctuations in U.S. macroeconomic time series', in J.B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics, Volume 1A*, Elsevier, Amsterdam.
- Whelan, K. (2000), 'A guide to the use of chain aggregated NIPA data', Finance and Economic Discussion Series Paper #2000-35, Federal Reserve Board.

**Table 1: Summary Statistics and Unit Root Tests for Log Real GNP,  $y_t$ ,  
and Log Real Consumption of Non-Durables and Services  $c_t$   
(1952:Q1-2003:Q2)**

	<b>Summary Statistics</b>	
	<b>Mean</b>	<b>Std. Deviation</b>
$100 * \Delta y_t$	0.81	0.97
$100 * \Delta c_t$	0.81	0.49

	<b>Augmented Dickey Fuller Tests<sup>11</sup></b>	
	<b>Dickey Fuller <math>t</math>-Statistic</b>	<b>5% Critical Value</b>
$y_t$	-2.43	-3.43
$c_t$	-1.33	-3.43
$y_t - c_t$	-3.85	-2.88

<sup>11</sup> The Augmented Dickey Fuller equations were estimated with lag length chosen using the BIC, with a maximum of four lags considered. One lag was chosen for each series. Tests for log real GNP and log real consumption of non-durables and services included a time trend and constant in the test regression. Tests for the log GNP / consumption ratio included a constant in the test regression.

**Table 2: Maximum Likelihood Estimates**  
(1952:Q1 – 2003:Q2, Standard Errors in Parentheses)

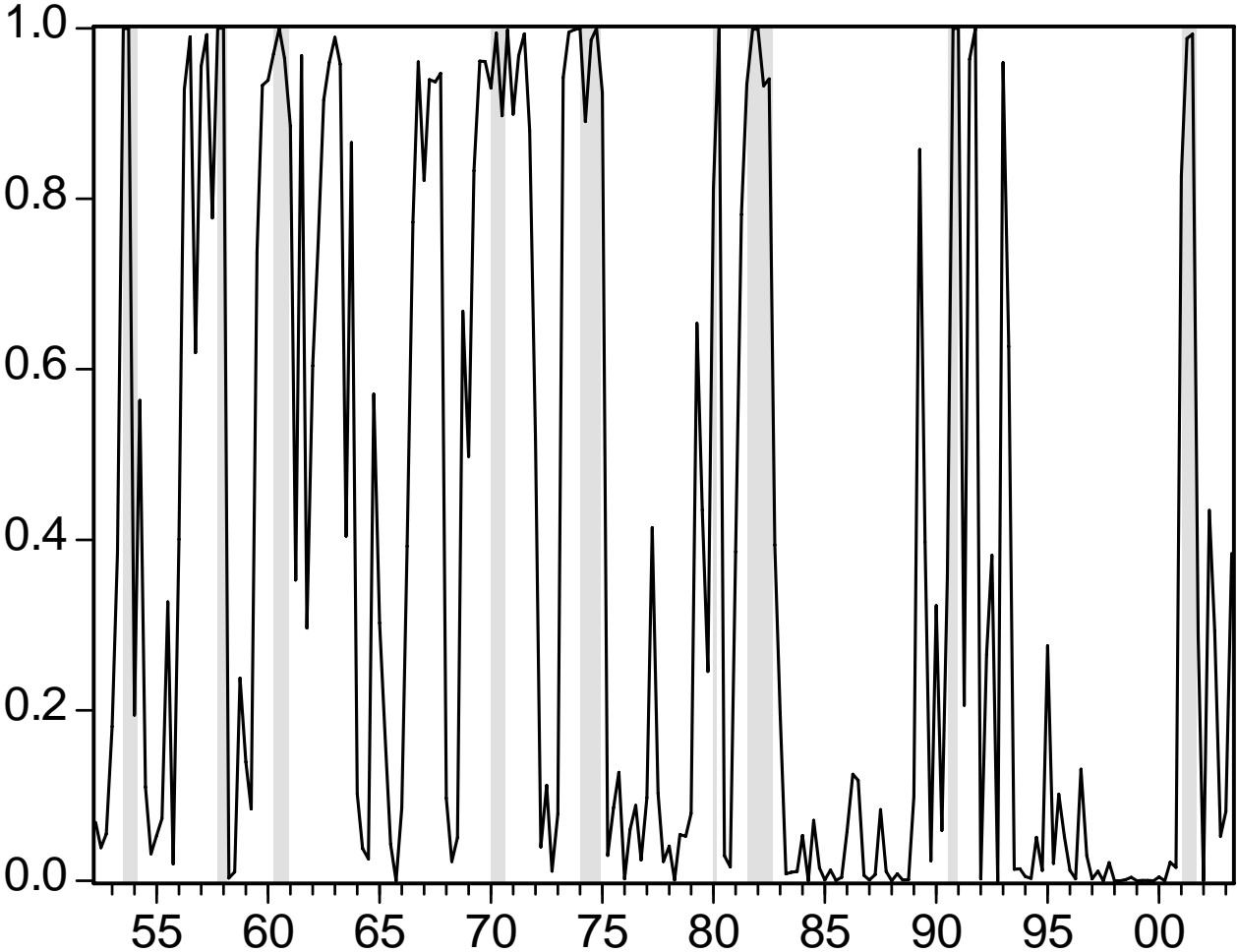
<b>Parameter</b>	<b>Estimate</b>			
$P(S_t^P = 0, S_t^T = 0   S_{t-1}^P = 0, S_{t-1}^T = 0)$	0.88			
	(0.04)			
$P(S_t^P = 0, S_t^T = 1   S_{t-1}^P = 0, S_{t-1}^T = 0)$	0.00			
	(0.00)			
$P(S_t^P = 1, S_t^T = 0   S_{t-1}^P = 0, S_{t-1}^T = 0)$	0.12			
	(0.04)			
$P(S_t^P = 0, S_t^T = 0   S_{t-1}^P = 0, S_{t-1}^T = 1)$	0.00			
	(0.00)			
$P(S_t^P = 0, S_t^T = 1   S_{t-1}^P = 0, S_{t-1}^T = 1)$	0.13			
	(0.18)			
$P(S_t^P = 1, S_t^T = 0   S_{t-1}^P = 0, S_{t-1}^T = 1)$	0.67			
	(0.17)			
$P(S_t^P = 0, S_t^T = 0   S_{t-1}^P = 1, S_{t-1}^T = 0)$	0.19			
	(0.07)			
$P(S_t^P = 0, S_t^T = 1   S_{t-1}^P = 1, S_{t-1}^T = 0)$	0.24			
	(0.09)			
$P(S_t^P = 1, S_t^T = 0   S_{t-1}^P = 1, S_{t-1}^T = 0)$	0.57			
	(0.12)			
$P(S_t^P = 0, S_t^T = 0   S_{t-1}^P = 1, S_{t-1}^T = 1)$	1.00			
	(0.00)			
$P(S_t^P = 0, S_t^T = 1   S_{t-1}^P = 1, S_{t-1}^T = 1)$	0.00			
	(0.00)			
$P(S_t^P = 1, S_t^T = 0   S_{t-1}^P = 1, S_{t-1}^T = 1)$	0.00			
	(0.00)			
$\phi_1, \phi_2$	0.66	-0.03		
	(0.15)	(0.09)		
$\Psi_{y1}, \Psi_{y2}, \Psi_{c1}, \Psi_{c2}$	1.46	-0.51	1.23	-0.28
	(0.07)	(0.07)	(0.13)	(0.12)
$\sigma_{\omega_y}, \sigma_{\omega_c}$	0.17	0.00		
	(0.06)	(0.05)		
$\sigma_\varepsilon, \sigma_\varepsilon^k, \sigma_v, \sigma_v^k$	0.63	0.33	0.33	0.22
	(0.07)	(0.05)	(0.03)	(0.02)
$\tau_y, \tau_c$	-1.46	-0.93		
	(0.19)	(0.13)		
$\mu_0, \mu_1, \mu^k$	1.26	0.72	-0.47	
	(0.07)	(0.08)	(0.08)	
$\gamma_x, \gamma_z$	1.00	-0.24		
	(0.01)	(0.06)		
<b>Log Likelihood</b>	<b>-313.25</b>			

**Table 3: Transition Probability Matrix**

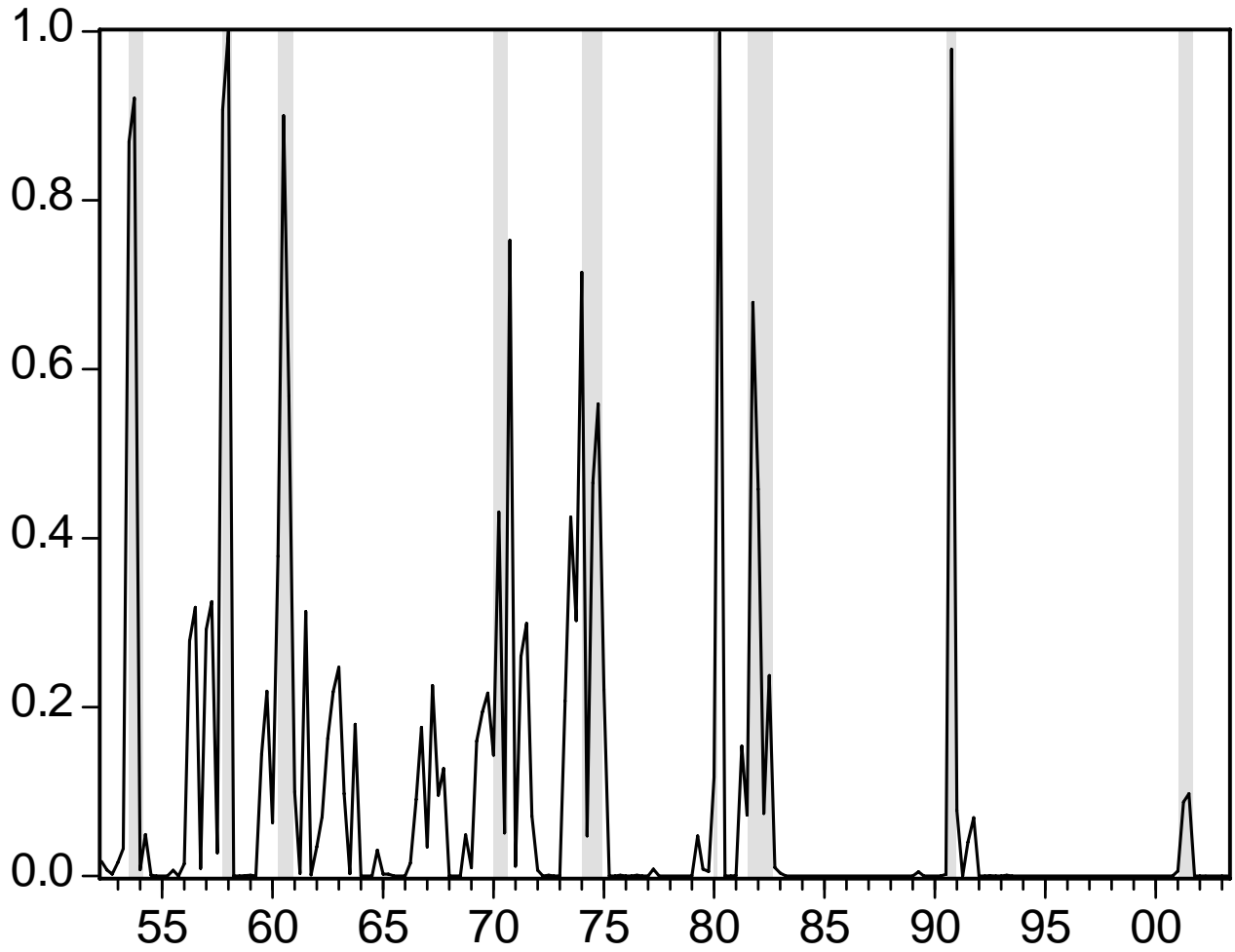
	$(S_{t-1}^P = 0, S_{t-1}^T = 0)$	$(S_{t-1}^P = 0, S_{t-1}^T = 1)$	$(S_{t-1}^P = 1, S_{t-1}^T = 0)$	$(S_{t-1}^P = 1, S_{t-1}^T = 1)$
$(S_t^P = 0, S_t^T = 0)$	0.88	0	0.19	1
$(S_t^P = 0, S_t^T = 1)$	0	0.13	0.24	0
$(S_t^P = 1, S_t^T = 0)$	0.12	0.67	0.57	0
$(S_t^P = 1, S_t^T = 1)$	0	0.2	0	0

**Figure 1: Filtered Probability that  $S_t^P = 1$  or  $S_t^T = 1$ ,  $P(S_t^P = 1 \cup S_t^T = 1 | t)$**

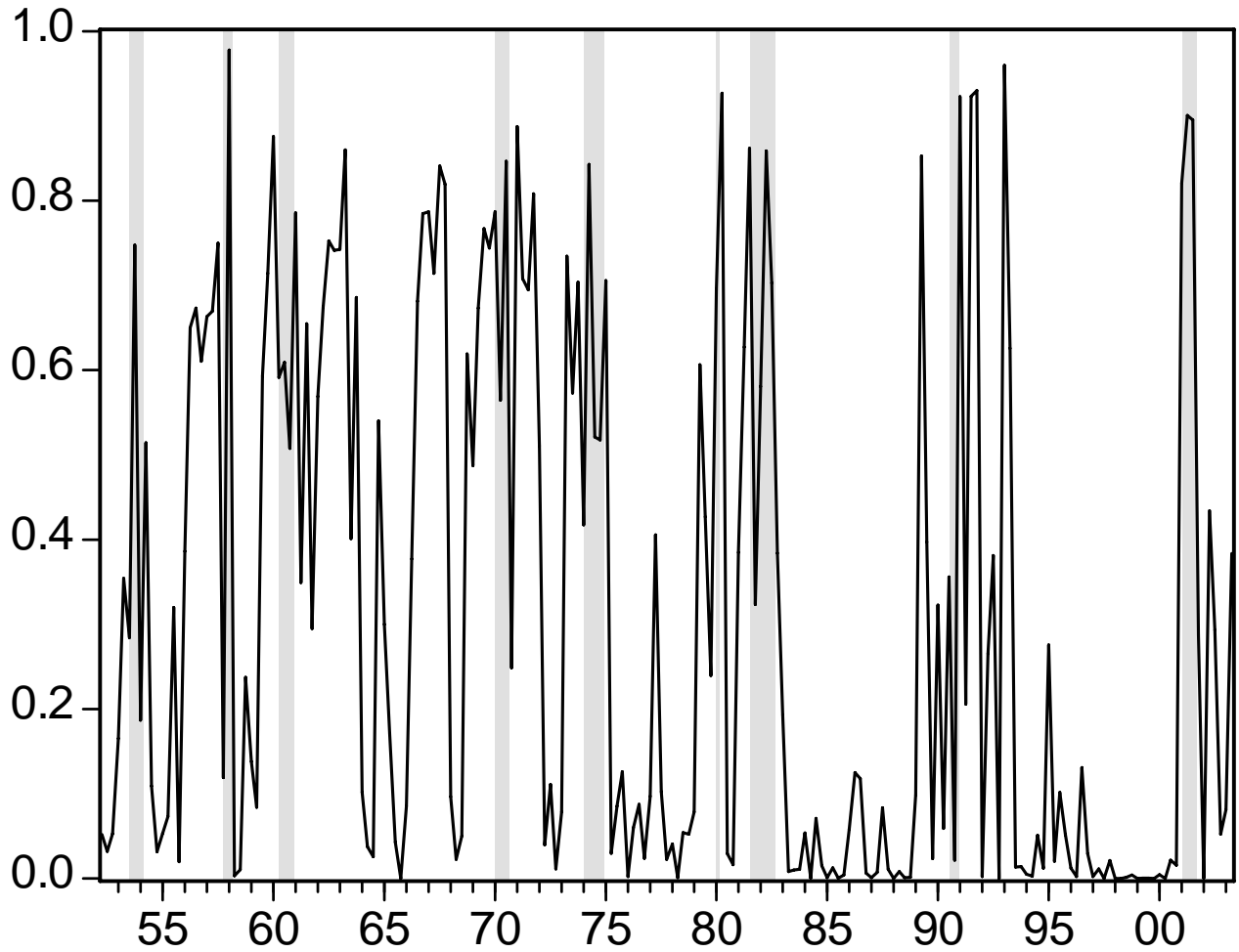
**(1952:Q1 – 2003:Q2, Shaded Areas Indicate NBER Recession Dates)**



**Figure 2: Filtered Probability that  $S_t^T = 1$ ,  $P(S_t^T = 1|t)$**   
(1952:Q1 – 2003:Q2, Shaded Areas Indicate NBER Recession Dates)



**Figure 3: Filtered Probability that  $S_t^P = 1$ ,  $P(S_t^P = 1 | t)$   
(1952:Q1 – 2003:Q2, Shaded Areas Indicate NBER Recession Dates)**



## Appendix: State Space Representation

In this appendix we present the state-space representation used for estimation of the model given in equations 1-6. The state-space representation is written for the case where all transitory dynamics are AR(2).

Observation Equation:

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} \mu_1^* S_t^P + \mu_0^* (1 - S_t^P) \\ \gamma_x (\mu_1^* S_t^P + \mu_0^* (1 - S_t^P)) \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 & -1 & 0 & 0 \\ \gamma_z & -\gamma_z & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} z_t \\ z_{t-1} \\ e_{y,t} \\ e_{y,t-1} \\ e_{c,t} \\ e_{c,t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ \gamma_x v_t \end{bmatrix}$$

Transition Equation:

$$\begin{bmatrix} z_t \\ z_{t-1} \\ e_{y,t} \\ e_{y,t-1} \\ e_{c,t} \\ e_{c,t-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tau_y S_t^T \\ 0 \\ \tau_c S_t^T \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{y,1} & \psi_{y,2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_{c,1} & \psi_{c,2} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{t-1} \\ z_{t-2} \\ e_{y,t-1} \\ e_{y,t-2} \\ e_{c,t-1} \\ e_{c,t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \omega_{y,t} \\ 0 \\ \omega_{c,t} \\ 0 \end{bmatrix}$$

The covariance matrix of the disturbance vector in the observation equation is given by:

$$E \left( \begin{bmatrix} v_t \\ \gamma_x v_t \end{bmatrix} \begin{bmatrix} v_t & \gamma_x v_t \end{bmatrix} \right) = \begin{bmatrix} 1 & \gamma_x \\ \gamma_x & \gamma_x^2 \end{bmatrix} \sigma_v^{*2}$$

Finally, we have the covariance matrix of the disturbance vector in the transition equation:

$$E \left( \begin{bmatrix} \varepsilon_t \\ 0 \\ \omega_{y,t} \\ 0 \\ \omega_{c,t} \\ 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t & 0 & \omega_{y,t} & 0 & \omega_{c,t} & 0 \end{bmatrix} \right) = \begin{bmatrix} \sigma_\varepsilon^{*2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\omega_y}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\omega_c}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$