WORKING PAPER SERIES

The Transition from a Pay-As-You-Go to a Fully-Funded Social Security System: Is There a Role for Social Insurance?

Patricia S. Pollard
Rowena A. Pecchenino

Working Paper 1997-022A
<http://research.stlouisfed.org/wp/1997/97-022.pdf>

FEDERAL RESERVE BANK OF ST. LOUIS

Research Division
411 Locust Street
St. Louis, MO 63102

The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Federal Reserve Bank of St. Louis Working Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.

Photo courtesy of The Gateway Arch, St. Louis, MO. www.gatewayarch.com

The Transition From A Pay-As-You-Go To A Fully-Funded Social Security System: Is There A Role For Social Insurance?

May 1998

Abstract

This paper develops a model to examine the effects of introducing a partially or fully privatized social security system into an overlapping generations model with an extant pay-as-you-go system. Privatization is introduced through mutual funds that provide an actuarially fair annuity contract as part of a portfolio of assets that bears a random rate of return. We find that in the presence of this investment risk maintaining a portion of the pay-as-you-go system through a means-tested minimum retirement income guarantee may be welfare improving.

Keywords: Social-Security, Annuities, Aging

JEL Classification: D9, E6, H2, H5

Patricia S. Pollard
Senior Economist
Federal Reserve Bank of St. Louis
411 Locust Street
St. Louis, MO 63102
pollard@stls.frb.org

Rowena A. Pecchenino
Professor
Department of Economics
Michigan State University
East Lansing, MI 48824
rowenap@pilot.msu.edu

We would like to thank the seminar participants at Michigan State University, the Universite' de Montreal and Oakland University for their comments.

I. Introduction

Recent suggestions for Social Security reform vary from doing nothing and hoping for the best to total privatization. Consensus has been building for an intermediate approach characterized by partial privatization. While partial privatization means different things to different people (Passell, 1998), what is generally agreed upon is that, as President Clinton said in his public address in Kansas City, “Social Security must provide a benefit that people can count on. Regardless of the ups and downs of the economy or the financial markets, we have to provide a solid and dependable foundation of retirement security” (Stevenson, 1998).

In this paper we develop a model that allows us to examine the effects of introducing partially or fully privatized Social Security into an overlapping generations model with an extant pay-as-you-go Social Security system. In our model the privatized portion of Social Security is comprised of mutual funds which provide an actuarially fair annuity contract¹ as part of a portfolio of assets which bears a random rate of return.² We examine whether in the presence of investment risk individual and social welfare can be improved by phasing out the current pay-as-you-go Social Security system and replacing it, all or in part, with a fully-funded system in which the privatized portion of Social Security is at least partially annuitized, and the pay-as-you-go benefits may be means tested. Fundamentally, we seek to answer the question put to President Clinton that if some of Social Security’s funds are placed in private accounts “are we really socially secure and not taking out that security piece” (Stevenson, 1998), that is, is there a role for social insurance?

¹ The pensions we propose are actuarially fair annuities, à la Sheshinski and Weiss (1981), since previous work (Eichenbaum and Peled, 1987; Sheshinski and Weiss, 1981; Abel, 1986; Eckstein *et al*, 1985a, b; and Townley and Boadway, 1988) suggests that social welfare can be improved by increasing the availability of actuarially fair annuity contracts.

² Because the agents in our model are ex ante equivalent, there are no adverse selection nor moral hazard problems.

II. The Model

The model is an extension of Pecchenino and Pollard (1997). Consider a Diamond (1965) style overlapping generations, infinite-horizon economy comprised of identical two-period lived agents, perfectly competitive firms, annuity markets, and a government. A new generation is born at each date. For simplicity we assume that there is no population growth, and that at each date N agents are born. Without loss of generality assume N is unity. This assumption enables us to concentrate attention on the age-dependency ratio, the ratio of retirees to workers, rather than on fertility trends.³

Agents in this model, as in Eckstein, et al. (1985a), are not altruistic: the old do not care for the young and the young do not care for the old. Agents' preferences are defined over consumption alone. Agents in the first period of their lives, the young, are endowed with one unit of labor that they supply inelastically to firms. They divide their wages between their own current consumption, saving (comprised of either an annuity, direct holdings of capital, or both) for consumption when old, and payment of Social Security taxes, τ , quoted as a proportion of their wages. Agents in the final period of their lives, the old, supply their savings inelastically to firms and consume their Social Security benefits and their accumulated savings. An agent dies at the onset of old age with probability $(1-p(t))$ and lives throughout old age with probability $p(t)$. If an agent dies "young", his unannuitized wealth is bequeathed to his children.⁴

³ We have also modeled the aged dependency ratio assuming a growing (or shrinking) population of young agents and a population of old agents whose life expectancy is indexed by p , as in the current model. Since this extension did not change the qualitative results, we assume that the population of young agents is constant at unity.

⁴ This assumption of unintentional rather than altruistic bequests is consistent with empirical findings by numerous researchers: see Hurd (1990), and Auerbach *et al.* (1992), as well as the empirical findings of Altonji *et al.* (1992) that parents and their adult children are not altruistically linked. But, other researchers find an operative bequest motive (Hamermesh and Menchik, 1987; Hurd, 1995), at least among the wealthy. Laitner and Juster (1996) find support for intergenerational altruism but note that it is not the major explanation for saving. Since there is not consensus on this issue, we will maintain the assumption of unintentional bequests.

Agents save for retirement by purchasing mutual funds comprised of direct holdings of capital as well as actuarially fair annuity contracts. Agents face two forms of risk: life span and investment. Investment risk is modeled as follows. There are many possible, observationally equivalent, mutual funds in which an agent can invest. The funds hold identical portfolios, but with probability $\pi(t)$ the mutual fund will earn a high return, and with probability $(1-\pi(t))$ the fund will earn a low return. The low-return funds earnings are a fraction, $(1-\varepsilon(t))$, of those of the high-return funds. The earnings reduction is modeled as a deadweight loss.⁵ This structure gives us two groups of old agents: the rich old and the poor old. There are other interpretations of why agents who were identical in youth may have different incomes in old age. They include catastrophic health shocks, etc. What is important for our purposes is that agents cannot affect the probability of the event, and so our results are generated neither by moral hazard nor adverse selection.

Let the representative member of generation t 's preferences be represented by

$$U(t) = \max \ln c_t(t) + p(t)[\pi(t)\ln c_t(t+1)^H + (1-\pi(t))\ln c_t(t+1)^L]$$

where $c_t(t)$ is consumption by a member of generation t when young, $c_t(t+1)^H$ [$c_t(t+1)^L$] is consumption by a member of generation t when old if his saving were placed in a high-return (low-return) fund, and $p(t)$ is the probability that an agent born at date t will live throughout old age.

The representative agent at time t takes as given the wage, $w(t)$, bequests, $B(t)$, the return on saving when old, $r(t+1)$, the tax rate, $\tau(t)$, Social Security benefits, $T(t+1)$, and the government non-Social Security pension plan. The government licenses mutual funds to provide actuarially

⁵ We also modeled investment risk such that agents would invest in observationally equivalent portfolios, some which would bear high returns, others which would bear low returns, the return differential depending on the, unobservable, differences in the quality of the capital held. Firms would produce using a diversified portfolio of capital, and earn the average return. This alternative modeling strategy, while removing the dead weight loss, does not affect the qualitative results.

fair annuities, but restricts the allocation of the mutual fund between annuitized and unannuitized assets so that at most $\gamma(t)$ percent of the fund can be annuitized. Because fund returns are random, only $\pi(t)$ percent of all, *ex ante*, observationally equivalent funds will earn high returns. The government can tax Social Security benefits at the rate $\sigma(t+1)$, and this tax can be applied differentially. Further, the pay-as-you-go Social Security benefit formula can take one of two forms: it can be stated as a replacement rate and/or as an income guarantee. These different formulas are discussed in more detail below.

The representative agent chooses saving, $s(t)$, to maximize

$$(1) \max \ln c_t(t) + p(t)[\pi(t)\ln c_t(t+1)^H + (1-\pi(t))\ln c_t(t+1)^L]$$

subject to

$$(2) \quad c_t(t) = w(t)(1-\tau(t)) - s(t) + B(t)$$

$$(3a) \quad c_t(t+1)^H = (1 + r(t+1) + \alpha(t+1))s(t) + T^H(t+1)(1-\sigma(t+1))$$

$$(3b) \quad c_t(t+1)^L = (1 + r(t+1) + \alpha(t+1))(1-\epsilon(t))s(t) + T^L(t+1)$$

$T^H(t+1)$ and $T^L(t+1)$ will differ if the program includes an income guarantee, and only the Social Security benefits of the well-off old meet the means test and so are taxed. Constraint (2) encompasses the assumption that bequests are allocated equally across all members of a generation, as in Hubbard and Judd (1987). This assumption ensures that bequests do not induce a nontrivial wealth distribution onto this economy comprised of representative agents, and does not introduce bequest risk into the model. Also, the return on saving/mutual fund investments in constraints (3a) and (3b) is stated as the sum of the return to direct holdings of capital, $1+r(t+1)$, and the excess return, prorated over all saving (the entire mutual fund), of holding $\gamma(t)$ percent of saving as an annuity, $\alpha(t+1)$. Since, as Yaari (1965) and Shenshinski and Weiss (1981) show, agents without a bequest motive would prefer to annuitize all their wealth, they will annuitize

their wealth up to the legal restriction. Further, even if annuity contracts are not actuarially fair, they will be purchased so long as their returns dominate those on direct holdings of capital.

Substituting constraints (2), (3a) and (3b) into the objective function (1) and maximizing yields the first-order condition for an interior solution

$$(4) \quad \frac{-1}{w(t)(1-\tau(t)) - s(t) + B(t)} + \frac{p(t)\pi(t)(1+r(t+1)+\alpha(t+1))}{s(t)(1+r(t+1)+\alpha(t+1)) + T^H(t+1)(1-\sigma(t+1))} + \frac{p(t)(1-\pi(t))(1+r(t+1)+\alpha(t+1))(1-\varepsilon(t))}{s(t)(1+r(t+1)+\alpha(t+1))(1-\varepsilon(t)) + T^L(t+1)} = 0$$

The firms are perfectly competitive profit maximizers that produce output using the Cobb-Douglas production function $Y(t) = A(t)K(t)^\beta N(t)^{1-\beta}$, $\beta \in [0,1]$. $K(t)$ is the capital stock at t , $N(t)$ is employment at t , and $A(t) > 0$ is a productivity scalar. Capital depreciates fully in the production process. The production function can be written in intensive form as $y(t) = A(t)k(t)^\beta$, where $k(t)$ is the capital-labor ratio. Assume, because of external effects of aggregate capital on productivity, as suggested by Romer (1986), $A(t) = a(t)K(t)^\eta$, $a(t) > 0$, $\eta \geq 0$, so that the aggregate capital stock, $K(t)$, enters the technology as a constant from the perspective of current producers.⁶

The individual firm takes wages, rental rates, and the aggregate stock of capital as given. It hires labor and capital until their marginal products equal their factor prices

$$(5) \quad a(t)K(t)^\eta(1-\beta)k(t)^\beta = w(t)$$

$$(6) \quad a(t)K(t)^\eta\beta k(t)^{\beta-1} = R(t).$$

⁶ We have also modeled growth as being generated by exogenous, labor augmenting technological change. As the results were not qualitatively affected by this alternative assumption, we chose to analyze the simple endogenous growth version of the model.

Because of the assumptions of constant returns production technology and inelastic labor supply, (5) and (6) also define factor market clearing.

The government in this economy can impose Social Security taxes, $\tau(t)$, on the wages of the young at t , and may means test the Social Security benefits of the old and/or provide an income guarantee for the old. It also has the ability to control access to the market for actuarially fair annuity contracts. It has the incentive to do so since, as we show in Pecchenino and Pollard (1997), full annuitization is not, in general, socially optimal. Assume that the government licenses private firms to provide actuarially fair annuities as part of a mutual fund, but restricts the proportion of the fund that can be annuitized to $\gamma(t)$, $\gamma(t) \in [0,1]$. The funds are identical *ex ante*.

The government must maintain a balanced budget. If an agent's income when old exceeds a government specified minimum, then Social Security benefits are specified as a replacement rate on current wages. If an agent's income when old, including Social Security benefits, is less than the specified minimum income, because of bad returns on one's investments not because of profligacy, then the agent will receive a supplemental benefit, social insurance, so that his income when old is at the legal minimum. Thus, agents whose income exceeds the minimum get transfers equal to

$$T^H(t+1) = \xi(t)w(t+1)$$

and

$$T^L(t+1) = \xi(t)w(t+1)$$

if even unlucky agents' income exceeds the guaranteed income, while those agents whose income is less than the guarantee, which is specified as a fraction, ζ , of GNP, $a(t+1)k(t+1)^{\eta+\beta}$, receive a transfer of

$$T^L(t+1) = \zeta a(t+1)k(t+1)^{\eta+\beta} - (1+r(t+1) + \alpha(t+1))(1-\varepsilon(t))s(t).$$

The tax rate necessary to generate these funds solves

$$(7) \quad \tau(t+1)w(t+1) = p(t)(1-\sigma(t+1))\pi(t)T^H(t+1) + p(t)(1-\pi(t))T^L(t+1).$$

If an agent dies young the unannuitized portion of his wealth is distributed to his heirs

$$(8) \quad B(t+1) = (1-\gamma(t))(1-p(t))[\pi(t)(1+r(t+1))s(t) + (1-\pi(t))(1+r(t+1))(1-\varepsilon(t))s(t)] \\ = (1-\gamma(t))(1-p(t))(1+r(t+1))s(t)[1-\varepsilon(t)(1-\pi(t))]$$

where $(1-\gamma(t))$ is the percentage of saving not annuitized. The annuitized portion is distributed, pro rata, among the other holders of annuities managed by the same fund. The excess return on that annuity, before the dead-weight loss, is thus

$$(9) \quad \alpha(t+1) = \frac{\gamma(t)(1-p(t))(1+r(t+1))}{p(t)} ..$$

The goods market clears when demand for goods equals supply of goods. Goods market clearing is defined by

$$(10) \quad c_t(t) + p(t)[\pi(t-1)c_{t-1}(t)^H + (1-\pi(t-1))c_{t-1}(t)^L] + s(t) = w(t) + R(t)k(t).$$

Substituting equations (2), (3a,b), (7), (8) and (9) into (10) yields

$$(11) \quad s(t) = k(t+1)$$

where by arbitrage

$$(12) \quad R(t+1) = (1+r(t+1)).$$

III. Equilibrium

A competitive equilibrium for this economy is a sequence of prices $\{w(t), R(t), r(t), \alpha(t)\}_{t=1}^{\infty}$, a sequence of capital stocks, $\{k(t)\}_{t=1}^{\infty}$, $k(1) > 0$ given, a sequence of allocations, $\{c_t(t), c_t^H(t+1), c_t^L(t+1)\}_{t=1}^{\infty}$, and a sequence of taxes $\{\tau(t)\}_{t=1}^{\infty}$, such that given these

prices, allocations, and taxes agents' utility is maximized, firms' profits are maximized, the government budget constraint is satisfied, and markets clear.

The equilibrium is fully characterized by equations (2) - (9), (11) and (12). Solving for $k(t+1)$ as a function of $k(t)$ and parameters describes the dynamic path of the economy.

$$(13) \quad k(t+1) =$$

$$\frac{\theta}{1+\theta} \left[(1-\beta)(1-\tau(t)) + (1-\gamma(t-1))(1-p(t-1))\beta(1-\varepsilon(t-1)(1-\pi(t-1))) \right] a(t)k(t)^{\eta+\beta}$$

where

$$\theta = \frac{p(t)\pi(t)}{1 + \frac{\xi(t)(1-\beta)(1-\sigma(t+1))}{\beta \left[1 + \frac{\gamma(t)(1-p(t))}{p(t)} \right]}} + \frac{p(t)(1-\pi(t))\beta \left[1 + \frac{\gamma(t)(1-p(t))}{p(t)} \right] (1-\varepsilon(t))}{\xi}$$

and

$$\tau(t) = p(t-1)[(1-\sigma(t))\pi(t-1)\xi(t-1) + \frac{(1-\pi(t-1))}{(1-\beta)} \left[\xi - \beta \left(1 + \frac{\gamma(t-1)(1-p(t-1))}{p(t-1)} \right) (1-\varepsilon(t-1)) \right]].$$

IV. Comparative Steady-State/Balanced-Growth Results

In this section we compare the steady-state equilibrium, which requires that $\eta+\beta<1$, and balanced growth, which requires that $\eta+\beta=1$, behavior of economies subject to different government policies, with different expected longevity of the old, and with different levels of investment risk. These comparisons provide us with the intuition we use to construct transition paths from a pay-as-you-go to a partially or fully-funded Social Security system in the following section.

Straightforward differentiation of equation (13) yields the following results.

Result 1 - The Social Security Program: (i) Let $\xi(t) = \xi$ for all t . Then economies with higher Social Security replacement rates, higher ξ , have lower steady-state capital stocks (lower rates of balanced growth). (ii) Let $\zeta(t) = \zeta$ for all t . Then economies with higher income guarantees, higher ζ , have lower steady-state capital stocks (lower rates of balanced growth) if the guarantee is effective. (iii) Let $\sigma(t) = \sigma$ for all t . Then economies with greater means reductions to Social Security benefits, higher σ , have higher steady-state capital stocks (higher rates of balanced growth).

Higher replacement rates (income guarantees) require higher taxes on the young. This reduces their after tax income, thus their saving and so capital accumulation, and, thereby, growth.⁷ A policy of increasing replacement rates, however, can be Pareto improving if the economy is dynamically inefficient and individuals are saving too much relative to the social optimum in the absence of the pay-as-you-go Social Security scheme. The U.S. economy is, however, generally considered to be dynamically efficient, so higher replacement rates, while increasing the utility of the initial generation of old, are most likely to worsen rather than improve welfare in the sense of Pareto. Higher means tested taxes on Social Security benefits of the relatively well-off, however, reduce the Social Security tax on workers' income. Higher after tax income today and lower Social Security benefits tomorrow both induce a higher rate of saving, and thus capital accumulation. Introducing such a change in the system worsens the welfare of the current old to the potential benefit of future generations.

⁷ This result is very common in the literature. See, for example, King and Rebelo (1990), Rebelo (1991), and Saint-Paul (1992).

Result 2 - Mutual Funds: (i) Let $\gamma(t) = \gamma$ for all t . Then economies with higher rates of annuitization, higher γ , have higher or lower steady-state capital stocks (higher or lower rates of balanced growth). (ii) Let $\pi(t) = \pi$ for all t . Then economies with higher probabilities of high portfolio returns have higher steady-state capital stocks (higher rates of balanced growth). (iii) Let $\varepsilon(t) = \varepsilon$ for all t . Then economies with higher deadweight loss ratios, higher ε , have lower steady-state capital stocks (lower rates of balanced growth).

A higher rate of annuitization generates a higher return on one's annuities, which leads to a saving decreasing income effect and a saving increasing substitution effect. The higher rate of annuitization also leads to lower bequests, which reduce saving. The overall effect is ambiguous. Notice that if bequests received and taxes paid by an individual are held fixed, and the allowable annuitization rate rises, then saving rises. Such an increase in saving could offset, all or in part, a reduction in Social Security benefits. This dynamic tradeoff is examined in section V, below.

The increase in the probability of investing in a high-return portfolio reduces investment risk while increasing expected return. Further, the tax on workers' income falls, increasing the after-tax income of the young. The tax effect and the substitution effect dominate the effect of higher income when old, and saving rises. The higher the deadweight loss ratio, however, the higher the tax rate needed to guarantee a minimum income and the lower the expected bequest. Both reduce the income of the young and thus the incentive to save.

Result 3 - Demographics: Let $p(t) = p$ for all t . Then under all plans, economies with greater longevity/aged-dependency ratios, higher p , have higher or lower steady-state capital stocks (higher or lower rates of balanced growth).

Greater longevity has two effects. An increase in life expectancy increases one's motivation to save for a longer life. It also reduces one's bequests and increases one's taxes, which reduces the income out of which one can save. The net effect is ambiguous. However, in the absence of both a pay-as-you-go Social Security system and restrictions on investments in annuities, $\xi = \zeta = \sigma = 0$ and $\gamma = 1$, increases in longevity increase saving. Results 1 and 3 together suggest that phasing out the current Social Security system as the population ages could lead to higher rates of saving and growth.

V. The Transition from a Pay-As-You-Go to a Fully-Funded Social Security System

This section examines the effects of aging on growth and welfare by simulating the model developed in sections II and III. Moreover it also examines the welfare effects of policies that eliminate the pay-as-you-go Social Security system and replace it with a partially or fully-funded system in the presence of investment risk. In particular we determine the conditions under which completely eliminating the pay-as-you-go system is not optimal and there remains a social insurance component of the Social Security program.

We simulate the model by first creating a set of baselines by calibrating the economy to achieve a 2% growth rate per year. Each period, t , is 25 years, roughly equal to the time span of a generation. The parameters for the economy used in the baseline simulation, given in Table 1, are based on empirical estimates for the U.S. economy. The replacement rate, ξ , is based on the replacement rate for individuals retiring in 1990 who earned the average wage while working. The degree of annuitization, γ , reflects the value of private pension funds as a percentage of U.S.

household net wealth.⁸ The aged-dependency ratio, p , was set equal to the current ratio of the population aged 65 and over to the population between the ages of 20 and 64. The value for β reflects capital's share in output. The means-test parameter, σ , is, approximately, the average income tax rate for high income individuals. Making use of the balanced growth restriction determines the value for η used in the simulations.

Table 1 Baseline Parameter Values for the Simulations	
Parameter	Value
ξ	0.43
γ	0.165
β	0.30
η	0.70
p	0.20
σ	0.30

Fourteen possible combinations of π , the probability of investing in a high-return fund, and ε , the reduction in asset earnings resulting from investing in a low-return fund are considered. The first 12 combinations, listed in Table 2, are based on a comparison of the average annual rates of return on 406 mutual funds and the S&P 500 index, over the 10 year period 1986-1996. The last two combinations were included to examine the effects of combining a high probability of investing in a low-return fund with a high earnings reduction from doing so. These two combinations allow us to address the concerns that under a fully-funded system many individuals will make poor investment decisions resulting in low returns.

For each of the 14 combinations the minimum income guarantee percentage, ζ , was varied from 0 - ζ^U , the endogenous upper bound, by increments of .05. The upper bound for ζ

⁸ See Auerbach, Kotlikoff, and Weil (1992). The results presented in this section are invariant to small changes in the value of γ .

was set at the point where the income of the unlucky old equals the income of the lucky old. If ζ were set any higher the income of the unlucky old incorporating the income guarantee would be above the income of the lucky old. Given the parameter values, the path of $a(t)$ in the production function is chosen to produce the desired 2% per annum growth rate.

Table 2 Values for ε and π in Plan C		
Variation	ε	π
1	0.00	0.13
2	0.10	0.25
3	0.20	0.36
4	0.25	0.41
5	0.30	0.48
6	0.40	0.57
7	0.50	0.71
8	0.60	0.93
9	0.70	0.98
10	0.75	0.99
11	0.80	0.99
12	0.90	0.99
13	0.90	0.10
14	0.90	0.50

After calculating the baselines (one for each of the various π and ε combinations), we introduce aging into the population and re-simulate the model, keeping the $a(t)$ at their baseline values. Specifically, we assumed that each generation through those born at $t = j-1$ had a 20% probability of living through old-age. For the generations born in periods j , $j+1$, and $j+2$, the probability of living through old-age increased to .238, .37, and .406, respectively. These

probabilities correspond to the projected aged-dependency ratios for the U.S. in 2015, 2040, and 2065.⁹

Result 4 - Aging and Social Insurance: As the population ages the minimum value for ζ for which the income guarantee is binding falls. Likewise the maximum feasible value for ζ also falls.

As the proportion of the population who live into retirement increases the marginal rate of return on annuities declines. This makes it more likely that the unlucky old agents' income will fall below the minimum income guarantee. Because the decline in the marginal rate of return on annuities also affects the income of the lucky old, it lowers the maximum feasible value for ζ . Table 3 illustrates this result for variations 2 - 14. In variation 1 the reduction in asset earnings if one invests in a low-return fund is zero. Thus, the income guarantee is never effective in this variation.

Table 3				
ζ above which minimum income guarantee binds				
Variation	Baseline	Aging		
		t = j+1	t = j+2	t = j+3
2	.749	.714	.647	.636
3	.699	.668	.608	.599
4	.675	.645	.589	.580
5	.650	.622	.570	.562
6	.600	.576	.532	.524
7	.550	.530	.493	.487
8	.500	.484	.455	.450
9	.450	.439	.416	.413
10	.426	.416	.397	.394
11	.401	.393	.378	.375
12-14	.351	.347	.339	.338
Maximum feasible value for ζ				

⁹ These projections are based on the Board of Trustees of the Federal Old-Age Disability and Survivors Trust Fund (1996).

All variations	.799	.759	.685	.673
----------------	------	------	------	------

Result 5 - Aging and the Current Regime: Holding the replacement rate, ξ , fixed, an aging population generates an increase in the Social Security tax rate. The growth paths of the aging economy and the lifetime utility paths dominate the baseline.

Result 5 holds replacement rates constant and adjusts taxes to ensure that the pay-as-you-go system is in actuarial balance at each date. If the replacement rate is unchanged, increasing the population of retirees relative to workers raises the tax rate required to finance the Social Security benefits. This increase in the tax rate occurs even though an increase in p raises the growth rate of the model economy.

As stated earlier, if wealth is only partially annuitized, $\gamma < 1$, the net effect of an increase in life span on growth is ambiguous. Result 5 indicates that given the parameters in our model, the negative effect on saving of reduced bequests and a lower marginal return on saving are likely to be dominated by the positive effect on saving of an increase in expected longevity. The increase in growth, by raising consumption, raises lifetime utility.

After incorporating the effects of an aging population into the model we examined the possibility of phasing out the pay-as-you-go system and replacing it with a fully funded system. As the previous section indicates, a policy of raising the annuitization rate, γ , and lowering the replacement rate, ξ , may increase saving and thus growth. While such a policy will benefit future generations it will not be welfare enhancing unless the current generation is made better off.

Result 6 - From PAYG to Full-Funding: Increasing γ at $t=j$ and setting $\xi=0$ at $t=j+1$ lowers the minimum values for ζ for which the income guarantee is effective in variations 9-14 and may

raise or lower the minimum values for ζ in variations 2-8. In all variations the maximum feasible values for ζ may increase or decrease.

Increasing the annuitization rate, γ , ceteris paribus, raises the minimum and maximum feasible values for the income guarantee percentage, ζ , while lowering the replacement rate, ξ , has the opposite effects. When $\xi=0$ and $\epsilon \geq .7$ (variations 8-14) the latter effect dominates with respect to the minimum values for ζ for all values of γ . For all other variations, the result depends both on the values of the dead-weight loss percentage, ϵ , and γ . When γ is high the minimum values for ζ rises, with the crucial level of γ falling as ϵ falls.

The maximum feasible values for ζ depend solely on the level of γ in each period. Relative to the maximum feasible values for ζ shown in Table 3, setting $\xi=0$ and raising γ increases ζ when γ is high and reduces ζ when γ is low. These results imply that when $\epsilon \leq .6$ a policy restricting the annuitization of wealth may reduce the need for an income guarantee, if such a policy is Pareto improving. Result 7 examines this issue.

Result 7 - Annuitization Rates and the Transition: It is always possible to find a γ such that increasing γ at $t=j$ and setting $\xi=0$ at $t=j+1$ generates a Pareto superior path for some values of ζ in all variations of the aging economy path.

Raising γ in one period and setting $\xi=0$ in the next period raises the long-run growth rate of the economy and results in a long-run increase in lifetime utility. Restricting our attention to Pareto optimal utility paths for all generations, reduces the range of feasible policies. Table 4 illustrates the Pareto optimal combinations of γ and ζ . As ϵ increases, the extent to which γ must

be raised for the transition path to be Pareto optimal declines. Holding ε constant, the minimum value for γ falls as the probability of investing in a low-return fund, $(1-\pi)$, rises.

Table 4 also indicates that in designing a Pareto optimal transition there may be a trade-off between the annuitization rate and the minimum income guarantee. That is, a low annuitization rate may necessitate a high minimum income guarantee. This is particularly true when there is a high level of investment risk. For example, in variation 14, it is possible to design a Pareto optimal transition policy that increases the annuitization rate to 20 percent if the minimum income guarantee is set at 30 percent of per capita GNP. A policy raising the annuitization rate to 50 percent allows the minimum income guarantee to be reduced to 20 percent.

Table 4 Minimum Values for γ and ζ in a Pareto Improving Transition Path		
Variation	γ	ζ
1-6	.50	0
7	.40	.40
	.50	0
8-12	.40	0
13	.20	.30
	.30	.25
	.40	.20
	.70	.15
14	.20	.35
	.30	.30
	.40	.25
	.50	.20
	.70	.15
	1.00	0

It is not possible in any of the variations to determine a single γ and ζ combination that maximizes lifetime utility across all generations. For a given ζ the generation born at $t=j$ will always prefer full annuitization. In general future generations would prefer a lower annuitization rate. This is because each generation prefers to receive bequests from the previous generation but

has no desire to leave bequests to the next generation. However, when investment risk is high, as in variations 13 and 14 all generations prefer full annuitization. Figures 1a and 1b illustrate these results showing the difference between the lifetime utility paths in the transition economy and the aging economy when γ is .5 or 1. Figure 1a depicts variation 3 with $\zeta=.05$. The generation born at $t=j$ prefers full annuitization while all future generations prefer the lower rate of annuitization. Figure 1b depicts variation 13 when $\zeta=.20$. In this case all generations prefer full annuitization.

Generations also may differ in their policy preferences over ζ for a given γ . Thus, while all generations in variations 13 and 14 prefer $\gamma=1$, the generation born at $t=j$ prefers a high ζ while future generations prefer a low ζ . These results are illustrated for variation 13 in Figure 2. Thus in none of the variations is it possible to choose a single policy (a γ and ζ combination) that is optimal for all generations.

Figures 1 and 2 also highlight the differences in the magnitude of the gains from privatization that accrue to the various generations. Generation j , the generation of workers at the time of the transition, gain the least from privatization. In all of the figures, lifetime utility for this group increases by less than 2 percent. Within a generation the utility gains more than double and continue to increase for future generations

Result 8 - Risk and Social Insurance: As investment risk rises the likelihood that a nonzero Social Security tax will be maintained even with $\xi=0$ increases.

In our model Social Security benefits serve two purposes. They provide retirement income for all the elderly and they provide insurance against low income for the unlucky elderly - those who invested in low-return funds. Setting $\xi=0$ eliminates the first aspect of Social

Security, but as long as the social insurance role is operative $\tau > 0$ is possible. When there is no risk of lower returns, variation 1, the insurance role is unnecessary and thus $\tau = 0$. For all other variations the insurance aspect comes into play if ζ is above some critical value. This value is negatively related to ε . For example, in variation 6, as shown in the left-hand side of Figure 3, if $\zeta \leq .3$ then $\tau = 0$. In variation 12, as shown on the right-hand side of Figure 3 $\tau > 0$ for values of ζ as low as 10 percent.

For a given ζ , increasing γ results in a lower τ as raising the annuitization rate increases the rate of return on investments for the unlucky old and thereby increases the likelihood that the income guarantee will not be needed. Thus, as shown in Figure 3, for variation 6, if $\zeta = .4$, $\tau = 0$ in all periods if there is full annuitization.

The highest tax rates for any ζ occur when a high probability of investing in a low-return fund is combined with a sharp reduction in the rate of return, variations 13 and 14. As shown in Figure 4 in variation 13, if $\gamma = .5$ and $\zeta = .2$ the tax rate is 3.4 percent in period $j+1$ and reaches its long-run value of 7.5 percent in $j+3$.¹⁰ Once again, increasing γ lowers the tax rate, so that when $\gamma = 1$ and $\zeta = .2$ the tax rate is 2.3 percent in period $j+1$ and reaches its long-run value of 6.6 percent in $j+3$. In variation 14, if $\gamma = .5$ and $\zeta = .2$ the tax rate is 1.9 percent in period $j+1$ and reaches its long-run value of 4.2 percent in $j+3$.

In all variations the long-run tax rate with an aging population and $\xi = 0$ is below the tax rate with an aging population that maintains the baseline replacement rate. Thus, even when the proportion of unlucky investors is high and the reduction in investment returns as a result of being unlucky is substantial a privatized system with a minimum income guarantee lowers the

¹⁰ If the replacement rate is unchanged ($\xi = .43$) the model shows the tax rate rising to a long-run value of 16.9 percent in period $j+3$.

tax burden and hence enhances economic growth and lifetime utility relative to the existing pay-as-you go system.

VI. Conclusions

It is well established that in a dynamically efficient economy with overlapping generations a pay-as-you-go Social Security system produces lower growth and welfare than a fully funded system.¹¹ Recently papers by Kotlikoff et al. (1997) and Pecchenino and Pollard (1997) have shown that it is possible to design a strategy that replaces the pay-as-you-go system with a fully-funded system in a Pareto optimal way. In this paper we examine whether the introduction of investment risk changes these results. We find that while it is still possible to eliminate the retirement income aspect of the existing pay-as-you-go system and replace it with a fully-funded pension system, it may be Pareto optimal to maintain the social insurance aspect of the current system. That is, in the face of adequate investment risk a minimum income guarantee financed in a pay-as-you-go manner may be needed. The success of this plan relies crucially on individuals placing a proportion their retirement saving in actuarially fair (or at least rate of return dominant) annuity contracts. Such contracts increase the return to saving, inducing higher saving and offsetting the loss of Social Security benefits and bequests. Unannuitized saving does not have this property.

Our analysis also points out a pitfall of moving from one system to another: initial generations see very small improvements in their welfare, and thus, in the presence of transactions costs, which we do not model, may be unwilling to give up what they know for an untried alternative. But, discussions of Social Security reform have shied away from the most

¹¹ See for example Samuelson (1975), Blanchard and Fisher (1989) and Saint-Paul (1992).

straightforward remedy of all: an immediate increase in taxes. In fact, President Clinton has largely taken consideration of tax increases out of the debate (Stevenson, 1998). Such a plan, however, would make clear the costs of maintaining the current system and, perhaps, make people and their Congressional representatives, more willing to make the difficult choices now.

References

- Abel, A. B., 1986, "Capital accumulation and uncertain lifetimes with adverse selection," *Econometrica* 54, 1079-97.
- Altonji, J. G., F. Hayashi, and L. J. Kotlikoff, 1992, "Is the extended family altruistically linked? Direct tests using micro data," *American Economic Review* 82, 1177-98.
- Auerbach, A. J., L. J. Kotlikoff, and D. N. Weil, 1992, "The increasing annuitization of the elderly - estimates and implications for intergenerational transfers, inequality, and national saving," NBER Working Paper No. 4182.
- Blanchard, O.J. and S. Fischer, 1989, *Lectures on Macroeconomics*, Cambridge: MIT Press.
- Board of Trustees of the Federal Old-Age Disability and Survivors Trust Fund (1996).
- Diamond, P. A., 1965, "National debt in a neoclassical growth model," *American Economic Review* 55, 1126-50.
- Eckstein, Z., M. Eichenbaum, and D. Peled, 1985a, "Uncertain lifetimes and the welfare enhancing properties of annuity markets and Social Security," *Journal of Public Economics* 26, 303-26.
- Eckstein, Z., M. Eichenbaum, and D. Peled, 1985b, "The distribution of wealth and welfare in the presence of incomplete annuity markets," *Quarterly Journal of Economics* 100, 789-806.
- Eichenbaum, M., and D. Peled, 1987, "Capital accumulation and annuities in an adverse selection economy," *Journal of Political Economy* 95, 334-54.
- Hamermesh, D. S. and P. L. Menchik, 1987, "Planned and Unplanned Bequests," *Economic Inquiry* 25, 55-66.
- Hubbard, R. G., and K. L. Judd, 1987, "Social security and individual welfare: precautionary saving, borrowing constraints, and the payroll tax," *American Economic Review* 77, 630-46.
- Hurd, M. D., 1990, "Research on the elderly: economic status, retirement and consumption and saving," *Journal of Economic Literature* 28, 565-637.
- Hurd, M. D., 1995, "Using subjective survival probabilities in life cycle models," mimeo., Department of Economics, SUNY Stony Brook.
- King, R. G., and S. Rebelo, 1990, "Public policy and economic growth: developing neoclassical implications," *Journal of Political Economy* 98, S126-50.
- Kotlikoff, L., K. Smetters and J. Walliser, 1997, "The economic impact of privatizing Social Security," paper presented at the 1997 Kiel Week Conference held at the Kiel Institute of World Economics.

- Laitner, J. and F.T. Juster, 1996, "New Evidence on Altruism: A Study of TIAA-CREF Retirees," *American Economic Review* 86, 893-908.
- Passell, P., 1998, "Partial privatization of Social Security looks very possible," *New York Times*, April 9, D2.
- Pecchenino, R., and P. Pollard, 1997, "The effects of annuities, bequests, and aging in an overlapping generations model of endogenous growth," *Economic Journal* 107, 26-46.
- Rebelo, S., 1991, "Long-run policy analysis and long-run growth," *Journal of Political Economy* 99, 500-21.
- Romer, P., 1986, "Increasing returns and long-run growth," *Journal of Political Economy* 94, 1002-37.
- Saint-Paul, G., 1992, "Fiscal policy in an endogenous growth model," *Quarterly Journal of Economics* 107, 1243-59.
- Samuelson, P., 1975, "Optimum Social Security in a life-cycle growth model," *International Economic Review* 16, 539-44.
- Sheshinski, E., and Y. Weiss, 1981, "Uncertainty and optimal Social Security systems," *Quarterly Journal of Economics* 96, 189-206.
- Social Security Advisory Council, 1996, *Report of the 1994-1996 Advisory Council on Social Security*, Volumes I and II, Washington, D.C.: Social Security Administration.
- Stevenson, R., 1998, "President presses debate on change in Social Security," *New York Times*, April 8, A1.
- Townley, P. G. C., and R. W. Boadway, 1988, "Social security and the failure of annuity markets," *Journal of Public Economics* 35, 75-96.
- Yaari, M. E., 1965, "Uncertain lifetime, life insurance, and the theory of the consumer," *Review of Economic Studies* 32, 137-50.

Figure 1a

Lifetime Utility -- Percent Deviation from Baseline

Variation 3 - $\zeta=.05$

Figure 1b

Lifetime Utility -- Percent Deviation from Baseline

Variation 13 - $\zeta=.20$

Figure 2

Lifetime Utility -- Percent Deviation from Baseline

Variation 13 - $\gamma=.5$

Figure 3. Social Security Tax Rate when $\xi_t = 0$

Variation 6

$\gamma=.5$

Variation 12

$\gamma=.5$

$\gamma=.75$

$\gamma=.75$

$\gamma=1$

$\gamma=1$

— $t=i+1$

..... $t=i+2$

..... $t=i+3$

Figure 4. Social Security Tax Rate when $\xi_t = 0$

Variation 13

$\gamma=.5$

Variation 14

$\gamma=.5$

$\gamma=.75$

$\gamma=.75$

$\gamma=1$

$\gamma=1$

— $t=j+1$

..... $t=j+2$

-.-.- $t=j+3$