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The Cyclical Relationship between Output and Prices: An Analysis in the Frequency Domain

Michael Pakko

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FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

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Abstract

Recent research showing a negative correlation between output and prices has brought into question the conventional wisdom that prices are procyclical. However, this finding has been shown to be sensitive to the sample period considered. This paper examines the relationship in the frequency domain: the covariance of output and prices is decomposed into spectral components to investigate whether the differences in the price-output relationship across sample periods reflect changes in the importance of various frequencies embedded within the correlations, or whether they reflect more fundamental changes in the entire spectral relationship. Some implications for model evaluation are also considered.

Keywords: Inflation, Business Cycles, Spectrum, Linear Filters

JEL Classification: E32, E31

Michael R. Pakko
Economist
Federal Reserve Bank of St. Louis
411 Locust Street
St. Louis, MO 63102
pakko@stls.frb.org

The Cyclical Relationship Between Output and Prices: An Analysis in the Frequency Domain

1. Introduction

The cyclical behavior the price level and its implications for evaluating macroeconomic models have recently been subject to controversy. Dating at least as far back as Burns and Mitchell (1946), it has been taken as an established point of fact that the price level is procyclical. However, a number of studies have recently found that detrended measures of output and prices have displayed a negative correlation over the postwar period. Kydland and Prescott (1990) found both the CPI and the GNP deflator to be negatively correlated with real GNP. This finding was confirmed and shown to be robust to alternative detrending methods by Cooley and Ohanian (1991). Moreover, Backus and Kehoe (1992), Chadha and Prasad (1994), Fiorito and Kollintzas (1994), and Kim (1996) have shown that price fluctuations have also been countercyclical in a number of other countries over recent years.

These findings been interpreted to have important implications for the evaluation of macroeconomic models. Cooley and Ohanian (1991) concluded that "much of the emphasis on developing models that feature a positive relationship between output and prices may have been unnecessary." Kydland and Prescott (1990) stated the case even more forcefully: "any theory in which procyclical prices figure crucially in accounting for postwar business cycle fluctuations is doomed to failure." On the surface, the finding lends support to flexible price, supply-driven models: In such a model, endogenous procyclical money demand fluctuations arising from supply-shock induced movements in output produce inverse movements in the price level.

The observation that prices are countercyclical has not proven to be ubiquitous, however. Table 1 presents cross correlations for GNP and the implicit deflator for subperiods of 1875 to 1994, illustrating that sign of the price-output correlation is sensitive to the sample period examined.[1] As Cooley and Ohanian reported, prices appear strongly procyclical only during the interwar era, while evidence for other periods is mixed.[2] Backus and Kehoe (1991) and Smith (1992) have confirmed that this pattern also characterizes the history of price cyclicality for a number of other countries. Table 1 also shows that the method used to detrend the raw data has an effect on measured correlations [as found by Chadha and Prasad (1993), and Serletis and Krause (1996)]. Finally, Table 1 confirms the caveat of Wolf (1991) that price countercyclicality in the U.S. during the postwar era is only significant for the period since 1973.

The cyclical behavior of prices has also not proven to be a particularly useful criterion for evaluating classes of macro models. Hall (1995) and Judd and Trehan (1995) have demonstrated that demand-driven models with sluggish nominal price adjustment can generate countercylical price behavior. Chadha and Prasad (1993)

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[1] Standard errors are calculated using the normally distributed Fisher Z statistic:

$$Z = \frac{\sqrt{T-3}}{2} \log[\frac{1+\rho}{1-\rho}]$$

[2] In both Cooley and Ohanian (1991) and Smith (1992), evidence of countercyclical prices in the pre-war period is evident when the data are linearly detrended.

show that alternative demand-driven and supply driven models are both capable of generating countercyclical prices. Gavin and Kydland (1996) suggest that endogenous money supply movements can significantly effect the relationship between real and nominal variables, providing a channel through which changes in the behavior of the monetary authority can affect the price-output correlation.

Although the validity of evaluating macro models on the basis of their implications for price cyclicality in any simple way has been shown lacking, the observed patterns of price cyclicality present a challenge to macroeconomists. The data suggests that a model of price-output dynamics should be capable of generating both procyclical *and* countercyclical prices.

This paper seeks to uncover additional information about the dynamic relationship between the price level and output by examining the relationship in the frequency domain: Cospectra of output and prices for various sample periods are used to decompose correlations into their constituent frequency components.[3] The purpose of this decomposition is to investigate whether the differences in price-output correlations across sample periods reflect shifts in the relative importance of various frequencies embedded within the correlation, or whether they reflect more fundamental changes in the entire spectral relationship. Some implications for model evaluation are also discussed.

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[3] The analysis is similar to that of den Haan (1995) in that it examines the issue of output-price comovement at various frequencies. Rather than examining independent correlations at different frequencies, however, the analysis in this paper provides a direct decomposition of the importance of different frequencies to the overall correlation coefficient.

2. Data and Methodology

The benefit of applying spectral methods to the analysis of comovement among economic time series derives from the spectral representation theorem, which states that any real valued, covariance stationary process can be represented as the weighted sum of orthogonal periodic components:

$$y_t = \int_0^{\pi} [\alpha(\omega)\cos(\omega t) + \beta(\omega)\sin(\omega t)]\, d\omega \ . \tag{1}$$

The spectral representation theorem can be applied to find the population spectrum of a vector of time series in terms of a Fourier sum:

$$s(\omega) = \frac{1}{2\pi} \sum_{k=0}^{\infty} \Gamma_k \, e^{-i\omega k} \tag{2}$$

where Γ_k is the kth order autocovariance matrix of the vector of time series. Note that the spectrum incorporates all information about the variance-covariance structure of the time series under consideration. In fact, the autocovariance matrix can be recovered from the spectrum by using the Fourier integral:

$$\Gamma_k = \int_{-\pi}^{\pi} e^{i\omega k} \, s(\omega)\, d\omega \tag{3}$$

For the special case of k=0,

$$\Gamma_0 = \int_{-\pi}^{\pi} s(\omega)\, d\omega \tag{4}$$

that is, the spectrum integrates to the contemporaneous cross-covariance matrix. Consequently, the real-valued components of the off-diagonal elements of the spectrum, known as the cospectra, reflect the contribution of comovements at various frequencies to the overall covariance between two series.

This paper analyzes the cospectra of output and prices over various sample periods, where the data consist of quarterly time series on GNP and the implicit deflator over the period from 1875 to 1994. Historical data are from Balke and Gordon (1984), updated by splicing the series to recently published observations. The cospectra are estimated using standard procedures, approximated at a discrete set of frequencies $\omega_j = 2\pi j/T$, $j=1,\ldots,T/2$, where T is the number of observations.[4] The raw cospectra, $\hat{c}(\omega)$, are then smoothed using a variant of the centered, weighted-average kernel:

$$\tilde{c}(\omega_j) = \sum_{m=-h}^{h} \frac{h+1-|m|}{(1+h)^2}\, \hat{c}(\omega_{j+m}), \tag{5}$$

with the bandwidth parameter, *h*, chosen to smooth the cospectrum without altering obvious peaks in the raw estimates.[5]

In previous studies of the output-price relationship, correlations have been shown to be sensitive to the method of detrending. Consequently, this paper analyses series that have been prefiltered using either the Hodrick-Prescott filter or a first-difference filter.[6] The reason that these two detrending techniques yield different

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[4] For details of estimation procedures for frequency domain analysis, see Harvey (1993) or Hamilton (1994).

[5] The variant of the kernel used in this paper involves using j rather than h for j<h. The purpose of this modification is to leave intact the property imposed on the data by both the first difference and HP filters that there is zero power at the zero frequency.

[6] Cogley and Nason (1995) find that HP filtering can generate spurious cyclical relationships when time series are difference stationary, providing another reason

implications for correlation results is clear when they are analyzed in the frequency domain. Figure 1 illustrates transfer functions of the two filters -- which show the extent to which the variance of the raw data series are transferred to the filtered series at various frequencies. The first difference filter has the effect of amplifying high-frequency fluctuations, while dampening low frequency movements. The HP filter acts more like a high-pass filter, eliminating low frequency movements while leaving high-frequency components intact.[7]

If we take the range of "business cycle frequencies" to encompass periodicities of between 6 and 32 quarters (.0625π to .333π), both filters create some distortions in the measurement of cyclical relationships.[8] The first-difference filter boosts the importance of higher-frequency moments within this range, while both filters diminish the importance of some of the lower-frequency range.

3. Cospectra of Output and Prices

Figures 2-4 show the cospectra of output and prices for the sample periods under consideration.

Figure 2 illustrates the spectral relationship for the entire sample period, 1875-1994. The upper panel, which shows the cospectrum for HP filtered output and prices, reveals that the positive covariance is predominantly attributable to co-movement at the

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for looking at both HP and first-difference filtered data.

[7] The properties of various filters are explored in Singleton (1988), King and Rebelo (1994) and Christiano and den Haan (1995).

[8] Citing Burns and Mitchell (1946), Baxter and King (1995) use this definition of the periodicity of business cycles.

frequencies toward the very low end of the business cycle range. An additional positive contribution appears in a range of frequencies centered at about 0.2π (a periodicity of 10 quarters). Small negative contributions come from frequencies of 0.05π (below the business cycle frequency range) and at about 0.14π (14 quarters).

Within the range of business cycle frequencies, the first-differenced data generate a cospectrum that has the same general features as the HP filtered data. However, the cospectrum for first-differenced data also highlights comovement at higher frequencies.

To the extent that the relationship between output and prices has changed over time, the cospectrum in Figure 2 represents a summation of cospectra for various subperiods, with some elements amplifying one another and others canceling each other out. Figures 3a-3c examine the cospectra for three main subperiods within the sample.

The cospectrum for the prewar period (1875-1914) in Figure 3a displays some features similar to those of the full sample period. A positive contribution to the covariance appears toward the very low end of the frequency range, followed by a negative spike at a frequency of about 0.1π (periodicity 20 quarters). The wide, higher-frequency peak provides a much more substantial contribution to the overall covariance in the prewar data than it does on average for the entire sample period.

The first-differenced data show an even more important positive contribution in the higher frequencies with the same wide peak at around 0.2π as seen for the HP-filtered data. A small negative contribution to the overall covariance at just under 0.1π also matches the HP-filtered cospectrum.

Figure 3b shows the cospectra for the interwar period (1920-1940). As might be expected from the analysis of Cooley and Ohanian (1991), Backus and Kehoe (1992) and others, the cospectrum between output and prices during the interwar period is dramatically different than for the prewar period. For both the HP-filtered and first-differenced data, the cospectrum is uniformly positive, with strong contributions from frequencies across the entire business-cycle range. The largest contribution to the overall covariance is concentrated at or below the very low end of the range.

Figure 3c shows that the cospectra for the postwar period (1950-1994) reflect a relationship which is more similar to that of the prewar period than of the interwar years. A broad peak in the upper end of the business-cycle frequency range contributes positive components to the overall covariance and a sharp spike at the low end of the frequency range (at or below the business cycle frequency, depending on detrending method) contributes negative elements. As shown in Table 1, the lower-frequency negative components dominate in this sample period, resulting in a negative correlation between output and prices.

Wolf (1991) finds that the negative output price correlation in the postwar period is only apparent in the latter part of the sample. To investigate this feature of the data, Figures 4a and 4b examine the cospectra for two subperiods of the postwar era. Figure 4a shows that the overall shape of the cospectrum for the first half of the period matches that of the entire postwar sample. However, the higher-frequency positive components of the spectrum are more prominent, contributing to a positive overall correlation. Figure 4b shows that the higher-frequency positive peak disappears in the period since 1973, with a negative, low-frequency peak completely dominating the overall relationship.

4. Some Implications for Model Evaluation

It has been noted that a simple correlation between output and prices does not necessarily provide a useful criterion for discriminating among classes of macro-models. For example, both Hall (1993) and Judd and Trehan (1995) have shown that demand-driven models with Keynesian features can generate negative correlations between output and prices. In addition, Gavin and Kydland (1995) have suggested that endogenous monetary policy responses to supply shocks can generate price-output correlations with either positive or negative signs, depending on the particular form of the policy reaction-function.

In the general context of model evaluation, several papers have recently advocated the use of spectral methods to compare theory and data.[9] Indeed, as shown in the preceding section, the co-spectrum gives a richer description of the dynamic relationship between output and prices than does a simple correlation coefficient.

In this section I use the spectral approach to evaluate the implications of two particular models, focussing on the price-output relationship. The data analysis in the previous section suggests that negative components of the covariance are focussed at low frequencies, while positive comovements are dominant at higher frequencies

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[9] See, for example, Watson (1993) and, more recently, Diebold, Ohanian and Berkowitz (1997).

(particularly so for the postwar period).[10] A successful model-based explanation for the pattern of output and price comovements over time should be able to account for both the frequency pattern of the cospectrum and plausible parameter changes altering the relative importance of comovements at various frequencies.

A Demand-Driven Model with Keynesian Features

The model of Judd and Trehan (1995) is a simple, parameterized Phillips curve model in which exogenous shocks originate in the monetary authority's policy reaction function. The model consists of four equations: aggregate demand, a Phillips curve, money demand, and the policy rule. (Appendix A describes the specific structure of the model.)

The model's dynamics imply a hump-shaped response of output to a demand shock in conjunction with a protracted price-level response (shown in Figure 5), which imparts an overall negative correlation between the two variables: "[O]utput and prices first move together, but then move in opposite directions as inflation continues to rise for a time while output falls back to its trend."[11] The correlations between output and the price level generated by this model are -0.58 for HP filtered data, and -0.42 for first differenced data.[12]

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[10] This finding is consistent with the analysis of den Haan (1995).

[11] Judd and Trehan (1995), p. 795.

[12] The correlations of HP filtered and first-differenced data for the models evaluated here are population moments calculated by frequency-domain methods described in King and Rebelo (1993). Cospectra are constructed using the same approach.

Note that the protracted nature of the relationship shown in Figure 5 suggests that the cospectrum of output and prices should show a negative contribution at low frequencies. This is, in fact, the case. Figure 6 shows that the cospectra generated by the model have one sharp spike at a frequency slightly below $.05\pi$ -- below the range of cyclical frequencies. Although the negative correlation is dominated by very low frequencies, the cospectra for the demand-driven model is not too far different from that displayed by the data in the post-1973 period. However, it is not clear how changes in the model parameters could generate the positive correlations found for other sample periods, nor would changes in parameter values be likely to have a straightforward economic interpretation.

A Shopping-Time Monetary Model with Endogenous Policy

Gavin and Kydland (1995) have recently suggested that changes in the parameters of the monetary authority's policy-reaction function can account for in real-nominal relationships over different sample periods. In this section, I evaluate implications for the spectral relationship between output and prices for a similar model.

The model is a standard neoclassical growth model, with demand for money motivated by a shopping-time function that depends on the ratio of real money balances to consumption:

$$S_t = \omega_1 [\frac{M_t/P_t}{C_t}]^{\omega_2} \qquad (6)$$

with $\omega_1 > 0$, $\omega_2 < 0$. [See Appendix B for the full model specification].[13]

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[13] The shopping-time framework is described in McCallum and Goodfriend (1987). Lucas (1994) shows that this type of model generates a general equilibrium

The money stock follows a law of motion which depends on lagged values of money and output and an exogenous shock:

$$M_{t+1} = M_t^{\nu_M} Y_t^{\nu_Y} \epsilon_{t+1} \ . \tag{7}$$

Using standard methods, the model is log-linearly approximated, calibrated, and solved for optimal decision rules and laws of motion for endogenous state variables.[14] These relationships are then used to calculate impulse-response functions, implied second-moments, and the cospectrum of output and the price level.

Figure 7 illustrates the basic model dynamics in terms of the impulse-response functions of output and the price level following a positive productivity shock. In this initial case, money does not respond endogenously ($\nu_M = \nu_Y = 0$). As one might expect, the result is an inverse relationship: the rise in output generates an increase in money demand and a consequent fall in the price level. This gives rise to a correlation between output and prices of -0.85 (HP filtered). The cospectrum of output and prices in the lower panel of Figure 7 shows that the relationship is dominated by comovements at the very low end of the business cycle frequency range -- not unlike the relationship observed in the data for the second part of the post-war sample period.

By altering the parameters of the money reaction function, equation (7), the sign of the price-output correlation can be reversed. Figure 8 shows impulse-response

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relationship between money, output and interest rates that is consistent with conventional money demand specifications.

[14] The solution algorithms are those used in, e.g., King, Plosser and Rebelo (1988).

functions and the cospectrum for a calibration with procyclical money ($\nu_Y = 1.0$)[15]. This results in generally positive comovement of output and prices: the correlation coefficient is 0.83 (HP filtered). As shown in the lower panel of Figure 8, the cospectrum for output and prices is almost a mirror image of the constant-money growth case from Figure 7: The positive comovement is concentrated in the very low end of the business cycle frequency range.[16] Although the model with procyclical money generates a positive correlation between output and prices, the spectral shape of the relationship fails to match the empirical finding that important contributions to a positive output-price correlation are present at higher frequencies (particularly in the early post-war sample period).

A second way to generate positive comovement between output and prices in this model is through shocks to the money supply process. The shopping-time specification for money demand implies that positive monetary shocks, for example, give rise to positive real effects as well as price level movements. Figure 9 illustrates this relationship. As shown in the top panel, the impulse-response functions for output and the price level move very closely together in response to monetary shocks. As a result, the spectral relationship between the two variables is spread over a broader frequency band. Although the peak of the cospectrum is at a fairly low frequency, the cospectrum remains relatively high throughout and beyond the high-

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[15] For the this particular exercise, the parameter governing policy responses to lagged money, ν_M, is set to 0.5.

[16] The long-run convergence of output and prices to their steady state level implies a tiny negative contribution to the overall correlation at very low frequencies (barely visible in the graphical presentation of the cospectrum in Figure 8).

frequency end of the business-cycle range. Interestingly, this pattern roughly matches the cospectral shape characterizing the data during the inter-war period.

The analysis of empirical cospectra in Section 2 indicated that changes in the sign of the price-output correlation are associated with relatively greater or lesser importance of the high-frequency positive comovements compared to lower-frequency negative comovements (particularly in the post-war period). The analysis of productivity shocks and money shocks in the shopping-time model suggests that a combination of shocks could yield the type of bimodal cospectrum characterizing the data. Figure 10 confirms the relevance of this possibility: persistent productivity shocks give rise to a low-frequency negative relationship while the positive comovement induced by money shocks dominates in the upper end of the business cycle frequency range.[17]

5. Concluding Comments

The cospectra of the price-output relationship examined in this paper display some general characteristics which appear to be robust to differences in sample periods and detrending methods. With the notable exception of the interwar period, negative components of the cospectrum of output and prices tend to appear at fairly low frequencies, with positive contributions emerging from the higher frequencies within the business-cycle range. The differences observed in raw correlation

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[17] Parameter values for the two-shock calibration are as follows: $\nu_Y=0$ and $\nu_M=0.7$, productivity shocks follow an AR(1) process with a coefficient of 0.95. The standard deviation of productivity shocks is about 30 times that of money shocks.

coefficients over different sample periods appear to be largely due to greater or lesser prominence of these lower- versus higher-frequency components.

Differences between correlations calculated using HP filtered data and first-differenced data are at least partly attributable to the first-difference filter's tendency to amplify very high frequency movements (those at seasonal and higher frequencies). Within the frequency range associated with business cycle fluctuations, the two detrending methods yield similar cospectra.

Although a single cyclical relationship cannot be used to distinguish or discriminate among competing models, the spectral decomposition of the price-output relationship illustrated in this paper can be brought to bear on some theoretical issues that have arisen in the context of this controversial relationship. The model evaluation exercises in this paper suggest that it can be difficult to account for the changing real-nominal covariance structure by altering parameter values in a single-shock model. However, the presence in a model of at least two shocks -- with one contributing to a negative low frequency comovement and one accounting for a higher-frequency positive comovement -- qualitatively matches the spectral structure of the data in a plausible way.

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Appendix A:

Model Specification from Judd and Trehan (1995)

Aggregate Demand:

$$\Delta y_t = 0.03 + 0.15\,\Delta y_{t-1} + 0.41\,(\Delta m_{t-1} - \Delta p_{t-1})$$

Phillips Curve:

$$\Delta p_t = -0.001 + 0.02\,(y_t - y^f) + 0.28\Delta p_{t-1} + 0.30\Delta p_{t-2} + 0.25\Delta p_{t-3} + 0.05\Delta p_{t-4}$$

Money Demand:

$$\Delta m_t = -0.08 - 0.09\,m_{t-1} + 0.09\,p_{t-1} + 0.10\,y_{t-1} - 0.14\,R_{t-1} + 0.70\Delta m_{t-1} + 0.17\Delta p_t - 0.07\Delta y_t - 0.26\Delta R_t$$

Policy Rule:

$$\Delta R_t = 0.08\,(\Delta p_{t-1} + \Delta y_{t-1}) - A_t$$

Appendix B:

A Shopping-Time Monetary Model With Endogenous Policy

$$\max \sum_{t=0}^{\infty} \beta^t \, (C_t^{\theta} L_t^{1-\theta})^{1-\sigma}/(1-\sigma)$$

Subject to:

$$A_t K_t^{\alpha} L_t^{1-\alpha} + \frac{M_t + V_t}{P_t} = C_t + K_{t+1} - (1-\delta)K_t + \frac{M_{t+1}}{P_t}$$

$$L_t + N_t + \omega_1 [\frac{(M_t + V_t)/P_t}{C_t}]^{\omega_2} = 1$$

With:

$$M_{t+1} = M_t^{\nu_M} Y_t^{\nu_Y} \epsilon_{t+1}$$

Parameter Values:

Risk aversion (σ)	2.0	Capital Share (α)	.30
Time preference (β)	.99	Depreciation (δ)	.025
Consumption Share (θ)	.3365	(Set to yield steady state work effort of .30)	
Shopping Time (ω_1 , ω_2)	.036, -.8	(Set to yield steady state shopping time of .015)	

Table 1: Contemporaneous Correlations Between Output and Prices

	HP Filter	First Difference
Full Sample 1875:1 - 1994:4	0.218 (0.046)	0.189 (0.046)
Prewar 1875:1 - 1914:4	0.270 (0.079)	0.185 (0.079)
Interwar 1920:1 - 1940:4	0.395 (0.109)	0.396 (0.109)
Postwar 1950:1 - 1994:4	-0.307 (0.075)	-0.105 (0.075)
Postwar A 1950:1 - 1972:4	0.169 (0.104)	0.125 (0.104)
Postwar B 1973:1 - 1994:4	-0.653 (0.107)	-0.196 (0.107)

(Standard errors in parentheses)

Figure 1

Transfer Functions for the First-Difference and Hodrick-Prescott Filters

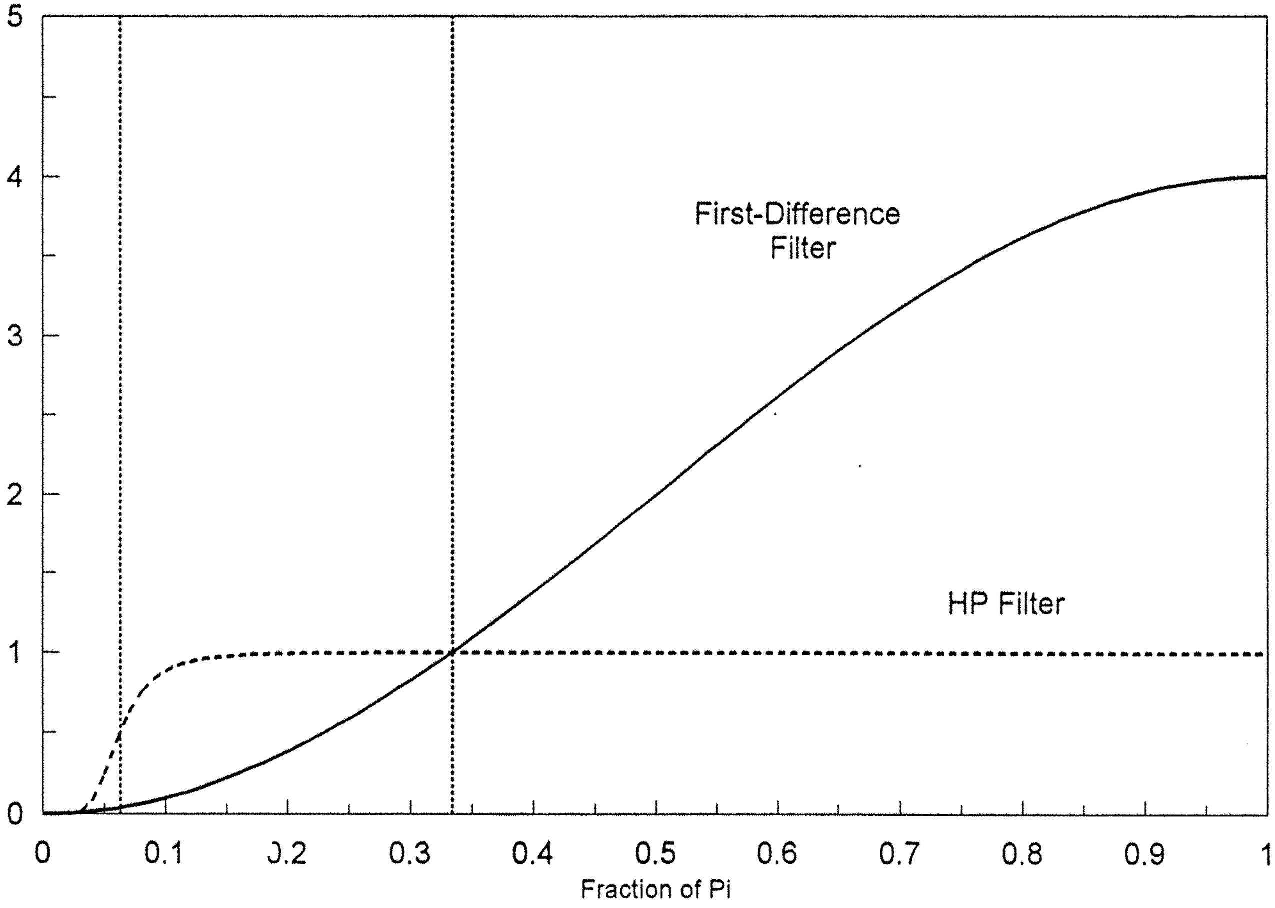
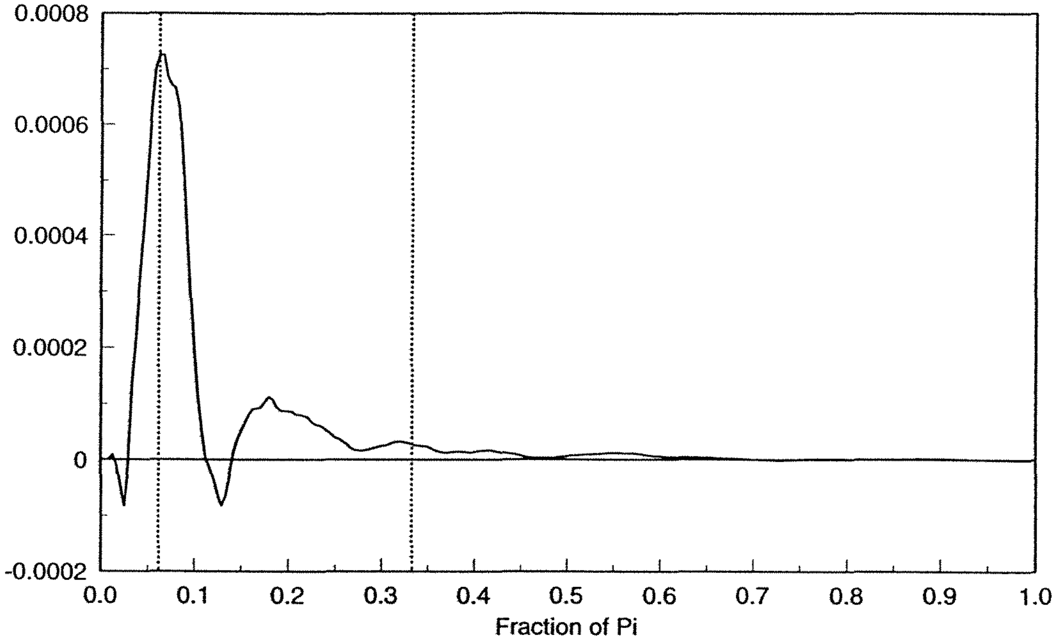


Figure 2

Co-Spectrum of HP Filtered Data
Full Sample



Co-Spectrum of FD Filtered Data
Full Sample

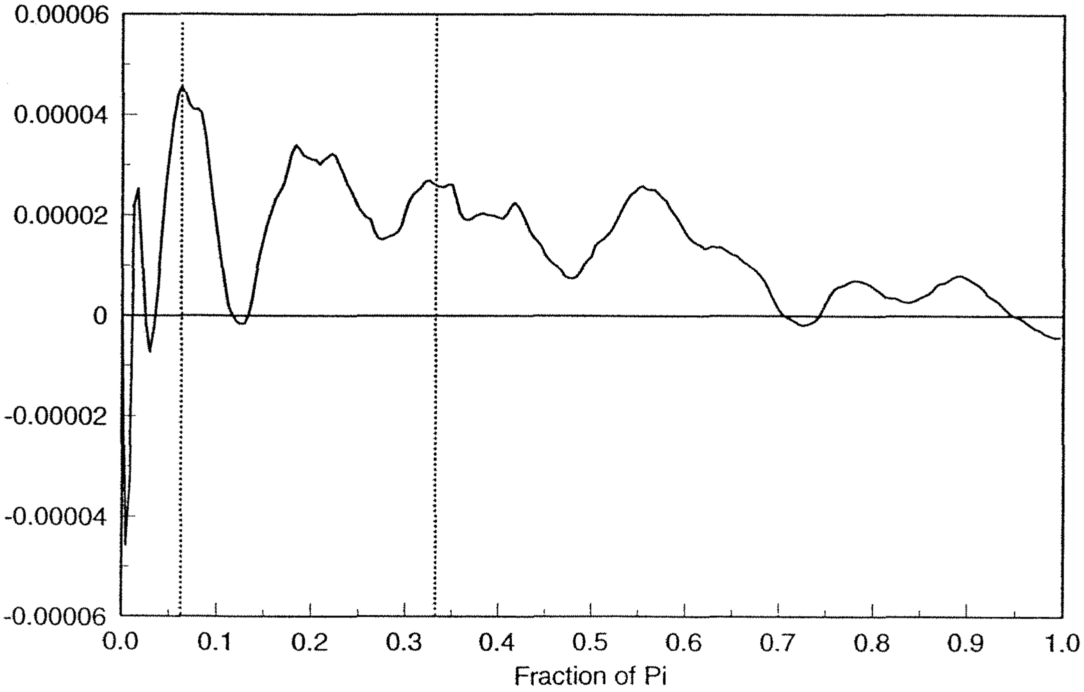
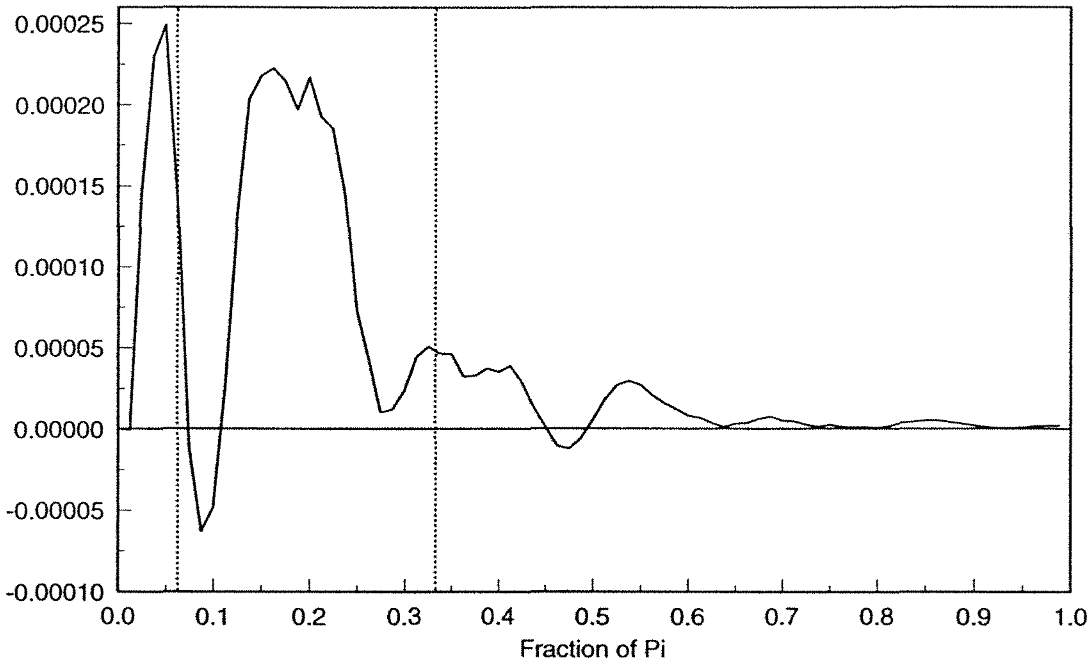


Figure 3a


Co-Spectrum of HP Filtered Data
Prewar

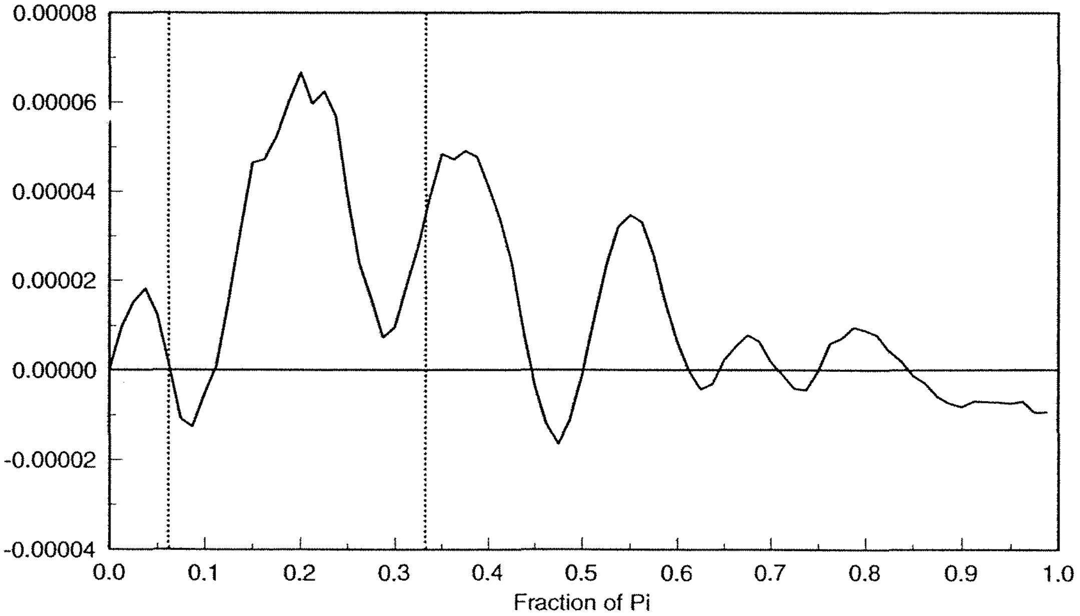
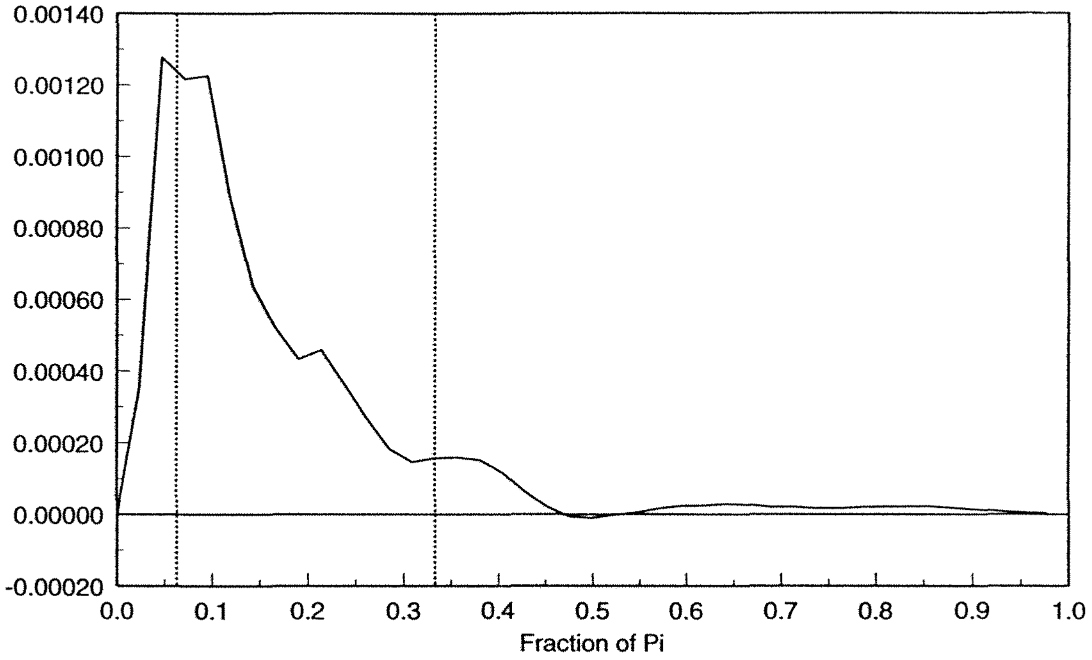

Co-Spectrum of FD Filtered Data
Prewar

Figure 3b

Co-Spectrum of HP Filtered Data
Interwar



Co-Spectrum of FD Filtered Data
Interwar

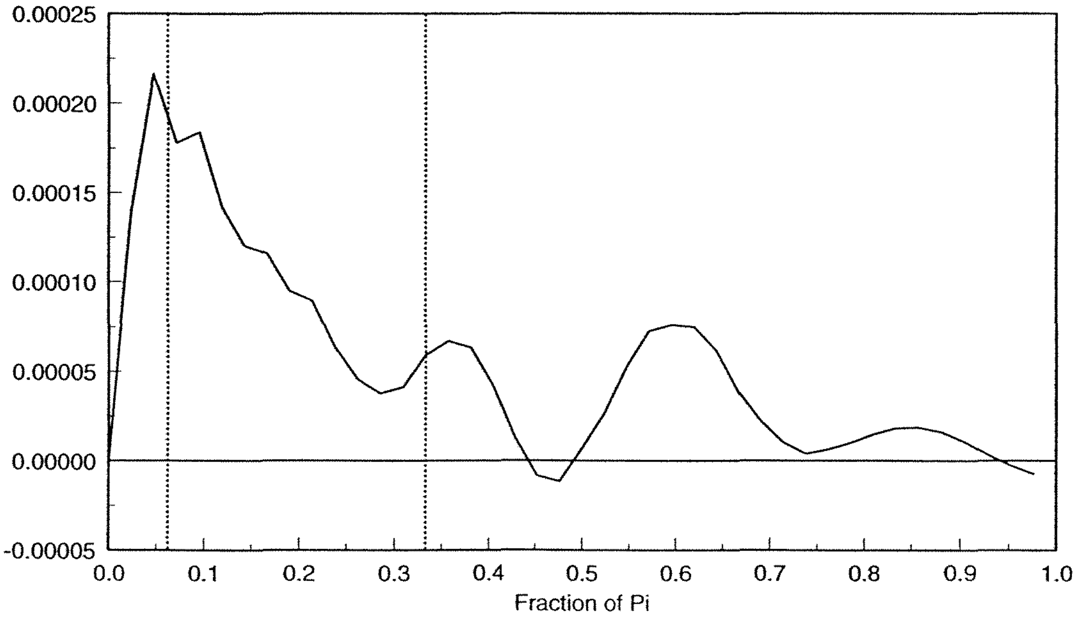
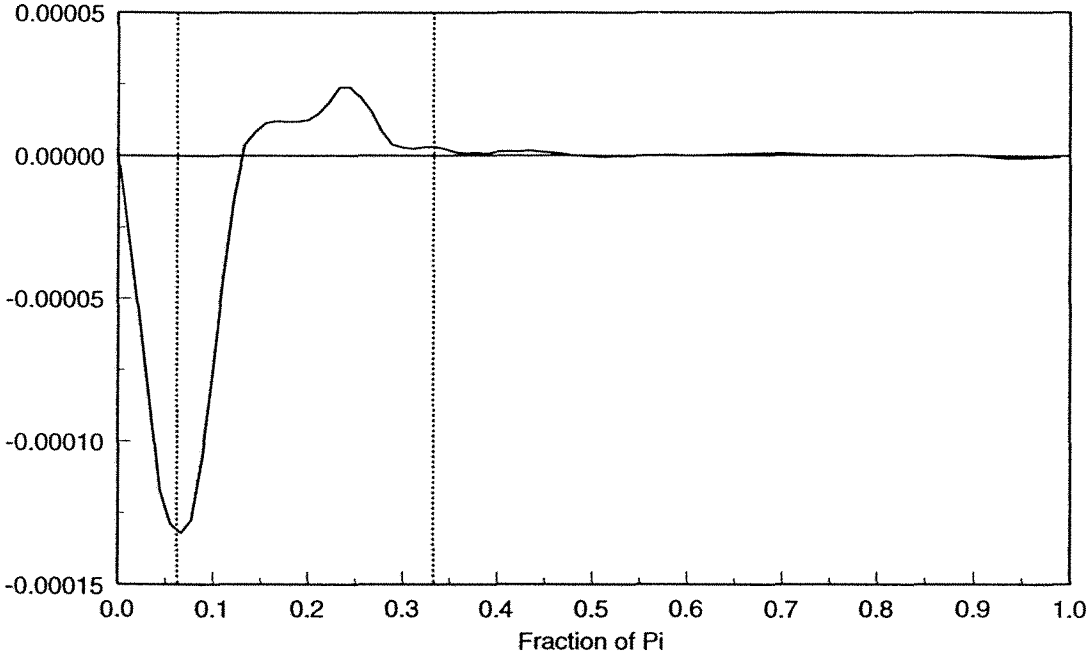


Figure 3c

Co-Spectrum of HP Filtered Data
Postwar



Co-Spectrum of FD Filtered Data
Postwar

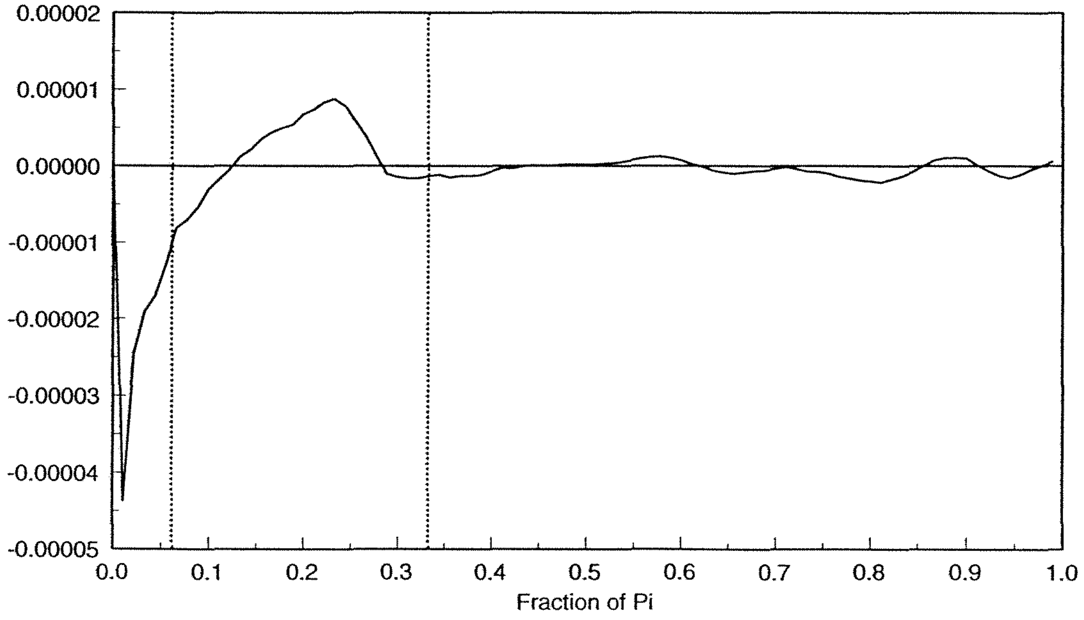
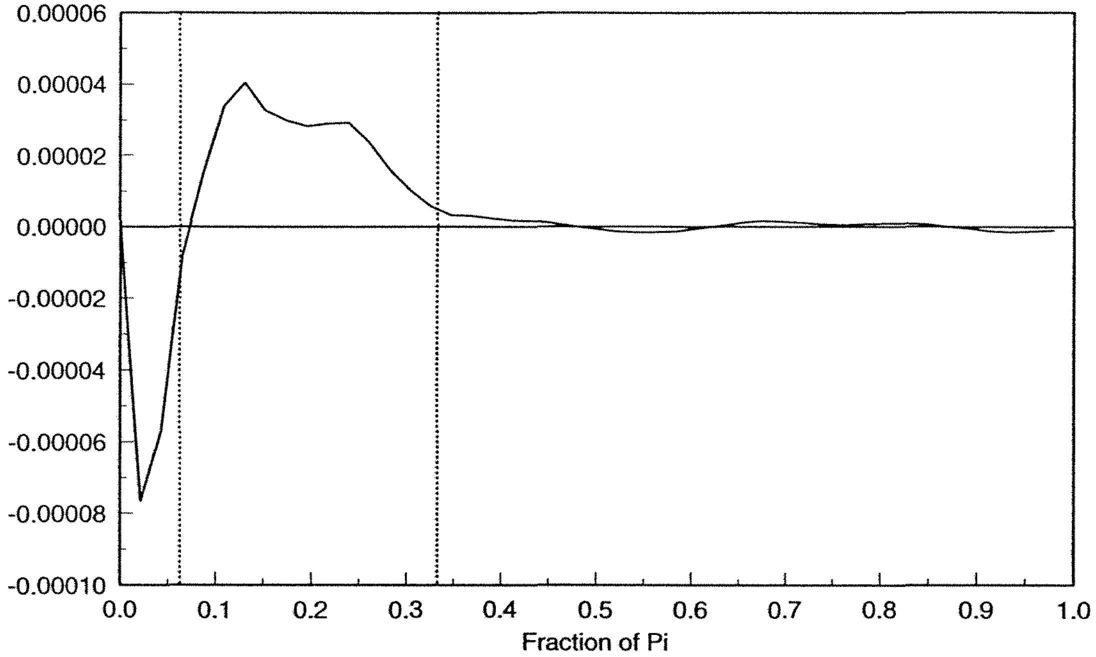
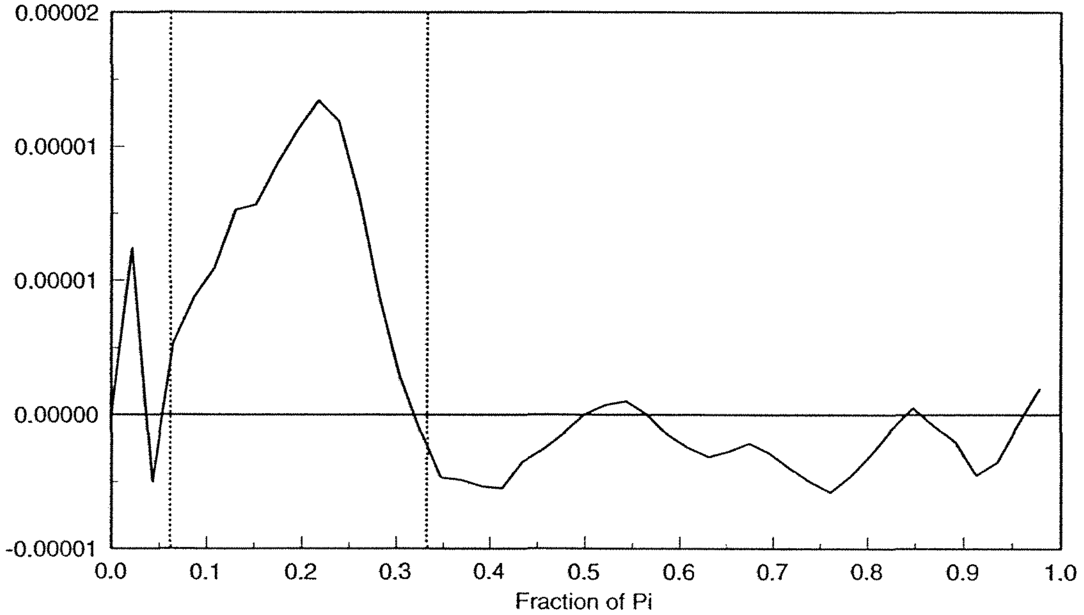


Figure 4a

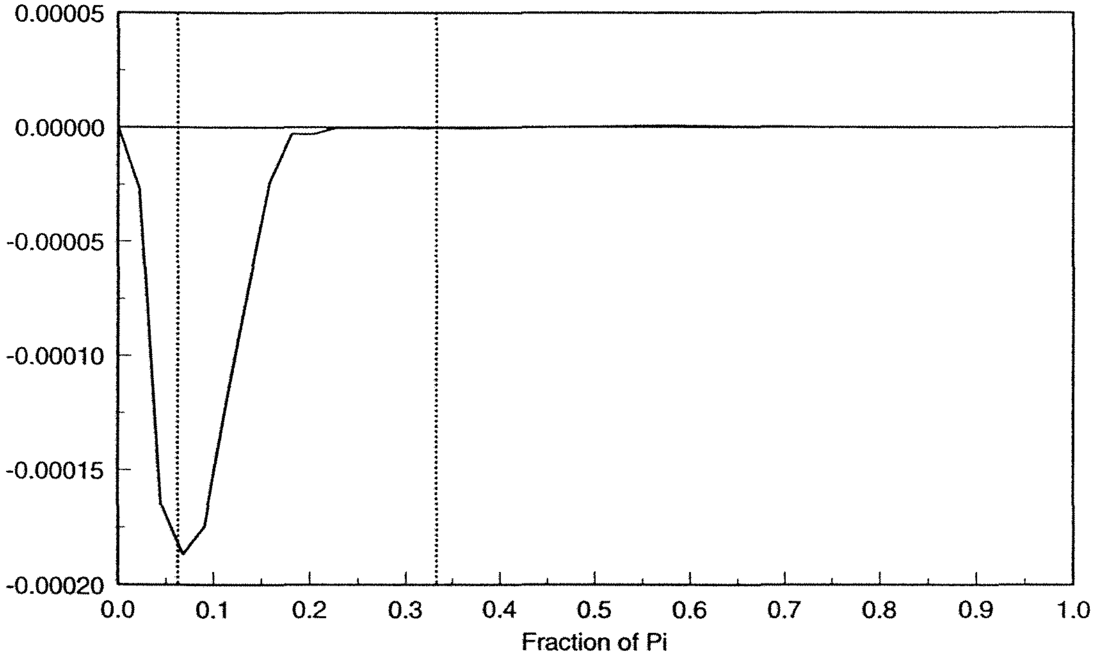


Co-Spectrum of HP Filtered Data
Postwar A

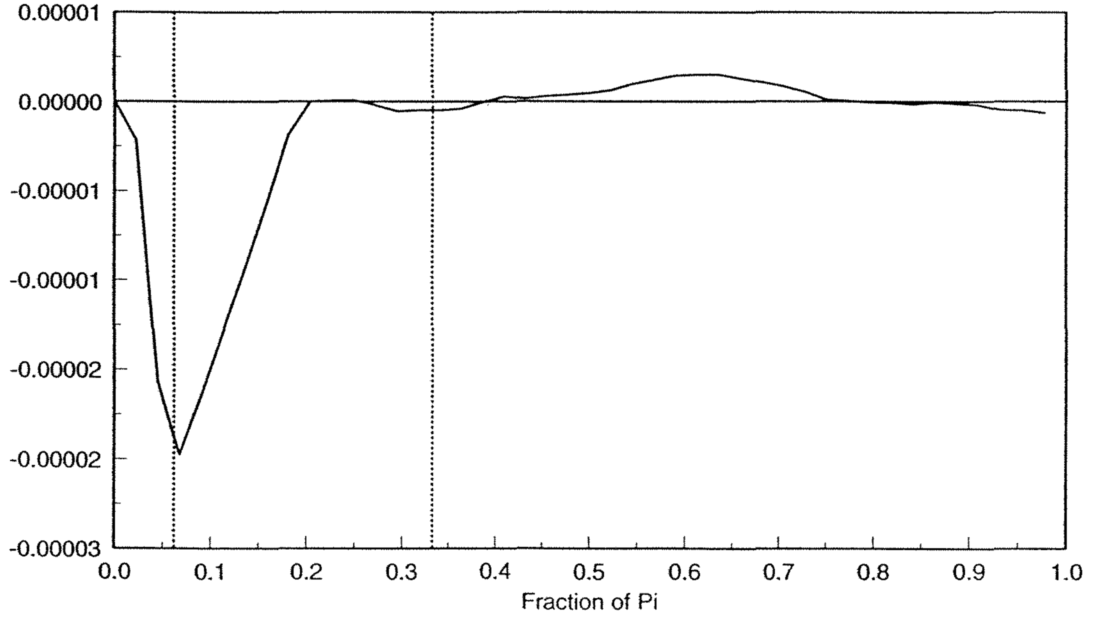


Co-Spectrum of FD Filtered Data
Postwar A

Figure 4b



Co-Spectrum of HP Filtered Data
Postwar B

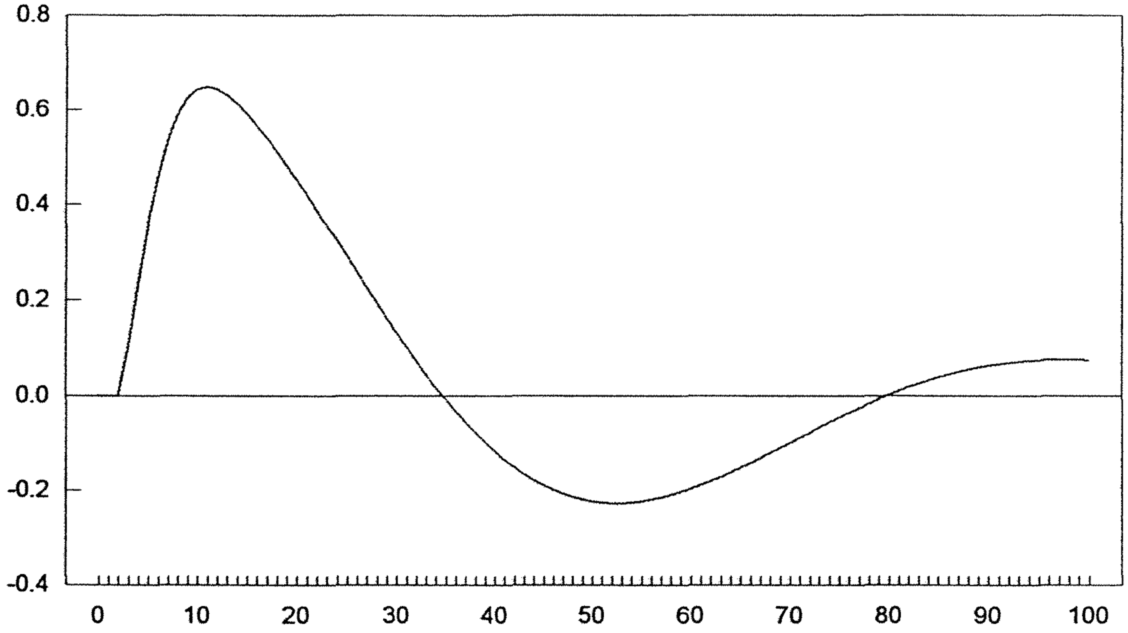


Co-Spectrum of FD Filtered Data
Postwar B

Figure 5

Impulse-Responses to a Monetary Policy Shock in the Demand-Driven Keynesian Model

Output



Price Level

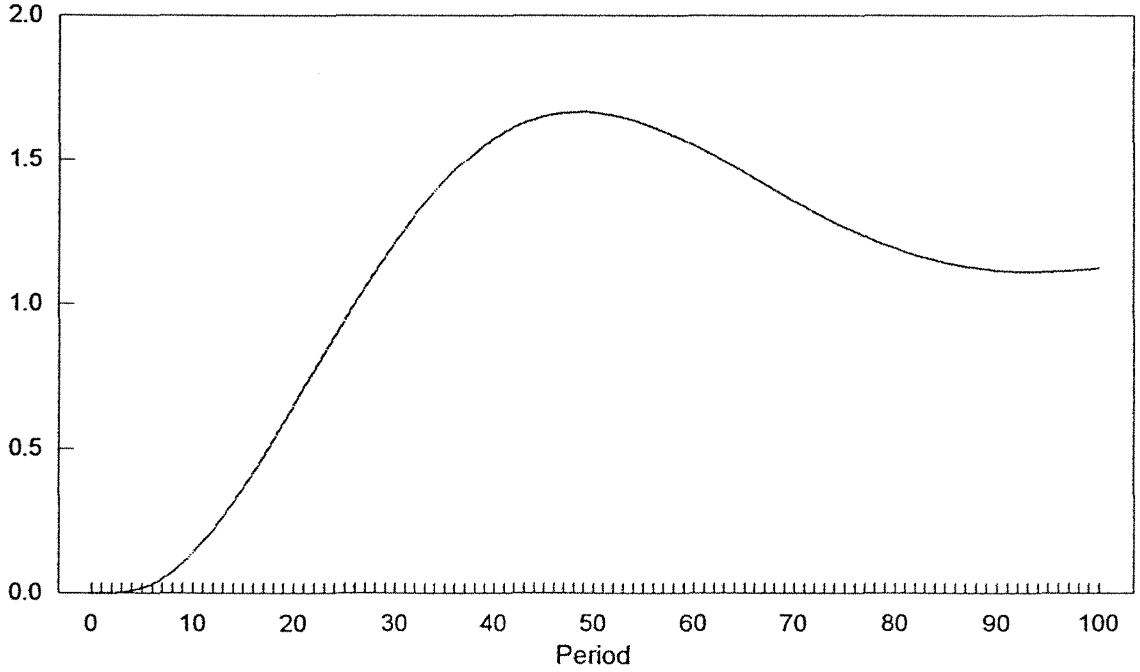
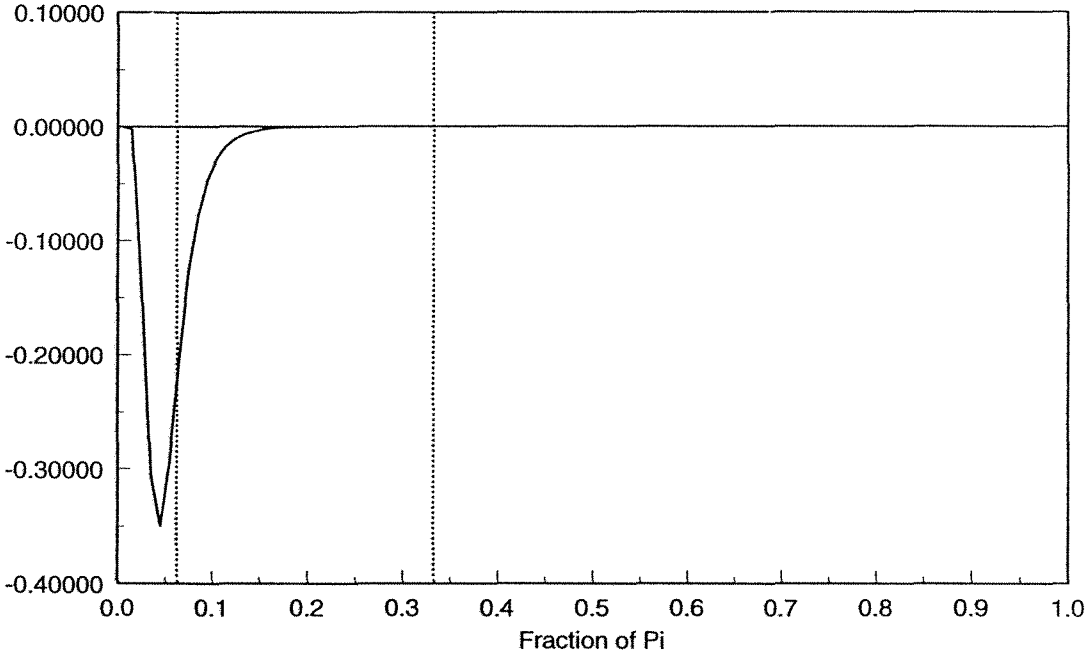


Figure 6

Co-Spectrum for a Demand Driven Keynesian Model

HP Filtered Data



FD Filtered Data

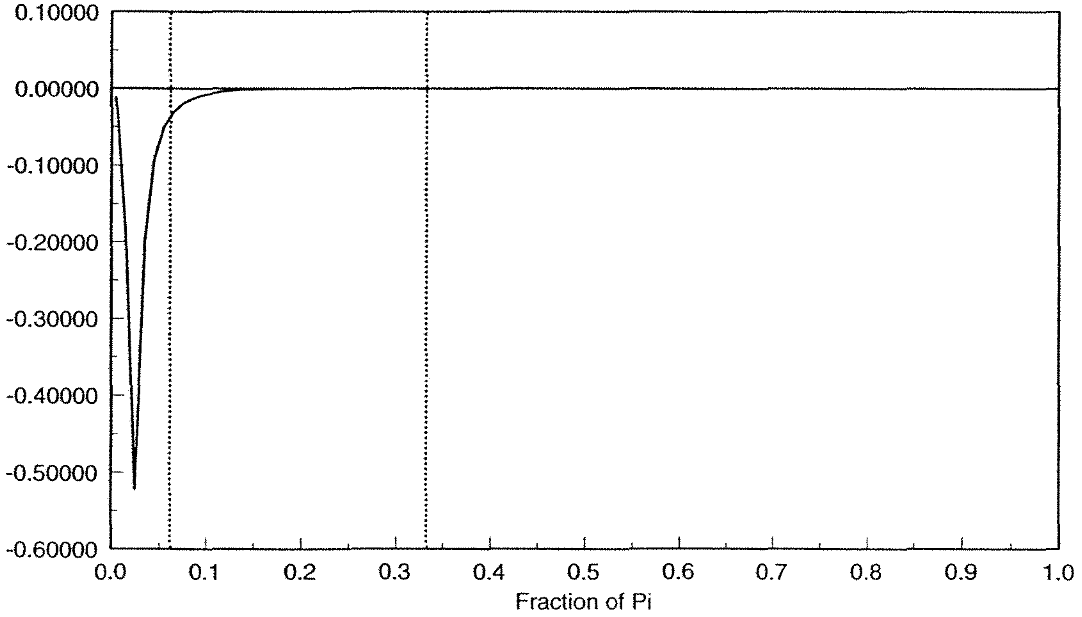
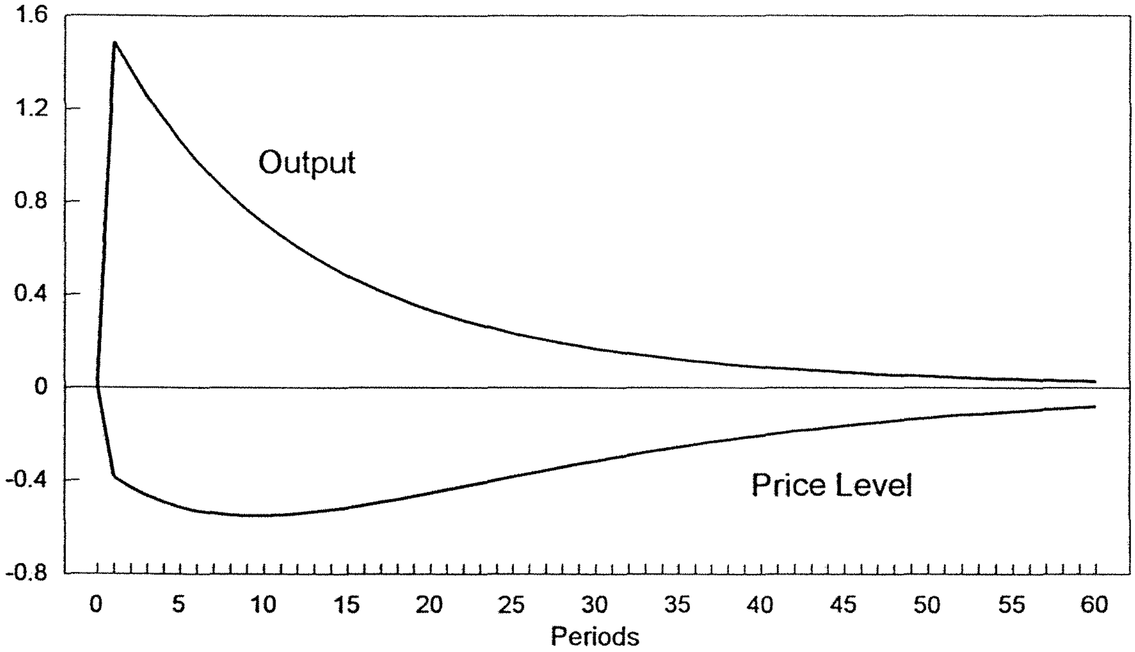


Figure 7

Productivity Shocks in the Shopping-Time Monetary Model With Constant Money Growth

Responses to a Positive Productivity Shock



Cospectrum of Output and Price Level
HP Filtered

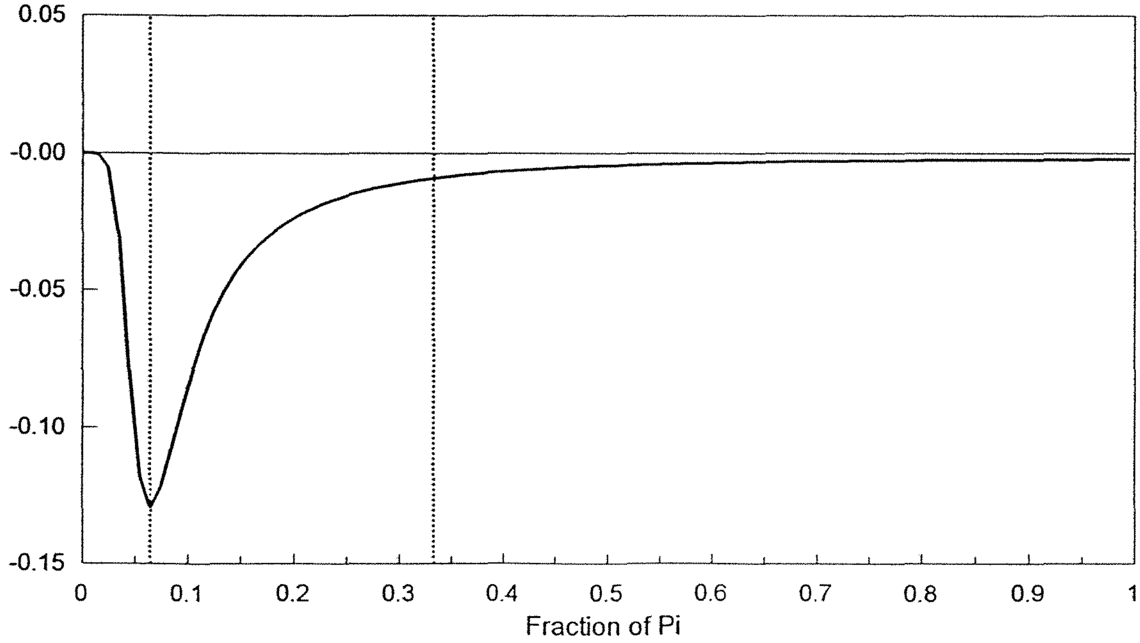
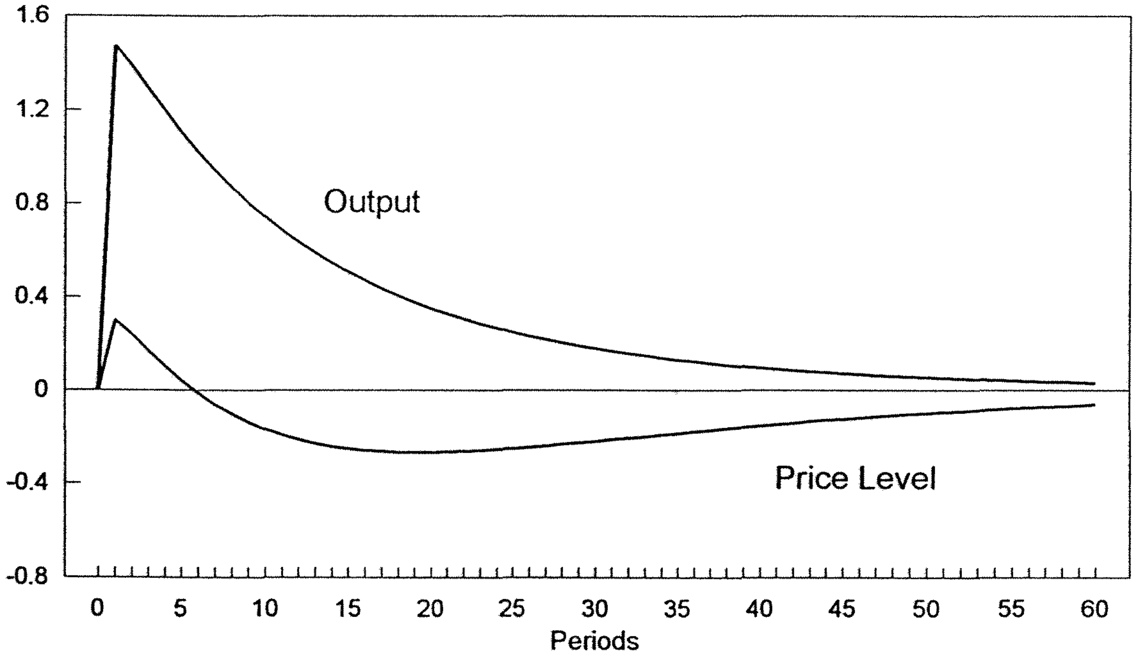


Figure 8

Productivity Shocks in the Shopping-Time Monetary Model With Procyclical Money

Response to a Positive Productivity Shock



Cospectrum of Output and Price Level
HP Filtered

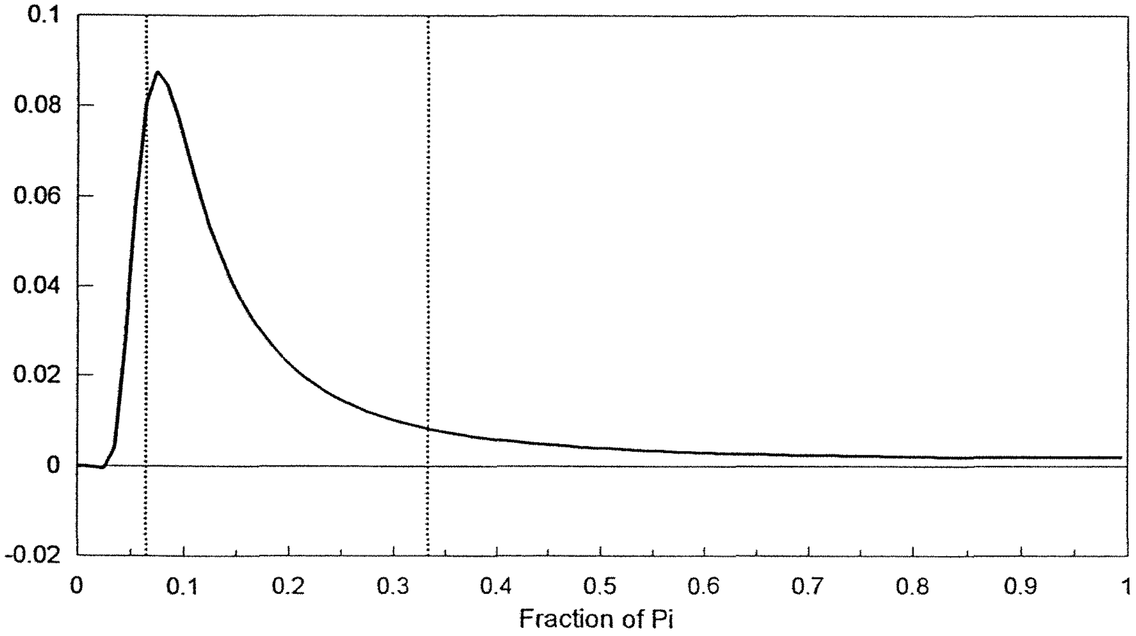
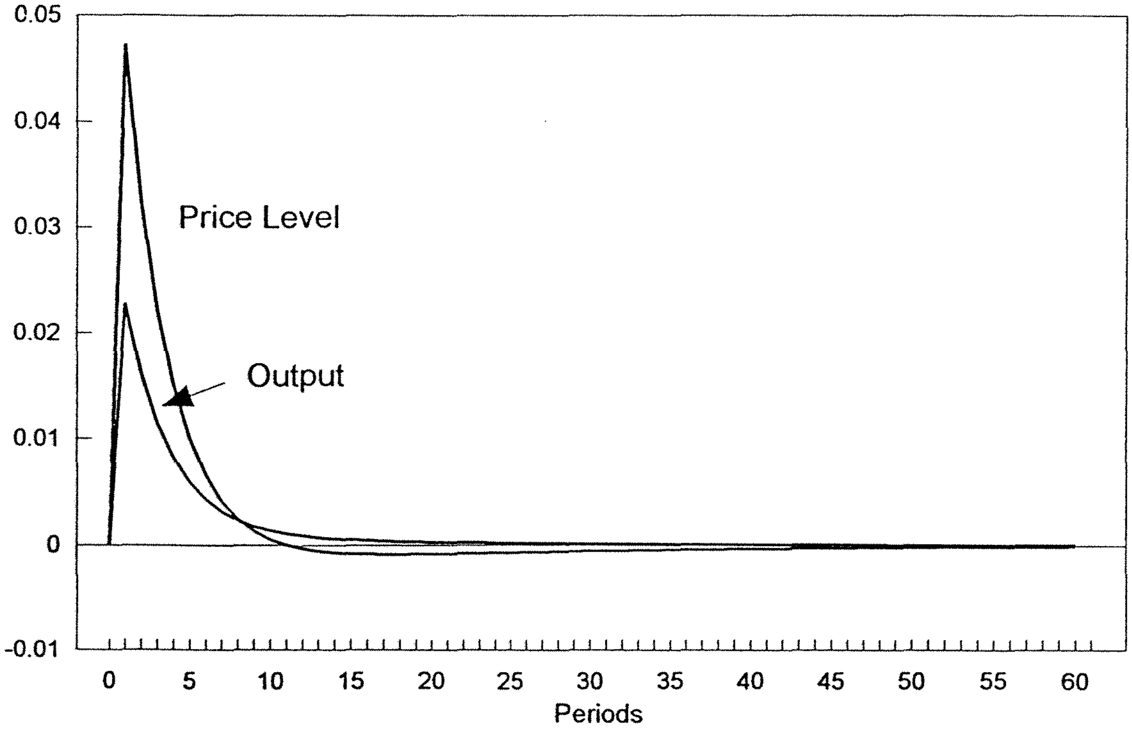


Figure 9

Money Shocks in the Shopping-Time Monetary Model

Response to a Positive Money Shock



Cospectrum of Output and Price Level

HP Filtered

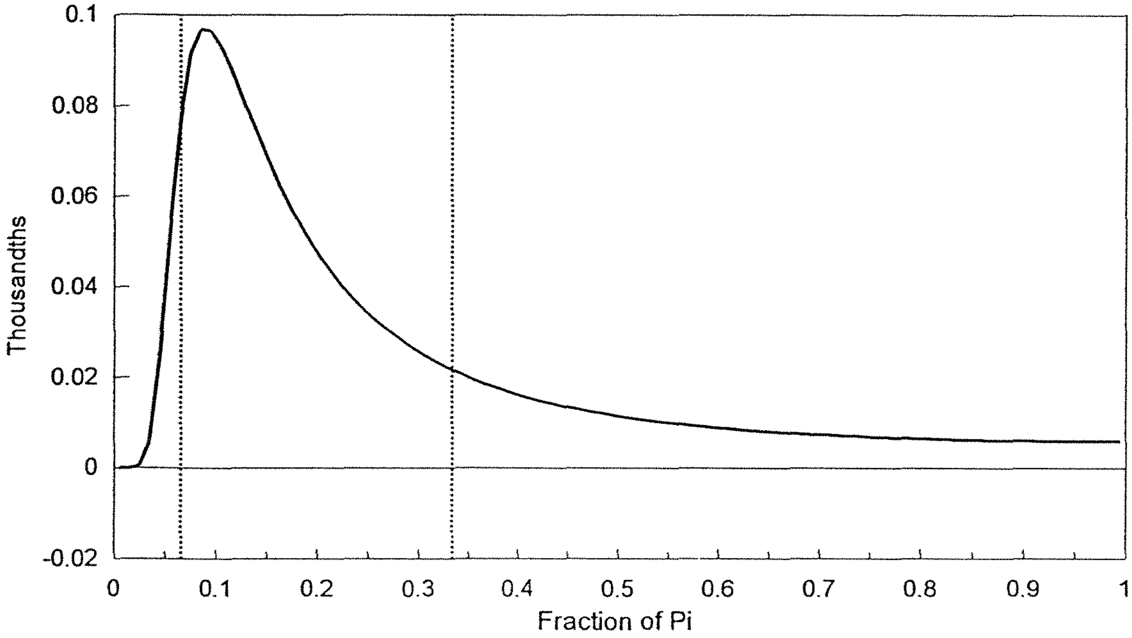


Figure 10

Two Shocks in the Shopping-Time Monetary Model

Cospectrum of Output and Price Level
HP Filtered