WORKING PAPER SERIES

Tariffs and Asset Market Structure: Some Basic Comparative Dynamics

Michael R. Pakko

Working Paper 1995-009A
http://research.stlouisfed.org/wp/1995/95-009.pdf

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

Published 6/95

TARIFFS AND ASSET MARKET STRUCTURE: SOME BASIC COMPARATIVE DYNAMICS

ABSTRACT

Stockman and Dellas (1986) demonstrated that in the presence of complete international asset markets, the relative welfare implications of a small tariff are reversed from standard trade theory. This paper examines the robustness of that result to change in preference parameters and asset market structure. For nearly all values of substitution elasticity and risk aversion, the reversal remains. For very low risk aversion, however, equilibrium outcomes resemble Lerner or Metzler tariff paradoxes. In the latter case, the tariff-imposing country is made better-off. Implications of asset market incompleteness are considered in the form of a bonds-only regime, in which the inability to trade directly across states induces intertemporal substitution. A permanent tariff change effects relative consumption of the two countries as predicted in standard trade theory. A temporary tariff change results in a wealth redistribution, with the tariff-imposing country generally running a current account surplus.

KEYWORDS:. Assets, Current account, Risk Pooling, Trade Tariffs

JEL CLASSIFICATION: F11, F13, F40

Michael R. Pakko
Economist
Federal Reserve Bank of St. Louis
411 Locust Street
St. Louis, MO 63102
pakko@stls.frb.org

Tariffs and Asset Market Structure: Some Basic Comparative Dynamics

1. Introduction

In models of international exchange, explicit consideration of asset trade can lead to results with are significantly different than the comparative statics in traditional trade theory.[1] One remarkable example of this type of difference was described by Stockman and Dellas (1986). They demonstrated that in a two country exchange model, the presence of complete asset markets results in relative welfare effects of tariff changes which are opposite to those predicted in static trade theory. When agents are able to insure against "political risk", the imposition of a small tariff by the one country can result in an *ex post* allocation in which that country's consumption falls -- precisely opposite to the outcome found in the well-known "optimal" tariff analysis.

In this paper, I examine the sensitivity of the Stockman/Dellas result to changes in the preference parameters and asset market structure of the model. The illustrative example in Stockman and Dellas incorporates an assumption that preferences are separable between domestic goods and imports.[2] This paper examines a slightly more general form for preferences in order to investigate the robustness of the basic result.

For nearly all feasible parameter values of the model examined in this paper, the *ex post* preference reversal phenomenon of Stockman and Dellas remains.

---

[1] Examples of the significance of asset trade are described in Stockman (1987).

[2] This separability assumption is equivalent to restricting the intertemporal elasticity of substitution to be identical to the elasticity of substitution between domestic goods and imports. In the terminology of Feeney and Jones (1992), this amounts to assuming equal degrees of aggregate and compositional risk aversion.

For very low values of aggregate risk aversion, however, it is possible for the equilibrium outcomes to resemble Lerner and Metzler tariff paradoxes. In the latter case, the home country *is* made better-off by the imposition of a small tariff on imports.

The implications of asset market incompleteness are also considered in the form of a bonds-only regime. In such a regime, the inability to trade directly across states induces agents to substitute intertemporally. A permanent change in tariff rates engenders no current account dynamics and affects relative consumption of the two countries as predicted in static trade theory. On the other hand, a temporary tariff disturbance results in a wealth redistribution which can imply differing short-run and long-run responses. With the exception of the extreme parameter values associated with the Metzler-type paradox, the tariff-imposing country always runs a current account surplus, accumulating a larger share of world wealth. Hence, asset market restrictions imply a modified form of the traditional trade-theory result that the imposition of a small tariff can raise domestic welfare; even a temporary tariff increase raises consumption in the tariff-imposing country, if not in the short-run at least in the longer-run.

The paper is organized as follows: In section 2 I describe the basic framework and characterize the baseline equilibrium associated with a deterministic version of the model. Equilibrium dynamics under complete markets are compared to the traditional static trade theory results in Section 3. Section 4 explores the implications of restricting asset-trade to a bonds-only regime, and describes the effects of the resulting current account dynamics. Section 5 concludes the paper.

2. Model Structure and Baseline Allocations

In this section, I present the basic model and characterize the equilibrium of a static deterministic version. This solution will be subsequently used as the baseline for examining equilibrium dynamics.

The model consists of two countries, each inhabited by a single representative agent and specialized in production: The home country produces good X, while the foreign country produces good Y. In order to focus exclusively on the dynamic effects of tariff fluctuations, output levels will be assumed constant throughout.

Preferences of the two agents are assumed to be identical, with a tariff wedge distorting allocations toward domestic consumption. In particular, both agents have preferences for consumption of x-goods and y-goods at time t:

$$U(c_{xt},c_{Yt}) = \left(\frac{1}{1-\gamma}\right)\left[\left(\frac{1}{2}\right) c_{xt}^{\;1-\delta} + \left(\frac{1}{2}\right) c_{Yt}^{\;1-\delta}\right]^{\frac{1-\gamma}{1-\delta}}$$

where the elasticity of substitution between x-goods and y-goods is $1/\delta$ and γ represents the coefficient of relative risk aversion. Time separability is assumed so that $1/\gamma$ represents the intertemporal substitution elasticity. Following Feeney and Jones (1994), the parameter δ will also be referred to as a coefficient of risk aversion with respect to *compositional* risk.

Output levels are normalized so that both countries have equal real incomes. The first order conditions determining allocations are

$$U_X(c_X,c_Y)/U_Y(c_X,c_Y) = \pi(1+\bar{\tau}) ,$$

$$U_X(c_X^*,c_Y^*)/U_Y(c_X^*,c_Y^*) = \pi/(1+\bar{\tau}) ,$$

where π is the world relative price of the y-good in terms of the x-good. The model's dynamics will be evaluated relative to the baseline equilibrium characterized by tariff rates taking on their unconditional expected value, $\tau=\tau^*=\bar{\tau}$.

Given the normalizations and symmetry assumptions which have been made, the baseline equilibrium will be symmetric with $c_X=c^*_Y$, $c_Y=c^*_X$ and $\pi=1$. Let s represent the fraction of each country's output consumed domestically; e.g., $s=c_X/X=c_Y/Y$. For the given functional form assumed for preferences, $s=(1+\bar{\tau})^{1/\delta}/\ [1+(1+\bar{\tau})^{1/\delta}]$.[3]

The existence of a tariff wedge implies that expenditure shares evaluated at domestic prices will differ from the distribution shares s and (1-s). Let θ represent the expenditure share on domestic goods; e.g., $\theta=c_X/[c_X+\pi(1+\bar{\tau})c_Y]$. This parameter is also a function of the steady-state tariff and the elasticity of substitution: $\theta = (1+\bar{\tau})^{1/\delta}/[(1+\bar{\tau})+(1+\bar{\tau})^{1/\delta}]$. The parameter θ can also be interpreted as a utility-share parameter, $\theta=c_X^{\,1-\delta}/(c_X^{\,1-\delta}+\ c_Y^{\,1-\delta})$, and will be useful in that regard for the evaluation of aggregate consumption dynamics considered in the next section.

Of course, $s>\theta$ as long as the tariff is positive, with the relationship between the two parameters summarized by

$$\left(\frac{s}{1-s}\right) = \left(\frac{\theta}{1-\theta}\right)(1+\bar{\tau}) .$$

For reference, Figure 1 illustrates the values of s, 1-s, θ and 1-θ as a function of the compositional risk aversion parameter, δ.

---

[3] The share parameter s also represents the domestic-good expenditure share evaluated at world relative prices, $c_X/(c_X\ +\ \pi c_Y)$.

3 Dynamic Trade Distortions and Risk Pooling

In traditional static trade theory, a small tariff increase can result in an increase in consumption (real income) for the tariff-imposing country, as the world price of imported goods falls in response to reduced domestic demand. As demonstrated by Stockman and Dellas (1986), the existence of a complete contingent claims market can reverse the welfare implications of a tariff, with a home tariff lowering domestic consumption.

In this section I explore the differences between these two outcomes and the conditions under which they will occur. That is, I compare the equilibrium dynamics in the extreme asset-market regimes of complete markets and "portfolio autarky", with the latter term referring essentially to a dynamic interpretation of a traditional static trade model.

These dynamics will be described in terms of proportional deviations from the baseline allocations described above. Tariff rates in each country are assumed to be the only stochastic element of the model. Formally, let $\omega=(1+\tau)$ and $\omega^*=(1+\tau^*)$ represent the gross domestic and foreign tariff rates, which are assumed to follow stationary AR(1) processes:

$$\hat{\omega}_t = \rho\hat{\omega}_{t-1} + \varepsilon_t \quad \text{and} \quad \hat{\omega}^*_t = \rho\hat{\omega}^*_{t-1} + \varepsilon^*_t,$$

where the "hat" variables refer to proportional deviations from the expected values, e.g. $\hat{\omega}_t = \ln(\omega_t\text{-}\bar{\omega})$, where $\bar{\omega} = 1+\bar{\tau}$. The disturbances ε and ε^* are distributed normally with zero means and identical variances. Note that tariff fluctuations are not transmitted across countries intertemporally, but can be contemporaneously correlated when $\text{cov}(\varepsilon_X,\varepsilon_Y)\neq 0$. In the two extreme cases of asset market structure examined in this section, the parameter ρ is irrelevant due to the time-separable nature of the optimization problems; however, ρ will be important in evaluating the dynamics of restricted asset markets examined in Section 3.

Portfolio Autarky:

In the absence of asset trade, analysis of the effects of tariff-rate changes represents an application of some basic comparative statics results from trade theory. It will be presented here in a form that differs somewhat from the usual analysis, so as to make the analysis directly comparable to the complete asset-market dynamics to be described later.

Agents maximize utility at time t subject to budget constraints

$$X_t + T_t = c_{Xt} + \pi_t(1+\tau_t)c_{Yt} \tag{1}$$

for the home agent and

$$\pi_t Y_t + T^*_t = c^*_{Xt}(1+\tau^*_t) + \pi_t c^*_{Yt} \tag{1*}$$

for the foreign agent, where T and T* are lump-sum transfers of tariff revenue to the residents of the tariff-imposing country. Agents take these transfers as given in their maximization problems, since their value depends on aggregate decisions which are beyond the control of individual agents.

First-order conditions for these maximization problems imply that marginal rates of substitution are set equal to domestic relative prices:

$$U_{Yt}/U_{Xt} = \pi_t(1+\tau_t) \ , \tag{2}$$

$$U^*_{Yt}/U^*_{Xt} = \pi^*_t/(1+\tau_t) \ . \tag{2*}$$

Solutions for consumption levels and the world relative price can be found by using equations (1), (2) and one of the two commodity market equilibrium conditions,

$$c_{xt} + c^*_{xt} = X_t \ , \tag{3a}$$

$$c_{Yt} + c^*_{Yt} = Y_t \ . \tag{3b}$$

Expressing these solutions in terms of proportional deviations from the baseline deterministic version of the model [e.g., $\hat{c}_X = dc_{xt}/c_{xt}$]:

$$\hat{c}_X = \frac{1-s}{2}\left\{\frac{1}{A}(\hat{\omega} - \hat{\omega}^*) + \frac{1}{\delta}(\hat{\omega} + \hat{\omega}^*)\right\} \tag{4a}$$

$$\hat{c}_Y = \frac{s}{2}\left\{\frac{1}{A}(\hat{\omega} - \hat{\omega}^*) - \frac{1}{\delta}(\hat{\omega} + \hat{\omega}^*)\right\} \tag{4b}$$

$$\hat{c}^*_X = -\frac{s}{2}\left\{\frac{1}{A}(\hat{\omega} - \hat{\omega}^*) + \frac{1}{\delta}(\hat{\omega} + \hat{\omega}^*)\right\} \tag{4a*}$$

$$\hat{c}^*_Y = -\frac{1-s}{2}\left\{\frac{1}{A}(\hat{\omega} - \hat{\omega}^*) - \frac{1}{\delta}(\hat{\omega} + \hat{\omega}^*)\right\} \tag{4b*}$$

where $A = 2s(1-\delta)+\delta$. From the definition of s, it can be verified that A is always positive.

Aggregate consumption is defined by the CES aggregator function nested within the utility function:[4]

$$C = \left[\left(\frac{1}{2}\right) c_{xt}^{1-\delta} + \left(\frac{1}{2}\right) c_{Yt}^{1-\delta}\right]^{1/(1-\delta)} .$$

Movements of aggregate consumption levels can therefore be approximated by using

[4] The expression for aggregate consumption can also be written as $C = p_x c_x + p_y c_y$, where p_x and p_y are utility-denominated prices (marginal utilities) of X and Y.

$\hat{C} = \theta\hat{c}_X + (1-\theta)\hat{c}_Y$, so

$$\hat{C} = \frac{1}{2}\left[\frac{K}{A}(\hat{\omega} - \hat{\omega}^*) + \frac{\theta - s}{\delta}(\hat{\omega} + \hat{\omega}^*)\right], \tag{5}$$

$$\hat{C}^* = \frac{1}{2}\left[-\frac{K}{A}(\hat{\omega} - \hat{\omega}^*) + \frac{\theta - s}{\delta}(\hat{\omega} + \hat{\omega}^*)\right]. \tag{5*}$$

where $K = \theta(1-s)+s(1-\theta)$. [K is illustrated as a function of δ in the lower panel of Figure 1.]

In the consumption solutions (4) and (5), the effects of tariff changes are expressed in two distinct terms. The second term reflects the distortionary effects that arise when either country raises its tariff rate. This term implies unambiguously lower consumption for both countries. The first terms in (4) and (5) capture the effects of *relative* tariff-rate changes. In the present setting, these effects are reflected in changes of the world relative price, π,

$$\hat{\pi} = -\frac{s}{A}(\hat{\omega} - \hat{\omega}^*) . \tag{6}$$

While the pure distortionary effect of a tariff lowers utility for both countries, the terms-of-trade effect favors the tariff-imposing country. The terms-of-trade effect dominates as long as $K/A > (s-\theta)/\delta$. I will assume that this condition holds; that is, that the beneficial terms-of trade effects stemming from a tariff do not exceed the loss due to the lower trade volume that results.[5] Regardless of whether or not a tariff increases the absolute level of welfare for the tariff-imposing country, it will always raise domestic consumption *relative* to

[5] This condition simply requires that the baseline tariff rate $\bar{\tau}$ not exceed the "optimum" level.

foreign consumption:

$$\widehat{\left(\frac{C}{C^*}\right)} = (\hat{C} - \hat{C}^*) = \frac{K}{A} (\hat{\omega} - \hat{\omega}^*) \ . \tag{7}$$

These dynamics can be illustrated in an Edgeworth-box diagram, where the relevant contract curve is a distorted relationship summarizing the set of consumption allocations for which marginal rates of substitution differ by the total tariff wedge. Figure 2 provides an example. The steady-state equilibrium is given by point S. If both countries simultaneously increase their tariff rates, the distorted contract curve bows out toward the endowment point but the world terms of trade remain unchanged. At point B, both countries suffer a decline in utility as a result of the distortion. If only the home country raises its tariff rate, the terms of trade move in favor of the home country, moving the equilibrium along the new contract curve to a point such as H. A foreign tariff results in equilibrium F, where the relative price of Y rises and the foreign country is made better off.[6]

Finally, consider the effect of tariff rate changes on the composition of agents' consumption bundles. Deviations in the ratio of X to Y consumed are:

$$(\hat{c}_X - \hat{c}_Y) = \frac{1-2s}{2A} (\hat{\omega} - \hat{\omega}^*) + \frac{1}{2\delta} (\hat{\omega} + \hat{\omega}^*) \ , \tag{8}$$

$$(\hat{c}^*_X - \hat{c}^*_Y) = \frac{1-2s}{2A} (\hat{\omega} - \hat{\omega}^*) - \frac{1}{2\delta} (\hat{\omega} + \hat{\omega}^*) \ . \tag{8*}$$

Equal increases in both tariffs [captured by the second terms in equations (8)] distort the composition of consumption bundles in opposite directions, increasing

---

6 Points F, H and B are associated with a common *product* of tariff wedges, $(1+\tau)(1+\tau^*)$=constant.

the c_X/c_Y ratio and lowering the c^*_X/c^*_Y ratio. Responses to changes in relative tariff rates [the first terms in equations (8)] give rise to movement along the distorted contract curve, changing the X/Y-consumption ratio in the same direction for both agents. This increases the compositional distortion for one agent and dampens it for the other.

The role of δ as a risk aversion parameter can be seen by considering the effect of a small change in its value on the variability of aggregate consumption and the composition of the consumption bundle. Using equations (5) and (8),

$$\mathrm{var}(\hat{C}) = \frac{1}{4}\left[\left(\frac{K}{A}\right)^2(\sigma^2 - \sigma_{12}) + \left(\frac{\theta\text{-}s}{\delta}\right)^2(\sigma^2 + \sigma_{12})\right] \tag{9}$$

$$\mathrm{var}(\hat{c}_X - \hat{c}_Y) = \frac{1}{4}\left[\left(\frac{1-2s}{A}\right)^2(\sigma^2 - \sigma_{12}) + \left(\frac{1}{\delta}\right)^2(\sigma^2 + \sigma_{12})\right] \tag{10}$$

where $\sigma^2 = \mathrm{var}(\omega) = \mathrm{var}(\omega^*)$ and $\sigma_{12} = \mathrm{cov}(\omega,\omega^*)$. A small increase in δ clearly lowers the variability of C and c_X/c_Y due to the second term (smaller volume-of-trade effects). At the same time, however, a small increase in δ reduces the magnitude of A, implying larger shifts along a given contract curve (greater terms-of-trade effects).[7] This effect tends to raise var(C) *and* var(c_X/c_Y). Hence, terms-of-trade changes tend *not* to act as a compositional risk pooling mechanism in this framework.[8]

---

[7] This analysis can be more formally demonstrated using numerical calculation of the derivatives of Δ and var(π) [not reported].

[8] The terms-of-trade effects tend to dominate for the case of var(C), while the volume-of-trade effects dominates for var(X/Y). So even though the effect of δ on A tends to raise var(c_x,c_y), the total effect of an increase in compositional risk aversion is to reduce the variability of the X/Y-consumption ratio, at the expense of increasing the variability of aggregate consumption.

As pointed out by Cole and Obstfeld (1991), relative endowment disturbances change the terms of trade in a way that tends to pool consumption risk. For the special case where $\delta=\gamma=1$, the introduction of asset trade has no effect on equilibrium allocations. As will be shown below, this correspondence does not hold when tariff distortions are the stochastic element of the model.

Complete Asset Markets:

Now consider an environment in which agents can trade in a complete array of state-contingent securities prior to the realization of tariff rates. This allows the pooling of tariff risk and the achievement of constrained Pareto-optimal allocations.

Agents now maximize *expected* utility subject to resource constraints

$$\int [q_X(X - c_X) - q_Y(1+\tau)c_Y + T]\, dF(\omega,\omega^*) = 0$$

for the home agent and

$$\int [q_Y(Y - c_Y^*) - q_X(1+\tau^*)c_X^* + T^*]\, dF(\omega,\omega^*) = 0$$

for the foreign agent; where q_X and q_Y denote the prices of assets which pay one unit of X or Y for a particular tariff realization, and $F(\omega,\omega^*)$ is the joint distribution function for the tariff rates.

First-order conditions for these problems imply that for each possible state of the world:

$$U_X = q_X\lambda \qquad U_X^* = q_X(1+\tau^*)\lambda^* \qquad (11a,11a^*)$$

$$U_Y = q_Y(1+\tau)\lambda \qquad U_Y^* = q_Y\lambda^* \qquad (11b,11b^*)$$

where λ and λ^* are the multipliers associated with the agents' resource constraints.

Note that it will still be true that marginal rates of substitution are set equal to distorted domestic relative prices: for $\pi = q_Y/q_X$,

$$U_Y/U_X = \pi(1+\tau) \quad \text{and} \quad U^*_Y/U^*_X = \pi/(1+\tau^*).$$

Of course, this must be true since allocations must clear the goods market as well as the asset market. Contingent claims trade introduces a new requirement to the equilibrium allocations:

$$(1+\tau^*)\left(\frac{U_X}{U^*_X}\right) = \left(\frac{1}{1+\tau}\right)\left(\frac{U_Y}{U^*_Y}\right) = \varphi\ , \tag{12}$$

where φ denotes the (constant) ratio of the shadow values of wealth $\varphi=\lambda/\lambda^*$. Given the endowment normalization and the symmetric joint distribution of tariff rates, these multipliers will be equal and their ratio φ will be one. Equations (12) demonstrate how asset trade links marginal utilities of each good separately across countries, with the relationship distorted by the presence of tariffs.

For any particular wealth distribution (summarized by φ), equations (12) and the two commodity-market equilibrium conditions (3) can be solved for equilibrium allocations. The terms of trade, π, can then be calculated as the ratio of (distorted) marginal utilities. Expressed as proportional deviations from the steady-state, these solutions are identical to equations (4), with A replaced by $\Delta = 2K(\delta-\gamma)-\delta$:

$$\hat{c}_X = \frac{1-s}{2}\left\{\frac{1}{\Delta}(\hat{\omega} - \hat{\omega}^*) + \frac{1}{\delta}(\hat{\omega} + \hat{\omega}^*)\right\}\ , \tag{13a}$$

$$\hat{c}_Y = \frac{s}{2}\left\{\frac{1}{\Delta}(\hat{\omega} - \hat{\omega}^*) - \frac{1}{\delta}(\hat{\omega} + \hat{\omega}^*)\right\}\ . \tag{13b}$$

Using (3), expressions for foreign consumptions can be found as $c^*_X = -[s/(1-s)]c_X$ and $c^*_Y = -[(1-s)/s]c_Y$.

The second terms in equations (13) are identical to the second terms in (4), illustrating that the pure distortionary effects of tariff wedges represent unpoolable risk (due to reduction in the volume of trade).

The differences between the two regimes are summarized by the differences between Δ and A, which determine the position of equilibrium along the distorted contract curve (as represented by the terms of trade). Since agents are now actively pooling risk, the coefficient of relative risk aversion, γ, is now relevant to the equilibrium outcome of changes in relative tariff rates.

With the exception of an extreme case described below, Δ is always negative (opposite in sign to A). Hence, a relative increase in the home country's tariff rate *lowers* domestic consumption and *raises* foreign consumption. For the special case of $\delta=\gamma=1$, $\Delta=-A=-1$, so portfolio autarky and complete markets have exactly opposite effects on aggregate consumption levels. In Figure 2, a home tariff is associated with point F while a foreign tariff is associated with point H.

Aggregate consumption deviations and relative X/Y consumption ratios will be identical in form to expressions (4) and (5), with A replaced by Δ. The variances of C and c_X/c_Y will be represented by modified forms of (9) and (10). Since the second terms of (9) and (10) are the same in both regimes, they have the same effects on var(C) and var(c_X/c_Y): greater aversion to compositional risk results in equilibrium dynamics with lower values for both variances.

In contrast to portfolio autarky, increase in δ will be associated with smaller movements along the contract curve, tending to reduce var(c_X/c_Y) further.[9] Graphically, a higher value of δ implies that points F and H in Figure 2 lie

[9] That is, increases in δ result in lower values of Δ (increasing the absolute value of $\Delta<0$).

closer to point B. It is in this sense that asset trade enhances the ability of agents to insure against compositional risk. Note that the decrease in the magnitude of shifts along the contract curve also tends to lower the variance of aggregate consumption, even though an increase in δ represents a decline in the magnitude of aggregate risk aversion relative to compositional risk aversion.

Since aggregate risk aversion is now relevant, it is also of interest to examine the effects of changes in γ on the variability of aggregate consumption and its composition. A change in γ has no effect on the variability attributable to the second terms of (9) and (10) [since those terms represent undiversifiable risk]. Taking the derivative of the first terms with respect to γ, it turns out that a small increase aggregate risk aversion results in lower var(C) *and* $\text{var}(c_X, c_Y)$.[10] The fact that both variances are affected in the same manner again demonstrates the property that aggregate consumption risk and compositional consumption risk cannot be directly traded-off against each other as is the case for a stochastic endowments model.[11]

Thus far, I have proceeded under the assumption of $\Delta < 0$, which implies that the relative consumption of the home agent unambiguously declines with the imposition of a home tariff. It is possible for Δ to be positive, however, implying that the home country benefits from a domestic tariff under either of the regimes. Formally:

$$\Delta > 0 \text{ iff } \gamma/\delta < (K - \tfrac{1}{2})/K .$$

---

[10] That is, $\partial\Delta/\partial\gamma = -2K < 0$. Increases in γ result in a higher absolute value of $\Delta < 0$, implying smaller shifts along the contract curve.

[11] See Pakko (1994).

Since $K > \frac{1}{2}$ for $\delta > 1$ [see Figure 1, Panel B], this possibility cannot be ruled out for very low values of γ. Although these conditions are rather stringent, this analysis shows that the *ex post* preference reversal phenomenon described by Stockman and Dellas does not always hold when $\gamma \neq \delta$. It will be demonstrated below that if Δ and A are both positive, $\Delta < \mathrm{A}$, so the complete-markets solution has home consumption rising by *more* than would be the case without asset trade.

Tariff Paradoxes:

Two possibilities for paradoxical outcomes are prominent in the traditional trade-theory literature on tariff changes. The first, attributed to Lerner (1936), involves the possibility that a tariff on imports raises the world price of imports. This outcome requires that the tariff revenue be transferred to an agent (e.g. the public sector) which has a high marginal propensity to consume imports. In the model considered here, where tariff revenue is rebated to the representative agent, this paradox clearly can not occur in portfolio autarky.

The second paradox, attributed to Metzler (1949), is that the world price of the import good will fall by such a large magnitude that the domestic price of imports falls. This outcome requires elasticity of foreign import demand is smaller than the home country's marginal propensity to consume its export commodity. As with the Lerner paradox, it can be verified that this situation can not occur in the framework considered in this chapter: With identical underlying preferences, a tariff always raises the domestic price of imports. Jones (1985) demonstrates that even if the foreign import demand elasticity is low, the Metzler paradox is ruled out when substitution effects in the home country are also small (as is the case here).

While these paradoxes cannot occur in portfolio autarky, they *are* possible in the complete-markets regime. Both require that γ be small relative to δ. As described above, the values of γ and δ determine the position of equilibrium along the distorted contract curve. Smaller values of γ, for example, are generally associated with larger deviations in relative consumption levels. The lower is γ, the greater is the decline in the home country's utility in response to a tariff on Y-imports.

To explain the emergence of paradox-type outcomes, it will be helpful to consider a simple state-contingent transfer scheme which could be used to replicate the constrained Pareto-optimal allocations achieved with complete markets. Such a transfer scheme is illustrated in Figure 3.

Point H illustrates the response to a home-country tariff for the benchmark case $\gamma=\delta=1$. This outcome could be achieved by arranging a transfer of income from the home agent to the foreign agent of magnitude T^0 (measured in units of x-goods) and then allowing free trade in commodities.

For $\gamma<1$, the complete-markets allocation involves a further decline in home consumption. For very low values of γ the equilibrium will be a point like L, which is associated with an increase in the world relative price of the y-good: a Lerner paradox outcome. Note that this allocation is achievable through a large transfer of purchasing power (T^L) to the foreign agent, who has a relatively higher propensity to consume the home country's imported good.

More formally, the conditions leading to this outcome can be derived by considering the determinants of terms-of-trade dynamics explicitly. In the complete markets setting, fluctuations in the world relative price of Y are given by

$$\hat{\pi} = \frac{1}{2}\left[\frac{(1-2s)\delta}{\Delta} - 1\right](\hat{\omega} - \hat{\omega}^*) \ . \qquad (14)$$

A tariff imposed by the home country will raise the world price of Y if the bracketed expression in (14) is positive. Obviously, this condition cannot hold for $\Delta>0$, but can occur for $\Delta<0$ as long as $\gamma/\delta<[K-(1-s)]/K$. Thus, the condition for a Lerner-type paradox is:

$$\frac{K - \frac{1}{2}}{K} < \frac{\gamma}{\delta} < \frac{K - (1-s)}{K} \ .$$

As shown above, very small values of γ/δ can be associated with $\Delta>0$ [as long as $\delta>1$], implying a welfare-improvement for the tariff-imposing country. This is the situation in which a Metzler-type paradox can occur, and is illustrated as point M in Figure 3. Note that point M is associated with a transfer of purchasing toward the *home* country (of magnitude T^M). Because the steady-state tariff wedge implies a high marginal propensity to consume the x-good on the part of the home agent, this increase in purchasing power tends to raise the relative price of the x-good, so depressing the price of the y-good that the home-country domestic price of Y falls.

Formally, a tariff imposed by the home-country will result in a lower domestic import price if $\hat{\pi} + \hat{\omega} < 0$, requiring $\delta(2s-1)/\Delta > 1$. This can clearly not happen for $\Delta<0$, but implies the condition $\gamma/\delta > (K-s)/K$ when $\Delta>0$. Since $(K-s)<0$, $\Delta>0$ is a necessary and sufficient condition for the Metzler-type paradox. (Recall that $\delta>1$ is, in turn, a necessary condition for $\Delta>0$).

To relate these paradoxical outcomes to the concepts of aggregate and compositional risk, consider the following sequence of comparative statics results: Beginning from the benchmark case of $\gamma=\delta=1$, a decline in aggregate risk aversion induces larger shifts along the contract curve. For γ low enough, the Lerner-type paradox occurs. In the limit as $\gamma \to 0$, the variability of consumption is maximized as $\Delta \to 0$. From that point, an increase in *compositional* risk aversion δ raises the value of Δ to a positive number (lowering the variability of c_X/c_Y), allowing the outcome of the Metzler-type paradox.

Because the Metzler-type paradox cannot occur under portfolio autarky but will always occur with complete markets for $\Delta > 0$, it is clear that the rise in domestic consumption associated with $\Delta > 0$ will always exceed the magnitude of the increase realized in the absence of asset markets.

The conditions underlying $\Delta > 0$ are rather extreme, so it is unlikely that this outcome is anything more than a theoretical possibility. In the following section, I will assume that γ is high enough to rule out this paradoxical case.

4. Incomplete Markets and Current Account Dynamics

The previous section examined extreme cases of asset market structure. In reality, neither of these extremes is likely to be literally true. In this section I examine the implications of trade in a limited menu of assets. In particular, I derive the dynamics which occur when countries are free to borrow and lend, but have no ability to trade in assets which are contingent on tariff realizations.

The main result of this section is that a temporary increase in import tariffs results in an allocation which can be expressed as a weighted average of the complete-markets and portfolio autarky cases. Consequently, current consumption may either rise or fall in the tariff-imposing country, but a current account surplus implies that *future* consumption will be unambiguously higher. This outcome recovers a version of the standard trade-theory result that a small import tariff raises domestic consumption; if not in the short-run, at least in the long-run. Moreover since a current account surplus is associated with an increase in the domestic price of importables, the classic Harberger-Laursen-Metzler effect is violated, demonstrating that the *source* of a relative price change is crucial to the validity of that effect.

Optimization Problems:

The setting for this version of the model is one in which agents are unable to trade in assets before the realization of tariff shocks, but can borrow and lend in order to smooth consumption once the current state is revealed.

Let B_t (B_t^*) denote the bond holdings of the home (foreign) agent brought forward into the period. Maximization problems can be defined recursively in terms of value functions of the form:

$$V(B_t) = \max \left\{ U(c_{xt}, c_{Yt}) + \beta E_t V(B_{t+1}) \right\}$$

where the maximization problem on the right-hand side is subject to intertemporal budget constraints

$$X_t + T_t + B_t = c_{xt} + \pi_t(1+\tau_t)c_{Yt} + q_t B_{t+1} \tag{15}$$

for the home agent, and

$$\pi_t Y_t + T_t^* + B_t^* = c_{xt}^*(1+\tau_t^*) + \pi_t c_{Yt}^* + q_t B_{t+1}^* \tag{15*}$$

for the foreign agent, where q_t represents the current price (denominated in x-goods) of discount bonds (the reciprocal of the gross one-period interest rate). Again, T and T* represent lump-sum transfers of tariff revenue.

First-order conditions for consumption demands are the same as those derived for complete markets (11), with λ_t (instead of λ), representing the current shadow value of wealth associated with the budget constraints. As in the complete markets case, these conditions can be combined to yield:

$$(1+\tau_t^*)\left(\frac{U_{xt}}{U_{xt}^*}\right) = \left(\frac{1}{1+\tau_t}\right)\left(\frac{U_{Yt}}{U_{Yt}^*}\right) = \varphi_t\ , \tag{16}$$

which is identical to (12) except that the ratio of shadow prices $\varphi_t = \lambda_t/\lambda_t^*$ is no longer constant over time, but depends on the current state of the world (which now includes the relative wealth distribution summarized by B_t and B_t^*).

Equilibrium Solutions:

Equations (16), together with the two commodity market equilibrium conditions (3), can be solved for c_{xt}, c_{Yt}, c_{xt}^*, and c_{Yt} as functions of the tariff rates, ω and ω^*, and the ratio of shadow prices φ_t. Expressed in terms of proportional deviations from the deterministic baseline equilibrium:[12]

$$\hat{c}_{xt} = \frac{1-s}{2}\left\{\frac{1}{\Delta}\,[2\hat{\varphi}_t + (\hat{\omega}_t - \hat{\omega}_t^*)] + \frac{1}{\delta}(\hat{\omega}_t + \hat{\omega}_t^*)\right\} \tag{17a}$$

$$\hat{c}_{Yt} = \frac{s}{2}\left\{\frac{1}{\Delta}\,[2\varphi_t + (\hat{\omega}_t - \hat{\omega}_t^*)] - \frac{1}{\delta}(\hat{\omega}_t + \hat{\omega}_t^*)\right\} \tag{17b}$$

again with $c_X^* = -[s/(1-s)]c_X$ and $c_Y^* = -[(1-s)/s]c_Y$.

First-order conditions with respect to the intertemporal borrowing/lending decision are $q_t = \beta E(\lambda_{t+1})/\lambda_t$ and $q_t = \beta E(\lambda_{t+1}^*)/\lambda_t^*$. In the complete contingent

[12] In the present context where relative wealth evolves over time in response to borrowing and lending, these deviations are perhaps more accurately described as representing proportional deviations from the original wealth distribution.

claims regime, marginal utilities were equalized across countries in all states of the world. In this restricted setting, interest rate parity implies that the expected *growth rates* of marginal utility are equalized. In effect, agents are using intertemporal trade as a partial substitute for inter-state trade. Ignoring the convexity term associated with the ratio of expected values (i.e. assuming approximate certainty equivalence), interest rate parity implies

$$\hat{\varphi}_t = \hat{\varphi}_{t+1} \, . \tag{18}$$

The asset market equilibrium condition, $B_t + B_t^* = 0$, implies that one of the two budget constraints (15) is redundant. Expressing the home budget constraint in terms of proportional deviations and substituting the consumption expressions (17),

$$\hat{B}_t - \beta\hat{B}_{t+1} = (1\text{-}s)\frac{A}{\Delta}\,\hat{\varphi}_t + \frac{(1\text{-}s)}{2}\left[\frac{\Delta - A}{\Delta}\right](\hat{\omega}_t - \hat{\omega}_t^*) \, . \tag{19}$$

Equations (18) and (19) represent a difference equation system in $\hat{B}$ and $\hat{\varphi}$, which can be solved recursively to yield:

$$\hat{\varphi}_t = \frac{\Delta}{A}\left(\frac{1\text{-}\beta}{1\text{-}s}\right)\hat{B}_t + \frac{1}{2}\left(\frac{1\text{-}\beta}{1\text{-}\rho\beta}\right)\left[\frac{\Delta - A}{\Delta}\right](\hat{\omega}_t - \hat{\omega}_t^*) \tag{20}$$

$$\hat{B}_{t+1} - \hat{B}_t = \frac{(1\text{-}s)}{2}\left(\frac{1\text{-}\rho}{1\text{-}\rho\beta}\right)\left[\frac{\Delta - A}{\Delta}\right](\hat{\omega}_t - \hat{\omega}_t^*) \tag{21}$$

Equation (20) summarizes the effects of the change in the interest rate on the ratio of marginal utilities, and (21) demonstrates the effect of this change on current account dynamics.

Finally, the solution for φ_t given in (20) can be substituted into equations (17) to yield:

$$\hat{c}_{xt} = \frac{(1-s)}{2}\left\{\left[\frac{\beta(1-\rho)}{1-\rho\beta}\right]\left(\frac{1}{\Delta}\right) + \left[\frac{1-\beta}{1-\rho\beta}\right]\left(\frac{1}{A}\right)\right\}(\hat{\omega}_t - \hat{\omega}_t^*) + \frac{(1-s)}{2\delta}(\hat{\omega}_t + \hat{\omega}_t^*) + \left(\frac{1-\beta}{A}\right)\hat{B}_t \tag{22a}$$

$$\hat{c}_{Yt} = \frac{s}{2}\left\{\left[\frac{\beta(1-\rho)}{1-\rho\beta}\right]\left(\frac{1}{\Delta}\right) + \left[\frac{1-\beta}{1-\rho\beta}\right]\left(\frac{1}{A}\right)\right\}(\hat{\omega}_t - \hat{\omega}_t^*) + \frac{s}{2\delta}(\hat{\omega}_t + \hat{\omega}_t^*) + \frac{s}{1-s}\left(\frac{1-\beta}{A}\right)\hat{B}_t \tag{22b}$$

Each of equations (22) consists of three terms. The first corresponds to shifts along a given contract curve in response to changes in *relative* tariff rates. In this case, the shifts reflect elements of both the complete-markets and portfolio autarky regimes. The second term, which is still unchanged from the previous analyses, reflects the undiversifiable risk due to shifts in the contract curve itself. The third term, which is unique to this particular regime, reflects accumulated changes in the wealth distribution brought about by international borrowing and lending in previous periods.

When the wealth distribution is equal to its initial value ($\hat{B}_t=0$), equations (22) can be written as:

$$\hat{c}_{xt} = \left[\frac{\beta(1-\rho)}{1-\rho\beta}\right]\hat{c}^C_{xt} + \left[\frac{1-\beta}{1-\rho\beta}\right]\hat{c}^A_{xt} \tag{22a$'$}$$

$$\hat{c}_{Yt} = \left[\frac{\beta(1-\rho)}{1-\rho\beta}\right]\hat{c}^C_{Yt} + \left[\frac{1-\beta}{1-\rho\beta}\right]\hat{c}^A_{Yt} \tag{22b$'$}$$

where the superscripts A and C denote the portfolio autarky and complete markets solutions, respectively. The coefficients in equations (22′) sum to one, so the bonds-only allocations are *weighted averages* of the two extreme cases of asset market structure. For permanent changes in tariff rates ($\rho=1$), the bonds-only allocations are identical to portfolio autarky. For purely temporary changes ($\rho=0$), allocations are more closely associated with the complete-market outcomes. Since tariffs have opposite welfare effects under the two extreme regimes (as long as $\Delta<0$), the weighted average nature of the allocations implies that current consumption may either rise or fall in the tariff-imposing country, depending on the (expected) persistence of the shock and the magnitudes of aggregate and compositional risk aversion.

As long as the tariff change is not permanent, an increase in the tariff rate leads to a current account surplus for the tariff-imposing country [as can be verified by examining (21)]. Hence, a domestic import tariff will always result in higher domestic consumption. This outcome is immediate and permanent for permanent tariff-rate changes, and will occur in the long-run (if not the short-run) for temporary changes.

This result is qualitatively the same as that described for a small-country case by Razin and Svensson (1983). However, the magnitude of current account changes will be greater in the large-country case considered here. Razin and Svensson explain the current account movement by pointing out that an increase in the domestic price of importables gives rise to both intra-temporal and intertemporal substitution effects: A tariff raises the demand for domestic goods over foreign goods, and by increasing the price of current consumption relative to future consumption, it also lowers current consumption and results in a current account surplus.

An additional effect is present in the large country case considered here. Prior to engaging in any borrowing or lending, the imposition of a tariff raises the real income of the tariff-imposing country (as under portfolio autarky). The increase in real income has a positive wealth effect, but because it is temporary consumption demand will rise by less than the income increase. This results in a larger current account improvement than implied by the intertemporal price change alone.

This additional channel is reminiscent of the Harberger-Laursen-Metzler effect, by which an increase in the domestic price of importables is postulated to reduce domestic saving as a result of declining real income. In this case, however, the effect works in the opposite direction. In the small country analysis of Svensson and Razin (1983), the wealth effect and intertemporal price effects have opposing influences, with a relatively stronger wealth effect required to generate the typical HLM effect. Here, both effects conspire to increase saving in the tariff-imposing country, unambiguously improving the current account. This analysis demonstrates that partial equilibrium analysis (or even general equilibrium analysis for a small country) can be misleading. The *source* of the change in the domestic price ratio is crucial to evaluating the validity of the HLM effect.

5. Conclusions

This paper has considered the risk sharing arrangements that emerge when trading partners are subject to tariff risk. The differences between extreme asset-market regimes of complete markets and portfolio autarky have been described in terms of aggregate and compositional risk aversion. In particular, complete markets enhance the ability of agents to pool compositional risk associated with tariff distortions.

While the reversal phenomenon described by Stockman and Dellas (1986) holds for almost all feasible parameter values, it does not necessarily hold when aggregate risk aversion is very low. Complete-market analogs to Lerner and Metzler tariff paradoxes are also possible.

When asset markets are incomplete, the inability to trade directly across states induces agents to substitute intertemporally. The equilibrium dynamics of such a restricted asset-market setting can be expressed as a weighted average of the extreme responses under portfolio autarky and complete markets. With the exception of the extreme parameter values associated with the Metzler-type paradox, a country imposing a temporary tariff increase tends to run a current account surplus, accumulating a larger share of world wealth. Hence, asset market restrictions imply a modified form of the traditional trade-theory result that the imposition of a small tariff can raise domestic welfare.

References

Backus, David K. (1989), "Interpreting the Comovements in the Trade Balance and the Terms of Trade," *Working Paper #441*, Federal Reserve Bank of Minneapolis.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992b), "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve Revisited", *Discussion Paper 65*, Institute for Empirical Macroeconomics.

Barari, Mahua, and Harvey E. Lapan (1993), "Stochastic Trade Policy With Asset Markets: The Role of Tariff Structure," *Journal of International Economics* **35**, 317-333.

Baxter, Marianne and Mario J. Crucini (1992), "Business Cycles and the Asset Structure of Foreign Trade", *Discussion Paper 65*, Institute for Empirical Macroeconomics.

Cole, Harold (1988), "Financial Structure and International Trade," *International Economic Review* **29**, 237-59.

Cole, Harold, and Maurice Obstfeld (1991), "Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?" *Journal of Monetary Economics* **28**, 3-24.

Edwards, Sebastian (1988), "Temporary Terms of Trade Disturbances, the Real Exchange Rate and the Current Account," NBER Working Paper No. 2629.

Feeney, Joanne and Ronald W. Jones (1994), "Risk Aversion and International Markets: Does Asset Trade Smooth Real Income?" *Review of International Economics* **2**, 13-26.

Hagiwara, May (1994), "Volatility in the Terms of Trade with Non-Identical Preferences," *Journal of International Money and Finance* **13**, 319-41.

Harberger, Arnold C. (1950), "Currency Depreciation, Income, and the Balance of Trade," *Journal of Political Economy* **58**, 47-60.

Jones, Ronald W. (1965), "The Structure of Simple General Equilibrium Models," *Journal of Political Economy* **73**, 557-72.

Jones, Ronald W. (1985), "Income Effects and Paradoxes in the Theory of International Trade," *Economic Journal* **95**, 330-44.

Jones, Ronald W. (1969), "Tariffs and Trade in General Equilibrium: A Comment," *The American Economic Review* **59**, 418-24.

Kollmann, Robert (1990), "World Business Cycles and Incomplete International Asset Markets," Manuscript, University of Chicago.

Laursen, Svend, and Lloyd A. Metzler (1950), "Flexible Exchange Rates and the Theory of Employment," *Review of Economics and Statistics* **32**, 281-99.

Lerner, A. P. (1936), "The Symmetry between Import and Export Taxes," *Economica* **3**, 67-84.

Lucas, Robert E. (1982), "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics* **10**, 335-59.

Metzler, Lloyd A. (1949), "Tariffs, the Terms of Trade, and the Distribution of National Income," *Journal of Political Economy* **57**, 1-29.

Obstfeld, Maurice (1982), "Aggregate Spending and the Terms of Trade: Is There a Laursen-Metzler Effect?" *Quarterly Journal of Economics* **97**, 251-70.

Ostry, Jonathan D. (1988), "The Balance of Trade, Terms of Trade, and Real Exchange Rate: An Intertemporal Optimizing Framework," *IMF Staff Papers* **37**, 541-73.

Ostry, Jonathan D. and Andrew K. Rose (1992), "An Empirical Evaluation of the Macroeconomic Effects of Tariffs," *Journal of International Money and Finance* **11**, 63-79.

Pakko, Michael R. (1994), "Reconciling Low Cross-Country Consumption Correlations with International Risk Sharing," Working Paper No. 94-019A, Federal Reserve Bank of St. Louis.

Persson, Torsten and Lars E. O. Svensson (1985), "Current Account Dynamics and the Terms of Trade: Harberger-Laursen-Metzler Two Generations Later," *Journal of Political Economy* **95**, 43-65.

Razin Assaf and Lars E.O. Svensson (1983), "Trade Taxes and the Current Account," *Economics Letters* **13**, 55-57.

Sadorsky, Perry (1994), "The Behavior of U.S. Tariff Rates: Comment," *The American Economic Review* **84**, 1097-1103.

Stockman, Alan C. (1987), "Some Interactions between Goods Markets and Asset Markets in Open Economies," in S. Arndt and D. Richardson (eds.) *Real-Financial Linkages Among Open Economies*, Cambridge, MA: MIT Press.

Stockman, Alan C. and Harris Dellas (1986), "Asset Markets, Tariffs, and Political Risk," *Journal of International Economics* **21**, 199-214.

Svensson, Lars E. O., and Assaf Razin (1983), "The Terms of Trade and the Current Account: The Harberger-Laursen-Metzler Effect," *Journal of Political Economy* **91**, 97-125.

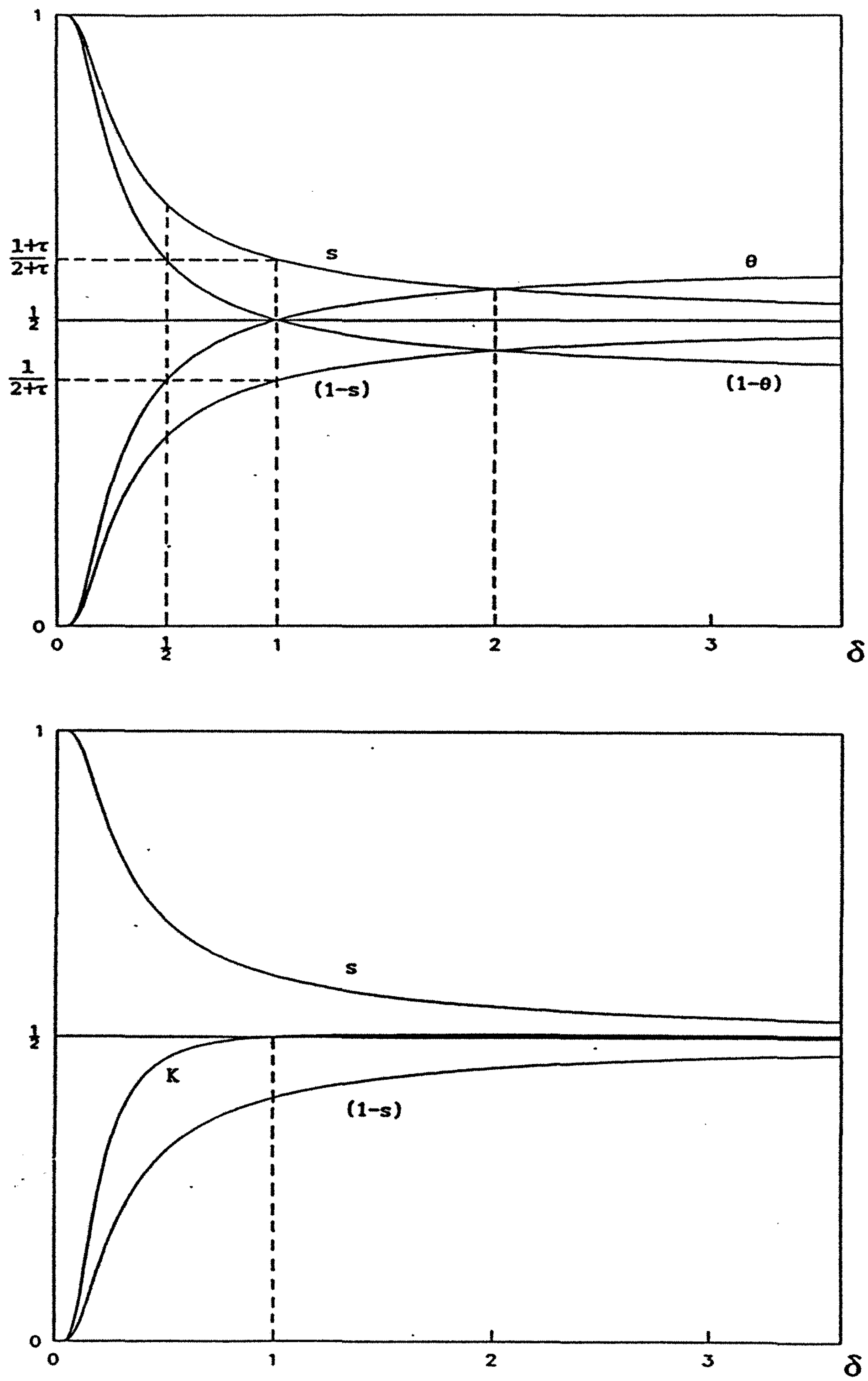

Figure 1:

Share Parameters and Substitution Elasticities

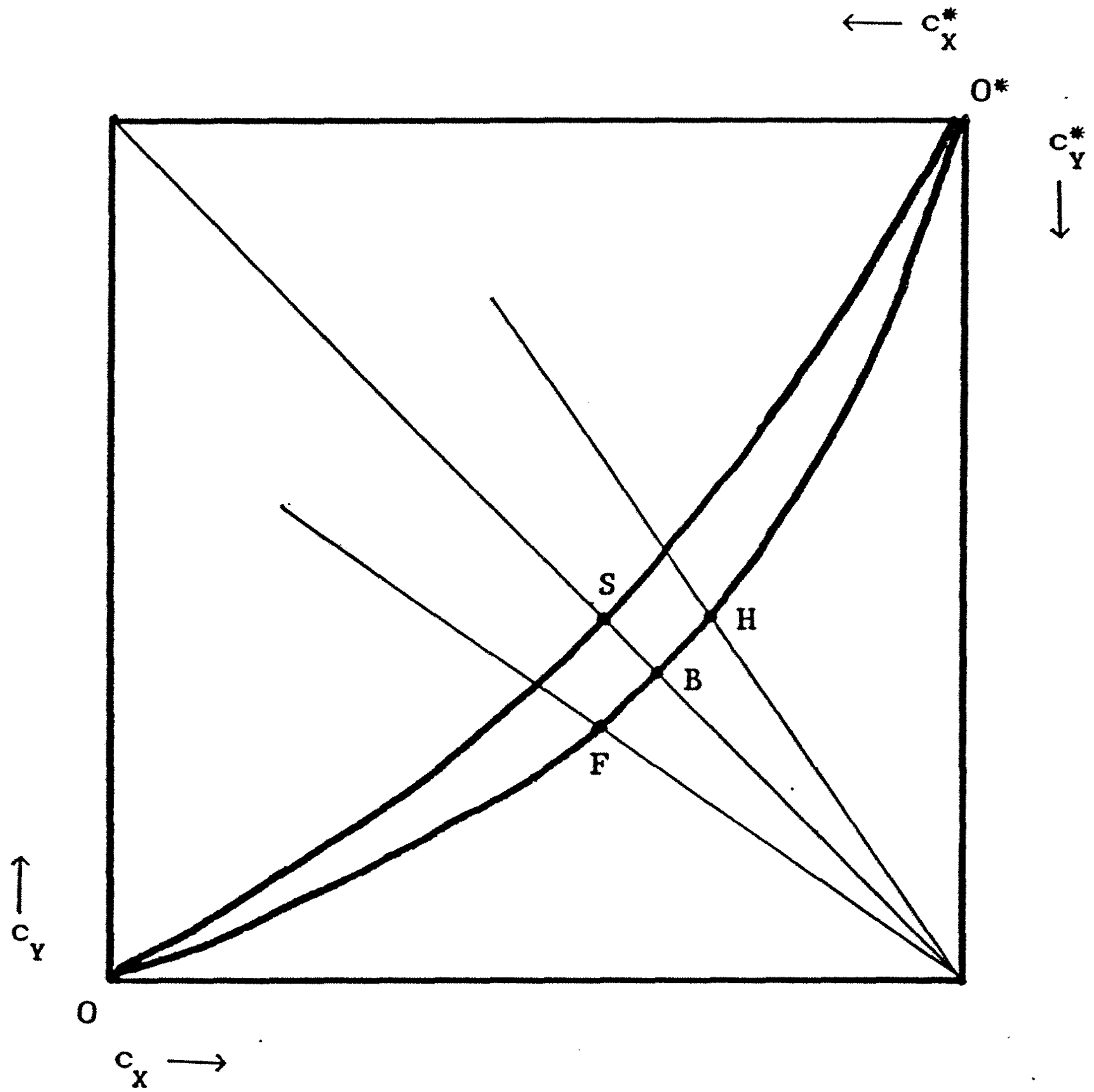

Figure 2:

Tariff Dynamics Under Portfolio Autarky

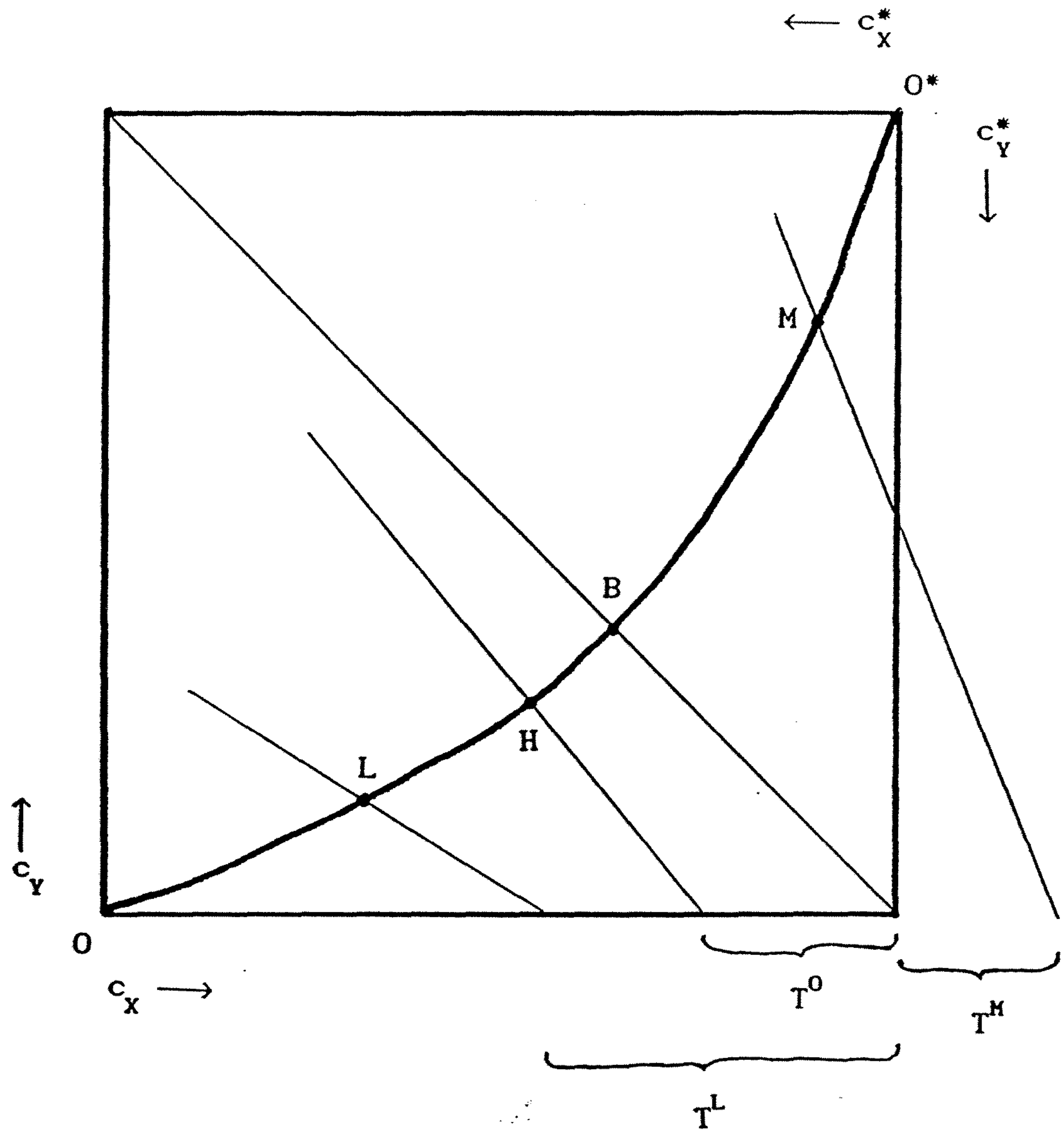

Figure 3:

Tariff Paradoxes