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Compound Volatility Processes in EMS Exchange Rates

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ABSTRACT

This paper introduces a compound GARCH/markov switching model to add flexibility to the GARCH model in order to model the volatilities of exchange rates in target zones subject to realignments. The compound volatility model endogenizes the weights given to realignments (and all other shocks) in the GARCH process. Previous GARCH applications to EMS exchange rates took polar positions by arbitrarily placing full or zero weight on realignment shocks. Markov switching in the student-*t* degrees-of-freedom parameter is shown to make the difference between rejection and acceptance of goodness-of-fit tests for four of the six EMS currencies studied.

KEYWORDS: Conditional heteroskedasticity, exchange rate target zone

JEL CLASSIFICATION: F31, C22

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Introduction

The Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) defines fluctuation bands around central parities for each member's exchange rate with the European Currency Unit (Ecu). For most of its history, the ERM target zones have been narrow 2.5 percent banks. The bands were widened substantially to 15 percent for all but the 'core' countries in response to speculative attacks in September 1992 and August 1993. The speculative attacks even forced Britain and Italy to suspend their participation in the ERM in September 1992. The switch to wider banks responded to the belief that narrow bands were making weaker currencies excessively vulnerable to speculation and, paradoxically, increasing exchange-rate uncertainty.

This paper re-examines the complex volatility processes behind exchange rate target zones subject to realignment. Summary statistics show that bilateral EMS exchange rate have the most leptokurtic (fat-tailed) distributions among high-frequency financial data [Vlaar and Palm (1993)]. We would expect the kurtosis, which is proportional to the "variance of the variance," to be high, as most changes in the exchange rate have to be very small to stay within what have generally been narrow 2.5 percent fluctuation bands in terms of an EMS currency relative to the European Currency Unit (Ecu), whereas realignments and speculative attacks can bring much larger changes in the exchange rates.

This article presents a volatility model which blends the widely-used model of generalized autoregressive conditional heteroscedasticity (GARCH) with a markov switching model. When applied to weekly EMS exchange rates, the compound volatility model permits time-varying variances, skewness, and kurtosis. One difficulty in modeling the conditional distributions of EMS exchange rates is that a realignment or even a large movement *within* the band possibly represents a discontinuous shift in the distribution. Widely used generalized autoregressive conditional heteroskedasticity (GARCH) models are not robust to discontinuous shifts in volatility [Nelson (1990)]. On the other hand, Krugman's (1991) theoretical model suggests that, when it is near the center of the band, the exchange rate should behave much like a free-floating rate in its reaction to news about fundamentals.

Thus, at times it would be convenient to use a GARCH-type model to capture changes in volatility, since GARCH models are believed to be adept at capturing changes in volatility caused by variation in the rate of information arrival [Lamoureux and Lastrapes (1990)]. Furthermore, Nelson (1992) has shown that GARCH models still obtain consistent variance estimates under certain types of misspecification, though not discontinuous shifts in the distribution.

The compound GARCH/markov switching volatility model presented here can address several features of EMS exchange rates. First, not all "jumps" in the level of the exchange rate (not necessarily limited to realignments) are assumed to be of the same size. The explained change in the level of the exchange rate can lie anywhere between an upper and lower bound, making the size of any given jump endogenous. Second, the model, unlike other models with a mean-variance relationship [Engel and Hakkio (1993)], does not impose that all periods of high variance are also periods of high skewness. In the compound variance process, the variance can be high due to large dispersion in the GARCH process, without a switch in the markov process. This distinction will be made more concrete in the next section where the model is presented. Third, the model follows much of the GARCH literature by allowing the innovations to be student-t rather than normal, yet differs from previous work by allowing the number of degrees of freedom in the student-t innovations to change over time. Fourth, the weight placed on last period's squared residual in the GARCH process depends on the state of the markov process. Thus, the model endogenizes the extent to which a realignment (or any lagged squared residual) is considered an innovation in the GARCH process. Previous models, in contrast, have taken the polar positions that realignments either are or are not innovations to the GARCH process.

Several previous applications of GARCH models to EMS exchange rates "dummy out" the realignments to avoid mixing discontinuous changes in the distribution with the GARCH process [Koedijk, Stork and De Vries (1992)]. Three problems with such a procedure are that not all realignments need be jumps; conversely, not all jumps need be realignments – they may also occur within the band; and not all realignments have identical impacts on the conditional variance in their aftermath. Another approach is the jump-diffusion model of Jorion (1988), Nieuwland et al. (1991) and Vlaar and Palm (1993), where in practice the conditional variance is increased each period by a constant amount due to the possibility of discontinuous jumps in the exchange rate. Another distinction between jump and markov processes is that the jumps occur independently across time, whereas markov processes can be positively serially correlated. Given the well-known clustering of volatile exchange-rate changes, a markov process is an attractive modeling device to combine with the widely-used GARCH process.

The next section presents the GARCH/markov switching volatility model for EMS exchange rates. The model is then applied to weekly data beginning in 1980 for EMS exchange rates vis-à-vis the Deutsche mark. The exchange rate bands in the ERM are formally defined in terms of a currency basket, the European Currency Unit or Ecu, but the Deutsche mark has never been devalued against the Ecu, nor has the D-mark's target value relative to the Ecu ever increased by less than another EMS currency relative to the Ecu. The subsequent section presents results for six ERM exchange rates and the Austrian schilling-Deutsche mark rate, since Austria has been a *de facto* EMS member by maintaining a tight exchange rate band with the Deutsche mark.

II A GARCH/Markov switching volatility model

The volatility model assumes a student-t error distribution with n_t degrees of freedom in the dependent variable y:

$$\begin{aligned} y_t &= \mu_t + \epsilon_t \\ & \epsilon_t \sim \text{student-}t(\text{mean} = 0, n_t, h_t) \\ & n_t > 2 \end{aligned} \tag{1}$$

with error variance

$$\sigma_t^2 = h_t \frac{n_t}{n_t - 2}. \tag{2}$$

The parameter n_t is the degrees-of-freedom parameter and a standard student-t random variable has variance equal to $\frac{n_t}{n_t-2}$ and kurtosis equal to $3\left(\frac{n_t-2}{n_t-4}\right)$. Thus, n_t can be called the "shape" parameter, because it defines the degree of leptokurtosis in the conditional distribution. The distribution converges to the standard normal as the degrees-of-freedom parameter approaches infinity. The time-varying parameter h_t scales the variance of ϵ_t for a given value of the shape parameter n_t. We call h_t the "dispersion" parameter, because it scales the variance up and down without affecting the thickness of the tails or leptokurtic shape of the conditional density. We then specify the three parameters of the conditional density (μ_t, h_t, n_t) as time-varying functions of one or more stochastic processes in order to model how the conditional density might change over time. We differ from the existing literature by not assuming that the conditional variance follows a GARCH process; instead the dispersion parameter h_t is assumed to follow a GARCH process. Note that if the shape parameter n_t were constant, this distinction would be immaterial.

In order to permit discrete shifts in the conditional distribution, we model the conditional mean, μ_t, and the shape parameter, n_t, as parameters subject to markov switching; they are assumed to be functions of a discrete, unobserved state variable that follows a first-order markov process:

$$\mu_t = \mu_l S_t + \mu_h(1 - S_t) \tag{3}$$

$$n_t = n_l S_t + n_h(1 - S_t) \tag{4}$$

$$2 < n_l < n_h$$

$$S_t \in \{0, 1\}$$

$$\text{Prob.}(S_t = 0 \mid S_{t-1} = 0) = p$$

$$\text{Prob.}(S_t = 1 \mid S_{t-1} = 1) \;=\; q$$

Equations (3) and (4) imply that the conditional mean, variance and kurtosis (provided it exists, i.e., that $n_t > 4$) undergo discrete shifts when the state variable switches. We place no prior restrictions on whether μ_l is greater or less than μ_h; μ_l is simply the conditional mean associated with the low degree-of-freedom state (where $n_t = n_l$), and its value can be greater than or less than the conditional mean associated with the high degree-of-freedom state, μ_h. Nevertheless, if the low degree-of-freedom state were to correspond to periods surrounding realignments, then we would expect that $\mu_l > \mu_h$, because realignments have always meant that the foreign-currency price of one Deutsche mark increases. Moreover, by tying the conditional mean to the same state variable as the conditional variance and kurtosis, we can generate a skewed distribution for the dependent variable y, even if the errors ϵ are symmetric.

The dispersion parameter, h_t, is assumed to follow a GARCH(1,1) process with an adjustment to account for the fact that the expected value of the squared residual is equal to σ_t^2, not h_t:

$$h_t \;=\; \gamma + \alpha\epsilon_{t-1}^2\left(\frac{n_{t-1}-2}{n_{t-1}}\right) + \beta h_{t-1} \tag{5}$$

Since $h_t = \sigma_t^2\left(\frac{n_t-2}{n_t}\right)$, the lagged squared residual in the GARCH process is also down-weighted by the factor $\left(\frac{n_t-2}{n_t}\right)$, which carries the interesting implication that the persistence term in the GARCH process, $\alpha\left(\frac{n_t-2}{n_t}\right)+\beta$, is time-varying such that the persistence decreases when $n_t = n_l$. In this way discrete shifts in n_t not only directly induce discrete shifts in the conditional variance, they also bring about shifts in the persistence of the GARCH dispersion.

In short, the key feature of the GARCH/Markov switching model is the relaxation of the assumption of a constant student-t shape parameter n, thereby permitting the conditional variance to be the product of *two* stochastic processes: a GARCH process for the dispersion parameter, h_t, and a discrete markov-switching process for the shape parameter, n_t. The idea behind this specification is that the variance can have the persistence associated with GARCH processes throughout periods when the discrete-valued shape parameter n remains essentially unchanged, yet the variance will be subject to potentially large discrete shifts whenever n changes. A secondary feature of this specification is that markov switching in n induces discrete shifts in the persistence of the GARCH process, as it reduces the weight given to shocks drawn from the more leptokurtic state. Previous studies that assume GARCH processes and student-t error distributions have held the degrees-of-freedom parameter constant [Baillie and Bollerslev (1989); Hsieh (1989)]. One exception is Hansen (1994), but his model assumes the variance follows a GARCH process, so the second moment is not subject to discrete shifts as the degrees-of-freedom parameter changes.

Estimation issues

This model builds on the work of Cai (1994) and Hamilton and Susmel (1994), who added markov switching to ARCH processes. The extension to GARCH processes is complicated by the presence of at least one moving-average coefficient (in our case β) in the GARCH process. Here I discuss how methods described in Kim (1994) can be applied to make estimation feasible. For estimation it is convenient to define $v = \frac{1}{n}$ and re-write equation (5) as

$$h_t = \gamma + \alpha\epsilon_{t-1}^2(1 - 2v_{t-1}) + \beta h_{t-1} \tag{6}$$

because in practice we estimate v_l and v_h as parameters in order to test whether they are significantly different from zero, i.e., whether the conditional densities are normal. We also introduce superscripts to indicate that a parameter is a function of contemporaneous and lagged values of the markovian state variable S. For example,

$$\mu_t^{(s_t)} = \mu_l S_t + \mu_h(1 - S_t) \qquad \text{when} \qquad S_t = s_t.$$

The autoregressive nature of equation (6) implies that h_t is a function of all past values of the state variable S_t. Using the superscript notation, we elaborate equation (6), making explicit the dependence on past values of S_t:

$$h_t^{(s_{t-1},s_{t-2},\ldots,s_1)} = \gamma + \alpha \left(\epsilon_{t-1}^{(s_{t-1})}\right)^2 \left(1 - 2v_{t-1}^{(s_{t-1})}\right) + \beta h_{t-1}^{(s_{t-2},\ldots,s_1)} \tag{7}$$

Clearly it is not practical to examine all of the possible sequences of past values of the state variable when evaluating the likelihood function for a sample of more than a thousand observations, as the number of cases to consider exceeds 1000 by the time $t = 10$. Kim (1994) addresses this problem by introducing a collapsing procedure that greatly facilitates evaluation of the likelihood function at the cost of introducing a degree of approximation that does not appear to distort the calculated likelihood by much. The absence of lagged conditional variance terms in ARCH processes enables Cai (1994) and Hamilton and Susmel (1994) to estimate ARCH-Markov switching models, as opposed to the GARCH/Markov switching model used here, in a straightforward way without any approximation.

The collapsing procedure of Kim (1994), when applied to a GARCH process, calls for treating the conditional dispersion, h_t, as a function of at most the most recent M values of the state variable S. For the filtering to be accurate, Kim notes that when h is p-order autoregressive, then M should be at least $p + 1$. In the GARCH(1,1) case $p = 1$, so we would have to keep track of M^2 or four cases, based on the two most recent values of a binary state variable. However, h_t is not a function of S_t, so, even though $M = 2$, h is treated as a function of only S_{t-1}:

$$h_t^{(0)} = h_t(S_{t-1} = 0)$$

$$h_t^{(1)} = h_t(S_{t-1} = 1)$$

Denoting φ_t as the information available through time t, we keep the number of cases to two by integrating out S_{t-1} before plugging lagged h into the GARCH equation:

$$\begin{aligned} \hat{h}_t &= \text{Prob.}(S_{t-1} = 0 \mid \varphi_t) h_t^{(0)} \\ &\quad + \text{Prob.}(S_{t-1} = 1 \mid \varphi_t) h_t^{(1)} \end{aligned} \tag{8}$$

This method of collapsing of $h_t^{(0)}$ and $h_t^{(1)}$ onto $\hat{h}_t$ at every observation gives us a tractable GARCH equation which is approximately equal to the exact GARCH equation from equation (7):

$$h_t^{(j)} = \gamma + \alpha \left(\epsilon_{t-1}^{(j)}\right)^2 \left(1 - 2v_{t-1}^{(j)}\right) + \beta \hat{h}_{t-1}, \tag{9}$$

where $j \in \{0, 1\}$ corresponds with $S_{t-1} \in \{0, 1\}$. Note that the collapsing procedure integrates out the first lag of the state variable, S_{t-1}, from the GARCH dispersion function, h_t, at the right point in the filtering process to prevent the conditional density from becoming a function of a growing number of past values of the state variable.

With equation (9) defining $h_t^{(j)}$, the log-likelihood function is

$$\begin{aligned} lnL_t^{(i,j)} &= ln\Gamma(.5(n_t^{(i)}+1)) - ln\Gamma(.5n_t^{(i)}) - .5ln(\pi n_t^{(i)} h_t^{(j)}) \\ &\quad -.5(n_t^{(i)}+1)ln\left(1+\frac{\left(\epsilon_t^{(i)}\right)^2}{h_t^{(j)} n_t^{(i)}}\right) \end{aligned} \tag{10}$$

where $i \in \{0,1\}$ corresponds with $S_t \in \{0,1\}$, $j \in \{0,1\}$ corresponds with $S_{t-1} \in \{0,1\}$ and Γ is the gamma function. The function maximized is the log of the expected likelihood or

$$\sum_{t=1}^{T} ln\left(\sum_{i=0}^{1}\sum_{j=0}^{1} \text{Prob.}(S_t = i, S_{t-1} = j \mid \varphi_{t-1}) L_t^{(i,j)}\right) \tag{11}$$

as in Hamilton (1990).

III Results for six bilateral EMS exchange rates

The compound GARCH/markov switching volatility model was estimated for weekly (Friday-to-Friday) log-differences in the exchange rates of six EMS currencies with the Deutsche mark from January 1980 through January 1995. Data come from the Federal Reserve Board's H.13 release. The rates are all in units of domestic currency per D-mark. The countries are Denmark, the Netherlands, France, Belgium, Italy and Ireland.[1] Results

[1]The sample includes post-September 1992 data for the Italian lira after it left the ERM.

for the Austrian schilling/D-mark rate are included for comparison purposes, because the schilling was also pegged to Deutsche mark throughout the sample period, but was not subject to official target zones or formal realignments until January 1995 when it joined the EMS.

The parameters in the compound GARCH/markov switching model of EMS exchange rates, from equations (5)-(9), are $(h_0, \alpha, \beta, \frac{1}{n_h}, \frac{1}{n_l}, p, q, \mu_h, \mu_l)$. Several starting values were used to check that the estimates did not converge to sub-optimal local maxima. Table 1 presents the parameter estimates and shows that the differences between the estimated degrees-of-freedom parameters are largest for the Netherlands, Denmark and Italy. For these countries the conditional distributions appear to switch between one that is nearly normal and one that is highly leptokurtic. The conditional distributions for France, Belgium and Austria are quite leptokurtic in both states. Ireland, in contrast, has a degrees-of-freedom parameter that is essentially greater than four in both states. With the exception of Belgium, all currencies have $\mu_l > 0$, which suggests that the currencies depreciate on average relative to the D-mark in the most leptokurtic state. Finally, Austria's exchange rate does not show evidence of state switching.

It is also interesting to note how the weights placed on lagged squared residuals in the GARCH process vary with the degrees-of-freedom parameter. Two rows near the bottom of Table 1 give the weights placed on lagged squared residuals in the GARCH dispersion from equation (6), $\alpha(1 - 2v_t)$. For the Netherlands, France, Denmark and Italy, squared residuals in the low degree-of-freedom state receive very low weights. Figures 1a-f show the movement of the reciprocal of the degrees-of-freedom parameter, where dashed vertical lines mark dates of realignments. The realignments of the French franc, Danish kroner and Italian lira often occur in the highly leptokurtic state, but the occurrences of the leptokurtic state are certainly not limited to dates of realignments. Thus there are many other dates where the model places a low weight on a lagged squared resiudal in the GARCH equation. For this reason, the markov-switching method of endogenizing the weights on squared residuals in the GARCH process offers a desirable alternative to simply dummying out the squared residuals that correspond to realignments.

The sum $\alpha(1-2v_t)+\beta$ represents the persistence of the GARCH dispersion and its values are in two rows at the bottom of Table 1. The only case where this sum is explosive is $\alpha(1-2v_h)+\beta = 1.02$ for the Dutch guilder, although the sum is not statistically significant from unity. Besides, the dispersion process will not stay in an explosive state indefinitely, because it will switch to a state where the persistence is .926. More importantly, the variance can undergo discrete shifts that allow it to change much more rapidly than the dispersion. Taking the Danish kroner as an example, the variance decreases by about 84 percent when the degrees-of-freedom parameter switches from n_l to n_h, holding the dispersion constant:

$$\left(\frac{\frac{n_h}{n_h-2}}{\frac{n_l}{n_l-2}}\right) = .157$$

In this way, the "variance of the variance" that the model can explain is quite large even within the space of a few observations (weeks). Without markov switching in the variance, the conditional variance would not be subject to discrete shifts and would adjust more gradually.

To illustrate the importance of switching in the degrees-of-freedom parameter, I estimated the model with n_l constrained to equal n_h. A standard likelihood ratio test would be valid only if we were certain that markov switching in the mean was significant. Otherwise, the test is nonstandard in that the transition probabilities might not be identified under the null [Hansen (1992)]. For this reason we do not rely solely on the likelihood ratio to examine the benefits of allowing n to switch. A goodness-of-fit test is included to test the specification, because it is not subject to the problems affecting the likelihood ratio test. The test is the same one Vlaar and Palm (1993) used, because it is valid for non-*i.i.d.* observations. The 786 observations are separated into 50 groups based on the probability of observing a value smaller than the actual residual. If the model's density function fits the data well, these probabilities should be uniformly distributed between zero and one. Following Vlaar and Palm (1993),

$$n_i = \sum_{t=1}^{T} I_{it} \qquad \text{where} \qquad \begin{aligned} I_{it} &= 1 \qquad \text{if} \quad \frac{(i-1)}{50} < EF(\epsilon_t, \hat{\theta}) \leq \frac{i}{50} \\ &= 0, \qquad \text{otherwise.} \end{aligned}$$

The expected value of the cumulative density function, F, is taken across the two states that might have held at each time.

The goodness-of-fit test statistic equals $T/50\sum_{i=1}^{50}(n_i - T/50)^2$ and is distributed χ^2_{49} under the null. Table 2 contains the maximized log-likelihoods for the model with and without the restriction on n_t and the corresponding goodness-of-fit test statistics. The goodness-of-fit test statistics improve dramatically when the degrees-of-freedom parameter is allowed to switch. Only Austria passes the goodness-of-fit test when n is held constant, but the Austrian model did not show any improvement in the likelihood function when n was allowed to switch. Four of the six EMS countries pass the goodness-of-fit test at the one-percent level when n is allowed to switch. Only France and Ireland fail, but their goodness-of-fit statistics still show improvement when n is allowed to switch.

Summary and Conclusions

This article introduces a GARCH/Markov switching volatility model that is well-suited to intra-EMS exchange rate data. Markov switching in the student-t degrees-of-freedom parameter has the desired effect of endogenously changing the weight placed on last period's squared residual in the GARCH process. Previous research in the GARCH literature had debated whether or not to dummy EMS realignments out of GARCH conditional variances, but markov switching provides a more satisfactory and flexible solution. More importantly, markov switching introduces potentially large discrete shifts in the conditional variance in this model, so the volatility can return to relatively normal levels within a few weeks following a large jump, such as a realignment.

Goodness-of-fit tests show that GARCH models of EMS exchange rates with markov switching in the mean and constant degrees-of-freedom parameters are all easily rejected. Only Austria, a newcomer to the EMS, passes this test. By allowing the student-t degrees-of-freedom parameter to switch also, four of the six EMS currencies pass the specification test.

Research into the effect of fat-tailed shocks on expected utility [Geweke (1993)] suggests that leptokurtosis has not received consideration in proportion to its potential welfare and asset-pricing effects. The model developed in this article permits time-varying conditional kurtosis, which could play a role in explaining variation in asset prices.

Table 1: Parameter Estimates for GARCH/Markov Switching Model							
parameter	**HFL**	**BLF**	**FF**	**DKR**	**LIT**	**IRL**	**AUT**
γ	9.8E-6 (7.7E-6)	9.4E-4 (3.2E-4)	3.2E-3 (1.4E-3)	.002 (.001)	9.8E-4 (5.0E-4)	.645 (.306)	1.1E-3 (3.1E-4)
α	.121 (.035)	1.29 (.838)	.847 (.581)	.114 (.058)	.120 (.029)	.153 (.063)	1.23 (.406)
β	.906 (.022)	.359 (.079)	.382 (.188)	.857 (.064)	.870 (.027)	.668 (.112)	.339 (.075)
$v_h = \frac{1}{n_h}$	.029 (.089)	.412 (.056)	.365 (.068)	.024 (.058)	.109 (.036)	.096 (.073)	.365 (.042)
$v_l = \frac{1}{n_l}$	.416 (.060)	.465 (.039)	.499 (2.1E-7)	.425 (.030)	.456 (.020)	.253 (.059)	.365 (.042)
μ_h	-.007 (.003)	6.0E-4 (.003)	-.022 (.007)	-.031 (.016)	.010 (.010)	-1.54 (.229)	8.8E-5 (2.5E-3)
μ_l	.003 (.005)	-.364 (.150)	.129 (.023)	.059 (.023)	.705 (.071)	1.29 (.172)	8.8E-5 (2.5E-3)
p	.910 (.044)	.994 (.007)	.880 (.037)	.946 (.037)	.949 (.012)	.764 (.057)	n.a.
q	.894 (.056)	.989 (.015)	.551 (.096)	.946 (.054)	.429 (.094)	.809 (.043)	n.a.
$\alpha(1-2v_h)$	.114	.227	.229	.109	.094	.124	.332
$\alpha(1-2v_l)$	.020	.090	.002	.017	.011	.076	.332
$\alpha(1-2v_h)+\beta$	1.02	.586	.588	.966	.964	.792	.671
$\alpha(1-2v_l)+\beta$	.926	.449	.384	.874	.881	.744	.671
Log-Lik.	883.1	320.9	176.6	34.5	-272.6	-1861.0	545.4

Note: Standard errors are in parentheses.
All exchange rates are relative to Deutsche mark.
HFL=Dutch guilder; BLF= Belgian franc; FF=French franc
DKR= Danish kroner; LIT= Italian lira
IRL= Irish pound; AUT= Austrian schilling

Table 2: Goodness-of-fit tests								
		HFL	**BLF**	**FF**	**DKR**	**LIT**	**IRL**	**AUT**
Log-Lik.	$n_l \neq n_h$	883.1	320.9	176.6	34.5	-272.6	-1861.0	545.4
Log-Lik.	$n_l = n_h$	879.1	320.1	170.9	28.0	-300.8	-1862.1	545.4
G. fit	$n_l \neq n_h$	64.5	68.7	82.9	46.8	64.5	83.0	47.1
		(.068)	(.033)	(.002)	(.563)	(.068)	(.002)	(.550)
G. fit	$n_l = n_h$	155.1	109.4	86.0	117.6	262.1	91.0	47.1
		(.000)	(.000)	(.001)	(.000)	(.000)	(.000)	(.550)

Goodness-of-fit statistics are χ^2_{49} under null
p-values are in parentheses.
1% critical value for χ^2_{49} is 74.9.

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Reciprocal of Student-t Deg. of Freedom for France/Germany

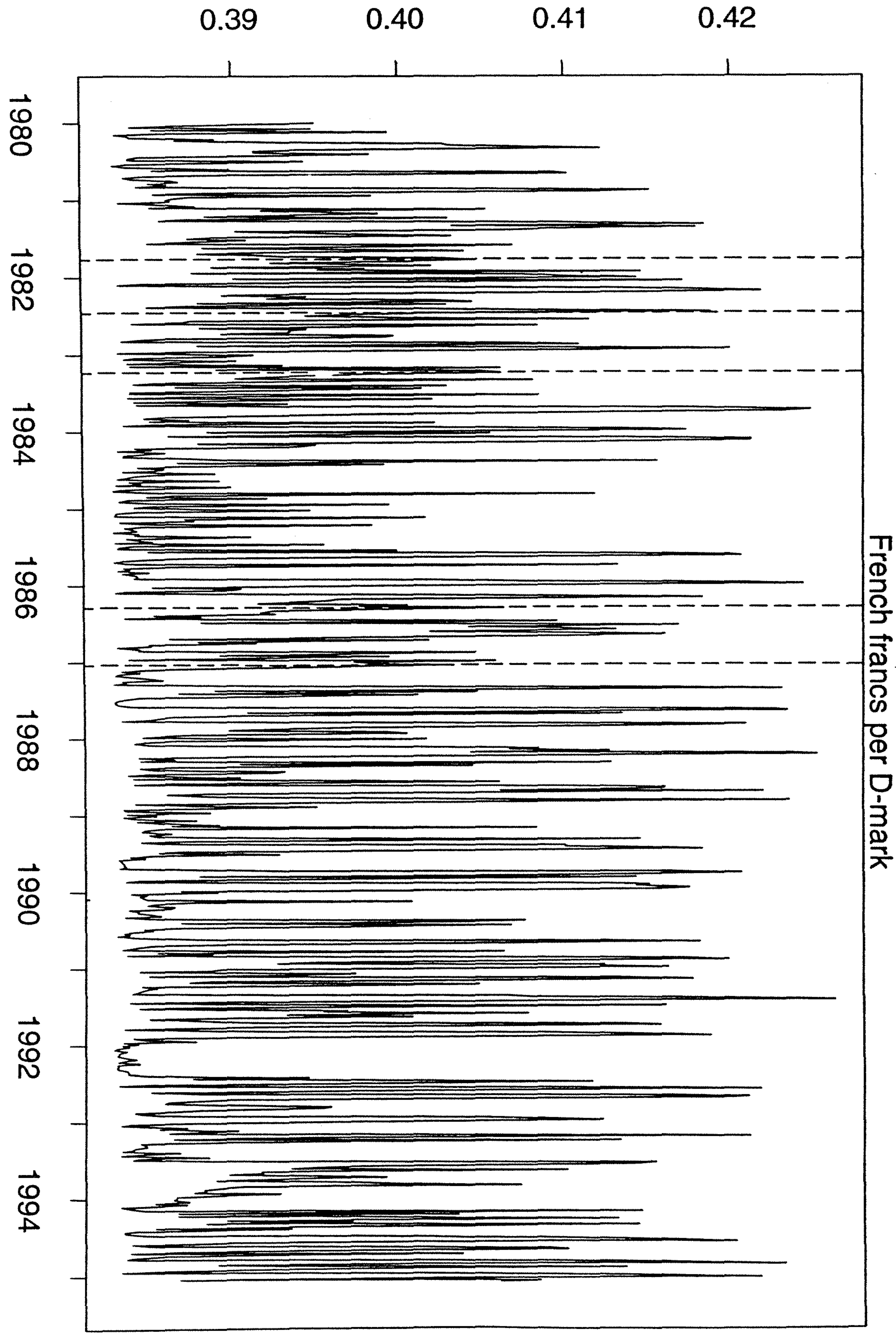
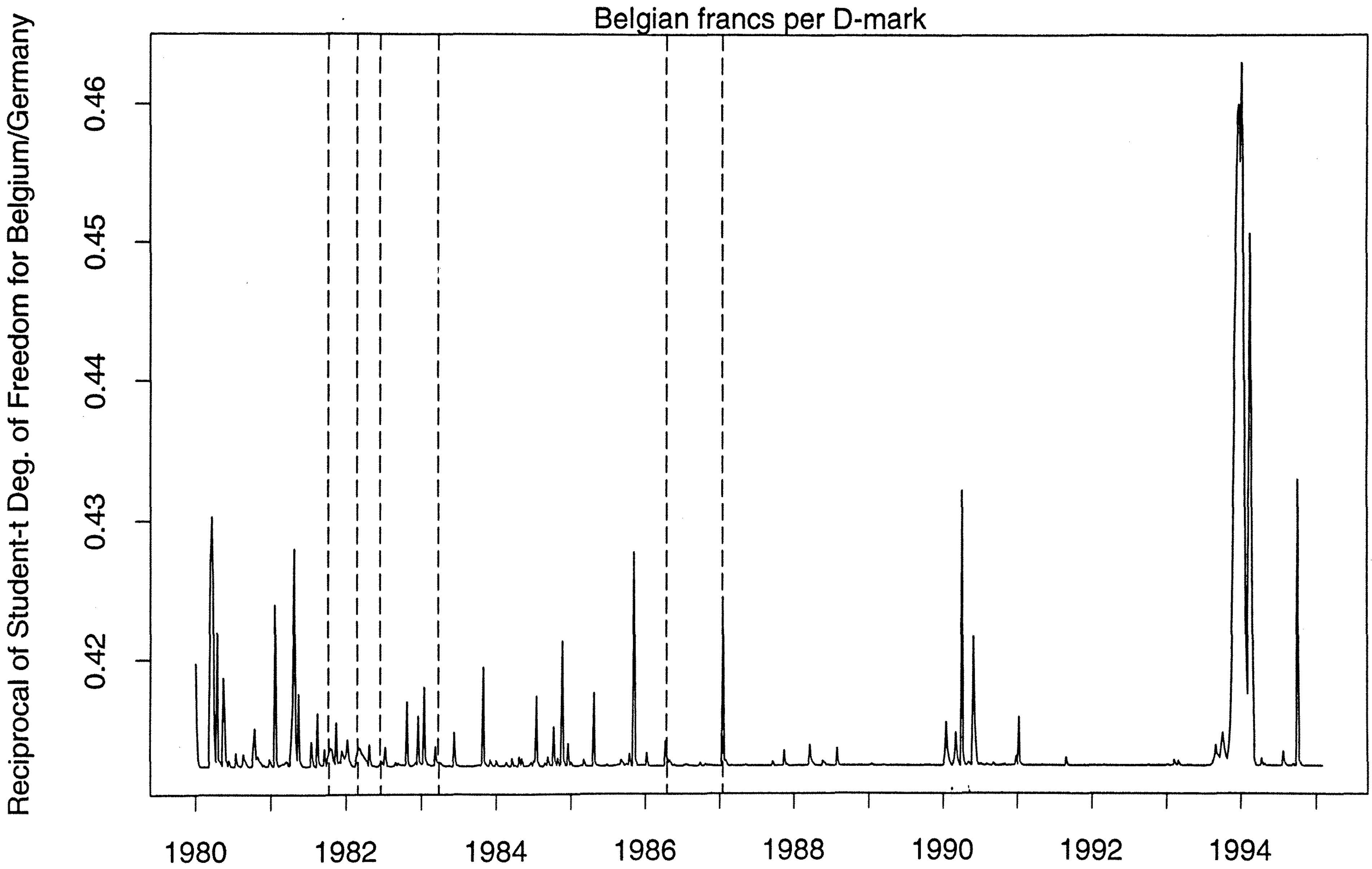


Figure 1a

Figure 1b


Belgian francs per D-mark
Reciprocal of Student-t Deg. of Freedom for Belgium/Germany
0.42
0.43
0.44
0.45
0.46
1980
1982
1984
1986
1988
1990
1992
1994

Reciprocal of Student-t Deg. of Freedom for Denmark/Germany

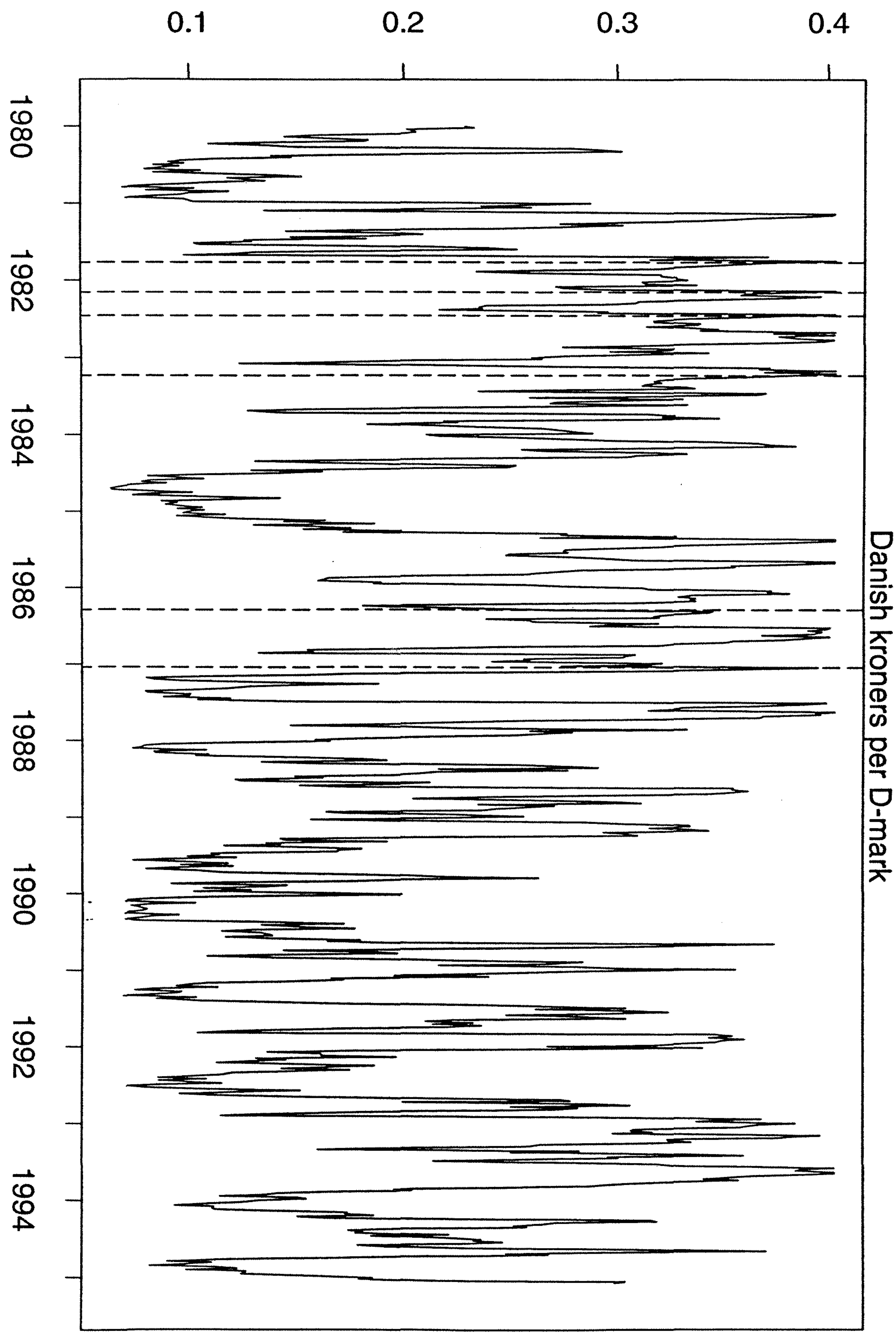
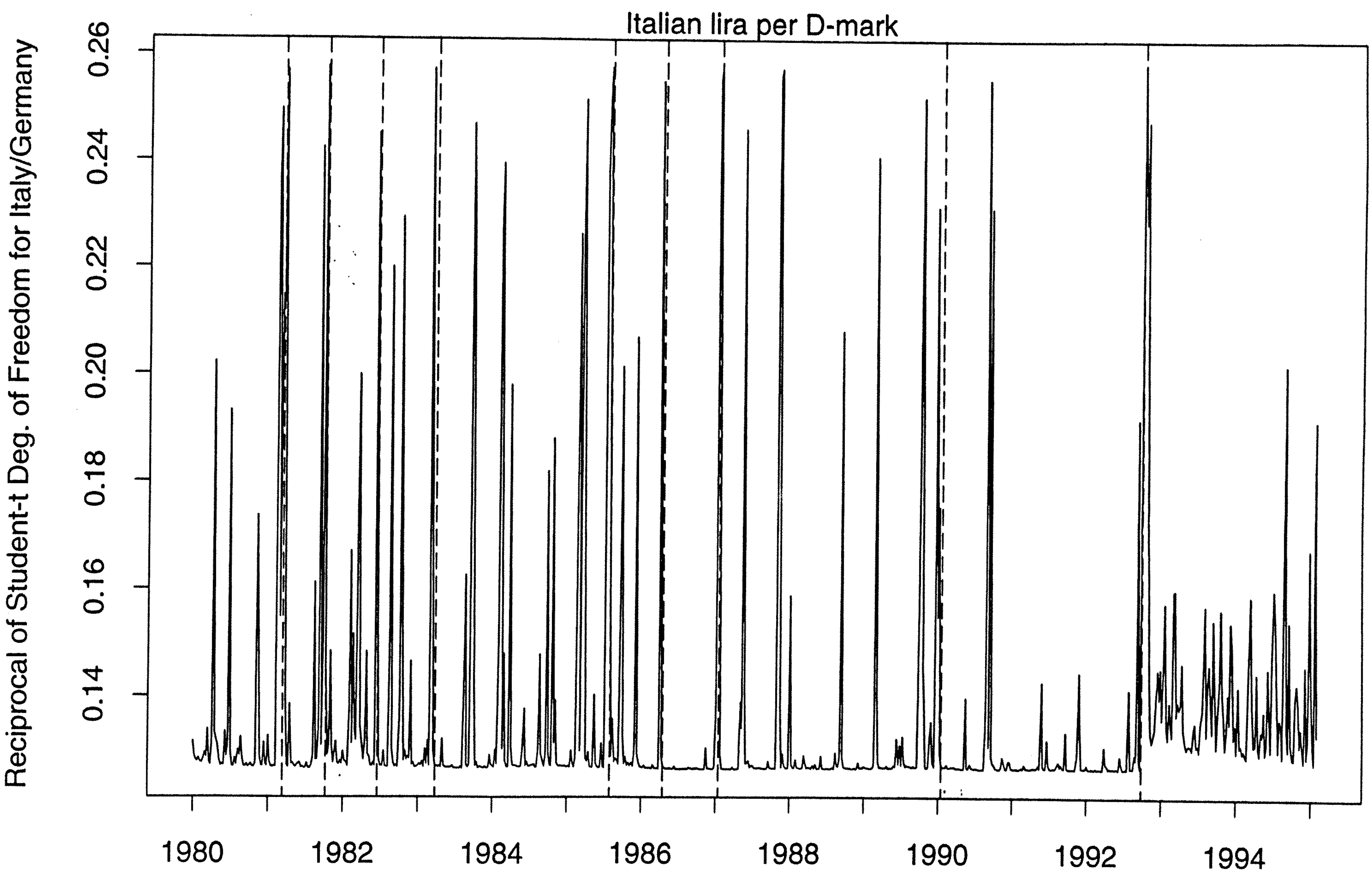


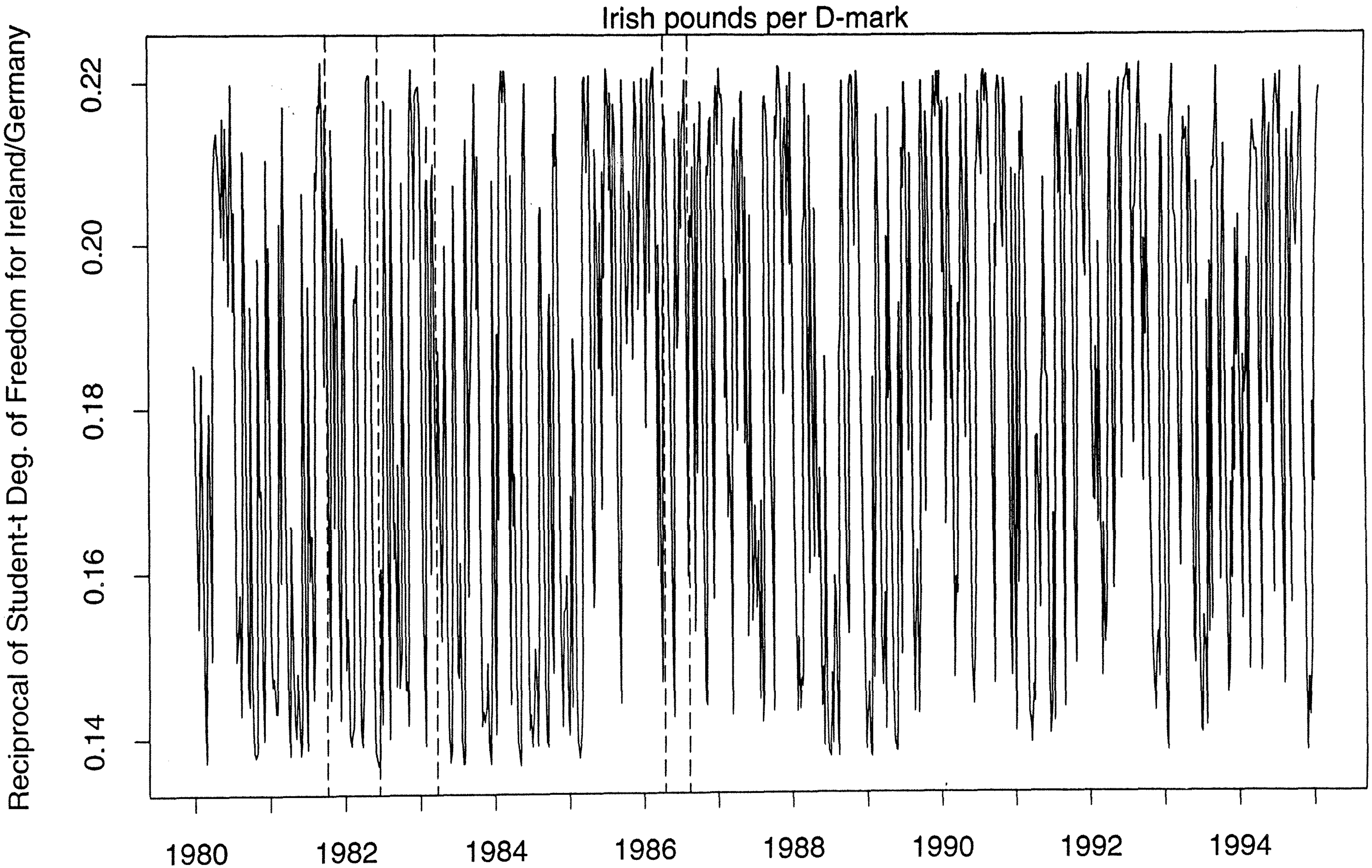
Figure 1c

Figure 1d



Italian lira per D-mark
Reciprocal of Student-t Deg. of Freedom for Italy/Germany
0.14
0.16
0.18
0.20
0.22
0.24
0.26
1980
1982
1984
1986
1988
1990
1992
1994

Figure 1f



Irish pounds per D-mark
Reciprocal of Student-t Deg. of Freedom for Ireland/Germany
0.14
0.16
0.18
0.20
0.22
1980
1982
1984
1986
1988
1990
1992
1994