



WORKING PAPER SERIES

Monetary Steady States in a Low Real Interest Rate Economy

James B. Bullard
Steven H. Russell

Working Paper 1994-012C
<http://research.stlouisfed.org/wp/1994/94-012.pdf>

PUBLISHED: Journal of Monetary Economics, as
"An Empirically Plausible Model of Low Real Interest Rates
and Unbacked Government Debt", December 1999.

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Federal Reserve Bank of St. Louis Working Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.

Photo courtesy of The Gateway Arch, St. Louis, MO. www.gatewayarch.com

AN EMPIRICALLY PLAUSIBLE MODEL OF LOW REAL INTEREST RATES AND UNBACKED GOVERNMENT DEBT

December 1998

ABSTRACT

We study the properties of an overlapping generations model with many-period-lived agents, neoclassical production and capital accumulation, labor-leisure decisions, population growth, and technological progress. We demonstrate that a plausibly calibrated version of this model has “monetary steady states” — Samuelson-case steady states with large real stocks of unbacked government debt. These steady states can duplicate a number of important features of U.S. postwar data, including three phenomena that challenge other sorts of calibrated models: the low average real interest rate on U.S. government debt, the government’s success in reducing the debt/GDP ratio without running large budget surpluses and the relatively high ratio of net saving to output.

KEYWORDS: Overlapping generations, risk-free rate puzzle

JEL CLASSIFICATION: D5, E4

James Bullard
Federal Reserve Bank of St. Louis
411 Locust Street
St. Louis, MO 63102
bullard@stls.frb.org
TEL (314) 444-8576
FAX (314) 444-8731

Steven Russell
Department of Economics
Indiana University-Purdue University
at Indianapolis
523 Cavanaugh Hall
425 University Blvd.
Indianapolis, IN 46202-5140
TEL (317) 274-0420
FAX (317) 274-2347
shrusse@iupui.edu

We thank Roger Farmer, Peter Rangazas, two anonymous referees and an associate editor for helpful comments.

1 Introduction

1.1 Overview

The overlapping generations (OLG) model has been widely used to study a variety of questions in macroeconomic theory and policy. Some of the most interesting properties of the OLG model are associated with its ability to produce steady-state equilibria in which agents hold unbacked government liabilities such as fiat currency or unfunded debt, even when there are no transactions frictions and even when the government does not interfere with the operation of the market for privately-issued liabilities. These equilibria are commonly described as “monetary steady states” — although the unbacked government liabilities are not necessarily best interpreted as fiat money and will not be given that interpretation here. Monetary steady states can arise when OLG models are specified to produce what Gale (1973) calls the *Samuelson case* in which there is excess demand for privately issued assets at the “golden rule” real interest rate (equal to the output growth rate). This shortage of privately issued assets creates a potential demand for unbacked government liabilities. In the alternative *classical case* there is excess supply of privately issued assets at the golden-rule real rate and there is no role for unbacked debt.¹

Until recently, most research involving OLG models was conducted using highly stylized specifications. In the vast majority of cases the agents lived for only two periods, and in most cases the models also abstracted from production or capital. In recent years, however, macroeconomists have become increasingly interested in general equilibrium models that are “realistic” in the sense that [1] the decision problems facing agents in the models are similar in nature, scale and timing to the problems facing agents in actual economies, and [2] the properties of the data generated by the models can be directly compared to the properties of actual macroeconomic data. Pioneers in the effort to develop such models include Kydland and Prescott (1982) and Auerbach and Kotlikoff (1987). Auerbach and Kotlikoff extend Diamond’s (1965) nonstochastic overlapping-generations model of production and capital by introducing households who live for many periods and a much more elaborate public sector. Auerbach and Kotlikoff calibrate their model and use it to study the effects of alternative fiscal policies. Their calibration strategy produces classical-case equilibria — as do the strategies used in the extensive literature that has grown out of their work.

We believe that applied theorists working in macroeconomics and public finance should also be interested in calibrated OLG models that produce Samuelson-case equilibria. An important difference between Samuelson-case and classical-case economies is that in the Samuelson case the existence of a demand for unbacked government liabilities allows the government to influence real interest rates and other real vari-

¹Government liabilities are said to be unbacked if they can be rolled over forever without the need for primary (net of interest) budget surpluses to cover any part of the interest or principal.

ables by changing the growth rate of the nominal supply of these liabilities. These changes have real effects because they produce direct shifts in the real supply of funds available to private borrowers. Thus, the predictions of models that support Samuelson-case equilibria are potentially consistent with the widely held belief that changes in monetary and fiscal policy can have significant and persistent real effects.

In the area of fiscal policy, Samuelson-case models predict that pay-as-you-go social security schemes may be welfare improving — a prediction that may help explain why the U.S. and most other developed countries have constructed such schemes. Samuelson-case models can produce significant departures from Ricardian equivalence, and they may consequently have the potential to explain persistent changes in the levels of real interest rates and investment such as those that occurred in the U.S. during the 1970s and 1980s — a period when there were also persistent changes in the nature of fiscal policy and the size of the national debt relative to GDP. In addition, Samuelson-case models can support equilibria that display a variety of different sorts of endogenous expectations-driven fluctuations — deterministic and stochastic cycles, chaotic and sunspot dynamics, etc. — that may help explain business cycle phenomena.²

In the realm of monetary policy, multiperiod/calibrated Samuelson-case models may provide an applied-theoretical framework that is consistent with recent empirical evidence of significant departures from long run superneutrality.³ In particular, these models may help explain both the apparent success of Federal Reserve policy in reducing the long-run inflation rate and the increase in the level of real interest rates that seems to have accompanied this effort. In recent work with two-period OLG models, Espinosa and Russell (1998a,b) show that specifications that support equilibria with substantial stocks of unbacked debt can reverse the “unpleasant arithmetic” of Sargent and Wallace (1981) and Wallace (1984), allowing sustained monetary tightening to produce both a higher real interest rate and a lower rate of inflation. Bullard and Russell (1997b) extend this analysis to a multiperiod/calibrated model and show that in specifications with realistic debt-GDP ratios, moderate decreases in the inflation rate can indeed produce significant increases in real interest rates.

1.2 Monetary steady states in realistic OLG models

In this paper we lay the groundwork for a research program that uses multiperiod overlapping generations models, parameterized to produce Samuelson-case equilibria, to study the real effects of monetary and fiscal policy. We do this by demonstrating

²The literature on endogenous fluctuations in OLG and other models has been surveyed by Boldrin and Woodford (1990) and Guesnerie and Woodford (1992).

³See King and Watson (1992), Weber (1994), Bullard and Keating (1995) and Ahmed and Rogers (1996). There is also a large empirical literature on the Fisher relationship which typically finds that nominal interest rates rise less than one-for-one with increases in the inflation rate.

that plausible calibrations of these models can produce steady-state equilibria that support relatively low real interest rates — rates substantially below the output growth rate — and large real stocks of unbacked government debt. While we suspect that many economists who have worked with OLG models have long believed that they can be extended in the manner we describe, our paper is the first we know of to demonstrate conclusively that this is the case.

Providing this demonstration requires us to confront a variety of important calibration issues. A distinguishing feature of OLG models is that the value of almost every parameter influences whether or not a particular specification has monetary steady states. As a result, monetary steady states have sometimes been described as “fragile.” This being the case, we think it is essential to establish that it is possible to obtain monetary steady states without choosing parameter values that are outside the range of published estimates, and that the existence of these steady states is robust to fairly large changes in the values of these parameters. It also seems important to identify the parameters that are critical in the sense that changes in their values that would leave them well inside the range of empirical estimates might rule out monetary steady states.

1.3 Empirical puzzles

In addition to demonstrating the empirical plausibility of Samuelson-case equilibria and monetary steady states, our model can help provide explanations for several empirical phenomena that have proved challenging for other calibrated models.

1.3.1 Real interest rates

One empirical regularity that has emerged as quite puzzling is the relatively low level of real interest rates on risk-free securities. In the United States, the before-tax *ex post* real interest rate on short-term government debt averaged a bit less than one percent during the postwar period, while the average after-tax real rate was slightly below zero percent. (The after-tax real interest rate is the appropriate empirical analogue of the real interest rate facing savers in a formal model.) During the same period the average annual growth rate of real GDP was more than three percent. The same relationship between the risk-free real interest rate and the real growth rate seems to have prevailed in other times and places — in the U.S. and U.K. for at least the last century, for example, and in western Europe and Japan during the postwar period.⁴

⁴For the U.S., Mehra and Prescott (1985) find that during 1889-1978 the average U.S. risk-free real interest rate was 0.8 percent, before taxes. They also report that U.S. *per capita* consumption grew at an average rate of 1.8 percent during this period. Given the average population growth rate, the implied growth rate of total consumption is approximately 3 percent — a figure consistent with real growth rate estimates from other sources. Siegel (1992) finds that the average risk-free real rate in the U.K. during this period was slightly lower than the average U.S. rate. Barro (1990) uses data on short-term interest rates in nine OECD countries, including the U.S., Canada, Japan and most of the major Western European economies, to construct a GDP-weighted “world real interest rate”

Explaining risk-free rates this low has been a serious problem for macroeconomists. Nonstochastic versions of standard representative-agent infinite-horizons models do not have steady states in which the real interest rate is lower than the output growth rate. In stochastic versions of these models it is possible for agents' aversion to risk to drive the real interest rate on risk-free assets below the output growth rate. However, Kocherlakota's (1996) recent survey of the "risk-free rate" literature indicates that in plausibly calibrated specifications investigators can produce safe real rates low enough to match targets drawn from the data only by assuming either that agents are extremely risk averse or that there are very serious imperfections in the credit and insurance markets.⁵ In most cases, moreover, achieving a realistically low mean for the risk-free rate requires accepting an unrealistically high variance.⁶

Calibrated versions of overlapping generations models are usually nonstochastic and are virtually always parameterized to produce classical-case equilibria. As a result, the real interest rates they produce are higher than the output growth rate — usually, substantially higher. For example, the model employed by Auerbach and Kotlikoff (1987) has an output growth rate of one percent and a real interest rate of 6.7 percent. In the baseline calibration of our model, however, the output growth rate is 3.2 percent and the steady-state real interest rate is zero percent.⁷

for 1959-1988. The average *ex post* world real rate for this period was 1.7 percent, before taxes.

⁵Recent work on this subject includes Díaz-Giménez and Prescott (1997) who construct an infinite horizon model with both aggregate and idiosyncratic production shocks and close down all private insurance markets. Their baseline calibration produces a steady state with a risk-free real interest rate of zero percent. However, the model does not include population or productivity growth, which tend to drive the risk-free rate upward (see note 6 below). It also abstracts from physical capital, which has been shown to be a powerful tool allowing agents to self-insure against risk. Aiyagari and McGrattan (1995), for example, use a model that includes physical capital to study the optimum quantity of government debt. They subject their agents to idiosyncratic earnings risk and close down private insurance markets. They find that government debt can provide a useful supplement to physical capital as an insurance device and that the optimal debt-output ratio may have been close to the U.S. postwar average. However, the welfare gains from government debt are quite small, and their real interest rate of 4.5 percent substantially exceeds their output growth rate of 1.85 percent.

⁶Models in the risk-free rate literature typically abstract from population growth, while models in the saving rate and related literatures (see below) typically abstract from productivity growth. Thus, both literatures underestimate the difficulty of duplicating the observed difference between the risk-free rate and the real growth rate. In addition, most papers in these literatures use observed *before-tax* real rates as their targets or reference standards, although in models without return taxes the appropriate targets are after-tax rates (see above). In nonstochastic representative agent models it is the after-tax real interest rate that is bounded below by the output growth rate.

⁷There has been a limited amount of work with stochastic general equilibrium versions of calibrated multi-period Diamond models. Ríos-Rull (1994) introduces aggregate production risk in order to study the business cycle characteristics of life cycle models. His baseline steady state produces a risk-free real interest rates close to 6 percent, whether or not he closes down contingent claims markets. İmrohoroglu *et al.* (1995) study the welfare effects of social security in a model that includes idiosyncratic mortality and employment risk and closes down the annuities and insurance markets. Their baseline steady state produces a real interest rate of 0.4 percent, which is low in an absolute sense but is only 0.8 percent lower than their real growth rate of 1.2 percent. Interestingly, they find that there are substantial welfare benefits from pay-as-you-go social security schemes. There is no productivity growth in their baseline calibration. Their alternative case that does include productivity growth produces a real interest rate that exceeds the output growth rate

Empirical estimates of the average real rate of return on capital are almost always higher than the average output growth rate. As a result, researchers using nonstochastic models with a single real interest rate must make a nontrivial choice about which real return rate to target. In our view, the principal contribution of our analysis is to demonstrate the viability of a modeling strategy that targets the observed average risk-free rate rather than the observed average risky return rate. We show in Section 4.1, however, that our results are robust to the introduction of frictions that drive the marginal product of capital above the output growth rate, as long as we keep the rates of return facing households and the government close to the observed risk-free real rate.

1.3.2 Saving and lifetime consumption

Calibrated versions of overlapping models have had some difficulty producing realistic levels of saving and wealth relative to income. This situation has produced an extensive literature that attempts to augment the models in ways that might increase agents' saving rates. The largest branch of this literature introduces a variety of stochastic features — supplemented, in most cases, by incomplete credit/insurance markets — in order to induce precautionary savings. Alternative strategies for increasing saving and wealth involve the introduction of liquidity constraints, bequests and related motives, and human capital. Our findings suggest that reparameterizing the standard nonstochastic OLG model to support Samuelson-case equilibria may solve the saving and wealth problem, making more radical respecifications unnecessary for this purpose. In our baseline steady state both the saving rate and the wealth-output ratio come quite close to matching estimates based on postwar-U.S. data.

A related problem with standard calibrated life cycle models is that the level of a household's consumption tends to increase close to linearly with the age of the household. Observed age-consumption profiles, in contrast, are hump-shaped, peaking in late middle age and declining fairly rapidly thereafter. A number of researchers have asserted that nonstochastic models cannot generate realistically humped-shaped profiles. In our model, however, the combination of low real interest rates and endogenous labor supply produces age-consumption profiles with empirically plausible humps.

1.3.3 Public debt

At the end of the Second World War the U.S. national debt exceeded annual U.S. output. During the ensuing half-century the federal government did not run large primary surpluses for any extended period, and on average its budget was balanced: during 1948-1994, the average annual net-of-interest surplus amounted to 0.0 percent and eliminates the welfare gains from social security.

of GDP. Thus, it is essentially accurate to describe the entire World War II debt as having been rolled over for the last 50 years. Despite the absence of significant debt retirement, in 1994 the U.S. debt-GDP ratio was less than two-thirds the size of the ratio in 1948.

In nonstochastic versions of standard representative agent models, or overlapping generations models calibrated to produce classical-case equilibria, it is impossible for the government to roll over a large debt without running substantial net-of-interest surpluses on average. In stochastic versions of these models maintaining large permanent debts without running average surpluses is theoretically possible, but quite difficult in practice — more difficult, broadly speaking, than producing equilibria with relatively low real interest rates.⁸ In the baseline steady state of our nonstochastic model, however, the debt-output ratio is equal to the U.S. postwar average and the government finances a small primary deficit by rolling this debt over each period.

1.4 Outline

In the Section 2 of this paper we lay out the model that will provide the basis for much of our analysis, and we derive and discuss the conditions for the existence of monetary steady states. In the third section we calibrate the model and compute its steady-state equilibria. We describe the characteristics of these equilibria and compare them to various features of U.S. postwar data. We also investigate of the sensitivity of our results to changes in the model's parameters.

In Section 4 we investigate the sensitivity of our results to the introduction of simple credit market frictions that drive the marginal product of capital above the government bond rate. We also discuss two issues that are raised by our analysis: dynamic efficiency and familial altruism. The fifth section presents some concluding remarks.

2 The model

2.1 The environment

We begin by assuming that time is infinite in both directions, with economic activity occurring at discrete dates $t = \dots -1, 0, 1, \dots$. A finite number of agents are born at each date t ; each of these “members of generation t ” lives for n periods. Members of a particular generation are identical, and members of different generations are identical except for their birthdates. We assume that the number of agents in each generation grows at gross rate $\psi \geq 1$ per period, so that the total population grows at the same rate. The population of generation 1 is normalized to unity.

There is a single good in the model. This good can be consumed or used as an

⁸See Bohn (1993).

input in production, in which case it is called capital. Capital produced during a period cannot be used in production until the following period, at which point it begins to depreciate at a *net* rate of $\delta \in [0, 1]$ per period. The production process is standard Cobb-Douglas, with exogenous technological progress:

$$Y(t) = F_t(K(t), L(t)) = \lambda^{(t-1)(1-\alpha)} K(t)^\alpha L(t)^{1-\alpha}. \quad (1)$$

Here $Y(t)$ is aggregate output at date t , and $\lambda \geq 1$ is the gross rate of technological progress. Aggregate employment of effective labor is denoted $L(t)$, and $K(t)$ represents the amount of capital used in production at date t . We will use $k(t) \equiv K(t)/L(t)$ to denote the ratio of capital to effective labor. The parameter $\alpha \in [0, 1]$ determines the capital share of output.

The production technology is available to an arbitrary number of perfectly competitive firms; in equilibrium, each of these firms earns zero profits at each date. The firms rent capital and hire effective labor from agents at rental and wage rates that are equal, in equilibrium, to the marginal products of the respective inputs. Thus the real rental rate on capital is $r(t) = \lambda^{(t-1)(1-\alpha)} \alpha k(t)^{\alpha-1}$, while the real wage is $w(t) = \lambda^{(t-1)(1-\alpha)} (1 - \alpha) k(t)^\alpha$.

Each member of generation t is endowed with $\bar{\ell}$ units of time per period of life. Each period, agents divide their total time endowment between time spent enjoying leisure and time spent supplying labor. The portion of the time endowment that a member of generation t allocates to leisure at date $t+j$ is denoted $\ell_t(t+j)$, $j = 0, \dots, n-1$, so that $l_t(t+j) \equiv \bar{\ell} - \ell_t(t+j)$ is the amount of time allocated to supplying labor. The productivity of an agent in the i^{th} period of life is denoted e_i , $i = 1, \dots, n$. These productivity endowments are intended to represent the amounts of fully-internalized human capital that agents have accumulated during their lives. The amount of *effective* labor an agent supplies at date t is the product of the amount of time allocated to labor and the value of the productivity endowment at that date, given the agent's age. At date $t+j$, the income of a member of generation t is the product of the real wage at that date and his supply of effective labor: $w(t+j) e_{j+1} l_t(t+j)$, $j = 0, \dots, n-1$.

There is a government that endures forever. The only role of the government is to collect real revenue by issuing unbacked liabilities and exchanging them for consumption goods. After it has been collected, government revenue disappears from the model. Its value at date t is

$$G(t) = \frac{H(t) - H(t-1)}{P(t)}, \quad (2)$$

where $H(t)$ represents the nominal stock of unbacked liabilities outstanding at the end of date t , and $P(t)$ represents the date t price of a unit of the consumption good in units of these liabilities. The government is assumed to issue unbacked

liabilities so that their nominal stock grows at a constant rate $\theta \geq 1$; that is, $H(t) = \theta H(t-1)$. Thus, when $\theta = 1$ the nominal stock of unbacked liabilities is constant, and government revenue is zero at each date.

We will let $s_t(t+j-1)$ denote the total real quantity of assets held by a member of generation t at the end of his j^{th} period of life. There are three types of assets available to agents: one-period consumption loans, unbacked government liabilities, and physical capital. The gross real rate of return on consumption loans extended at date t is denoted $R(t)$, and the rate on unbacked liabilities purchased at date t is $P(t)/P(t+1)$. Capital purchased at date t can be carried into date $t+1$, rented to firms, and then sold, net of depreciation; in equilibrium, the resulting gross rate of return is $1 + r(t+1) - \delta$. The possibility of arbitrage implies that in any perfect-foresight competitive equilibrium where agents purchase positive amounts of physical capital and government liabilities, we will have $P(t)/P(t+1) = R(t) = r(t+1) + (1 - \delta)$.

We assume agents have perfect foresight regarding future interest and wage rates. Under this assumption, the budget constraints facing a member of generation t can be written

$$c_t(t) + s_t(t) = w(t) e_1 l_t(t) \quad (3)$$

$$c_t(t+j) + s_t(t+j) = w(t+j) e_{j+1} l_t(t+j) + R(t+j-1) s_t(t+j-1) \quad (4)$$

for $j = 1, \dots, n-2$, and

$$c_t(t+n-1) = w(t+n-1) e_n l_t(t+n-1) + R(t+n-1) s_t(t+n-1) \quad (5)$$

where $c_t(t+j) \geq 0$, $j = 0, \dots, n-1$, and $0 \leq l_t(t+j) \leq \bar{\ell}$, $j = 0, \dots, n-1$. The first three constraints can be combined as follows:

$$c_t(t) + \sum_{i=1}^{n-1} c_t(t+i) \prod_{j=0}^{i-1} R(t+j)^{-1} \leq w(t) e_1 l_t(t) + \sum_{i=1}^{n-1} w(t+i) e_{i+1} l_t(t+i) \prod_{j=0}^{i-1} R(t+j)^{-1}. \quad (6)$$

Agents choose quantities of consumption and leisure at each date in order to maximize

$$U = \sum_{i=0}^{n-1} \beta^i u(c_t(t+i), \ell_t(t+i)), \quad (7)$$

where

$$u(c, \ell) = \frac{[c^\eta \ell^{(1-\eta)}]^{(1-\gamma)}}{1-\gamma}, \quad (8)$$

subject to the above constraints. We assume $\gamma > 0$, $\eta \in [0, 1]$, and $\beta > 0$. The parameter γ governs the curvature of the period utility function for consumption and leisure and the parameter η governs the willingness of an agent to substitute current leisure for current consumption.

The parameter β is a time discount factor, and can be defined as $\beta \equiv 1/(1 + \rho)$, where $\rho > -1$ is agents' pure rate of time preference. The parameters ρ and γ interact to determine agents' willingness to defer consumption. An alternative time-preference measure that captures this interaction is

$$\hat{\rho} \equiv 1 - (1 + \rho)^{-\frac{1}{\gamma}}. \quad (9)$$

To understand the meaning of this measure, imagine an agent who has a constant (flat) pattern of efficiency endowments, receives a constant real wage, and is confronted by a net real interest rate of zero. If this agent has negative time preference, he will choose a consumption bundle in which his consumption grows at a positive rate from the first period to the second period, and vice-versa. The *effective time-preference rate* $\hat{\rho}$ is the opposite of this desired consumption growth rate.

2.2 Steady-state equilibria

The aggregate net real end-of-period asset holdings of the members of the n generations that are alive at date t are denoted

$$A(t) = \sum_{j=0}^{n-2} \psi^{t-j} s_{t-j}(t). \quad (10)$$

If no agent chooses to retire (provide zero units of labor time) at any date, both aggregate asset demand at date t and the aggregate supply of effective labor at that date can be expressed as functions of the $R(t+i)$, $i = 2-n, \dots, n-2$. This can be accomplished by solving the maximization problem just described, assuming an interior optimum, and using the fact that the generations are identical except for birthdates and population size. While we assume there is an interior optimum for the purposes of the present discussion, when we compute equilibria numerically we will always take account of retirement possibilities.

Dynamic paths for this economy can be characterized by two equations:

$$H(t)/P(t) = A(t) - K(t+1) \quad (11)$$

$$H(t) = \theta H(t-1). \quad (12)$$

Equilibrium paths can consequently be described as solutions to

$$A(t) - K(t+1) = \theta R(t-1) [A(t-1) - K(t)]. \quad (13)$$

This condition requires that the aggregate real supply of unbacked liabilities at date t , which consists of the *ex post* real value of the liabilities agents purchased at date $t-1$ plus the real value of the new liabilities issued by the government, must equal the aggregate real demand for liabilities at date t . It also imposes perfect foresight, in that the *ex post* value of the liabilities agents purchased at date $t-1$ is equal to

the value the agents expected at that date. Given the other equilibrium conditions, and the perfect foresight requirement, condition (1) ensures that the goods market clears at each date.

Because $K(t) = k(t)L(t)$, where $k(t)$ can be written as a function of the real interest rate at date $t-1$, condition (1) can be expressed as a univariate nonlinear difference equation in the real interest rate. Stationary solutions can be found by imposing $R(t) = R \forall t$. In a stationary equilibrium, both aggregate savings and the aggregate capital stock grow at gross rate $\lambda\psi$. Thus equilibria occur at values of R such that $A(1) - \lambda\psi K(1) = 0$, where $A(t) = (\lambda\psi)^{t-1}A(1)$ and $K(t) = (\lambda\psi)^{t-1}K(1)$. An equilibrium at a value of R that produces $A(1) - \lambda\psi K(1) = 0$ will be called a *nonmonetary steady state*, and the equilibrium gross real rate of interest will be denoted R_{nmss} . Unbacked liabilities are not held in steady states of this type. There is also a monetary steady state at $R = \lambda\psi\theta^{-1}$, provided $A(1) - \lambda\psi K(1)$ is positive at this value of R . We will call a steady state of this type a *monetary steady state*, and the associated gross real interest rate will be denoted R_{mss} . In this steady state, unbacked liabilities are held and valued, and their real stock grows at gross rate $\lambda\psi$ per period.

For the purpose of characterizing the steady states that can arise in a particular specification of the model, it is useful to define an aggregate asset-demand function that presumes that the real interest rate is date-invariant, and nets out assets that represent holdings of physical capital. We will refer to this function as the “aggregate savings function,” because it plays the same role in our analysis of this model that the aggregate savings function plays in the analysis of a two-period OLG model. The date-1 aggregate savings function is

$$\mathbf{S}_1[R] \equiv A(1)[R] - K(2)[R] \quad (14)$$

and the date- j function is $\mathbf{S}_j[R] = (\psi\lambda)^{j-1}\mathbf{S}_1[R]$. The condition

$$A(1) - \lambda\psi K(1) |_{R=\lambda\psi} > 0, \quad (15)$$

which can be rewritten as $\mathbf{S}_1[\lambda\psi] > 0$, is necessary and sufficient for the existence of a monetary steady state in this model.

While it is possible for a specification of the model to have many nonmonetary steady states, in the calibrated specifications we study the aggregate savings function is strictly increasing in R , and there is consequently only one equilibrium of this type. For the moment, we will focus on monetary steady states in which the nominal stock of unbacked liabilities is constant ($\theta = 1$), so that $R_{mss} = \lambda\psi$. As long as the aggregate savings function is increasing, the existence of such a monetary steady state is a necessary and sufficient condition for the existence of monetary steady states in which $\theta > 1$. In particular, if there is a monetary steady state when $\theta = 1$,

then there will also be a monetary steady state associated with each $\theta \in (1, \hat{\theta})$, where $\hat{\theta}$ solves $R_{nmss} = \lambda\psi/\theta$.⁹

3 Monetary steady states

3.1 Calibration

Our analysis of the model centers around a *baseline case* in which we choose empirically plausible values for each of the model’s parameters and compute the resulting monetary steady state.

The value of n controls the number of periods in an agent’s life. We follow Auerbach and Kotlikoff (1987) by setting $n = 55$ for most of our analysis and by thinking of a single period as a year. We think of the first period of an agent’s life as corresponding to the first year of a person’s working life — roughly age 21, so that the last year of an agent’s life corresponds to age 75. There is no uncertainty about this terminal age. We adopt an interpretation of a “period” under which changing the value of n does not change the *length* of an agent’s life: instead, it changes the number of periods into which a life of fixed real-time length is divided. When we change the value of n we adjust the values of a number of other parameters so that their annualized values remain constant. One consequence of this interpretation is that the steady-state values of the endogenous variables are essentially invariant to our choice of n , once n is large enough to make our discrete-time model reasonably good approximation of a continuous-time model. (See Section 3.3.1 below.)

The preference parameter η controls the willingness of agents to substitute consumption for leisure by supplying labor, and thus controls the elasticity of their labor supply. In addition, there is a close but not exact correspondence between the value of η and the average share of an agent’s time endowment that is devoted to supplying labor. On the basis of this correspondence we set η to 0.22. This setting comes close to producing our estimate of the average fraction of the year that a full-time worker devotes to labor, which is 0.229. We obtain this estimate by subtracting weekends, holidays and two five-day weeks of vacation from the total number of days in a calendar year, dividing by three to represent an eight-hour working day, and dividing the result by the number of days in a year.¹⁰ In the baseline case, we do not force agents to retire at any age, though they may choose to retire voluntarily.

As we have noted, the value of γ determines the curvature of agents’ period utility

⁹In Section 3.3 we use the test $R_{nmss} < \lambda\psi$ to determine whether or not there is a monetary steady state. When the aggregate savings function is upward-sloping, this test is equivalent to condition (15).

¹⁰The standard value for η in the calibration literature is 1/3, which is based on a 14 hour day. However, utility functions like ours imply that the marginal disutility from working all available hours is minus infinity. Since some people do work 14 or more hours per day, we think an interpretation based on a 24 hour day is more defensible. Our calculation also assumes a 100 percent labor force participation rate. If we used a more realistic 65 percent participation rate our agents would devote only 14.8 percent of their available time to work.

function $u(c, \ell)$. This parameter consequently influences the willingness of agents to substitute consumption (or leisure) across periods in response to changes in the real rate of interest: $\sigma \equiv [1 - \eta(1 - \gamma)]^{-1}$ is the intertemporal elasticity of substitution in consumption. In recent years, the value of this elasticity has been the subject of a large empirical literature. The only consensus this literature has reached is that σ is probably between zero and unity. In the absence of such a consensus we choose a baseline-case value for γ which, when combined with our other baseline parameter choices, produces an unbacked-debt-to-GDP ratio that matches the postwar average. The choice for γ produced by this procedure is 4.2. Given our baseline choice of η , this γ -value produces a consumption-substitution elasticity of approximately 0.59. This value is quite close to a recent σ -estimate by Attanasio and Weber (1995). These authors, who identify and address a number of problems with earlier estimates, obtain an estimate of 0.56.

The question of empirically appropriate assumptions about time preference is also quite unsettled. Our choice for ρ is based on Hurd (1989), who estimates the *difference* between the risk-free real rate of return facing agents and their pure rate of time preference. His favored estimate of this parameter is 0.041. Because our target for the average after-tax, risk-free real interest rate in the postwar U.S. is zero, we have chosen $\rho = -0.041$ in our baseline case.¹¹ Given our choice of γ , this choice yields an effective time preference rate of -0.01 . The value of ρ must also be adjusted for the choice of n ; this is done by raising the associated discount factor β to the power $55/n$, and using the reversion formula $\rho = (1 - \beta)/\beta$.

The efficiency-endowment pattern we employ is based on estimates of productivity by age reported by Hansen (1993). The underlying pattern is a weighted average of Hansen's estimated patterns for men and women; we smooth it by fitting a second-degree polynomial. The smoothed pattern is

$$e(j) = \exp [-1.46 + 0.070731 j - 0.00075 j^2] , \quad (16)$$

$j = 21, 22, \dots, 76$. We construct the endowment patterns used in the model for alternative values of n by dividing the interval $[21, 76]$ into n subintervals of equal length, and setting e_i , the endowment received in the i^{th} period of life, equal to the integral of this endowment function over the i^{th} subinterval.

The parameter α determines the share of gross output that is paid out in the form of returns to capital. We follow Auerbach and Kotlikoff (1987) and many subsequent studies by choosing a value of 0.25. The appropriate value of α depends, to some extent, on the breadth of the modelers' empirical definitions of "capital" and "labor," and on the extent to which they believe that other forms of capital (such as government physical capital, R&D capital, and uninternalized human capital) are

¹¹For a discussion of the empirical plausibility of negative time preference see section 3.3.4 below.

substitutes for private tangible capital. The gross population growth rate ψ is set at 1.017, which is the average U.S. labor force growth rate during the postwar period. The gross rate of technological progress λ is set equal to 1.015; this produces a steady-state net output growth rate of 3.2 percent, the postwar average. If we use a different value for n , these parameters are adjusted by raising them to the power $55/n$. The net depreciation rate δ is set at 0.1. Values of δ must also be adjusted for n ; if we denote the annual depreciation rate $\hat{\delta}$, the adjustment is $\delta(n) = 1 - (1 - \hat{\delta})^{55/n}$.

The remaining parameter is θ , the gross rate at which the government increases the nominal stock of unbacked liabilities. We have set the baseline net growth rate of the nominal stock of unbacked debt at 3.2 percent ($\theta = 1.032$), a value that produces a net real interest rate of zero in the monetary steady state. The value of θ can also be adjusted for different choices of n by raising it to the power $55/n$. We chose zero as a target risk-free interest rate based on U.S. postwar data. The most widely-used empirical proxy for the risk-free real interest rate is the *ex post* real yield on a three-month Treasury bills. Our estimate of the average value of this yield during 1948-1994 is 1.0 percent. We estimate the average after-tax safe real rate by applying an estimate of the average marginal tax rate on interest income for each quarter to the net nominal yield for that quarter.¹² This procedure produces an average rate of -0.4 percent. For the purpose of constructing our baseline case we rounded this estimate up to zero percent.

3.2 Features of the baseline steady state

Under our baseline calibration the aggregate saving function, which was defined in equation (14), is strictly increasing in the gross real interest rate R . This finding is robust to parameter changes of the types described in the next subsection. The saving function is negative at low values of R and is positive at higher values. There is consequently a unique nonmonetary steady state at which the value of the function is zero. The associated gross real interest rate $R_{nmss} \doteq 0.996$ is lower than the gross output growth rate $\psi\lambda \doteq 1.032$. As a result, for each $\theta \in [1, \psi\lambda R_{nmss}^{-1})$ there is a unique monetary steady state involving $R_{mss} = \psi\lambda\theta^{-1}$. In our baseline monetary steady state we set $\theta = \psi\lambda$, which produces $R_{mss} = 1$.

3.2.1 Real interest rates and debt

Perhaps the single most striking feature of our baseline steady state, other than the fact that it is a *monetary* steady state, is that the equilibrium real interest rate is more than three percent lower the equilibrium output growth rate. As we have noted, the steady-state values of the real interest rate and the output growth rate are close to our estimates of the average after-tax real interest rate on risk-free credit

¹²We thank Professor Joseph Peek of Boston College for providing us with marginal tax rate estimates.

instruments and the average real GDP growth rate, respectively, in the U.S. during the postwar period. To the best of our knowledge, no other plausibly calibrated general equilibrium model has produced this combination of steady-state values.

Another unique feature of our baseline steady state is that the ratio of the stock of unbacked liabilities to output is 0.59, which is our estimate of the average ratio of the U.S. national debt to GDP during the period 1948-1994.¹³ Our figure for the national debt is federal debt held by the public, including the social security system, plus base money outstanding. (The latter is essentially equal to federal debt held by the Federal Reserve Banks.) Our use of this figure reflects our maintained assumption that the entire national debt has been and remains unbacked, both in reality and in the expectations of U.S. households. While we could not claim to be able to prove that this assumption is correct, it is quite consistent with the facts about the history of the debt that we presented in our introduction. In our baseline steady state, the government uses bond seigniorage revenue to finance a permanent net-of-interest deficit that amounts to 1.9 percent of annual output.¹⁴

3.2.2 Consumption and labor supply

Our baseline steady state produces reasonable labor supply behavior on the part of the households. As we have indicated, households devote an average of about 22 percent of their time to supplying labor. Although they retire (supply zero labor) only in their final period of life, they are semi-retired during the last fifth of their lives, working only 7.6 percent of the time. Over the first four-fifths of their lifetimes, in contrast, they work for an average of 30 percent of the available hours.

An important difference between overlapping-generations and infinite-horizon models is that in OLG models agents' lifetime consumption growth rates can be quite different from the growth rate of aggregate *per capita* consumption, even when all agents are identical within and across generations. Laitner (1992) notes that the empirical evidence supports approximate equality between the lifetime consumption growth rate and the aggregate growth rate of *per capita* consumption. In many calibrated OLG models, however, the high real interest rates facing households produce consumption growth rates well in excess of the average *per capita* growth rate. Auerbach and Kotlikoff, for example, obtain an average lifetime consumption growth rate of roughly 2.5 percent in a model with a *per capita* consumption growth rate of zero percent. In our baseline steady state, however, the average lifetime consumption consumption of 1.8 percent is quite close to the *per capita* consumption growth rate

¹³There is a sense in which this is true by construction, since we choose γ in order to reproduce this ratio. However, our baseline parameterization is unique in allowing us to do this while keeping σ in the middle of the empirically plausible range. In other calibrated models, reproducing the observed bonds-output ratio would either be impossible or would require values of σ well in excess of unity.

¹⁴Bond seigniorage, which is conceptually similar to currency seigniorage, is the revenue the government can earn from maintaining a real stock of outstanding debt when the real interest rate on its bonds is lower than the output growth rate. See Miller and Sargent (1984).

of 1.5 percent. (The latter is necessarily equal to the exogenous productivity growth rate.)

Evidence from cross-sectional surveys of consumption behavior suggests that profiles of consumption by age are humped-shaped. The consumption of middle-aged households is substantially higher than that of young households, and after late middle age the consumption of older people begins to decrease relative to that of their middle-aged contemporaries. However, conventional nonstochastic life-cycle models, such as the Auerbach-Kotlikoff (1987) model, produce age-consumption profiles that are close to linear. Engen (1992) argues that nonstochastic models are incapable of producing humped-shaped age-consumption profiles.

Our baseline steady state produces a humped shape age-consumption profile. Consumption is highest among individuals who are roughly 50 years of age; their consumption is approximately 50 percent higher than the consumption of the youngest agents (age 21) and 25 percent higher than the consumption of the oldest ones (age 75). This profile is quite similar to the age-consumption profiles obtained by Engen (1992) in a model that includes both mortality risk and earnings uncertainty. Both these model-generated profiles are broadly similar to the empirical consumption profile Engen constructs using data from the Consumer Expenditures Survey.¹⁵ The empirical profile peaks at about the same age but has a more dramatic hump: peak middle-age consumption is roughly 1.7 times higher than the consumption of new labor force entrants, and the consumption of 75-year-olds is about equal to that of 21-year-olds.

The source of our hump-shaped consumption profile is a combination of three features of our model: a humped-shaped labor-efficiency profile, elastic labor supply, and low real interest rates. The first feature makes the opportunity cost of labor relatively high later in life, the second feature allows agents to respond by substituting into leisure, and the third feature allows substitution into leisure to produce a decline in consumption.¹⁶

3.2.3 Saving and wealth

As we noted in the introduction, calibrated versions of nonstochastic OLG models have had difficulty generating enough savings to match empirical estimates of saving rates and wealth-output ratios. Auerbach and Kotlikoff (1987), for example, obtain a steady-state saving rate of 3.7 percent. In our baseline case, however, the saving rate is approximately 8.1 percent. This is actually a bit higher than our estimate of the

¹⁵Engen's empirical profile is quite similar to the profile constructed by Attanasio and Weber (1995). It is important to note that in a growing economy the age-consumption profile at a particular date can look quite different from a time series of a household's consumption over its lifetime.

¹⁶In a model without endogenous labor supply the age-consumption profile is necessarily linear. Auerbach and Kotlikoff (1987) have endogenous labor supply and a humped-shaped labor efficiency profile. They obtain a nearly-linear age-consumption profile because of their high real interest rate.

U.S. saving rate (the ratio of private saving less depreciation to GDP), which averaged of 7.2 percent during 1948-1994. Technological progress, which is absent from the Auerbach-Kotlikoff model and many other models in the saving-rate literature, plays an important role in generating high saving rates in our model.¹⁷

Our baseline steady state produces a wealth-output ratio of 3.1. Laitner (1992) presents evidence supporting a target value of 3.15 for the ratio of aggregate wealth to GDP. His calculation includes the market value of government debt, which does not appear in most other wealth-output calculations but is an important component of total wealth in our model. Laitner goes on to conduct a partial equilibrium analysis in which he varies the exogenous real interest rate. His nonstochastic model cannot generate plausible wealth-income ratios unless it is augmented by bequest motives and/or liquidity constraints and unless the real interest rate is at least as high as the output growth rate.

Our baseline case produces a lifetime wealth profile similar to the profiles produced by other nonstochastic life-cycle models. Young households accumulate assets fairly rapidly, especially in early middle age, and then draw down these assets even more rapidly starting around age 60. Studies based on cross-section evidence have often found that the elderly continue to accumulate assets until the end of their lives — a finding that has stimulated interest in models with bequest motives. However, theoretical analysis conducted by Hurd (1987,1989) indicates that most elderly people should have declining wealth trajectories even with a bequest motive. Hurd's empirical analysis indicates that most elderly people do dissave fairly rapidly as they age, and he finds little or no evidence of bequest motives.¹⁸ (For additional discussion of bequest and related motives see Section 4.3 below.)

The capital-output ratio in the baseline steady state is 2.5, which is a standard figure in the literature on calibrated general equilibrium models. The range of values in the literature is quite broad, although values lower than 2 or higher than 3.5 are unusual. Prescott (1986) advocates a target capital-output ratio of 2.6.¹⁹

¹⁷There is a large literature that attempts to explain saving rates by introducing stochastic features that induce precautionary saving. Most of this literature uses partial equilibrium models with real interest rates that are exogenous and relatively high: examples include Engen (1992) and Hubbard, Skinner and Zeldes (1994). Recent work by Rangazas and Lord (1998) indicates that general equilibrium versions of these models are unlikely to succeed in generating realistically high saving rates.

¹⁸Hurd (1987) provides brief survey of previous empirical evidence for and against the proposition that the elderly tend to dissave as they age.

¹⁹More recent work by Cooley and Prescott (1995) identifies the target ratio as 3.3. The target ratio increased because they broadened their empirical definition of capital stock to include both government capital and consumer durables.

3.3 Robustness of monetary steady states

3.3.1 Overview

In this subsection we report the results of some simple experiments in which we vary a single parameter across a range of alternative values, holding all the other parameters at their baseline values (except θ , which is not relevant for this purpose). We compute the associated value of the nonmonetary steady state gross real interest rate R_{nmss} and compare this value to the associated value of the gross output growth rate (which changes only when we change ψ or λ — see below). In each set of experiments, it turns out that R_{nmss} increases or decreases monotonically with the value of the parameter in question. In each case but the first (which involves changing the value of n), moreover, the relationship is close to linear.

We begin by investigating how the “annualized” value of the nonmonetary gross real interest rate R_{nmss} — that is, the raw value raised to the power $n/55$ — depends on the value of n , the number of periods in agents’ lives, as n is varied from 2 to 220. The annualized nonmonetary real rate starts out fairly high and declines rapidly at first, but settles down to a long, slow and apparently asymptotic decline after n reaches 10-15 periods. The nonmonetary rate is always below the annualized output growth rate $\psi\lambda$. Thus, in our baseline case there are monetary steady states regardless of the value of n , and for reasonably large values the properties of these steady states are essentially invariant to n . However, these results suggest that the properties of small- n specifications are not a reliable guide to the properties of specifications in which n is fairly large. When $n = 2$, for example, the annualized gross real interest rate in the nonmonetary steady state is 1.03, which is only 0.2 percent lower than the annualized output growth rate. The corresponding value for our baseline case (in which $n = 55$) is 0.996. Thus, the $n = 2$ specification has monetary states only for values of θ quite close to unity and cannot deliver annualized real interest rates substantially below the real growth rate. These experiments indicate that calibration exercises using small- n OLG models may produce misleading results.

The rest of this section reports the results of calibration experiments in which n is fixed at 55.

3.3.2 Critical parameters

Rate of time preference. The nonmonetary real interest rate increases with the pure rate of time preference. In the baseline case, values of ρ in excess of -0.006 fail to produce monetary steady states. Given our baseline choice of η , a pure time preference rate of -0.006 yields an effective time preference rate of -0.001 . Thus, it is essentially accurate to describe our baseline specification as one in which monetary steady states exist if and only if the time preference rate is negative. It is important to note, however, that the finding that positive time preference rates do not support

monetary steady states is *not* a general implication of our model: it is simply a characteristic of our baseline parameterization. If we were to reduce our baseline value of γ to unity — a value often used in real-business-cycle studies — then there would be monetary steady states for values of ρ as high as +0.012. However, the pure time preference rate is the clearest example of a parameter that is critical in the sense that many values which are widely regarded as empirically plausible rule out monetary steady states.

Elasticity of intertemporal substitution in consumption. The nonmonetary real rate also increases in the value of γ , a parameter which controls resistance to intertemporal substitution. Monetary steady states do not exist for γ -values higher than 10, which correspond to a substitution elasticities lower than 1/3. The results of this experiment may be somewhat misleading, however. As we noted in section 2.1, increasing γ without adjusting ρ produces agents who are also very resistant to deferring consumption. If we modify the above experiment by adjusting ρ in order to keep the effective time preference rate $\hat{\rho}$ fixed at its baseline value of -1 percent, then it turns out that R_{nmss} is no longer increasing in γ . For example, if we choose $\gamma = 42$, which produces $\sigma = 0.1$, then there is still a monetary steady at $R = 1$, and in order to preserve the baseline debt-GDP ratio we must actually reduce $\hat{\rho}$ slightly (to -0.8 percent). The associated value of ρ is -0.28 . While this value may seem low, there is nothing implausible about the way these agents value current vs. future consumption: their average lifetime consumption growth rate remains 1.8 percent.

3.3.3 Other parameters

Productivity and technology growth rates. The nonmonetary steady-state real interest rate increases with the rate of technological progress. However, increasing λ also increases the steady-state output growth rate — a fact that broadens the range of λ -values consistent with the existence of monetary steady states. There are monetary steady states for any rate of technological progress lower than 7.2 percent. Increasing the gross population growth rate ψ also increases both the output growth rate and value of R_{nmss} . In this case, however, the interest rate rises more slowly than the growth rate, so that the range of values of θ consistent with the existence of a monetary steady state broadens markedly as ψ increases. Reducing ψ has the opposite effect, but it is not very pronounced. When ψ is reduced to unity (a constant population), for example, R_{nmss} falls to 0.994 and remains substantially below the implied gross output growth rate of 1.015.

Capital share of output and labor share of time. Increases in the value of the capital share parameter α tends to increase R_{nmss} , but the gradient is relatively low. A capital share of 0.5, which is twice the baseline value, produces $R_{nmss} = 1.023$. Since the baseline gross output growth rate is 1.032, this capital share still leaves

scope for monetary steady states. Increasing η , the labor share of time, has similar effects. An η of 0.5, which is more than twice the baseline value, also produces $R_{nmss} = 1.023$.

Depreciation rate. Reducing the depreciation rate tends to increase the real interest rate in the nonmonetary steady state. However, unless the value of δ is quite close to zero — less than 0.002 — R_{nmss} continues to fall short of the gross output growth rate.

Age of forced retirement. In our baseline case agents work relatively little in the last ten years of life but retire completely only in their very last period. The date of retirement is fairly sensitive to the interest rate the agent faces in the monetary steady state. If we choose $\theta = 1$ in our baseline case, so that the net real interest rate in the monetary steady state rises from zero to 3.2 percent, then agents retire during the last 11 periods of life. We also conducted experiments in which we imposed retirement beginning at figurative ages from 62 to 76 so that agents must retire for 14, 13, ..., 2, 1, 0 years. Reducing the age of forced retirement tends to increase saving and decrease R_{nmss} , but the effect is fairly weak. Forced retirement at age 62 decreases R_{nmss} by only 1.1 percentage points.

Labor-efficiency profile. Finally, we investigate the effects of changing the pattern of labor-efficiency endowments. Generally speaking, steeper patterns (patterns in which productivity tends to be higher later in life) tend to produce higher non-monetary real interest rates and vice-versa. Our strategy for constructing a family of steeper or flatter patterns starts by defining an augmentation factor f . This factor will be larger than unity if we want a steeper pattern and between zero and unity if we want a flatter one. To obtain a revised pattern, we multiply the efficiency endowment in the last period of an agent's life by the value of the augmentation factor, and multiply endowments in earlier periods by a number obtained by interpolating linearly between unity and f , depending on the current age of the agent. Increasing the steepness of the profile tends to increase R_{nmss} , but the effect is much less dramatic than we expected. Unless f exceeds 4.3, a value that produces an extremely steep efficiency profile, there will be monetary steady states for some values of θ . Using the alternative efficiency-endowment patterns employed by Auerbach and Kotlikoff (1987) makes it marginally easier to obtain monetary steady states.

3.3.4 Interpretation

We think the results of these experiments demonstrate that our baseline monetary steady state is quite robust, in the sense that most of the parameters of the model can be varied within broad ranges without ruling out monetary steady states. In most cases, the ranges in question include the values of most published estimates of the

respective parameters.²⁰ One noteworthy exception is the rate of time preference. Unless we are willing to make fairly large changes in the baseline values of other parameters — such as increasing γ and/or decreasing η in order to increase the elasticity of intertemporal substitution — we cannot have monetary steady states with positive rates of time preference.

As Bailey and Olson (1980) point out, nothing in the neoclassical theory of intertemporal consumer choice rules out negative time preference rates. As a result, the sign of ρ (or $\hat{\rho}$) must be regarded as an empirical question. Bailey and Olson pronounce in favor of positive time preference, arguing that agents with negative time preference rates would choose implausibly high consumption growth rates and would consequently have implausibly low consumption levels when young. Our results provide a counterexample to this claim, however. In our baseline steady state the average lifetime consumption growth rate is only 1.8 percent, and a household's consumption in the first period of life is only 25 percent lower than the current level of average *per capita* consumption. As we have seen, both values are consistent with empirical estimates of age-consumption profiles.

A large number of empirical studies have attempted to estimate the parameters of agents' utility functions using aggregate time series or pooled cross-sectional and time series data. Many of these studies have produced negative estimates of the rate of time preference. Examples include MaCurdy (1981), Hayashi (1982), Hansen and Singleton (1983), Mankiw, Rotemberg and Summers (1985), Eichenbaum, Hansen and Singleton (1988), Hotz, Kydland and Sedlacek (1988), Hurd (1989), Singleton (1990) and Epstein and Zin (1991). These estimates all reflect the same basic feature of the data: the observed real interest rates on risk-free assets (which are low) and the observed growth rates of individual and/or aggregate consumption (which are substantially positive) are difficult to reconcile with positive rates of time preference.

4 Issues

4.1 Credit market frictions and the rate of return on capital

A wealth of empirical evidence indicates that there is a large gap between the average real interest rate on risk-free government debt, which is relatively low (see above), and the average real return rates on diversified portfolios of risky private liabilities, which are relatively high. The evidence also suggests that the average output growth rate

²⁰Larch (1993) also investigates the question of whether Auerbach-Kotlikoff-style models can support nonmonetary steady states in which the real interest rate is lower than the real growth rate. His model does not include depreciation, endogenous labor supply or realistic labor efficiency profiles, and he rules out negative time preference rates *a priori*. As we have seen, all these modeling decisions tend to work against steady states with relatively low real interest rates. As a result, Larch concludes that plausible parameter choices rule out steady states of this type. He does not study monetary steady states and he does not compare the endogenous features of the steady states his model generates to the characteristics of actual economies.

lies between these two average real return rates. The source of this “equity premium” is one of the most profoundly unsettled questions in modern macroeconomics.²¹ In standard nonstochastic models there is no equity premium: in equilibrium all assets yield a common rate of return. Researchers using these models must choose between a parameter-choice strategy that delivers a relatively high common return rate, on the order of the average real rate of return on equity, or a strategy that delivers a relatively low common rate, on the order of the average real government bill yield. They have almost invariably chosen the former strategy, constructing models in which the common asset return rate exceeds the output growth rate.

As we have indicated, we view the principal contribution of this paper as demonstrating that the alternative low-common-return-rate strategy is quite viable, has a number of important advantages, and is likely to provide a fruitful path for further research. In this section, however, we present a brief analysis of the question of whether our results are robust to the introduction of simple credit market frictions that drive a wedge between the real government bond rate and the real rate of return on capital — a wedge wide enough to push the real capital return rate above the output growth rate. The frictions we introduce are taxes on capital returns and costs of financial intermediation. These frictions probably account for part of the observed equity premium, and they are fairly easy to incorporate in nonstochastic models (at least in stylized forms).

At one level, the question we have just posed has a clear and almost immediate answer. We know that holding the rate of return facing savers fixed and introducing frictions that drive the marginal product of capital upward will reduce the demand for capital and increase the residual demand for government debt. And as long as we structure our frictions so that the real interest rate paid by the government stays at empirically plausible levels, the government will continue to be able to roll its debt over. Finally, since the real interest rate facing savers is the key determinant of the implications of the model for consumption behavior, a respecification of the model along the lines just described should leave these implications largely unaffected.

The only remaining question involves the size of the government debt. If the rest of the baseline parameterization is unaltered, then driving the marginal product of capital far above the output growth rate will reduce the demand for capital dramatically enough to drive the bonds-output ratio to implausibly high levels. This situation will also endanger the empirical plausibility of the baseline values of capital-related variables such as the capital-output ratio and the net saving rate. Is it possible to find a plausible alternative parameterization that solves this problem without disrupting the other appealing features of the baseline steady state?

²¹The basic reference is Mehra and Prescott (1985). The equity premium literature has recently been surveyed by Kocherlakota (1996).

We will show that the answer to this question is “yes.” The reparameterization strategy we have developed involves increasing the capital share parameter α without changing any other parameters from their baseline values. Increasing α tends to increase the demand for capital without affecting the consumption-related features of the steady state. Although our baseline value for α is standard in the literature on life-cycle models, real business cycle studies typically use substantially higher values. As a result, we suspect that few economists will regard the relatively moderate increases in α that we are about to describe as empirically implausible.

The tax and intermediation assumptions behind the experiments described below work as follows. We begin by assuming that all private credit is channeled through zero-profits financial intermediaries who issue deposits to savers; government debt, in contrast, may be held by savers directly. The intermediaries face a proportional intermediation cost c . We will think of c as a proxy for the costs of dealing with the diversification and information problems associated with default risk, and possibly also as a proxy for the premium for bearing undiversifiable risk.

Poterba (1997) estimates the average after-tax rate of return on U.S. corporate capital during 1959-1996 as 3.9 percent. We will round this figure to 4 percent. When we include intermediation costs in our experiments we choose a value for c of 0.03 because this value implies that the value of R^i , the gross after-tax return rate received by the intermediaries from the firms, is 1.04. Díaz-Giménez *et al.* (1992) find that financial intermediation services account for 5-7 percent of total output. In both experiments where we introduce intermediation costs the ratio of total intermediation costs to output falls into this range.

The gross real interest rate the intermediaries pay on their deposits, which is denoted R^b , will be equal to $R^i - c$. Note that R^b will also be equal to the gross real interest rate on government debt.

Net nominal interest paid by government or private borrowers to savers is taxed at a proportional “personal interest income” tax rate τ^p . To nominalize the returns we will use a shadow inflation rate of 4 percent, which is close to the postwar average. Thus, R^d , the gross real rate of return received by savers, will be related to R^b by the formula $R^d = (1 - \tau^p)R^b + \tau^p R^m$. Here R^m , the gross real rate of return on “money,” is equal to $(1 + \pi)^{-1}$, where π is the net inflation rate. We will set τ^p at the level necessary to produce the observed one-percent difference between the average before- and after-tax yields on short-term government debt. This level turns out to be 20 percent ($\tau^p = 0.2$). We will set R^d at unity so that savers continue to receive a net after-tax real interest rate of zero percent.

Next, we impose a second nominal return tax on the interest paid by private borrowers (almost exclusively firms) to the intermediaries. We will call this tax rate τ^c . We think of τ^c as a proxy for other direct and indirect taxes on capital income,

including corporate profits taxes, capital gains taxes and historical cost depreciation. The formula describing the relationship between R^i and R , the gross real interest rate paid by private borrowers, is $R^i = (1 - \tau^c)R^i + \tau^c R^m$. Note that $R = 1 + r - \delta$, where r is the rental rate on capital and is equal to the marginal product of capital. The real interest rate relevant to agents' consumption and saving decisions is R^d , while the real rate relevant to firms' investment decisions, and also to the determination of the real wage rate, is R . We assume that the real revenue raised by these taxes leaves the economy — the same assumption we have made about the revenue from bond seigniorage. In the second and third experiments described below we target R at 1.05 — a figure that substantially exceeds our baseline gross output growth rate of 1.032. In the fourth experiment we target R at 1.07 — a value which is very close to Mehra and Prescott's (1985) estimate of the average real rate of return on equity in the U.S. during the last century, and also to Cooley and Prescott's (1995) estimate of the average real return rate on U.S. business capital during the postwar period.

In our first experiment we set the personal interest income tax rate at 20 percent ($\tau^p = 0.2$) but set both the capital income tax rate and the proportional intermediation cost at zero. These choices duplicate the observed spread between the before- and after-tax real government bond rates but do not produce any spread between the before-tax real government bond rate and the marginal product of capital, which remains substantially below the output growth rate. We then increase the capital share parameter α to a level necessary to keep the bonds-output ratio at its baseline value of 0.59; this turns out to require $\alpha = 0.27$.

In our second experiment we add a capital income tax at a rate necessary to drive the marginal product of capital up to 5 percent, keeping the interest income tax rate at 20 percent and the intermediation cost at zero. The required capital income tax rate is 45 percent ($\tau^c = 0.45$). In our third experiment we add intermediation costs ($c = 0.03$) and reduce the capital income tax rate to the level consistent with $R = 1.05$; this requires a much more moderate tax rate of 11.3 percent ($\tau^c = 0.113$). In each case, we keep the bonds-output ratio at its baseline level by increasing α to a value slightly greater than 0.33. The fourth experiment is similar to the third except that the target return rate on capital is 7 percent ($R = 1.07$). In this experiment we leave c at 0.03: the required values of τ^c and α are 0.277 and 0.358, respectively.

Table 1 displays the values of key endogenous variables in the steady states associated with these experiments, along with their values for the baseline steady state. In the table, the symbols B/Y and K/Y stand for the bonds-output ratio and the capital-output ratio, respectively. The symbol S/Y stands for the net saving rate, while $\Delta c/c$ represents agents' average lifetime consumption growth rate and " l -share" represents the average labor share of agents' time endowments. The next two symbols represent summary measures of the hump-shapedness of the lifetime consumption

Table 1					
Variable	Baseline	Case 1	Case 2	Case 3	Case 4
R^d	1.00	1.00	1.00	1.00	1.00
R^b	1.00	1.01	1.01	1.01	1.01
R^i	1.00	1.01	1.01	1.04	1.04
R	1.00	1.01	1.05	1.05	1.07
B/Y	0.59	0.59	0.59	0.59	0.59
K/Y	2.50	2.45	2.21	2.21	2.10
S/Y (net)	0.0806	0.0792	0.0711	0.0713	0.0678
$\frac{\Delta c}{c}$	0.0183	0.0185	0.0185	0.0186	0.0186
l -share	0.223	0.223	0.223	0.223	0.223
c_{50}/c_{21}	1.50	1.50	1.50	1.51	1.51
c_{50}/c_{75}	1.26	1.25	1.25	1.25	1.25
cK/Y	—	—	—	0.0663	0.0630

Table 1: Results from alternative experiments.

profile: c_{50}/c_{21} stands for the ratio of the consumption of middle-aged (50-year-old) agents to that of new labor force entrants (21-year-olds) at the same date, while c_{50}/c_{75} stands for the ratio of middle-aged consumption at a given date to the consumption of agents who have reached the end of their lives (75-year-olds) at that date. Finally, cK/Y stands for the ratio of intermediation costs to output, which is relevant only in Cases 3 and 4.

The column for Case 1 indicates that this relatively limited reparameterization has virtually no effect on the key endogenous variables. The columns for Cases 2 and 3 indicate that the effects of these experiments are not large either, even though they produce steady states in which the net marginal product of capital substantially exceeds the output growth rate. In each case, the new steady state delivers slightly smaller values of the capital-output ratio and the net saving rate and does not have significant effects on any other variables. The decline in the capital-output ratio may slightly reduce the empirical plausibility of the steady state, but the decline in the net saving rate probably has the opposite effect. Case 4 is very similar to Cases 2 and 3: the capital-output ratio and the net saving rate are a bit lower, but both values remain quite plausible. On balance we think these last three alternative steady states look remarkably similar to our baseline steady state, and we view them as demonstrating that our results are robust to respecifications that drive the marginal product of capital above the output growth rate.

4.2 Dynamic inefficiency

One interesting question raised by Cases 2-4 is whether these steady states are dynamically efficient. “Dynamic efficiency” is a term that is often used to characterize some of the distinctive features of Samuelson-case equilibria. Most economists associate dynamic efficiency with overaccumulation of capital — that is, with equilibria in which the stock of capital has reached a level at which the marginal rate at which resources can be transferred from the present into the future through capital accumulation (the marginal product of capital) is lower than the rate at which they could be transferred via intergenerational exchanges (the output growth rate). In steady states of this type the cost of maintaining the capital stock at its current level relative to output (gross investment) exceeds the returns the capital stock provides (gross capital income).

In Cases 2 and 4, the steady states are not dynamically inefficient in this “Type A” sense: in each case, the marginal product of capital exceeds the output growth rate and gross capital income exceeds gross investment.²² On the other hand, these equilibria remain inefficient in the “Type B” sense that by holding government bonds, agents are engaging in intergenerational transfers at a rate that is lower than the transfer rate permitted by the physical environment (the growth rate of output), and that is also lower than the (pretax) rate of return on physical assets.²³ On the margin, replacing government bonds with either direct intergenerational transfers or physical capital would permit an increase in the consumption of every agent across steady states. It is this sort of inefficiency that is relevant to our analysis, because the low real interest rate on government debt allows the government to issue and roll over a large stock of unbacked debt. From the viewpoint of applied macroeconomic theory, it is the ability of dynamically inefficient equilibria to support unbacked debt that gives rise to most of their unusual and interesting features.²⁴

²²The Case 3 equilibrium is dynamically inefficient. Since we have assumed that there is no way for the returns to capital to be distributed to households without incurring the intermediation cost, the rate of return on capital that is relevant to dynamic efficiency calculations is the marginal product of capital less the intermediation cost rate, which we will call the received rate of return on capital (RRK). In Case 3 the marginal product of capital exceeds the output growth rate but the RRK falls slightly short of it. In Case 2, where there are no intermediation costs, the RRK is well above the output growth rate, and in Case 4 the RRK exceeds the output growth rate despite the intermediation costs.

²³One can think of this type of inefficiency as resulting from bad tax policy on the part of the government. It is similar to the inefficiency described by Freeman (1987) in his analysis of optimal reserve requirements. In Freeman’s economy, however, the rate of return on capital was high under *laissez faire* and bad tax policy created a demand for unbacked government liabilities. In our economies there is a demand for unbacked debt under *laissez faire* and bad tax policy is responsible for the high rate of return on capital.

²⁴For example, the results obtained by Espinosa and Russell (1998a,b) regarding the long-run real effects of monetary policy depend critically on the ability of their model to produce equilibria with unbacked government debt. In the latter paper they show that their results are robust to the inclusion of an intermediation cost that drives the marginal product of capital above the output growth rate.

In stochastic models, interest rate comparisons do not provide reliable tests of dynamic efficiency. A steady state can be dynamically inefficient when the average real return rate on capital is higher than the average output growth rate, or dynamically efficient when the average real interest rate on risk-free debt is lower than the average output growth rate. Abel *et al.* (1989) test for dynamic efficiency by comparing “gross profits” (gross capital income) to gross investment for the U.S. and a number of other developed countries over the past 60 years. These calculations indicate that gross profits have exceeded gross investment in every country in every year. They conclude that the U.S. economy has been dynamically efficient.

The empirical calculations presented by Abel *et al.* implicitly define “gross profits” as the total returns to capital *paid by firms* — a definition that does not make any adjustment for taxes, intermediation costs, or other frictions that might reduce the amount of income received by households. An economy with intermediation costs can have a steady state in which gross capital income before intermediation costs exceeds gross investment in every period, even though gross capital income after intermediation costs is smaller than gross investment in every period. In this case the economy would pass the Abel *et al.* test but would nevertheless be Type A inefficient (as in Case 3; see note 22 above). Similarly, an economy with taxes on capital income can have a steady state in which gross capital income always exceeds gross investment, but in which differential taxes on returns paid by firms and the government produce a level/distribution of the real interest rate on government debt low enough to allow debt to be rolled over forever. In this case, the economy would pass the Abel *et al.* test but would be Type B inefficient. Cases 2 and 4 are examples of nonstochastic economies of this type. For this reason and others, we do not think the evidence presented by Abel *et al.* (1989) endangers the empirical relevance of our analysis.²⁵

4.3 Familial altruism

Empirical evidence indicates that many households transfer substantial quantities of assets or goods to younger or older family members, either during their lives (*inter vivos* transfers) or via bequests, without requiring compensation that has a clear market value. These transfers seem inconsistent with the assumption that agents have selfish preferences. Most attempts to explain private intergenerational transfers are based on the notion of familial altruism — that is, on the assumption that households care about the welfare, consumption or income, etc., of family members from different generations.

²⁵ Another problem with the Abel *et al.* test is that the link between capital income and investment in formal models and empirical definitions of these variables is not very clear. Abel *et al.* (1989) use a broad empirical definition of capital income and a narrow definition of gross investment. Alternative plausible definitions of these concepts could change the results of their calculations substantially. For a more elaborate discussion of this question see Bullard and Russell (1997a).

A relatively extreme familial-altruism assumption postulates that the utility of a household's direct descendants appears in the household's utility function. This "Becker-Barro" [Becker (1974), Barro (1974)] altruism has the well-known effect of converting an overlapping generations model into what amounts to a representative-agent infinite horizon model. As long as the transfer motive is uniformly active, so that transfers occur at each date, each member of a dynastic sequence of households makes the same decisions that would be prescribed by a hypothetical dynastic planner who could transfer resources across consecutive generations of family members at a rate equal to the output growth rate. As a result, models peopled by Becker-Barro altruists do not have Samuelson-case equilibria.

Most alternative theories of altruistic, quasi-altruistic, or non-altruistic intergenerational transfers are not inconsistent with the existence of Samuelson-case equilibria. This point has most often been made in connection with the closely related question of Ricardian equivalence.²⁶ Interest in Ricardian equivalence and related questions has produced a large literature on the empirical plausibility of Becker-Barro altruism. The findings of this literature suggest that most bequests and transfers are either non-altruistic or are driven by other forms of altruism. For example, Hurd (1987,1989) finds that most bequests seem to be accidental, resulting from lifespan uncertainty and incomplete annuities markets rather than any deliberate attempt on the part of agents to provide funds for their descendants. There are also a large number of papers that test and reject key predictions of the Barro-Becker theory using data on *inter vivos* transfers (which are clearly deliberate) and/or apparently deliberate bequests.²⁷

There is a substantial theoretical literature on non-Becker-Barro transfer motives. Typically, introducing these motives into overlapping generations models increases the supply of capital and makes it easier to obtain Samuelson-case steady states. The logic behind this is simple: in a steady state, deliberate bequests and gifts represent capital passed from generation to generation, so additional capital must be accumulated in order to provide them. In addition, if agents are reasonably risk averse then introducing lifetime uncertainty and accidental bequests tends to promote capital accumulation by generating precautionary saving — as in Huggett (1996).

²⁶See Abel (1985) concerning accidental bequests, Bernheim *et al.* (1985) regarding bequests driven by exchange motives, Andreoni (1989) for "warm glow" transfer motives and Abel (1987) and Kimball (1987) regarding altruistic transfers from children to parents. O'Connell and Zeldes (1993) generalize the Abel-Kimball analyses in a way that rules out dynamic efficiency when agents have positive time preference, but not otherwise.

²⁷With regard to bequests, see Menchik (1980), Bernheim *et al.* (1985) and Wilhelm (1996). Wilhelm's work is particularly noteworthy because he studies a sample of very wealthy households, and it is these households who seem to account for high percentage of the bequests that do appear to be deliberate. For *inter vivos* transfers see Cox (1987) and Cox and Rank (1992) and Altonji *et al.* (1997) In addition, Altonji *et al.* (1992) find that within extended families the distribution of consumption is independent of the distribution of household resources — a finding that is inconsistent with the Becker-Barro theories of both *inter vivos* transfers and bequests.

The belief that intergenerational transfer motives tend to increase aggregate saving has led a number of authors, including Kolikoff and Summers (1981) and Auerbach and Kotlikoff (1987), to argue that introducing these motives into standard life cycle models may allow them to generate empirically plausible saving rates. However, bequests and gifts have a wealth effect on the consumption of recipients that tends to partly offset their direct effect on the supply of capital. Rangazas and Lord (1991) and Lord (1992) find that the effect is strong enough to make it impossible for bequest-augmented life-cycle models to generate realistic saving rates. As we have noted, our low-real-interest rate approach to specifying life-cycle models allows us to produce empirically plausible saving rates without introducing intergenerational transfer motives.

While the jury on the empirical plausibility of Becker-Barro altruism may still be out, the large body of evidence against this particular motive for intergenerational transfers provides abundant justification for continued research using calibrated models from which it is absent — including and especially models that have Samuelson-case equilibria. Models with selfish preferences and certain lifetimes are natural starting point for this research, but it will eventually have to confront the empirical evidence on the nature and scale of private intergenerational transfers. For this reason and others, we think that introducing uncertain lifetimes, warm glow bequests and/or other, more complicated transfer motives into analyses such as ours would be an interesting and useful extension of our research.

5 Concluding remarks

In this paper we have specified an overlapping-generations model with features that capture a large number of important characteristics of empirical economies. These features include agents who live for many periods, neoclassical production, capital accumulation, nontrivial labor-leisure decisions, life-cycle productivity changes, population growth, and technological progress. We have calibrated this model using parameter values that are standard in the calibration literature and/or have substantial empirical support. Our analysis of this plausibly calibrated version of the model demonstrates that it can support “monetary steady states” — Samuelson-case steady states in which the government can maintain a large real stock of unbacked debt. These steady states can replicate a large number of important long-run features of U.S. postwar data, including the relatively low real interest rates on risk-free debt, the decline in the U.S. debt-GDP ratio despite the absence of large or persistent primary surpluses, the relatively high ratio of net savings to GDP, and the humped shape of the average household’s age-consumption profile. We have shown that the existence of these monetary steady states is robust to relatively large changes in the values of most of the parameters of the model. We have also demonstrated that the

properties of our baseline steady state are also robust to the addition of credit market frictions that drive the rate of return on capital above the output growth rate.

References

- [1] Abel, A. (1985) "Precautionary Saving and Accidental Bequests." *American Economic Review* 75: 777-791.
- [2] Abel, A. (1987) "Operative Gift and Bequest Motives." *American Economic Review* 77: 1037-1047.
- [3] Abel, A., N. G. Mankiw, L. Summers and R. Zeckhauser. (1989) "Assessing Dynamic Efficiency: Theory and Evidence." *Review of Economic Studies* 56: 1-20.
- [4] Ahmed, S. and J. Rogers. (1996) "Long-Term Evidence on the Tobin and Fisher Effects: A New Approach." Working paper, Federal Reserve Board.
- [5] Aiyagari, S. R. and E. McGrattan. (1995) "The Optimum Quantity of Debt." Federal Reserve Bank of Minneapolis Staff Report 203, December.
- [6] Altonji, J., F. Hayashi and L. Kotlikoff. (1992) "Is the Extended Family Altruistically Linked? Direct Tests Using Micro Data." *American Economic Review* 82: 1177-1198.
- [7] Altonji, J., F. Hayashi and L. Kotlikoff. (1997) "Parental Altruism and Inter Vivos Transfers: Theory and Evidence." *Journal of Political Economy* 105: 1121-1166.
- [8] Andreoni, J. (1989) "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence." *Journal of Political Economy* 97: 1447-1458.
- [9] Attanasio, O. and G. Weber. (1995) "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey." *Journal of Political Economy* 103: 1121-1157.
- [10] Auerbach, A. and L. Kotlikoff. (1987) *Dynamic Fiscal Policy*. Cambridge University Press.
- [11] Bailey, M. and M. Olson. (1981) "Positive Time Preference." *Journal of Political Economy* 89: 1-25.
- [12] Barro, R. (1974) "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82: 1095-1117.
- [13] Barro, R. and X. Sala-i-Martin. (1990) "World Real Interest Rates." In O. Blanchard and S. Fischer, eds., *NBER Macroeconomics Annual 1990*. MIT Press.
- [14] Becker, G. (1974) "A Theory of Social Interactions." *Journal of Political Economy* 82: 1063-1093.
- [15] Bernheim, B. D., A. Schleifer and L. Summers. (1985) "The Strategic Bequest Motive." *Journal of Political Economy* 93: 1045-1076.
- [16] Boldrin, M. and M. Woodford. (1990) "Equilibrium Models Displaying Endoge-

- nous Fluctuations and Chaos: A Survey.” *Journal of Monetary Economics* 25: 189-222.
- [17] Bohn, Henning. “The Sustainability of Budget Deficits in a Stochastic Economy.” *Journal of Money, Credit and Banking* 27: 257-271.
 - [18] Bullard, J. and J. Keating. (1995) “The Long-Run Relationship Between Inflation and Output in Postwar Economies.” *Journal of Monetary Economics* 36: 477-496.
 - [19] Bullard, J. and S. Russell. (1997a) “An Empirically Plausible Model of Low Real Interest Rates and Unbacked Government Debt.” Working Paper, Federal Reserve Bank of St. Louis and IUPUI.
 - [20] Bullard, J. and S. Russell. (1997b) “How Costly is Sustained Low Inflation for the U.S. Economy?” Working Paper, Federal Reserve Bank of St Louis and IUPUI.
 - [21] Cooley, T. and E. Prescott (1995) “Economic Growth and Business Cycles.” In T. Cooley, ed., *Frontiers of Business Cycle Research*. Princeton University Press.
 - [22] Cox, D. (1987) “Motives for Private Income Transfers.” *Journal of Political Economy* 95: 508-546.
 - [23] Cox, D. and M. Rank. (1992) “Inter-Vivos Transfers and and Intergenerational Exchange.” *Review of Economics and Statistics* 74: 305-314.
 - [24] Diamond, P. (1965) “National Debt in a Neoclassical Growth Model.” *American Economic Review* 55: 1126-1150.
 - [25] Díaz-Giménez, J. and E. Prescott. (1997) “Real Returns on Government Debt: A General Equilibrium Quantitative Exploration.” *European Economic Review* 41: 115-137.
 - [26] Díaz-Giménez, J., E. Prescott, T. Fitzgerald and F. Alvarez. (1992) “Banking in a Computable General Equilibrium Model.” *Journal of Economic Dynamics and Control* 16: 533-559.
 - [27] Eichenbaum, M., L. Hansen and K. Singleton. (1988) “A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice Under Uncertainty.” *Quarterly Journal of Economics* 103: 51-78.
 - [28] Engen, E. (1992) “Consumption and Saving in a Life-Cycle Model with Stochastic Earnings and Mortality Risk.” Working Paper, UCLA.
 - [29] Epstein, L. and S. Zin. (1991) “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis.” *Journal of Political Economy* 99: 263-286.
 - [30] Espinosa, M. and S. Russell. (1998a) “Can Higher Inflation Reduce Real Interest Rates in the Long Run?” *Canadian Journal of Economics* 31 (forthcoming).
 - [31] Espinosa, M. and S. Russell. (1998b) “Conventional Monetary-Policy Wisdom

- in the Diamond Model.” Working Paper, Federal Reserve Bank of Atlanta and IUPUI.
- [32] Freeman, S. (1987) “Optimal Reserve Requirements.” *Journal of Monetary Economics* 19: 307-314.
 - [33] Gale, David. (1973) “Pure Exchange Equilibria of Dynamic Economic Models.” *Journal of Economic Theory* 6: 12-36.
 - [34] Guesnerie, R. and M. Woodford. (1992) “Endogenous Fluctuations.” Chapter 6 of J-J. Laffont, ed., *Advances in Economic Theory*, Vol. II. Cambridge University Press.
 - [35] Hansen, G. (1993) “The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked.” *Journal of Applied Econometrics* 8: 71-80.
 - [36] Hansen, L. and K. Singleton. (1983) “Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns.” *Journal of Political Economy* 91: 249-265.
 - [37] Hayashi, F. (1982) “The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables.” *Journal of Political Economy* 90: 895-916.
 - [38] Hotz, V., F. Kydland and G. Sedlacek. (1988) “Intertemporal Preferences and Labor Supply.” *Econometrica* 56: 335-360.
 - [39] Hubbard, R. G., J. Skinner and S. Zeldes. (1994) “The Importance of Precautionary Motives for Explaining Individual and Aggregate Saving.” *Carnegie-Rochester Conference Series on Public Policy* 40: 59-125.
 - [40] Huggett, M. (1993) “The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies.” *Journal of Economic Dynamics and Control* 17: 953-969.
 - [41] Huggett, M. (1996) “Wealth Distribution in Life-Cycle Economies.” *Journal of Monetary Economics* 38: 469-494.
 - [42] Hurd, M. (1987) “Savings of the Elderly and Desired Bequests.” *American Economic Review* 77: 298-312.
 - [43] Hurd, M. (1989) “Mortality Risk and Bequests.” *Econometrica* 57: 779-813.
 - [44] İmrohoroglu, A., S. İmrohoroglu and D. Joines. (1995) “A Life Cycle Analysis of Social Security.” *Economic Theory* 6: 83-114.
 - [45] Kimball, M. (1987) “Making Sense of Two-Sided Altruism.” *Journal of Monetary Economics* 20: 301-326.
 - [46] Kotlikoff, L. and L. Summers. (1981) “The Role of Intergenerational Transfers in Aggregate Capital Accumulation.” *Journal of Political Economy* 89: 706-732.
 - [47] King, R. and M. Watson (1992) “Testing Long-Run Neutrality.” NBER Working Paper No. 4156.
 - [48] Kocherlakota, N. (1996) “The Equity Premium: It’s Still a Puzzle.” *Journal of Economic Literature* 34: 42-71.

- [49] Kydland, F. and E. Prescott. (1982) "Time to Build and Aggregate Fluctuations." *Econometrica* 50: 1345-1370.
- [50] Laitner, J. (1992) "Random Earnings Differences, Lifetime Liquidity Constraints, and Altruistic Intergenerational Transfers." *Journal of Economic Theory* 58: 135-170.
- [51] Larch, M. (1993) "Dynamically Inefficient Equilibria in the Auerbach-Kotlikoff Model." *Empirical Economics* 18: 159-172.
- [52] Lord, W. (1992) "Saving, Wealth, and the Exchange-Bequest Motive." *Canadian Journal of Economics* 25: 743-753.
- [53] Lord, W. and P. Rangazas. (1991) "Savings and Wealth in Models with Altruistic Bequests." *American Economic Review* 81: 289-296.
- [54] Lord, W. and P. Rangazas. (1998) "Capital Accumulation and Taxation in a General Equilibrium Model with Risky Human Capital." *Journal of Macroeconomics* (forthcoming).
- [55] MaCurdy, T. (1981) "An Empirical Model of Labor Supply in a Life Cycle Setting." *Journal of Political Economy* 89: 1059-1085.
- [56] Mankiw, N., J. Rotemberg and L. Summers. (1985) "Intertemporal Substitution in Macroeconomics." *Quarterly Journal of Economics* 100: 225-251.
- [57] Mehra, R. and E. Prescott. (1985) "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15: 145-161.
- [58] Menchik, P. (1980) "Primogeniture, Equal Sharing and the U.S. Distribution of Wealth." *Quarterly Journal of Economics* 94: 299-316.
- [59] Miller, P. and T. Sargent. (1984) "A Reply to Darby." Federal Reserve Bank of Minneapolis *Quarterly Review* 8 (Spring): 21-26.
- [60] O'Connell, S. and S. Zeldes. (1993) "Dynamic Efficiency in the Gifts Economy." *Journal of Monetary Economics* 31: 363-379.
- [61] Poterba, J. (1997) "The Rate of Return to Corporate Capital and Factor Shares: New Estimates Using Revised National Income Accounts and Capital Stock Data." NBER Working Paper No. 6263.
- [62] Prescott, E. (1986) "Theory Ahead of Business Cycle Measurement." Federal Reserve Bank of Minneapolis *Quarterly Review* 10 (Fall): 9-22.
- [63] Ríos-Rull, J-V. (1994) "On the Quantitative Importance of Market Incompleteness." *Journal of Monetary Economics* 34: 463-496.
- [64] Sargent, T. and N. Wallace. (1981) "Some Unpleasant Monetarist Arithmetic." Federal Reserve Bank of Minneapolis *Quarterly Review* 5 (Fall): 1-17.
- [65] Siegel, J. (1992) "The Real Rate of Interest from 1800-1990." *Journal of Monetary Economics* 29: 227-252.
- [66] Singleton, K. (1990) "Specification and Estimation of Intertemporal Asset Pricing Models." In B. Friedman and F. Hahn, eds., *Handbook of Monetary Eco-*

nomics, Vol. I. North-Holland.

- [67] Wallace, N. (1984) "Some of the Choices for Monetary Policy." Federal Reserve Bank of Minneapolis *Quarterly Review* 8 (Winter): 15-24.
- [68] Weber, A. (1994) "Testing Long-Run Neutrality: Empirical Evidence for the G7-Countries, With Special Emphasis on Germany." *Carnegie-Rochester Conference Series on Public Policy* 41: 67-117.
- [69] Wilhelm, M. (1996) "Bequest Behavior and the Effect of Heirs' Earnings: Testing the Altruistic Model of Bequests." *American Economic Review* 86: 874-892.